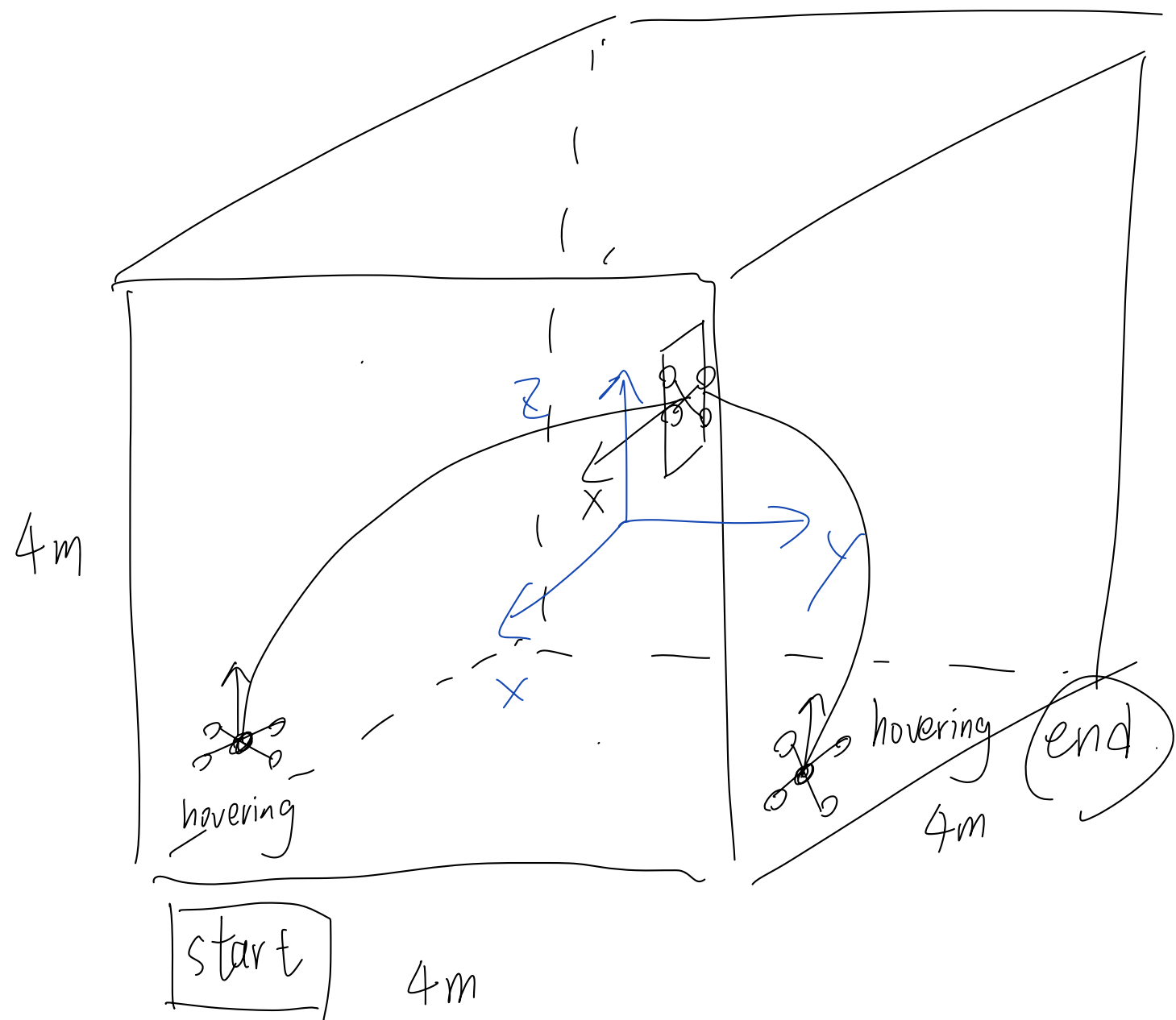
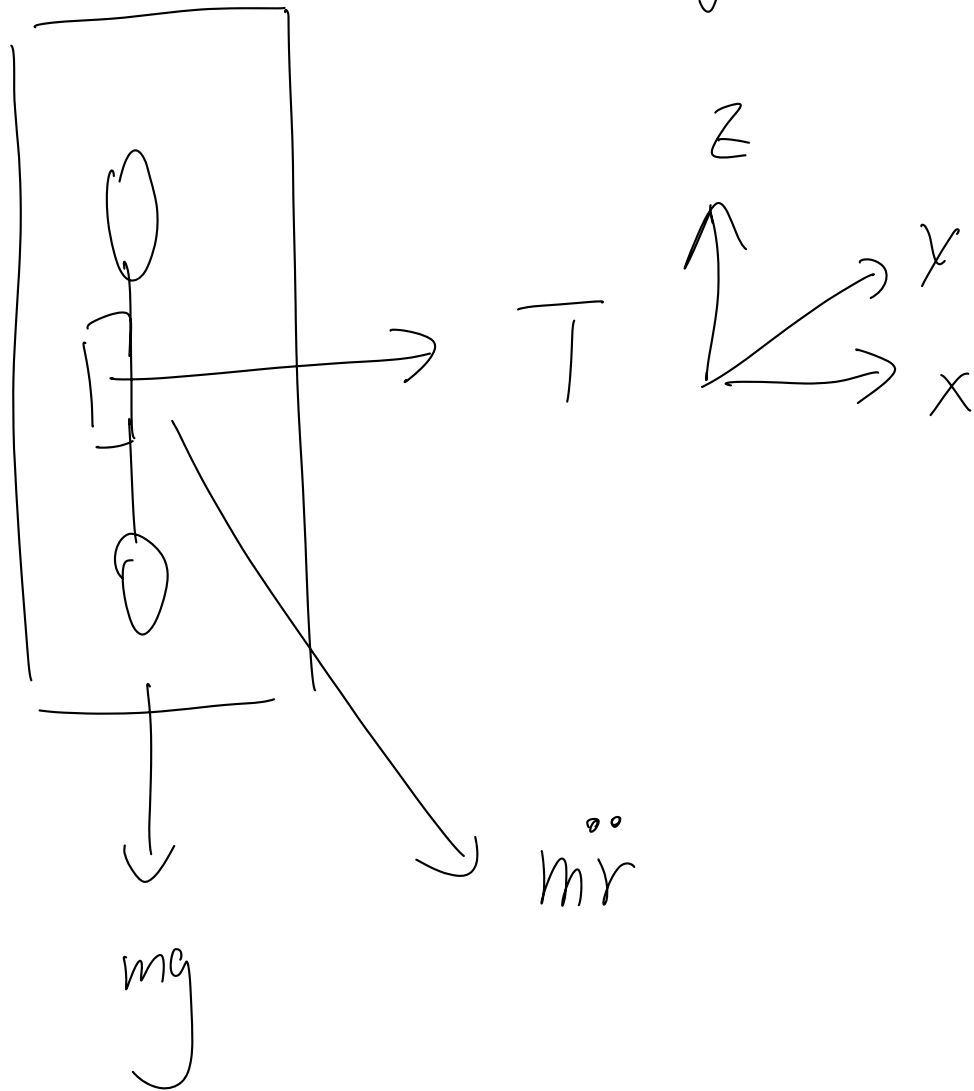


Set up



Dynamics at the gate $\rightarrow (0, 0, 0)$
at origin



$$m \ddot{r}_f^x = T$$

$$\ddot{r}_f^x = \frac{1}{m} T$$

$$\ddot{r}_f^y = 0$$

$$m \ddot{r}_f^z = -mg$$

$$\ddot{r}_f^z = -g$$

Boundary Conditions.

hovering

$$1. \vec{r}_0 \text{ known}$$

$$2. \dot{\vec{r}}_0 = \vec{0}$$

$$3. \ddot{\vec{r}}_0 = \vec{0}$$

$$4. \vec{\omega}_0 = \vec{0} \rightarrow \ddot{\vec{r}}_0 = \vec{0}$$

$$5. \vec{\alpha}_0 = \vec{0}$$

at gate

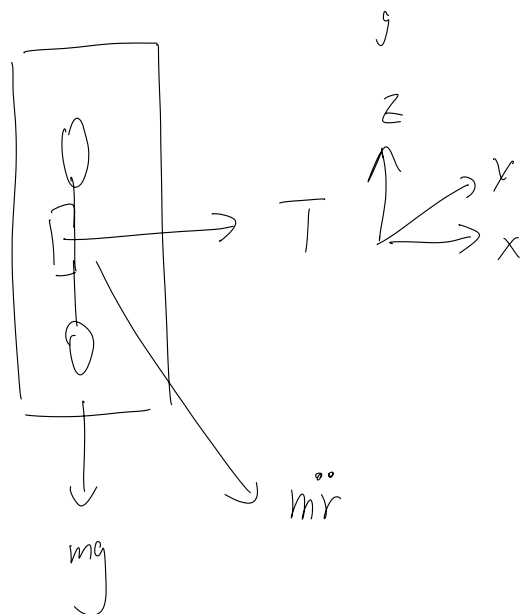
$$6. \vec{r}_f \text{ known}$$

$$7. \dot{\vec{r}}_f = (0, \dot{y}, 0)$$

$$8. \ddot{\vec{r}}_f = \left(\frac{T}{m}, 0, -g \right)$$

$$9. \vec{\omega}_f = \vec{0} \rightarrow \ddot{\vec{r}}_f = \vec{0}$$

$$10. \vec{\alpha}_f = \dots$$



minimum snap trajectory

→ use 7th-order polynomial

$$\vec{r}(t) = \sum_{i=0}^7 \vec{a}_i t^i$$

$$\dot{\vec{r}}(t) = \sum_{i=0}^7 i \vec{a}_i t^{i-1}$$

$$\ddot{\vec{r}}(t) = \sum_{i=0}^7 i(i-1) \vec{a}_i t^{i-2}$$

$$\dddot{\vec{r}}(t) = \sum_{i=0}^7 i(i-1)(i-2) \vec{a}_i t^{i-3}$$

Set up matrix

$$\begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 2 & & & & & \\ & & & 6 & & & & \\ & 1 & t_1 & t_1^2 & \dots & t_1^7 \\ & & 1 & 2t_1 & \dots & 7t_1^6 \\ & & & 2 & \dots & 42t_1^5 \\ & & & & 6 & \dots & 210t_1^4 \end{bmatrix} \begin{bmatrix} \vec{a}_0 \\ \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \\ \vec{a}_4 \\ \vec{a}_5 \\ \vec{a}_6 \\ \vec{a}_7 \end{bmatrix} = \begin{bmatrix} \vec{r}_0 \\ \vec{v}_0 \\ \vec{a}_0 \\ \vec{j}_0 \\ \vec{r}_1 \\ \vec{v}_1 \\ \vec{a}_1 \\ \vec{j}_1 \end{bmatrix}$$

$$a : 8 \times 3$$

$$1 \times 8$$

$$t : \begin{bmatrix} 1 & t & \dots & t^7 \end{bmatrix}$$

$$\vec{r}(t) = \begin{matrix} n \times 8 \\ [t] \end{matrix} * \begin{matrix} 8 \times 3 \\ [a] \end{matrix} = \begin{matrix} n \times 3 \end{matrix}$$

$$\begin{bmatrix} \vdots & \vdots & \vdots \\ x & y & z \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

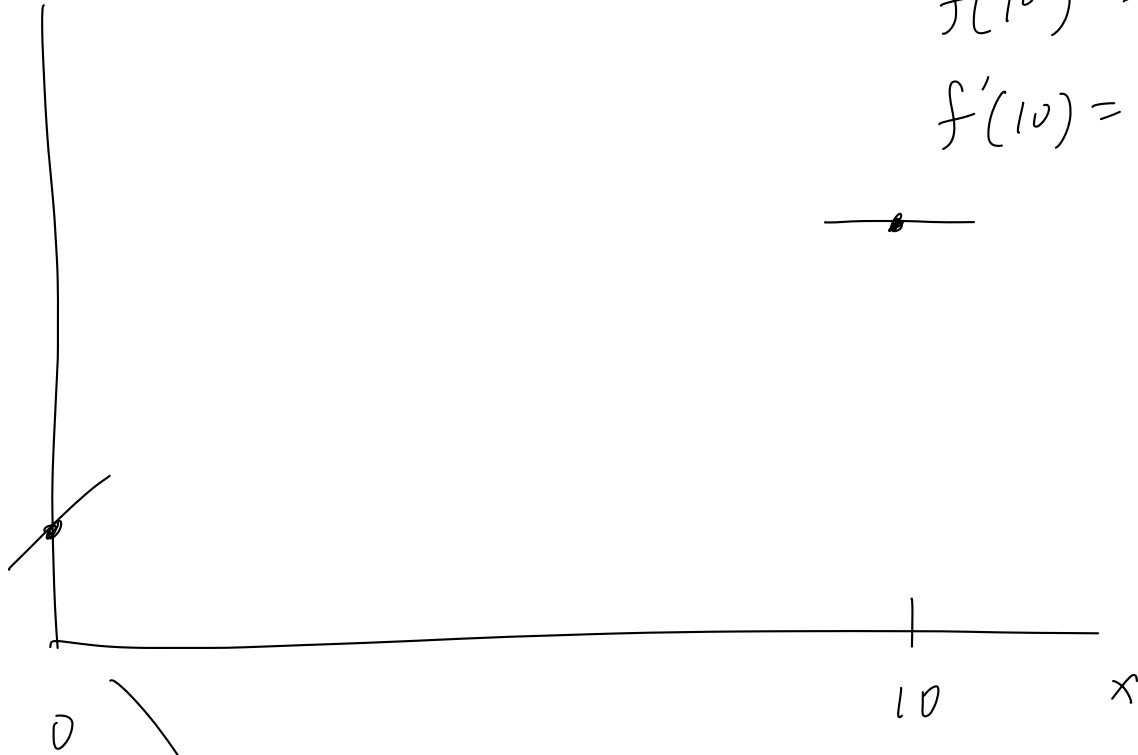
Simple example (3rd-order polynomial, 4 BCs)

$$f(x) = ax^3 + bx^2 + cx + d$$

$f(x)$

$$f(10) = 5$$

$$f'(10) = 0$$



$$f(0) = 1$$

$$f'(0) = 1$$

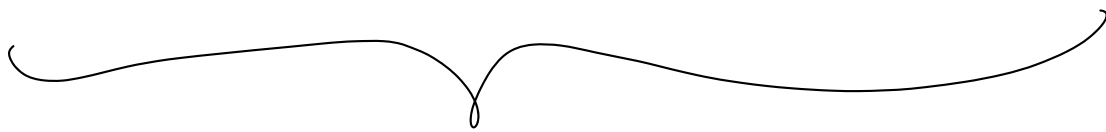
$$f'(x) = 3ax^2 + 2bx + c$$

$$f(0)=0 \rightarrow a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d = 1$$

$$f'(0)=1 \rightarrow 3a \cdot 0^2 + 2b \cdot 0 + c = 1$$

$$f(10)=5 \rightarrow a \cdot 10^3 + b \cdot 10^2 + c \cdot 10 + d = 5$$

$$f'(10)=0 \rightarrow 3a \cdot 10^2 + 2b \cdot 10 + c = 0$$



$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 10^3 & 10^2 & 10 & 1 \\ 3 \cdot 10^2 & 2 \cdot 10 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0.002 \\ -0.08 \\ 1 \\ 1 \end{bmatrix}$$

$$f(x) = 0.002x^3 - 0.08x^2 + x + 1$$

