3-2-1.시그마의 뜻과 기본 성질_천재(류희찬)

개념check

[∑의 뜻과 기본 성질]

• $\sum_{k=1}^{n} a_k$: 수열 $\{a_n\}$의 첫째항부터 제n항까지의 합

⇨ $\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n$

[∑의 기본 성질]

① $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

② $\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

③ $\sum_{k=1}^{n} \alpha a_k = c\sum_{k=1}^{n} a_k$ (단, c는 상수)

④ $\sum_{k=1}^{n} c = cn$ (단, c는 상수)

기본문제

[문제]

1. $7+9+11+\cdots+25=\sum_{k=p}^{q}(2k+1)$에 대하여 $p+q$의 값은?

① 11 ② 12
③ 13 ④ 14
⑤ 15

[예제]

2. $\sum_{k=1}^{n} a_k = 5n+2$, $\sum_{k=1}^{n} b_k = -3n+10$일 때, $\sum_{k=1}^{6}(3a_k+4b_k-2)$의 값은?

① 51 ② 52
③ 53 ④ 54
⑤ 62

[문제]

3. $\sum_{k=1}^{10} a_k = 5$, $\sum_{k=1}^{10} b_k = 7$일 때, 다음 식의 값을 구하시오.

$$\sum_{k=1}^{10}(3a_k+6b_k)+\sum_{k=1}^{10}(2a_k-3b_k+2)$$

① 64 ② 65
③ 66 ④ 67
⑤ 68

[문제]

4. 다음 식을 합의 기호 ∑을 이용하여 바르게 나타낸 것은?

$$2+(2+4)+(2+4+6)+(2+4+6+8)+(2+4+6+8+10)$$

① $\sum_{k=1}^{5} 2k$ ② $\sum_{k=1}^{15} 2k$
③ $\sum_{k=1}^{5} n(n-1)$ ④ $\sum_{j=1}^{5}\left(\sum_{i=1}^{j} 2i\right)$
⑤ $\sum_{j=1}^{5}\left\{\sum_{i=1}^{j} n(n+1)\right\}$

평가문제

[스스로 확인하기]

5. $\sum_{k=1}^{20} a_k = 10$, $\sum_{k=1}^{40} a_k = 30$, $\sum_{k=1}^{20} b_k = 5$, $\sum_{k=1}^{40} b_k = 20$일 때, $\sum_{k=21}^{40}(4a_k-5b_k)$의 값은?

① 5 ② 10
③ 15 ④ 20
⑤ 25

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[스스로 확인하기]

6. $\sum_{k=1}^{5} a_k = 7$, $\sum_{k=1}^{5} a_k^2 = 11$일 때, $\sum_{k=1}^{5} (a_k+1)^2$의 값은?

① 20 ② 25
③ 30 ④ 35
⑤ 40

[스스로 확인하기]

7. $\sum_{k=1}^{n} a_k = 5n$, $\sum_{k=1}^{n} b_k = 3n-3$일 때,
$\sum_{k=1}^{10} (a_k+k^2+3k) - \sum_{k=1}^{10} (k+1)^2 + \sum_{k=1}^{10} (2b_k-k)$의 값은?

① 88 ② 90
③ 92 ④ 94
⑤ 96

유사문제

8. $\sum_{k=1}^{10} a_k = 20$, $\sum_{k=1}^{10} b_k = 14$일 때, $\sum_{k=1}^{10} (3a_k-5b_k+4)$의 값은?

① -10 ② 0
③ 30 ④ 70
⑤ 90

9. $\sum_{k=1}^{5} a_k = 10$, $\sum_{k=1}^{5} a_k{}^2 = 50$일 때, $\sum_{k=1}^{5} (a_k-2)^2$의 값은?

① 15 ② 20
③ 25 ④ 30
⑤ 35

10. $\sum_{k=1}^{n} a_k = 8n$, $\sum_{k=1}^{n} b_k = 4n$일 때, $\sum_{k=1}^{10} (2a_k-b_k+2^k)$의 값은?

① 2158 ② 2160
③ 2162 ④ 2164
⑤ 2166

11. $\sum_{k=1}^{10} a_k = 5$, $\sum_{k=1}^{10} b_k = 7$일 때 $\sum_{k=1}^{10} (3a_k-b_k+1)$의 값은?

① 15 ② 16
③ 17 ④ 18
⑤ 19

12. $\sum_{k=1}^{10} a_k = 5$, $\sum_{k=1}^{10} a_k{}^2 = 10$일 때, $\sum_{k=1}^{10} (3a_k-2)^2$의 값은?

① 30 ② 40
③ 50 ④ 60
⑤ 70

13. 수열 $\{a_n\}$, $\{b_n\}$에 대하여
$\sum_{k=1}^{10} a_k = 10$, $\sum_{k=1}^{20} a_k = 20$, $\sum_{k=1}^{10} b_k = 20$, $\sum_{k=1}^{20} b_k = 30$일 때, $\sum_{k=11}^{20} (2a_k+b_k)$의 값은?

① 10 ② 20
③ 30 ④ 40
⑤ 50

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14. $\sum_{k=1}^{10} a_k = 2$**일 때,** $\sum_{k=1}^{10} (a_k+1)^2 - \sum_{k=1}^{10} a_k^2$**의 값은?**

① 8 ② 10
③ 12 ④ 14
⑤ 16

15. $\sum_{k=1}^{15} a_k = 3$, $\sum_{k=1}^{15} b_k = 5$**일 때,** $\sum_{k=1}^{15} (5a_k + 2b_k - 1)$**의 값은?**

① 5 ② 10
③ 15 ④ 20
⑤ 25

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정답 및 해설

1) [정답] ⑤

[해설] $7+9+11+\cdots+25$

$=(2\times3+1)+(2\times4+1)+\cdots+(2\times12+1)$

$=\sum_{k=3}^{12}(2k+1)$

$\therefore\ p+q=15$

2) [정답] ②

[해설] $\sum_{k=1}^{n}a_k=5n+2,\ \sum_{k=1}^{n}b_k=-3n+10$일 때,

$\sum_{k=1}^{6}(3a_k+4b_k-2)$

$=3\sum_{k=1}^{6}a_k+4\sum_{k=1}^{6}b_k-\sum_{k=1}^{6}2$

$=3\times(5\times6+2)+4\times\{(-3)\times6+10\}-2\times6$

$=52$

3) [정답] ③

[해설] $\sum_{k=1}^{10}(3a_k+6b_k)+\sum_{k=1}^{10}(2a_k-3b_k+2)$

$=\sum_{k=1}^{10}(5a_k+3b_k+2)$

$=5\sum_{k=1}^{10}a_k+3\sum_{k=1}^{10}b_k+\sum_{k=1}^{10}2$

$=5\times5+3\times7+2\times10=66$

4) [정답] ④

[해설] $2+(2+4)+(2+4+6)+(2+4+6+8)+(2+4+6+8+10)$

$=\sum_{k=1}^{1}2k+\sum_{k=1}^{2}2k+\sum_{k=1}^{3}2k+\sum_{k=1}^{4}2k+\sum_{k=1}^{5}2k$

$=\sum_{m=1}^{5}\left(\sum_{k=1}^{m}2k\right)$

$=\sum_{j=1}^{5}\left(\sum_{i=1}^{j}2i\right)$

5) [정답] ①

[해설] $\sum_{k=1}^{20}a_k=10,\ \sum_{k=1}^{40}a_k=30$에서 $\sum_{k=21}^{40}a_k=20$

$\sum_{k=1}^{20}b_k=5,\ \sum_{k=1}^{40}b_k=20$에서 $\sum_{k=21}^{40}b_k=15$

$\therefore\ \sum_{k=21}^{40}(4a_k-5b_k)=4\sum_{k=21}^{40}a_k-5\sum_{k=21}^{40}b_k$

$=4\times20-5\times15=5$

6) [정답] ③

[해설] $\sum_{k=1}^{5}(a_k+1)^2=\sum_{k=1}^{5}(a_k^2+2a_k+1)$

$=\sum_{k=1}^{5}a_k^2+2\sum_{k=1}^{5}a_k+\sum_{k=1}^{5}1$

$=11+2\times7+5=30$

7) [정답] ④

[해설] $\sum_{k=1}^{10}(a_k+k^2+3k)-\sum_{k=1}^{10}(k+1)^2+\sum_{k=1}^{10}(2b_k-k)$

$=\sum_{k=1}^{10}\{(a_k+k^2+3k)-(k^2+2k+1)+(2b_k-k)\}$

$=\sum_{k=1}^{10}(a_k+2b_k-1)$

$=\sum_{k=1}^{10}a_k+2\sum_{k=1}^{10}b_k-\sum_{k=1}^{10}1$

$=(5\times10)+2(3\times10-3)-1\times10=94$

8) [정답] ③

[해설] $\sum_{k=1}^{10}(3a_k-5b_k+4)=3\cdot20-5\cdot14+4\cdot10=30$

9) [정답] ④

[해설] $\sum_{k=1}^{5}(a_k-2)^2=\sum_{k=1}^{5}({a_k}^2-4a_k+4)$

$=\sum_{k=1}^{5}{a_k}^2-4\sum_{k=1}^{5}a_k+\sum_{k=1}^{5}4$

$=50-4\times10+5\times4=30$

10) [정답] ⑤

[해설] $\sum_{k=1}^{10}(2a_k-b_k+2^k)=2\sum_{k=1}^{10}a_k-\sum_{k=1}^{10}b_k+\frac{2(2^{10}-1)}{2-1}$

$=2\cdot80-40+2046=2166$

11) [정답] ④

[해설] $\sum_{k=1}^{10}(3a_k-b_k+1)=3\cdot5-7+10=18$

12) [정답] ⑤

[해설] $\sum_{k=1}^{10}(3a_k-2)^2=\sum_{k=1}^{10}(9{a_k}^2-12a_k+4)$

$=9\cdot10-12\cdot5+4\cdot10=70$

13) [정답] ③

[해설] $\sum_{k=11}^{20}(2a_k+b_k)=\sum_{k=1}^{20}(2a_k+b_k)-\sum_{k=1}^{10}(2a_k+b_k)$

$=(2\cdot20+30)-(2\cdot10+20)=30$

14) [정답] ④

[해설] $\sum_{k=1}^{10}(a_k+1)^2-\sum_{k=1}^{10}a_k^2=\sum_{k=1}^{10}(2a_k+1)$

$=2\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1=2\times2+10=14$

15) [정답] ②

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[해설] $\sum_{k=1}^{15}(5a_k+2b_k-1)$

$=5\sum_{k=1}^{15}a_k+2\sum_{k=1}^{15}b_k-\sum_{k=1}^{15}1$

$=5\times3+2\times5-1\times15=10$

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