Exercises(10)

November 14, 2023

1. Let (S, d) and (T, ρ) be two metric spaces. Assume that $f, g : S \to T$ are two functions.

   (a) (8 points) Prove that f is continuous on S if and only if

   $$f(\overline{A}) \subseteq \overline{f(A)} \text{ for every subset } A \text{ of } S.$$

   (b) (6 points) Let f and g be continuous on S and let $E \subseteq S$ with $\overline{E} = S$. If $g(p) = f(p)$ for all $p \in E$, prove that $g(p) = f(p)$ for all $p \in S$.

2. (8 points) Consider a compact set $B \subseteq \mathbb{R}^n$ and let $f : B \to \mathbb{R}^m$ be continuous and one-to-one. Then prove that $f^{-1} : f(B) \to B$ is continuous.

3. (8 points) Let f be a continuous function on $\mathbb{R}$ to $\mathbb{R}$ which is strictly increasing (in the sense that if $x_1 < x_2$, then $f(x_1) < f(x_2)$). Prove that $f^{-1} : f(\mathbb{R}) \to \mathbb{R}$ is continuous and strictly increasing.