# Exercises(6)

October 17, 2023

1. (10 points) Consider the metric space $\mathbb{Q}$ of rational numbers with the Euclidean metric of $\mathbb{R}$. Let $S = (\sqrt{2}, \pi) \cap \mathbb{Q}$. Show that $S$ is a closed and bounded subset of $\mathbb{Q}$ which is not compact.

2. (4 points) Let $V_n \subseteq M$ be open sets such that $\overline{V}_n$ is compact, $V_n \neq \emptyset$, and $\overline{V}_n \subseteq V_{n-1}$. Prove $\bigcap_{n=1}^{\infty} V_n \neq \emptyset$.

3. (8 points) Suppose that $G = \{G_\alpha\}$ is an open covering of a compact subset $K$ of $\mathbb{R}^n$. Show that there exists a strictly positive number $\lambda$ such that if $x, y$ belong to $K$ and $\|x - y\| < \lambda$, then there is a set in $G$ containing both $x$ and $y$.

4. (8 points) Let $M$ be complete and $A \subseteq M$ be totally bounded. Show that $\bar{A}$ is compact.