

## Topics for Test #2, ISyE 6644 Online Masters in Analytics, Summer 2019

1. I've tried my best to make this list as complete as possible, but I may have missed a topic or two. That being said, you are responsible for everything that we did in class or homework.
2. As GT students, you are expected to formulate problems and solution strategies which are more than mere rote regurgitation of material you learned in class. Thus, you shouldn't be surprised if some questions cover natural extensions of material from class.
3. I'll supply all necessary tables, e.g.,  $N(0,1)$ ,  $t$ , and  $\chi^2$ , but you can feel free to use your own.
4. **Summary:** You'll be responsible for everything up to the continuous version of the Acceptance-Rejection method (Week 7 / HW 7); see below.
5. **Study Hint:** The material from Test 1 will receive less emphasis than the more-recent stuff; see Practice Test 2.

OK, so off we go! Here's the list of all of the stuff we've done so far (don't be scared!)

### 1. Intro Material

- a. Definition of simulation
- b. Advantages and disadvantages of simulation
- c. History of simulation
- d. Typical questions and applications

### 2. Calculus, Probability, and Statistics Review

- a. **Calculus [not really responsible for this material, except I might make you search for a zero]**
  - i. Basic definitions
  - ii. Derivatives
  - iii. Solving for zeros
  - iv. Integration
  - v. Numerical integration
- b. Probability Preliminaries
  - i. Conditional probability
  - ii. Independent events
  - iii. Definition of random variable
  - iv. Discrete RV's and probability mass function
  - v. Continuous RV's and probability density function
  - vi. Cumulative distribution function
- c. Simulating RV's (first pass)
  - i. Discrete uniform distribution
  - ii. General discrete distribution

- iii. Inverse Transform Theorem for continuous RV's
  - iv. Exponential (and other) continuous distributions via IVT.
  - v. Generating  $U(0,1)$ 's via desert island algorithm, including walk-through of pseudo-code.
- d. Expected Values
  - i. Definition
  - ii. Discrete and continuous examples of expected value
  - iii. Law of the Unconscious Statistician
  - iv. Moments, central moments, variance, standard deviation
  - v. Discrete and continuous examples of LOTUS
  - vi. Moment generating function
  - vii. Examples and properties of mgf's
- e. Functions of a RV
  - i. Discrete examples
  - ii. Continuous examples
  - iii. IVT methods (again) with examples
  - iv. Relationship with LOTUS
- f. Jointly distributed RV's
  - i. Definition of joint cdf
  - ii. Marginal cdf's
  - iii. Joint and marginal pmf's
  - iv. Joint and marginal pdf's
  - v. Examples for discrete and continuous cases
  - vi. Independent RV's
  - vii. Conditional pmf's and pdf's
  - viii. **Conditional expectation [this won't be on the test]**
  - ix. **Double expectation  $E(E(Y|X)) = EY$ , including examples [this won't be on the test]**
- g. Covariance and correlation
  - i. Definitions
  - ii. Relationship between independence and correlation
  - iii. Examples
  - iv. Miscellaneous properties (e.g.,  $\text{Var}(X+Y)$ , bounds on correlation, etc.)
- h. Probability distributions
  - i. Discrete distributions
    - 1. Bernoulli
    - 2. Binomial
    - 3. Geometric
    - 4. Poisson (including discussion on Poisson processes)
  - ii. Continuous distributions
    - 1. Uniform

2. Exponential (including memoryless property)
  3. Erlang, Gamma distributions
  4. Triangular
  5. Normal (including Standard Normal)
  6. Other sampling distributions (including chi-square, t, F, and various relationships with each other)
- i. Limit theorems
    - i. Linear combinations of independent normal (including distribution of sample mean)
    - ii. Convergence in distribution
    - iii. Law of Large Numbers
    - iv. Central Limit Theorem for independent and identically distributed data.
    - v. Examples
  - j. Statistics Tidbits [don't worry about this stuff... until Test 3!!! 😊 ]
    - i. Properties of sample mean and sample variance
    - ii. Confidence intervals for the mean and variance

### 3. Hand Simulations

- a. Monte Carlo integration
- b. Determining  $\pi$  via simulation (dart tossing on a circle and sphere)
- c. Single-server queue (including FIFO and LIFO service disciplines)
- d. (s,S) inventory system
- e. Simulating RV's (repeats some material from the Prob/Stats review)
- f. Spreadsheet simulation (e.g., stock portfolio in Excel)

### 4. General Simulation Principles

- a. Steps in a simulation study
- b. List of various simulation definitions (e.g., event, system state, simulation clock, etc.)
- c. Event-Scheduling vs. Process Interaction modeling approaches
- d. How are events processed?
- e. Future events list + extended example
- f. Simulation languages – what to look for

### 5. Arena

- a. Layout of Arena screen (panels, modules, etc.)
- b. Basic Process template: CREATE-PROCESS-DISPOSE modules
- c. SEIZE-DELAY-RELEASE inside of the PROCESS module.
- d. Resource, Schedule, Queue, Entity, and other spreadsheets
- e. ASSIGN module
- f. DECIDE module – probabilistic and conditional routing
- g. Simple examples, e.g. (partial list),

- i. Single-server queue
  - ii. Parallel servers
  - iii. Schedules for servers
  - iv. Multiple arrival streams
- h. Displays, graphics, etc.
- i. BATCH and SEPARATE modules
- j. Run set-up and control
- k. More-sophisticated queueing networks (e.g., two-channel manufacturing example, call center example)
  - i. Advanced Process modules (e.g., SEIZE, DELAY, RELEASE modules)
  - ii. Some primitive blocks (e.g., QUEUE)
  - iii. Use of “pretend” customers
  - iv. Nonhomogeneous Poisson arrivals
  - v. Use of resource sets, including how to prioritize servers
  - vi. Use of submodels
- l. Inventory processes
- m. Crazy examples such as re-entrant queues [this won't be on the test]
- n. SMARTS files and other Rockwell examples [this won't be on the test]
- o. Manufacturing systems (you're only responsible for the basics)
  - i. Advanced Transfer modules
  - ii. Sequences of customer visitation locations
  - iii. Advanced sets of sequences [this won't be on the test]

## 6. Uniform Random Number Generation

- a. Overview – desirable properties of a pseudo-random number generator
- b. Some generators we won't use, e.g.,
  - i. PRN's from tables
  - ii. Midsquare
- c. Linear congruential generators
  - i. Cycling
  - ii. 16807 desert island generator (again)
  - iii. RANDU (a bad generator)
- d. Tausworthe generator
- e. Combined generators
  - i. L'Ecuyer's generator of cycle length  $2^{191}$
  - ii. Mersenne Twister
- f. Some theoretical considerations, e.g., from Knuth's book
- g. Statistical tests for randomness
  - i. Goodness-of-fit tests
    - 1. Chi-squared
    - 2. Kolmogorov-Smirnov

- ii. Runs tests for independence
  1. Runs up and down
  2. Runs above and below the mean
  3. Autocorrelation test [this won't be on the test]

## 7. Random Variate Generation

- a. Inverse Transform Theorem (yet again)
  - i. Proof
  - ii. Discrete example adaptations
  - iii. Continuous examples
    1. Easy ones such as Exponential, Weibull, etc.
    2. Slightly harder examples such as Triangle distribution
    3. Normal distribution, both exact and approximate methods
  - iv. Special case methods, e.g., Geometric
  - v. Empirical distributions [this won't be on the test]
- b. Convolution method
  - i. Binomial
  - ii. Triangle
  - iii. Erlang
  - iv. CLT
    1. Desert island sum of Uniforms to generate Normal
    2. Normal approximation to Poisson (including continuity correction)
  - v. Cauchy
    1. Cauchy's add up to another Cauchy
    2. IVT method
    3. Ratio of two Normals
- c. Acceptance-Rejection methods
  - i. Trivial Uniform example
  - ii. Some discussion on general method
  - iii. Proof of the general method [don't expect to see this on the final, unless I'm in a really bad mood!]
  - iv. Examples involving polynomial and half-normal (continuous distribution) p.d.f.'s
  - v. Poisson distribution [not on this test... stay tuned for Test 3!]