계산시 참고사항

1. 곱셈공식

: 제곱근을 문자로 생각하고 다항식의 곱셈공식과 같은 방법으로 계산한다.

(1) $(a+b)^2=a^2+2ab+b^2$, $(a-b)^2=a^2-2ab+b^2$

(2) $(a+b)(a-b)=a^2-b^2$

(3) $(x+a)(x+b)=x^2+(a+b)x+ab$

(4) $(ax+b)(cx+d)=acx^2+(ad+bc)x+bd$

2. 곱셈공식을 이용한 분모의 유리화

: 분모가 두 개의 항으로 되어 있는 무리수일 때, $(a+b)(a-b)=a^2-b^2$을 이용하여 분모를 유리화한다.

$$\frac{c}{\sqrt{a}+\sqrt{b}}=\frac{c(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}=\frac{c(\sqrt{a}-\sqrt{b})}{a-b}$$

3. 식의 값 구하기

(1) 주어진 조건을 식에 직접 대입하여 식의 값을 구한다.

(2) 주어진 조건을 변형하여 식의 값을 구한다.

곱셈공식을 이용한 분모의 유리화

◪ 다음 분수를 유리화하여라.

1. $\frac{1}{\sqrt{2}+1}$

2. $\frac{3}{\sqrt{5}-\sqrt{2}}$

3. $\frac{2-\sqrt{3}}{2+\sqrt{3}}$

4. $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

5. $\frac{2}{\sqrt{5}+1}$

6. $\frac{\sqrt{2}}{2-\sqrt{3}}$

7. $\frac{1}{3-2\sqrt{2}}$

8. $\frac{\sqrt{3}}{2\sqrt{3}-3}$

9. $\frac{2\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

10. $\frac{4\sqrt{3}}{3-\sqrt{3}}$

11. $\frac{\sqrt{5}+2}{\sqrt{5}-2}$

12. $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

13. $\frac{3-\sqrt{5}}{3+\sqrt{5}}$

14. $\frac{2(\sqrt{5}+\sqrt{3})}{\sqrt{5}-\sqrt{3}}$

15. $\frac{2-\sqrt{2}}{3+2\sqrt{2}}$

16. $\frac{\sqrt{27}-4}{\sqrt{3}-2}$

17. $\frac{1}{1+\sqrt{2}}$

18. $\frac{1}{2-\sqrt{3}}$

19. $\frac{1}{\sqrt{5}+2}$

20. $\frac{4}{1+\sqrt{5}}$

21. $\frac{2}{\sqrt{3}-\sqrt{5}}$

22. $\frac{2}{\sqrt{8}-\sqrt{6}}$

23. $\frac{1}{3+2\sqrt{2}}$

24. $\frac{1}{5-2\sqrt{6}}$

25. $\frac{1}{2\sqrt{2}+\sqrt{7}}$

26. $\frac{1}{3\sqrt{3}-2\sqrt{7}}$

27. $\frac{3}{3-2\sqrt{3}}$

28. $\frac{6}{2\sqrt{6}-3\sqrt{2}}$

29. $\frac{2}{5\sqrt{2}+4\sqrt{3}}$

30. $\frac{12}{2\sqrt{3}-3\sqrt{2}}$

31. $\frac{2-\sqrt{5}}{2+\sqrt{5}}$

32. $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

33. $\frac{4}{2\sqrt{5}+3}$

34. $\frac{2}{3\sqrt{3}-2\sqrt{7}}$

35. $\frac{\sqrt{10}+3}{\sqrt{10}-3}$

36. $\frac{3}{3\sqrt{2}+2\sqrt{6}}$

37. $\frac{\sqrt{7}}{3-\sqrt{7}}$

38. $\frac{1}{\sqrt{6}+\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

39. $\frac{2}{\sqrt{3}+1}-\frac{4}{\sqrt{3}-1}$

곱셈공식을 이용한 식의 계산

◪ 곱셈 공식을 이용하여 다음 식을 계산하여라.

40. $(\sqrt{3}+\sqrt{5})^2$

41. $(2\sqrt{3}+1)^2$

42. $(\sqrt{5}-4)^2$

43. $(\sqrt{3}-\sqrt{6})^2$

44. $(2\sqrt{7}-\sqrt{2})^2$

45. $(\sqrt{7}+\sqrt{11})(\sqrt{7}-\sqrt{11})$

46. $(3\sqrt{2}+4)(3\sqrt{2}-4)$

47. $(-5+\sqrt{6})(-5-\sqrt{6})$

48. $(\sqrt{5}-6)(\sqrt{5}+2)$

49. $(\sqrt{3}+2)(\sqrt{3}+5)$

50. $(\sqrt{2}+3\sqrt{5})(\sqrt{2}-\sqrt{5})$

51. $(2\sqrt{7}+1)(\sqrt{7}-3)$

52. $(3\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

53. $(2+3\sqrt{5})(3-\sqrt{5})$

54. $(\sqrt{6}+4)(2\sqrt{6}-3)$

55. $\frac{1}{\sqrt{27}}\times\sqrt{15}-\frac{(2+\sqrt{5})^2}{3}$

56. $(\sqrt{6}+2)^2-(3+2\sqrt{2})(3-2\sqrt{2})$

57. $(\sqrt{2}+\sqrt{3})^2-(\sqrt{3}+1)(\sqrt{3}-1)$

식의 값 구하기

◪ 다음 식의 값을 구하여라.

58. $x=\sqrt{2}+3$일 때, x^2-6x의 값

59. $x=3-2\sqrt{2}$일 때, $x^2-6x+11$의 값

60. $x=\frac{1}{3-2\sqrt{2}}$일 때, x^2-6x-2의 값

61. $x=\frac{1}{2-\sqrt{5}}$일 때, x^2+4x-5의 값

62. $x=2+\sqrt{3}$일 때, x^2-4x의 값

63. $x=2\sqrt{5}-3$일 때, x^2+6x+1의 값

64. $x=\frac{1}{\sqrt{2}+1}$일 때, x^2-x+1의 값

65. $x=\frac{1}{\sqrt{5}+2}$일 때, x^2-2x+3의 값

66. $x=2-\sqrt{3}$일 때, $x^2-4x+10$의 값

67. $x=\sqrt{7}-4$일 때, x^2+8x-5의 값

68. $x=\sqrt{6}-2$일 때, x^2+4x-3의 값

69. $x=2\sqrt{5}-3$일 때, x^2+6x+3의 값

70. $x=5-2\sqrt{2}$일 때, $x^2-10x+10$의 값

71. $x=4+\sqrt{3}$일 때, x^2-8x-7의 값

72. $x=\frac{1}{2-\sqrt{3}}$일 때, x^2+2x-5의 값

73. $x+y=\sqrt{5}$, $xy=1$일 때, x^2+y^2의 값

74. $a-b=3\sqrt{2}$, $a^2+b^2=8$일 때, ab의 값

75. $x-\frac{1}{x}=\sqrt{3}$일 때, $x^2+\frac{1}{x^2}$의 값

76. $x+\frac{1}{x}=\sqrt{6}$일 때, $\left(x-\frac{1}{x}\right)^2$의 값

◪ $x=\frac{1}{3-2\sqrt{2}}$, $y=\frac{1}{3+2\sqrt{2}}$ 일 때, 다음 식의 값을 구하여라.

77. $x+y$

78. xy

79. x^2+y^2

80. $\frac{y}{x}+\frac{x}{y}$

◪ $x=\frac{1}{\sqrt{2}-\sqrt{3}}$, $y=\frac{1}{\sqrt{2}+\sqrt{3}}$ 일 때, 다음 식의 값을 구하여라.

81. $x+y$

82. xy

83. x^2+y^2

◪ $x=2+\sqrt{5}$, $y=2-\sqrt{5}$ 일 때, 다음 식의 값을 구하여라.

84. $x+y$

85. xy

86. x^2+y^2

87. $\frac{1}{x}+\frac{1}{y}$

88. $\frac{y}{x}+\frac{x}{y}$

89. x^2+xy+y^2

90. x^2+y^2+5xy

◪ $x=\sqrt{5}$, $y=\sqrt{7}$ 일 때, 다음 식의 값을 구하여라.

91. $(x+y)(x-y)$

92. $x(y+1)-y(x+1)$

93. $(2x+y)(2x-y)$

94. $(x+y)^2-(x-y)^2$

95. $(2x+y)(2x-y)-2y^2$

96. $(x+y)^2-3xy$

97. $(x+1)(y+1)(x-1)(y-1)$

98. $x(x+y)-x(x-y)-xy$

99. $(x+4y)^2-8xy-(3x+y)(3x-y)$

◪ $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$일 때, 다음 식의 값을 구하여라.

100. $x+y$

101. xy

102. $x^2-10x+10$

◪ 다음 유리수가 되도록 유리수 a의 값을 정하여라.

103. $-3+(a+2)\sqrt{3}$

104. $2\sqrt{3}+6-a\sqrt{3}+3a$

105. $6\sqrt{5}-4a-2+2a\sqrt{5}$

106. $\frac{a+\sqrt{2}}{3\sqrt{2}+1}$

107. $\frac{4-a\sqrt{2}}{5-2\sqrt{2}}$

108. $(4+a\sqrt{5})(2-\sqrt{5})$

109. $(a\sqrt{5}+3)(2\sqrt{5}-1)$

110. $\sqrt{3}(\sqrt{3}+a)-\sqrt{12}(2+\sqrt{3})$

111. $(4-3\sqrt{5})(a+2\sqrt{5})$

112. $(3a+2\sqrt{11})(9-3\sqrt{11})$

113. $(\sqrt{3}+a)(2\sqrt{3}-6)$

114. $(6+2\sqrt{3})(a-\sqrt{3})$

115. $(\sqrt{2}-4)^2-a(7-4\sqrt{2})$

정답 및 해설

1) $\sqrt{2}-1$

2) $\sqrt{5}+\sqrt{2}$

⇨ $\frac{3}{\sqrt{5}-\sqrt{2}}=\frac{3(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}=\sqrt{5}+\sqrt{2}$

3) $7-4\sqrt{3}$

⇨ $\frac{2-\sqrt{3}}{2+\sqrt{3}}=\frac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})}=7-4\sqrt{3}$

4) $4+\sqrt{15}$

⇨ $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$
$=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$

5) $\frac{\sqrt{5}-1}{2}$

⇨ $\frac{2}{\sqrt{5}+1}=\frac{2(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}=\frac{2(\sqrt{5}-1)}{4}=\frac{\sqrt{5}-1}{2}$

6) $2\sqrt{2}+\sqrt{6}$

⇨ $\frac{\sqrt{2}}{2-\sqrt{3}}=\frac{\sqrt{2}(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}=2\sqrt{2}+\sqrt{6}$

7) $3+2\sqrt{2}$

⇨ $\frac{1}{3-2\sqrt{2}}=\frac{3+2\sqrt{2}}{(3-2\sqrt{2})(3+2\sqrt{2})}=3+2\sqrt{2}$

8) $2+\sqrt{3}$

⇨ $\frac{\sqrt{3}}{2\sqrt{3}-3}=\frac{\sqrt{3}(2\sqrt{3}+3)}{(2\sqrt{3}-3)(2\sqrt{3}+3)}=\frac{6+3\sqrt{3}}{3}=2+\sqrt{3}$

9) $\sqrt{3}+1$

⇨ $\frac{2\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\frac{2\sqrt{2}(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$
$=\frac{4\sqrt{3}+4}{4}=\sqrt{3}+1$

10) $2\sqrt{3}+2$

⇨ $\frac{4\sqrt{3}}{3-\sqrt{3}}=\frac{4\sqrt{3}(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})}=\frac{12\sqrt{3}+12}{6}=2\sqrt{3}+2$

11) $9+4\sqrt{5}$

⇨ $\frac{\sqrt{5}+2}{\sqrt{5}-2}=\frac{(\sqrt{5}-2)^2}{(\sqrt{5}-2)(\sqrt{5}+2)}=5+4\sqrt{5}+4=9+4\sqrt{5}$

12) $3-2\sqrt{2}$

⇨ $\frac{\sqrt{2}-1}{\sqrt{2}+1}=\frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}$
$=2-2\sqrt{2}+1=3-2\sqrt{2}$

13) $\frac{7-3\sqrt{5}}{2}$

⇨ $\frac{3-\sqrt{5}}{3+\sqrt{5}}=\frac{(3-\sqrt{5})^2}{(3+\sqrt{5})(3-\sqrt{5})}=\frac{9-6\sqrt{5}+5}{4}$
$=\frac{14-6\sqrt{5}}{4}=\frac{7-3\sqrt{5}}{2}$

14) $8+2\sqrt{15}$

⇨ $\frac{2(\sqrt{5}+\sqrt{3})}{\sqrt{5}-\sqrt{3}}=\frac{2(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$
$=\frac{2(5+2\sqrt{15}+3)}{2}=8+2\sqrt{15}$

15) $10-7\sqrt{2}$

⇨ $\frac{2-\sqrt{2}}{3+2\sqrt{2}}=\frac{(2-\sqrt{2})(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})}$
$=6-4\sqrt{2}-3\sqrt{2}+4=10-7\sqrt{2}$

16) $-1-2\sqrt{3}$

⇨ $\frac{\sqrt{27}-4}{\sqrt{3}-2}=\frac{(3\sqrt{3}-4)(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)}$
$=\frac{9+6\sqrt{3}-4\sqrt{3}-8}{-1}=-1-2\sqrt{3}$

17) $-1+\sqrt{2}$

⇨ $\frac{1}{1+\sqrt{2}}=\frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})}=-1+\sqrt{2}$

18) $2+\sqrt{3}$

⇨ $\frac{1}{2-\sqrt{3}}=\frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}=2+\sqrt{3}$

19) $\sqrt{5}-2$

⇨ $\frac{1}{\sqrt{5}+2}=\frac{-\sqrt{5}+2}{(\sqrt{5}+2)(-\sqrt{5}+2)}=\sqrt{5}-2$

20) $-1+\sqrt{5}$

⇨ $\frac{4}{1+\sqrt{5}}=\frac{4(1-\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})}=-1+\sqrt{5}$

21) $-\sqrt{3}-\sqrt{5}$

⇨ $\frac{2}{\sqrt{3}-\sqrt{5}}=\frac{2(\sqrt{3}+\sqrt{5})}{(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})}=-\sqrt{3}-\sqrt{5}$

22) $2\sqrt{2}+\sqrt{6}$

⇨ $\frac{2}{\sqrt{8}-\sqrt{6}}$
$=\frac{2(\sqrt{8}+\sqrt{6})}{(\sqrt{8}-\sqrt{6})(\sqrt{8}+\sqrt{6})}=\sqrt{8}+\sqrt{6}=2\sqrt{2}+\sqrt{6}$

23) $3-2\sqrt{2}$

⇨ $\frac{1}{3+2\sqrt{2}}=\frac{3-2\sqrt{2}}{(3+2\sqrt{2})(3-2\sqrt{2})}=3-2\sqrt{2}$

24) $5+2\sqrt{6}$

⇨ $\frac{1}{5-2\sqrt{6}}=\frac{5+2\sqrt{6}}{(5-2\sqrt{6})(5+2\sqrt{6})}=5+2\sqrt{6}$

25) $2\sqrt{2}-\sqrt{7}$

⇨ $\frac{1}{2\sqrt{2}+\sqrt{7}}=\frac{2\sqrt{2}-\sqrt{7}}{(2\sqrt{2}+\sqrt{7})(2\sqrt{2}-\sqrt{7})}$
$=2\sqrt{2}-\sqrt{7}$

26) $-3\sqrt{3}-2\sqrt{7}$

⇨ $\frac{1}{3\sqrt{3}-2\sqrt{7}}=\frac{3\sqrt{3}+2\sqrt{7}}{(3\sqrt{3}-2\sqrt{7})(3\sqrt{3}+2\sqrt{7})}$
$=-3\sqrt{3}-2\sqrt{7}$

27) $-3-2\sqrt{3}$

⇨ $\frac{3}{3-2\sqrt{3}}=\frac{3(3+2\sqrt{3})}{(3-2\sqrt{3})(3+2\sqrt{3})}$
$=\frac{3(3+2\sqrt{3})}{-3}=-3-2\sqrt{3}$

28) $2\sqrt{6}+3\sqrt{2}$

⇨ $\frac{6}{2\sqrt{6}-3\sqrt{2}}=\frac{6(2\sqrt{6}+3\sqrt{2})}{(2\sqrt{6}-3\sqrt{2})(2\sqrt{6}+3\sqrt{2})}$
$=\frac{6(2\sqrt{6}+3\sqrt{2})}{24-18}=2\sqrt{6}+3\sqrt{2}$

29) $5\sqrt{2}-4\sqrt{3}$

⇨ $\frac{2}{5\sqrt{2}+4\sqrt{3}}=\frac{2(5\sqrt{2}-4\sqrt{3})}{(5\sqrt{2}+4\sqrt{3})(5\sqrt{2}-4\sqrt{3})}$
$=5\sqrt{2}-4\sqrt{3}$

30) $-4\sqrt{3}-6\sqrt{2}$

⇨ $\frac{12}{2\sqrt{3}-3\sqrt{2}}=\frac{12(2\sqrt{3}+3\sqrt{2})}{(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+3\sqrt{2})}$
$=-2(2\sqrt{3}+3\sqrt{2})=-4\sqrt{3}-6\sqrt{2}$

31) $-9+4\sqrt{5}$

⇨ $\frac{2-\sqrt{5}}{2+\sqrt{5}}=\frac{(2-\sqrt{5})^2}{(2+\sqrt{5})(2-\sqrt{5})}=-(9-4\sqrt{5})$
$=-9+4\sqrt{5}$

32) $\frac{5-\sqrt{21}}{2}$

⇨ $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}=\frac{(\sqrt{7}-\sqrt{3})^2}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})}=\frac{10-2\sqrt{21}}{4}$
$=\frac{5-\sqrt{21}}{2}$

33) $\frac{8\sqrt{5}-12}{11}$

⇨ $\frac{4}{2\sqrt{5}+3}=\frac{4(2\sqrt{5}-3)}{(2\sqrt{5}+3)(2\sqrt{5}-3)}=\frac{8\sqrt{5}-12}{11}$

34) $-6\sqrt{3}-4\sqrt{7}$

⇨ $\frac{2}{3\sqrt{3}-2\sqrt{7}}=\frac{2(3\sqrt{3}+2\sqrt{7})}{(3\sqrt{3}-2\sqrt{7})(3\sqrt{3}+2\sqrt{7})}$
$=-6\sqrt{3}-4\sqrt{7}$

35) $19+6\sqrt{10}$

36) $\frac{-3\sqrt{2}+2\sqrt{6}}{2}$

⇨ $\frac{3}{3\sqrt{2}+2\sqrt{6}}=\frac{3(3\sqrt{2}-2\sqrt{6})}{(3\sqrt{2}+2\sqrt{6})(3\sqrt{2}-2\sqrt{6})}$
$=\frac{-3\sqrt{2}+2\sqrt{6}}{2}$

37) $\frac{3\sqrt{7}+7}{2}$

38) $-\sqrt{2}$

39) $-3-\sqrt{3}$

40) $8+2\sqrt{15}$

⇨ $(\sqrt{3}+\sqrt{5})^2=3+2\sqrt{15}+5=8+2\sqrt{15}$

41) $13+4\sqrt{3}$

⇨ $(2\sqrt{3}+1)^2=12+4\sqrt{3}+1=13+4\sqrt{3}$

42) $21-8\sqrt{5}$

⇨ $(\sqrt{5}-4)^2=5-8\sqrt{5}+16=21-8\sqrt{5}$

43) $9-6\sqrt{2}$

⇨ $(\sqrt{3}-\sqrt{6})^2=3-6\sqrt{2}+6=9-6\sqrt{2}$

44) $30-4\sqrt{14}$

⇨ $(2\sqrt{7}-\sqrt{2})^2=28-4\sqrt{14}+2=30-4\sqrt{14}$

45) -4

⇨ $(\sqrt{7}+\sqrt{11})(\sqrt{7}-\sqrt{11})=7-11=-4$

46) 2

⇨ $(3\sqrt{2}+4)(3\sqrt{2}-4)=18-16=2$

47) 19

⇨ $(-5+\sqrt{6})(-5-\sqrt{6})=25-6=19$

48) $-7-4\sqrt{5}$

⇨ $(\sqrt{5}-6)(\sqrt{5}+2)=5+2\sqrt{5}-6\sqrt{5}-12=-7-4\sqrt{5}$

49) $13+7\sqrt{3}$

⇨ $(\sqrt{3}+2)(\sqrt{3}+5)=3+5\sqrt{3}+2\sqrt{3}+10=13+7\sqrt{3}$

50) $-13+2\sqrt{10}$

⇨ $(\sqrt{2}+3\sqrt{5})(\sqrt{2}-\sqrt{5})$
$=2-\sqrt{10}+3\sqrt{10}-15=-13+2\sqrt{10}$

51) $11-5\sqrt{7}$

⇨ $(2\sqrt{7}+1)(\sqrt{7}-3)=14-6\sqrt{7}+\sqrt{7}-3=11-5\sqrt{7}$

52) $7+2\sqrt{6}$

53) $-9+7\sqrt{5}$

⇨ $(2+3\sqrt{5})(3-\sqrt{5})=6-2\sqrt{5}+9\sqrt{5}-15=-9+7\sqrt{5}$

54) $5\sqrt{6}$

⇨ $(\sqrt{6}+4)(2\sqrt{6}-3)=12-3\sqrt{6}+8\sqrt{6}-12=5\sqrt{6}$

55) $-3-\sqrt{5}$

⇨ (주어진 식)

$=\frac{1}{3\sqrt{3}}\times\sqrt{15}-\frac{9+4\sqrt{5}}{3}=\frac{\sqrt{5}}{3}-\frac{9+4\sqrt{5}}{3}$

$=\frac{-9-3\sqrt{5}}{3}=-3-\sqrt{5}$

56) $9+4\sqrt{6}$

⇨ (주어진 식)

$=6+4\sqrt{6}+4-(9-8)=10+4\sqrt{6}-1=9+4\sqrt{6}$

57) $3+2\sqrt{6}$

58) -7

⇨ $x=\sqrt{2}+3$에서 $x-3=\sqrt{2}$이므로 양변을 각각 제곱하면
$(x-3)^2=(\sqrt{2})^2,\ x^2-6x+9=2$
$\therefore\ x^2-6x=-7$

59) 10

⇨ $x=3-2\sqrt{2}$에서 $x-3=-2\sqrt{2}$이므로 양변을 각각 제곱하면
$(x-3)^2=(-2\sqrt{2})^2,\ x^2-6x+9=8$
$\therefore\ x^2-6x=-1\quad\therefore\ x^2-6x+11=10$

60) -3

⇨ $x=\frac{1}{3-2\sqrt{2}}=\frac{3+2\sqrt{2}}{(3-2\sqrt{2})(3+2\sqrt{2})}=3+2\sqrt{2}$에서
$x-3=2\sqrt{2}$이므로 양변을 각각 제곱하면
$(x-3)^2=(2\sqrt{2})^2,\ x^2-6x+9=8$
$\therefore\ x^2-6x=-1\quad\therefore\ x^2-6x-2=-3$

61) -4

⇨ $x=\frac{1}{2-\sqrt{5}}=\frac{2+\sqrt{5}}{(2-\sqrt{5})(2+\sqrt{5})}=-2-\sqrt{5}$에서
$x+2=-\sqrt{5}$이므로 양변을 각각 제곱하면
$(x+2)^2=(-\sqrt{5})^2$
$x^2+4x+4=5,\ x^2+4x=1$
$\therefore\ x^2+4x-5=-4$

62) -1

⇨ $x=2+\sqrt{3}$에서 $x-2=\sqrt{3}$이므로 양변을 각각 제곱하면 $(x-2)^2=(\sqrt{3})^2,\ x^2-4x+4=3$
$\therefore\ x^2-4x=-1$

63) 12

⇨ $x=2\sqrt{5}-3$에서 $x+3=2\sqrt{5}$이므로 양변을 각각 제곱하면 $(x+3)^2=(2\sqrt{5})^2,\ x^2+6x+9=20$
$\therefore\ x^2+6x=11\quad\therefore\ x^2+6x+1=12$

64) $5-3\sqrt{2}$

⇨ $x=\frac{1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}=\sqrt{2}-1$
$\therefore\ x^2-x+1=(\sqrt{2}-1)^2-(\sqrt{2}-1)+1$
$=3-2\sqrt{2}-\sqrt{2}+1+1=5-3\sqrt{3}$

65) $16-6\sqrt{5}$

⇨ $x=\frac{1}{\sqrt{5}+2}=\frac{\sqrt{5}-2}{(\sqrt{5}+2)(\sqrt{5}-2)}=\sqrt{5}-2$
$\therefore\ x^2-2x+3=(\sqrt{5}-2)^2-2(\sqrt{5}-2)+3$
$=9-4\sqrt{5}-2\sqrt{5}+4+3=16-6\sqrt{5}$

66) 9

⇨ $x=2-\sqrt{3}$에서 $x-2=-\sqrt{3}$이므로
$(x-2)^2=(-\sqrt{3})^2,\ x^2-4x+4=3,$
$x^2-4x=-1$
$\therefore x^2-4x+10=-1+10=9$

67) -14

⇨ $x=\sqrt{7}-4$에서 $x+4=\sqrt{7}$이므로
$(x+4)^2=(\sqrt{7})^2,\ x^2+8x+16=7,$
$x^2+8x=-9$
$\therefore x^2+8x-5=-9-5=-14$

68) -1

⇨ $x=\sqrt{6}-2$에서 $x+2=\sqrt{6}$이므로
$(x+2)^2=(\sqrt{6})^2,\ x^2+4x+4=6,\ x^2+4x=2$
$\therefore x^2+4x-3=2-3=-1$

69) 14

⇨ $x=2\sqrt{5}-3$에서 $x+3=2\sqrt{5}$이므로
$(x+3)^2=(2\sqrt{5})^2,\ x^2+6x+9=20,$
$x^2+6x=11$
$\therefore x^2+6x+3=11+3=14$

70) -7

⇨ $x=5-2\sqrt{2}$에서 $x-5=-2\sqrt{2}$이므로

$(x-5)^2=(-2\sqrt{2})^2$, $x^2-10x+25=8$,

$x^2-10x=-17$

$\therefore x^2-10x+10=-17+10=-7$

71) -20

⇨ $x=4+\sqrt{3}$에서 $x-4=\sqrt{3}$이므로

$(x-4)^2=(\sqrt{3})^2$, $x^2-8x+16=3$

$x^2-8x=-13$

$\therefore x^2-8x-7=-13-7=-20$

72) $6+6\sqrt{3}$

⇨ $x=\dfrac{1}{2-\sqrt{3}}=\dfrac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}=2+\sqrt{3}$

$\therefore x^2+2x-5=(2+\sqrt{3})^2+2(2+\sqrt{3})-5$

$=7+4\sqrt{3}+4+2\sqrt{3}-5=6+6\sqrt{3}$

73) 3

⇨ $x^2+y^2=(x+y)^2-2xy=(\sqrt{5})^2-2\times1=3$

74) -5

⇨ $(a-b)^2=a^2-2ab+b^2$에서

$(3\sqrt{2})^2=8-2ab$

$18=8-2ab$ $\therefore ab=-5$

75) 5

⇨ $x^2+\dfrac{1}{x^2}=\left(x-\dfrac{1}{x}\right)^2+2=(\sqrt{3})^2+2=5$

76) 2

⇨ $\left(x-\dfrac{1}{x}\right)^2=\left(x+\dfrac{1}{x}\right)^2-4=(\sqrt{6})^2-4=2$

77) 6

⇨ $x=\dfrac{1}{3-2\sqrt{2}}=\dfrac{3+2\sqrt{2}}{(3-2\sqrt{2})(3+2\sqrt{2})}=3+2\sqrt{2}$

$y=\dfrac{1}{3+2\sqrt{2}}=\dfrac{3-2\sqrt{2}}{(3+2\sqrt{2})(3-2\sqrt{2})}=3-2\sqrt{2}$

$\therefore x+y=(3+2\sqrt{2})+(3-2\sqrt{2})=6$

78) 1

⇨ $xy=(3+2\sqrt{2})(3-2\sqrt{2})=9-8=1$

79) 34

⇨ $x^2+y^2=(x+y)^2-2xy=6^2-2\times1=34$

80) 34

⇨ $\dfrac{y}{x}+\dfrac{x}{y}=\dfrac{x^2+y^2}{xy}=34$

81) $-2\sqrt{2}$

⇨ $x=\dfrac{1}{\sqrt{2}-\sqrt{3}}=\dfrac{\sqrt{2}+\sqrt{3}}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})}$

$=-\sqrt{2}-\sqrt{3}$

$y=\dfrac{1}{\sqrt{2}+\sqrt{3}}=\dfrac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}=-\sqrt{2}+\sqrt{3}$

$\therefore x+y=(-\sqrt{2}-\sqrt{3})+(-\sqrt{2}+\sqrt{3})=-2\sqrt{2}$

82) -1

⇨ $x=-\sqrt{2}-\sqrt{3}$, $y=-\sqrt{2}+\sqrt{3}$이므로

$\therefore xy=(-\sqrt{2}-\sqrt{3})(-\sqrt{2}+\sqrt{3})=2-3=-1$

83) 10

⇨ $x=-\sqrt{2}-\sqrt{3}$, $y=-\sqrt{2}+\sqrt{3}$이므로

$\therefore x^2+y^2=(x+y)^2-2xy=(-2\sqrt{2})^2-2\times(-1)$

$=8+2=10$

84) 4

⇨ $x+y=(2+\sqrt{5})+(2-\sqrt{5})=4$

85) -1

⇨ $xy=(2+\sqrt{5})(2-\sqrt{5})=-1$

86) 18

⇨ $x^2+y^2=(x+y)^2-2xy=4^2-2\times(-1)=18$

87) -4

⇨ $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{x+y}{xy}=\dfrac{4}{-1}=-4$

88) -18

⇨ $\dfrac{y}{x}+\dfrac{x}{y}=\dfrac{x^2+y^2}{xy}=\dfrac{18}{-1}=-18$

89) 17

⇨ $x^2+xy+y^2=x^2+y^2+xy=18-1=17$

90) 13

⇨ $x^2+y^2+5xy=(x+y)^2+3xy=4^2+3\times(-1)=13$

91) -2

⇨ $(x+y)(x-y)=x^2-y^2=(\sqrt{5})^2-(\sqrt{7})^2$

$=5-7=-2$

92) $\sqrt{5}-\sqrt{7}$

⇨ $x(y+1)-y(x+1)=xy+x-xy-y=x-y=\sqrt{5}-\sqrt{7}$

93) 13

⇨ $(2x+y)(2x-y)=4x^2-y^2=4(\sqrt{5})^2-(\sqrt{7})^2=13$

94) $4\sqrt{35}$

⇨ $(x+y)^2-(x-y)^2=x^2+2xy+y^2-(x^2-2xy+y^2)$

$=4xy=4\sqrt{5}\sqrt{7}=4\sqrt{35}$

95) -1

⇨ $(2x+y)(2x-y)-2y^2=4x^2-y^2-2y^2=4x^2-3y^2$
$=4\times(\sqrt{5})^2-3\times(\sqrt{7})^2=20-21=-1$

96) $12-\sqrt{35}$

⇨ $(x+y)^2-3xy=x^2+y^2+2xy-3xy=x^2+y^2-xy$
$=(\sqrt{5})^2+(\sqrt{7})^2-\sqrt{5}\sqrt{7}=5+7-\sqrt{35}=12-\sqrt{35}$

97) 24

⇨ $(x+1)(y+1)(x-1)(y-1)$
$=(x+1)(x-1)(y+1)(y-1)=(x^2-1)(y^2-1)$
$=\{(\sqrt{5})^2-1\}\{(\sqrt{7})^2-1\}=4\times6=24$

98) $\sqrt{35}$

⇨ $x(x+y)-x(x-y)-xy=x^2+xy-x^2+xy-xy$
$=xy=\sqrt{5}\sqrt{7}=\sqrt{35}$

99) 79

⇨ $(x+4y)^2-8xy-(3x+y)(3x-y)$
$=x^2+8xy+16y^2-8xy-(9x^2-y^2)$
$=-8x^2+17y^2$
$=-8\times(\sqrt{5})^2+17\times(\sqrt{7})^2$
$=79$

100) 10

⇨ $x=\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\dfrac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$
$=(\sqrt{3}-\sqrt{2})^2=5-2\sqrt{6}$
$y=\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\dfrac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$
$=(\sqrt{3}+\sqrt{2})^2=5+2\sqrt{6}$
$\therefore\ x+y=(5-2\sqrt{6})+(5+2\sqrt{6})=10$

101) 1

⇨ $x=5-2\sqrt{6}$, $y=5+2\sqrt{6}$ 이므로
$\therefore\ xy=(5-2\sqrt{6})(5+2\sqrt{6})=25-24=1$

102) 9

⇨ $x=5-2\sqrt{6}$, $y=5+2\sqrt{6}$ 이므로
$\therefore\ x^2-10x+10=(x-5)^2-15=(5-2\sqrt{6}-5)^2-15$
$=24-15=9$

103) -2

⇨ $(a+2)\sqrt{3}=0$이므로 $a+2=0$ $\therefore a=-2$

104) 2

⇨ (주어진 식)$=(2-a)\sqrt{3}+6+3a$가 유리수가 되려면
$2-a=0$ $\therefore a=2$

105) -3

⇨ (주어진 식)$=(6+2a)\sqrt{5}-4a-2$가 유리수가 되려면
$6+2a=0$ $\therefore a=-3$

106) $\dfrac{1}{3}$

⇨ $\dfrac{a+\sqrt{2}}{3\sqrt{2}+1}=\dfrac{(a+\sqrt{2})(3\sqrt{2}-1)}{(3\sqrt{2}+1)(3\sqrt{2}-1)}=\dfrac{6-a+(3a-1)\sqrt{2}}{17}$
유리수가 되려면 $3a-1=0$ $\therefore a=\dfrac{1}{3}$

107) $\dfrac{8}{5}$

⇨ $\dfrac{4-a\sqrt{2}}{5-2\sqrt{2}}=\dfrac{(4-a\sqrt{2})(5+2\sqrt{2})}{(5-2\sqrt{2})(5+2\sqrt{2})}=\dfrac{20-4a+(8-5a)\sqrt{2}}{17}$
유리수가 되려면 $8-5a=0$이므로 $a=\dfrac{8}{5}$

108) 2

⇨ (주어진 식)$=8-5a+(-4+2a)\sqrt{5}$가 유리수가 되려면
$-4+2a=0$ $\therefore a=2$

109) 6

⇨ $(a\sqrt{5}+3)(2\sqrt{5}-1)=10a-3+(6-a)\sqrt{5}$가
유리수가 되려면 $6-a=0$ $\therefore a=6$

110) 4

⇨ $\sqrt{3}(\sqrt{3}+a)-\sqrt{12}(2+\sqrt{3})$
$=3+a\sqrt{3}-4\sqrt{3}-6=-3+(a-4)\sqrt{3}$
유리수가 되려면 $a-4=0$ $\therefore a=4$

111) $\dfrac{8}{3}$

112) 2

113) 3

114) 3

115) 2