Exercises(7)

October 24, 2023

1. Let $F = \{F_1, F_2, \ldots\}$ be a countable collection of connected compact sets in $\mathbb{R}^n$ such that $F_{k+1} \subseteq F_k$ for each $k \geq 1$. Prove that the intersection $\bigcap_{k=1}^{\infty} F_k$ is connected and closed.

2. Let (S, d) be an connected metric space which is not bounded. Prove that for every a in S and every $r > 0$, the set $\{x : d(x, a) = r\}$ is nonempty.

3. Prove that if A is connected in a metric space M and $A \subseteq B \subseteq \mathrm{cl}(A)$, then B is connected.

4. Show that $A \subseteq \mathbb{R}$ is connected if and only if it is an interval (an interval is a set of the form $[a, b]$, (a, b), $[a, b)$, $(a, b]$, where a or b can be $\pm\infty$ on an open end of the interval).