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Projected d-wave superconducting state: a fermionic projected entangled pair state study

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Published: Phys.Rev. B 107, 125128

1. Motivation

- The Projected Bardeen–Cooper–Schrieffer (BCS) state is important in the studies of strong correlated electrons.
- The tensor network is a powerful and promising in the investigation of strong correlated system. Though, the tensor network representation of states is required.
- A systematic approach to convert projected BCS state into two-dimensional tensor network is highly desired.

2. Method

2.1 Sketch

Fermionic quadratic Hamiltonian H_{BCS}

↓ Optimization

Correlation matrix Γ

↓ Translation

Local tensor A^f_{uldr}

↓ Projection

Tensor network representation of $P_G\,|\text{BCS}\rangle$

↓ Contraction

Physics of $P_G\,|\text{BCS}\rangle$

2.2 Details

1. A fermionic quadratic Hamiltonian.

$$H_{\text{BCS}} = \sum_{\mathbf{k}} \begin{pmatrix} f_{\mathbf{k}\uparrow} & f^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & \xi_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} f^\dagger_{\mathbf{k}\uparrow} \\ f_{-\mathbf{k}\downarrow} \end{pmatrix} \tag{1}$$

2. Optimize $\langle H_{\text{BCS}}\rangle$ for the Gaussian fPEPS state [4].

3. Correlation matrix of the fiducial state $|A_{\mathbf{i}}\rangle$.

$$\Gamma_{\mu\nu} = \frac{i}{2}\langle A_{\mathbf{i}}|[\gamma_\mu,\gamma_\nu]|A_{\mathbf{i}}\rangle. \tag{2}$$

Here, γ is the Majorana fermions corresponding to physical f and virtual complex fermions u,l,d,r.

4. Translation can be implemented by solving the ground state of the fiducial hamiltonian

$$h = -\sum_{\mu\nu} i\Gamma_{\mu\nu}\gamma_\mu\gamma_\nu. \tag{3}$$

5. The ground state of Eq. (3) is the local tensor A^f_{uldr} of $|A_{\mathbf{i}}\rangle$

$$|A_{\mathbf{i}}\rangle = \sum_{f,u,l,d,r} A^f_{uldr}\,|f\rangle \otimes |uldr\rangle\,. \tag{4}$$

which can be visualized in Fig. 1(a).

6. Gutzwiller projection P_G is visualized in Fig. 1(b). Here, we include a fugacity term z thus

$$P_G = \prod_{\mathbf{i}} z^{(1-n_{\mathbf{i}\uparrow}-n_{\mathbf{i}\downarrow})/2}\left(1-n_{\mathbf{i}\uparrow}n_{\mathbf{i}\downarrow}\right). \tag{5}$$

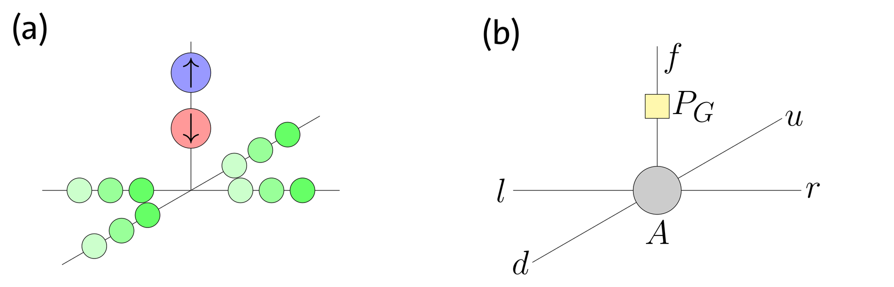
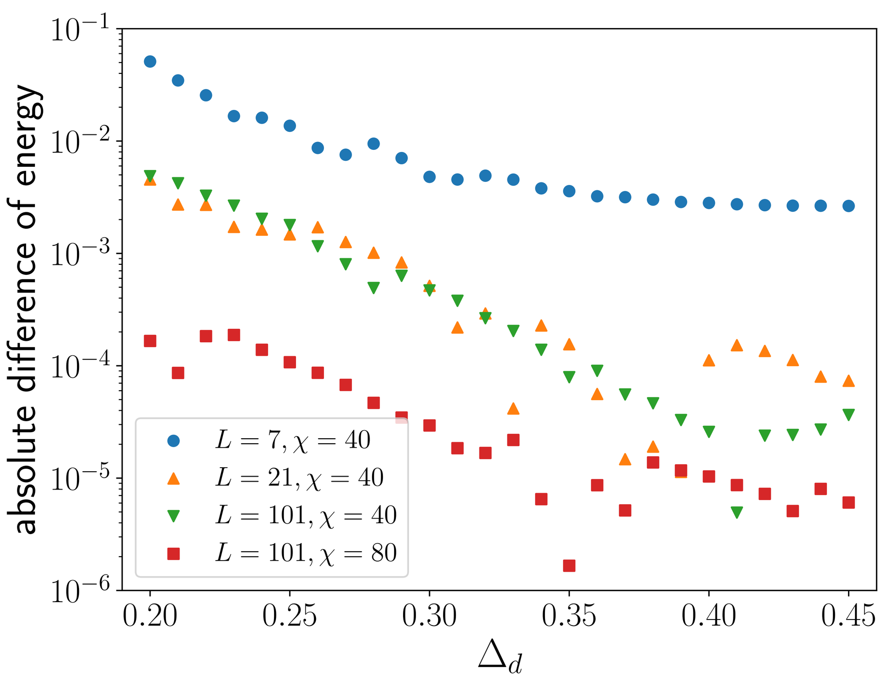


Figure 1: (a) Green dots: virtual fermions. Red and Blue dots: Physical fermions. (b) Grey dot: tensor A^f_{uldr}. Yellow square: tensor P_G.

7. Contraction: the tensor network is contracted in infinite two-dimensional lattice with variational uniform matrix product state algorithm.[1, 5]

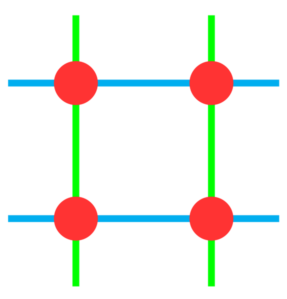
3. Result

3.1 Energy difference from tensor network contraction (infinite) and Gaussian fPEPS (finite)



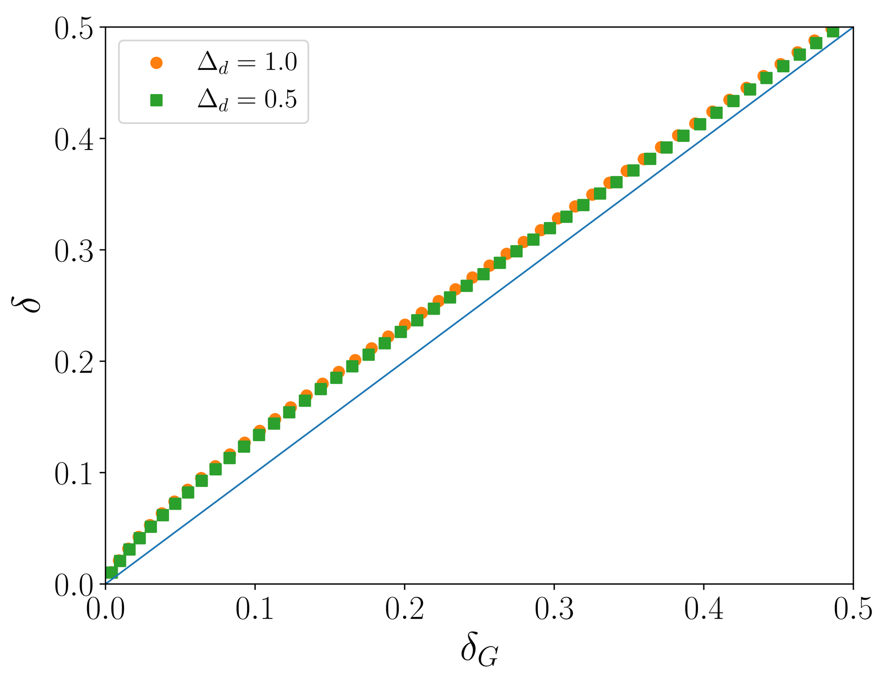
The difference is quite small and can be reduced systematically by enlarging the bond dimension χ kept in the contraction.

3.2 Order parameter from contraction



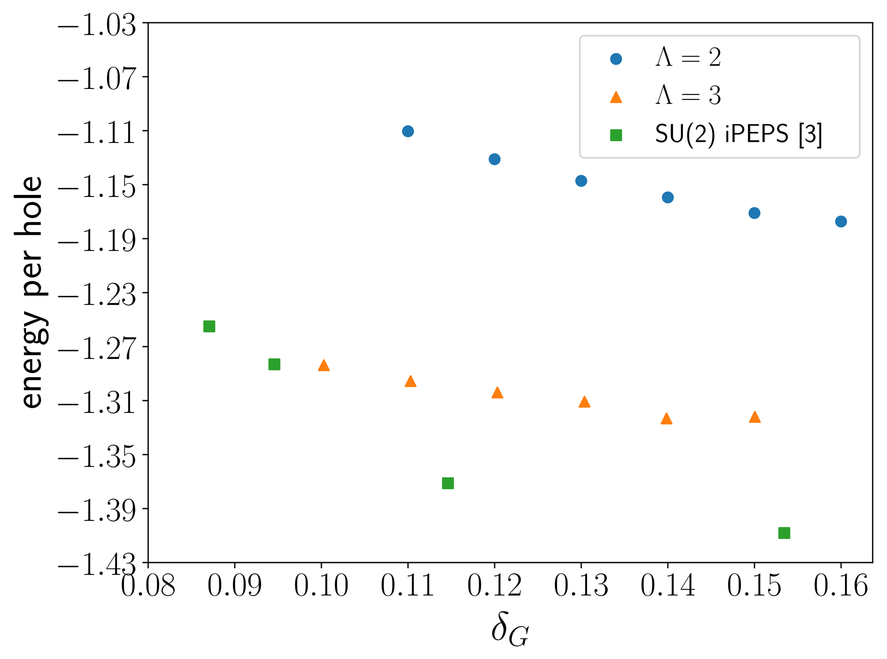
☑ uniform
☑ d-wave superconducting order
☒ magnetic order
☒ charge order

3.3 Hole density before δ and after δ_G Gutzwiller projection



The deviation of points from the blue line represents the error of the Gutzwiller approximation $z = 2\delta/(1+\delta)$ [2] for the hole fugacity.

3.4 Energy per hole of the t-J model with doping hole density δ_G



Projected d-wave BCS superconducting state is a competing low-energy state in t-J model.

4. Summary

- We have developed a systematic method to construct the fPEPS representation of projected BCS states and investigated the physical pairing state on an infinite square lattice.
  Code is available online:
- Our approach can also be applied to other classes of projected fermionic states.
- Partial Gutzwiller projection can also be straightforwardly implemented.

5. Outlook

- Obtaining initialization from mean-field theory in the study of fermionic two-dimensional tensor network (e.g. fermionic iPEPS).
- Estabilishing a bridge between variational monte carlo method and infinite tensor networks by contructing variational wavefunctions in infinite tensor networks.

6. Funding

This project is supported by the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant No. XDB30000000, the National Natural Science Foundation of China under Grant No. 11774398, and the Deutsche Forschungsgemeinschaft (DFG) through project A06 of SFB 1143 (project-id 247310070).

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