The University of Sydney
School of Computer Science

THE UNIVERSITY OF SYDNEY

COMP5313/COMP4313—Large Scale Networks S1 2025
Week 4 - Graphs

The goal of this tutorial is to review the notions of network, triadic closure, embeddedness and regions.

Exercise 1: Breadth-First Search

Answer the following questions based on the graph shown in Figure 1.

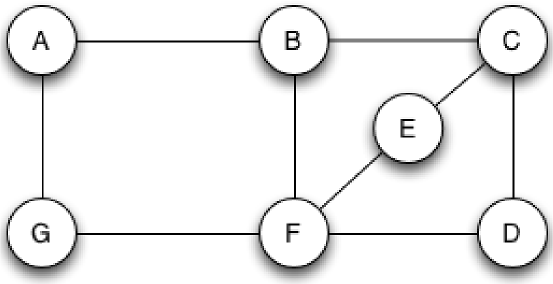

Figure 1: A network of 7 nodes

a) How many links does the network have?

Answer: 9.

b) Is this network directed or undirected?

Answer: Undirected

c) Is this network connected?

Answer: Yes.

d) In what order nodes get visited during a breadth-first search starting at node B?

Answer:
B, A, C, F, G, D, E. Nodes at the same level can be visited in an arbitrary order.

Duration: 5 min

Exercise 2: Triadic closure

What are the nodes that violate the strong tradic closure property in the graph of Figure 2? Explain why.

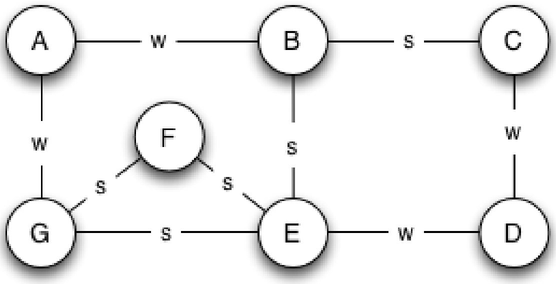

Figure 2: A network of 7 nodes

Answer:
E and B.

- E because it has strong ties with B and F that are not neighbors and
- B because C and E are not neighbors.

Duration: 5 min

Exercise 3: Embeddedness vs. Betweenness

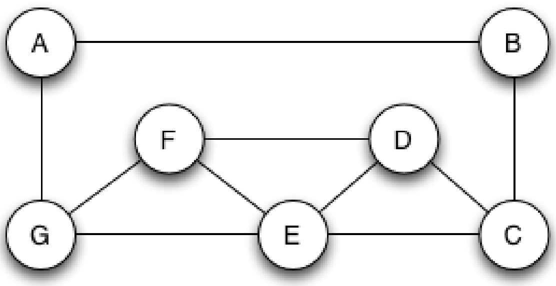

Figure 3: Another sample network of 7 nodes

(a) What are the embeddedness and betweenness of edges (A,B) and (D,F) in the graph of Figure 3?

Answer:
The embeddedness of (A,B) is 0 as A and B have no neighbours in common. The embeddedness of (D,F) is 1 as D and F have only one neighbor in common, E.

The betweenness of (A,B) is 3.667 as it is the sum of:

- the flow from A to B
- the flow from A to C
- the flow from B to G
- $\frac{1}{3}$ of the flow from B to F
- $\frac{1}{3}$ of the flow from D to A

The betweenness of (D,F) is 2.667 as it is the sum of:

- the flow from D to F
- $\frac{1}{2}$ of the flow from C to F
- $\frac{1}{2}$ of the flow from D to G
- $\frac{1}{3}$ of the flow from B to F
- $\frac{1}{3}$ of the flow from A to D

(b) Is (A,B) a bridge or a local bridge? What is its span? Explain.

Answer: (A,B) is a local bridge because A and B have no neighbor in common, but this is not a bridge as A and B remain connected despite its removal. Its span is 4 as its removal would increase the distance between A and B to 4.

Duration: 15 min

Exercise 4: Partitioning

What are the regions at level 2 obtained with the Girvan-Newman method applied to the graph in Figure 3? Explain.

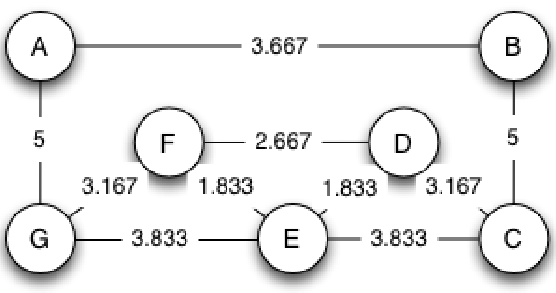

Figure 4: Betweenness of all edges of the graph

Answer:
Edge (A,G) has a betweenness of 5 as it is the sum of:

- $\frac{1}{3}$ of the flow from B to F
- the flow between A and E
- the flow between A and F
- the flow between A and G
- the flow between B and G
- $\frac{2}{3}$ of the flow from A to D

Symmetrically, edge (B,C) has a betweenness of 5. Edge (E,G) has a betweenness of 3.833 (the flow between C and G, $\frac{1}{3}$ of flow between A and D, $\frac{1}{2}$ of flow between D and G, the flow between E and G and the flow between A and E). Symmetrically, (C,E) has a betweenness of 3.833.

Edge (C,D) has a betweenness of 3.167 (the flow between B and D, the flow between C and D, $\frac{1}{2}$ of flow between C and F, $\frac{1}{3}$ flow between B and F, $\frac{1}{3}$ flow between A and D). Symmetrically, (F,G) has a betweenness of 3.167.

At iteration 1, we have two regions: {A, B} and {C, D, E, F, G}. These are the level 1 regions. At iteration 2, we remove {E,C} and {E,G}. Still we will have two regions: {A, B} and {C, D, E, F, G}. At iteration 3, we remove {G,F}, {F,D}, and {D,C}, We have regions {A,B}, {G}, {F,E,D}, and {C}. These are the level 2 regions.

Finally, at the end, please also illustrate the betweenness of all edges via NetworkX.

Example code: After each step re-build the graph and repeat

```
>>> import networkx as NX
>>> g = NX.Graph()
>>> g.add_nodes_from(['A','B','C','D','E','F','G'])
>>> g.add_edges_from([('A','B'),('A','G'),('B','C')
,('G','E'),('E','C'),('G','F'),('D','C'),('F','E'),('E','D'),('F','D')])
>>> print(NX.edge_betweenness_centrality(g, normalized=False))
```

Duration: 25 min