Exercises(2)

September 19, 2023

1. (10 points) Show that the following statement is true:

   Every Cauchy sequence converges in $\mathbb{R} \Rightarrow \mathbb{R}$ has the least-upper-bound property.

2. (a) (5 points) Show that every sequence in $\mathbb{R}$ either has an increasing subsequence or a decreasing subsequence.

   (b) (5 points) Use (a) to prove the Bolzano-Weierstrass Theorem for sequences in $\mathbb{R}$.

3. (5 points) Determine the convergence or divergence of the sequence $\{x_n\}$, where

$$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} \text{ for } n \in \mathbb{N}.$$

4. (5 points) Show that the sequence $\{x_n\}$ in $\mathbb{R}$ diverges to ∞ if

$$\liminf_{n\to\infty} x_n = \infty.$$