Exercises(5)

October 12, 2023

1. (10 points) Definition: Let X be a metric space and let $E \subseteq X$. E is dense if $\overline{E} = X$.

   Let (X, d) be a metric space which every infinite subset has an accumulation point. Prove that X contains a countable dense subset.

2. (8 points) Prove that every open set in $\mathbb{R}$ is the union of an at most countable collection of disjoint segments.

3. (6 points) Let A_n be subsets of a metric space (M, d), $A_{n+1} \subseteq A_n$, and $A_n \neq \emptyset$, but assume that $\bigcap_{n=1}^{\infty} A_n = \emptyset$. Let $A = \bigcap_{n=1}^{\infty} \overline{A}_n$. Show that $A \subseteq A_1'$

4. (6 points) For $A \subseteq M$, a metric space, prove that

$$\partial A = (A \cap \overline{M \setminus A}) \cup (\overline{A} \setminus A).$$