Set up

4m

4m

hovering

start

z

x

y

x

hovering

end

4m
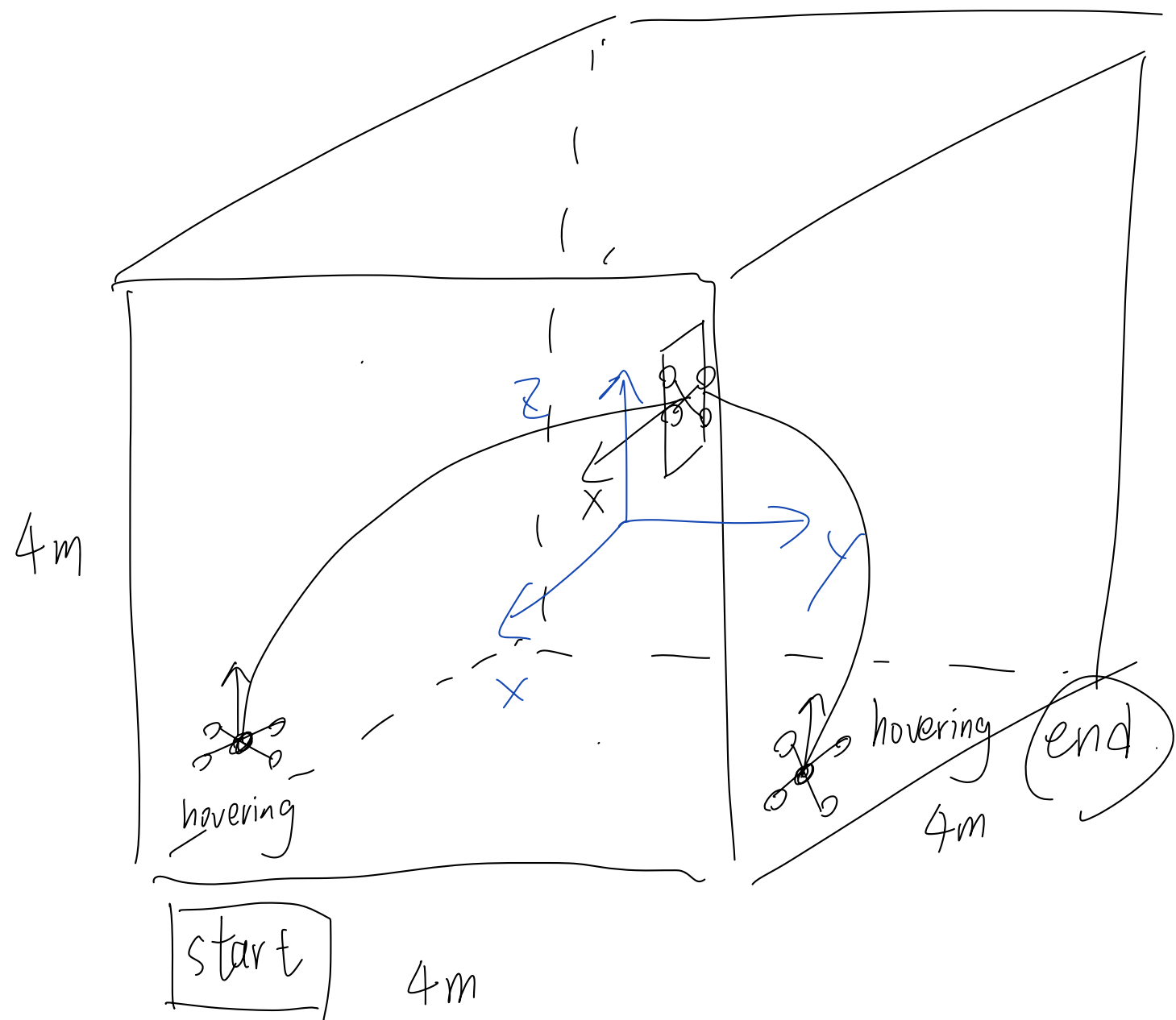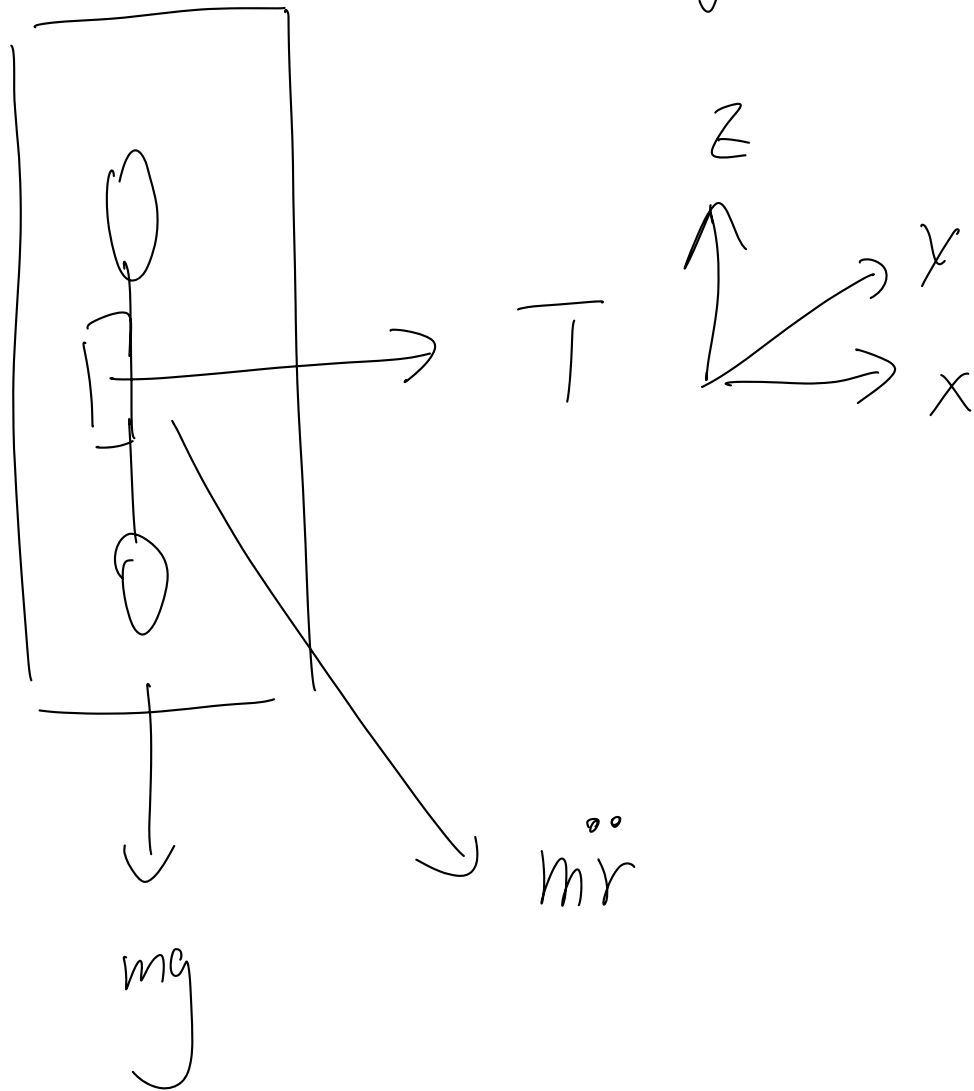
Dynamics at the gate $\rightarrow$ $(0, 0, 0)$

at origin

$m\ddot{r}_f^x = T$

$\ddot{r}_f^y = 0$

$m\ddot{r}_f^z = -mg$

$\ddot{r}_f^x = \frac{1}{m}T$

$\ddot{r}_f^z = -g$

Boundary Conditions.

hovering
$$
\begin{cases}
1. & \vec{r}_0 \quad \text{Known} \\
2. & \dot{\vec{r}}_0 = \vec{0} \\
3. & \ddot{\vec{r}}_0 = \vec{0} \\
4. & \vec{w}_0 = \vec{0} \\
5. & \vec{\alpha}_0 = \vec{0}
\end{cases}
\qquad \longrightarrow \qquad \dddot{\vec{r}}_0 = \vec{0}
$$

at gate
$$
\begin{cases}
6. & \vec{r}_f \quad \text{Known} \\
7. & \dot{\vec{r}}_f = \left(0, \dot{y}, 0 \right) \\
8. & \ddot{\vec{r}}_f = \left(\dfrac{T}{m}, 0, -g \right) \\
9. & \vec{w}_f = \vec{0} \quad \longrightarrow \quad \dddot{\vec{r}}_f = \vec{0} \\
10. & \vec{\alpha}_f = \ldots
\end{cases}
$$
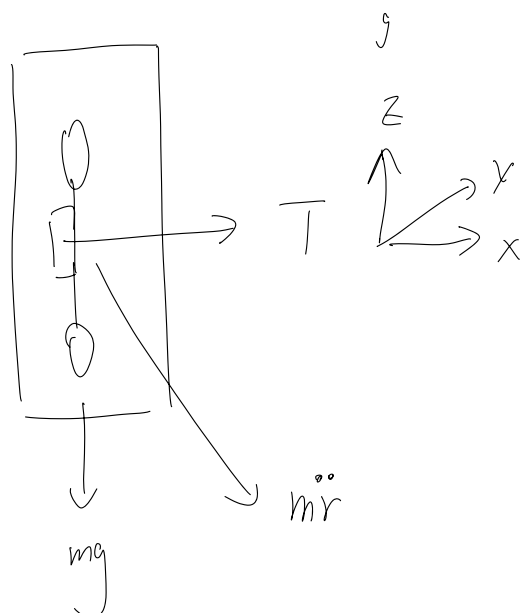
minimum snap trajectory

→ use 7^{-th} - order polynomial

$$\vec{r}(t) = \sum_{i=0}^{7} \vec{a}_i t^i$$

$$\dot{\vec{r}}(t) = \sum_{i=0}^{7} i\, \vec{a}_i t^{i-1}$$

$$\ddot{\vec{r}}(t) = \sum_{i=0}^{7} i(i-1)\, \vec{a}_i t^{i-2}$$

$$\dddot{\vec{r}}(t) = \sum_{i=0}^{7} i(i-1)(i-2)\, \vec{a}_i t^{i-3}$$

Set up matrix

$$\begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 2 & & & & & \\ & & & 6 & & & & \\ 1 & t_1 & t_1^2 & \cdots & & & & t_1^7 \\ & 1 & 2t_1 & \cdots & & & & 7t_1^6 \\ & & 2 & \cdots & & & & 42t_1^5 \\ & & & 6 & \cdots & & & 210\, t_1^4 \end{bmatrix} \begin{bmatrix} \vec{a}_0 \\ \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \\ \vec{a}_4 \\ \vec{a}_5 \\ \vec{a}_6 \\ \vec{a}_7 \end{bmatrix} = \begin{bmatrix} \vec{r}_0 \\ \vec{v}_0 \\ \vec{a}_0 \\ \dddot{\vec{r}}_0 \\ \vec{r}_1 \\ \vec{v}_1 \\ \vec{a}_1 \\ \vec{j}_1 \end{bmatrix}$$

$$a: \quad 8 \times 3$$

$$t: \quad \overset{n \times 8}{\left[\begin{array}{cccc} 1 & t & \cdots & t^7 \end{array} \right]}$$

$$\vec{r}(t) = \overset{n \times 8}{[t]} * \overset{8 \times 3}{[a]} = \overset{n \times 3}{\begin{bmatrix} \vdots & \vdots & \vdots \\ x & y & z \\ \vdots & \vdots & \vdots \end{bmatrix}}$$

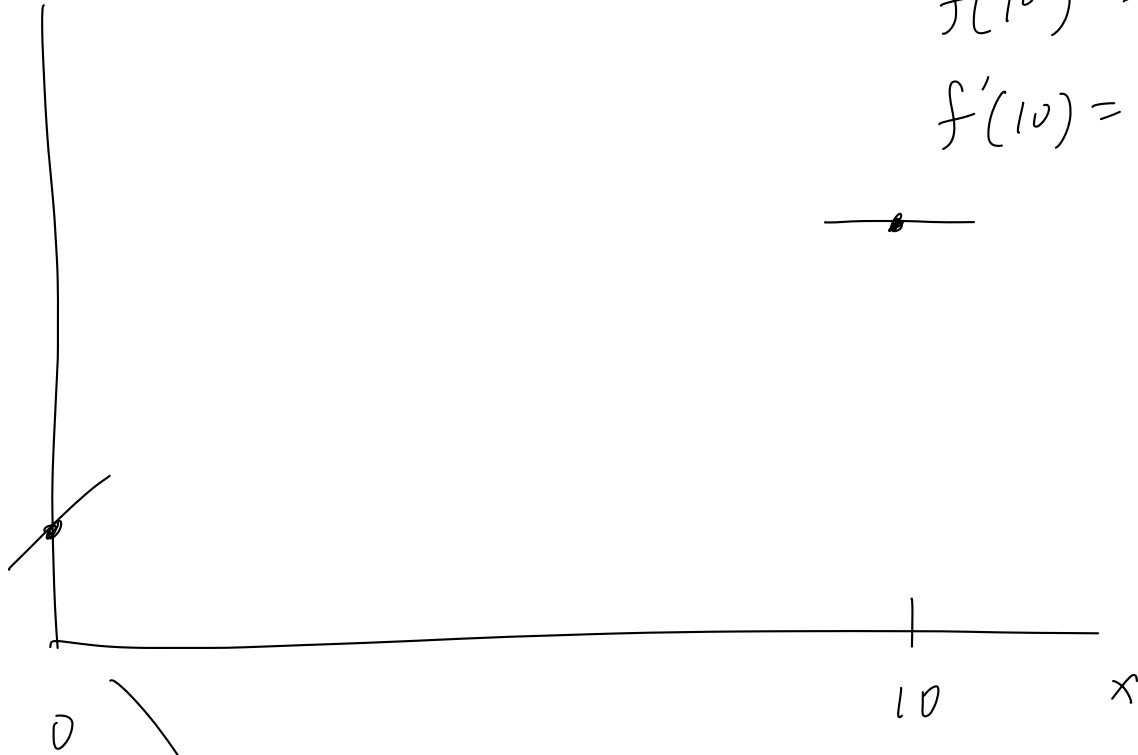Simple example (3rd - order polynomial, 4 BCs)

$$f(x) = ax^3 + bx^2 + cx + d$$

f(x)

$$f(10) = 5$$
$$f'(10) = 0$$

10   x

0

$$f(0) = 1$$
$$f'(0) = 1$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$f(0) = 0 \rightarrow a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d = 1$$

$$f'(0) = 1 \rightarrow 3a \cdot 0^2 + 2b \cdot 0 + c = 1$$

$$f(10) = 5 \rightarrow a \cdot 10^3 + b \cdot 10^2 + c \cdot 10 + d = 5$$
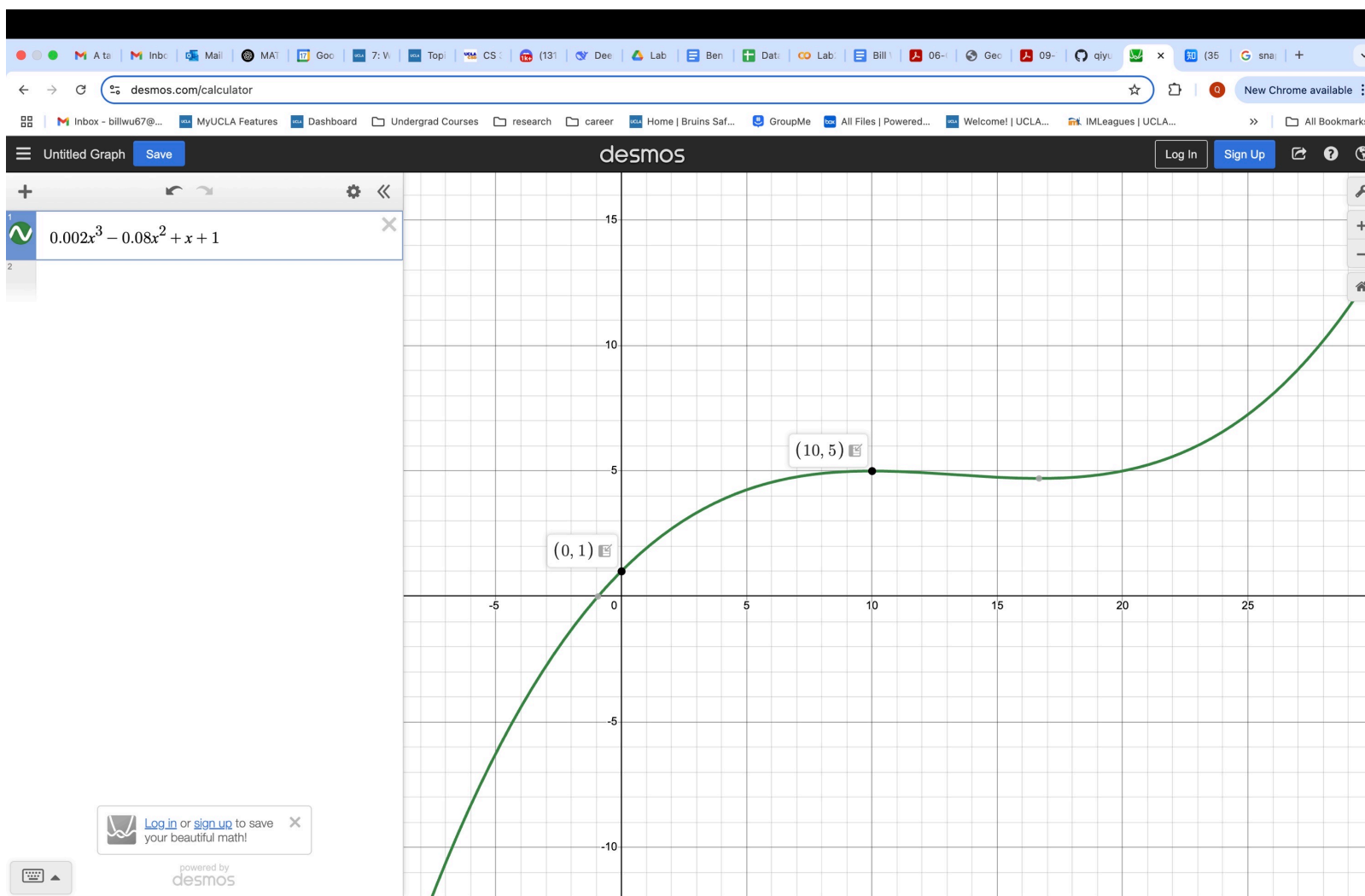
$$f'(10) = 0 \rightarrow 3a \cdot 10^2 + 2b \cdot 10 + c = 0$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 10^3 & 10^2 & 10 & 1 \\ 3 \cdot 10^2 & 2 \cdot 10 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0.002 \\ -0.08 \\ 1 \\ 1 \end{bmatrix}$$

$$f(x) = 0.002 x^3 - 0.08 x^2 + x + 1$$

desmos

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$0.002x^3 - 0.08x^2 + x + 1$

$(10, 5)$

$(0, 1)$

Minimum Snap Trajectory: $r(t)$

snap is: $\ddddot{r}$

want to minimize $\int_0^T (\ddddot{r})^2 \, dt$

this is called the Lagrangian,

i.e. $L = \ddddot{r}^2$

solve the Euler-Lagrange equations

$$\cancel{\frac{\partial L}{\partial r}} - \frac{d}{dt}\left(\cancel{\frac{\partial L}{\partial \dot r}}\right) + \frac{d^2}{dt^2}\left(\cancel{\frac{\partial L}{\partial \ddot r}}\right) - \frac{d^3}{dt^3}\left(\cancel{\frac{\partial L}{\partial \dddot r}}\right) + \frac{d^4}{dt^4}\left(\frac{\partial L}{\partial \ddddot r}\right) = 0$$

$L = 2(\ddddot r)$ so $\frac{\partial L}{\partial r}, \frac{\partial L}{\partial \dot r}, \frac{\partial L}{\partial \ddot r}, \frac{\partial L}{\partial \dddot r} = 0$.

$$\frac{d^4}{dt^4}\left(\frac{\partial L}{\partial \ddddot r}\right) = 0$$

$$\frac{d^4}{dt^4}\left(2\ddddot r\right) = 0$$

$$2 r^{(8)} = 0.$$

the 8th derivative of $r(t)$ must vanish