Homework 1 of Introduction to Analysis (I), Honor Class

AM15 黃琦翔 111652028

2023/9/19

1. For any Cauchy sequence $\{x_n\}_{n=1}^{\infty}$, for all $\varepsilon > 0$, $\exists N \in \mathbb{N}$ s.t. for all $n, m > N$, $|x_n - x_m| \leq \varepsilon$

   Then, since for any convergence sequence, it is Cauchy sequence

2. (a) If we can find infinite many points labeled by $n_1, n_2, \cdots$ s.t. $\{x_{n_k}\}_{k=1}^{\infty}$ is decreasing, thus $\{x_n\}_{n=1}^{\infty}$ have a decreasing subsequence.

   If not, then we only can find finite points s.t. $\{x_{n_k}\}_{k=1}^{l}$ is decreasing. Thus, we name the last element of the decreasing sequence as N. Thus, we can say that $n'_1 = N + 1$, since n'_1 is not in decreasing sequence, implies that $\exists n'_2 > n'_1 \Rightarrow x_{n'_2} \geq x_{n'_1}$. And doing the same way, we can find $n'_1 < n'_2 < n'_3 < \cdots$ s.t. $\{x_{n'_i}\}_{i=1}^{\infty}$ is a increasing subsequence.

   Then, every sequence in $\mathbb{R}$ either has an increasing subsequence or a decreasing subsequence

   (b) Since every sequence have either increasing or decreasing subsequence and $\{x_n\}$ is bounded, then by monotone convergence theorem, the subsequence converges.