COMP5313/COMP4313 - Large Scale Networks

Week 08: Information Cascades

Lijun Chang

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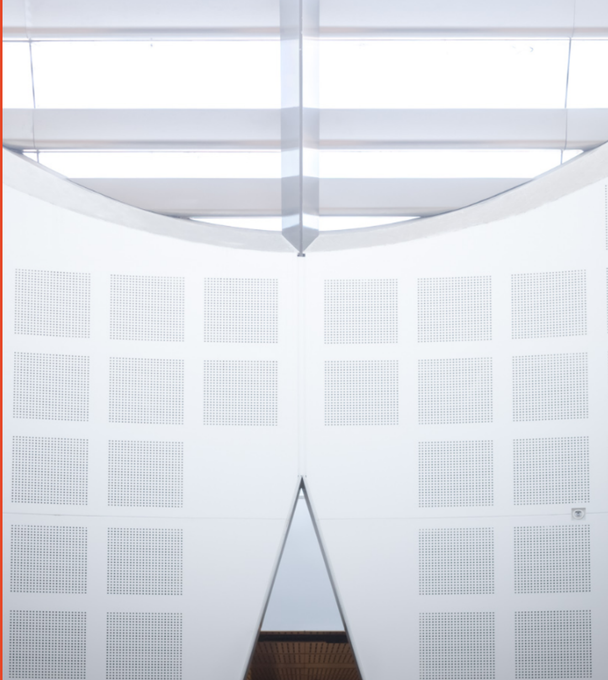
THE UNIVERSITY OF SYDNEY



Introduction

- Previously, we have studied social network analysis and information network analysis
  - We analyze the structure of social networks and information networks
- Now, let us look at network dynamics
  - Information cascade
    - How/why people influence each other's behavior and decisions
  - Power law
  - Small world
  - P2P network

Information Cascade

- There are many situations where people are influenced by others
  - e.g., the products they buy, the technology they use
- We will consider some of the reasons why such influence occurs
- We will see that there are many settings in which it is rational for an individual to imitate the choices/behaviours of others
  - even if the individual's own information suggests a different choice

Outline

Following the Crowd

- Suppose you are choosing a restaurant in an unfamiliar town [1]
  - Based on your own research you intend to go to restaurant A
  - Upon arrival you see that no one is eating at A but many people are eating at B
  - If you believe that others have similar tastes and they have some information, then the information that you can infer from their choices may be more powerful than your own private information

$\Longrightarrow$ herding or information cascade has occurred

[1] A. V. Banerjee. A simple model of herd behavior. The quarterly journal of economics, vol. 107, no. 3, 1992.

Following the Crowd

- Consider the experiment of Milgram, Bickman, and Berkowitz in the 1960s [2]
  - Groups of 1 to 15 stand on a street corner and stare up into the sky
  - How many passersby stopped and also looked up at the sky?

- Observations:
  - With 1 person looking up
    $\Longrightarrow$ very few passersby stopped
  - With 5 persons looking up
    $\Longrightarrow$ more passersby stopped, but most still ignored
  - With 15 persons looking up
    $\Longrightarrow$ 45% of passersby stopped and also stared into the sky

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[2] S. Milgram, L. Bickman, L. Berkowitz. Note on the drawing power of crowds of different size. Journal of personality and social psychology, vol. 13, no. 2, 1969.

Following the Crowd

- Information cascade is likely to occur when people make decisions sequentially, with later people watching the actions of earlier people and inferring what the earlier people might know from their actions.
- In a cascade, individuals are imitating the behavior of others.
  - This is not mindless imitation, but the result of drawing rational inferences from limited information.

Following the Crowd

- There are two possible explanations:
  1. There is a social force for conformity that grows stronger as the group conforming to the activity becomes larger, or
  2. Information cascade: As more people look up, there is a better reason to look up (since those looking up may know something you ignore)

- Information cascade is part of the explanation for many types of imitation in social settings
  - Fashion and fads
  - Voting for popular candidates
  - The self-reinforcing success of books placed highly on best-seller list

Following the Crowd

- There are two rational reasons for imitation
  - Informational effects
    - The actions of some affect indirectly your decision (by information propagation)
    - Example of the restaurant we have seen before
  - Direct-benefit effects
    - The actions of some affect directly the payoff of others
    - Example of fax machines: useless if no one else owns one
    - Whether there are others who own a fax machine, helps deciding whether to buy one

- Some decisions exhibit both information and direct benefits
  - Example of the iPhone: learning from other's decision (fashion) and benefiting from compatibility (facetime)

Outline

Following the Crowd

Herding Experiment

Model of Decision-Making under Uncertainty

A General Cascade Model

Herding Experiment

- Anderson and Holt created a simple herding experiment that contains the ingredients we discussed: [3]
  1. There is a decision to be made (e.g., vote, eat in a restaurant, etc.)
  2. Every person take this decision one after another (sequentially) and each can observe the decisions taken earlier
  3. Each person has some private information that helps guide their decision
  4. A person cannot directly observe the private information of someone else

[3] L. R. Anderson and C. A. Holt. Information cascades in the laboratory. The American economic review, 1997

Herding Experiment

- Consider a classroom with many participating students
- The experimenter puts an urn at the front of the room with three marbles hidden in it
- She announces that there is a 50% chance the urn contains two red marbles and one blue marble, and 50% chance it contains two blue marbles and one red marble
- One by one, each student draws a marble from the urn, looks at the color and places it back in the urn without showing it to the rest of the class
- The student announces whether he guesses that the urn is majority red or majority blue

Herding Experiment

- An urn contains 3 items:
  - 1 red, 2 blue with 50% probability
  - 2 red, 1 blue with 50% probability
- Is the content majoritarily red or majoritarily blue?
- One by one, come and pick one item
  - look at its color without showing it to the rest of the class
  - You should write your guess of the majority color ("red" or "blue") on the white board
- Assume that all the students reason correctly about what to do when it is their turn to guess, using everything they heard so far

Herding Experiment

- What will likely happen during the experiment?
  1. 1st student conveys perfect information
     $\Longrightarrow$ red leads to guessing red and blue leads to guessing blue
     $\Longrightarrow$ everyone knows that the guess is also the color seen
  2. 2nd student conveys perfect information
     $\Longrightarrow$ 'red, red' leads to guessing red and 'blue, blue' to guessing blue
     $\Longrightarrow$ assume other cases lead the 2nd student to guess the color he sees
  3. 3rd student may convey no information
     $\Longrightarrow$ guesses red if 'red, red, $*$' or blue if 'blue, blue, $*$'; otherwise, he guesses the color he sees
  4. Next students may be exactly in the same state conveying no information
     $\Longrightarrow$ guesses red if 'red, red, $*$, $*$' or blue if 'blue, blue, $*$, $*$' and so on

Quiz

- If the first two students see red and blue, will the 3rd student convey perfect information?

Herding Experiment

- Information cascade occurred in this experiment, when the same color was seen by the first two students
  - However, no one is under the illusion that students pick the same color again and again
- Information cascade can lead to non-optimal general outcome
  - Suppose we have a majoritarily red content
  - Probability that blue is seen by the first student is $1/3$
  - Probability that blue is seen by the first two students is $1/9$
  - This error is not alleviated with more students participating

Outline

Following the Crowd

Herding Experiment

Model of Decision-Making under Uncertainty

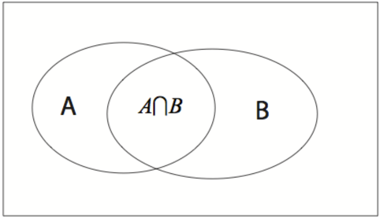
A General Cascade Model

Probability of Events

- Let's try to derive a model to reason which decision to take:
  - What is the probability this urn is majority-red, given the item I just drew and the guesses I've heard?
- Given an event A, we denote its probability of occurring as $\mathbf{Pr}[A]$
  - From the experiments seen before, an event could be: "the first student draws a blue item" or "the urn is majoritarily blue"

Review of Conditional Probability

- We therefore imagine a large sample space, in which each point in the sample space consists of a particular realization for each of these random outcomes



- The unit area rectangle in the figure represents the sample space of all possible outcomes and the event A is then a region in this sample space
- The overlapping area between A and B corresponds to the joint event where both A and B occurred

Review of Conditional Probability

- Conditional probability
  - We may have to reason about the probability of an event A given the event B
  - This is the conditional probability of A given B denoted by $\mathbf{Pr}[A \mid B]$

- Going back to the previous figure:
  - We are in B and we want the probability that it is also in A
  - We can think of this as the fraction of the area of B occupied by $A \cap B$:

$$\mathbf{Pr}[A \mid B] = \frac{\mathbf{Pr}[A \cap B]}{\mathbf{Pr}[B]}$$

  - By symmetry we have

$$\mathbf{Pr}[B \mid A] = \frac{\mathbf{Pr}[B \cap A]}{\mathbf{Pr}[A]} = \frac{\mathbf{Pr}[A \cap B]}{\mathbf{Pr}[A]}$$

Bayes' Rule

- Rewriting the previous equations we have:

$$\mathbf{Pr}[A \mid B] \cdot \mathbf{Pr}[B] = \mathbf{Pr}[A \cap B] = \mathbf{Pr}[B \mid A] \cdot \mathbf{Pr}[A]$$

- Dividing by $\mathbf{Pr}[B]$ leads to the Bayes' Rule:

$$\mathbf{Pr}[A \mid B] = \frac{\mathbf{Pr}[A] \cdot \mathbf{Pr}[B \mid A]}{\mathbf{Pr}[B]}$$

- Using this equation we can obtain $\mathbf{Pr}[A \mid B]$, i.e., the posterior probability of A given B, from $\mathbf{Pr}[A]$, i.e., the prior probability of A

Spam Filter Example

- Bayes' Rule is a fundamental way to decide whether an email is spam [4]
- Suppose you receive a piece of e-mail whose subject line contains the phrase "check this out" (a popular phrase among spammers), what is the chance the message is spam?

$$\mathbf{Pr}[\text{message is spam} \mid \text{subject contains ``check this out''}]$$

- Let us simplify the name of these events

$$\mathbf{Pr}[\text{spam} \mid \text{``check this out''}]$$

[4] J. Goodman, G. V. Cormack, D. Heckerman. Spam and the ongoing battle for the inbox. Communications of the ACM, vol. 50, no. 2, 2007.

Spam Filter Example

- To determine the probability $\mathbf{Pr}[\text{spam} \mid \text{``check this out''}]$. Suppose:
  - 40% of all your emails are spam, $\mathbf{Pr}[\text{spam}] = 0.4$
  - 1% of all spam messages contain the phrase "check this out"

$$\mathbf{Pr}[\text{``check this out''} \mid \text{spam}] = 0.01$$

  - 0.4% of all non-spam messages contains the phrase "check this out"

$$\mathbf{Pr}[\text{``check this out''} \mid \text{not spam}] = 0.004$$

Spam Filter Example

- Bayes' rule gives us:

$$\mathbf{Pr}[\text{spam}|\text{“check this out”}] = \frac{\mathbf{Pr}[\text{spam}] \cdot \mathbf{Pr}[\text{“check this out”} \mid \text{spam}]}{\mathbf{Pr}[\text{“check this out”}]}$$

- For the denominator we have:

$$\begin{aligned}\mathbf{Pr}[\text{“check this out”}] =& \mathbf{Pr}[\text{spam}] \cdot \mathbf{Pr}[\text{“check this out”} \mid \text{spam}] + \\ & \mathbf{Pr}[\text{not spam}] \cdot \mathbf{Pr}[\text{“check this out”} \mid \text{not spam}] \\ =& 0.4 \times 0.01 + 0.6 \times 0.004\end{aligned}$$

- Dividing the numerator $0.4 \times 0.01 = 0.004$ by the denominator, we get the result: $\mathbf{Pr}[\text{spam} \mid \text{“check this out”}] = \frac{0.004}{0.0064} = \frac{5}{8} = 0.625$

Claim

This email is more likely to be a spam than not to be a spam.

Revisit Herding Experiment

- An urn contains 3 items:
  - 1 red, 2 blue with 50% probability
  - 2 red, 1 blue with 50% probability
- Is the content majoritarily red or majoritarily blue?
- One by one, come and pick one item
  - look at its color without showing it to the rest of the class
  - You should write your guess of the majority color ("red" or "blue") on the white board
- Assume that all the students reason correctly about what to do when it is their turn to guess, using everything they heard so far

Bayes' Rule in the Herding Experiment

- For the student to guess majority-blue in the herding experiment, we need:

$$\mathbf{Pr}[\text{majority-blue} \mid \text{what she has seen and heard}] > \frac{1}{2}$$

- Or guess majority-red otherwise. If the two probabilities are the same, then it doesn't matter what she guesses
- The prior probability of majoritarily blue and the prior probability of majoritarily red content are both $\frac{1}{2}$

$$\mathbf{Pr}[\text{majority-blue}] = \mathbf{Pr}[\text{majority-red}] = \frac{1}{2}$$

- Based on the composition of the two kinds of urns,

$$\mathbf{Pr}[\text{blue} \mid \text{majority-blue}] = \mathbf{Pr}[\text{red} \mid \text{majority-red}] = \frac{2}{3}$$

Bayes' Rule in the Herding Experiment

- Now, let us suppose that the first student draws a blue item, he can use Bayes' rule to calculate:

$$\mathbf{Pr}[\text{majority-blue} \mid \text{blue}] = \frac{\mathbf{Pr}[\text{majority-blue}] \cdot \mathbf{Pr}[\text{blue} \mid \text{majority-blue}]}{\mathbf{Pr}[\text{blue}]}$$

- The numerator is $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$. The urn is either majority-blue or majority-red:

$$\begin{aligned}\mathbf{Pr}[\text{blue}] =& \mathbf{Pr}[\text{majority-blue}] \cdot \mathbf{Pr}[\text{blue} \mid \text{majority-blue}] + \\ & \mathbf{Pr}[\text{majority-red}] \cdot \mathbf{Pr}[\text{blue} \mid \text{majority-red}] \\ =& \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}\end{aligned}$$

- The answer $\mathbf{Pr}[\text{blue}] = \frac{1}{2}$ makes sense given that the roles of blue and red in this experiment are symmetric

Bayes' Rule in the Herding Experiment

- Dividing numerator by denominator, we get:

$$\mathbf{Pr}[\text{majority-blue} \mid \text{blue}] = \frac{1/3}{1/2} = \frac{2}{3}$$

- Since this conditional probability is greater than $\frac{1}{2}$, we get the intuitive result that the first student should guess a majoritarily blue content when he sees a blue item.
- The calculation is similar for the second student

Bayes' Rule in the Herding Experiment

- Let us focus on the case where the first two students have announced guesses of blue and the third student draws a red item
- What the third student wants to know is

$$\mathbf{Pr}[\text{majority-blue} \mid \text{blue,blue,red}]$$

- Using Bayes' rule we get

$$\mathbf{Pr}[\text{majority-blue} \mid \text{blue,blue,red}] = \frac{\mathbf{Pr}[\text{majority-blue}] \cdot \mathbf{Pr}[\text{blue,blue,red} \mid \text{majority-blue}]}{\mathbf{Pr}[\text{blue,blue,red}]}$$

- Since the draws from the urn are independent:

$$\mathbf{Pr}[\text{blue,blue,red} \mid \text{majority-blue}] = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$$

Bayes' Rule in the Herding Experiment

- To determine the probability, as usual we consider the two different ways this sequence could have happened - the urn is either majority-blue or majority-red:

$$\begin{aligned}\mathbf{Pr}[\text{blue,blue,red}] =& \mathbf{Pr}[\text{majority-blue}] \cdot \mathbf{Pr}[\text{blue,blue,red} \mid \text{majority-blue}] + \\ & \mathbf{Pr}[\text{majority-red}] \cdot \mathbf{Pr}[\text{blue,blue,red} \mid \text{majority-red}] \\ =& \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{6}{54} = \frac{1}{9}\end{aligned}$$

- Finally, we get the result:

$$\mathbf{Pr}[\text{majority-blue} \mid \text{blue,blue,red}] = \frac{\frac{4}{27} \cdot \frac{1}{2}}{\frac{1}{9}} = \frac{2}{3}$$

- Therefore the third student should guess majority-blue (from which she will have $\frac{2}{3}$ chance of being correct)

Outline

A General Cascade Model

- Now, we consider a general cascade model that covers the herding experiment
- A group of people making decision sequentially, i.e., one after the other
- A decision is the choice of
  - Either accepting or rejecting some option
  - **Example:** decision about whether to adopt a new technology
- The general cascade model has three ingredients:
  - States of the World
  - Payoffs
  - Signals

A General Cascade Model

1. States of the World: initial state (B/G) where the option is bad (B) or good (G)
   - $\mathbf{Pr}[\text{G}] = 1 - \mathbf{Pr}[\text{B}] = p$
   - **Example:** majoritarily red or majoritarily blue in the herding experiment

2. Payoffs: each individual receives a payoff depending on her decision
   - Reject the option => payoff $= 0$
   - Accept a bad option => payoff $v_b < 0$
   - Accept a good option => payoff $v_g > 0$
   - Assume that $v_g p + v_b(1 - p) = 0$
   - **Example:** guessing red if accepting the option that the urn is majoritarily red has a positive expected payoff

Quiz

- If accepting the option that the urn is majoritarily blue has a negative expected payoff, which color should we guess?

A General Cascade Model

1. States of the World: initial state (B/G)
2. Payoffs: individual receives a payoff depending on her decision
3. Signals:
   - Before making decision, one receives a *private signal suggesting* whether the option is good or bad
   - High signal H suggests that the option is good
   - Low signal L suggests that the option is bad
   - $\mathbf{Pr}[\mathsf{H} \mid \mathsf{G}] = q > \frac{1}{2}$, $\mathbf{Pr}[\mathsf{L} \mid \mathsf{G}] = 1 - q < \frac{1}{2}$
   - $\mathbf{Pr}[\mathsf{L} \mid \mathsf{B}] = q > \frac{1}{2}$, $\mathbf{Pr}[\mathsf{H} \mid \mathsf{B}] = 1 - q < \frac{1}{2}$

States

Signals		B	G
	L	q	$1-q$
	H	$1-q$	q

   - **Example:** red is a high signal, $\mathbf{Pr}[\mathsf{H} \mid \mathsf{G}] = \mathbf{Pr}[\text{red} \mid \text{majority-red}] = q = \frac{2}{3}$

A General Cascade Model

Making decisions about accepting or rejecting the option

1. Based solely on their own private signal:
   - If a person gets a high signal, the expected payoff shifts from

$$v_g \mathbf{Pr}[\mathsf{G}] + v_b \mathbf{Pr}[\mathsf{B}] = 0 \quad \text{to} \quad v_g \mathbf{Pr}[\mathsf{G} \mid \mathsf{H}] + v_b \mathbf{Pr}[\mathsf{B} \mid \mathsf{H}]$$

   - We compute the expected payoff:

$$\begin{aligned}
\mathbf{Pr}[\mathsf{G} \mid \mathsf{H}] &= \frac{\mathbf{Pr}[\mathsf{G}] \cdot \mathbf{Pr}[\mathsf{H} \mid \mathsf{G}]}{\mathbf{Pr}[\mathsf{H}]} \\
&= \frac{\mathbf{Pr}[\mathsf{G}] \cdot \mathbf{Pr}[\mathsf{H} \mid \mathsf{G}]}{\mathbf{Pr}[\mathsf{G}] \cdot \mathbf{Pr}[\mathsf{H} \mid \mathsf{G}] + \mathbf{Pr}[\mathsf{B}] \cdot \mathbf{Pr}[\mathsf{H} \mid \mathsf{B}]} \\
&= \frac{pq}{pq + (1-p)(1-q)} > p
\end{aligned}$$

   - Last inequality follows from $pq + (1-p)(1-q) < pq + (1-p)q = q$ and $q > \frac{1}{2}$
   - This makes sense, as a high signal is more likely to occur if the option is good

A General Cascade Model

Making decisions about accepting or rejecting the option

2. Based on multiple signals:
   - Individuals receive a sequence S of a high signals and b low signals
   - The posterior probability $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}]$ is greater than the prior one $\mathbf{Pr}[\mathsf{G}]$ if $a > b$
   - The posterior probability $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}]$ is lower than the prior one $\mathbf{Pr}[\mathsf{G}]$ if $a < b$
   - The two probabilities $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}]$ and $\mathbf{Pr}[\mathsf{G}]$ are equal if $a = b$

Claim

Individuals should accept the option when they get more high signals than low signals, and reject it when they get more low signals than high ones, they are indifferent if they receive the same number of each signal

Proving the Claim

- Compare $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}]$ with $\mathbf{Pr}[\mathsf{G}] = p$

$$\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}] = \frac{\mathbf{Pr}[\mathsf{G}] \cdot \mathbf{Pr}[\mathsf{S} \mid \mathsf{G}]}{\mathbf{Pr}[\mathsf{S}]}$$

- Signals are generated independently, so $\mathbf{Pr}[\mathsf{S} \mid \mathsf{G}] = q^a(1-q)^b$
- S can occur if the option is either good or bad:

$$\begin{aligned}\mathbf{Pr}[\mathsf{S}] &= \mathbf{Pr}[\mathsf{G}] \cdot \mathbf{Pr}[\mathsf{S} \mid \mathsf{G}] + \mathbf{Pr}[\mathsf{B}] \cdot \mathbf{Pr}[\mathsf{S} \mid \mathsf{B}] \\ &= pq^a(1-q)^b + (1-p)(1-q)^a q^b\end{aligned}$$

- We thus obtain:

$$\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}] = \frac{pq^a(1-q)^b}{pq^a(1-q)^b + (1-p)(1-q)^a q^b}$$

Proving the Claim

- Compare $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}]$ with $\mathbf{Pr}[\mathsf{G}] = p$

$$\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}] = \frac{pq^a(1-q)^b}{pq^a(1-q)^b + (1-p)(1-q)^a q^b}$$

- If we replace the second term in the denominator by $(1-p)q^a(1-q)^b$ then we would have:

$$\frac{pq^a(1-q)^b}{q^a(1-q)^b} = p$$

Proving the Claim

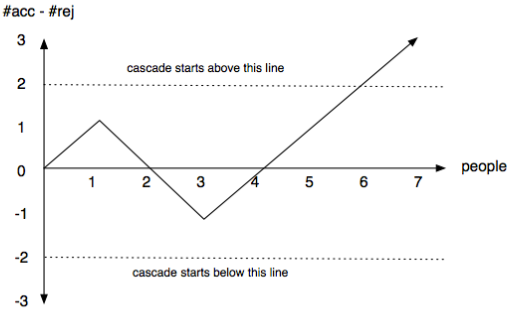
- Replacing $(1-p)(1-q)^a q^b$ by $(1-p)q^a(1-q)^b$
- Would this make the denominator smaller or larger?
  - If $a > b$, then this replacement makes the denominator larger because $q > \frac{1}{2}$ and we have fewer factors of $1-q$ and more of q so $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}] > p = \mathbf{Pr}[\mathsf{G}]$
  - If $a < b$, then the argument is symmetric, $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}] < p = \mathbf{Pr}[\mathsf{G}]$
  - Finally, if $a = b$, then this replacement keeps the value of the denominator the same and $\mathbf{Pr}[\mathsf{G} \mid \mathsf{S}] = p = \mathbf{Pr}[\mathsf{G}]$

Implication for Sequential Decision Making

The general cascade model suggests that

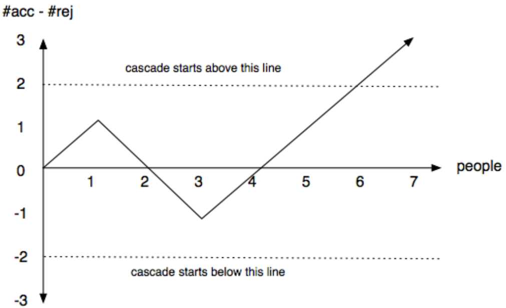
Individuals should accept the option when they get more high signals than low signals, and reject it when they get more low signals than high ones, they are indifferent if they receive the same number of each signal

- Thus, as long as the number of acceptances differs from the number of rejections by at most one, each person in sequence is simply following their own private signal in deciding what to do. But once the difference reaches two or over, a cascade takes over.



Implication for Sequential Decision Making

- Probability of cascade converges to 1 as N grows to infinity:
  - probability of no cascade $\leq$ probability no 3 consecutive people receiving the same signal
  - probability that 3 specific consecutive people receives the same signals is $q^3 + (1-q)^3$
  - probability that among N people no 3 consecutive ones receive the same signals is $\leq (1 - q^3 - (1-q)^3)^{N/3}$



Conclusion

Implication for Sequential Decision Making

- Cascades are likely to occur
- Cascades can be based on very little information
  - Because people ignore their information once a cascade started
- Cascade can be wrong
  - If accepting the option is a bad decision but the signals (e.g., first two) were misleading

Reading

- Reading for this week
  - Chapter 16 of the textbook