Homework 15 of Introduction to Analysis (I), Honor Class

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1. (a) For any $y \in [c,d]$, we can find a partition $P_y =$

   (b) $F'(y) = \lim_{h\to 0} \frac{F(y+h)-F(y)}{h} = \int_a^b \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h} dx = \int_a^b \frac{\partial f}{\partial y}(x,y)dx.$

2. (a) $g'(x) = \int_0^1 -2(t^2+1)x\frac{e^{-x^2(t^2+1)}}{t^2+1}dt = \int_0^1 -2xe^{-x^2(t^2+1)}dt.$

   $f'(x) = e^{-x^2}(\int_0^x e^{-t^2}dt)$

   Since $f'(x)+g'(x) = 0$ for all x, $f(x)+g(x) = f(0)+g(0) = g(0) = \int_0^1 \frac{1}{t^2+1}dt = \int_0^{\frac{\pi}{4}} du = \frac{\pi}{4}.$

   (b) Since $g(x) \to 0$ as $x \to \infty$, $f(x) \to \frac{\pi}{4}$. Then, $\lim_{x\to\infty} \int_0^x e^{-t^2}dt = \lim_{x\to\infty} \sqrt{f(x)} = \sqrt{\frac{\pi}{4}} = \frac{\sqrt{\pi}}{2}.$