Homework 5 of Introduction to Analysis (I), Honor Class

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1.

2. Let U be any non-empty open set $\in \mathbb{R}$. Since U is open, U contains rational numbers. Then, for every U, we call it U_x, for a $x \in \mathbb{Q} \cap U$. Thus, we can number the open sets by $\mathbb{Q}$ which is countable.

3. For $x \in A$, since $\cap_{n=1}^{\infty} A_n = \emptyset$, $A = \cap_{n=1}^{\infty} \bar{A}_n = (\cap_{n=1}^{\infty} A_n) \cup (\cap_{n=1}^{\infty} A'_n) = \cap_{n=1}^{\infty} A'_n$.

   Thus, $x \in A'_1$.

4. First, we want to proof $\overline{M-A} \supseteq (\overline{A} - A)$.

   Since $\overline{M-A} = (M-A)' \cup (M-A)$ and $\overline{A} \subseteq M \implies (\overline{A} - A) \subseteq (M-A) \subseteq \overline{M-A}$.

   $(A \cap \overline{M-A}) \cup (\overline{A} - A) = (A \cup (\overline{A} - A)) \cap (\overline{M-A} \cup (\overline{A} - A)) = \overline{A} \cap (\overline{M-A}) = \partial A$