



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

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3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 / 무리수 e 와 자연로그(1) 무리수 e : x 의 값이 0에 한없이 가까워질 때, $(1+x)^{\frac{1}{x}}$ 의 값은 일정한 값에 수렴하는데 그 값을 e 라 하고, e 는 무리수이며 그 값은 2.71828182845904...이다.

$$\Leftrightarrow \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

(2) 자연로그 : 무리수 e 를 밑으로 하는 로그 $\log_e x$ 를 자연로그라 하고, 간단히 $\ln x$ 와 같이 나타낸다.

$$\Leftrightarrow \log_e x = \ln x$$

■ 다음 값을 구하여라.

1. $\ln 1$

2. $\ln e$

3. $\ln e^3$

4. $e^{\ln \frac{1}{3}}$

5. $\ln \sqrt{e}$

6. $\ln 3e$

7. $e^{\ln 3}$

8. $\ln \frac{1}{10e}$

9. $\ln \frac{1}{\sqrt{e}}$

10. $\ln \frac{1}{e^3} + 5$

11. $\ln e^5$

12. $e^{\frac{1}{2} \ln 4}$

13. $e^{\ln \sqrt{2}}$

14. $e^{\ln \sqrt{8}}$

■ 다음 등식을 만족하는 x 의 값을 구하여라.

15. $\ln x = 3$

16. $\ln x = -1$

17. $e^x = \frac{1}{3}$

18. $e^x = 2$

19. $e^{2x} = \frac{1}{9}$

20. $\ln x = -\frac{1}{2}$

02 / $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$ 꼴의 극한

■ 다음 극한값을 구하여라.

21. $\lim_{x \rightarrow 0} (1-x)^{\frac{1}{2x}}$

22. $\lim_{x \rightarrow 0} (1-4x)^{-\frac{1}{2x}}$

23. $\lim_{x \rightarrow 0} (1+2x)^{\frac{1}{x}}$

24. $\lim_{x \rightarrow 0} \left(1 + \frac{x}{2}\right)^{\frac{3}{x}}$

25. $\lim_{x \rightarrow 0} (1-2x)^{\frac{1}{x}}$

26. $\lim_{x \rightarrow 0} (1-3x)^{-\frac{5}{6x}}$

27. $\lim_{x \rightarrow 0} (1+x)^{\frac{6}{x}}$

28. $\lim_{x \rightarrow 0} (1-4x)^{\frac{1}{x}}$

29. $\lim_{x \rightarrow 0} (1-x)^{\frac{1}{x}}$

30. $\lim_{x \rightarrow 0} \left(1 + \frac{x}{2}\right)^{-\frac{3}{x}}$

31. $\lim_{x \rightarrow 0} (1+2x)^{\frac{1}{3x}}$

32. $\lim_{x \rightarrow 0} \left(1 + \frac{x}{4}\right)^{\frac{20}{x}}$

$$33. \lim_{x \rightarrow 0} (1+4x)^{\frac{2}{x}}$$

$$34. \lim_{x \rightarrow 0} (1+4x)^{\frac{1}{x}}$$

$$35. \lim_{x \rightarrow 0} (1+3x)^{\frac{1}{x}}$$

$$36. \lim_{x \rightarrow 0} (1+x)^{\frac{2}{x}}$$

$$37. \lim_{x \rightarrow 0} \left(1 - \frac{x}{3}\right)^{\frac{1}{3x}}$$

$$38. \lim_{x \rightarrow 0} \left(1 - \frac{3x}{2}\right)^{-\frac{8}{x}}$$

$$39. \lim_{x \rightarrow 0} (1+3x)^{\frac{2}{x}}$$

03 $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ 꼴의 극한

■ 다음 극한값을 구하여라.

$$40. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{3x}$$

$$41. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{-x}$$

$$42. \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$$

$$43. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x}\right)^{6x}$$

$$44. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{-4x}$$

$$45. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{2x}$$

$$46. \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^{\frac{x}{10}}$$

$$47. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^x$$

$$48. \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^x$$

$$49. \lim_{x \rightarrow \infty} \left(1 + \frac{7}{x}\right)^{2x}$$

$$50. \lim_{x \rightarrow \infty} \left(\frac{x+1}{x}\right)^{2x}$$

$$51. \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1}\right)^{x-1}$$

$$52. \lim_{x \rightarrow \infty} \left\{\frac{3x}{3x+1}\right\}^{2x}$$

$$53. \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1}\right)^x$$

$$54. \lim_{x \rightarrow \infty} \left\{\left(1 + \frac{1}{3x}\right)\left(1 + \frac{1}{5x}\right)\right\}^{15x}$$

$$55. \lim_{x \rightarrow \infty} \left\{\left(1 + \frac{1}{2x}\right)\left(1 + \frac{1}{3x}\right)\right\}^{6x}$$

$$56. \lim_{x \rightarrow \infty} \left\{\left(1 + \frac{1}{x}\right)\left(1 + \frac{2}{x}\right)\left(1 + \frac{3}{x}\right) \cdots \left(1 + \frac{7}{x}\right)\right\}^x$$

$$57. \lim_{x \rightarrow \infty} \left\{\frac{1}{2}\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x+1}\right)\left(1 + \frac{1}{x+2}\right) \cdots \left(1 + \frac{1}{2x}\right)\right\}^x$$

$$58. \lim_{x \rightarrow \infty} \left\{\frac{1}{3}\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x+1}\right)\left(1 + \frac{1}{x+2}\right) \cdots \left(1 + \frac{1}{3x}\right)\right\}^{2x}$$

$$59. \lim_{x \rightarrow -\infty} \left(1 - \frac{1}{x}\right)^{2x}$$

$$60. \lim_{x \rightarrow -\infty} \left(1 - \frac{2}{x}\right)^{2x}$$

$$61. \lim_{x \rightarrow -\infty} \left(1 - \frac{2}{x}\right)^{3x}$$

$$62. \lim_{x \rightarrow -\infty} \left(1 - \frac{1}{6x}\right)^{-4x}$$

$$63. \lim_{x \rightarrow -\infty} \left(1 - \frac{1}{4x}\right)^{-12x}$$

$$64. \lim_{x \rightarrow -\infty} \left(1 - \frac{2}{3x}\right)^{\frac{x}{2}}$$

$$65. \lim_{x \rightarrow -\infty} \left(1 - \frac{1}{2x}\right)^{-8x}$$

$$66. \lim_{x \rightarrow -\infty} \left(\frac{x-1}{x}\right)^x$$



정답 및 해설

1) 0

2) 1

3) 3

$$\Rightarrow \ln e^3 = 3 \ln e = 3$$

4) $\frac{1}{3}$

$$\Rightarrow e^{\ln \frac{1}{3}} = \left(\frac{1}{3}\right)^{\ln e} = \frac{1}{3}$$

5) $\frac{1}{2}$

$$\Rightarrow \ln \sqrt{e} = \frac{1}{2} \ln e = \frac{1}{2}$$

6) $1 + \ln 3$

$$\Rightarrow \ln 3e = \ln 3 + \ln e = 1 + \ln 3$$

7) 3

$$\Rightarrow e^{\ln 3} = 3^{\ln e} = 3$$

8) $-1 - \ln 10$

$$\Rightarrow \ln \frac{1}{10e} = -\ln 10e = -1 - \ln 10$$

9) $-\frac{1}{2}$

$$\Rightarrow \ln \frac{1}{\sqrt{e}} = \ln \left(e^{-\frac{1}{2}}\right) = -\frac{1}{2} \cdot \ln e = -\frac{1}{2}$$

10) 2

$$\Rightarrow \ln \frac{1}{e^3} + 5 = \ln e^{-3} + 5 = -3 + 5 = 2$$

11) 5

$$\Rightarrow \ln e^5 = 5 \ln e = 5 \cdot 1 = 5$$

12) 2

$$\Rightarrow e^{\frac{1}{2} \ln 4} = e^{\ln 4^{\frac{1}{2}}} = e^{\ln 2} = 2^{\ln e} = 2$$

13) $\sqrt{2}$

$$\Rightarrow e^{\ln \sqrt{2}} = (\sqrt{2})^{\ln e} = \sqrt{2}$$

14) $2\sqrt{2}$

$$\Rightarrow e^{\ln \sqrt{8}} = (\sqrt{8})^{\ln e} = \sqrt{8} = 2\sqrt{2}$$

15) e^3

$$\Rightarrow \ln x = 3 \therefore x = e^3$$

16) $\frac{1}{e}$

$$\Rightarrow \ln x = -1 \therefore x = e^{-1} = \frac{1}{e}$$

17) $-\ln 3$

$$\Rightarrow e^x = \frac{1}{3} \therefore x = \ln \left(\frac{1}{3}\right) = -\ln 3$$

18) $\ln 2$

$$\Rightarrow e^x = 2 \therefore x = \ln 2$$

19) $-\ln 3$

$$\Rightarrow e^{2x} = \frac{1}{9} \text{에서 } 2x = \ln \frac{1}{9}, 2x = -2 \ln 3 \\ \therefore x = -\ln 3$$

20) $\frac{\sqrt{e}}{e}$

$$\Rightarrow \ln x = -\frac{1}{2} \therefore x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} = \frac{\sqrt{e}}{e}$$

21) $\frac{1}{\sqrt{e}}$

$$\Rightarrow \lim_{x \rightarrow 0} (1-x)^{\frac{1}{2x}} = \lim_{x \rightarrow 0} \left\{ (1-x)^{-\frac{1}{x}} \right\}^{-\frac{1}{2}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

22) e^2

$$\Rightarrow \lim_{x \rightarrow 0} (1-4x)^{-\frac{1}{2x}} = \lim_{x \rightarrow 0} \left\{ (1-4x)^{-\frac{1}{4x}} \right\}^2 = e^2$$

23) e^2

$$\Rightarrow \lim_{x \rightarrow 0} (1+2x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left\{ (1+2x)^{\frac{1}{2x}} \right\}^2 = e^2$$

24) $e^{\frac{3}{2}}$

$$\Rightarrow \lim_{x \rightarrow 0} \left(1 + \frac{x}{2}\right)^{\frac{3}{x}} = \lim_{x \rightarrow 0} \left\{ \left(1 + \frac{x}{2}\right)^{\frac{2}{x}} \right\}^{\frac{3}{2}} = e^{\frac{3}{2}}$$

25) $\frac{1}{e^2}$

$$\Rightarrow \lim_{x \rightarrow 0} (1-2x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left\{ (1-2x)^{-\frac{1}{2x}} \right\}^{-2} = e^{-2} = \frac{1}{e^2}$$

26) $e^{\frac{5}{2}}$

$$\Rightarrow \lim_{x \rightarrow 0} (1-3x)^{-\frac{5}{6x}} = \lim_{x \rightarrow 0} \left\{ (1-3x)^{-\frac{1}{3x}} \right\}^{\frac{5}{2}} = e^{\frac{5}{2}}$$

27) e^6

$$\Rightarrow \lim_{x \rightarrow 0} (1+x)^{\frac{6}{x}} = \lim_{x \rightarrow 0} \left\{ (1+x)^{\frac{1}{x}} \right\}^6 = e^6$$

$$28) \frac{1}{e^4}$$

$$\Rightarrow \lim_{x \rightarrow 0} (1-4x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left\{ (1-4x)^{-\frac{1}{4x}} \right\}^{-4} = e^{-4} = \frac{1}{e^4}$$

$$29) \frac{1}{e}$$

$$\Rightarrow \lim_{x \rightarrow 0} (1-x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left\{ (1-x)^{-\frac{1}{x}} \right\}^{-1} = e^{-1} = \frac{1}{e}$$

$$30) e^{-\frac{3}{2}}$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(1 + \frac{x}{2} \right)^{-\frac{3}{x}} = \lim_{x \rightarrow 0} \left\{ \left(1 + \frac{x}{2} \right)^{\frac{2}{x}} \right\}^{-\frac{3}{2}} = e^{-\frac{3}{2}}$$

$$31) e^{\frac{2}{3}}$$

$$\Rightarrow \lim_{x \rightarrow 0} (1+2x)^{\frac{1}{3x}} = \lim_{x \rightarrow 0} \left\{ (1+x)^{\frac{1}{2x}} \right\}^{\frac{2}{3}} = e^{\frac{2}{3}}$$

$$32) e^5$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(1 + \frac{x}{4} \right)^{\frac{20}{x}} = \lim_{x \rightarrow 0} \left\{ \left(1 + \frac{x}{4} \right)^{\frac{4}{x}} \right\}^5 = e^5$$

$$33) e^8$$

$$\Rightarrow \lim_{x \rightarrow 0} (1+4x)^{\frac{1}{4x} \times \frac{8x}{x}} = e^8$$

$$34) e^4$$

$$\Rightarrow \lim_{x \rightarrow 0} (1+4x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left((1+4x)^{\frac{1}{4x}} \right)^4 = e^4$$

$$35) e^3$$

$$\Rightarrow \lim_{x \rightarrow 0} (1+3x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left\{ (1+3x)^{\frac{1}{3x}} \right\}^3 = e^3$$

$$36) e^2$$

$$\Rightarrow \lim_{x \rightarrow 0} (1+x)^{\frac{2}{x}} = \lim_{x \rightarrow 0} \left\{ (1+x)^{\frac{1}{x}} \right\}^2 = e^2$$

$$37) e^{-\frac{1}{9}}$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(1 - \frac{x}{3} \right)^{\frac{1}{3x}} = \lim_{x \rightarrow 0} \left\{ \left(1 - \frac{x}{3} \right)^{-\frac{3}{x}} \right\}^{-\frac{1}{3 \times 3}} = e^{-\frac{1}{9}}$$

$$38) e^{12}$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(1 - \frac{3x}{2} \right)^{-\frac{8}{x}} = \lim_{x \rightarrow 0} \left\{ \left(1 - \frac{3x}{2} \right)^{-\frac{2}{3x}} \right\}^{4 \times 3} = e^{12}$$

$$39) e^6$$

$$\Rightarrow \lim_{x \rightarrow 0} (1+3x)^{\frac{2}{x}} = \lim_{x \rightarrow 0} \left\{ (1+3x)^{\frac{1}{3x}} \right\}^6 = e^6$$

$$40) e^3$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^{3x} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{x} \right)^x \right\}^3 = e^3$$

$$41) \frac{1}{\sqrt{e}}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x} \right)^{-x} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{2x} \right)^{2x} \right\}^{-\frac{1}{2}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

$$42) e^2$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^x = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{2}{x} \right)^{\frac{x}{2}} \right\}^2 = e^2$$

$$43) e^2$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x} \right)^{6x} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{3x} \right)^{3x} \right\}^2 = e^2$$

$$44) \frac{1}{e^2}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x} \right)^{-4x} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{2x} \right)^{2x} \right\}^{-2} = e^{-2} = \frac{1}{e^2}$$

$$45) e$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x} \right)^{2x} = e$$

$$46) \sqrt{e}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x} \right)^{\frac{x}{10}} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{5}{x} \right)^{\frac{x}{5}} \right\}^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$$

$$47) \sqrt{e}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x} \right)^x = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{2x} \right)^{2x} \right\}^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$$

$$48) e^5$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x} \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x} \right)^{\frac{x}{5} \cdot 5} = e^5$$

$$49) e^{14}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{7}{x} \right)^{2x} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{7}{x} \right)^{\frac{x}{7}} \right\}^{2 \times 7} = e^{14}$$

$$50) e^2$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(\frac{x+1}{x} \right)^{2x} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{x} \right)^x \right\}^2 = e^2$$

$$51) e^2$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1} \right)^{x-1} = \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x-1} \right)^{\frac{x-1}{2} \cdot 2} = e^2$$

$$52) e^{-\frac{2}{3}}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left\{ \frac{3x}{3x+1} \right\}^{2x} = \lim_{x \rightarrow \infty} \left\{ \frac{1}{1 + \frac{1}{3x}} \right\}^{3x \times \frac{2}{3}} = e^{-\frac{2}{3}}$$

$$53) e^2$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1} \right)^x &= \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x-1} \right)^x \\ &= \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x-1} \right)^{x-1} \left(1 + \frac{2}{x-1} \right) \\ &= \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x-1} \right)^{\frac{x-1}{2} \times 2} \times \left(1 + \frac{2}{x-1} \right) \\ &= e^2 \times 1 = e^2 \end{aligned}$$

$$54) e^8$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{3x} \right) \left(1 + \frac{1}{5x} \right) \right\}^{15x} \\ &= \lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x} \right)^{15x} \left(1 + \frac{1}{5x} \right)^{15x} \\ &= \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{3x} \right)^{3x} \right\}^5 \left\{ \left(1 + \frac{1}{5x} \right)^{5x} \right\}^3 = e^5 \times e^3 = e^8 \end{aligned}$$

$$55) e^5$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{2x} \right) \left(1 + \frac{1}{3x} \right) \right\}^{6x} \\ &= \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x} \right)^{2x \times 3} \left(1 + \frac{1}{3x} \right)^{3x \times 2} \\ &= e^3 \times e^2 = e^5 \end{aligned}$$

$$56) e^{28}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{x} \right) \left(1 + \frac{2}{x} \right) \left(1 + \frac{3}{x} \right) \cdots \left(1 + \frac{7}{x} \right) \right\}^x \\ &= \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x \times \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^x \times \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x} \right)^x \\ &\quad \times \cdots \times \lim_{x \rightarrow \infty} \left(1 + \frac{7}{x} \right)^x \\ &= e \times e^2 \times e^3 \times \cdots \times e^7 = e^{1+2+3+\cdots+7} = e^{28} \end{aligned}$$

$$57) \sqrt{e}$$

$$\begin{aligned} \Rightarrow \frac{1}{2} \left(1 + \frac{1}{x} \right) \left(1 + \frac{1}{x+1} \right) \left(1 + \frac{1}{x+2} \right) \cdots \left(1 + \frac{1}{2x} \right) \\ &= \frac{1}{2} \cdot \frac{x+1}{x} \cdot \frac{x+2}{x+1} \cdot \frac{x+3}{x+2} \cdots \frac{2x+1}{2x} \\ &= \frac{1}{2} \cdot \frac{2x+1}{x} = \frac{2x+1}{2x} \end{aligned}$$

따라서

$$\begin{aligned} (\text{주어진 식}) &= \lim_{x \rightarrow \infty} \left(\frac{2x+1}{2x} \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x} \right)^x \\ &= \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{2x} \right)^{2x} \right\}^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e} \end{aligned}$$

$$58) e^{\frac{2}{3}}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \left\{ \frac{1}{3} \left(\frac{x+1}{x} \right) \left(\frac{x+2}{x+1} \right) \cdots \left(\frac{3x+1}{3x} \right) \right\}^{2x} \\ &= \lim_{x \rightarrow \infty} \left\{ \frac{1}{3} \left(\frac{3x+1}{x} \right) \right\}^{2x} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x} \right)^{3x \times \frac{2}{3}} = e^{\frac{2}{3}} \end{aligned}$$

$$59) \frac{1}{e^2}$$

$$\begin{aligned} \Rightarrow -x = t \text{로 놓으면 } x \rightarrow -\infty \text{ 일 때 } t \rightarrow \infty \text{ 이므로} \\ \lim_{x \rightarrow -\infty} \left(1 - \frac{1}{x} \right)^{2x} &= \lim_{t \rightarrow \infty} \left(1 + \frac{1}{t} \right)^{-2t} \\ &= \lim_{t \rightarrow \infty} \left\{ \left(1 + \frac{1}{t} \right)^t \right\}^{-2} \\ &= e^{-2} = \frac{1}{e^2} \end{aligned}$$

$$60) e^{-4}$$

$$\begin{aligned} \Rightarrow x = -t \text{로 놓으면 } x \rightarrow -\infty \text{ 일 때 } t \rightarrow \infty \text{ 이므로} \\ \lim_{x \rightarrow -\infty} \left(1 - \frac{2}{x} \right)^{2x} &= \lim_{t \rightarrow \infty} \left(1 + \frac{2}{t} \right)^{-2t} \\ &= \lim_{t \rightarrow \infty} \left\{ \left(1 + \frac{2}{t} \right)^{\frac{t}{2}} \right\}^{-4} = e^{-4} \end{aligned}$$

$$61) e^{-6}$$

$$\begin{aligned} \Rightarrow x = -t \text{로 놓으면 } x \rightarrow -\infty \text{ 일 때 } t \rightarrow \infty \text{ 이므로} \\ \lim_{x \rightarrow -\infty} \left(1 - \frac{2}{x} \right)^{3x} &= \lim_{t \rightarrow \infty} \left(1 + \frac{2}{t} \right)^{-3t} \\ &= \lim_{t \rightarrow \infty} \left\{ \left(1 + \frac{2}{t} \right)^{\frac{t}{2}} \right\}^{-6} = e^{-6} \end{aligned}$$

$$62) e^{\frac{2}{3}}$$

$$\begin{aligned} \Rightarrow x = -t \text{로 놓으면 } x \rightarrow -\infty \text{ 일 때 } t \rightarrow \infty \text{ 이므로} \\ \lim_{x \rightarrow -\infty} \left(1 - \frac{1}{6x} \right)^{-4x} &= \lim_{t \rightarrow \infty} \left\{ \left(1 + \frac{1}{6t} \right)^{6t} \right\}^{\frac{4}{6}} = e^{\frac{2}{3}} \end{aligned}$$

$$63) e^3$$

$$\begin{aligned} \Rightarrow x = -t \text{로 놓으면 } x \rightarrow -\infty \text{ 일 때 } t \rightarrow \infty \text{ 이므로} \\ \lim_{x \rightarrow -\infty} \left(1 - \frac{1}{4x} \right)^{-12x} &= \lim_{t \rightarrow \infty} \left\{ \left(1 + \frac{1}{4t} \right)^{4t} \right\}^3 = e^3 \end{aligned}$$

$$64) e^{-\frac{1}{3}}$$

$$\Rightarrow x = -t \text{로 놓으면 } x \rightarrow -\infty \text{ 일 때 } t \rightarrow \infty \text{ 이므로}$$

$$\begin{aligned}\lim_{x \rightarrow -\infty} \left(1 - \frac{2}{3x}\right)^{\frac{x}{2}} &= \lim_{t \rightarrow \infty} \left(1 + \frac{2}{3t}\right)^{-\frac{t}{2}} \\ &= \lim_{t \rightarrow \infty} \left\{ \left(1 + \frac{2}{3t}\right)^{\frac{3t}{2}} \right\}^{-\frac{1}{3}} = e^{-\frac{1}{3}}\end{aligned}$$

65) e^4 $\Rightarrow x = -t$ 로 놓으면 $x \rightarrow -\infty$ 일 때 $t \rightarrow \infty$ 이므로

$$\begin{aligned}\lim_{x \rightarrow -\infty} \left(1 - \frac{1}{2x}\right)^{-8x} &= \lim_{t \rightarrow \infty} \left(1 + \frac{1}{2t}\right)^{8t} \\ &= \lim_{t \rightarrow \infty} \left\{ \left(1 + \frac{1}{2t}\right)^{2t} \right\}^4 = e^4\end{aligned}$$

66) $\frac{1}{e}$

$$\Rightarrow \lim_{x \rightarrow -\infty} \left(\frac{x-1}{x}\right)^x = \lim_{x \rightarrow -\infty} \left\{ \left(1 - \frac{1}{x}\right)^{-x} \right\}^{-1} = e^{-1}$$