Data Analytics in Business

Linear Regression

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Steps in Regression Analysis

Lessons in this Module

A. Steps in Regression Analysis
B. Linear Regression Example
C. Notation
D. R^2, Adjusted R^2
E. Simple Regression (One Predictor Variable) Using R
F. Multiple Regression
G. R^2, Adjusted R^2 from Multiple Regression
H. Prediction

Steps in Regression Analysis

1. Statement of the problem
2. Using regression for
    - Diagnostic,
    - Predictive, or
    - Prescriptive analytics?
3. Selection of potentially relevant response and explanatory variables
4. Data collection
    - Internal data external data, purchased data, experiments, etc.

Adapted from Chatterjee, S., & Hadi, A. S. (2013). *Regression Analysis by Example* (5th ed.). Somerset: Wiley.

Steps in Regression Analysis (cont'd)

5. Choice of fitting method
    - Ordinary least squares (OLS),
    - Generalized least squares,
    - Maximum likelihood,
    - Etc.
6. Model fitting
7. Model validation (diagnostics)
8. Refine the model & iterate from step 3
9. Use of the model

Business Examples

Y - Dependent Variable	X - Independent Variable(s)
1. Used car price	odometer reading, age of car, condition
2. Sales	advertisement spending
3. Time taken to repair a product	experience of technician in years
4. Product added to shopping cart?	ratings, price
5. Starting salary of new employee	work experience, years of education
6. Sale price of house	square feet, # of bedrooms, location
7. Will customer default?	credit balance, income, age
8. Will customer churn?	length of contract, age of customer

Quiz (True/False)

- Could a variable, say price, be either a dependent or an independent variable?

  Answer: **TRUE**. Depends on the purpose of your model; see examples #1 and #4 in the previous slide.

- A variable that takes binary values (pass/fail or true/false) cannot be a dependent variable.

  Answer: **FALSE**. We do use 0/1 dependent variables in logistic regression models; #7 in the previous slide is one example.

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Linear Regression Example

Linear Regression: A Sample Problem

- Assume that you need to sell your house.
- You want to predict the listing price based on how other houses are listed in the market.
- How would you approach this task?
- A typical approach is to ask realtors
  - Realtors often will use "comparables" (i.e., recent sales of houses in your neighborhood) and somehow come up with a suggested sale price.
- However, you want to be more analytical in your approach
  - You have access to recent actual home sales in your city.
  - You'd like to know what are the impacts of factors such as lotsize, # of bedrooms, # of bathrooms, etc., on the price.
  - Could you use linear regression to help you get a "better" estimate of the listing price?

Use Housing dataframe in Ecdat package in R

- This data set is a sample of the real estate transactions in one city
- It is a cross-section of 546 home prices (from 1987) in the city of Windsor in Canada.
- Alternatively, you could collect house prices from websites or scrape them from the web

str(Housing)

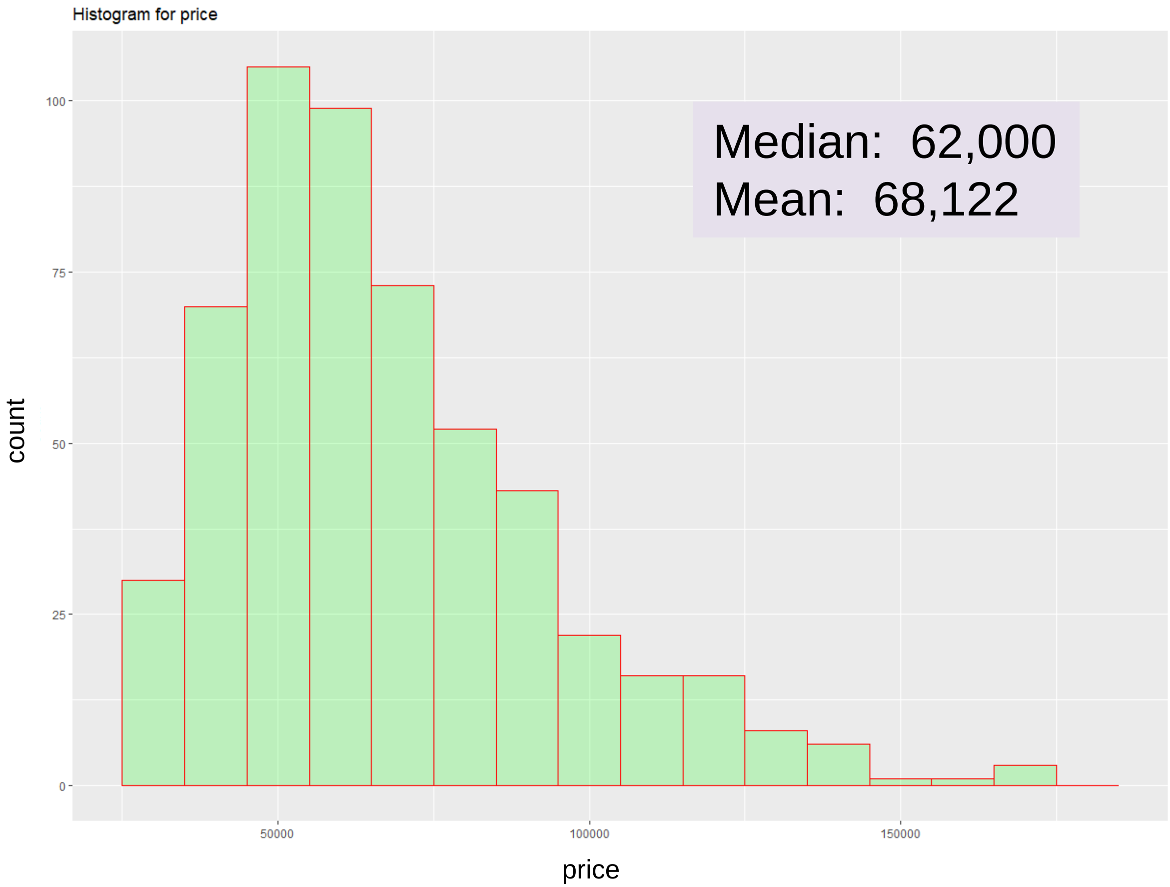
- 'data.frame': 546 obs. of 12 variables:
- $ price: num 42000 38500 49500 60500 61000 66000 66000 69000 83800 88500 ...
- $ lotsize: num 5850 4000 3060 6650 6360 4160 3880 4160 4800 5500 ...
- $ bedrooms: num 3 2 3 3 2 3 3 3 3 3 ...
- $ bathrms: num 1 1 1 1 1 1 2 1 1 2 ...
- $ stories: num 2 1 1 2 1 1 2 3 1 4 ...
- $ driveway: Factor w/ 2 levels "no","yes": 2 2 2 2 2 2 2 2 2 2 ...
- $ recroom: Factor w/ 2 levels "no","yes": 1 1 1 2 1 2 1 1 2 2 ...
- $ fullbase: Factor w/ 2 levels "no","yes": 2 1 1 1 1 2 2 1 2 1 ...
- $ gashw: Factor w/ 2 levels "no","yes": 1 1 1 1 1 1 1 1 1 1 ...
- $ airco: Factor w/ 2 levels "no","yes": 1 1 1 1 1 2 1 1 1 2 ...
- $ garagepl: num 1 0 0 0 0 0 2 0 0 1 ...
- $ prefarea: Factor w/ 2 levels "no","yes": 1 1 1 1 1 1 1 1 1 1 ...

The First 10 Records in Housing

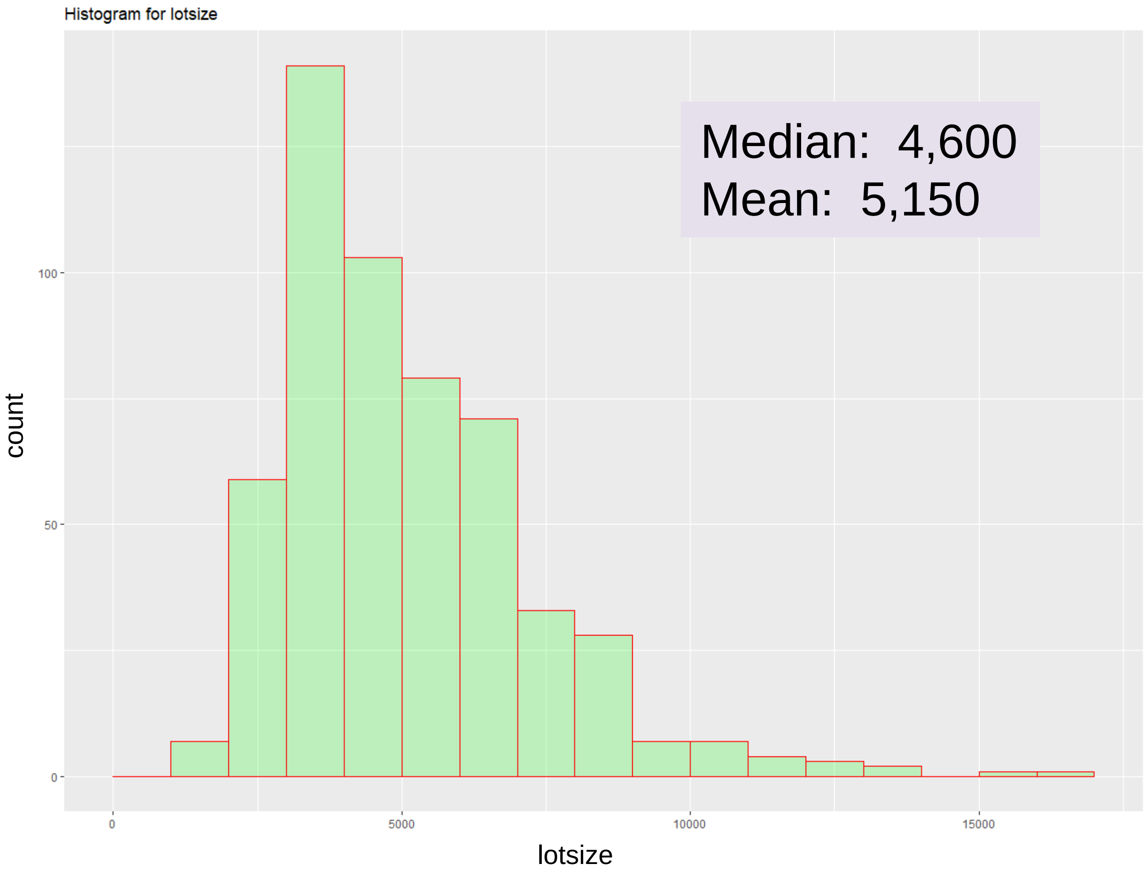
Housing Dataset in the *Ecdat* package in R

price	lotsize	bedrooms	bathrms	stories	driveway	recroom	fullbase	gashw	airco	garagepl	prefarea
42000	5850	3	1	2	yes	no	yes	no	no	1	no
38500	4000	2	1	1	yes	no	no	no	no	0	no
49500	3060	3	1	1	yes	no	no	no	no	0	no
60500	6650	3	1	2	yes	yes	no	no	no	0	no
61000	6360	2	1	1	yes	no	no	no	no	0	no
66000	4160	3	1	1	yes	yes	yes	no	yes	0	no
66000	3880	3	2	2	yes	no	yes	no	no	2	no
69000	4160	3	1	3	yes	no	no	no	no	0	no
83800	4800	3	1	1	yes	yes	yes	no	no	0	no
88500	5500	3	2	4	yes	yes	no	no	yes	1	no

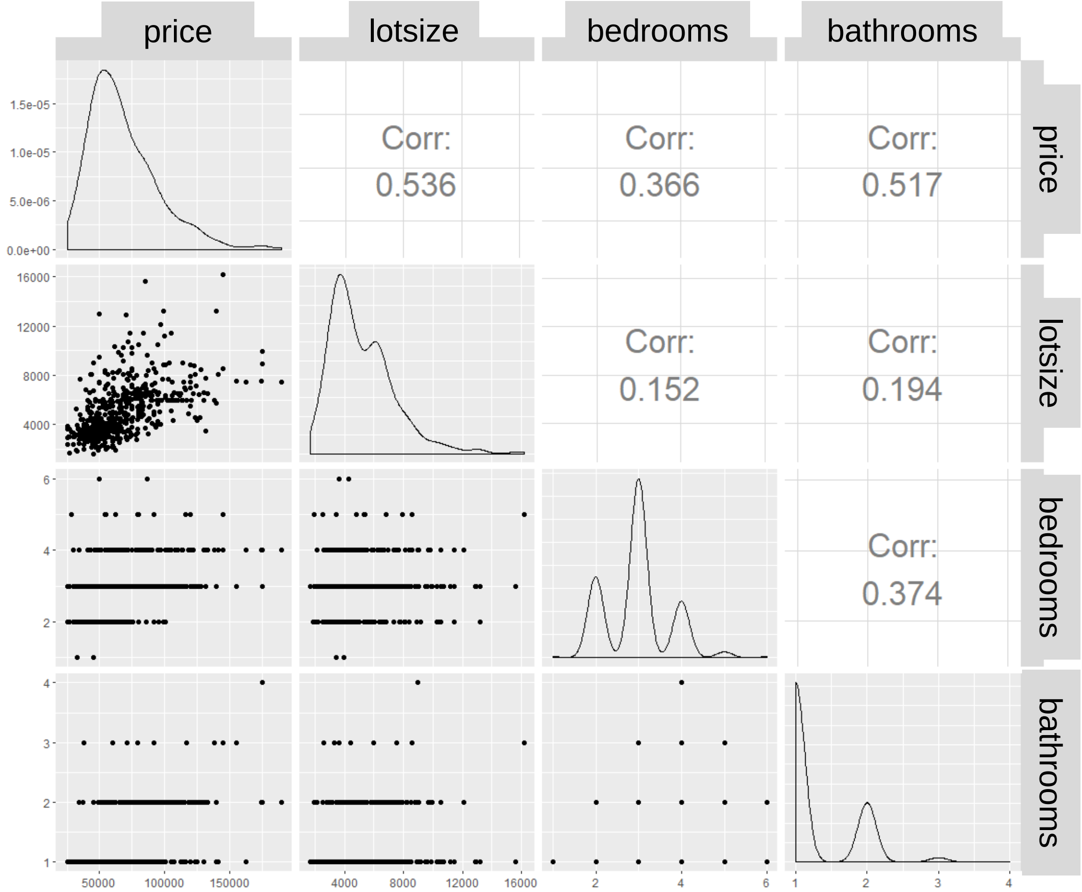
Histogram of House Prices



Histogram of lotsize



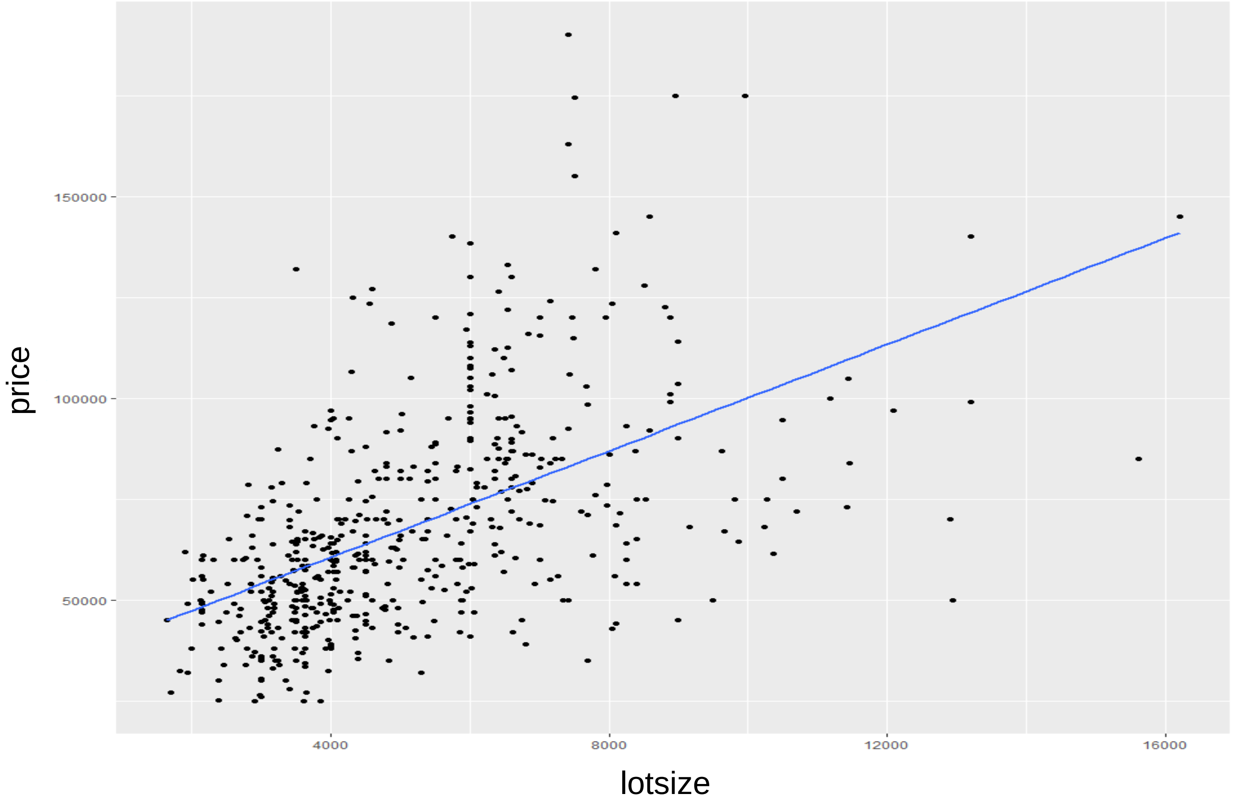
Correlation Matrix



Correlation Matrix

Scatter Plot

Scatter Plot of price (*y*) against lotsize (*x*), including the linear regression line



Quiz (True/False)

- The mean of a variable that has a right-skewed distribution is smaller than the median.

  Answer: **FALSE**.

- The correlation coefficient can capture the strength of both linear and non-linear relationships.

  Answer: **FALSE**.

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Linear Regression

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Notation

Linear Regression: Notation

Notation	Meaning
$i = 1,2,\ldots,n$	*i* refers to the *i*th observation or record in a data set of records (typically a sample of the population)
$(x_{11}, x_{21}, \ldots, x_{p1})$, $(x_{12}, x_{22}, \ldots, x_{p2})$, $\ldots$, $(x_{1n}, x_{2n}, \ldots, x_{pn})$	*n observations of the p explanatory variables*
$y_1, y_2, \ldots, y_n$	*n* observations of the dependent variable
$\bar{y}$	Mean value of the dependent (*y*) variable
$\bar{x}_\mathrm{k}$	Mean value of the x_kth explanatory (independent) variable

Linear Regression: Notation (cont'd)

Notation	Meaning
$\beta_0, \beta_1, \ldots, \beta_p$	Parameters of the regression line for the entire population
$b_0, b_1 \ldots, b_p$	Estimates of the β parameters obtained by fitting the regression to the sample data
ε_i	Error term for the *i*th observation in the population
e_i	Error term for the *i*th observation in the sample
$\hat{y}_i$	Estimated value of *y* for the *i*th observation in a sample. This is obtained by evaluating the regression function at x_i

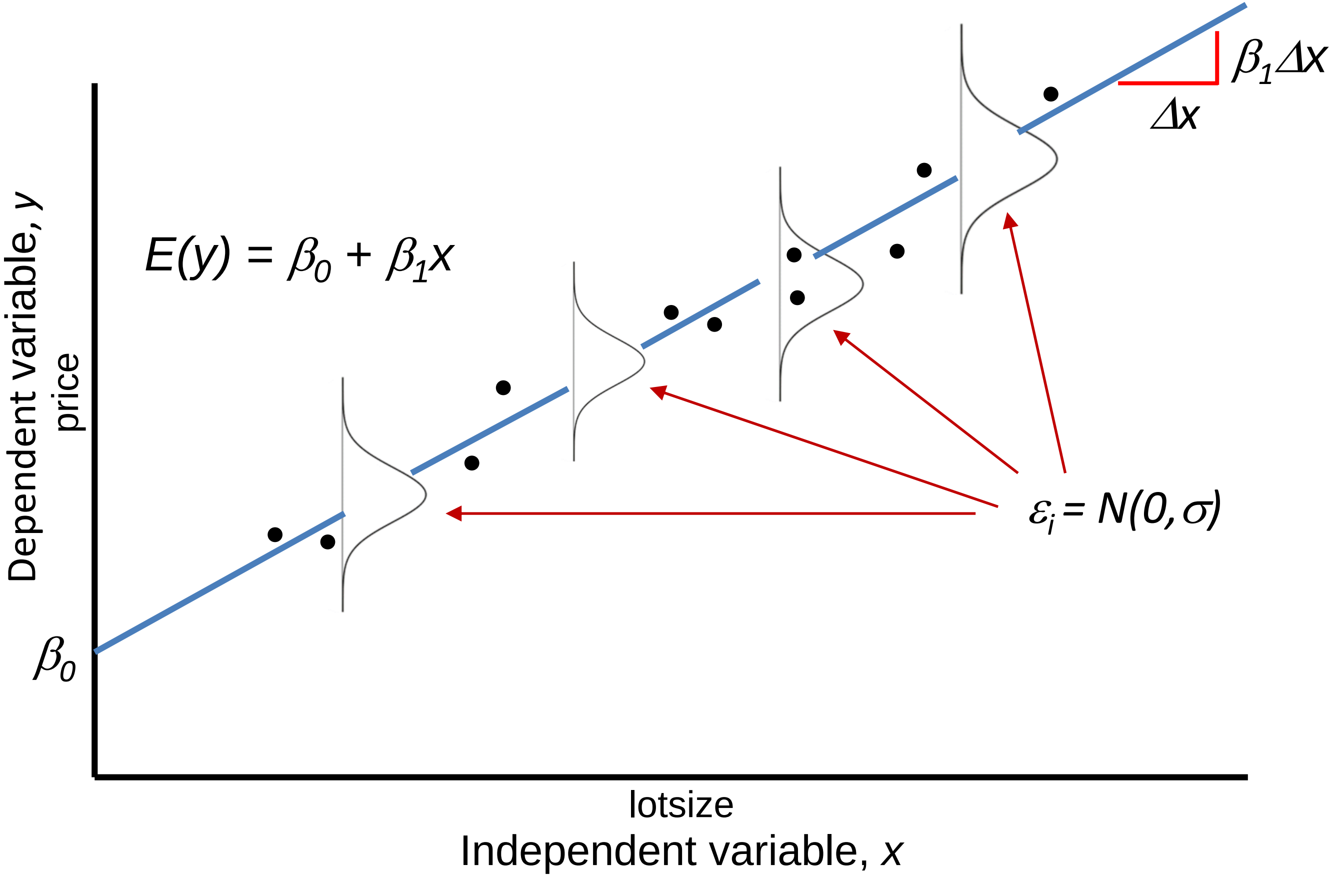
Simple Linear Regression

- We observe the data in the Housing dataset (which is a sample)
- We want to build a model for the **population**:

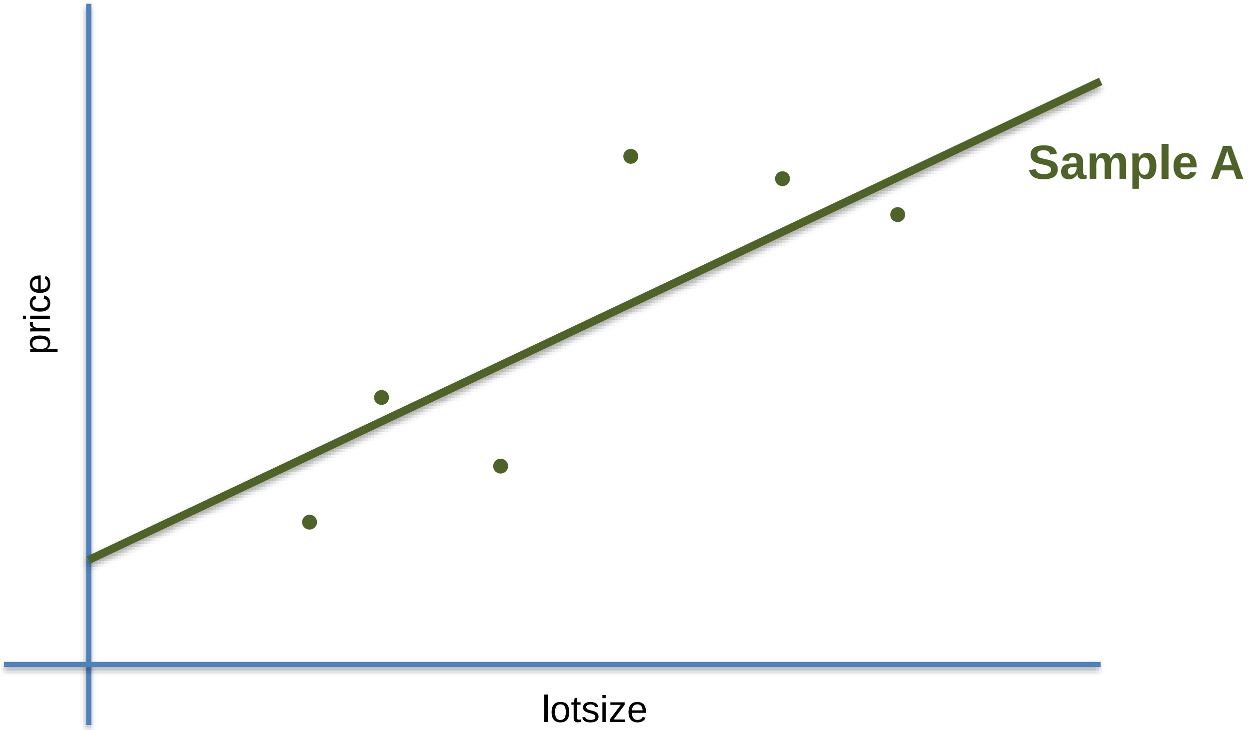
  $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ (which is the valid relation)
- ε_i are independent and identically distributed (i.i.d.) random variables, which are normally distributed with mean 0 and standard deviation σ
- However, we do not know β_0, β_1, or σ; so we need to estimate them based on the **sample** in the Housing dataset
- Using this sample, we are going to build a model

  $Y_i = b_0 + b_1 X_i + e_i$

Population model, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$



Estimates of Slope and Intercept Depend on the Sample Being Used



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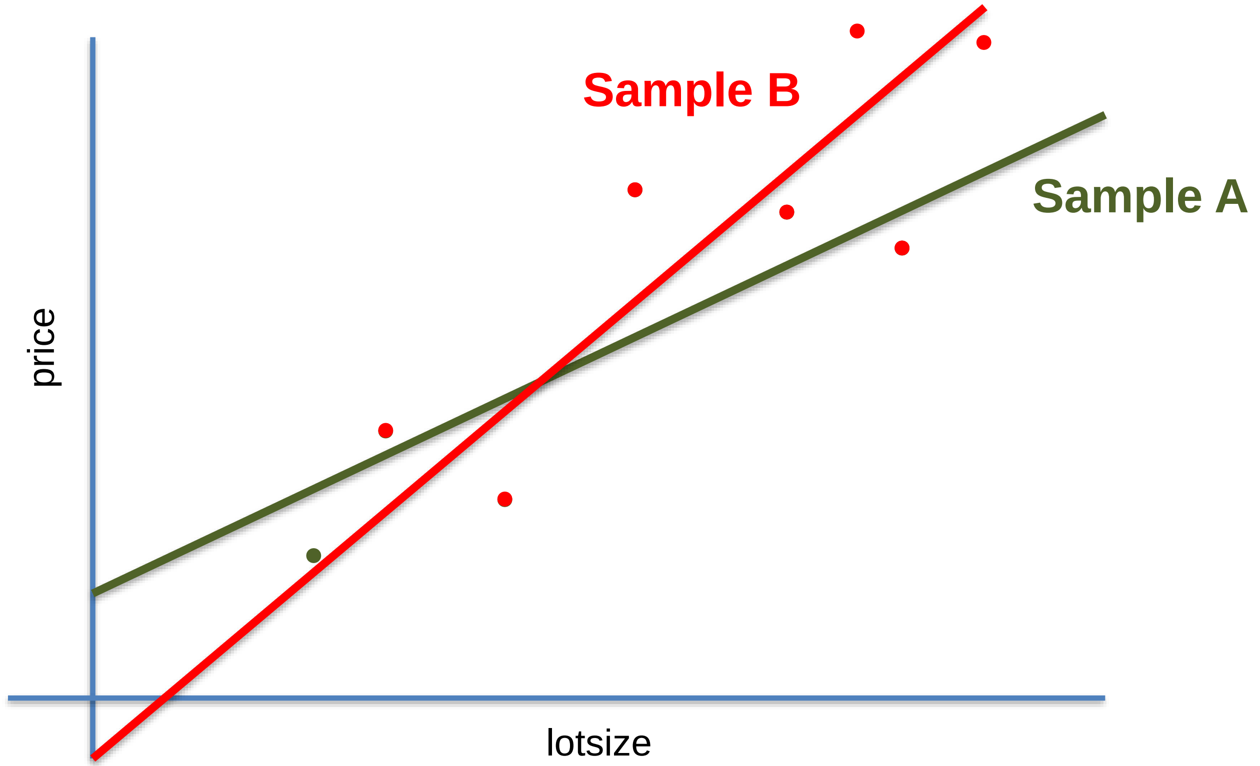
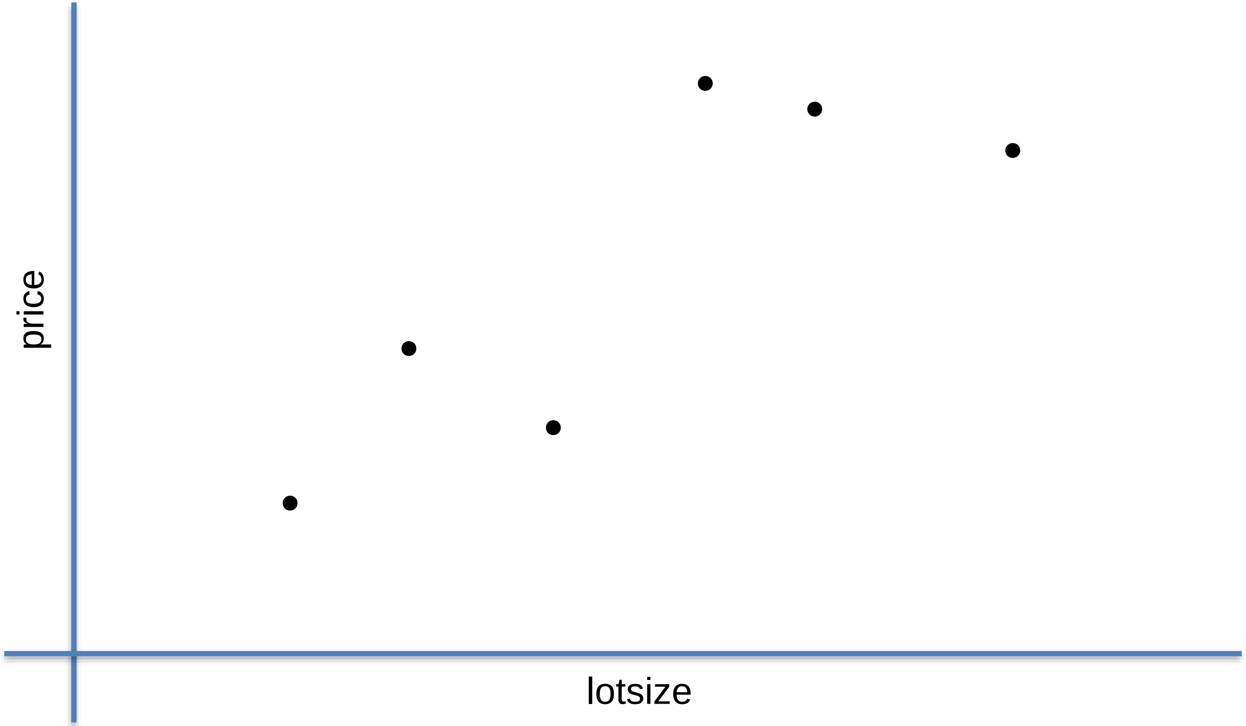


Use Ordinary Least Squares (OLS) to fit the line $\hat{y} = b_0 + b_1 x$



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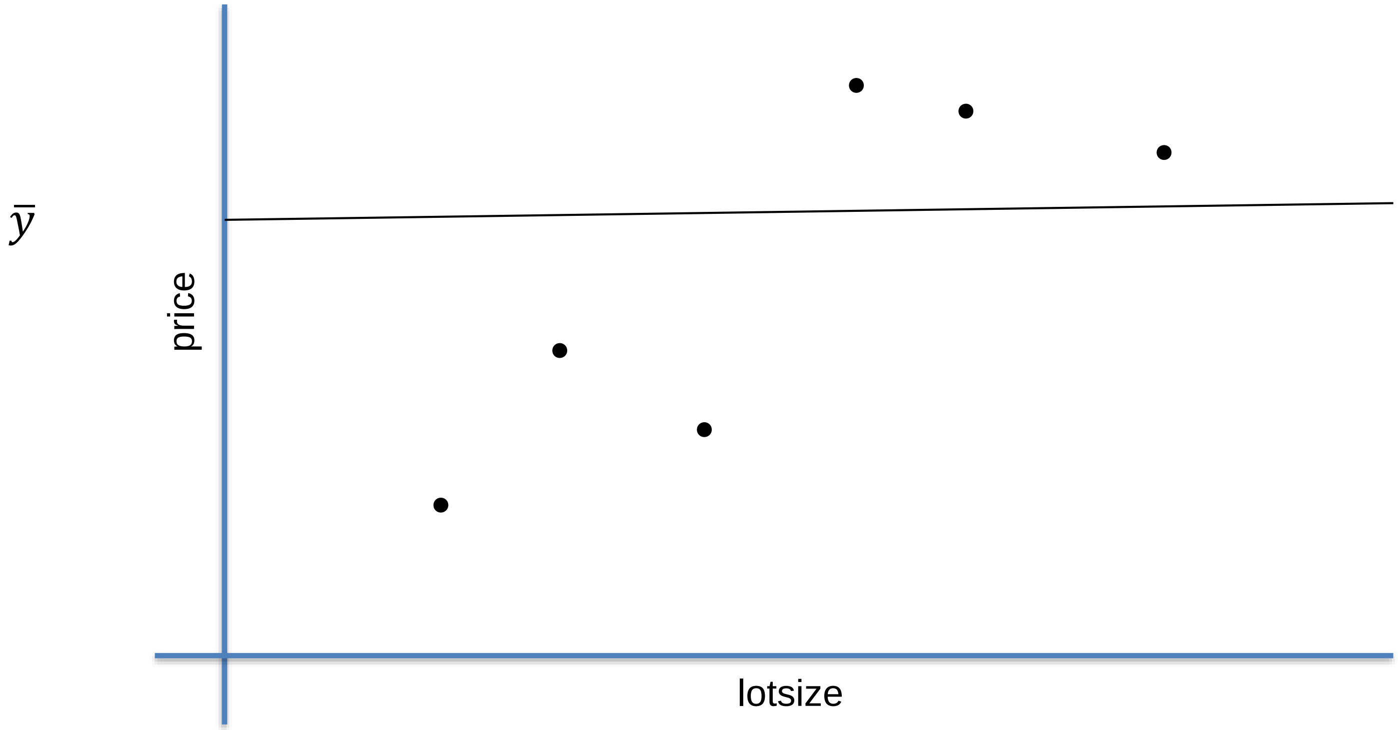

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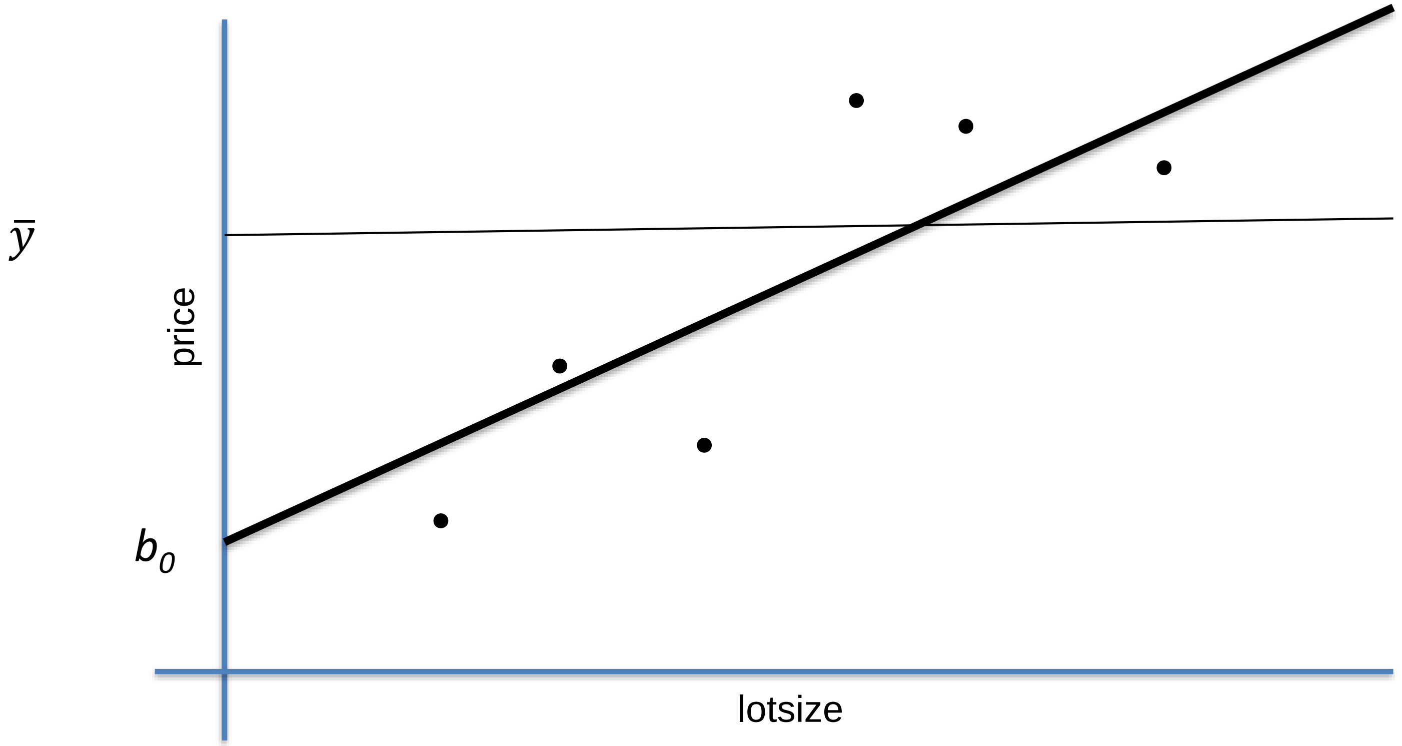

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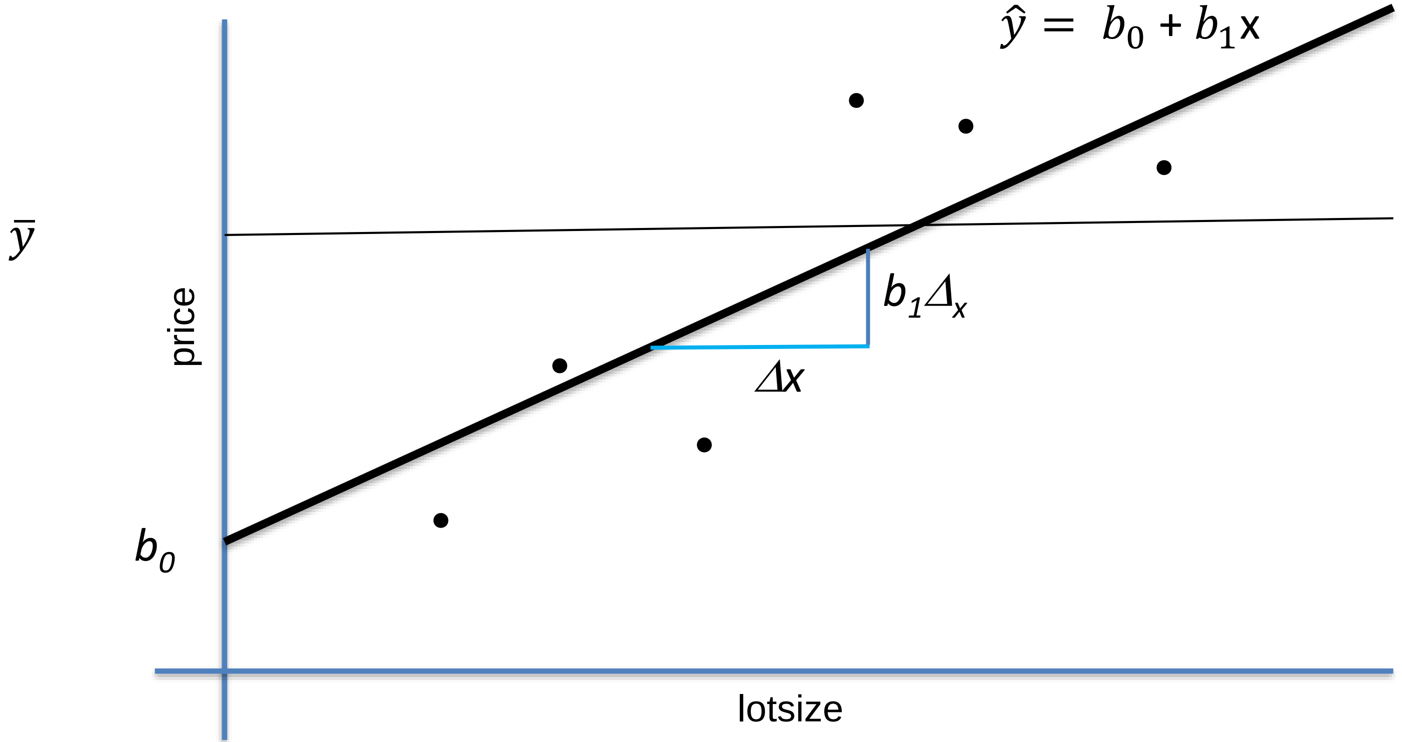
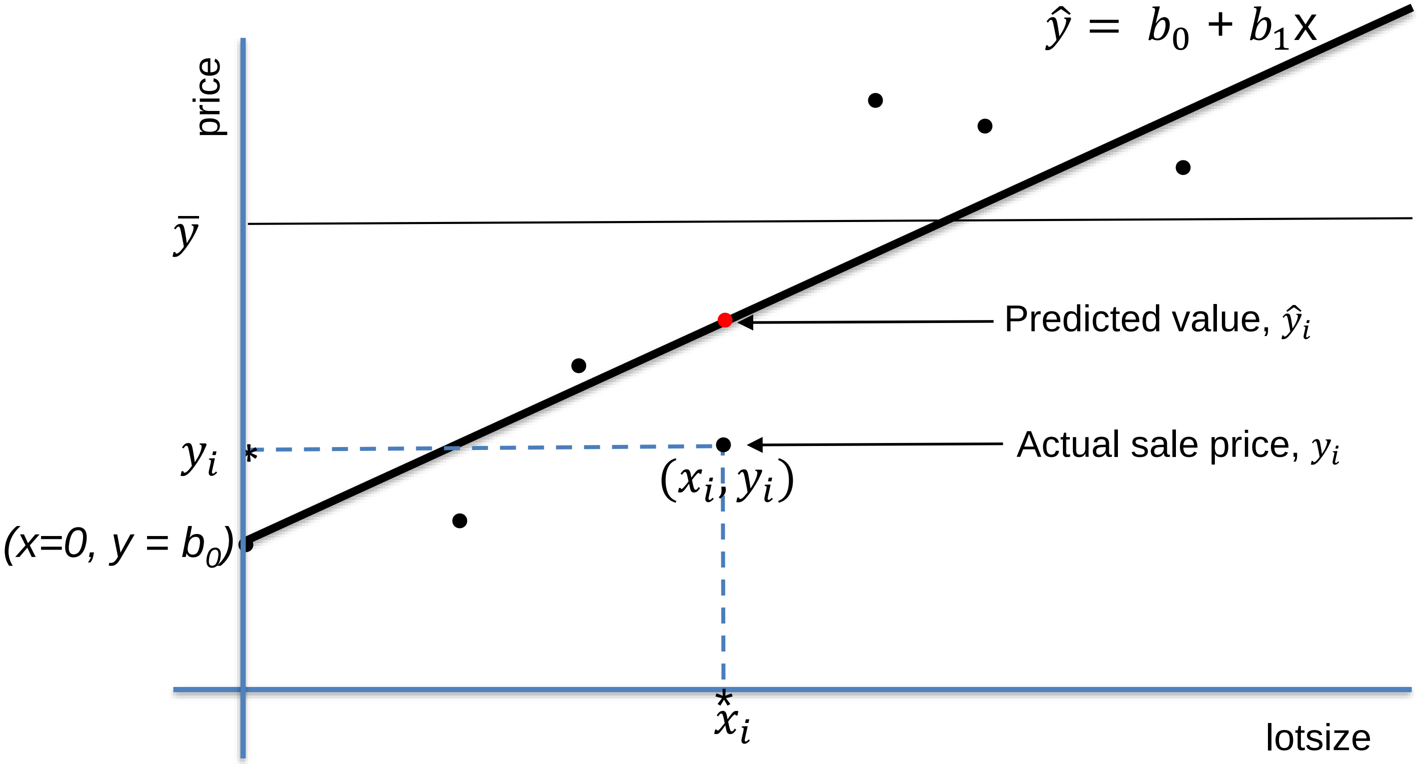
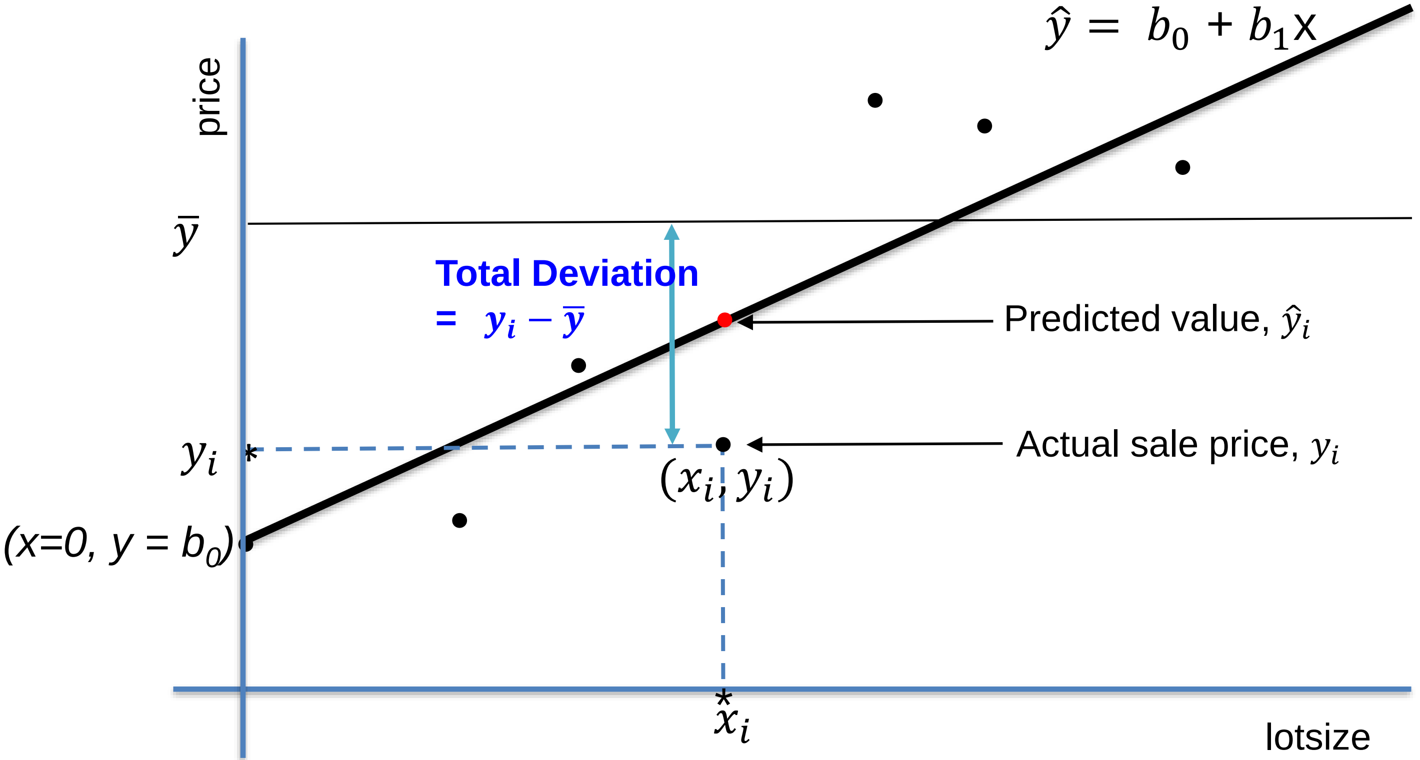


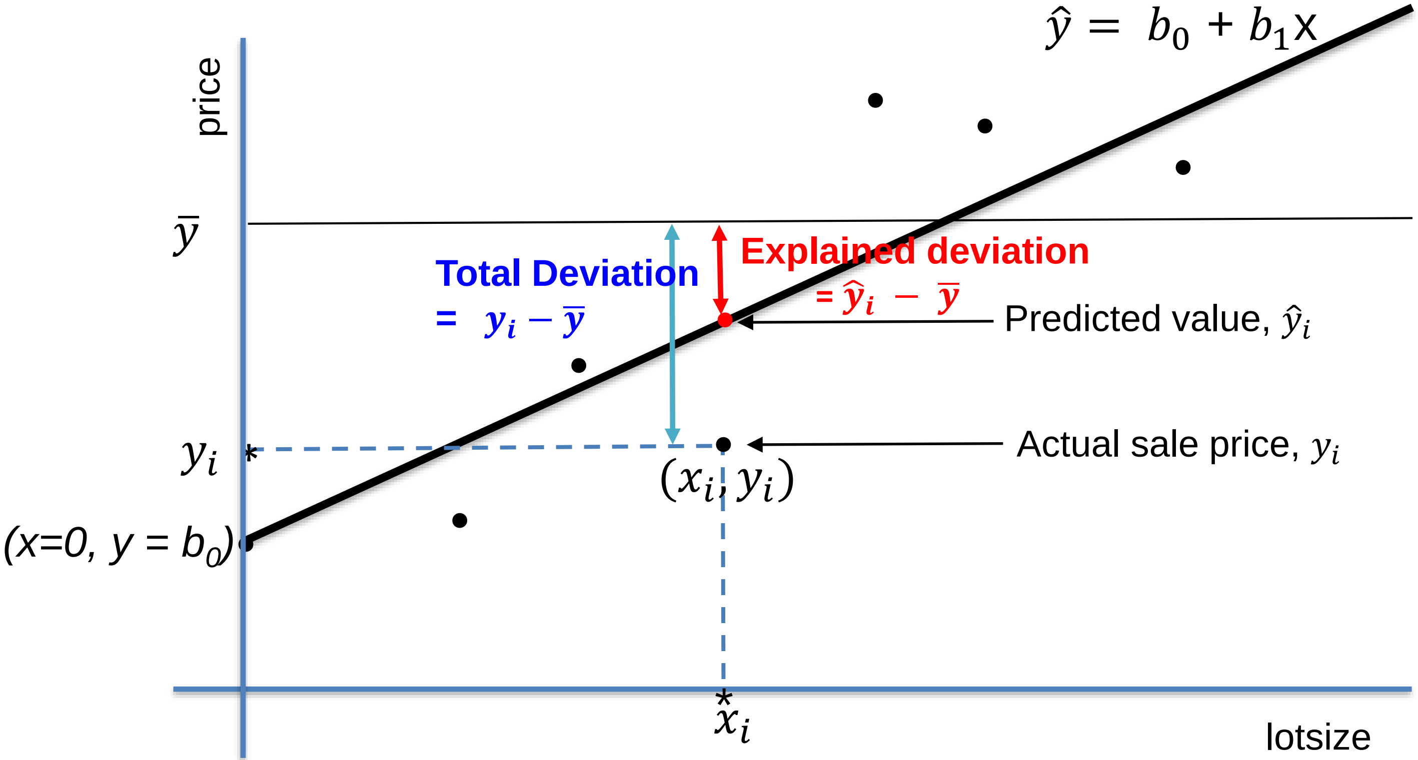
Total Deviation = Explained Deviation + Unexplained Deviation



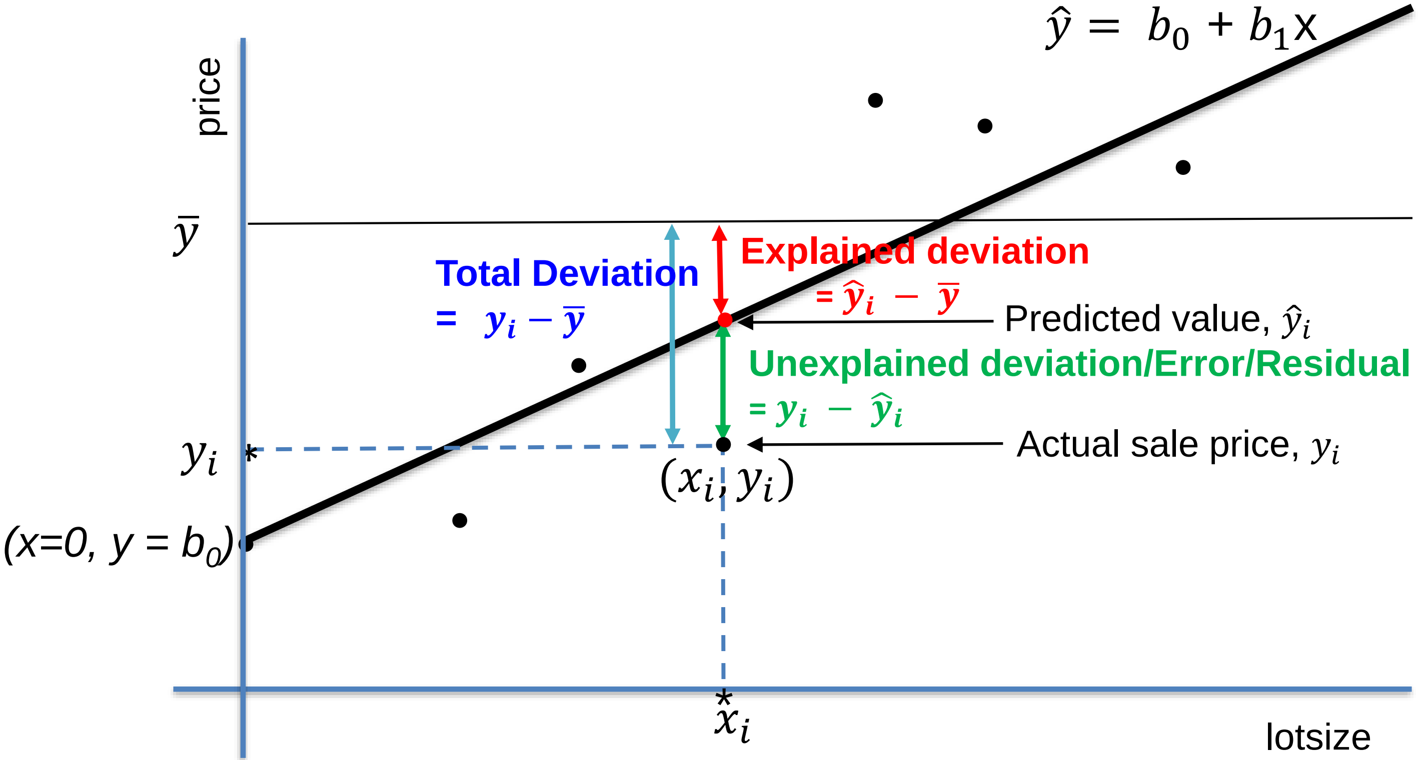
Total Deviation = Explained Deviation + Unexplained Deviation



Total Deviation = Explained Deviation + Unexplained Deviation



Total Deviation = Explained Deviation + Unexplained Deviation



Quiz (True/False)

- The total deviation at observation $(x_i\ ,\ y_i)$ is $\boldsymbol{y_i - \bar{y}}$.

  Answer: **TRUE**

- In OLS, the estimates of slope and intercept do not depend on the sample being used.

  Answer: **FALSE**

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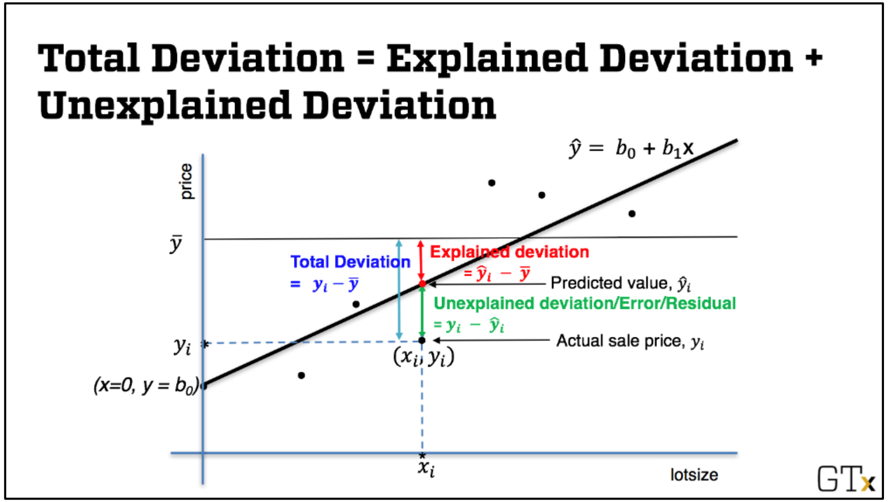
R^2, Adjusted R^2

Regression (Ordinary Least Squares): Sum of Squared Errors (SSE)

Regression (OLS) determines the line that minimizes the Sum of Squared Errors.

- i.e., b_0 and b_1 are determined such that they minimize:

$$SSE = \sum_i (y_i - \hat{y}_i)^2 = \sum_i (y_i - (b_0 + b_1 x_i))^2$$



Summing the Deviations

$\sum_i (y_i - \bar{y})^2$	=	$\sum_i (y_i - \hat{y}_i)^2$	+	$\sum_i (\bar{y} - \hat{y}_i)^2$
SST	=	SSE	+	SSR
Total Sum of Squares	=	Sum of Squared Errors	+	Sum of Squares Regression

Regression Output R^2 and Adjusted R^2

Coefficient of determination (R^2)

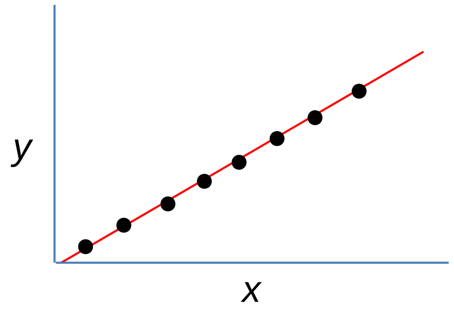
- A measure of the overall strength of the relationship between the dependent variable (Y) and independent variables (X)
- $R^2 = 1 - (SSE/SST) = SSR/SST$

  = Explained deviation (SSR)/Total Deviation (SST)
- $R^2 \rightarrow$ how much of the variation in Y (from the mean) has been explained

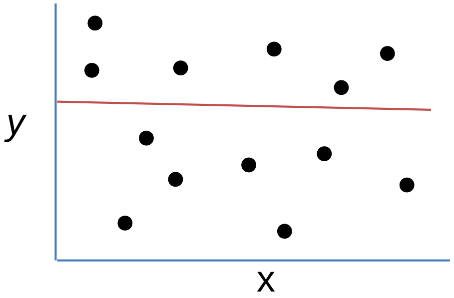
Adjusted R^2

- Adding a penalty for the number of independent variables (p)
- Adjusted $R^2 = 1 - \{SSE/(n - p - 1)\}/\{SST/(n - 1)\}$

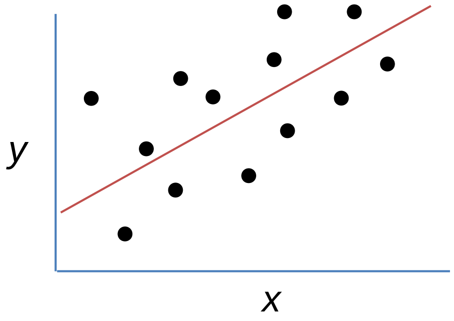
R^2



$R^2 = 1$,
X accounts for all *Y* variation



$R^2 = 0$,
X accounts for none of the *Y* variation



$R^2 = 0.75$,
X accounts for most of the *Y* variation

Quiz (True/False)

- $R^2 = 0$, implies that *X* values account for all of the variation in the *Y* values

  Answer: **FALSE**. $R^2 = 0$ implies that *X* values account for none of the variation in the *Y* values

- R^2 can take any value from – infinity to + infinity

  Answer: **FALSE**. It can take on values between 0 and 1

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Linear Regression

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Simple Regression (One Predictor Variable) Using R

Regression Output: Simple Linear Regression

lm(formula = price ~ lotsize, data = Housing)

Residuals:

Min	1Q	Median	3Q	Max
-69551	-14626	-2858	9752.	106901

Unexplained deviation/Error/Residual
$= y_i - \hat{y}_i$

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.414e+04	2.491e+03	13.7	<2e-16 ***
lotsize	6.599e+00	4.458e-01	14.8	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 22570 on 544 degrees of freedom
Multiple R-squared: 0.2871, Adjusted R-squared: 0.2858
F-statistic: 219.1 on 1 and 544 DF, p-value: < 2.2e-16

Regression Output: Coefficients

b_0 and b_1 are estimates of the true parameters β_0 and β_1

H_0: the parameter is zero, H_1: The parameter is not zero

lm(formula = price ~ lotsize, data = Housing)

Residuals:

Min	1Q	Median	3Q	Max
-69551	-14626	-2858	9752	106901

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Multiple R-squared: 0.2871, Adjusted R-squared: 0.2858

F-statistic: 219.1 on 1 and 544 DF, p-value: < 2.2e-16

Regression Output: t-values for Coefficients

p value: the probability of finding a t value of this size if the null hypothesis is true

H_0: the parameter is zero

```
lm(formula = price ~ lotsize, data = Housing)

Residuals:
   Min       1Q      Median    3Q     Max
 -69551   -14626    -2858     9752   106901

Coefficients:
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```

Interpreting Coefficients

	Estimate
(Intercept)	3.414e+04 ***
lotsize	6.599e+00 ***

$b_0 = 34{,}140$

Intercept of the regression line with the y-axis (when lotsize is zero). Not useful.

$b_1 = 6.599$

An increase of 1,000 square feet is associated with an increase of the sale price of a house by $6,599, keeping all else constant (*ceteris paribus*)

Regression Output: Sum of Squares

Analysis of Variance Table

	Df	Sum Sq
lotsize	1	1.1156e+11
Residuals	544	2.7704e+11

SSR = 1.1156e+11

SSE = 2.7704e+11

SST = SSR + SSE = 3.886e+11

Regression Output: R^2

```
lm(formula = price ~ lotsize, data = Housing)

Residuals:
   Min       1Q      Median     3Q      Max
-69551   -14626     -2858     9752   106901

Coefficients:
              Estimate     Std. Error    t value    Pr(>|t|)
(Intercept)   3.414e+04    2.491e+03     13.7       <2e-16 ***
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Multiple R-squared:  0.2871,     Adjusted R-squared:  0.2858

F-statistic: 219.1 on 1 and 544 DF,  p-value: < 2.2e-16
```

Regression Output R^2 and Adjusted R^2

SSR = 1.1156e+11

SSE = 2.7704e+11

SST = SSR + SSE = 3.886e+11

Multiple R-squared: 0.2871, Adjusted R-squared: 0.2858

R^2 = 1 – (SSE/SST) = SSR/SST = 1.1156e+11/3.886e+11

= 0.2871

Note: $\sqrt{R^2} = \sqrt{0.2871}$ = 0.536 (which is the correlation coefficient between price and lotsize)

Adjusted R^2 = 1 – {SSE/(n – p – 1)}/{SST/(n – 1)}

= 1 – {(2.7704e+11/(546 – 1 – 1)} / {3.886e+11/(546 – 1)}

= 0.2858

F-test that the model is significant (H_0: $b_1 = 0$)

SSR = 1.1156e+11

SSE = 2.7704e+11

SST = SSR + SSE = 3.886e+11

If p is the number of independent variables, The F statistic

= (SSR/p)/ (SSE/n – p – 1) = (R^2/p) / (1 – R^2)/(n – p – 1)

The value of Prob(F) is the probability that H_0 is true (i.e., $b_1 = 0$).

For this model, p = 1,

F = (0.2871/1) / (1 – 0.2871)/(546 – 1 – 1) = 219.1

with (1,544) degrees of freedom.

F statistic: 219.1 with(1, 544) DF, p-value: < 2.2e-16. Hence H_0 is rejected.

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Multiple Regression

Multiple Linear Regression, with *p* Explanatory Variables

- **Regression coefficients**:

  $b_0, b_1, \ldots, b_p$ are estimates of $\beta_0, \beta_1, \ldots, \beta_p$
- **Prediction** for Y at x_i

  $\hat{y}_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + \ldots + b_p x_{pi}$
- **Residual**:

  $e_i = y_i - \hat{y}_i$

Goal: choose $b_0, b_1, \ldots, b_p$ to minimize the sum of squared errors

$$SSE = \sum_i (y_i - \hat{y}_i)^2 =$$

$$\sum_i (y_i - (b_0 + b_1 x_{1i} + \ \ldots b_p x_{pi}))^2$$

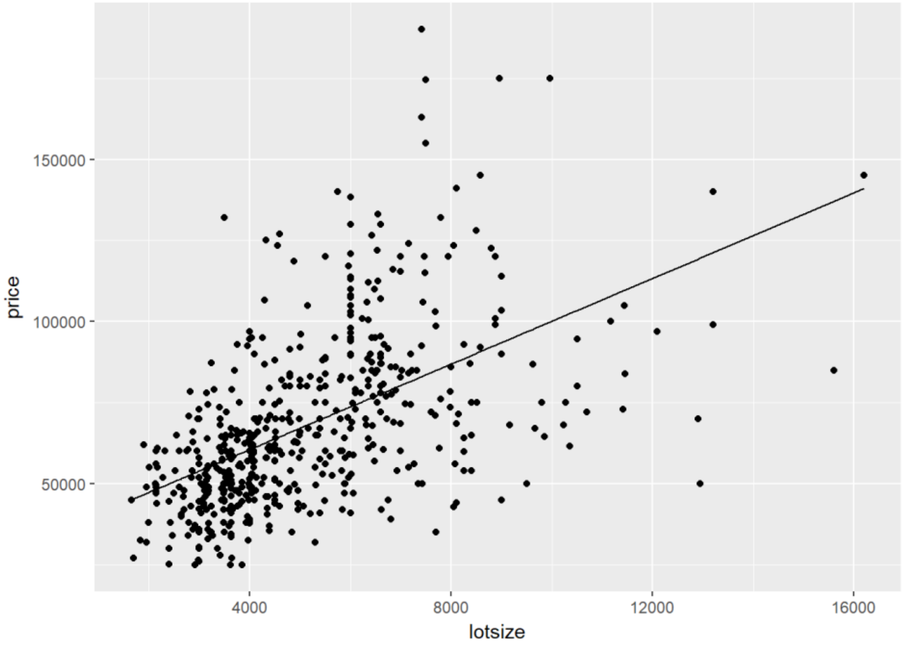
Using R to estimate a Linear Model

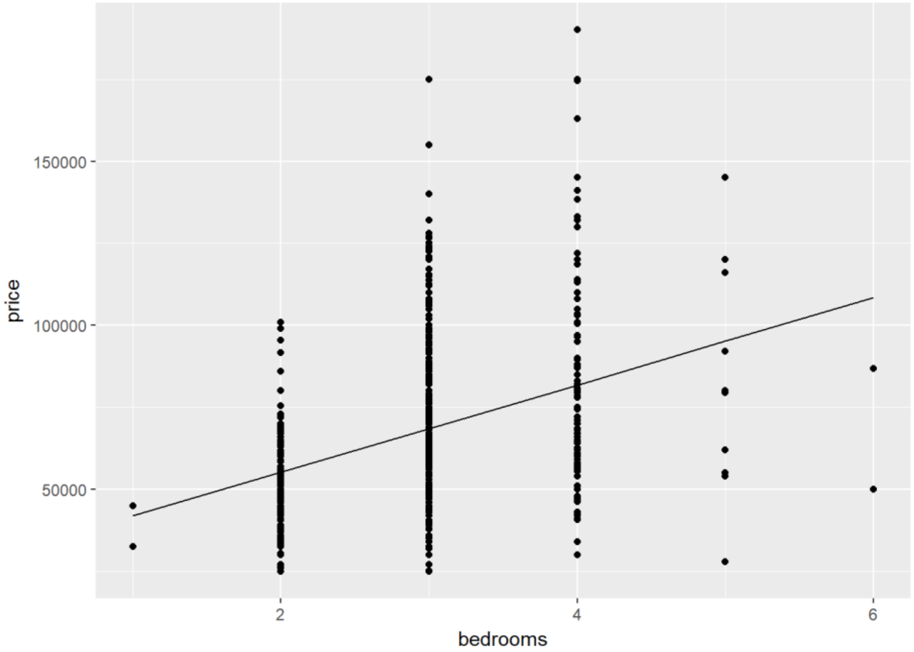
Using the *Housing* Dataset in the *Ecdat* package in R

Adding Bedrooms to the Analysis

price	lotsize	bedrooms
42,000	5,850	3
38,500	4,000	2
49,500	3,060	3
60,500	6,650	3
61,000	6,360	2
66,000	4,160	3
66,000	3,880	3
69,000	4,160	3
83,800	4,800	3
88,500	5,500	3
90,000	7,200	3
30,500	3,000	2
27,000	1,700	3
36,000	2,880	3
37,000	3,600	2

Visualize (Plots)





Do the slopes make sense?

Regression Output

lm(formula = price ~ lotsize + bedrooms, data = Housing)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.613e+03	4.103e+03	1.368	0.172
lotsize	6.053e+00	4.243e-01	14.265	< 2e-16 ***
bedrooms	1.057e+04	1.248e+03	8.470	2.31e-16 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 21230 on 543 degrees of freedom

Multiple R-squared: 0.3703, Adjusted R-squared: 0.3679

F-statistic: 159.6 on 2 and 543 DF, p-value: < 2.2e-16

Regression Output: Coefficients

b_0, b_1, …, b_p* are estimates of the true parameters *β_0, β_1, …, β_p

H_0: the parameter is zero, H_1: The parameter is not zero

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lotsize	6.053e+00	4.243e-01	14.265	< 2e-16 ***
bedrooms	1.057e+04	1.248e+03	8.470	2.31e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21230 on 543 degrees of freedom
Multiple R-squared: 0.3703, Adjusted R-squared: 0.3679
F-statistic: 159.6 on 2 and 543 DF, p-value: < 2.2e-16

Regression Output: Standard Error of the Coefficients

Similar to Standard Deviation

lm(formula = price ~ lotsize + bedrooms, data = Housing)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.613e+03	4.103e+03	1.368	0.172
lotsize	6.053e+00	4.243e-01	14.265	< 2e-16 ***
bedrooms	1.057e+04	1.248e+03	8.470	2.31e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21230 on 543 degrees of freedom
Multiple R-squared: 0.3703, Adjusted R-squared: 0.3679
F-statistic: 159.6 on 2 and 543 DF, p-value: < 2.2e-16

Regression Output: t-values for Coefficients

lm(formula = price ~ lotsize + bedrooms, data = Housing)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.613e+03	4.103e+03	1.368	0.172
lotsize	6.053e+00	4.243e-01	14.265	< 2e-16 ***
bedrooms	1.057e+04	1.248e+03	8.470	2.31e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21230 on 543 degrees of freedom
Multiple R-squared: 0.3703, Adjusted R-squared: 0.3679
F-statistic: 159.6 on 2 and 543 DF, p-value: < 2.2e-16

Interpreting Coefficients

	Estimate
(Intercept)	5.613e+03
lotsize	6.053e+00
Bedrooms	1.057e+04

b_0 = 5613

Intercept of the regression line with the y-axis (when all x's are zero). Not useful

b_1 = 6.053

An increase of 1,000 square feet is associated with an increase of the sale price of a house by $6,053, keeping all else constant

b_2 = 10570

An additional bedroom is associated with an increase of the sale price of a house by $10,570, keeping all else constant

Data Analytics in Business

Linear Regression

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R^2, Adjusted R^2 from Multiple Regression

Regression Output: Sum of Squares

Analysis of Variance Table

	Df	Sum Sq
lotsize	1	1.1156e+11
bedrooms	1	3.2329e+10
Residuals	543	2.4472e+11

SSR = (1.1156 + 0.32329) e+11

SSE = 2.4472e+11

SST = SSR + SSE = 3.88609e+11

Regression Output: R^2

```
lm(formula = price ~ lotsize + bedrooms, data = Housing)
Coefficients:
              Estimate     Std. Error   t value   Pr(>|t|)
(Intercept)   5.613e+03    4.103e+03    1.368     0.172
lotsize       6.053e+00    4.243e-01    14.265    < 2e-16 ***
bedrooms       1.057e+04   1.248e+03    8.470     2.31e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21230 on 543 degrees of freedom
Multiple R-squared:  0.3703,   Adjusted R-squared:  0.3679
F-statistic: 159.6 on 2 and 543 DF,  p-value: < 2.2e-16
```

Regression Output R^2 and Adjusted R^2

SSR = (1.1156 + 0.32329) e+11

SSE = 2.4472e+11

SST = SSR + SSE = 3.88609e+11

Multiple R-squared: 0.3703, Adjusted R-squared: 0.3679

R^2 = 1 – SSE/SST = SSR/SST = 1.43889e+11/3.88609e+11

= 0.3703

Adjusted R^2 = 1 – {SSE/(n – p – 1)}/{SST/(n – 1)}

= 1 – ((2.4472e+11/(546 – 2 – 1)} / {3.88609e+11/(546 – 1)}

= 0.3679

F-test of the overall significance of the model (H_0: $b_1 = b_2 = 0$)

SSR = (1.1156 + 0.32329) e+11

SSE = 2.4472e+11

SST = SSR + SSE = 3.88609e+11

The F statistic

= (SSR/p) / (SSE/(n – p – 1)) = (R^2/p) / ((1 – R^2)/(n – p – 1))

The value of Prob(F) is the probability that H_0 is true.

For this example, F = (0.3703/2)/(1 – 0.3703)/(546 – 2 – 1) = 159.6

F-statistic: 159.6 on 2 and 543 DF, p-value: < 2.2e-16. Hence H_0 is rejected.

Simple vs. Multiple Regression

- For the Simple Regression we got:

  Multiple R-squared: 0.2871, Adjusted R-squared: 0.2858

- For the Multiple Regression we got:

  Multiple R-squared: 0.3703, Adjusted R-squared: 0.3679

- As you add variables, R-square will not decrease

Comparing the Two Models

n: number of observations

p: number of variables (do not count the intercept)

Bigger model 2 which has (more) p_2 variables

Smaller model 1 which has (fewer) p_1 variables

We want to determine whether model 2 gives a *significantly* better fit to the data. Then use the F statistic shown below

- $F(p_2 - p_1,\ n - p_2 - 1)$
- F test statistic is calculated as

$$F = \frac{(R_2^2 - R_1^2)/(p_2 - p_1)}{(1 - R_2^2)/(n - p_2 - 1)}$$

Quiz (True/False)

- In general, adding more variables decreases the overall R-Square value of the multiple regression.

  Answer: **FALSE**.

- In the regression output shown below, a p-value of < 2e-16 *** means that there is not much evidence for the coefficient of lotsize to be different from zero.

  Answer: **FALSE**.

lm(formula = price ~ lotsize + bedrooms, data = Housing)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.613e+03	4.103e+03	1.368	0.172
lotsize	6.053e+00	4.243e-01	14.265	< 2e-16 ***
bedrooms	1.057e+04	1.248e+03	8.470	2.31e-16 ***

GTx

Data Analytics in Business

Linear Regression

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Predictions (Interpolation)

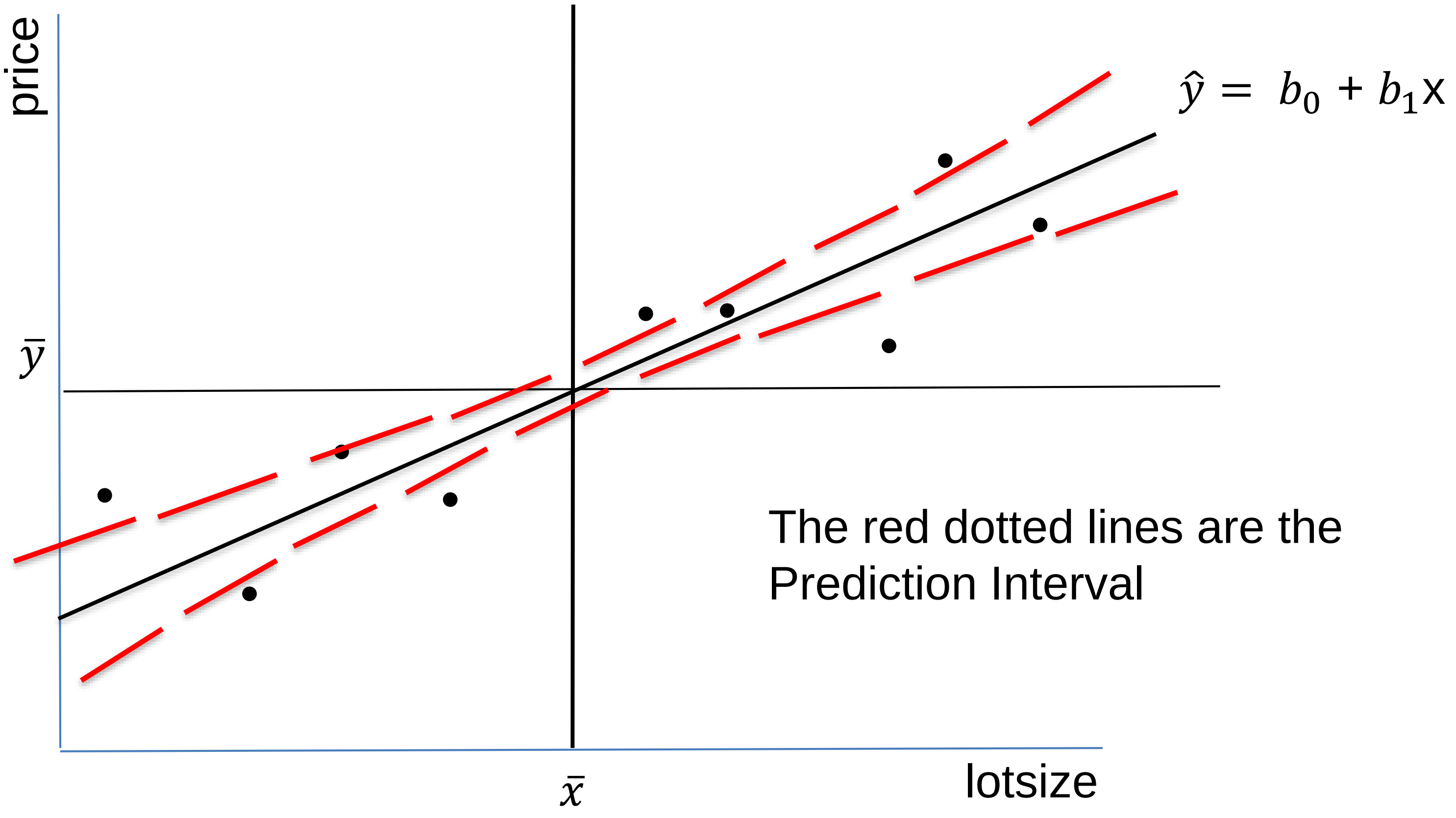
Predictions

You know of several methods for prediction, including:

- Decision Trees
- Random Forests
- Neural Nets
- Etc.

We are interested in interpolation here.

Prediction Interval



The red dotted lines are the Prediction Interval

Prediction Interval

What does this mean?

- Narrowest at $(\bar{X}, \bar{Y})$
- It gets wider the further away X is from $(\bar{X})$
- Need to be careful about when to use prediction

Making Predictions (Interpolation)

```
ab.lm <- lm(formula = price ~ lotsize + bedrooms, data = Housing)
```

Coefficients:

	Estimate	Std. Error
(Intercept)	5.613e+03	4.103e+03
lotsize	6.053e+00	4.243e-01
bedrooms	1.057e+04	1.248e+03

For a new data point for a house with a lotsize of 3000 sq ft and 2 bedrooms, we can use the predict function in R.

```
newdata = data.frame(lotsize=3000, bedrooms = 2)
predict(ab.lm, newdata, interval = "predict")
```

fit	lwr	upr
44906.37	3077.091	86735.65

The expected predicted value is $44,906.37
and the 95% Prediction Interval is ($3,077.091, $86,735.65)

Evaluation of Prediction Methods

- Sometimes, regression is used for prediction.
- You already have learned about metrics to evaluate prediction performance in other courses.
- You could also use training sets and evaluation sets with regression models.
- Use Confusion Matrices to develop metrics (sensitivity, specificity, accuracy, etc.) to evaluate the prediction performance of your regression model.

Recap of this Module

A. Steps in Regression Analysis
B. Linear Regression Example
C. Notation
D. R^2, Adjusted R^2
E. Simple Regression (One Predictor Variable) Using R
F. Multiple Regression
G. R^2, Adjusted R^2 from Multiple Regression
H. Prediction