Homework 1 of Introduction to Analysis (I), Honor Class

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1. For any Cauchy sequence $\{x_n\}_{n=1}^{\infty}$, for all $\varepsilon > 0$, $\exists N \in \mathbb{N}$ s.t. for all $n, m > N$, $|x_n - x_m| \leq \varepsilon$

   Then, since for any convergence sequence, it is Cauchy sequence

2. (a) If we can find infinite many points labeled by $n_1, n_2, \cdots$ s.t. $\{x_{n_k}\}_{k=1}^{\infty}$ is decreasing, thus $\{x_n\}_{n=1}^{\infty}$ have a decreasing subsequence.

   If not, then we only can find at most $l \in \mathbb{N}, l < \infty$ ponints as $n_1, n_2, \cdots, n_l$ s.t. $\{x_{n_k}\}_{k=1}^{l}$ is decreasing. Thus, for $m > n_l$, $\{x_n\}_{n=m}^{\infty}$ have to be non-decreasing, Thus, $\{x_n\}_{n=1}^{\infty}$ have increasing subsequence.

   Then, every sequence in $\mathbb{R}$ either has an increasing subsequence or a decreasing subsequence

   (b) Since every sequence have either increasing or decreasing subsequence and $\{x_n\}$ is bounded, then by monotone convergence theorem, the sequence converges.