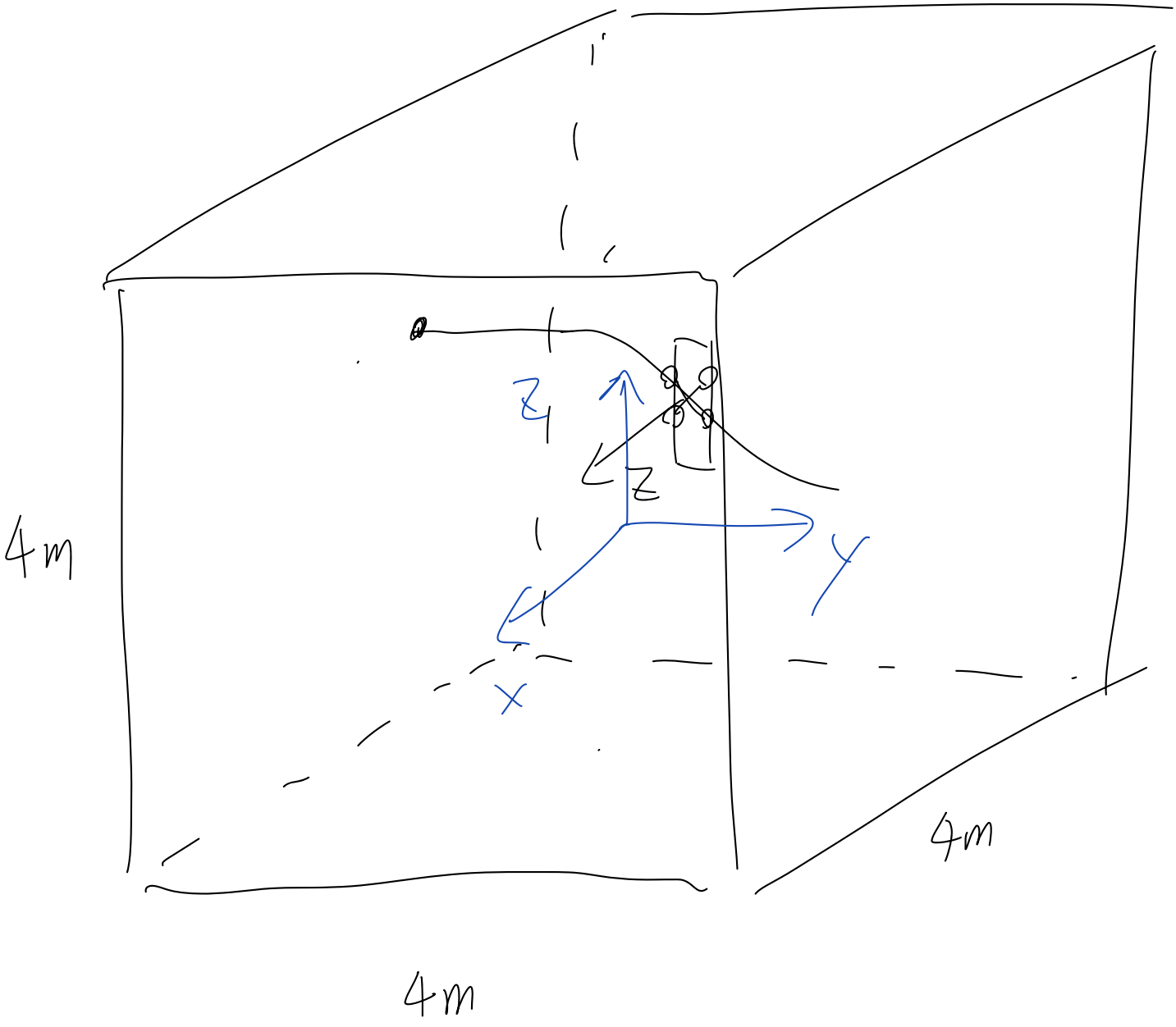
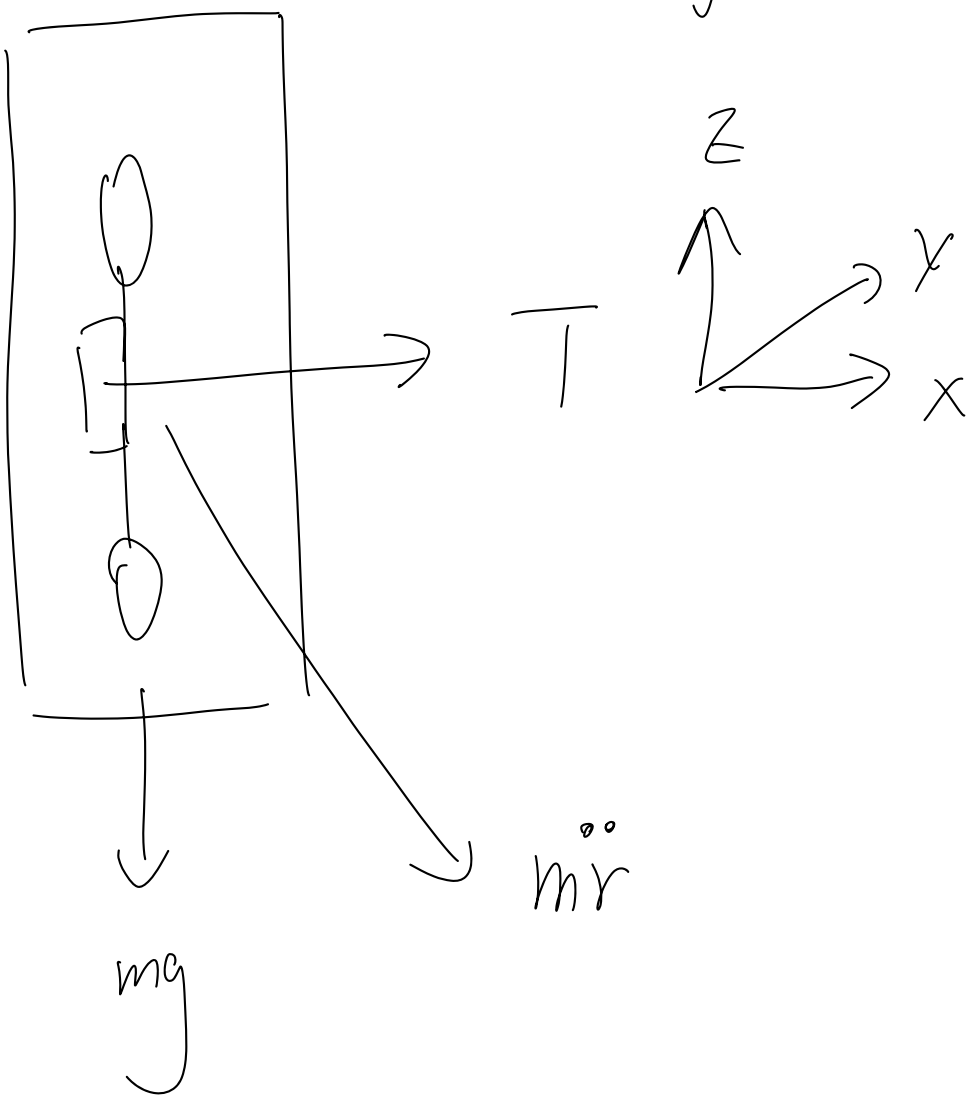
Set up



Dynamics at the gate → (0, 0, 0)

at origin



$$m\ddot{r}_f^x = T$$

$$\ddot{r}_f^x = \frac{1}{m}T$$

$$\ddot{r}_f^y = 0$$

$$m\ddot{r}_f^z = -mg$$

$$\ddot{r}_f^z = -g$$

Initial & final condition.

1. $\vec{r}_0 = (x_0, y_0, z_0)$
2. $\dot{\vec{r}}_0 = (0, 0, 0)$
3. $\ddot{\vec{r}}_0 = (0, 0, 0)$
4. $\vec{r}_f = (x_f, y_f, z_f)$
5. $\dddot{\vec{r}}_f = \left(\frac{I}{m}, 0, -g\right)$

$\begin{cases} 6a\ \ddddot{\vec{r}}_f \quad \text{for } x \& y & \dddot{x} = \dddot{y} = 0. \\ 6b.\ \dddot{\vec{r}}_f. \quad \text{for } z & \ddot{z} = 0. \end{cases}$

$$\vec{r}(t) = \vec{a}_5 t^5 + \vec{a}_4 t^4 + \vec{a}_3 t^3 + \cancel{\vec{a}_2 t^2} + \cancel{\vec{a}_1 t} + \vec{a}_0 \qquad (7)$$

from (1), (2), (3) $\rightarrow$ $\vec{a}_0 = \vec{r}_0$, $\vec{a}_1 = \vec{a}_2 = 0$

(7) becomes

$$\vec{r}(t) = \vec{a}_5 t^5 + \vec{a}_4 t^4 + \vec{a}_3 t^3 + \vec{r}_0 \qquad (8)$$

$$\dot{\vec{r}}(t) = 5\vec{a}_5 t^4 + 4\vec{a}_4 t^3 + 3\vec{a}_3 t^2 \qquad (9)$$

$$\ddot{\vec{r}}(t) = 20\vec{a}_5 t^3 + 12\vec{a}_4 t^2 + 6\vec{a}_3 t \qquad (10)$$

$$\dddot{\vec{r}}(t) = 60\vec{a}_5 t^2 + 24\vec{a}_4 t + 6\vec{a}_3 \qquad (11)$$

for x and y

use (8), (10), (11), with (4), (5), (6a)

$$\begin{bmatrix} t_f^5 & t_f^4 & t_f^3 \\ 20t_f^3 & 12t_f^2 & 6t_f \\ 60t_f^2 & 24t_f & 6 \end{bmatrix} \begin{bmatrix} \vec{a}_5 \\ \vec{a}_4 \\ \vec{a}_3 \end{bmatrix} = \begin{bmatrix} \vec{r}_f - \vec{r}_0 \\ \ddot{\vec{r}}_f \\ \dddot{\vec{r}}_f \end{bmatrix}$$

for z

use (8), (9), (10), w/ (4), (5), (6b)

$$\begin{bmatrix} t_f^5 & t_f^4 & t_f^3 \\ 5t_f^4 & 4t_f^3 & 3t_f^2 \\ 20t_f^3 & 12t_f^2 & 6t_f \end{bmatrix} \begin{bmatrix} a_5 \\ a_4 \\ a_3 \end{bmatrix} = \begin{bmatrix} \vec{r}_f - \vec{r}_0 \\ \dot{\vec{r}}_f \\ \ddot{\vec{r}}_f \end{bmatrix}$$

$$m\vec{a} = \vec{T} - m\vec{g}$$

$$\vec{T} = m(\vec{a} + \vec{g})$$