



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

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3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 / 함수의 극한에 대한 성질두 함수 $f(x)$, $g(x)$ 에서 $\lim_{x \rightarrow a} f(x) = \alpha$, $\lim_{x \rightarrow a} g(x) = \beta$ (α, β 는 실수)일 때,

(1) $\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x) = c\alpha$ (단, c 는 상수)

(2) $\lim_{x \rightarrow a} \{f(x) + g(x)\} = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x) = \alpha + \beta$

(3) $\lim_{x \rightarrow a} \{f(x) - g(x)\} = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x) = \alpha - \beta$

(4) $\lim_{x \rightarrow a} f(x)g(x) = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x) = \alpha\beta$

(5) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{\alpha}{\beta}$ (단, $\beta \neq 0$)

■ 두 함수 $f(x)$, $g(x)$ 에 대하여 $\lim_{x \rightarrow a} f(x) = -2$, $\lim_{x \rightarrow a} g(x) = 3$ 일 때, 다음 극한값을 구하여라. (단, a 는 상수이다.)

1. $\lim_{x \rightarrow a} 3f(x)$

2. $\lim_{x \rightarrow a} \{2f(x) + g(x)\}$

3. $\lim_{x \rightarrow a} f(x)g(x)$

4. $\lim_{x \rightarrow a} \{f(x)\}^2$

5. $\lim_{x \rightarrow a} \frac{g(x)}{f(x)}$

6. $\lim_{x \rightarrow a} \frac{f(x) + 2}{2g(x) - 1}$

■ 두 함수 $f(x)$, $g(x)$ 에 대하여 $\lim_{x \rightarrow a} f(x) = 3$, $\lim_{x \rightarrow a} g(x) = -2$ 일 때, 다음 극한값을 구하여라. (단, a 는 상수이다.)

7. $\lim_{x \rightarrow a} 4f(x)$

8. $\lim_{x \rightarrow a} \{f(x) - 3g(x)\}$

9. $\lim_{x \rightarrow a} f(x)g(x)$

10. $\lim_{x \rightarrow a} \{g(x)\}^2$

11. $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$

12. $\lim_{x \rightarrow a} \frac{4f(x) + 1}{g(x) + 3}$

■ 다음 극한값을 구하여라.

13. $\lim_{x \rightarrow 1} (2x + 5)$

14. $\lim_{x \rightarrow 2} (x^2 - 2x + 4)$

15. $\lim_{x \rightarrow 3} (x - 2)(x^2 + 3)$

16. $\lim_{x \rightarrow 3} (x - 1)(2x^2 - 3x - 1)$

17. $\lim_{x \rightarrow 2} \frac{2x - 1}{x + 1}$

18. $\lim_{x \rightarrow 3} \frac{x^2 - 4}{x + 1}$

19. $\lim_{x \rightarrow -2} \frac{x - 1}{2x^2 + 1}$

20. $\lim_{x \rightarrow 1} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

02 함수의 극한값의 계산

(1) $\frac{0}{0}$ 꼴

① 유리식인 경우 \Rightarrow 분모, 분자를 인수분해 한 다음 약분하여 극한값을 구한다.

② 무리식인 경우 \Rightarrow 분모, 분자 중 $\sqrt{\quad}$ 가 있는 쪽을 먼저 유리화 한 후 약분하여 극한값을 구한다.

(2) $\frac{\infty}{\infty}$ 꼴 : 분모의 최고차항으로 분자, 분모를 각각 나눈다.

(3) $\infty - \infty$ 꼴

① 다항식인 경우 \Rightarrow 최고차항으로 묶는다.

② 무리식인 경우 \Rightarrow 분모를 1로 보고 분자를 유리화 한다.

(4) $\infty \times 0$ 꼴 : $\infty \times c$, $\frac{c}{\infty}$, $\frac{0}{\infty}$, $\frac{\infty}{\infty}$ (c 는 상수) 꼴로 변형한다.

■ 다음 극한값을 구하여라.

21. $\lim_{x \rightarrow 0} \frac{x(x^2 + 2)}{x}$

22. $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x + 1}$

23. $\lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1}$

24. $\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x - 1}$

25. $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1}$

26. $\lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+2)}{x^2-1}$

$$27. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$$

$$28. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$29. \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$$

$$30. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$$

$$31. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$$

$$32. \lim_{x \rightarrow -1} \frac{x^3 - x^2 - x + 1}{x^2 - 1}$$

$$33. \lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$$

$$34. \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

$$35. \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+4} - 2}$$

$$36. \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1} - 1}$$

$$37. \lim_{x \rightarrow 3} \frac{2x - 6}{\sqrt{x+1} - 2}$$

$$38. \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$$

$$39. \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x+3} - 2}$$

$$40. \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+7} - 3}$$

$$41. \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - 3}{x - 2}$$

▣ 다음 극한을 조사하여라.

$$42. \lim_{x \rightarrow 0} \frac{|x|}{x}$$

$$43. \lim_{x \rightarrow 1+} \frac{x - 1}{|x - 1|}$$

$$44. \lim_{x \rightarrow 1-} \frac{x - 1}{|x - 1|}$$

$$45. \lim_{x \rightarrow 1^+} \frac{x^2 - 1}{|x - 1|}$$

$$46. \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{|x - 1|}$$

$$47. \lim_{x \rightarrow 1} \frac{x^2 - 1}{|x - 1|}$$

$$48. \lim_{x \rightarrow -2} \frac{x^2 + 2x}{x + 2}$$

$$49. \lim_{x \rightarrow -2} \frac{x^2 + 2x}{|x + 2|}$$

▣ 다음 극한을 조사하여라.

$$50. \lim_{x \rightarrow \infty} \frac{x}{x + 1}$$

$$51. \lim_{x \rightarrow -\infty} \frac{4x - 1}{3x + 2}$$

$$52. \lim_{x \rightarrow \infty} \frac{3x + 1}{x + 1}$$

$$53. \lim_{x \rightarrow \infty} \frac{2x - 1}{x - 1}$$

$$54. \lim_{x \rightarrow -\infty} \frac{x}{x + 1}$$

$$55. \lim_{x \rightarrow -\infty} \frac{-2x + 5}{x + 3}$$

$$56. \lim_{x \rightarrow \infty} \frac{x^2 - 1}{3x^3 + x - 1}$$

$$57. \lim_{x \rightarrow \infty} \frac{x^2 + x + 2}{2x^2 - 3}$$

$$58. \lim_{x \rightarrow -\infty} \frac{x^2 - x + 2}{2x^2 + 3}$$

$$59. \lim_{x \rightarrow \infty} \frac{(2x + 3)(2x - 1)}{x^2 + 1}$$

$$60. \lim_{x \rightarrow \infty} \frac{(3x + 5)(2x - 1)}{x^2 + 1}$$

$$61. \lim_{x \rightarrow \infty} \frac{(x + 1)(2x - 1)}{3x^2 + x - 1}$$

$$62. \lim_{x \rightarrow \infty} \frac{2x^2 + 3}{4x + 3}$$

$$63. \lim_{x \rightarrow \infty} \frac{x+1}{3x^2+x+1}$$

■ 다음 극한값을 구하여라.

$$64. \lim_{x \rightarrow \infty} \frac{2x-1}{5x+\sqrt{x^2+1}}$$

$$65. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1}+x}{2x+3}$$

$$66. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}+x}{2x-3}$$

$$67. \lim_{x \rightarrow \infty} \frac{x}{\sqrt{1+x^2}-1}$$

$$68. \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2+3}-4}$$

$$69. \lim_{x \rightarrow \infty} \frac{4x-1}{3x+\sqrt{x^2+1}}$$

$$70. \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2+3}+4}$$

$$71. \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2}}$$

$$72. \lim_{x \rightarrow -\infty} \frac{3x+2}{\sqrt{9x^2-1}}$$

$$73. \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2+1}}{x+1}$$

$$74. \lim_{x \rightarrow -\infty} \frac{2x}{\sqrt{x^2+1}-2}$$

$$75. \lim_{x \rightarrow -\infty} \frac{x+1}{\sqrt{x^2+x}-x}$$

■ 다음 극한을 조사하여라.

$$76. \lim_{x \rightarrow \infty} (x^2-2x+3)$$

$$77. \lim_{x \rightarrow \infty} (-2x^3+4x^2-3x+1)$$

$$78. \lim_{x \rightarrow -\infty} (x^3+3x^2+2x-1)$$

$$79. \lim_{x \rightarrow \infty} (\sqrt{x^2+1}-x)$$

$$80. \lim_{x \rightarrow \infty} (\sqrt{x^2+x}-x)$$

$$81. \lim_{x \rightarrow \infty} (\sqrt{x^2+2x} - x)$$

$$82. \lim_{x \rightarrow \infty} (\sqrt{x^2+4x} - x)$$

$$83. \lim_{x \rightarrow \infty} (\sqrt{x^2+6x} - x)$$

$$84. \lim_{x \rightarrow -\infty} (\sqrt{x^2-2x} + x)$$

$$85. \lim_{x \rightarrow \infty} (\sqrt{x^2-3x} - \sqrt{x^2+3x})$$

$$86. \lim_{x \rightarrow \infty} (\sqrt{x^2+2x+2} - \sqrt{x^2-2x-2})$$

■ 다음 극한값을 구하여라.

$$87. \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x+1} - 1 \right)$$

$$88. \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x-1} + 1 \right)$$

$$89. \lim_{x \rightarrow 0} \frac{1}{x} \left\{ 1 - \frac{1}{(x+1)^2} \right\}$$

$$90. \lim_{x \rightarrow 0} \frac{1}{x} \left\{ 1 - \frac{1}{(x-1)^2} \right\}$$

$$91. \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x+\sqrt{2}} - \frac{1}{\sqrt{2}} \right)$$

$$92. \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{3}-x} - \frac{1}{\sqrt{3}} \right)$$

$$93. \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{5}-x} - \frac{1}{\sqrt{5}} \right)$$

$$94. \lim_{x \rightarrow 2} \frac{1}{x-2} \left(2x - \frac{5x+2}{x+1} \right)$$

$$95. \lim_{x \rightarrow \infty} x \left(1 - \frac{\sqrt{x+2}}{\sqrt{x}} \right)$$

$$96. \lim_{x \rightarrow \infty} x \left(1 - \frac{\sqrt{2x+1}}{\sqrt{2x}} \right)$$



정답 및 해설

1) -6

$$\Rightarrow \lim_{x \rightarrow a} 3f(x) = 3 \lim_{x \rightarrow a} f(x) = 3 \cdot (-2) = -6$$

2) -1

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} \{2f(x) + g(x)\} &= 2 \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x) \\ &= 2 \cdot (-2) + 3 = -1 \end{aligned}$$

3) -6

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} f(x)g(x) &= \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) \\ &= (-2) \cdot 3 = -6 \end{aligned}$$

4) 4

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} \{f(x)\}^2 &= \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} f(x) \\ &= (-2) \cdot (-2) = 4 \end{aligned}$$

5) $-\frac{3}{2}$

$$\Rightarrow \lim_{x \rightarrow a} \frac{g(x)}{f(x)} = \frac{\lim_{x \rightarrow a} g(x)}{\lim_{x \rightarrow a} f(x)} = -\frac{3}{2}$$

6) 0

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} \frac{f(x)+2}{2g(x)-1} &= \frac{\lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} 2}{2 \lim_{x \rightarrow a} g(x) - \lim_{x \rightarrow a} 1} \\ &= \frac{(-2)+2}{2 \cdot 3 - 1} = \frac{0}{5} = 0 \end{aligned}$$

7) 12

$$\Rightarrow \lim_{x \rightarrow a} 4f(x) = 4 \lim_{x \rightarrow a} f(x) = 4 \times 3 = 12$$

8) 9

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} \{f(x) - 3g(x)\} &= \lim_{x \rightarrow a} f(x) - 3 \lim_{x \rightarrow a} g(x) \\ &= 3 - 3 \times (-2) = 9 \end{aligned}$$

9) -6

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} f(x)g(x) &= \lim_{x \rightarrow a} f(x) \times \lim_{x \rightarrow a} g(x) = 3 \times (-2) = -6 \end{aligned}$$

10) 4

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} \{g(x)\}^2 &= \lim_{x \rightarrow a} g(x) \times \lim_{x \rightarrow a} g(x) \\ &= (-2) \times (-2) = 4 \end{aligned}$$

11) $-\frac{3}{2}$

$$\Rightarrow \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = -\frac{3}{2}$$

12) 13

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow a} \frac{4f(x)+1}{g(x)+3} &= \frac{\lim_{x \rightarrow a} \{4f(x)+1\}}{\lim_{x \rightarrow a} \{g(x)+3\}} \\ &= \frac{4 \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} 1}{\lim_{x \rightarrow a} g(x) + \lim_{x \rightarrow a} 3} \\ &= \frac{4 \times 3 + 1}{-2 + 3} = 13 \end{aligned}$$

13) 7

$$\Rightarrow \lim_{x \rightarrow 1} (2x+5) = 2 \cdot 1 + 5 = 7$$

14) 4

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 2} (x^2 - 2x + 4) &= \lim_{x \rightarrow 2} x^2 - 2 \lim_{x \rightarrow 2} x + \lim_{x \rightarrow 2} 4 \\ &= 2^2 - 2 \cdot 2 + 4 = 4 \end{aligned}$$

15) 12

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 3} (x-2)(x^2+3) &= \lim_{x \rightarrow 3} (x-2) \cdot \lim_{x \rightarrow 3} (x^2+3) \\ &= 1 \cdot 12 = 12 \end{aligned}$$

16) 16

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 3} (x-1)(2x^2-3x-1) \\ &= (3-1) \cdot (2 \cdot 3^2 - 3 \cdot 3 - 1) = 2 \cdot 8 = 16 \end{aligned}$$

17) 1

$$\Rightarrow \lim_{x \rightarrow 2} \frac{2x-1}{x+1} = \frac{2 \cdot 2 - 1}{2 + 1} = \frac{3}{3} = 1$$

18) $\frac{5}{4}$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{x^2-4}{x+1} = \frac{\lim_{x \rightarrow 3} (x^2-4)}{\lim_{x \rightarrow 3} (x+1)} = \frac{5}{4}$$

19) $-\frac{1}{3}$

$$\Rightarrow \lim_{x \rightarrow -2} \frac{x-1}{2x^2+1} = \frac{\lim_{x \rightarrow -2} (x-1)}{\lim_{x \rightarrow -2} (2x^2+1)} = \frac{-3}{9} = -\frac{1}{3}$$

20) $\sqrt{2}$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = \frac{\sqrt{2}-0}{1} = \sqrt{2}$$

21) 2

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x(x^2+2)}{x} = \lim_{x \rightarrow 0} (x^2+2) = 2$$

22) -3

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x + 1} &= \lim_{x \rightarrow -1} \frac{(x-2)(x+1)}{x+1} \\ &= \lim_{x \rightarrow -1} (x-2) = -3\end{aligned}$$

23) -5

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1} &= \lim_{x \rightarrow -1} \frac{(x+1)(2x-3)}{x+1} \\ &= \lim_{x \rightarrow -1} (2x-3) = -5\end{aligned}$$

24) 5

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x - 1} &= \lim_{x \rightarrow 1} \frac{(x-1)(x+4)}{x-1} \\ &= \lim_{x \rightarrow 1} (x+4) = 5\end{aligned}$$

25) 4

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1} &= \lim_{x \rightarrow 1} \frac{(x+3)(x-1)}{x-1} \\ &= \lim_{x \rightarrow 1} (x+3) = 4\end{aligned}$$

26) 2

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 2)}{x^2 - 1} &= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 2)}{(x+1)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{x^2 + x + 2}{x+1} = 2\end{aligned}$$

27) 4

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} &= \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{x-2} \\ &= \lim_{x \rightarrow 2} (x+2) = 4\end{aligned}$$

28) 6

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} &= \lim_{x \rightarrow 3} \frac{(x+3)(x-3)}{x-3} \\ &= \lim_{x \rightarrow 3} (x+3) = 6\end{aligned}$$

29) $\frac{3}{2}$

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} &= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+1)} \\ &= \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x+1} = \frac{3}{2}\end{aligned}$$

30) 12

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} &= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{x-2} \\ &= \lim_{x \rightarrow 2} (x^2 + 2x + 4) = 12\end{aligned}$$

31) 3

$$\Rightarrow \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 + 2x + 4}{x+2} = \frac{12}{4} = 3$$

32) -2

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow -1} \frac{x^3 - x^2 - x + 1}{x^2 - 1} &= \lim_{x \rightarrow -1} \frac{(x-1)^2(x+1)}{(x-1)(x+1)} \\ &= \lim_{x \rightarrow -1} (x-1) = -2\end{aligned}$$

33) 4

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} &= \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{x-4} \\ &= \lim_{x \rightarrow 4} (\sqrt{x}+2) = 4\end{aligned}$$

34) $\frac{1}{6}$

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9} &= \lim_{x \rightarrow 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} \\ &= \lim_{x \rightarrow 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x \rightarrow 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}\end{aligned}$$

35) 4

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+4}-2} &= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+4}+2)}{(\sqrt{x+4}-2)(\sqrt{x+4}+2)} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+4}+2)}{x} \\ &= \lim_{x \rightarrow 0} (\sqrt{x+4}+2) = 4\end{aligned}$$

36) 2

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1}-1} &= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+1}+1)}{x} = \lim_{x \rightarrow 0} (\sqrt{x+1}+1) = 2\end{aligned}$$

37) 8

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 3} \frac{2x-6}{\sqrt{x+1}-2} &= \lim_{x \rightarrow 3} \frac{2(x-3)(\sqrt{x+1}+2)}{(\sqrt{x+1}-2)(\sqrt{x+1}+2)} \\ &= \lim_{x \rightarrow 3} \frac{2(x-3)(\sqrt{x+1}+2)}{x-3} = \lim_{x \rightarrow 3} 2(\sqrt{x+1}+2) = 8\end{aligned}$$

38) $\frac{1}{4}$

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3} &= \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} \\ &= \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}\end{aligned}$$

39) 8

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 1} \frac{x^2-1}{\sqrt{x+3}-2} &= \lim_{x \rightarrow 1} \frac{(x^2-1)(\sqrt{x+3}+2)}{(\sqrt{x+3}-2)(\sqrt{x+3}+2)} \\ &= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)(\sqrt{x+3}+2)}{x-1} \\ &= \lim_{x \rightarrow 1} (x+1)(\sqrt{x+3}+2) = 2 \times 4 = 8\end{aligned}$$

40) 24

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 2} \frac{x^2-4}{\sqrt{x+7}-3} &= \lim_{x \rightarrow 2} \frac{(x^2-4)(\sqrt{x+7}+3)}{(\sqrt{x+7}-3)(\sqrt{x+7}+3)} \\ &= \lim_{x \rightarrow 2} \frac{(x-2)(x+2)(\sqrt{x+7}+3)}{x-2} \\ &= \lim_{x \rightarrow 2} (x+2)(\sqrt{x+7}+3) = 4 \cdot 6 = 24\end{aligned}$$

41) $\frac{2}{3}$

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 2} \frac{\sqrt{x^2+5}-3}{x-2} &= \lim_{x \rightarrow 2} \frac{(\sqrt{x^2+5}-3)(\sqrt{x^2+5}+3)}{(x-2)(\sqrt{x^2+5}+3)} \\ &= \lim_{x \rightarrow 2} \frac{x^2-4}{(x-2)(\sqrt{x^2+5}+3)} \\ &= \lim_{x \rightarrow 2} \frac{x+2}{\sqrt{x^2+5}+3} = \frac{2}{3}\end{aligned}$$

42) 존재하지 않는다.

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 0+} \frac{|x|}{x} &= \lim_{x \rightarrow 0+} \frac{x}{x} = 1 \\ \lim_{x \rightarrow 0-} \frac{|x|}{x} &= \lim_{x \rightarrow 0-} \frac{-x}{x} = -1 \\ \text{즉, } \lim_{x \rightarrow 0+} \frac{|x|}{x} &\neq \lim_{x \rightarrow 0-} \frac{|x|}{x} \text{ 이므로} \\ \text{극한 } \lim_{x \rightarrow 0} \frac{|x|}{x} &\text{는 존재하지 않는다.}\end{aligned}$$

43) 1

$$\begin{aligned}\Rightarrow x > 1 \text{ 일 때, } |x-1| &= x-1 \text{ 이므로} \\ \lim_{x \rightarrow 1+} \frac{x-1}{|x-1|} &= \lim_{x \rightarrow 1+} \frac{x-1}{x-1} = 1\end{aligned}$$

44) -1

$$\begin{aligned}\Rightarrow x < 1 \text{ 일 때, } |x-1| &= -(x-1) \text{ 이므로} \\ \lim_{x \rightarrow 1-} \frac{x-1}{|x-1|} &= \lim_{x \rightarrow 1-} \frac{x-1}{-(x-1)} = -1\end{aligned}$$

45) 2

$$\begin{aligned}\Rightarrow x > 1 \text{ 일 때, } |x-1| &= x-1 \text{ 이므로} \\ \lim_{x \rightarrow 1+} \frac{x^2-1}{|x-1|} &= \lim_{x \rightarrow 1+} \frac{x^2-1}{x-1} = \lim_{x \rightarrow 1+} (x+1) = 2\end{aligned}$$

46) -2

$$\begin{aligned}\Rightarrow x < 1 \text{ 일 때, } |x-1| &= -(x-1) \text{ 이므로} \\ \lim_{x \rightarrow 1-} \frac{x^2-1}{|x-1|} &= \lim_{x \rightarrow 1-} \frac{x^2-1}{-(x-1)} = \lim_{x \rightarrow 1-} \{-(x+1)\} = -2\end{aligned}$$

47) 존재하지 않는다.

$$\begin{aligned}\Rightarrow \lim_{x \rightarrow 1+} \frac{x^2-1}{|x-1|} &= \lim_{x \rightarrow 1+} \frac{x^2-1}{x-1} = \lim_{x \rightarrow 1+} (x+1) = 2, \\ \lim_{x \rightarrow 1-} \frac{x^2-1}{|x-1|} &= \lim_{x \rightarrow 1-} \frac{x^2-1}{-(x-1)} = \lim_{x \rightarrow 1-} \{-(x+1)\} = -2 \\ \text{즉, } \lim_{x \rightarrow 1+} \frac{x^2-1}{|x-1|} &\neq \lim_{x \rightarrow 1-} \frac{x^2-1}{|x-1|} \text{ 이므로}\end{aligned}$$

극한 $\lim_{x \rightarrow 1} \frac{x^2-1}{|x-1|}$ 은 존재하지 않는다.

48) -2

$$\Rightarrow x \neq -2 \text{ 일 때, } \frac{x^2+2x}{x+2} = x \text{ 이므로}$$

$$\lim_{x \rightarrow -2+} \frac{x^2+2x}{x+2} = \lim_{x \rightarrow -2+} x = -2,$$

$$\lim_{x \rightarrow -2-} \frac{x^2+2x}{x+2} = \lim_{x \rightarrow -2-} x = -2$$

$$\therefore \lim_{x \rightarrow -2} \frac{x^2+2x}{x+2} = -2$$

49) 존재하지 않는다.

$$\Rightarrow \lim_{x \rightarrow -2+} \frac{x^2+2x}{|x+2|} = \lim_{x \rightarrow -2+} x = -2,$$

$$\lim_{x \rightarrow -2-} \frac{x^2+2x}{|x+2|} = \lim_{x \rightarrow -2-} (-x) = 2$$

$$\text{즉, } \lim_{x \rightarrow -2+} \frac{x^2+2x}{|x+2|} \neq \lim_{x \rightarrow -2-} \frac{x^2+2x}{|x+2|} \text{ 이므로}$$

극한 $\lim_{x \rightarrow -2} \frac{x^2+2x}{|x+2|}$ 는 존재하지 않는다.

50) 1

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x}{x+1} = \lim_{x \rightarrow \infty} \frac{1}{1+\frac{1}{x}} = 1$$

51) $\frac{4}{3}$

$$\Rightarrow \lim_{x \rightarrow -\infty} \frac{4x-1}{3x+2} = \lim_{t \rightarrow \infty} \frac{-4t-1}{-3t+2} = \lim_{t \rightarrow \infty} \frac{-4-\frac{1}{t}}{-3+\frac{2}{t}} = \frac{4}{3}$$

52) 3

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{3x+1}{x+1} = \lim_{x \rightarrow \infty} \frac{3+\frac{1}{x}}{1+\frac{1}{x}} = 3$$

53) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{2x-1}{x-1} = \lim_{x \rightarrow \infty} \frac{2-\frac{1}{x}}{1-\frac{1}{x}} = 2$$

54) 1

$$\Rightarrow \lim_{x \rightarrow -\infty} \frac{x}{x+1} = \lim_{t \rightarrow \infty} \frac{-t}{-t+1} = \lim_{t \rightarrow \infty} \frac{-1}{-1+\frac{1}{t}} = 1$$

55) -2

$$\Rightarrow \lim_{x \rightarrow -\infty} \frac{-2x+5}{x+3} = \lim_{t \rightarrow \infty} \frac{2t+5}{-t+3} = \lim_{t \rightarrow \infty} \frac{2+\frac{5}{t}}{-1+\frac{3}{t}} = -2$$

56) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^2-1}{3x^3+x-1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{1}{x^3}}{3 + \frac{1}{x^2} - \frac{1}{x^3}} = 0$$

57) $\frac{1}{2}$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^2+x+2}{2x^2-3} = \lim_{x \rightarrow \infty} \frac{1 + \frac{1}{x} + \frac{2}{x^2}}{2 - \frac{3}{x^2}} = \frac{1}{2}$$

58) $\frac{1}{2}$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow -\infty} \frac{x^2-x+2}{2x^2+3} &= \lim_{t \rightarrow \infty} \frac{t^2+t+2}{2t^2+3} \\ &= \lim_{t \rightarrow \infty} \frac{1 + \frac{1}{t} + \frac{2}{t^2}}{2 + \frac{3}{t^2}} = \frac{1}{2} \end{aligned}$$

59) 4

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \frac{(2x+3)(2x-1)}{x^2+1} &= \lim_{x \rightarrow \infty} \frac{4x^2+4x-3}{x^2+1} \\ &= \lim_{x \rightarrow \infty} \frac{4 + \frac{4}{x} - \frac{3}{x^2}}{1 + \frac{1}{x^2}} = 4 \end{aligned}$$

60) 6

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \frac{(3x+5)(2x-1)}{x^2+1} &= \lim_{x \rightarrow \infty} \frac{6x^2+7x-5}{x^2+1} \\ &= \lim_{x \rightarrow \infty} \frac{6 + \frac{7}{x} - \frac{5}{x^2}}{1 + \frac{1}{x^2}} = 6 \end{aligned}$$

61) $\frac{2}{3}$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \frac{(x+1)(2x-1)}{3x^2+x-1} &= \lim_{x \rightarrow \infty} \frac{2x^2+x-1}{3x^2+x-1} \\ &= \lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x} - \frac{1}{x^2}}{3 + \frac{1}{x} - \frac{1}{x^2}} = \frac{2}{3} \end{aligned}$$

62) ∞

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{2x^2+3}{4x+3} = \lim_{x \rightarrow \infty} \frac{2x + \frac{3}{x}}{4 + \frac{3}{x}} = \infty$$

63) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x+1}{3x^2+x+1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{3 + \frac{1}{x} + \frac{1}{x^2}} = 0$$

64) $\frac{1}{3}$

$$\lim_{x \rightarrow \infty} \frac{2x-1}{5x+\sqrt{x^2+1}} = \lim_{x \rightarrow \infty} \frac{2 - \frac{1}{x}}{5 + \sqrt{1 + \frac{1}{x^2}}} = \frac{1}{3}$$

65) 1

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1}+x}{2x+3} = \lim_{x \rightarrow \infty} \frac{\sqrt{1 - \frac{1}{x^2}} + 1}{2 + \frac{3}{x}} = \frac{1+1}{2} = 1$$

66) 1

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}+x}{2x-3} = \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{x^2}} + 1}{2 - \frac{3}{x}} = \frac{1+1}{2} = 1$$

67) 1

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x}{\sqrt{1+x^2}-1} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{1}{x^2}+1} - \frac{1}{x}} = 1$$

68) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2+3}-4} = \lim_{x \rightarrow \infty} \frac{2}{\sqrt{1 + \frac{3}{x^2}} - \frac{4}{x}} = 2$$

69) 1

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{4x-1}{3x+\sqrt{x^2+1}} = \lim_{x \rightarrow \infty} \frac{4 - \frac{1}{x}}{3 + \sqrt{1 + \frac{1}{x^2}}} = \frac{4}{3+1} = 1$$

70) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2+3}+4} = \lim_{x \rightarrow \infty} \frac{2}{\sqrt{1 + \frac{3}{x^2}} + \frac{4}{x}} = 2$$

71) -1

$\Rightarrow x = -t$ 로 놓으면 $x \rightarrow -\infty$ 일 때 $t \rightarrow \infty$ 이므로

$$\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2}} = \lim_{t \rightarrow \infty} \frac{-t}{\sqrt{t^2}} = -1$$

72) -1

$\Rightarrow -x = t$ 라 하면 $x \rightarrow -\infty$ 일 때 $t \rightarrow \infty$ 이므로

$$\lim_{x \rightarrow -\infty} \frac{3x+2}{\sqrt{9x^2-1}} = \lim_{t \rightarrow \infty} \frac{-3t+2}{\sqrt{9t^2-1}} = \lim_{t \rightarrow \infty} \frac{-3 + \frac{2}{t}}{\sqrt{9 - \frac{1}{t^2}}} = -1$$

73) -2 $\Rightarrow -x=t$ 라 하면 $x \rightarrow -\infty$ 일 때 $t \rightarrow \infty$ 이므로

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2+1}}{x+1} = \lim_{t \rightarrow \infty} \frac{\sqrt{4t^2+1}}{-t+1} = \lim_{t \rightarrow \infty} \frac{\sqrt{4+\frac{1}{t^2}}}{-1+\frac{1}{t}} = -2$$

74) -2 $\Rightarrow x=-t$ 로 놓으면 $x \rightarrow -\infty$ 일 때 $t \rightarrow \infty$ 이므로

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{2x}{\sqrt{x^2+1}-2} &= \lim_{t \rightarrow \infty} \frac{-2t}{\sqrt{t^2+1}-2} \\ &= \lim_{t \rightarrow \infty} \frac{-2}{\sqrt{1+\frac{1}{t^2}}-\frac{2}{t}} = -2 \end{aligned}$$

75) $-\frac{1}{2}$ $\Rightarrow x=-t$ 로 놓으면 $x \rightarrow -\infty$ 일 때 $t \rightarrow \infty$ 이므로

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{x+1}{\sqrt{x^2+x}-x} &= \lim_{t \rightarrow \infty} \frac{-t+1}{\sqrt{t^2-t}+t} \\ &= \lim_{t \rightarrow \infty} \frac{-1+\frac{1}{t}}{\sqrt{1-\frac{1}{t}}+1} = -\frac{1}{2} \end{aligned}$$

76) ∞

$$\Rightarrow \lim_{x \rightarrow \infty} (x^2-2x+3)$$

$$= \lim_{x \rightarrow \infty} x^2 \left(1 - \frac{2}{x} + \frac{3}{x^2} \right) = \infty$$

77) $-\infty$

$$\Rightarrow \lim_{x \rightarrow \infty} (-2x^3+4x^2-3x+1)$$

$$= \lim_{x \rightarrow \infty} x^3 \left(-2 + \frac{4}{x} - \frac{3}{x^2} + \frac{1}{x^3} \right) = -\infty$$

78) $-\infty$

$$\Rightarrow \lim_{x \rightarrow -\infty} (x^3+3x^2+2x-1)$$

$$= \lim_{x \rightarrow -\infty} x^3 \left(1 + \frac{3}{x} + \frac{2}{x^2} - \frac{1}{x^3} \right) = -\infty$$

79) 0

$$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2+1}-x) = \lim_{x \rightarrow \infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x^2+1}+x} = 0$$

80) $\frac{1}{2}$

$$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2+x}-x)$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$$

$$= \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+x}+x} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{2}$$

81) 1

$$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2+2x}-x)$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+2x}-x)(\sqrt{x^2+2x}+x)}{\sqrt{x^2+2x}+x}$$

$$= \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2+2x}+x} = \lim_{x \rightarrow \infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = 1$$

82) 2

$$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2+4x}-x)$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+4x}-x)(\sqrt{x^2+4x}+x)}{\sqrt{x^2+4x}+x}$$

$$= \lim_{x \rightarrow \infty} \frac{4x}{\sqrt{x^2+4x}+x}$$

$$= \lim_{x \rightarrow \infty} \frac{4}{\sqrt{1+\frac{4}{x}}+1} = \frac{4}{1+1} = 2$$

83) 3

$$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2+6x}-x)$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+6x}-x)(\sqrt{x^2+6x}+x)}{\sqrt{x^2+6x}+x}$$

$$= \lim_{x \rightarrow \infty} \frac{6x}{\sqrt{x^2+6x}+x} = \lim_{x \rightarrow \infty} \frac{6}{\sqrt{1+\frac{6}{x}}+1} = \frac{6}{1+1} = 3$$

84) 1 $\Rightarrow -x=t$ 로 놓으면 $x \rightarrow -\infty$ 일 때, $t \rightarrow \infty$ 이므로

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2-2x}+x) = \lim_{t \rightarrow \infty} (\sqrt{t^2+2t}-t)$$

$$= \lim_{t \rightarrow \infty} \frac{(\sqrt{t^2+2t}-t)(\sqrt{t^2+2t}+t)}{\sqrt{t^2+2t}+t}$$

$$= \lim_{t \rightarrow \infty} \frac{2t}{\sqrt{t^2+2t}+t}$$

$$= \lim_{t \rightarrow \infty} \frac{2}{\sqrt{1+\frac{2}{t}}+1} = 1$$

85) -3

$$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2-3x}-\sqrt{x^2+3x})$$

$$= \lim_{x \rightarrow \infty} \frac{x^2-3x-x^2-3x}{\sqrt{x^2-3x}+\sqrt{x^2+3x}}$$

$$= \lim_{x \rightarrow \infty} \frac{-6x}{\sqrt{x^2-3x}+\sqrt{x^2+3x}}$$

$$= \lim_{x \rightarrow \infty} \frac{-6}{\sqrt{1-\frac{3}{x}}+\sqrt{1+\frac{3}{x}}} = -3$$

86) 2

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2+2x+2} - \sqrt{x^2-2x-2}) \\
&= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+2x+2} - \sqrt{x^2-2x-2})(\sqrt{x^2+2x+2} + \sqrt{x^2-2x-2})}{\sqrt{x^2+2x+2} + \sqrt{x^2-2x-2}} \\
&= \lim_{x \rightarrow \infty} \frac{4x+4}{\sqrt{x^2+2x+2} + \sqrt{x^2-2x-2}} \\
&= \lim_{x \rightarrow \infty} \frac{4 + \frac{4}{x}}{\sqrt{1 + \frac{2}{x} + \frac{2}{x^2}} + \sqrt{1 - \frac{2}{x} - \frac{2}{x^2}}} = 2
\end{aligned}$$

87) -1

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x+1} - 1 \right) = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{-x}{x+1} \\
&= \lim_{x \rightarrow 0} \frac{-1}{x+1} = -1
\end{aligned}$$

88) -1

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x-1} + 1 \right) = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{1+(x-1)}{x-1} \\
&= \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x}{x-1} \\
&= \lim_{x \rightarrow 0} \frac{1}{x-1} = -1
\end{aligned}$$

89) 2

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \left\{ 1 - \frac{1}{(x+1)^2} \right\} \\
&= \lim_{x \rightarrow 0} \frac{1}{x} \left(1 - \frac{1}{x+1} \right) \left(1 + \frac{1}{x+1} \right) \\
&= \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x}{x+1} \cdot \frac{x+2}{x+1} \\
&= \lim_{x \rightarrow 0} \frac{x+2}{(x+1)^2} = 2
\end{aligned}$$

90) -2

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \left\{ 1 - \frac{1}{(x-1)^2} \right\} \\
&= \lim_{x \rightarrow 0} \frac{1}{x} \left(1 - \frac{1}{x-1} \right) \left(1 + \frac{1}{x-1} \right) \\
&= \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x-2}{x-1} \cdot \frac{x}{x-1} = \lim_{x \rightarrow 0} \frac{x-2}{(x-1)^2} = -2
\end{aligned}$$

91) $-\frac{1}{2}$

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x + \sqrt{2}} - \frac{1}{\sqrt{2}} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{-x}{\sqrt{2}(x + \sqrt{2})} \\
&= \lim_{x \rightarrow 0} \frac{-1}{\sqrt{2}(x + \sqrt{2})} = -\frac{1}{2}
\end{aligned}$$

92) $\frac{1}{3}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{3}-x} - \frac{1}{\sqrt{3}} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x}{\sqrt{3}(\sqrt{3}-x)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{3 - \sqrt{3}x} = \frac{1}{3}$$

93) $\frac{1}{5}$

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{5}-x} - \frac{1}{\sqrt{5}} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x}{\sqrt{5}(\sqrt{5}-x)} \\
&= \lim_{x \rightarrow 0} \frac{1}{5 - \sqrt{5}x} = \frac{1}{5}
\end{aligned}$$

94) $\frac{5}{3}$

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow 2} \frac{1}{x-2} \left(2x - \frac{5x+2}{x+1} \right) \\
&= \lim_{x \rightarrow 2} \frac{1}{x-2} \cdot \frac{2x^2-3x-2}{x+1} \\
&= \lim_{x \rightarrow 2} \frac{1}{x-2} \cdot \frac{(2x+1)(x-2)}{x+1} \\
&= \lim_{x \rightarrow 2} \frac{2x+1}{x+1} = \frac{5}{3}
\end{aligned}$$

95) -1

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow \infty} x \left(1 - \frac{\sqrt{x+2}}{\sqrt{x}} \right) \\
&= \lim_{x \rightarrow \infty} x \cdot \frac{\sqrt{x} - \sqrt{x+2}}{\sqrt{x}} \\
&= \lim_{x \rightarrow \infty} x \cdot \frac{(\sqrt{x} - \sqrt{x+2})(\sqrt{x} + \sqrt{x+2})}{\sqrt{x}(\sqrt{x} + \sqrt{x+2})} \\
&= \lim_{x \rightarrow \infty} \frac{-2x}{x + \sqrt{x^2+2x}} \\
&= \lim_{x \rightarrow \infty} \frac{-2}{1 + \sqrt{1 + \frac{2}{x}}} = \frac{-2}{1+1} = -1
\end{aligned}$$

96) $-\frac{1}{4}$

$$\begin{aligned}
&\Rightarrow \lim_{x \rightarrow \infty} x \left(1 - \frac{\sqrt{2x+1}}{\sqrt{2x}} \right) \\
&= \lim_{x \rightarrow \infty} \frac{x(\sqrt{2x} - \sqrt{2x+1})}{\sqrt{2x}} \\
&= \lim_{x \rightarrow \infty} \frac{x(\sqrt{2x} - \sqrt{2x+1})(\sqrt{2x} + \sqrt{2x+1})}{\sqrt{2x}(\sqrt{2x} + \sqrt{2x+1})} \\
&= \lim_{x \rightarrow \infty} \frac{-x}{2x + \sqrt{4x^2+2x}} = \lim_{x \rightarrow \infty} \frac{-1}{2 + \sqrt{4 + \frac{2}{x}}} \\
&= \frac{-1}{2+2} = -\frac{1}{4}
\end{aligned}$$