Homework 10 of Introduction to Analysis (I), Honor Class

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1. (a)

   ($\Longrightarrow$) Since f is continuous, then for $x \in \bar{A}$, we can find a $\{x_i \mid x_i \in A\}$ converges to x. Thus, for all $\varepsilon > 0$, exists $\delta > 0$ s.t. $d(x_i, x) < \delta \implies \rho(f(x_i), f(x)) < \varepsilon$.

   Then, for $y \in f(\bar{A})$, $D(y, \varepsilon) \cap f(A) \neq \emptyset$ for all $\varepsilon > 0$. Thus, $y \in \overline{f(A)}$.

   ($\Longleftarrow$) Since $f(\bar{A}) \subseteq \overline{f(A)}$,

   (b)