



◇「콘텐츠산업 진흥법」제33조에 의한 표시

1) 제작연월일 : 2018-07-26

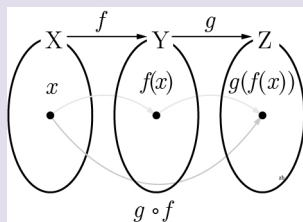
2) 제작자 : 교육지대(주)

3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

## 01 합성함수

두 함수  $f: X \rightarrow Y$ ,  $g: Y \rightarrow Z$ 에 대하여  
 $(g \circ f)(x) = g(f(x))$



■ 두 함수  $f(x) = 3x - 1$ ,  $g(x) = x^2 + 2$ 에 대하여 다음을 구하시오.

1.  $(g \circ f)(x)$

2.  $(f \circ f)(x)$

3.  $(g \circ g)(x)$

4.  $(f \circ g)(x)$

■ 두 함수  $f(x)$ ,  $g(x)$ 가 다음과 같을 때, 합성함수  $(g \circ f)(x)$ 를 구하여라.

5.  $f(x) = 3x$ ,  $g(x) = 2x - 3$

6.  $f(x) = 2x + 1$ ,  $g(x) = 3x - 2$

7.  $f(x) = x - 3$ ,  $g(x) = 2x^2 + 1$

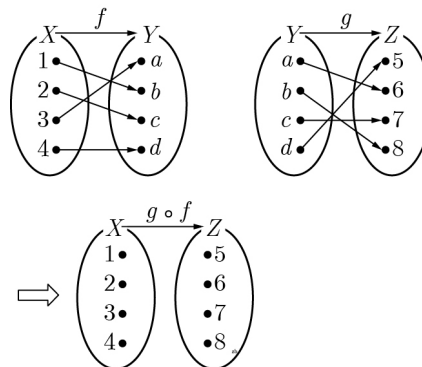
8.  $f(x) = 3x^2 - 1$ ,  $g(x) = 2x + 5$

9.  $f(x) = 2x + 1$ ,  $g(x) = 3x^2$

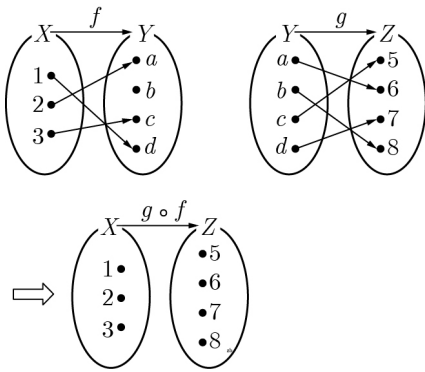
10.  $f(x) = -x + 3$ ,  $g(x) = x^2 - 4$

■ 두 함수  $f$ ,  $g$ 가 다음 그림과 같을 때, 합성함수  $g \circ f$ 를 그림으로 나타내어라.

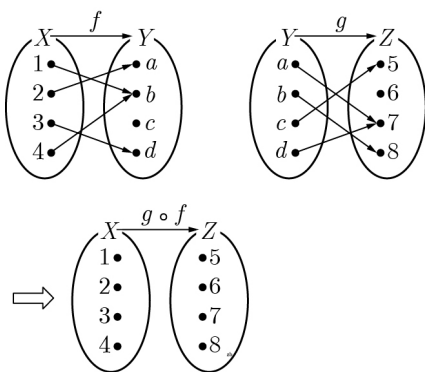
11.



12.



13.

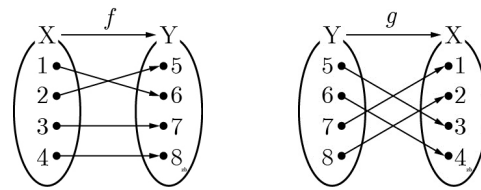


■ 두 함수  $f(x)$ ,  $g(x)$ 가 다음과 같을 때, 합성함수  $(g \circ f)(x)$ 를 구하여라.

14.  $f(x) = \frac{1}{2}x + 4$ ,  $g(x) = 2x - 3$

15.  $f(x) = 2x^2 - 1$ ,  $g(x) = 3x - 2$

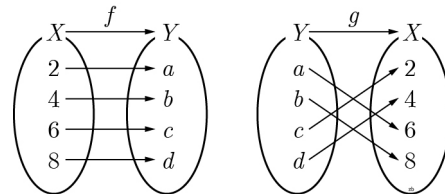
■ 두 함수  $f$ ,  $g$ 가 다음 그림과 같을 때, 다음을 구하시오.



16.  $(g \circ f)(2)$

17.  $(f \circ g)(5)$

■ 두 함수  $f$ ,  $g$ 가 그림과 같을 때, 다음 값을 구하여라.



18.  $(g \circ f)(4)$

19.  $(f \circ g)(c)$

20.  $(g \circ f)(6)$

21.  $(f \circ g)(b)$

■ 두 함수  $f(x) = -3x + 1$ ,  $g(x) = \frac{1}{2}x + 5$ 에 대하여 다음 값을 구하여라.

22.  $(g \circ f)(-1)$

23.  $(f \circ g)(2)$

24.  $(g \circ g)(0)$

25.  $(f \circ f)(3)$

■ 두 함수  $f(x) = 2x + 1$ ,  $g(x) = x^2$ 에 대하여 다음을 구하여라.

26.  $(g \circ f)(-2)$

27.  $(f \circ f)(5)$

28.  $(f \circ g)\left(\frac{1}{2}\right)$

29.  $(g \circ g)\left(-\frac{1}{2}\right)$

30.  $(g \circ f)(x)$

31.  $(f \circ f)(x)$

32.  $(f \circ g)(x)$

33.  $(g \circ g)(x)$

■ 세 함수  $f(x) = 2x - 3$ ,  $g(x) = 3x + 2$ ,  $h(x) = x^2 - 1$ 에 대하여 다음 값을 구하여라.

34.  $(f \circ f)(-1)$

35.  $(g \circ f)\left(-\frac{1}{2}\right)$

36.  $(f \circ g)(-2)$

37.  $(f \circ h)(2)$

38.  $(h \circ f)(2)$

39.  $(g \circ h)(-1)$

40.  $(h \circ g)\left(\frac{1}{3}\right)$

## 02 / 합성함수의 성질

세 함수  $f, g, h$ 에 대하여

(1)  $g \circ f \neq f \circ g$

(2)  $h \circ (g \circ f) = (h \circ g) \circ f$

(3)  $f: X \rightarrow X$ 일 때,  $f \circ I = I \circ f$  (단,  $I$ 는 항등함수)

■ 두 함수  $f(x) = 3x + 2$ ,  $g(x) = x^2 - 1$ 에 대하여 다음 물음에 답하여라.

41.  $(f \circ g)(x)$ 를 구하여라.

42.  $f \circ g = g \circ f$ 인지 확인하여라.

43.  $(g \circ f)(x)$ 를 구하여라.

■ 세 함수  $f(x) = x + 3$ ,  $g(x) = x^2 + 1$ ,  $h(x) = x - 2$ 에 대하여 다음 물음에 답하여라.

44.  $((h \circ g) \circ f)(x)$ 를 구하여라.

45.  $(h \circ g) \circ f = h \circ (g \circ f)$ 인지 확인하여라.

46.  $(h \circ (g \circ f))(x)$ 를 구하여라.

■ 다음 두 함수  $f, g$ 에 대하여  $f \circ g = g \circ f$ 가 성립할 때, 상수  $k$ 의 값을 구하여라.

47.  $f(x) = 2x$ ,  $g(x) = 3x + k$

48.  $f(x) = kx - 3$ ,  $g(x) = \frac{2}{3}x + 1$

49.  $f(x) = 3x + 1$ ,  $g(x) = -x + k$

50.  $f(x) = x + k$ ,  $g(x) = 3x + 10$

51.  $f(x) = 2x + k$ ,  $g(x) = -x + 3$

52.  $f(x) = 2x + 3$ ,  $g(x) = kx - 1$

53.  $f(x) = x + 1$ ,  $g(x) = kx + 2$

54.  $f(x) = 2x + 1$ ,  $g(x) = -x - k$

■ 양의 실수 전체에서 정의된 세 함수

$$f(x) = 3x - 1, \quad g(x) = x^2 + 2, \quad h(x) = -\frac{1}{2}x + 3$$

에 대하여 다음 값을 구하여라.

55.  $(g \circ f)(1) + (f \circ g)(1)$

56.  $(f \circ g \circ h)(2) - (g \circ h \circ f)(1)$

57. 세 함수  $f(x) = 2x^2 - 1$ ,  $g(x) = x + 3$ ,  
 $h(x) = \frac{1}{2}x + 1$ 에 대하여 다음 값을 구하여라.

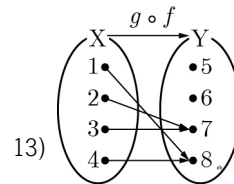
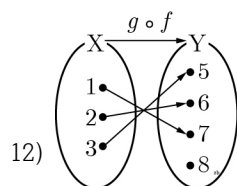
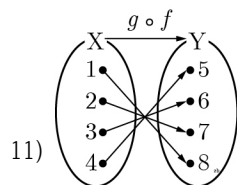
(1)  $(f \circ g)(3) - (g \circ h)(2)$

(2)  $(g \circ h \circ f)(1) - (g \circ f \circ h)(2)$



## 정답 및 해설

- 1)  $(g \circ f)(x) = 9x^2 - 6x + 3$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = (3x-1)^2 + 2 = 9x^2 - 6x + 3$
- 2)  $(f \circ f)(x) = 9x - 4$   
 $\Rightarrow (f \circ f)(x) = f(f(x)) = 3(3x-1) - 1 = 9x - 4$
- 3)  $(g \circ g)(x) = x^4 + 4x^2 + 6$   
 $\Rightarrow (g \circ g)(x) = g(g(x)) = (x^2+2)^2 + 2 = x^4 + 4x^2 + 6$
- 4)  $(f \circ g)(x) = 3x^2 + 5$   
 $\Rightarrow (f \circ g)(x) = f(g(x)) = 3(x^2+2) - 1 = 3x^2 + 5$
- 5)  $6x - 3$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = g(3x)$   
 $= 2(3x) - 3 = 6x - 3$
- 6)  $6x + 1$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = g(2x+1)$   
 $= 3(2x+1) - 2 = 6x + 1$
- 7)  $2x^2 - 12x + 19$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = g(x-3)$   
 $= 2(x-3)^2 + 1 = 2x^2 - 12x + 19$
- 8)  $6x^2 + 3$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = g(3x^2-1)$   
 $= 2(3x^2-1) + 5 = 6x^2 + 3$
- 9)  $12x^2 + 12x + 3$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = g(2x+1)$   
 $= 3(2x+1)^2 = 12x^2 + 12x + 3$
- 10)  $x^2 - 6x + 5$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = g(-x+3)$   
 $= (-x+3)^2 - 4 = x^2 - 6x + 5$



- 14)  $(g \circ f)(x) = x + 5$   
 $\Rightarrow (g \circ f)(x) = g(f(x)) = g\left(\frac{1}{2}x + 4\right)$   
 $= 2\left(\frac{1}{2}x + 4\right) - 3 = x + 5$
- 15)  $(g \circ f)(x) = 6x^2 - 5$   
 $\Rightarrow (g \circ f)(x) = g(f(x))$   
 $= g(2x^2 - 1)$   
 $= 3(2x^2 - 1) - 2 = 6x^2 - 5$
- 16) 3  
 $\Rightarrow (g \circ f)(2) = g(f(2)) = g(5) = 3$
- 17) 7  
 $\Rightarrow (f \circ g)(5) = f(g(5)) = f(3) = 7$
- 18) 8  
 $\Rightarrow (g \circ f)(4) = g(f(4)) = g(b) = 8$
- 19) a  
 $\Rightarrow (f \circ g)(c) = f(g(c)) = f(2) = a$
- 20) 2  
 $\Rightarrow (g \circ f)(6) = g(f(6)) = g(c) = 2$
- 21) d  
 $\Rightarrow (f \circ g)(b) = f(g(b)) = f(8) = d$
- 22) 7  
 $\Rightarrow f(-1) = -3 \cdot (-1) + 1 = 4$  이므로  
 $(g \circ f)(-1) = g(f(-1)) = g(4) = \frac{1}{2} \cdot 4 + 5 = 7$
- 23) -17  
 $\Rightarrow g(2) = \frac{1}{2} \cdot 2 + 5 = 6$  이므로  
 $(f \circ g)(2) = f(g(2)) = f(6) = -3 \cdot 6 + 1 = -17$
- 24)  $\frac{15}{2}$   
 $\Rightarrow g(0) = \frac{1}{2} \cdot 0 + 5 = 5$  이므로  
 $(g \circ g)(0) = g(g(0)) = g(5) = \frac{1}{2} \cdot 5 + 5 = \frac{15}{2}$
- 25) 25  
 $\Rightarrow f(3) = -3 \cdot 3 + 1 = -8$  이므로  
 $(f \circ f)(3) = f(f(3)) = f(-8) = -3 \cdot (-8) + 1 = 25$
- 26) 9

$$\Rightarrow f(-2) = 2 \cdot (-2) + 1 = -3 \text{ 이므로}$$

$$(g \circ f)(-2) = g(f(-2)) = g(-3) = (-3)^2 = 9$$

$$27) 23$$

$$\Rightarrow f(5) = 2 \cdot 5 + 1 = 11 \text{ 이므로}$$

$$(f \circ f)(5) = f(f(5)) = f(11) = 2 \cdot 11 + 1 = 23$$

$$28) \frac{3}{2}$$

$$\Rightarrow g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \text{ 이므로}$$

$$(f \circ g)\left(\frac{1}{2}\right) = f\left(g\left(\frac{1}{2}\right)\right) = f\left(\frac{1}{4}\right) = 2 \cdot \frac{1}{4} + 1 = \frac{3}{2}$$

$$29) \frac{1}{16}$$

$$\Rightarrow g\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$(g \circ g)\left(-\frac{1}{2}\right) = g\left(g\left(-\frac{1}{2}\right)\right) = g\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$30) 4x^2 + 4x + 1$$

$$\begin{aligned} \Rightarrow (g \circ f)(x) &= g(f(x)) \\ &= g(2x+1) = (2x+1)^2 = 4x^2 + 4x + 1 \end{aligned}$$

$$31) 4x+3$$

$$\begin{aligned} \Rightarrow (f \circ f)(x) &= f(f(x)) \\ &= f(2x+1) = 2(2x+1) + 1 = 4x+3 \end{aligned}$$

$$32) 2x^2 + 1$$

$$\Rightarrow (f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2 + 1$$

$$33) x^4$$

$$\Rightarrow (g \circ g)(x) = g(g(x)) = g(x^2) = (x^2)^2 = x^4$$

$$34) -13$$

$$\Rightarrow f(-1) = 2 \cdot (-1) - 3 = -5 \text{ 이므로}$$

$$\begin{aligned} (f \circ f)(-1) &= f(f(-1)) = f(-5) \\ &= 2 \cdot (-5) - 3 = -13 \end{aligned}$$

$$35) -10$$

$$\Rightarrow f\left(-\frac{1}{2}\right) = 2 \cdot \left(-\frac{1}{2}\right) - 3 = -4 \text{ 이므로}$$

$$(g \circ f)\left(-\frac{1}{2}\right) = g\left(f\left(-\frac{1}{2}\right)\right) = g(-4)$$

$$(g \circ f)\left(-\frac{1}{2}\right) = 3 \cdot (-4) + 2 = -10$$

$$36) -11$$

$$\Rightarrow g(-2) = 3 \cdot (-2) + 2 = -4 \text{ 이므로}$$

$$\begin{aligned} (f \circ g)(-2) &= f(g(-2)) = f(-4) \\ &= 2 \cdot (-4) - 3 = -11 \end{aligned}$$

$$37) 3$$

$$\Rightarrow h(2) = 2^2 - 1 = 3 \text{ 이므로}$$

$$(f \circ h)(2) = f(h(2)) = f(3) = 2 \cdot 3 - 3 = 3$$

$$38) 0$$

$$\Rightarrow f(2) = 2 \cdot 2 - 3 = 1 \text{ 이므로}$$

$$(h \circ f)(2) = h(f(2)) = h(1) = 1^2 - 1 = 0$$

$$39) 2$$

$$\Rightarrow h(-1) = (-1)^2 - 1 = 0 \text{ 이므로}$$

$$(g \circ h)(-1) = g(h(-1)) = g(0) = 3 \cdot 0 + 2 = 2$$

$$40) 8$$

$$\Rightarrow g\left(\frac{1}{3}\right) = 3 \cdot \left(\frac{1}{3}\right) + 2 = 3 \text{ 이므로}$$

$$(h \circ g)\left(\frac{1}{3}\right) = h\left(g\left(\frac{1}{3}\right)\right) = h(3) = 3^2 - 1 = 8$$

$$41) 3x^2 - 1$$

$$\begin{aligned} \Rightarrow (f \circ g)(x) &= f(g(x)) = f(x^2 - 1) \\ &= 3(x^2 - 1) + 2 = 3x^2 - 1 \end{aligned}$$

$$42) f \circ g \neq g \circ f$$

$$\Rightarrow 41, 42 \text{에서 } f \circ g \neq g \circ f \text{ 이다.}$$

$$43) 9x^2 + 12x + 3$$

$$\begin{aligned} \Rightarrow (g \circ f)(x) &= g(f(x)) = g(3x+2) \\ &= (3x+2)^2 - 1 = 9x^2 + 12x + 3 \end{aligned}$$

$$44) x^2 + 6x + 8$$

$$\begin{aligned} \Rightarrow (h \circ g)(x) &= h(g(x)) = h(x^2 + 1) \\ &= (x^2 + 1) - 2 = x^2 - 1 \\ \therefore ((h \circ g) \circ f)(x) &= (h \circ g)(f(x)) \\ &= (h \circ g)(x+3) \\ &= (x+3)^2 - 1 = x^2 + 6x + 8 \end{aligned}$$

$$45) (h \circ g) \circ f = h \circ (g \circ f)$$

$$\Rightarrow 44 \text{에서 } (h \circ g) \circ f = h \circ (g \circ f) = x^2 + 6x + 8$$

$$46) x^2 + 6x + 8$$

$$\begin{aligned} \Rightarrow (g \circ f)(x) &= g(f(x)) = g(x+3) \\ &= (x+3)^2 + 1 = x^2 + 6x + 10 \\ \therefore (h \circ (g \circ f))(x) &= h \circ ((g \circ f)(x)) \\ &= h(x^2 + 6x + 10) \\ &= (x^2 + 6x + 10) - 2 = x^2 + 6x + 8 \end{aligned}$$

$$47) 0$$

$$\begin{aligned} \Rightarrow (f \circ g)(x) &= f(g(x)) = f(3x+k) \\ &= 2(3x+k) = 6x+2k \\ (g \circ f)(x) &= g(f(x)) = g(2x) \\ &= 3(2x) + k = 6x+k \end{aligned}$$

$$f \circ g = g \circ f \text{ 이므로 } 2k = k \quad \therefore k = 0$$

$$48) 2$$

$$\Rightarrow (f \circ g)(x) = f(g(x)) = f\left(\frac{2}{3}x + 1\right)$$

$$\begin{aligned}
 &= k\left(\frac{2}{3}x+1\right)-3 = \frac{2}{3}kx+k-3 \\
 (g \circ f)(x) &= g(f(x)) = g(kx-3) \\
 &= \frac{2}{3}(kx-3)+1 = \frac{2}{3}kx-1 \\
 f \circ g &= g \circ f \text{이므로 } k-3=-1 \quad \therefore k=2
 \end{aligned}$$

$$\begin{aligned}
 49) \quad &-1 \\
 \Leftrightarrow (f \circ g)(x) &= f(g(x)) = f(-x+k) \\
 &= 3(-x+k)+1 = -3x+3k+1 \\
 (g \circ f)(x) &= g(f(x)) = g(3x+1) \\
 &= -(3x+1)+k \\
 &= -3x+k-1 \\
 f \circ g &= g \circ f \text{이므로 } 3k+1=k-1 \quad \therefore k=-1
 \end{aligned}$$

$$\begin{aligned}
 50) \quad &0 \\
 \Leftrightarrow (f \circ g)(x) &= (g(x)) = f(3x+10) = 3x+10+k \\
 (g \circ f)(x) &= g(f(x)) = g(x+k) = 3(x+k)+10 \\
 &= 3x+3k+10 \\
 f \circ g &= g \circ f \text{이므로 } 3x+10+k = 3x+3k+10 \\
 10+k &= 3k+10 \quad \therefore k=0
 \end{aligned}$$

$$\begin{aligned}
 51) \quad &-\frac{3}{2} \\
 \Leftrightarrow (f \circ g)(x) &= f(g(x)) = f(-x+3) \\
 &= 2(-x+3)+k = -2x+6+k \\
 (g \circ f)(x) &= g(f(x)) = g(2x+k) \\
 &= -(2x+k)+3 \\
 &= -2x-k+3 \\
 f \circ g &= g \circ f \text{이므로 } 6+k = -k+3 \\
 \therefore k &= -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 52) \quad &\frac{2}{3} \\
 \Leftrightarrow (f \circ g)(x) &= f(g(x)) = f(kx-1) \\
 &= 2(kx-1)+3 = 2kx+1 \\
 (g \circ f)(x) &= g(f(x)) = g(2x+3) = k(2x+3)-1 \\
 &= 2kx+3k-1 \\
 f \circ g &= g \circ f \text{이므로 } 2kx+1 = 2kx+3k-1 \\
 1 &= 3k-1 \quad \therefore k = \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 53) \quad &1 \\
 \Leftrightarrow (f \circ g)(x) &= f(g(x)) = f(kx+2) \\
 &= (kx+2)+1 = kx+3 \\
 (g \circ f)(x) &= g(f(x)) = g(x+1) \\
 &= k(x+1)+2 = kx+k+2 \\
 f \circ g &= g \circ f \text{이므로 } kx+3 = kx+k+2 \\
 3 &= k+2 \quad \therefore k=1
 \end{aligned}$$

$$\begin{aligned}
 54) \quad &2 \\
 \Leftrightarrow (f \circ g)(x) &= f(g(x)) = f(-x-k) \\
 &= 2(-x-k)+1 = -2x-2k+1 \\
 (g \circ f)(x) &= g(f(x)) = g(2x+1)
 \end{aligned}$$

$$\begin{aligned}
 &= -(2x+1)-k = -2x-1-k \\
 f \circ g &= g \circ f \text{이므로 } -2x-2k+1 = -2x-1-k \\
 -2k+1 &= -1-k \quad \therefore k=2
 \end{aligned}$$

$$\begin{aligned}
 55) \quad &14 \\
 \Leftrightarrow (g \circ f)(1) &+ (f \circ g)(1) = g(f(1)) + f(g(1)) \\
 &= g(2) + f(3) \\
 &= 6 + 8 = 14
 \end{aligned}$$

$$\begin{aligned}
 56) \quad &11 \\
 \Leftrightarrow (f \circ g \circ h)(2) &- (g \circ h \circ f)(1) \\
 &= (f \circ g)(h(2)) - (g \circ h)(f(1)) \\
 &= (f \circ g)(2) - (g \circ h)(2) \\
 &= f(g(2)) - g(h(2)) \\
 &= f(6) - g(2) \\
 &= 17 - 6 = 11
 \end{aligned}$$

$$\begin{aligned}
 57) \quad (1) \quad &66 \quad (2) \quad -\frac{11}{2} \\
 \Leftrightarrow (1) \quad (f \circ g)(3) &- (g \circ h)(2) \\
 &= f(g(3)) - g(h(2)) \\
 &= f(3+3) - g\left(\frac{1}{2} \cdot 2+1\right) = f(6) - g(2) \\
 &= (2 \cdot 6^2 - 1) - (2+3) = 71 - 5 = 66 \\
 (2) \quad (g \circ h \circ f)(1) &- (g \circ f \circ h)(2) \\
 &= g(h(f(1))) - (g(f(h(2)))) \\
 &= g(h(2 \cdot 1^2 - 1)) - g\left(f\left(\frac{1}{2} \cdot 2+1\right)\right) \\
 &= g(h(1)) - g(f(2)) \\
 &= g\left(\frac{1}{2} \cdot 1+1\right) - g(2 \cdot 2^2 - 1) = g\left(\frac{3}{2}\right) - g(7) \\
 &= \left(\frac{3}{2}+3\right) - (7+3) = -\frac{11}{2}
 \end{aligned}$$