Homework 8 of Introduction to Analysis (I), Honor Class

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1. (a) Suppose $\prod_{n=1}^{\infty} a_n = a \in \mathbb{R}^+$, $\exp(\sum_{n=1}^{\infty} \ln(a_n)) = \prod_{n=1}^{\infty} a_n = a$. Thus, $\sum_{n=1}^{\infty} ln(a_n) = \ln(a) \in (-\infty, \infty)$

   Therefore, $\prod_{n=1}^{\infty} a_n$ converges iff $\sum_{n=1}^{\infty} \ln(a_n)$ converges.

   (b) Suppose $\sum_{n=1}^{\infty} u_n$ convergs. $\prod_{n=1}^{\infty}(1+u_n) = \sum_{n=1}^{\infty} \ln(1+u_n)$

2. Counter example: $a_n =$, then $\sum_{n=1}^{\infty} \sqrt{\frac{a_n}{n}} =$