

Exercises(1)

September 12, 2023

1. (10 points) Let X be a non-empty set and let $f : X \rightarrow \mathbb{R}$ have bounded range in \mathbb{R} (this is, there is a real number M such that $|f(x)| \leq M$ for all $x \in X$). If $a \in \mathbb{R}$, show that

$$\sup\{a + f(x) : x \in X\} = a + \sup\{f(x) : x \in X\}.$$

2. (10 points) Let X be a non-empty set and let f and g be defined on X and have bounded ranges in \mathbb{R} . Show that

$$\begin{aligned} & \inf\{f(x) : x \in X\} + \inf\{g(x) : x \in X\} \\ & \leq \inf\{f(x) + g(x) : x \in X\} \\ & \leq \sup\{f(x) : x \in X\} + \sup\{g(x) : x \in X\}. \end{aligned}$$

Give examples to show that each inequality can be strict.

3. (10 points) Fix $b > 1, y > 0$, and prove that there is a unique real x such that $b^x = y$. (Hint: Let A be the set of all w such that $b^w < y$, and show that $x = \sup A$ satisfies $b^x = y$.)