01 무리수 e와 자연로그

(1) 무리수 e: x의 값이 0에 한없이 가까워질 때, $(1+x)^{\frac{1}{x}}$의 값은 일정한 값에 수렴하는데 그 값을 e라 하고, e는 무리수이며 그 값은 $2.71828182845904\cdots$이다.

⇨ $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e$

(2) 자연로그 : 무리수 e를 밑으로 하는 로그 $\log_e x$를 자연로그라 하고, 간단히 $\ln x$와 같이 나타낸다.

⇨ $\log_e x=\ln x$

◪ 다음 값을 구하여라.

1. $\ln 1$

2. $\ln e$

3. $\ln e^3$

4. $e^{\ln\frac{1}{3}}$

5. $\ln\sqrt{e}$

6. $\ln 3e$

7. $e^{\ln 3}$

8. $\ln\frac{1}{10e}$

9. $\ln\frac{1}{\sqrt{e}}$

10. $\ln\frac{1}{e^3}+5$

11. $\ln e^5$

12. $e^{\frac{1}{2}\ln 4}$

13. $e^{\ln\sqrt{2}}$

14. $e^{\ln\sqrt{8}}$

◪ 다음 등식을 만족하는 x의 값을 구하여라.

15. $\ln x=3$

16. $\ln x=-1$

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17. $e^x = \frac{1}{3}$

18. $e^x = 2$

19. $e^{2x} = \frac{1}{9}$

20. $\ln x = -\frac{1}{2}$

02 $\lim_{x \to 0}(1+x)^{\frac{1}{x}}$ 꼴의 극한

◪ 다음 극한값을 구하여라.

21. $\lim_{x \to 0}(1-x)^{\frac{1}{2x}}$

22. $\lim_{x \to 0}(1-4x)^{-\frac{1}{2x}}$

23. $\lim_{x \to 0}(1+2x)^{\frac{1}{x}}$

24. $\lim_{x \to 0}\left(1+\frac{x}{2}\right)^{\frac{3}{x}}$

25. $\lim_{x \to 0}(1-2x)^{\frac{1}{x}}$

26. $\lim_{x \to 0}(1-3x)^{-\frac{5}{6x}}$

27. $\lim_{x \to 0}(1+x)^{\frac{6}{x}}$

28. $\lim_{x \to 0}(1-4x)^{\frac{1}{x}}$

29. $\lim_{x \to 0}(1-x)^{\frac{1}{x}}$

30. $\lim_{x \to 0}\left(1+\frac{x}{2}\right)^{-\frac{3}{x}}$

31. $\lim_{x \to 0}(1+2x)^{\frac{1}{3x}}$

32. $\lim_{x \to 0}\left(1+\frac{x}{4}\right)^{\frac{20}{x}}$

33. $\lim_{x\to0}(1+4x)^{\frac{2}{x}}$

34. $\lim_{x\to0}(1+4x)^{\frac{1}{x}}$

35. $\lim_{x\to0}(1+3x)^{\frac{1}{x}}$

36. $\lim_{x\to0}(1+x)^{\frac{2}{x}}$

37. $\lim_{x\to0}\left(1-\frac{x}{3}\right)^{\frac{1}{3x}}$

38. $\lim_{x\to0}\left(1-\frac{3x}{2}\right)^{-\frac{8}{x}}$

39. $\lim_{x\to0}(1+3x)^{\frac{2}{x}}$

03 $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$ 꼴의 극한

◪ 다음 극한값을 구하여라.

40. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

41. $\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{-x}$

42. $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$

43. $\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^{6x}$

44. $\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{-4x}$

45. $\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{2x}$

46. $\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{\frac{x}{10}}$

47. $\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{x}$

48. $\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{x}$

49. $\lim_{x\to\infty}\left(1+\frac{7}{x}\right)^{2x}$

50. $\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^{2x}$

51. $\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^{x-1}$

52. $\lim_{x\to\infty}\left\{\frac{3x}{3x+1}\right\}^{2x}$

53. $\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^{x}$

54. $\lim_{x\to\infty}\left\{\left(1+\frac{1}{3x}\right)\left(1+\frac{1}{5x}\right)\right\}^{15x}$

55. $\lim_{x\to\infty}\left\{\left(1+\frac{1}{2x}\right)\left(1+\frac{1}{3x}\right)\right\}^{6x}$

56. $\lim_{x\to\infty}\left\{\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\cdots\left(1+\frac{7}{x}\right)\right\}^{x}$

57. $\lim_{x\to\infty}\left\{\frac{1}{2}\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\cdots\left(1+\frac{1}{2x}\right)\right\}^{x}$

58.

$\lim_{x\to\infty}\left\{\frac{1}{3}\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\cdots\left(1+\frac{1}{3x}\right)\right\}^{2x}$

59. $\lim_{x\to-\infty}\left(1-\frac{1}{x}\right)^{2x}$

60. $\lim_{x\to-\infty}\left(1-\frac{2}{x}\right)^{2x}$

61. $\lim_{x\to-\infty}\left(1-\frac{2}{x}\right)^{3x}$

62. $\lim_{x \to -\infty}\left(1-\frac{1}{6x}\right)^{-4x}$

63. $\lim_{x \to -\infty}\left(1-\frac{1}{4x}\right)^{-12x}$

64. $\lim_{x \to -\infty}\left(1-\frac{2}{3x}\right)^{\frac{x}{2}}$

65. $\lim_{x \to -\infty}\left(1-\frac{1}{2x}\right)^{-8x}$

66. $\lim_{x \to -\infty}\left(\frac{x-1}{x}\right)^{x}$

정답 및 해설

1) 0

2) 1

3) 3
⇨ $\ln e^3 = 3\ln e = 3$

4) $\frac{1}{3}$
⇨ $e^{\ln\frac{1}{3}} = \left(\frac{1}{3}\right)^{\ln e} = \frac{1}{3}$

5) $\frac{1}{2}$
⇨ $\ln\sqrt{e} = \frac{1}{2}\ln e = \frac{1}{2}$

6) $1+\ln 3$
⇨ $\ln 3e = \ln 3 + \ln e = 1 + \ln 3$

7) 3
⇨ $e^{\ln 3} = 3^{\ln e} = 3$

8) $-1-\ln 10$
⇨ $\ln\frac{1}{10e} = -\ln 10e = -1-\ln 10$

9) $-\frac{1}{2}$
⇨ $\ln\frac{1}{\sqrt{e}} = \ln\left(e^{-\frac{1}{2}}\right) = -\frac{1}{2}\cdot\ln e = -\frac{1}{2}$

10) 2
⇨ $\ln\frac{1}{e^3}+5 = \ln e^{-3}+5 = -3+5 = 2$

11) 5
⇨ $\ln e^5 = 5\ln e = 5\cdot 1 = 5$

12) 2
⇨ $e^{\frac{1}{2}\ln 4} = e^{\ln 4^{\frac{1}{2}}} = e^{\ln 2} = 2^{\ln e} = 2$

13) $\sqrt{2}$
⇨ $e^{\ln\sqrt{2}} = (\sqrt{2})^{\ln e} = \sqrt{2}$

14) $2\sqrt{2}$
⇨ $e^{\ln\sqrt{8}} = (\sqrt{8})^{\ln e} = \sqrt{8} = 2\sqrt{2}$

15) e^3
⇨ $\ln x = 3 \quad \therefore x = e^3$

16) $\frac{1}{e}$
⇨ $\ln x = -1 \qquad \therefore x = e^{-1} = \frac{1}{e}$

17) $-\ln 3$
⇨ $e^x = \frac{1}{3} \qquad \therefore x = \ln\left(\frac{1}{3}\right) = -\ln 3$

18) $\ln 2$
⇨ $e^x = 2 \qquad \therefore x = \ln 2$

19) $-\ln 3$
⇨ $e^{2x} = \frac{1}{9}$에서 $2x = \ln\frac{1}{9}$, $2x = -2\ln 3$
$\therefore\ x = -\ln 3$

20) $\frac{\sqrt{e}}{e}$
⇨ $\ln x = -\frac{1}{2} \quad \therefore x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} = \frac{\sqrt{e}}{e}$

21) $\frac{1}{\sqrt{e}}$
⇨ $\lim_{x\to 0}(1-x)^{\frac{1}{2x}} = \lim_{x\to 0}\left\{(1-x)^{-\frac{1}{x}}\right\}^{-\frac{1}{2}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

22) e^2
⇨ $\lim_{x\to 0}(1-4x)^{-\frac{1}{2x}} = \lim_{x\to 0}\left\{(1-4x)^{-\frac{1}{4x}}\right\}^2 = e^2$

23) e^2
⇨ $\lim_{x\to 0}(1+2x)^{\frac{1}{x}} = \lim_{x\to 0}\left\{(1+2x)^{\frac{1}{2x}}\right\}^2 = e^2$

24) $e^{\frac{3}{2}}$
⇨ $\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\frac{3}{x}} = \lim_{x\to 0}\left\{\left(1+\frac{x}{2}\right)^{\frac{2}{x}}\right\}^{\frac{3}{2}} = e^{\frac{3}{2}}$

25) $\frac{1}{e^2}$
⇨ $\lim_{x\to 0}(1-2x)^{\frac{1}{x}} = \lim_{x\to 0}\left\{(1-2x)^{-\frac{1}{2x}}\right\}^{-2} = e^{-2} = \frac{1}{e^2}$

26) $e^{\frac{5}{2}}$
⇨ $\lim_{x\to 0}(1-3x)^{-\frac{5}{6x}} = \lim_{x\to 0}\left\{(1-3x)^{-\frac{1}{3x}}\right\}^{\frac{5}{2}} = e^{\frac{5}{2}}$

27) e^6
⇨ $\lim_{x\to 0}(1+x)^{\frac{6}{x}} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^6 = e^6$

28) $\frac{1}{e^4}$

$\Rightarrow \lim_{x\to 0}(1-4x)^{\frac{1}{x}} = \lim_{x\to 0}\left\{(1-4x)^{-\frac{1}{4x}}\right\}^{-4} = e^{-4} = \frac{1}{e^4}$

29) $\frac{1}{e}$

$\Rightarrow \lim_{x\to 0}(1-x)^{\frac{1}{x}} = \lim_{x\to 0}\left\{(1-x)^{-\frac{1}{x}}\right\}^{-1} = e^{-1} = \frac{1}{e}$

30) $e^{-\frac{3}{2}}$

$\Rightarrow \lim_{x\to 0}\left(1+\frac{x}{2}\right)^{-\frac{3}{x}} = \lim_{x\to 0}\left\{\left(1+\frac{x}{x}\right)^{\frac{2}{x}}\right\}^{-\frac{3}{2}} = e^{-\frac{3}{2}}$

31) $e^{\frac{2}{3}}$

$\Rightarrow \lim_{x\to 0}(1+2x)^{\frac{1}{3x}} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{2x}}\right\}^{\frac{2}{3}} = e^{\frac{2}{3}}$

32) e^5

$\Rightarrow \lim_{x\to 0}\left(1+\frac{x}{4}\right)^{\frac{20}{x}} = \lim_{x\to 0}\left\{\left(1+\frac{x}{4}\right)^{\frac{4}{x}}\right\}^{5} = e^5$

33) e^8

$\Rightarrow \lim_{x\to 0}(1+4x)^{\frac{1}{4x}\times\frac{8x}{x}} = e^8$

34) e^4

$\Rightarrow \lim_{x\to 0}(1+4x)^{\frac{1}{x}} = \lim_{x\to 0}\left((1+4x)^{\frac{1}{4x}}\right)^4 = e^4$

35) e^3

$\Rightarrow \lim_{x\to 0}(1+3x)^{\frac{1}{x}} = \lim_{x\to 0}\left\{(1+3x)^{\frac{1}{3x}}\right\}^3 = e^3$

36) e^2

$\Rightarrow \lim_{x\to 0}(1+x)^{\frac{2}{x}} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^2 = e^2$

37) $e^{-\frac{1}{9}}$

$\Rightarrow \lim_{x\to 0}\left(1-\frac{x}{3}\right)^{\frac{1}{3x}} = \lim_{x\to 0}\left\{\left(1-\frac{x}{3}\right)^{-\frac{3}{x}}\right\}^{-\frac{1}{3\times 3}} = e^{-\frac{1}{9}}$

38) e^{12}

$\Rightarrow \lim_{x\to 0}\left(1-\frac{3x}{2}\right)^{-\frac{8}{x}} = \lim_{x\to 0}\left\{\left(1-\frac{3x}{2}\right)^{-\frac{2}{3x}}\right\}^{4\times 3} = e^{12}$

39) e^6

$\Rightarrow \lim_{x\to 0}(1+3x)^{\frac{2}{x}} = \lim_{x\to 0}\left\{(1+3x)^{\frac{1}{3x}}\right\}^6 = e^6$

40) e^3

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x} = \lim_{x\to\infty}\left\{\left(1+\frac{1}{x}\right)^{x}\right\}^3 = e^3$

41) $\frac{1}{\sqrt{e}}$

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{-x} = \lim_{x\to\infty}\left\{\left(1+\frac{1}{2x}\right)^{2x}\right\}^{-\frac{1}{2}} = e^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{e}}$

42) e^2

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x} = \lim_{x\to\infty}\left\{\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right\}^2 = e^2$

43) e^2

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^{6x} = \lim_{x\to\infty}\left\{\left(1+\frac{1}{3x}\right)^{3x}\right\}^2 = e^2$

44) $\frac{1}{e^2}$

$\Rightarrow$

$\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{-4x} = \lim_{x\to\infty}\left\{\left(1+\frac{1}{2x}\right)^{2x}\right\}^{-2} = e^{-2} = \frac{1}{e^2}$

45) e

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{2x} = e$

46) $\sqrt{e}$

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{\frac{x}{10}} = \lim_{x\to\infty}\left\{\left(1+\frac{5}{x}\right)^{\frac{x}{5}}\right\}^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$

47) $\sqrt{e}$

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{x}$

$= \lim_{x\to\infty}\left\{\left(1+\frac{1}{2x}\right)^{2x}\right\}^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$

48) e^5

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{x} = \lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{\frac{x}{5}\cdot 5} = e^5$

49) e^{14}

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{7}{x}\right)^{2x} = \lim_{x\to\infty}\left\{\left(1+\frac{7}{x}\right)^{\frac{x}{7}}\right\}^{2\times 7} = e^{14}$

50) e^2

⇨ $\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^{2x}=\lim_{x\to\infty}\left\{\left(1+\frac{1}{x}\right)^{x}\right\}^{2}=e^{2}$

51) e^2

⇨ $\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^{x-1}=\lim_{x\to\infty}\left(1+\frac{2}{x-1}\right)^{\frac{x-1}{2}\cdot 2}=e^{2}$

52) $e^{-\frac{2}{3}}$

⇨ $\lim_{x\to\infty}\left\{\frac{3x}{3x+1}\right\}^{2x}=\lim_{x\to\infty}\left\{\frac{1}{1+\frac{1}{3x}}\right\}^{3x\times\frac{2}{3}}=e^{-\frac{2}{3}}$

53) e^2

⇨ $\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{2}{x-1}\right)^{x}$

$=\lim_{x\to\infty}\left(1+\frac{2}{x-1}\right)^{x-1}\left(1+\frac{2}{x-1}\right)$

$=\lim_{x\to\infty}\left(1+\frac{2}{x-1}\right)^{\frac{x-1}{2}\times 2}\times\left(1+\frac{2}{x-1}\right)$

$=e^2\times 1=e^2$

54) e^8

⇨ $\lim_{x\to\infty}\left\{\left(1+\frac{1}{3x}\right)\left(1+\frac{1}{5x}\right)\right\}^{15x}$

$=\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^{15x}\left(1+\frac{1}{5x}\right)^{15x}$

$=\lim_{x\to\infty}\left\{\left(1+\frac{1}{3x}\right)^{3x}\right\}^{5}\left\{\left(1+\frac{1}{5x}\right)^{5x}\right\}^{3}=e^5\times e^3=e^8$

55) e^5

⇨ $\lim_{x\to\infty}\left\{\left(1+\frac{1}{2x}\right)\left(1+\frac{1}{3x}\right)\right\}^{6x}$

$=\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{2x\times 3}\left(1+\frac{1}{3x}\right)^{3x\times 2}$

$=e^3\times e^2=e^5$

56) e^{28}

⇨ $\lim_{x\to\infty}\left\{\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\cdots\left(1+\frac{7}{x}\right)\right\}^{x}$

$=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\times\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}\times\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}$

$\times\cdots\times\lim_{x\to\infty}\left(1+\frac{7}{x}\right)^{x}$

$=e\times e^2\times e^3\times\cdots\times e^7=e^{1+2+3+\cdots+7}=e^{28}$

57) $\sqrt{e}$

⇨ $\frac{1}{2}\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\cdot\cdots\cdot\left(1+\frac{1}{2x}\right)$

$=\frac{1}{2}\cdot\frac{x+1}{x}\cdot\frac{x+2}{x+1}\cdot\frac{x+3}{x+2}\cdot\cdots\cdot\frac{2x+1}{2x}$

$=\frac{1}{2}\cdot\frac{2x+1}{x}=\frac{2x+1}{2x}$

따라서

(주어진 식) $=\lim_{x\to\infty}\left(\frac{2x+1}{2x}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{x}$

$=\lim_{x\to\infty}\left\{\left(1+\frac{1}{2x}\right)^{2x}\right\}^{\frac{1}{2}}=e^{\frac{1}{2}}=\sqrt{e}$

58) $e^{\frac{2}{3}}$

⇨ $\lim_{x\to\infty}\left\{\frac{1}{3}\left(\frac{x+1}{x}\right)\left(\frac{x+2}{x+1}\right)\cdots\left(\frac{3x+1}{3x}\right)\right\}^{2x}$

$=\lim_{x\to\infty}\left\{\frac{1}{3}\left(\frac{3x+1}{x}\right)\right\}^{2x}=\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^{3x\times\frac{2}{3}}=e^{\frac{2}{3}}$

59) $\frac{1}{e^2}$

⇨ $-x=t$로 놓으면 $x\to-\infty$일 때 $t\to\infty$이므로

$\lim_{x\to-\infty}\left(1-\frac{1}{x}\right)^{2x}=\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{-2t}$

$=\lim_{t\to\infty}\left\{\left(1+\frac{1}{t}\right)^{t}\right\}^{-2}$

$=e^{-2}=\frac{1}{e^2}$

60) e^{-4}

⇨ $x=-t$로 놓으면 $x\to-\infty$일 때, $t\to\infty$이므로

$\lim_{x\to-\infty}\left(1-\frac{2}{x}\right)^{2x}=\lim_{t\to\infty}\left(1+\frac{2}{t}\right)^{-2t}$

$=\lim_{t\to\infty}\left\{\left(1+\frac{2}{t}\right)^{\frac{t}{2}}\right\}^{-4}=e^{-4}$

61) e^{-6}

⇨ $x=-t$로 놓으면 $x\to-\infty$일 때 $t\to\infty$이므로

$\lim_{x\to-\infty}\left(1-\frac{2}{x}\right)^{3x}=\lim_{t\to\infty}\left(1+\frac{2}{t}\right)^{-3t}$

$=\lim_{t\to\infty}\left\{\left(1+\frac{2}{t}\right)^{\frac{t}{2}}\right\}^{-6}=e^{-6}$

62) $e^{\frac{2}{3}}$

⇨ $x=-t$로 놓으면 $x\to-\infty$일 때 $t\to\infty$이므로

$\lim_{x\to-\infty}\left(1-\frac{1}{6x}\right)^{-4x}=\lim_{t\to\infty}\left\{\left(1+\frac{1}{6t}\right)^{6t}\right\}^{\frac{4}{6}}=e^{\frac{2}{3}}$

63) e^3

⇨ $x=-t$로 놓으면 $x\to-\infty$일 때 $t\to\infty$이므로

$\lim_{x\to-\infty}\left(1-\frac{1}{4x}\right)^{-12x}=\lim_{t\to\infty}\left\{\left(1+\frac{1}{4t}\right)^{4t}\right\}^{3}=e^{3}$

64) $e^{-\frac{1}{3}}$

⇨ $x=-t$로 놓으면 $x\to-\infty$일 때 $t\to\infty$이므로

$$\lim_{x\to-\infty}\left(1-\frac{2}{3x}\right)^{\frac{x}{2}}=\lim_{t\to\infty}\left(1+\frac{2}{3t}\right)^{-\frac{t}{2}}$$
$$=\lim_{t\to\infty}\left\{\left(1+\frac{2}{3t}\right)^{\frac{3t}{2}}\right\}^{-\frac{1}{3}}=e^{-\frac{1}{3}}$$

65) e^4

⇨ $x=-t$로 놓으면 $x\to-\infty$일 때 $t\to\infty$이므로

$$\lim_{x\to-\infty}\left(1-\frac{1}{2x}\right)^{-8x}=\lim_{t\to\infty}\left(1+\frac{1}{2t}\right)^{8t}$$
$$=\lim_{t\to\infty}\left\{\left(1+\frac{1}{2t}\right)^{2t}\right\}^{4}=e^4$$

66) $\frac{1}{e}$

⇨ $\lim_{x\to-\infty}\left(\frac{x-1}{x}\right)^{x}=\lim_{x\to-\infty}\left\{\left(1-\frac{1}{x}\right)^{-x}\right\}^{-1}=e^{-1}$

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