Homework 4 of Introduction to Analysis (I), Honor Class

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October 3, 2023

1. Since $A \subseteq \mathbb{R}^n$ is open, then for any point $x \in A$, exists some $\varepsilon > 0$ s.t. $B(x, \varepsilon) \subset A$.

2. (a) no, if $U = \{x\}$ is the only neighborhood of x, there isnot a point $p \in U,\ p \neq x$ s.t. $p \in A$.

   (b) Assume there exists a limit point x of A is not in A. Then, $x \in A^c$. We want to show that A is not close.

   For A^c, since all neighborhood U of x has intersection with A, then $U \nsubseteq A^c$. Thus, A^c is not open.

   Therefore, if A is close, A contains all its limit points.

3. (a) For $x \in \bar{A}_1 = A_1 \cup A_1'$, then we only need to show that the accumulation point x of $A_1 \implies \inf\{d(x,y) \mid y \in A_1\} = 0$.

   Assume $\inf\{d(x,y) \mid y \in A_1\} = \alpha > 0$, then $B(x,\varepsilon) \cap A_1 = \emptyset$ for $0 < \varepsilon < \alpha$. Implies that x is not an accumulation point of A_1.

   Thus, if $x \in \bar{A}_1$, $\inf\{d(x,y) \mid y \in A_1\} = 0$.

   (b) ($\subseteq$) For $x \in B_n$, $D(x,\varepsilon) \cap B_n \neq \emptyset$ for all $\varepsilon > 0$.
   $D(x,\varepsilon) \cap (\cup_{i=1}^n A_i) = \cup_{i=1}^n (D(x,\varepsilon) \cap A_i) \neq \emptyset \implies D(x,\varepsilon) \cap A_i \neq \emptyset$ for some $i \in [1,n]$. Thus, $x \in \bar{A}_i$ for any $\varepsilon > 0$ and some $i \in [1,n] \implies x \in \cup_{i=1}^n \bar{A}_i$.

   ($\supseteq$) For $x \in \cup_{i=1}^n \bar{A}_i$ and $\varepsilon > 0$, $D(x,\varepsilon) cap \bar{A}_i \neq \emptyset$ for some i. This implies that $D(x,\varepsilon) \cap (\cup_{i=1}^n A_i) = D(x,\varepsilon) \cap B_n \neq \emptyset \implies x \in \bar{B}_n$

   (c) For $x \in \cup_{i=1}^\infty A_i$, $D(x,\varepsilon) \cap (\cup_{i=1}^\infty A_i) \neq \emptyset$