01 정적분의 성질

① $\int_{a}^{a} f(x) = 0$

② $\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx$

③ $\int_{a}^{b} kf(x)dx = k\int_{a}^{b} f(x)dx$ (단, k는 실수)

④ $\int_{a}^{b} \{f(x) \pm g(x)\}dx = \int_{a}^{b} f(x)dx \pm \int_{a}^{b} g(x)dx$
(복부호동순)

⑤ $\int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx = \int_{a}^{b} f(x)dx$
(a, b, c의 대소 관계와 상관없이 항상 성립한다.)

◪ 다음 정적분의 값을 구하여라.

1. $\int_{1}^{3} x dx$

2. $\int_{0}^{3} x^2 dx$

3. $\int_{0}^{8} \sqrt[3]{x}\, dx$

4. $\int_{1}^{2} \frac{1}{x^3} dx$

5. $\int_{1}^{9} \frac{1}{\sqrt{x}} dx$

6. $\int_{1}^{3} (x+1)dx$

7. $\int_{0}^{4} \sqrt{x}(x-1)dx$

8. $\int_{0}^{3} (x^2+1)dx$

9. $\int_{1}^{6} \frac{1}{x} dx$

10. $\int_{0}^{\ln 2} e^x dx$

11. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x dx$

12. $\int_{0}^{\pi} (\sin x + 1)dx$

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13. $\int_{0}^{1}(3x^2+e^x)dx$

14. $\int_{0}^{\pi}\sin x dx$

15. $\int_{0}^{3}3^x dx$

16. $\int_{0}^{1}\frac{e^{2x}}{e^{x-1}}dx-\int_{0}^{1}\frac{1}{e^{x-1}}dx$

17. $\int_{0}^{2}\frac{3}{x^2+5x+4}dx$

18. $\int_{0}^{3}(2^x+4^x)dx$

19. $\int_{1}^{4}\frac{x-1}{\sqrt{x}+1}dx$

20. $\int_{1}^{4}(5x+3)\sqrt{x}\,dx$

21. $\int_{1}^{e}\frac{x+3}{x^2}dx$

22. $\int_{e}^{4e}\frac{3x+1}{x}dx$

23. $\int_{0}^{1}(3e^x+2^{x+1})dx$

24. $\int_{0}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+\sin x}dx$

25. $\int_{0}^{\frac{\pi}{3}}\frac{1}{1-\sin^2 x}dx$

26. $\int_{-1}^{1}\frac{1}{2x+3}dx$

27. $\int_{0}^{\frac{\pi}{4}}(\cos x+\sin x)dx$

28. $\int_{0}^{\frac{\pi}{3}} \sec x \tan x dx$

29. $\int_{0}^{\frac{\pi}{6}} (\sin x + \cos x)^2 dx$

30. $\int_{0}^{\frac{\pi}{2}} (x+1)\cos x dx$

31. $\int_{1}^{9} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$

32. $\int_{0}^{2} (e^x + 3^x) dx$

33. $\int_{0}^{1} \frac{1}{x^2+3x+2} dx$

34. $\int_{0}^{\frac{\pi}{2}} \frac{\sin^3 x}{1+\cos x} dx$

35. $\int_{1}^{3} \frac{8^x - 1}{2^x - 1} dx$

36. $\int_{0}^{2} \frac{27^x + 1}{3^x + 1} dx$

37. $\int_{0}^{4} \frac{5x+1}{2x^2+5x+2} dx$

◪ 다음 정적분의 값을 구하여라.

38. $\int_{0}^{2} (\sqrt{x} - 1) dx + \int_{0}^{2} (\sqrt{x} + 1) dx$

39. $\int_{2}^{8} \frac{1}{\sqrt{x+1}} dx - \int_{3}^{8} \frac{1}{\sqrt{y+1}} dy + \int_{0}^{2} \frac{1}{\sqrt{z+1}} dz$

40. $\int_{\sqrt{3}}^{3} \frac{x^4}{x^2+1} dx - \int_{\sqrt{3}}^{3} \frac{1}{x^2+1} dx$

41. $\int_{0}^{\ln 3}\frac{e^{3x}}{e^{2x}+e^{x}+1}dx+\int_{\ln 3}^{0}\frac{1}{e^{2x}+e^{x}+1}dx$

42. $\int_{0}^{\frac{\pi}{3}}(\sec x+1)^{2}dx-\int_{\frac{\pi}{3}}^{0}(\sec x-1)^{2}dx$

43. $\int_{0}^{\frac{\pi}{2}}(2\cos x-e^{2x})dx+\int_{0}^{\frac{\pi}{2}}(2\cos x+e^{2x})dx$

44. $\int_{-\pi}^{\pi}(1-\cos x)^{2}dx+\int_{-\pi}^{\pi}(2+\sin x)^{2}dx$

45. $\int_{0}^{\frac{\pi}{4}}(\sin x-\cos x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)dx$

46. $\int_{0}^{1}(2^{x}-1)dx+\int_{1}^{3}(2^{x}-1)dx$

47. $\int_{-1}^{0}(e^{x}+1)dx+\int_{0}^{-1}(e^{-x}+1)dx$

48. $\int_{0}^{1}(2^{x}+1)^{2}dx-\int_{1}^{0}(2^{x}-1)^{2}dx$

49. $\int_{0}^{\pi}(\cos x-x)dx+\int_{2\pi}^{\pi}(x-\cos x)dx$

◪ **다음 정적분의 값을 구하여라.**

50. $\int_{-2}^{2}|x|dx$

51. $\int_{0}^{4}|\sqrt{x}-1|dx$

52. $\int_{0}^{\pi}|\sin 2x|dx$

53. $\int_{-1}^{1}|e^{x}-1|dx$

54. $\int_{0}^{\frac{\pi}{2}} |\cos x - \sin x| dx$

55. $\int_{-1}^{1} e^{|x|} dx$

56. $\int_{-1}^{1} |x| e^{x} dx$

◪ 다음 물음에 답하여라.

57. 함수 $f(x)=\begin{cases} \cos x & (x \le 0) \\ e^{x} & (x > 0) \end{cases}$일 때, 정적분 $\int_{-\frac{\pi}{2}}^{\ln 2} f(x) dx$를 구하여라.

58. 함수 $f(x)=\begin{cases} 3^{x} & (x \le 0) \\ \cos x & (x > 0) \end{cases}$에 대하여 정적분 $\int_{-1}^{\frac{\pi}{2}} f(x) dx$를 구하여라.

59. 함수 $f(x)=\begin{cases} \sin x + 1 & (x \le 0) \\ \dfrac{1}{x+1} & (x > 0) \end{cases}$일 때, 정적분 $\int_{-\pi}^{1} f(x) dx$를 구하여라.

60. 함수 $f(x)=\begin{cases} \sqrt{x} & (0 \le x < 1) \\ \dfrac{1}{x} & (x \ge 1) \end{cases}$라 할 때, 정적분 $\int_{0}^{e} f(x) dx$를 구하여라.

61. 함수 $f(x)=\begin{cases} e^{-x} & (x \le 0) \\ \cos x - \sin x & (x > 0) \end{cases}$일 때, 정적분 $\int_{-1}^{\pi} f(x) dx$를 구하여라.

◪ 다음 알맞은 값을 구하여라.

62. 정적분 $\int_{-1}^{3} \frac{6x^{3}-5}{x} dx$의 값이 $\alpha - \ln\beta$라고 할 때, 두 상수 α, β의 합 $\alpha+\beta$의 값을 구하여라.

63. $\int_{-1}^{0} \frac{1}{x^{2}+5x+6} dx = \ln\frac{b}{a}$일 때, $a+b$의 값을 구하여라. (단, a, b는 서로소인 자연수)

64. 정적분 $\int_{0}^{2} \left| \frac{x-2}{x+1} \right| dx = a\ln 3 + b$일 때, 정수 a, b에 대하여 $a+b$의 값을 구하여라.

65. 함수 $f(x)=\ln x+\int_1^3 f(t)dt$에 대하여 $f(3)$의 값을 구하여라.

66. 함수 $f(x)$가 $f(x)=x+\int_1^e \frac{2f(t)}{t}dt$를 만족할 때, $f(e)$의 값을 구하여라.

02 우함수, 기함수의 정적분의 계산

(1) 함수 $f(x)$가 y축 대칭인 우함수이면

$$\int_{-a}^{a} f(x)dx=2\int_0^a f(x)dx$$

(2) 함수 $f(x)$가 원점 대칭인 기함수이면

$$\int_{-a}^{a} f(x)dx=0$$

◪ 다음 정적분의 값을 구하여라.

67. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x+\cos x)dx$

68. $\int_{-4}^{4} (e^x+e^{-x})dx$

69. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin x+\cos x)dx$

70. $\int_{-1}^{1} \frac{e^x+e^{-x}}{2}dx$

71. $\int_{-1}^{1} (2^x-2^{-x})dx$

72. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\tan x+\tan^2 x)dx$

정답 및 해설

1) 4

⇨ $f(x)=x$로 놓으면 정적분의 정의에서 $a=1$, $b=3$이므로

$\Delta x=\frac{3-1}{n}=\frac{2}{n}$, $x_k=1+k\Delta x=1+\frac{2k}{n}$

$f(x_k)=x_k=1+\frac{2k}{n}$

$$\therefore \int_1^3 x dx=\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)\cdot\frac{2}{n}$$
$$=\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)$$
$$=\lim_{n\to\infty}\left\{\frac{2}{n}\cdot n+\frac{4}{n^2}\cdot\frac{n(n+1)}{2}\right\}$$
$$=2+2=4$$

2) 9

⇨ $f(x)=x^2$로 놓으면 정적분의 정의에서 $a=0$, $b=3$이므로

$\Delta x=\frac{3-0}{n}=\frac{3}{n}$, $x_k=0+k\Delta x=\frac{3k}{n}$

$f(x_k)=x_k^2=\left(\frac{3k}{n}\right)^2=\frac{9k^2}{n^2}$

$$\therefore \int_0^3 x^2 dx=\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{9k^2}{n^2}\cdot\frac{3}{n}=\lim_{n\to\infty}\frac{27}{n^3}\sum_{k=1}^{n}k^2$$
$$=\lim_{n\to\infty}\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$
$$=9$$

3) 12

⇨ $\int_0^8 \sqrt[3]{x}\,dx=\int_0^8 x^{\frac{1}{3}}dx=\left[\frac{3}{4}x^{\frac{4}{3}}\right]_0^8$
$=12$

4) $\frac{3}{8}$

⇨ $\int_1^2 x^{-3}dx=\left[-\frac{1}{2}x^{-2}\right]_1^2=-\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}=\frac{3}{8}$

5) 4

⇨ $\int_1^9 \frac{1}{\sqrt{x}}dx=\left[2\sqrt{x}\,\right]_1^9=2(3-1)=4$

6) 6

⇨ $f(x)=x+1$로 놓으면 정적분의 정의에서 $a=1$, $b=3$이므로

$\Delta x=\frac{3-1}{n}=\frac{2}{n}$, $x_k=1+k\Delta x=1+\frac{2k}{n}$

$f(x_k)=\left(1+\frac{2k}{n}\right)+1=2+\frac{2k}{n}$

$$\therefore \int_1^3 (x+1)dx=\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{2k}{n}\right)\cdot\frac{2}{n}$$
$$=\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}\left(2+\frac{2k}{n}\right)$$
$$=\lim_{n\to\infty}\left\{\frac{4}{n}\cdot n+\frac{4}{n^2}\cdot\frac{n(n+1)}{2}\right\}$$
$$=4+2=6$$

7) $\frac{112}{15}$

⇨ $\int_0^4 \sqrt{x}(x-1)dx$

$=\int_0^4 x\sqrt{x}\,dx-\int_0^4 \sqrt{x}\,dx$

$=\left[\frac{2}{5}x^2\sqrt{x}\right]_0^4-\left[\frac{2}{3}x\sqrt{x}\right]_0^4$

$=\frac{64}{5}-\frac{16}{3}$

$=\frac{192-80}{15}$

$=\frac{112}{15}$

8) 12

⇨ $f(x)=x^2+1$로 놓으면 정적분의 정의에서 $a=0$, $b=3$이므로

$\Delta x=\frac{3-0}{n}=\frac{3}{n}$, $x_k=0+k\Delta x=\frac{3k}{n}$

$f(x_k)=\frac{9k^2}{n^2}+1$

$\therefore \int_0^3 (x^2+1)dx$

$$=\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2}+1\right)\cdot\frac{3}{n}$$
$$=\lim_{n\to\infty}\left\{\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}+n\cdot\frac{3}{n}\right\}$$
$$=9+3=12$$

9) ln6

⇨ $\int_1^6 \frac{1}{x}dx=\left[\ln|x|\right]_1^6=\ln 6-\ln 1=\ln 6$

10) 1

⇨ $\int_0^{\ln 2} e^x dx=\left[e^x\right]_0^{\ln 2}=2-1=1$

11) $\frac{1}{2}$

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⇨ $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 1-\frac{1}{2}=\frac{1}{2}$

12) $\pi+2$

⇨ $\int_0^{\pi}(\sin x+1)dx= [-\cos x+x]_0^{\pi}=1+1+\pi=\pi+2$

13) e

⇨ $\int_0^1(3x^2+e^x)dx=[x^3+e^x]_0^1=1+e-1=e$

14) 2

⇨ $\int_0^{\pi}\sin x dx=[-\cos x]_0^{\pi}=1-(-1)=2$

15) $\frac{26}{\ln 3}$

⇨ $\int_0^3 3^x dx=\left[\frac{3^x}{\ln 3}\right]_0^3=\frac{27}{\ln 3}-\frac{1}{\ln 3}=\frac{26}{\ln 3}$

16) e^2-2e+1

⇨ $\int_0^1\frac{e^{2x}}{e^{x-1}}dx-\int_0^1\frac{1}{e^{x-1}}dx$

$=\int_0^1(e^{x+1}-e^{-x+1})dx$

$=\left[\ e^{x+1}+e^{-x+1}\ \right]_0^1$

$=(e^{1+1}+e^{-1+1})-(e^{0+1}+e^{-0+1})$

$=e^2-2e+1$

17) $\ln 2$

⇨ $\frac{3}{x^2+5x+4}=\frac{3}{(x+1)(x+4)}=\frac{1}{x+1}-\frac{1}{x+4}$이므로

$\int_0^2\frac{3}{x^2+5x+4}dx=\int_0^2\left(\frac{1}{x+1}-\frac{1}{x+4}\right)dx$

$=\left[\ \ln|x+1|-\ln|x+4|\ \right]_0^2$

$=(\ln 3-\ln 6)-(\ln 1-\ln 4)$

$=\ln 2$

18) $\frac{77}{2\ln 2}$

⇨ $\int_0^3(2^x+4^x)dx=\left[\ \frac{2^x}{\ln 2}+\frac{4^x}{\ln 4}\ \right]_0^3$

$=\left(\frac{2^3}{\ln 2}+\frac{4^3}{2\ln 2}\right)-\left(\frac{2^0}{\ln 2}+\frac{4^0}{2\ln 2}\right)$

$=\frac{77}{2\ln 2}$

19) $\frac{5}{3}$

⇨ 주어진 정적분 식은

$\int_1^4\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}+1}dx$

$=\int_1^4\sqrt{x}-1\,dx=\left[\frac{2}{3}x^{\frac{3}{2}}-x\right]_1^4=\frac{5}{3}$

20) 76

⇨ $\int_1^4(5x\sqrt{x}+3\sqrt{x})dx=\int_1^4\left(5x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$=\left[2x^{\frac{5}{2}}+2x^{\frac{3}{2}}\right]_1^4=\{2\times(2^5)+2\times(2^3)\}-4$

$=2\times(32+8)-4=76$

21) $4-\frac{3}{e}$

⇨ $\int_1^e\frac{x+3}{x^2}dx=\int_1^e\left(\frac{1}{x}+\frac{3}{x^2}\right)dx$

$=\left[\ \ln|x|-\frac{3}{x}\ \right]_1^e$

$=\left(1-\frac{3}{e}\right)-(0-3)$

$=4-\frac{3}{e}$

22) $9e+2\ln 2$

⇨ $\frac{3x+1}{x}=3+\frac{1}{x}$이므로

$\int_e^{4e}\frac{3x+1}{x}dx=\int_e^{4e}\left(3+\frac{1}{x}\right)dx$

$=\left[\ 3x+\ln|x|\ \right]_e^{4e}$

$=(3\cdot 4e+\ln 4e)-(3\cdot e+\ln e)$

$=9e+2\ln 2$

23) $3e+\frac{2}{\ln 2}-3$

⇨ $\int_0^1(3e^x+2^{x+1})dx=\left[\ 3e^x+\frac{2^{x+1}}{\ln 2}\ \right]_0^1$

$=\left(3e^1+\frac{2^{1+1}}{\ln 2}\right)-\left(3e^0+\frac{2^{0+1}}{\ln 2}\right)$

$=3e+\frac{2}{\ln 2}-3$

24) $\frac{\pi}{2}-1$

⇨ $\cos^2x=1-\sin^2x$이므로

$\int_0^{\frac{\pi}{2}}\frac{\cos^2x}{1+\sin x}dx=\int_0^{\frac{\pi}{2}}\frac{1-\sin^2x}{1+\sin x}dx$

$=\int_0^{\frac{\pi}{2}}\frac{(1-\sin x)(1+\sin x)}{1+\sin x}dx$

$=\int_0^{\frac{\pi}{2}}(1-\sin x)dx$

$=\left[\ x+\cos x\ \right]_0^{\frac{\pi}{2}}$

$=\left(\frac{\pi}{2}+\cos\frac{\pi}{2}\right)-(0+\cos 0)$

$=\frac{\pi}{2}-1$

25) $\sqrt{3}$

⇨ $\frac{1}{1-\sin^2 x}=\frac{1}{\cos^2 x}=\sec^2 x$이므로

$\int_0^{\frac{\pi}{3}}\frac{1}{1-\sin^2 x}dx=\int_0^{\frac{\pi}{3}}\sec^2 x dx$

$=\left[\ \tan x\ \right]_0^{\frac{\pi}{3}}$

$=\tan\frac{\pi}{3}-\tan 0$

$=\sqrt{3}$

26) $\frac{1}{2}\ln 5$

⇨ $\int_{-1}^{1}\frac{1}{2x+3}dx=\left[\frac{1}{2}\ln|2x+3|\right]_{-1}^{1}$

$=\frac{1}{2}(\ln 5-\ln 1)=\frac{1}{2}\ln 5$

27) 1

⇨ $\int_0^{\frac{\pi}{4}}(\cos x+\sin x)dx=\left[\ \sin x-\cos x\ \right]_0^{\frac{\pi}{4}}$

$=\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)-(0-1)=1$

28) 1

⇨ $[\sec x]_0^{\frac{\pi}{3}}=\sec\left(\frac{\pi}{3}\right)-\sec(0)=2-1=1$

29) $\frac{\pi}{6}+\frac{1}{4}$

⇨ $\int_0^{\frac{\pi}{6}}(\sin^2 x+2\sin x\cos x+\cos^2 x)dx$

$=\int_0^{\frac{\pi}{6}}(1+\sin 2x)dx=\left[x-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}}$

$=\frac{\pi}{6}-\frac{1}{4}+\frac{1}{2}=\frac{\pi}{6}+\frac{1}{4}$

30) $\frac{\pi}{2}$

⇨

$\int_0^{\frac{\pi}{2}}(x+1)\cos x dx=[(x+1)\sin x]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\sin x dx$

$=\frac{\pi}{2}+1-[-\cos x]_0^{\frac{\pi}{2}}=\frac{\pi}{2}+1-1=\frac{\pi}{2}$

31) $\frac{64}{3}$

⇨ $\int_1^9\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=\left[\frac{2}{3}x\sqrt{x}+2\sqrt{x}\right]_1^9$

$=(18+6)-\left(\frac{2}{3}+2\right)$

$=\frac{64}{3}$

32) $e^2-1+\frac{8}{\ln 3}$

⇨ $\int_0^2(e^x+3^x)dx=\left[e^x+\frac{3^x}{\ln 3}\right]_0^2$

$=\left(e^2+\frac{9}{\ln 3}\right)-\left(1+\frac{1}{\ln 3}\right)$

$=e^2-1+\frac{8}{\ln 3}$

33) $\ln\frac{4}{3}$

⇨

$\int_0^1\frac{1}{x^2+3x+2}dx=\int_0^1\left(\frac{1}{x+1}-\frac{1}{x+2}\right)dx=[\ln|x+1|-\ln|x+2|]_0^1$

$=\ln 2-\ln 3-\ln 1+\ln 2=\ln\frac{4}{3}$

34) $\frac{1}{2}$

⇨

$\int_0^{\frac{\pi}{2}}\frac{\sin^3 x}{1+\cos x}dx=\int_0^{\frac{\pi}{2}}\frac{\sin x(1-\cos^2 x)}{1+\cos x}dx$

$=\int_0^{\frac{\pi}{2}}\sin x(1-\cos x)dx$

$=\int_0^{\frac{\pi}{2}}\left(\sin x-\frac{1}{2}\sin 2x\right)dx=\left[-\cos x+\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}}=\frac{1}{2}$

35) $\frac{36}{\ln 2}+2$

⇨

$\int_1^3\frac{8^x-1}{2^x-1}dx=\int_1^3(4^x+2^x+1)dx=\left[\frac{4^x}{\ln 4}+\frac{2^x}{\ln 2}+x\right]_1^3$

$=\frac{30}{\ln 2}+\frac{6}{\ln 2}+2=\frac{36}{\ln 2}+2$

36) $\frac{32}{\ln 3}+2$

⇨ $\int_0^2\frac{(3^x+1)(9^x-3^x+1)}{3^x+1}dx=\int_0^2 9^x-3^x+1\,dx$

$=\left[\frac{3^{2x}}{2\ln 3}-\frac{3^x}{\ln 3}+x\right]_0^2=\frac{32}{\ln 3}+2$

37) $2\ln 3$

⇨ $\int_0^4\frac{5x+1}{2x^2+5x+2}dx=\int_0^4\frac{3}{x+2}-\frac{1}{2x+1}dx$

$=\left[3\ln|x+2|-\frac{1}{2}\ln|2x+1|\right]_0^4$

$=3\ln6-\frac{1}{2}\ln9-3\ln2$

$=3(\ln2+\ln3)-\ln3-3\ln2=2\ln3$

38) $\frac{8\sqrt{2}}{3}$

⇨ $\int_0^2(\sqrt{x}-1)dx+\int_0^2(\sqrt{x}+1)dx$

$=\int_0^2(\sqrt{x}-1+\sqrt{x}+1)dx$

$=\int_0^2 2\sqrt{x}\,dx=\left[\frac{4}{3}x^{\frac{3}{2}}\right]_0^2$

$=\frac{8\sqrt{2}}{3}$

39) 2

⇨ $\int_2^8\frac{1}{\sqrt{x+1}}dx-\int_3^8\frac{1}{\sqrt{y+1}}dy+\int_0^2\frac{1}{\sqrt{z+1}}dz$

$=\int_2^8\frac{1}{\sqrt{x+1}}dx-\int_3^8\frac{1}{\sqrt{x+1}}dx+\int_0^2\frac{1}{\sqrt{x+1}}dx$

$=\left(\int_2^8\frac{1}{\sqrt{x+1}}dx+\int_8^3\frac{1}{\sqrt{x+1}}dx\right)+\int_0^2\frac{1}{\sqrt{x+1}}dx$

$=\int_2^3\frac{1}{\sqrt{x+1}}dx+\int_0^2\frac{1}{\sqrt{x+1}}dx$

$=\int_0^2\frac{1}{\sqrt{x+1}}dx+\int_2^3\frac{1}{\sqrt{x+1}}dx$

$=\int_0^3\frac{1}{\sqrt{x+1}}dx=[2\sqrt{x+1}]_0^3$

$=4-2=2$

40) 6

⇨ $\int_{\sqrt{3}}^3\frac{x^4}{x^2+1}dx-\int_{\sqrt{3}}^3\frac{1}{x^2+1}dx$

$=\int_{\sqrt{3}}^3\frac{x^4-1}{x^2+1}dx=\int_{\sqrt{3}}^3\frac{(x^2-1)(x^2+1)}{x^2+1}dx$

$=\int_{\sqrt{3}}^3(x^2-1)dx=\left[\frac{1}{3}x^3-x\right]_{\sqrt{3}}^3$

$=(9-3)-(\sqrt{3}-\sqrt{3})=6$

41) $2-\ln3$

⇨ $\int_0^{\ln3}\frac{e^{3x}}{e^{2x}+e^x+1}dx+\int_{\ln3}^0\frac{1}{e^{2x}+e^x+1}dx$

$=\int_0^{\ln3}\frac{e^{3x}}{e^{2x}+e^x+1}dx-\int_0^{\ln3}\frac{1}{e^{2x}+e^x+1}dx$

$=\int_0^{\ln3}\frac{e^{3x}-1}{e^{2x}+e^x+1}dx$

$=\int_0^{\ln3}\frac{(e^x-1)(e^{2x}+e^x+1)}{e^{2x}+e^x+1}dx$

$=\int_0^{\ln3}(e^x-1)dx$

$=\left[\ e^x-x\ \right]_0^{\ln3}=(e^{\ln3}-\ln3)-(e^0-0)$

$=2-\ln3$

42) $2\sqrt{3}+\frac{2}{3}\pi$

⇨ $\int_0^{\frac{\pi}{3}}(\sec x+1)^2dx-\int_{\frac{\pi}{3}}^0(\sec x-1)^2dx$

$=\int_0^{\frac{\pi}{3}}(\sec x+1)^2dx+\int_0^{\frac{\pi}{3}}(\sec x-1)^2dx$

$=\int_0^{\frac{\pi}{3}}\{(\sec x+1)^2+(\sec x-1)^2\}dx$

$=\int_0^{\frac{\pi}{3}}(2\sec^2x+2)dx=2\int_0^{\frac{\pi}{3}}(\sec^2x+1)dx$

$=2[\tan x+x]_0^{\frac{\pi}{3}}$

$=2\sqrt{3}+\frac{2}{3}\pi$

43) 4

⇨ $\int_0^{\frac{\pi}{2}}(2\cos x-e^{2x})dx+\int_0^{\frac{\pi}{2}}(2\cos x+e^{2x})dx$

$=\int_0^{\frac{\pi}{2}}\{(2\cos x-e^{2x})+(2\cos x+e^{2x})\}dx$

$=\int_0^{\frac{\pi}{2}}4\cos x dx$

$=[4\sin x]_0^{\frac{\pi}{2}}=4$

44) 12π

⇨ $\sin^2x+\cos^2x=1$이므로

$\int_{-\pi}^{\pi}(1-\cos x)^2dx+\int_{-\pi}^{\pi}(2+\sin x)^2dx$

$=\int_{-\pi}^{\pi}(1-2\cos x+\cos^2x)dx+\int_{-\pi}^{\pi}(4+4\sin x+\sin^2x)dx$

$=\int_{-\pi}^{\pi}(5-2\cos x+4\sin x+\cos^2x+\sin^2x)dx$

$=\left[\ 6x-2\sin x-4\cos x\ \right]_{-\pi}^{\pi}=12\pi$

45) 0

⇨ $\int_0^{\frac{\pi}{4}}(\sin x-\cos x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)dx$

$=\int_0^{\frac{\pi}{2}}(\sin x-\cos x)dx=[-\cos x-\sin x]_0^{\frac{\pi}{2}}$

$=-1+1=0$

46) $\frac{7}{\ln2}-3$

⇨ $\int_0^1 (2^x-1)dx+\int_1^3 (2^x-1)dx$

$=\int_0^3 (2^x-1)dx=\left[\frac{2^x}{\ln 2}-x\right]_0^3$

$=\left(\frac{8}{\ln 2}-3\right)-\frac{1}{\ln 2}=\frac{7}{\ln 2}-3$

47) $2-e-\frac{1}{e}$

⇨ $\int_{-1}^0 (e^x+1)dx+\int_0^{-1} (e^{-x}+1)dx$

$=\int_{-1}^0 (e^x+1)dx-\int_{-1}^0 (e^{-x}+1)dx$

$=\int_{-1}^0 (e^x-e^{-x})dx=\left[e^x+e^{-x}\right]_{-1}^0$

$=2-(e^{-1}+e)=2-e-\frac{1}{e}$

48) $\frac{3}{\ln 2}+2$

49) $-2\pi^2$

⇨ $\int_0^{\pi} (\cos x-x)dx+\int_{2\pi}^{\pi} (x-\cos x)dx$

$=\int_0^{\pi} (\cos x-x)dx+\int_{\pi}^{2\pi} (\cos x-x)dx$

$=\int_0^{2\pi} (\cos x-x)dx=\left[\sin x-\frac{1}{2}x^2\right]_0^{2\pi}$

$=-2\pi^2$

50) 4

⇨ $|x|$는 우함수이므로

$\int_{-2}^2 |x|dx=2\int_0^2 xdx=2\left[\frac{1}{2}x^2\right]_0^2=4$

51) 2

⇨ $\sqrt{x}-1=0$에서 $\sqrt{x}=1$ $\therefore x=1$

$|\sqrt{x}-1|=\begin{cases}\sqrt{x}-1 & (x \ge 1)\\ -\sqrt{x}+1 & (0 \le x < 1)\end{cases}$

$\therefore \int_0^4 |\sqrt{x}-1|dx$

$=\int_0^1 (-\sqrt{x}+1)dx+\int_1^4 (\sqrt{x}-1)dx$

$=\left[-\frac{2}{3}x\sqrt{x}+x\right]_0^1+\left[\frac{2}{3}x\sqrt{x}-x\right]_1^4$

$=\left(-\frac{2}{3}+1\right)+\left\{\left(\frac{2}{3}\times 4\sqrt{4}-4\right)-\left(\frac{2}{3}-1\right)\right\}$

$=2$

52) 2

⇨ $\int_0^{\pi} |\sin 2x|dx=\int_0^{\frac{\pi}{2}} \sin 2x dx+\int_{\frac{\pi}{2}}^{\pi} (-\sin 2x)dx$

$=\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}+\left[\frac{1}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\pi}$

$=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=2$

53) $e+\frac{1}{e}-2$

⇨ $\int_{-1}^1 |e^x-1|dx$

$=\int_{-1}^0 1-e^x dx+\int_0^1 e^x-1\,dx$

$=\left[x-e^x\right]_{-1}^0+\left[e^x-x\right]_0^1=e+\frac{1}{e}-2$

54) $2(\sqrt{2}-1)$

⇨ $\cos x-\sin x=0$에서 $x=\frac{\pi}{4}\left(\because 0 \le x \le \frac{\pi}{2}\right)$

$|\cos x-\sin x|=\begin{cases}\cos x-\sin x & \left(0 \le x < \frac{\pi}{4}\right)\\ -\cos x+\sin x & \left(\frac{\pi}{4} \le x \le \frac{\pi}{2}\right)\end{cases}$

$\therefore \int_0^{\frac{\pi}{2}} |\cos x-\sin x|dx$

$=\int_0^{\frac{\pi}{4}} (\cos x-\sin x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (-\cos x+\sin x)dx$

$=\left[\sin x+\cos x\right]_0^{\frac{\pi}{4}}+\left[-\sin x-\cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$=(\sqrt{2}-1)+(-1+\sqrt{2})$

$=2(\sqrt{2}-1)$

55) $2(e-1)$

⇨ $\int_{-1}^0 e^{-x}dx+\int_0^1 e^x dx=\left[-e^{-x}\right]_{-1}^0+\left[e^x\right]_0^1$

$=-1+e+e-1=2e-2$

56) $2-\frac{2}{e}$

⇨

$\int_{-1}^0 (-xe^x)dx+\int_0^1 (xe^x)dx$

$=\left[-xe^x\right]_{-1}^0+\int_{-1}^0 e^x dx+\left[xe^x\right]_0^1-\int_0^1 e^x dx$

$=-\frac{1}{e}+\left[e^x\right]_{-1}^0+e-\left[e^x\right]_0^1$

$=-\frac{1}{e}+1-\frac{1}{e}+e-e+1=2-\frac{2}{e}$

57) 2

⇨ $\int_{-\frac{\pi}{2}}^{\ln 2} f(x)dx=\int_{-\frac{\pi}{2}}^0 f(x)dx+\int_0^{\ln 2} f(x)dx$

$=\int_{-\frac{\pi}{2}}^0 \cos x dx+\int_0^{\ln 2} e^x dx$

$=\left[\sin x\right]_{-\frac{\pi}{2}}^0+\left[e^x\right]_0^{\ln 2}$

$=\{0-(-1)\}+(e^{\ln 2}-1)$

$=1+2-1=2$

58) $\dfrac{2}{3\ln 3}+1$

59) $\pi-2+\ln 2$

⇨ $\displaystyle\int_{-\pi}^{1} f(x)dx=\int_{-\pi}^{0} f(x)dx+\int_{0}^{1} f(x)dx$

$\displaystyle=\int_{-\pi}^{0}(\sin x+1)dx+\int_{0}^{1}\frac{1}{x+1}dx$

$=[-\cos x+x]_{-\pi}^{0}+[\ln(x+1)]_{0}^{1}$

$=\pi-2+\ln 2$

60) $\dfrac{5}{3}$

⇨ $\displaystyle\int_{0}^{e} f(x)dx=\int_{0}^{1}\sqrt{x}\,dx+\int_{1}^{e}\frac{1}{x}\,dx$

$\displaystyle=\left[\frac{2}{3}x^{\frac{3}{2}}\right]_{0}^{1}+[\ln|x|]_{1}^{e}=\frac{2}{3}+1=\frac{5}{3}$

61) $e-3$

⇨ $\displaystyle\int_{-1}^{\pi} f(x)dx=\int_{-1}^{0} f(x)dx+\int_{0}^{\pi} f(x)dx$

$\displaystyle=\int_{-1}^{0} e^{-x}dx+\int_{0}^{\pi}(\cos x-\sin x)dx$

$=[-e^{-x}]_{-1}^{0}+[\sin x+\cos x]_{0}^{\pi}$

$=(-1+e)+(-1-1)=e-3$

62) 299

⇨ $\displaystyle\int_{-1}^{3}\frac{6x^3-5}{x}dx$

$\displaystyle=\int_{-1}^{3}6x^2dx-\int_{-1}^{3}\frac{5}{x}dx$

$=\left[\ 2x^3\ \right]_{-1}^{3}-\left[\ 5\ln|x|\ \right]_{-1}^{3}$

$=\{2\cdot 3^3-2\cdot(-1)^3\}-(5\ln 3-0)$

$=56-5\ln 3$

$=56-\ln 3^5$

$\therefore \alpha=56,\ \beta=3^5=243$

따라서 구하는 값은 $\alpha+\beta=56+243=299$

63) 7

⇨ $\displaystyle\int_{-1}^{0}\frac{1}{x^2+5x+6}dx$

$\displaystyle=\int_{-1}^{0}\frac{1}{(x+2)(x+3)}dx$

$\displaystyle=\int_{-1}^{0}\left(\frac{1}{x+2}-\frac{1}{x+3}\right)dx$

$=[\ln|x+2|-\ln|x+3|]_{-1}^{0}$

$=(\ln 2-\ln 3)-(\ln 1-\ln 2)$

$=\ln\dfrac{4}{3}=\ln\dfrac{b}{a}$

따라서 $a=3,\ b=4$이므로 $a+b=7$

64) 1

⇨ $\displaystyle\int_{0}^{2}\left|\frac{x-2}{x+1}\right|dx=\int_{0}^{2}\left|1-\frac{3}{x+1}\right|dx$

$\displaystyle=-\int_{0}^{2}\left(1-\frac{3}{x+1}\right)dx=-[x-3\ln(x+1)]_{0}^{2}$

$=3\ln 3-2$

따라서 $a=3,\ b=-2$이므로 $a+b=1$

65) $2-2\ln 3$

⇨ $\displaystyle\int_{1}^{3} f(t)dt=a$라고 하자.

$f(x)=\ln x+a$

$\displaystyle\int_{1}^{3}(\ln x+a)dx=a$

$[x\ln x-x+ax]_{1}^{3}=a$

$3\ln 3-3+3a+1-a=a$ $\quad\therefore\ a=2-3\ln 3$

$\therefore\ f(3)=\ln 3+a=2-2\ln 3$

66) $2-e$

⇨ $\displaystyle\int_{1}^{e}\frac{2f(t)}{t}dt=k$라 하면

$\displaystyle k=\int_{1}^{e}\frac{2t+2k}{t}dt=\int_{1}^{e}2+\frac{2k}{t}dt$

$k=[2t+2k\ln t]_{1}^{e},\ k=2e-2+2k$

$\therefore k=2-2e$

$\therefore f(x)=x+2-2e,\ f(e)=2-e$

67) 2

⇨ $\sin x$는 기함수, $\cos x$는 우함수이므로

$\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin x+\cos x)dx=2\int_{0}^{\frac{\pi}{2}}\cos x dx$

$=2[\sin x]_{0}^{\frac{\pi}{2}}=2$

68) $2\left(e^4-\dfrac{1}{e^4}\right)$

⇨ $f(x)=e^x+e^{-x}$으로 놓으면

$f(-x)=e^{-x}+e^x=f(x)$

즉, $f(x)=e^x+e^{-x}$은 우함수이므로

$\displaystyle\int_{-4}^{4}(e^x+e^{-x})dx=2\int_{0}^{4}(e^x+e^{-x})dx$

$=2[e^x-e^{-x}]_{0}^{4}$

$=2\left(e^4-\dfrac{1}{e^4}\right)$

69) $\sqrt{2}$

⇨ $y=\sin x$는 기함수, $y=\cos x$는 우함수이므로

$\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(\sin x+\cos x)dx$

$\displaystyle=2\int_{0}^{\frac{\pi}{4}}\cos x dx\left(\because \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sin x\,dx=0\right)$

$=2[\sin x]_0^{\frac{\pi}{4}}=2\times\frac{\sqrt{2}}{2}=\sqrt{2}$

70) $e-\frac{1}{e}$

⇨ $f(x)=\frac{e^x+e^{-x}}{2}$이라 하면

$f(-x)=\frac{e^{-x}+e^x}{2}=f(x)$이므로 $f(x)$는 우함수이다.

$\therefore \int_{-1}^{1}\frac{e^x+e^{-x}}{2}dx=2\int_0^1\frac{e^x+e^{-x}}{2}dx$

$=\int_0^1(e^x+e^{-x})dx=[e^x-e^{-x}]_0^1$

$=\left(e-\frac{1}{e}\right)-(1-1)=e-\frac{1}{e}$

71) 0

⇨ $f(x)=2^x-2^{-x}$이라 하면

$f(-x)=2^{-x}-2^x=-(2^x-2^{-x})=-f(x)$

이므로 $f(x)$는 기함수이다.

$\therefore \int_{-1}^{1}(2^x-2^{-x})dx=0$

72) $2\left(1-\frac{\pi}{4}\right)$

⇨ $\tan x$는 기함수, $\tan^2 x$는 우함수이므로 주어진 적분은

$2\int_0^{\frac{\pi}{4}}\tan^2 x\,dx=2\int_0^{\frac{\pi}{4}}(\sec^2 x-1)dx$

$=2[\tan x-x]_0^{\frac{\pi}{4}}=2\left(1-\frac{\pi}{4}\right)$