01 $\sum$의 뜻과 기본 성질

(1) $\sum$의 뜻: 수열 $\{a_n\}$의 첫째항부터 제n항까지의 합을 기호 $\sum$를 사용하여 나타낸다.

⇨ $a_1+a_2+a_3+\cdots+a_n=\sum_{k=1}^{n} a_k=S_n$

(2) $\sum$의 기본 성질

① $\sum_{k=1}^{n}(a_k+b_k)=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$

② $\sum_{k=1}^{n}(a_k-b_k)=\sum_{k=1}^{n}a_k-\sum_{k=1}^{n}b_k$

③ $\sum_{k=1}^{n}ca_k=c\sum_{k=1}^{n}a_k$ (단, c는 상수)

④ $\sum_{k=1}^{n}c=cn$ (단, c는 상수)

◪ 다음을 합의 기호 $\sum$를 사용하여 나타내시오.

1. $3+3+3+3+3$

2. $6+6+6+6+6+6$

3. $2+4+6+\cdots+20$

4. $1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$

5. $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots+\frac{1}{2n}$

6. $1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{10}$

7. $2+4+8+\cdots+2^n$

8. $1+4+7+\cdots+46$

9. $1+2^2+3^2+\cdots+15^2$

10. $2+4+8+\cdots+1024$

11. $1+5+9+\cdots+45$

12. $1+4+7+\cdots+25$

13. $1+3+3^2+\cdots+3^9$

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14. $1 \cdot 10 + 2 \cdot 9 + 3 \cdot 8 + \cdots + 10 \cdot 1$

15. $3 \times 5 + 5 \times 7 + 7 \times 9 + \cdots + 49 \times 51$

◪ 다음을 합의 기호 $\sum$를 사용하지 않고 나타내어라.

16. $\sum_{k=1}^{7} 3^k$

17. $\sum_{k=5}^{12} k^2$

18. $\sum_{k=1}^{n} k^2$

19. $\sum_{i=1}^{5} 2^i$

20. $\sum_{k=1}^{5} 2k$

21. $\sum_{k=1}^{10} 3k$

22. $\sum_{k=1}^{7} (2k+1)$

23. $\sum_{k=3}^{8} \frac{1}{k(k+1)}$

24. $\sum_{k=3}^{10} (2k-1)(2k+1)$

25. $\sum_{k=4}^{10} (2k+1)$

26. $\sum_{k=1}^{10} (k+1)(k+2)$

27. $\sum_{k=1}^{10} (5k+1)$

28. $\sum_{k=1}^{n} k(k+2)$

◪ $\sum_{k=1}^{10} a_k = 3$, $\sum_{k=1}^{10} b_k = 2$일 때, 다음 식의 값을 구하여라.

29. $\sum_{k=1}^{10}(8a_k - 5b_k + 2)$

30. $\sum_{k=1}^{10}(4a_k + 3b_k - 2)$

31. $\sum_{k=1}^{10}(3a_k - 4b_k)$

32. $\sum_{k=1}^{10}(a_k + b_k)$

◪ $\sum_{k=1}^{10} {a_k}^2 = 7$, $\sum_{k=1}^{10} a_k = 3$일 때, 다음 식의 값을 구하여라.

33. $\sum_{k=1}^{10}(2a_k - 1)^2$

34. $\sum_{k=1}^{10}(a_k - 3)^2$

35. $\sum_{k=1}^{10}(3{a_k}^2 + a_k - 4)$

36. $\sum_{k=1}^{10}({a_k}^2 - 2a_k)$

◪ $\sum_{k=1}^{10} {a_k}^2 = 6$, $\sum_{k=1}^{10} a_k = 3$일 때, 다음 식의 값을 구하여라.

37. $\sum_{k=1}^{10}(3a_k - 2)^2$

38. $\sum_{k=1}^{10}(a_k + 2)^2$

39. $\sum_{k=1}^{10}(3{a_k}^2 - 2a_k + 1)$

40. $\sum_{k=1}^{10}({a_k}^2 + 3a_k)$

◪ $\sum_{k=1}^{10} {a_k}^2 = 4$, $\sum_{k=1}^{10} a_k = 2$일 때, 다음을 구하여라.

41. $\sum_{k=1}^{10}(3a_k - 2)^2$

42. $\sum_{k=1}^{10}(a_k - 2)^2$

43. $\sum_{k=1}^{10}(a_k + 1)^2$

◪ $\sum_{k=1}^{10} a_k = 100$, $\sum_{k=1}^{10} b_k = 50$일 때, 다음을 구하여라.

44. $\sum_{k=1}^{10} (a_k - 2b_k + 2)$

45. $\sum_{k=1}^{10} (a_k + b_k)$

◪ 다음 식의 값을 구하여라.

46. $\sum_{k=1}^{15} a_k = 40$, $\sum_{k=1}^{20} a_k = 55$일 때, $\sum_{k=16}^{20} a_k$

47. $\sum_{k=1}^{10} a_k = 2$일 때, $\sum_{k=1}^{10} (5a_k + 2)$

48. $\sum_{k=1}^{10} a_k = 2$, $\sum_{k=1}^{10} b_k = 3$일 때, $\sum_{k=1}^{10} (2a_k + 3b_k)$

49. $\sum_{k=1}^{10} a_k = 12$, $\sum_{k=1}^{10} b_k = 7$일 때, $\sum_{k=1}^{10} (4a_k - 3b_k + 2)$

50. $\sum_{k=1}^{10} a_k = 10$, $\sum_{k=1}^{20} a_k = 30$일 때, $\sum_{k=11}^{20} a_k$

51. $\sum_{k=1}^{6} a_k = 6$, $\sum_{k=1}^{6} {a_k}^2 = 30$일 때, $\sum_{k=1}^{6} (2a_k - 1)^2$

52. $\sum_{k=1}^{10} a_k = 7$, $\sum_{k=1}^{10} {a_k}^2 = 15$일 때, $\sum_{k=1}^{10} (a_k + 2)(a_k - 1)$

53. $\sum_{k=1}^{10} (2a_k + 1) = 30$, $\sum_{k=1}^{10} ({a_k}^2 + a_k + 1) = 50$일 때,
$\sum_{k=1}^{10} {a_k}^2$

54. $\sum_{k=1}^{10} a_k = 15$, $\sum_{k=1}^{10} {a_k}^2 = 30$, $\sum_{k=1}^{10} b_k = 10$일 때,
$\sum_{k=1}^{10} \{(a_k + 1)^2 + b_k\}$

55. $\sum_{k=1}^{10} a_k = 30$, $\sum_{k=1}^{10} b_k = 40$일 때, $\sum_{k=1}^{10} (3a_k - b_k + 2)$

56. $\sum_{k=1}^{10} a_k = 20$, $\sum_{k=1}^{10} b_k = 30$일 때, $\sum_{k=1}^{10} (3a_k - 2b_k + 1)$

57. $\sum_{n=1}^{10} a_n = 5$, $\sum_{n=1}^{10} b_n = 10$**일 때,** $\sum_{n=1}^{10}(2a_n + 3b_n - 1)$

58. $\sum_{k=1}^{30} a_k = 10$, $\sum_{k=1}^{30} b_k = 5$**일 때,** $\sum_{k=1}^{30}(3a_k - 2b_k + 6)$

59. $\sum_{k=1}^{20} a_k = 35$, $\sum_{k=11}^{20} a_k = 15$, $\sum_{k=1}^{10} b_k = 12$**일 때,**
$\sum_{k=1}^{10}(2a_k + 5b_k - 1)$

60. $\sum_{k=1}^{7} a_k = 10$, $\sum_{k=1}^{7} {a_k}^2 = 50$**일 때,** $\sum_{k=1}^{7}(3a_k - 5)^2$

61. $\sum_{k=1}^{n} a_k = 5$, $\sum_{k=1}^{n} b_k = -2$**일 때,** $\sum_{k=1}^{n}(3a_k + 7b_k)$

62. $\sum_{k=1}^{10} a_k = 10$, $\sum_{k=1}^{10} a_k^2 = 40$**일 때,** $\sum_{k=1}^{10}(2a_k + 1)^2$

◪ **다음을 구하여라.**

63. $\sum_{k=1}^{20}(k^2+1) - \sum_{k=1}^{19}(k^2+1)$

64. $\sum_{k=1}^{20} k^2 - \sum_{k=3}^{20} k^2$

65. $\sum_{k=1}^{20}(k^2+2) - \sum_{k=1}^{20}(k^2-2)$

정답 및 해설

1) $\sum_{k=1}^{5}3$

⇨ 3이 5개 있으므로 $3+3+3+3+3=\sum_{k=1}^{5}3$

2) $\sum_{k=1}^{6}6$

⇨ $6+6+6+6+6+6=\sum_{k=1}^{6}6$

3) $\sum_{k=1}^{10}2k$

4) $\sum_{k=1}^{n}\frac{1}{k}$

⇨ $1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}=\sum_{k=1}^{n}\frac{1}{k}$

5) $\sum_{k=1}^{n}\frac{1}{2k}$

⇨ $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots+\frac{1}{2n}=\sum_{k=1}^{n}\frac{1}{2k}$

6) $\sum_{k=1}^{10}\frac{1}{k}$

7) $\sum_{k=1}^{n}2^k$

⇨ $2+4+8+\cdots+2^n=\sum_{k=1}^{n}2^k$

8) $\sum_{k=1}^{16}(3k-2)$

⇨ $a_n=1+(n-1)\times3=3n-2$

$3n-2=46 \qquad \therefore\ n=16$

일반항이 $a_n=3n-2$이고, 첫째항부터 제16항까지의 합이므로

$1+4+7+\cdots+46=\sum_{k=1}^{16}(3k-2)$

9) $\sum_{k=1}^{15}k^2$

10) $\sum_{k=1}^{10}2^k$

11) $\sum_{k=1}^{12}(4k-3)$

⇨ $a_n=1+(n-1)\times4=4n-3$

$4n-3=45 \qquad \therefore\ n=12$

일반항이 $a_n=4n-3$이고, 첫째항부터 제12항까지의 합이므로

$1+5+9+\cdots+45=\sum_{k=1}^{12}(4k-3)$

12) $\sum_{k=1}^{9}(3k-2)$

⇨ $1+4+7+\cdots+25=\sum_{k=1}^{9}(3k-2)$

13) $\sum_{k=1}^{10}3^{k-1}$

⇨ $1+3+3^2+\cdots+3^9=\sum_{k=1}^{10}3^{k-1}$

14) $\sum_{k=1}^{10}k(11-k)$

15) $\sum_{k=1}^{24}(2k+1)(2k+3)$

⇨ $a_n=(2n+1)(2n+3)$

$2n+1=49 \qquad \therefore\ n=24$

일반항이 $a_n=(2n+1)(2n+3)$이고, 첫째항부터 제24항까지의 합이므로

$3\times5+5\times7+7\times9+\cdots+49\times51$

$=\sum_{k=1}^{24}(2k+1)(2k+3)$

16) $3+3^2+3^3+\cdots+3^7$

⇨ $\sum_{k=1}^{7}3^k=3+3^2+3^3+\cdots+3^7$

17) $5^2+6^2+7^2+\cdots+12^2$

⇨ $\sum_{k=5}^{12}k^2=5^2+6^2+7^2+\cdots+12^2$

18) $1^2+2^2+3^2+\cdots+n^2$

⇨ $\sum_{k=1}^{n}k^2=1^2+2^2+3^2+\cdots+n^2$

19) $2+4+8+16+32$

⇨ $\sum_{i=1}^{5}2^i=2^1+2^2+2^3+2^4+2^5$

$=2+4+8+16+32$

20) $2+4+6+8+10$

⇨ $\sum_{k=1}^{5}2k=2\cdot1+2\cdot2+2\cdot3+2\cdot4+2\cdot5$

$=2+4+6+8+10$

21) $3+6+9+\cdots+30$

⇨ $\sum_{k=1}^{10}3k=3\cdot1+3\cdot2+3\cdot3+\cdots+3\cdot10$

$=3+6+9+\cdots+30$

22) $3+5+7+\cdots+15$

⇨ $k=1,\ 2,\ 3,\ \cdots,\ 7$이므로

$\sum_{k=1}^{7}(2k+1)=3+5+7+\cdots+15$

23) $\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\cdots+\frac{1}{8\cdot 9}$

⇨ $\sum_{k=3}^{8}\frac{1}{k(k+1)}=\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\cdots\ \frac{1}{8\cdot 9}$

24) $5\cdot 7+7\cdot 9+9\cdot 11+\cdots+19\cdot 21$

⇨ $\sum_{k=3}^{10}(2k-1)(2k+1)$
$=5\cdot 7+7\cdot 9+9\cdot 11+\cdots+19\cdot 21$

25) $9+11+13+\cdots+21$

⇨ $\sum_{k=4}^{10}(2k+1)=9+11+13+\cdots+21$

26) $2\cdot 3+3\cdot 4+4\cdot 5+\cdots+11\cdot 12$

⇨ $\sum_{k=1}^{10}(k+1)(k+2)=2\cdot 3+3\cdot 4+4\cdot 5+\cdots+11\cdot 12$

27) $6+11+16+\cdots+51$

⇨ $\sum_{k=1}^{10}(5k+1)=6+11+16+\cdots+51$

28) $1\times 3+2\times 4+3\times 5+\cdots+n(n+2)$

⇨ $k=1,\ 2,\ 3,\ \cdots,\ n$이므로

$\sum_{k=1}^{n}k(k+2)=1\times 3+2\times 4+3\times 5+\cdots+n(n+2)$

29) 34

⇨ $\sum_{k=1}^{10}(8a_k-5b_k+2)=8\sum_{k=1}^{10}a_k-5\sum_{k=1}^{10}b_k+\sum_{k=1}^{10}2$
$=8\cdot 3-5\cdot 2+2\cdot 10=34$

30) -2

⇨ $\sum_{k=1}^{10}(4a_k+3b_k-2)=4\sum_{k=1}^{10}a_k+3\sum_{k=1}^{10}b_k-\sum_{k=1}^{10}2$
$=4\cdot 3+3\cdot 2-2\cdot 10=-2$

31) 1

⇨ $\sum_{k=1}^{10}(3a_k-4b_k)=3\sum_{k=1}^{10}a_k-4\sum_{k=1}^{10}b_k=3\cdot 3-4\cdot 2=1$

32) 5

⇨ $\sum_{k=1}^{10}(a_k+b_k)=\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}b_k=3+2=5$

33) 26

⇨ $\sum_{k=1}^{10}(2a_k-1)^2=\sum_{k=1}^{10}(4a_k{}^2-4a_k+1)$
$=4\sum_{k=1}^{10}a_k{}^2-4\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1$
$=4\times 7-4\times 3+10=26$

34) 79

⇨ $\sum_{k=1}^{10}(a_k-3)^2=\sum_{k=1}^{10}(a_k{}^2-6a_k+9)$
$=\sum_{k=1}^{10}a_k{}^2-6\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}9$
$=7-6\times 3+9\cdot 10=79$

35) -16

⇨ $\sum_{k=1}^{10}(3a_k{}^2+a_k-4)=3\sum_{k=1}^{10}a_k{}^2+\sum_{k=1}^{10}a_k-\sum_{k=1}^{10}4$
$=3\times 7+3-4\cdot 10=-16$

36) 1

⇨ $\sum_{k=1}^{10}(a_k{}^2-2a_k)=\sum_{k=1}^{10}a_k{}^2-2\sum_{k=1}^{10}a_k$
$=7-2\times 3=1$

37) 58

⇨ $\sum_{k=1}^{10}(3a_k-2)^2=\sum_{k=1}^{10}(9a_k{}^2-12a_k+4)$
$=9\sum_{k=1}^{10}a_k{}^2-12\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}4$
$=9\cdot 6-12\cdot 3+4\cdot 10=58$

38) 58

⇨ $\sum_{k=1}^{10}(a_k+2)^2=\sum_{k=1}^{10}(a_k{}^2+4a_k+4)$
$=\sum_{k=1}^{10}a_k{}^2+4\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}4$
$=6+4\cdot 3+4\cdot 10=58$

39) 22

⇨ $\sum_{k=1}^{10}(3a_k{}^2-2a_k+1)=3\sum_{k=1}^{10}a_k{}^2-2\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1$
$=3\cdot 6-2\cdot 3+10=22$

40) 15

⇨ $\sum_{k=1}^{10}(a_k{}^2+3a_k)=\sum_{k=1}^{10}a_k{}^2+3\sum_{k=1}^{10}a_k=6+3\cdot 3=15$

41) 52

⇨ $\sum_{k=1}^{10}(3a_k-2)^2=\sum_{k=1}^{10}(9a_k{}^2-12a_k+4)$
$=9\sum_{k=1}^{10}a_k{}^2-12\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}4$
$=9\times 4-12\times 2+4\times 10=52$

42) 36

⇨ $\sum_{k=1}^{10}(a_k-2)^2=\sum_{k=1}^{10}(a_k{}^2-4a_k+4)$

$=\sum_{k=1}^{10}a_k{}^2-4\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}4$

$=4-4\times2+4\times10=36$

43) 18

⇨ $\sum_{k=1}^{10}(a_k+1)^2=\sum_{k=1}^{10}(a_k{}^2+2a_k+1)$

$=\sum_{k=1}^{10}a_k{}^2+2\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1$

$=4+2\times2+1\times10=18$

44) 20

⇨ $\sum_{k=1}^{10}(a_k-2b_k+2)=\sum_{k=1}^{10}a_k-2\sum_{k=1}^{10}b_k+\sum_{k=1}^{10}2$

$=100-2\times50+2\times10=20$

45) 150

⇨ $\sum_{k=1}^{10}(a_k+b_k)=\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}b_k=100+50=150$

46) 15

⇨ $\sum_{k=16}^{20}a_k=\sum_{k=1}^{20}a_k-\sum_{k=1}^{15}a_k=55-40=15$

47) 30

⇨ $\sum_{k=1}^{10}(5a_k+2)=5\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}2=5\cdot2+2\cdot10=30$

48) 13

⇨ $\sum_{k=1}^{10}(2a_k+3b_k)=2\sum_{k=1}^{10}a_k+3\sum_{k=1}^{10}b_k=2\cdot2+3\cdot3=13$

49) 47

⇨ $\sum_{k=1}^{10}(4a_k-3b_k+2)=4\sum_{k=1}^{10}a_k-3\sum_{k=1}^{0}b_k+\sum_{k=1}^{10}2$

$=4\cdot12-3\cdot7+2\cdot10=47$

50) 20

⇨ $\sum_{k=11}^{20}a_k=\sum_{k=1}^{20}a_k-\sum_{k=1}^{10}a_k=30-10=20$

51) 102

⇨ $\sum_{k=1}^{6}(2a_k-1)^2=\sum_{k=1}^{6}(4a_k^2-4a_k+1)$

$=4\cdot30-4\cdot6+6=102$

52) 2

⇨ $\sum_{k=1}^{10}(a_k+2)(a_k-1)=\sum_{k=1}^{10}(a_k^2+a_k-2)$

$=15+7-2\cdot10=2$

53) 30

⇨ $\sum_{k=1}^{10}(2a_k+1)=30$에서

$2\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1=30,\ 2\sum_{k=1}^{10}a_k+10=30$

$\therefore\ \sum_{k=1}^{10}a_k=10$

$\sum_{k=1}^{10}(a_k{}^2+a_k+1)=\sum_{k=1}^{10}a_k{}^2+\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1$

$=\sum_{k=1}^{10}a_k{}^2+10+10=\sum_{k=1}^{10}a_k{}^2+20=50$

$\therefore\sum_{k=1}^{10}a_k{}^2=30$

54) 80

⇨ $\sum_{k=1}^{10}\{(a_k+1)^2+b_k\}=\sum_{k=1}^{10}\{a_k^2+2a_k+1+b_k\}$

$=30+2\cdot15+10+10=80$

55) 70

⇨ $\sum_{k=1}^{10}(3a_k-b_k+2)=3\cdot30-40+2\cdot10=70$

56) 10

⇨ $\sum_{k=1}^{10}(3a_k-2b_k+1)=3\cdot20-2\cdot30+10=10$

57) 30

⇨ $\sum_{n=1}^{10}(2a_n+3b_n-1)=2\cdot5+3\cdot10-10=30$

58) 200

⇨ $\sum_{k=1}^{30}(3a_k-2b_k+6)=3\cdot10-2\cdot5+6\cdot30=200$

59) 90

⇨ $\sum_{k=1}^{10}a_k=\sum_{k=1}^{20}a_k-\sum_{k=11}^{20}a_k=35-15=20$

$\therefore\sum_{k=1}^{10}(2a_k+5b_k-1)=2\cdot20+5\cdot12-10=90$

60) 325

⇨ $\sum_{k=1}^{7}(3a_k-5)^2=\sum_{k=1}^{7}(9a_k^2-30a_k+25)$

$=9\cdot50-30\cdot10+25\cdot7=325$

61) 1

⇨ $\sum_{k=1}^{n}a_k=5,\ \sum_{k=1}^{n}b_k=-2$이므로

$\sum_{k=1}^{n}(3a_k+7b_k)=3\sum_{k=1}^{n}a_k+7\sum_{k=1}^{n}b_k=15-14=1$

62) 210

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⇨ $\sum_{k=1}^{10} a_k = 10$, $\sum_{k=1}^{10} {a_k}^2 = 40$ 이므로

$$\sum_{k=1}^{10}(2a_k+1)^2 = \sum_{k=1}^{10}\left(4{a_k}^2+4a_k+1\right)$$
$$= 4\sum_{k=1}^{10}{a_k}^2 + 4\sum_{k=1}^{10}a_k + \sum_{k=1}^{10}1$$
$$= (4\times 40)+(4\times 10)+10$$
$$= 210$$

63) 401

⇨ $\sum_{k=1}^{20}(k^2+1) - \sum_{k=1}^{19}(k^2+1)$

$$= \sum_{k=20}^{20}(k^2+1) = 20^2+1 = 401$$

64) 5

⇨ $\sum_{k=1}^{20}k^2 - \sum_{k=3}^{20}k^2 = \sum_{k=1}^{2}k^2 = 1^2+2^2 = 5$

65) 80

⇨ $\sum_{k=1}^{20}(k^2+2) - \sum_{k=1}^{20}(k^2-2)$

$$= \sum_{k=1}^{20}\{(k^2+2)-(k^2-2)\} = \sum_{k=1}^{20}4 = 4\times 20 = 80$$