2D-Ising critical fluctuation

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1 Introduction

This project mainly investigate 2D-Ising system near the critical point. We use Markov Chain Monte Carlo method with Metropolis updates as dynamics of this system and extrapolate the result from finite size system to infinity size system by tools named Finite Size Scaling.
The 2D-Ising system is always a classical example in statistical physics. The most important property of this system near critical point is the space-correlation would be divergent and fluctuation with different span could be treatedd equivalently. In other words, all configurations are near equilibrium. As a result, we can do linear response analysis without any approximation. It is quite straightforward that the relaxtion behavior coule be related to information of fluctuation in critical points.
This project is coded with Julia and Python. The code is published on Github: https://github.com/qiyang-ustc/2d-Ising-Dynamics

2 Method

The model and the method in this report mainly follow the article given by Ito in 1993.https://arxiv.org/abs/cond-mat/9302009 In that article, Ito studied 3D-Ising model but did not related the result with fluctuation-dissipation theorem.

2.1 Dynamics

The dynamics in this project is given by Metropolis Method. Each steps, we pick up a spin sequentially

3 Theoretical analysis

The Hamiltonian of oußr system is: s_i could be $+1$ or -1.

$$\mathcal{H} = \sum_{\langle i,j \rangle} -J s_i s_j - \sum_i h s_i$$

We first prepare our system at $h \to \infty$, which is equivalent to say:

$$\mathcal{H} = \mathcal{H}_0 - Nhm\theta(-t)$$

where N is the number of total spins and m is order parameter $m = \sum s_i/N$. T is time. $\theta(t) = 1$ if $t > 0$ else $theta(t) = 0$ It is obvious that the expectation for order parameter is zero for non-disturbed state. And the system is prepare at $m(0) = 1$.
Now, we use linear response theory to analyze this system: when $t = 0$, The wight is

$$W(x, t = 0) = \frac{e^{-\beta H}}{Z} = \frac{e^{-\beta H_0} \times e^{\beta Nmh}}{\sum_s e^{-\beta H_0 + \beta Nmh}}$$

So,

$$= \frac{e^{-\beta H_0}(1 + \beta Nhm)}{Z} - \frac{e^{-\beta H_0}}{Z^2} e^{-\beta H_0} Nhm = W_0(x, 0) \times (1 + \beta Nh(m - \langle m \rangle))$$

We can conclude that:

$$\rho(t = 0) + \Delta\rho(t = 0) = W(x, 0) = W_0(x, 0)(1 + \beta Nh\delta m)$$

$$\Delta\rho(t) = \beta NH\delta m W_0(t)$$

If we donated the time-correlation of order parameter by $A(t)$ which means:

$$A(t) = \langle \delta m(t)\delta m(0)\rangle$$

Generalized susceptibility χ in linear response theory:

$$\langle \delta m(t)\rangle = \int_{\infty}^{t} \chi(t-t')\theta(-t')Nhdt'$$

Besides, by definition

$$\langle \delta m(t)\rangle = Tr(\Delta\rho(t)\hat{m}) = Tr(\Delta\rho(0)\hat{m}(t)) = \beta Nh\,\langle m(t)m(0)\rangle = \beta NhA(t)$$

Take derivatives of the equation:

$$\beta NhA(t) = \langle m(t)\rangle = \int_{-\infty}^{t} \chi(t-t')Nh\theta(-t')dt'$$

We can get:

$$\chi(t) = -\beta\dot{A}(t) \quad if \quad t > 0$$