Exercises (10) November 14, 2023

- 1. Let (S,d) and (T,ρ) be two metric spaces. Assume that $f,g:S\to T$ are two functions.
 - (a) (8 points) Prove that f is continuous on S if and only if

$$f(\overline{A}) \subseteq \overline{f(A)}$$
 for every subset A of S.

- (b) (6 points) Let f and g be continuous on S and let $E \subseteq S$ with $\overline{E} = S$. If g(p) = f(p) for all $p \in E$, prove that g(p) = f(p) for all $p \in S$.
- 2. (8 points) Consider a compact set $B \subseteq \mathbb{R}^n$ and let $f: B \to \mathbb{R}^m$ be continuous and one-to-one. Then prove that $f^{-1}: f(B) \to B$ is continuous.
- 3. (8 points) Let f be a continuous function on \mathbb{R} to \mathbb{R} which is strictly increasing (in the sense that if $x_1 < x_2$, then $f(x_1) < f(x_2)$). Prove that $f^{-1}: f(\mathbb{R}) \to \mathbb{R}$ is continuous and strictly increasing.