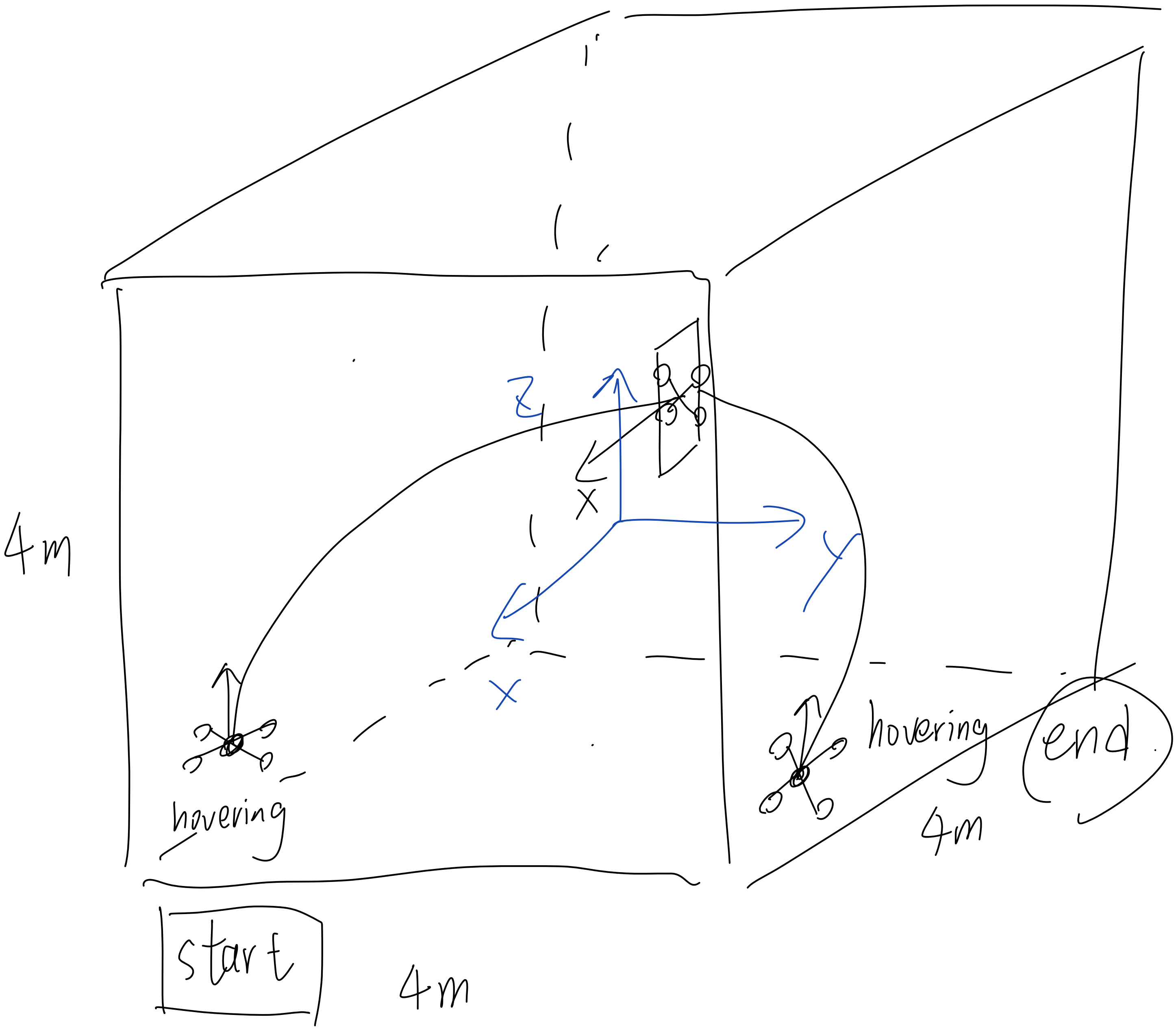
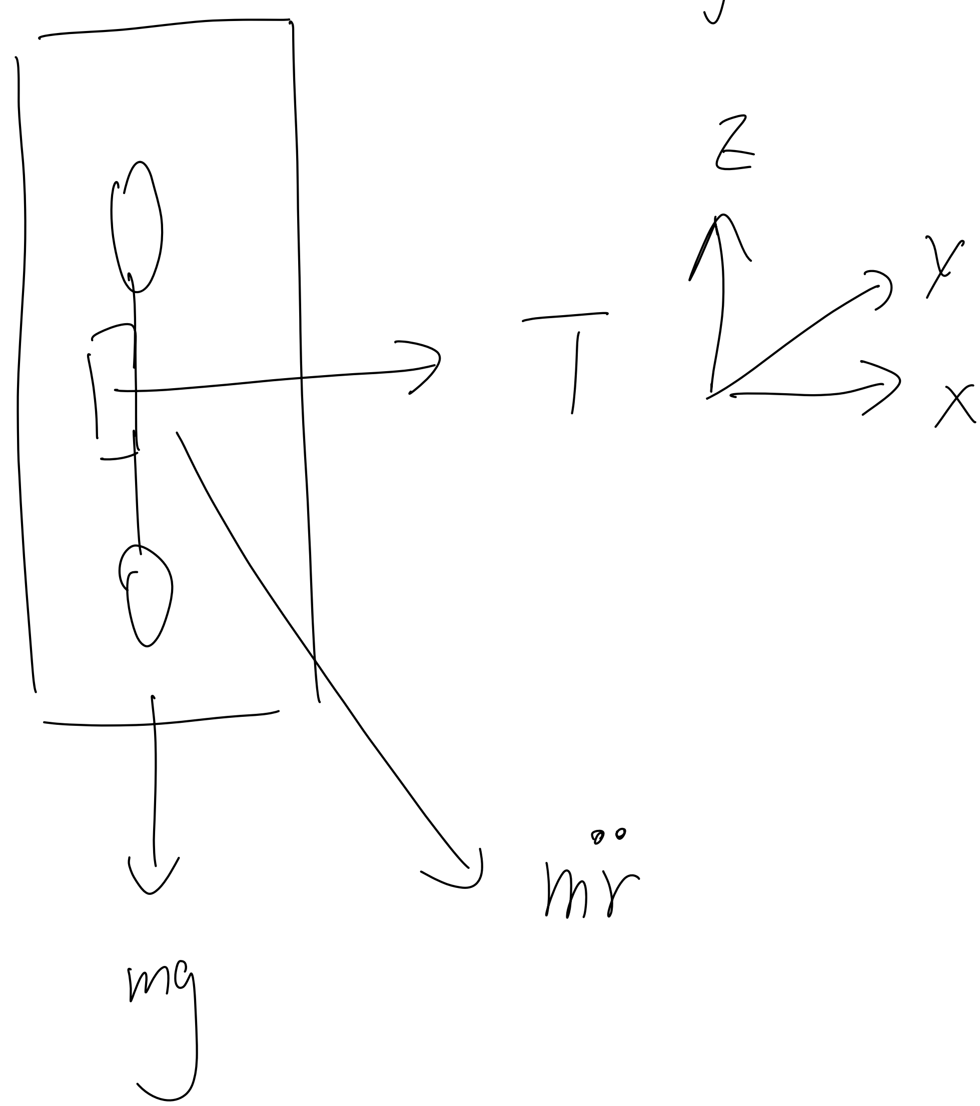
Set up



Dynamics at the gate → $(0, 0, 0)$

at origin



$m\ddot{r}_f^x = T$

$\ddot{r}_f^x = \frac{1}{m} T$

$\ddot{r}_f^y = 0$

$\cancel{m}\ddot{r}_f^z = -\cancel{m}g$

$\ddot{r}_f^z = -g$

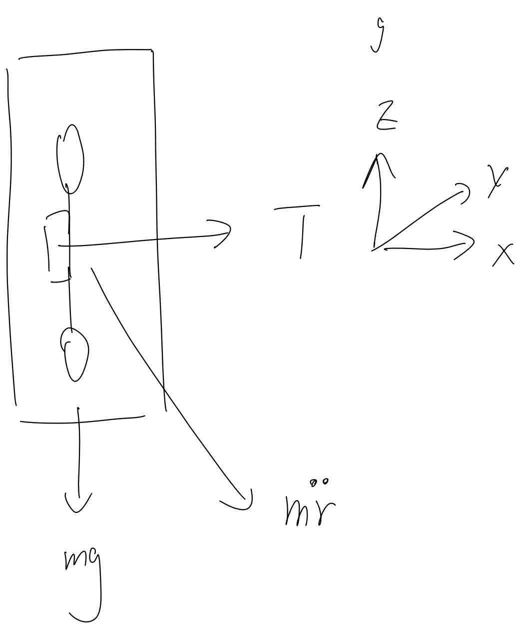
Boundary Conditions.

hovering

1. $\vec{r}_0$ Known
2. $\dot{\vec{r}}_0 = \vec{0}$
3. $\ddot{\vec{r}}_0 = \vec{0}$
4. $\vec{\omega}_0 = \vec{0} \longrightarrow \dddot{\vec{r}}_0 = \vec{0}$
5. $\vec{\alpha}_0 = \vec{0}$

at gate

6. $\vec{r}_f$ Known
7. $\dot{\vec{r}}_f = (0, \dot{y}, 0)$
8. $\ddot{\vec{r}}_f = (\frac{T}{m}, 0, -g)$
9. $\vec{\omega}_f = \vec{0} \longrightarrow \dddot{\vec{r}}_f = \vec{0}$
10. $\vec{\alpha}_f = \ldots$



minimum snap trajectory

→ use 7-th-order polynomial

$$\vec{r}(t) = \sum_{i=0}^{7} \vec{a}_i t^i$$

$$\dot{\vec{r}}(t) = \sum_{i=0}^{7} i \vec{a}_i t^{i-1}$$

$$\ddot{\vec{r}}(t) = \sum_{i=0}^{7} i(i-1) \vec{a}_i t^{i-2}$$

$$\dddot{\vec{r}}(t) = \sum_{i=0}^{7} i(i-1)(i-2) \vec{a}_i t^{i-3}$$

Set up matrix

$$\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 2 & & & \\ & & & 6 & & \\ 1 & t_1 & t_1^2 & & \cdots & t_1^7 \\ & 1 & 2t_1 & & \cdots & 7t_1^6 \\ & & 2 & & \cdots & 42t_1^5 \\ & & & 6 & \cdots & 210t_1^4 \end{bmatrix} \begin{bmatrix} \vec{a}_0 \\ \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \\ \vec{a}_4 \\ \vec{a}_5 \\ \vec{a}_6 \\ \vec{a}_7 \end{bmatrix} = \begin{bmatrix} \vec{r}_0 \\ \vec{V}_0 \\ \vec{a}_0 \\ \vec{j}_0 \\ \vec{r}_1 \\ \vec{V}_1 \\ \vec{a}_1 \\ \vec{j}_1 \end{bmatrix}$$

$a: \quad 8 \times 3$

$$t: \quad \overset{n \times 8}{\begin{bmatrix} 1 & t & \cdots & t^7 \end{bmatrix}}$$

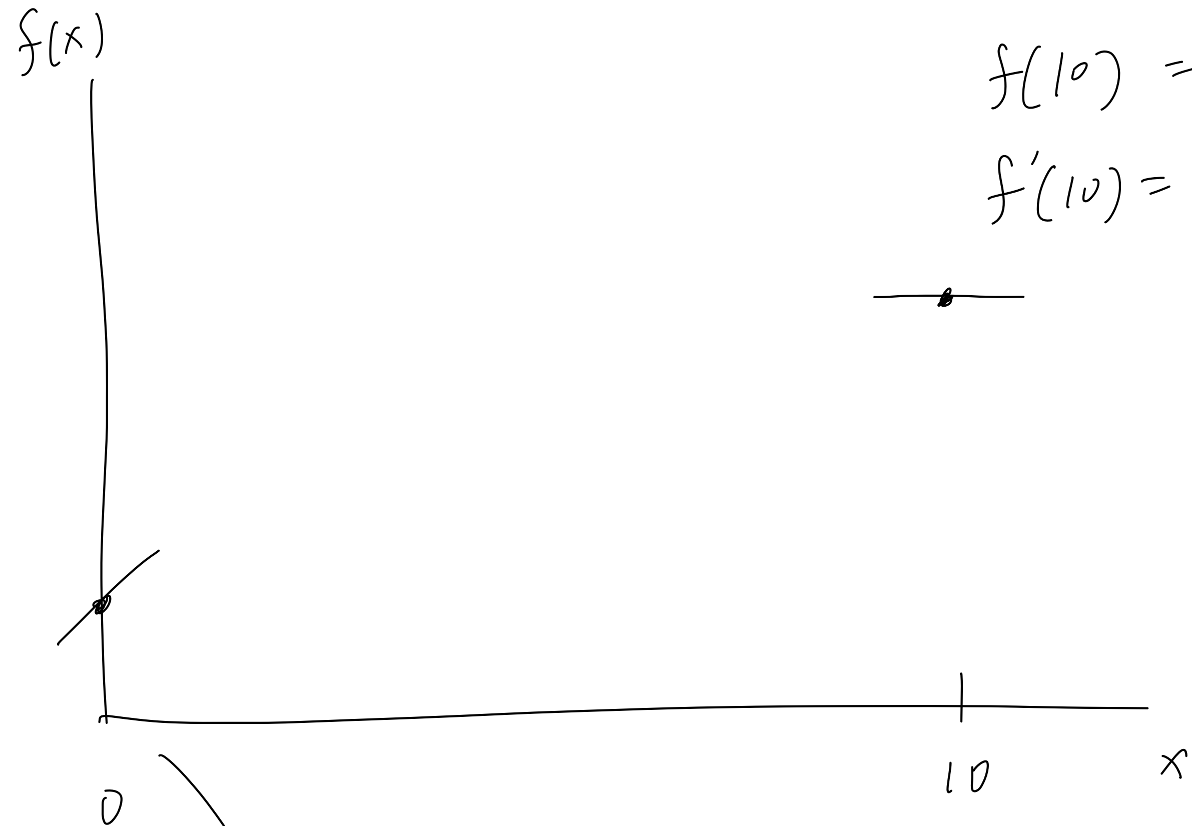
$$\vec{r}(t) = \underset{n \times 8}{\begin{bmatrix} t \end{bmatrix}} * \underset{8 \times 3}{\begin{bmatrix} a \end{bmatrix}} = \overset{n \times 3}{\begin{bmatrix} \vdots & \vdots & \vdots \\ x & y & z \\ \vdots & \vdots & \vdots \end{bmatrix}}$$

Simple example (3^{rd}- order polynomial, 4 BCs)

$$f(x) = ax^3 + bx^2 + cx + d$$

$f(10) = 5$

$f'(10) = 0$



$f(0) = 1$

$f'(0) = 1$

$$f'(x) = 3ax^2 + 2bx + c$$

$$f(0)=0 \rightarrow a\cdot 0^3 + b\cdot 0^2 + c\cdot 0 + d = 1$$

$$f'(0)=1 \rightarrow 3a\cdot 0^2 + 2b\cdot 0 + c = 1$$

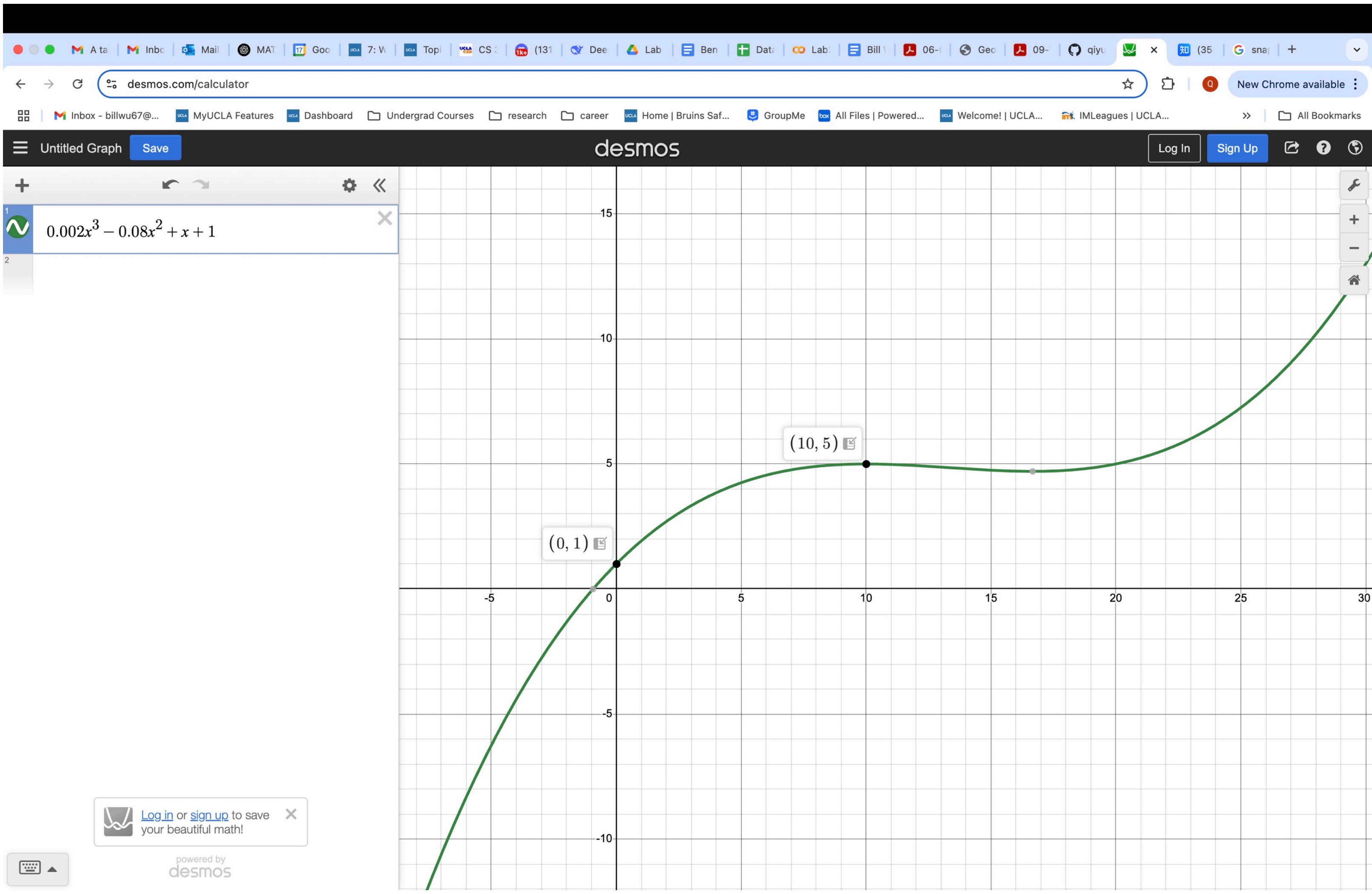
$$f(10)=5 \rightarrow a\cdot 10^3 + b\cdot 10^2 + c\cdot 10 + d = 5$$

$$f'(10)=0 \rightarrow 3a\cdot 10^2 + 2b\cdot 10 + c = 0$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 10^3 & 10^2 & 10 & 1 \\ 3\cdot 10^2 & 2\cdot 10 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0.002 \\ -0.08 \\ 1 \\ 1 \end{bmatrix}$$

$$f(x) = 0.002x^3 - 0.08x^2 + x + 1$$


desmos.com/calculator
Untitled Graph
Save
desmos
Log In
Sign Up
$0.002x^3 - 0.08x^2 + x + 1$
$(10, 5)$
$(0, 1)$
Log in or sign up to save your beautiful math!
powered by desmos

Minimum Snap Trajectory: $r(t)$

snap is: $\ddddot{r}$

want to minimize $\int_0^T (\ddddot{r})^2 \, dt$

this is called the Lagrangian,

i.e. $\mathcal{L} = \ddddot{r}^2$

solve the Euler-Lagrange equations

$$\cancel{\frac{\partial L}{\partial r}} - \cancel{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right)} + \cancel{\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{r}}\right)} - \cancel{\frac{d^3}{dt^3}\left(\frac{\partial L}{\partial \dddot{r}}\right)} + \frac{d^4}{dt^4}\left(\frac{\partial L}{\partial \ddddot{r}}\right) = 0$$

$L = 2(\ddddot{r})$ so $\frac{\partial L}{\partial r}, \frac{\partial L}{\partial \dot{r}}, \frac{\partial L}{\partial \ddot{r}}, \frac{\partial L}{\partial \dddot{r}} = 0.$

$$\frac{d^4}{dt^4}\left(\frac{\partial L}{\partial \ddddot{r}}\right) = 0$$

$$\frac{d^4}{dt^4}(2\ddddot{r}) = 0$$

$$2r^{(8)} = 0.$$

the 8^{th}- derivative of $r(t)$ must vanish

i.e. $r(t)$ is a 7-th order polynomial