

Exercises(4)

October 5, 2023

1. (10 points) Show that if $A \subseteq \mathbb{R}^n$ is open, then A is the union of a countable collection of open balls.
2. Define a **limit point** of a set A in a metric space (M, d) be a point $x \in M$ such that $U \cap A \neq \emptyset$ for every neighborhood U of x .
 - (a) (4 points) If x is an accumulation point of A , then show that x is a limit point of A . Is the converse true? (Justify your answer!)
 - (b) (4 points) Prove: A set is closed if and only if it contains all of its limit points.
3. Let A_1, A_2, \dots be subsets of a metric space (M, d) :
 - (a) (4 points) If $x \in A'_1$, prove that $\inf\{d(x, y) : y \in A_1\} = 0$.
 - (b) (4 points) If $B_n = \bigcup_{i=1}^n A_i$, prove that $B'_n = \bigcup_{i=1}^n A'_i$.
 - (c) (4 points) True or false: If $B = \bigcup_{i=1}^{\infty} A_i$, then $B' = \bigcup_{i=1}^{\infty} A'_i$. (Justify your answer!)