Homework 13 of Introduction to Analysis (I), Honor Class

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1. (a) Since $f(x) = \ln(x)$ is concave down, $\ln(uv) = \ln(u) + \ln(v) = \frac{1}{p}\ln(u^p) + \frac{1}{q}\ln(v^q) \leq \ln(\frac{u^p}{p} + \frac{v^q}{q})$.
   And since $\ln(x)$ is strictly increasing, $uv \leq \frac{u^p}{p} + \frac{v^q}{q}$.
   If $u^p = v^q$, $\frac{u^p}{p} + \frac{v^q}{q} = (\frac{1}{p} + \frac{1}{q})u^p = u^p = u^{p(\frac{1}{p}+\frac{1}{q})} = u \cdot u^{\frac{p}{q}} = uv$.

   (b) $\int_a^b \frac{(f(x))^p}{p} dx = \frac{1}{p}$ and $\int_a^b \frac{(g(x))^q}{q} = \frac{1}{q}$.
   Then, $\int_a^b \frac{(f(x))^p}{p} + \frac{(g(x))^q}{q} dx = \frac{1}{p} + \frac{1}{q} = 1$.
   And since $f(x)g(x) \leq \frac{(f(x)^p)}{p} + \frac{(g(x))^q}{q}$ for all $x \in [a,b]$ and f, g are Reimann integral, $\int_a^b f(x)g(x)dx$ exists and $\int_a^b f(x)g(x)dx \leq \int_a^b \frac{(f(x))^p}{p} + \frac{(g(x))^q}{q} dx = 1$.

   (c)

2.