

Exercises(11)

November 22, 2023

1. Let $f : A \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m$ be uniformly continuous. Show that
 - (i) (10 points) there is a function $g : \overline{A} \rightarrow \mathbb{R}^m$ is uniformly continuous with $f(x) = g(x)$ for all $x \in A$;
 - (ii) (5 points) if A is bounded, then $f(A)$ is bounded.
2. Let A be a nonempty subset of \mathbb{R}^n . Define a function $f_A : \mathbb{R}^n \rightarrow \mathbb{R}$ by the equation
$$f_A(x) = \inf\{\|x - y\| : y \in A\}$$
for each x in \mathbb{R}^n . The number $f_A(x)$ is called the distance from x to A .
 - (i) (5 points) Prove that f_A is uniformly continuous on \mathbb{R}^n .
 - (ii) (5 points) Prove that $\overline{A} = \{x : x \in \mathbb{R}^n \text{ and } f_A(x) = 0\}$.
3. (5 points) If f is one-to-one and continuous on (a, b) , prove that f must be strictly monotonic on (a, b) .