01 여러 가지 각에 대한 삼각함수의 성질

(1) $2n\pi+x$의 삼각함수 (단, n은 정수)
① $\sin(2n\pi+x)=\sin x$
② $\cos(2n\pi+x)=\cos x$
③ $\tan(2n\pi+x)=\tan x$

(2) $-x$의 삼각함수
① $\sin(-x)=-\sin x$
② $\cos(-x)=\cos x$
③ $\tan(-x)=-\tan x$

(3) $\frac{\pi}{2}\pm x$의 삼각함수
① $\sin\left(\frac{\pi}{2}+x\right)=\cos x$, $\sin\left(\frac{\pi}{2}-x\right)=\cos x$
② $\cos\left(\frac{\pi}{2}+x\right)=-\sin x$, $\cos\left(\frac{\pi}{2}-x\right)=\sin x$
③ $\tan\left(\frac{\pi}{2}+x\right)=-\frac{1}{\tan x}$, $\tan\left(\frac{\pi}{2}-x\right)=\frac{1}{\tan x}$

(4) $\pi\pm x$의 삼각함수
① $\sin(\pi+x)=-\sin x$, $\sin(\pi-x)=\sin x$
② $\cos(\pi+x)=-\cos x$, $\cos(\pi-x)=-\cos x$
③ $\tan(\pi+x)=\tan x$, $\tan(\pi-x)=-\tan x$

◪ 다음 θ에 대하여 $\sin\theta$, $\cos\theta$, $\tan\theta$ 의 값을 구하여라.

1. $\theta=\frac{5}{4}\pi$

2. $\theta=\frac{7}{6}\pi$

3. $\theta=\frac{2}{3}\pi$

4. $\theta=-\frac{\pi}{3}$

5. $\theta=-\frac{\pi}{4}$

◪ 다음 삼각함수의 값을 구하여라.

6. $\tan\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$

7. $\tan\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$

8. $\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

9. $\sin\frac{5}{4}\pi$

10. $\tan\frac{4}{3}\pi$

11. $\cos\frac{7}{6}\pi$

12. $\tan\frac{19}{3}\pi$

13. $\tan\left(-\frac{9}{4}\pi\right)$

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14. $\cos\frac{9}{4}\pi$

15. $\cos\frac{5}{4}\pi$

16. $\sin\frac{5}{6}\pi$

17. $\tan\frac{7}{3}\pi$

18. $\cos\frac{25}{6}\pi$

19. $\tan\frac{5}{6}\pi$

20. $\cos\frac{3}{4}\pi$

21. $\sin\frac{2}{3}\pi$

22. $\sin\frac{13}{6}\pi$

23. $\sin\left(-\frac{13}{6}\pi\right)$

24. $\sin\left(-\frac{\pi}{4}\right)$

25. $\cos\left(-\frac{\pi}{4}\right)$

26. $\sin\left(-\frac{\pi}{6}\right)$

27. $\tan\frac{11}{6}\pi$

28. $\sin 120^\circ$

29. $\tan 150^\circ$

30. $\cos 135^\circ$

31. $\sin 210^\circ$

32. $\tan 390^\circ$

33. $\tan 210^\circ$

34. $\cos 330^\circ$

35. $\sin 780^{\circ}$

36. $\cos 750^{\circ}$

37. $\sin 450^{\circ}$

38. $\cos 240^{\circ}$

◪ 다음 삼각함수의 값을 이용하여 $\sin(\pi+\theta)$, $\cos(\pi+\theta)$, $\tan(\pi+\theta)$의 값을 구하여라. $\left(단,\ 0<\theta<\frac{\pi}{2}\right)$

39. $\tan\theta=\frac{2}{3}$

40. $\cos\theta=\frac{4}{5}$

41. $\sin\theta=\frac{\sqrt{3}}{4}$

◪ 다음 삼각함수의 값을 이용하여 $\sin(\pi-\theta)$, $\cos(\pi-\theta)$, $\tan(\pi-\theta)$의 값을 구하여라. $\left(단,\ 0<\theta<\frac{\pi}{2}\right)$

42. $\tan\theta=\frac{\sqrt{6}}{3}$

43. $\cos\theta=\frac{4}{5}$

44. $\sin\theta=\frac{\sqrt{5}}{5}$

◪ 다음 삼각함수의 값을 이용하여 $\sin\left(\frac{\pi}{2}-\theta\right)$, $\cos\left(\frac{\pi}{2}-\theta\right)$, $\tan\left(\frac{\pi}{2}-\theta\right)$의 값을 구하여라. $\left(단,\ 0<\theta<\frac{\pi}{2}\right)$

45. $\tan\theta=\frac{\sqrt{2}}{2}$

46. $\cos\theta=\frac{12}{13}$

47. $\sin\theta=\frac{8}{17}$

◪ 다음 삼각함수의 값을 이용하여 $\sin\left(\frac{\pi}{2}+\theta\right)$, $\cos\left(\frac{\pi}{2}+\theta\right)$, $\tan\left(\frac{\pi}{2}+\theta\right)$의 값을 구하여라. $\left(단,\ 0<\theta<\frac{\pi}{2}\right)$

48. $\tan\theta=\frac{\sqrt{15}}{7}$

49. $\cos\theta=\frac{4}{5}$

50. $\cos\theta=\frac{5}{6}$

51. $\sin\theta=\frac{\sqrt{14}}{6}$

◪ 다음 식의 값을 구하여라.

52. $\sin\frac{5}{6}\pi+\cos\frac{4}{3}\pi$

53. $\cos\frac{7\pi}{6}-\tan\frac{2\pi}{3}$

54. $\sin\left(\frac{3}{2}\pi-\frac{\pi}{3}\right)+\tan\frac{7}{4}\pi$

55. $\sin\frac{7}{6}\pi+\cos\left(-\frac{8}{3}\pi\right)-\tan\left(-\frac{7}{4}\pi\right)$

56. $\sin\frac{2}{3}\pi+\cos\left(-\frac{13}{6}\pi\right)+\tan\frac{8}{3}\pi$

57. $\frac{1}{\sin\frac{\pi}{3}}-\cos\frac{3}{4}\pi+2\tan\left(-\frac{7}{6}\pi\right)$

58. $\sin\left(-\frac{13}{6}\pi\right)\cos\left(-\frac{2}{3}\pi\right)+\frac{1}{\sin300^\circ}\cdot\tan210^\circ$

◪ 다음 식을 간단히 하여라.

59. $\frac{\sin\left(\frac{3}{2}\pi-\theta\right)}{\cos\left(\frac{\pi}{2}+\theta\right)\cos^2\theta}-\frac{\sin(\pi+\theta)\tan(\pi-\theta)}{\cos\left(\frac{3}{2}\pi+\theta\right)}$

60. $\frac{\cos(\pi+\theta)}{\sin\left(\frac{3}{2}\pi+\theta\right)}+\frac{\tan\frac{19}{4}\pi}{\sin\frac{5}{2}\pi-\cos^2\left(\frac{5}{2}\pi-\theta\right)}$

◪ 다음 식의 값을 구하여라.

61. $\frac{\cos(\pi-\theta)\tan(\pi-\theta)}{\cos\left(\frac{\pi}{2}-\theta\right)}$

62. $\sin\left(\frac{5\pi}{2}-\theta\right)+\cos(-\pi+\theta)+\sin\left(\frac{3}{2}\pi+\theta\right)+\cos(-\theta)$

63. $\sin\left(\frac{\pi}{2}-\theta\right)-\sin(\pi-\theta)+\cos(\pi+\theta)+\cos\left(\frac{3\pi}{2}+\theta\right)$

64. $\sin^2\left(\frac{\pi}{2}+\theta\right)+\sin^2(\pi+\theta)$

65. $\dfrac{\sin(\pi+\theta)\tan^2(\pi-\theta)}{\cos\left(\frac{3}{2}\pi+\theta\right)}-\dfrac{\sin\left(\frac{3}{2}\pi-\theta\right)}{\sin\left(\frac{\pi}{2}+\theta\right)\cos^2\theta}$

66. $\dfrac{\sin\theta\sin\left(\frac{\pi}{2}+\theta\right)}{\tan\left(\frac{\pi}{2}+\theta\right)}+\cos\theta\tan\left(\frac{\pi}{2}-\theta\right)\cos\left(\frac{\pi}{2}+\theta\right)$

67. $\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)+\sin\left(\pi+\frac{\pi}{6}\right)-\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

◪ 다음 식의 값을 구하여라.

68. $\tan 1^\circ \tan 3^\circ \tan 5^\circ \ \cdots\ \tan 87^\circ \tan 89^\circ$

69. $\sin^2 5^\circ+\sin^2 10^\circ+\sin^2 15^\circ+\cdots+\sin^2 90^\circ$

70. $\tan 2^\circ\times\tan 4^\circ\times\cdots\times\tan 86^\circ\times\tan 88^\circ$

71. $\cos^2 0^\circ+\cos^2 10^\circ+\cos^2 20^\circ+\cdots+\cos^2 90^\circ$

72. $\cos^2 0^\circ+\cos^2 1^\circ+\cos^2 2^\circ+\cdots+\cos^2 360^\circ$

73. $\tan 1^\circ\ \tan 2^\circ\ \cdots\ \tan 8^\circ\ \tan 89^\circ$

74. $\cos^2 10^\circ+\cos^2 20^\circ+\cdots+\cos^2 70^\circ+\ \cos^2 80^\circ$

75. $\sin^2 1^\circ+\sin^2 2^\circ+\cdots+\sin^2 89^\circ+\sin^2 90^\circ$

76. $\sin^2 10^\circ+\sin^2 20^\circ+\sin^2 30^\circ+\cdots+\sin^2 80^\circ$

77. $\cos^2 1^\circ+\cos^2 2^\circ+\cdots+\cos^2 88^\circ+\ \cos^2 89^\circ$

78. $\sin^2 10^\circ+\sin^2 20^\circ+\cdots+\sin^2 80^\circ+\sin^2 90^\circ$

정답 및 해설

1) $\sin\frac{5}{4}\pi=-\frac{\sqrt{2}}{2}$, $\cos\frac{5}{4}\pi=-\frac{\sqrt{2}}{2}$, $\tan\frac{5}{4}\pi=1$

⇨ $\sin\frac{5}{4}\pi=\sin\left(\pi+\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

$\cos\frac{5}{4}\pi=\cos\left(\pi+\frac{\pi}{4}\right)=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

$\tan\frac{5}{4}\pi=\tan\left(\pi+\frac{\pi}{4}\right)=\tan\frac{\pi}{4}=1$

2) $\sin\frac{7}{6}\pi=-\frac{1}{2}$, $\cos\frac{7}{6}\pi=-\frac{\sqrt{3}}{2}$, $\tan\frac{7}{6}\pi=\frac{\sqrt{3}}{3}$

⇨ $\sin\frac{7}{6}\pi=\sin\left(\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

$\cos\frac{7}{6}\pi=\cos\left(\pi+\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$

$\tan\frac{7}{6}\pi=\tan\left(\pi+\frac{\pi}{6}\right)=\tan\frac{\pi}{6}=\frac{\sqrt{3}}{3}$

3) $\sin\frac{2}{3}\pi=\frac{\sqrt{3}}{2}$, $\cos\frac{2}{3}\pi=-\frac{1}{2}$, $\tan\frac{2}{3}\pi=-\sqrt{3}$

⇨ $\sin\frac{2}{3}\pi=\sin\left(\pi-\frac{\pi}{3}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\cos\frac{2}{3}\pi=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}=-\frac{1}{2}$

$\tan\frac{2}{3}\pi=\tan\left(\pi-\frac{\pi}{3}\right)=-\tan\frac{\pi}{3}=-\sqrt{3}$

4) $\sin\left(-\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$, $\cos\left(-\frac{\pi}{3}\right)=\frac{1}{2}$,

$\tan\left(-\frac{\pi}{3}\right)=-\sqrt{3}$

⇨ $\sin\left(-\frac{\pi}{3}\right)=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$

$\cos\left(-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$

$\tan\left(-\frac{\pi}{3}\right)=-\tan\frac{\pi}{3}=-\sqrt{3}$

5) $\sin\left(-\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$, $\cos\left(-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$,

$\tan\left(-\frac{\pi}{4}\right)=-1$

⇨ $\sin\left(-\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

$\cos\left(-\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$\tan\left(-\frac{\pi}{4}\right)=-\tan\frac{\pi}{4}=-1$

6) $-\sqrt{3}$

⇨ $\tan\left(\frac{\pi}{2}+\frac{\pi}{6}\right)=-\frac{1}{\tan\frac{\pi}{6}}=-\sqrt{3}$

7) $\sqrt{3}$

⇨ $\tan\left(\frac{\pi}{2}-\frac{\pi}{6}\right)=\frac{1}{\tan\frac{\pi}{6}}=\sqrt{3}$

8) $\frac{1}{2}$

⇨ $\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$

9) $-\frac{\sqrt{2}}{2}$

⇨ $\sin\frac{5}{4}\pi=\sin\left(\pi+\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

10) $\sqrt{3}$

⇨ $\tan\frac{4}{3}\pi=\tan\left(\pi+\frac{\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

11) $-\frac{\sqrt{3}}{2}$

⇨ $\cos\frac{7}{6}\pi=\cos\left(\pi+\frac{\pi}{6}\right)=-\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$

12) $\sqrt{3}$

⇨ $\tan\frac{19}{3}\pi=\tan\left(6\pi+\frac{\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

13) -1

⇨ $\tan\left(-\frac{9}{4}\pi\right)=-\tan\frac{9}{4}\pi=-\tan\left(2\pi+\frac{\pi}{4}\right)$

$=-\tan\frac{\pi}{4}=-1$

14) $\frac{\sqrt{2}}{2}$

⇨ $\cos\frac{9}{4}\pi=\cos\left(2\pi+\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

15) $-\frac{\sqrt{2}}{2}$

⇨ $\cos\frac{5}{4}\pi=\cos\left(\pi+\frac{\pi}{4}\right)=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

16) $\frac{1}{2}$

⇨ $\sin\frac{5}{6}\pi=\sin\left(\pi-\frac{\pi}{6}\right)=\sin\frac{\pi}{6}=\frac{1}{2}$

17) $\sqrt{3}$

⇨ $\tan\frac{7}{3}\pi=\tan\left(2\pi+\frac{\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

18) $\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\cos\frac{25}{6}\pi=\cos\left(4\pi+\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

19) $-\frac{\sqrt{3}}{3}$

$\Rightarrow$ $\tan\frac{5}{6}\pi=\tan\left(\pi-\frac{\pi}{6}\right)=-\tan\frac{\pi}{6}=-\frac{\sqrt{3}}{3}$

20) $-\frac{\sqrt{2}}{2}$

$\Rightarrow$ $\cos\frac{3}{4}\pi=\cos\left(\pi-\frac{\pi}{4}\right)=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

21) $\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin\frac{2}{3}\pi=\sin\left(\pi-\frac{\pi}{3}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

22) $\frac{1}{2}$

$\Rightarrow$ $\sin\frac{13}{6}\pi=\sin\left(2\pi+\frac{\pi}{6}\right)=\sin\frac{\pi}{6}=\frac{1}{2}$

23) $-\frac{1}{2}$

$\Rightarrow$ $\sin\left(-\frac{13}{6}\pi\right)=-\sin\frac{13}{6}\pi=-\sin\left(2\pi+\frac{\pi}{6}\right)$

$=-\sin\frac{\pi}{6}=-\frac{1}{2}$

24) $-\frac{\sqrt{2}}{2}$

$\Rightarrow$ $\sin\left(-\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

25) $\frac{\sqrt{2}}{2}$

$\Rightarrow$ $\cos\left(-\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

26) $-\frac{1}{2}$

$\Rightarrow$ $\sin\left(-\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

27) $-\frac{\sqrt{3}}{3}$

$\Rightarrow$ $\tan\frac{11}{6}\pi=\tan\left(2\pi-\frac{\pi}{6}\right)=\tan\left(-\frac{\pi}{6}\right)$

$=-\tan\frac{\pi}{6}=-\frac{\sqrt{3}}{3}$

28) $\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin 120^\circ=\sin(90^\circ+30^\circ)=\cos 30^\circ=\frac{\sqrt{3}}{2}$

29) $-\frac{\sqrt{3}}{3}$

$\Rightarrow$ $\tan 150^\circ=\tan(90^\circ+60^\circ)=-\frac{1}{\tan 60^\circ}=-\frac{\sqrt{3}}{3}$

30) $-\frac{\sqrt{2}}{2}$

$\Rightarrow$ $\cos 135^\circ=\cos(90^\circ+45^\circ)=-\sin 45^\circ=-\frac{\sqrt{2}}{2}$

31) $-\frac{1}{2}$

$\Rightarrow$ $\sin 210^\circ=\sin(180^\circ+30^\circ)=-\sin 30^\circ=-\frac{1}{2}$

32) $\frac{\sqrt{3}}{3}$

$\Rightarrow$ $\tan 390^\circ=\tan(360^\circ+30^\circ)=\tan 30^\circ=\frac{\sqrt{3}}{3}$

33) $\frac{\sqrt{3}}{3}$

$\Rightarrow$ $\tan 210^\circ=\tan(180^\circ+30^\circ)=\tan 30^\circ=\frac{\sqrt{3}}{3}$

34) $\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\cos 330^\circ=\cos(360^\circ-30^\circ)=\cos(-30^\circ)$

$=\cos 30^\circ=\frac{\sqrt{3}}{2}$

35) $\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin 780^\circ=\sin(360^\circ\times 2+60^\circ)=\sin 60^\circ=\frac{\sqrt{3}}{2}$

36) $\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\cos 750^\circ=\cos(2\times 360^\circ+30^\circ)=\cos 30^\circ=\frac{\sqrt{3}}{2}$

37) 1

$\Rightarrow$ $\sin 450^\circ=\sin(360^\circ+90^\circ)=\sin 90^\circ=1$

38) $-\frac{1}{2}$

$\Rightarrow$ $\cos 240^\circ=\cos(180^\circ+60^\circ)=-\cos 60^\circ=-\frac{1}{2}$

39) $\sin(\pi+\theta)=-\frac{2\sqrt{13}}{13}$, $\cos(\pi+\theta)=-\frac{3\sqrt{13}}{13}$

$\tan(\pi+\theta)=\frac{2}{3}$

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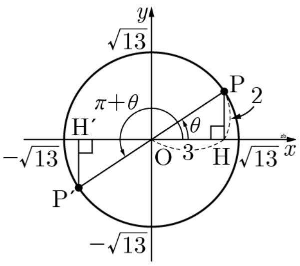
⇨ 

$\overline{OP}=\sqrt{3^2+2^2}=\sqrt{13}$이므로 $\sin(\pi+\theta)=-\dfrac{2\sqrt{13}}{\sqrt{13}}$

$\cos(\pi+\theta)=-\dfrac{3\sqrt{13}}{13}$, $\tan(\pi+\theta)=\dfrac{2}{3}$

40) $\sin(\pi+\theta)=-\dfrac{3}{5}$, $\cos(\pi+\theta)=-\dfrac{4}{5}$,

$\tan(\pi+\theta)=\dfrac{3}{4}$

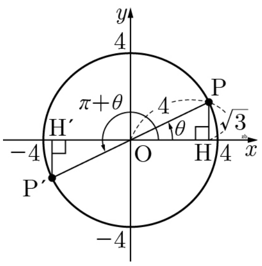
⇨ $\cos\theta=\dfrac{4}{5}$일 때 $\sin\theta=\dfrac{3}{5}$, $\tan\theta=\dfrac{3}{4}$이므로

$\sin(\pi+\theta)=-\dfrac{3}{5}$, $\cos(\pi+\theta)=-\dfrac{4}{5}$

$\tan(\pi+\theta)=\dfrac{3}{4}$

41) $\sin(\pi+\theta)=-\dfrac{\sqrt{3}}{4}$, $\cos(\pi+\theta)=-\dfrac{\sqrt{13}}{4}$

$\tan(\pi+\theta)=\dfrac{\sqrt{3}}{\sqrt{13}}=\dfrac{\sqrt{39}}{13}$

⇨ 

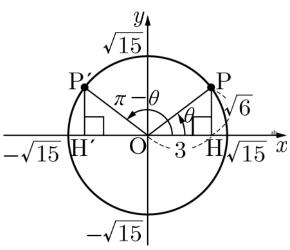
$\overline{OH}=\sqrt{4^2-\sqrt{3}^{\,2}}=\sqrt{13}$이므로

$\sin(\pi+\theta)=-\dfrac{\sqrt{3}}{4}$, $\cos(\pi+\theta)=-\dfrac{\sqrt{13}}{4}$

$\tan(\pi+\theta)=\dfrac{\sqrt{3}}{\sqrt{13}}=\dfrac{\sqrt{39}}{13}$

42) $\sin(\pi-\theta)=\dfrac{\sqrt{10}}{5}$, $\cos(\pi-\theta)=-\dfrac{\sqrt{15}}{5}$,

$\tan(\pi-\theta)=-\dfrac{\sqrt{6}}{3}$

⇨ 

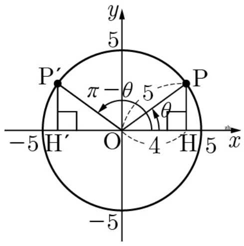
$\overline{OP}=\sqrt{3^2+\sqrt{6}^{\,2}}=\sqrt{15}$이므로

$\sin(\pi-\theta)=\dfrac{\sqrt{10}}{5}$, $\cos(\pi-\theta)=-\dfrac{\sqrt{15}}{5}$

$\tan(\pi-\theta)=-\dfrac{\sqrt{6}}{3}$

43) $\sin(\pi-\theta)=\dfrac{3}{5}$, $\cos(\pi-\theta)=-\dfrac{4}{5}$,

$\tan(\pi-\theta)=-\dfrac{3}{4}$

⇨ 

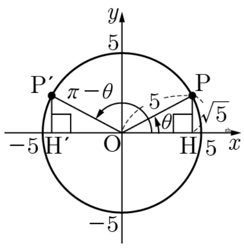
$\overline{HP}=\sqrt{5^2-4^2}=3$이므로

$\sin(\pi-\theta)=\dfrac{3}{5}$, $\cos(\pi-\theta)=-\dfrac{4}{5}$

$\tan(\pi-\theta)=-\dfrac{3}{4}$

44) $\sin(\pi-\theta)=\dfrac{\sqrt{5}}{5}$, $\cos(\pi-\theta)=-\dfrac{2\sqrt{5}}{5}$,

$\tan(\pi-\theta)=-\dfrac{1}{2}$

⇨ 

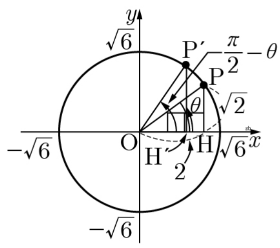
$\overline{OH}=\sqrt{5^2-\sqrt{5}^{\,2}}=2\sqrt{5}$이므로

$\sin(\pi-\theta)=\dfrac{\sqrt{5}}{5}$, $\cos(\pi-\theta)=-\dfrac{2\sqrt{5}}{5}$

$\tan(\pi-\theta)=-\dfrac{1}{2}$

45) $\sin\left(\dfrac{\pi}{2}-\theta\right)=\dfrac{\sqrt{6}}{3}$, $\cos\left(\dfrac{\pi}{2}-\theta\right)=\dfrac{\sqrt{3}}{3}$

$\tan\left(\dfrac{\pi}{2}-\theta\right)=\sqrt{2}$

⇨ 

$\overline{OP}=\sqrt{2^2+\sqrt{2}^{\,2}}=\sqrt{6}$이므로

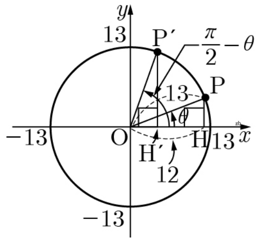
$\sin\left(\dfrac{\pi}{2}-\theta\right)=\dfrac{\sqrt{6}}{3}$

$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{\sqrt{3}}{3}$

$\tan\left(\frac{\pi}{2}-\theta\right)=\sqrt{2}$

46) $\sin\left(\frac{\pi}{2}-\theta\right)=\frac{12}{13}$, $\cos\left(\frac{\pi}{2}-\theta\right)=\frac{5}{13}$

$\tan\left(\frac{\pi}{2}-\theta\right)=\frac{12}{5}$

⇨ 

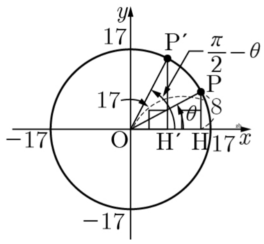
$\overline{HP}=\sqrt{13^2-12^2}=5$이므로

$\sin\left(\frac{\pi}{2}-\theta\right)=\frac{12}{13}$, $\cos\left(\frac{\pi}{2}-\theta\right)=\frac{5}{13}$

$\tan\left(\frac{\pi}{2}-\theta\right)=\frac{12}{5}$

47) $\sin\left(\frac{\pi}{2}-\theta\right)=\frac{15}{17}$, $\cos\left(\frac{\pi}{2}-\theta\right)=\frac{8}{17}$

$\tan\left(\frac{\pi}{2}-\theta\right)=\frac{15}{8}$

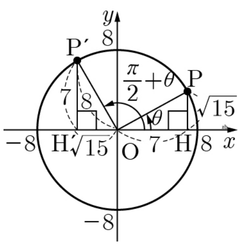
⇨ 

$\overline{OH}=\sqrt{17^2-8^2}=15$이므로 $\sin\left(\frac{\pi}{2}-\theta\right)=\frac{15}{17}$

$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{8}{17}$, $\tan\left(\frac{\pi}{2}-\theta\right)=\frac{15}{8}$

48) $\sin\left(\frac{\pi}{2}+\theta\right)=\frac{7}{8}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{15}}{8}$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\frac{7\sqrt{15}}{15}$

⇨ 

$\overline{OP}=\sqrt{7^2+\sqrt{15}^{\,2}}=8$이므로

$\sin\left(\frac{\pi}{2}+\theta\right)=\frac{7}{8}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{15}}{8}$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\frac{7\sqrt{15}}{15}$

49) $\sin\left(\frac{\pi}{2}+\theta\right)=\frac{4}{5}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{3}{5}$,

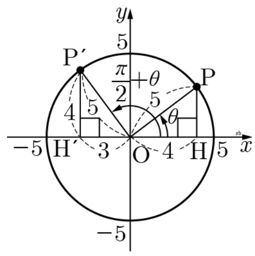
$\tan\left(\frac{\pi}{2}+\theta\right)=-\frac{4}{3}$

⇨ △POH와 △OP′H′이 합동이고,

$\overline{HP}=\sqrt{5^2-4^2}=3$이므로

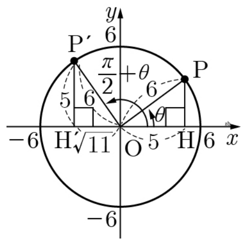
$\sin\left(\frac{\pi}{2}+\theta\right)=\frac{4}{5}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{3}{5}$

$\tan\left(\frac{\pi}{2}=\theta\right)=-\frac{4}{3}$



50) $\sin\left(\frac{\pi}{2}+\theta\right)=\frac{5}{6}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{11}}{6}$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\frac{5\sqrt{11}}{11}$

⇨ 

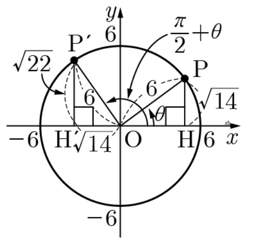
$\overline{HP}=\sqrt{6^2-5^2}=\sqrt{11}$이므로

$\sin\left(\frac{\pi}{2}+\theta\right)=\frac{5}{6}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{11}}{6}$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\frac{5\sqrt{11}}{11}$

51) $\sin\left(\frac{\pi}{2}+\theta\right)=\frac{\sqrt{22}}{6}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{14}}{6}$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{77}}{7}$

⇨ 

$\overline{OH}=\sqrt{6^2-\sqrt{14}^{\,2}}=\sqrt{22}$이므로

$\sin\left(\frac{\pi}{2}+\theta\right)=\frac{\sqrt{22}}{6}$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{14}}{6}$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\frac{\sqrt{77}}{7}$

52) 0

⇨ $\sin\frac{5}{6}\pi=\frac{1}{2}$, $\cos\frac{4}{3}\pi=-\frac{1}{2}$ 이므로 $\frac{1}{2}-\frac{1}{2}=0$이다.

53) $\frac{\sqrt{3}}{2}$

⇨ $\cos\frac{7}{6}\pi=-\frac{\sqrt{3}}{2}$, $\tan\frac{2}{3}\pi=-\sqrt{3}$

$\cos\frac{7\pi}{6}-\tan\frac{2\pi}{3}=-\frac{\sqrt{3}}{2}+\sqrt{3}=\frac{\sqrt{3}}{2}$

54) $-\frac{3}{2}$

⇨ $\sin\left(\frac{3}{2}\pi-\frac{\pi}{3}\right)=\sin\left\{2\pi-\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\right\}$

$=-\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)=-\cos\left(-\frac{\pi}{3}\right)=-\frac{1}{2}$

$\tan\frac{7}{4}\pi=\tan\frac{3}{4}\pi=-1$

$\therefore\ \sin\left(\frac{3}{2}\pi-\frac{\pi}{3}\right)+\tan\frac{7}{4}\pi=-\frac{1}{2}-1=-\frac{3}{2}$

55) -2

56) 0

⇨ $\sin\frac{2}{3}\pi=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\cos\left(-\frac{13}{6}\pi\right)=\cos\left(-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$\tan\frac{8}{3}\pi=\tan\left(-\frac{\pi}{3}\right)=-\tan\frac{\pi}{3}=-\sqrt{3}$

$\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}-\sqrt{3}=0$

57) $\frac{\sqrt{2}}{2}$

⇨ $\frac{1}{\sin\frac{\pi}{3}}=\frac{1}{\frac{\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\cos\frac{3}{4}\pi=-\frac{\sqrt{2}}{2}$

$\tan\left(-\frac{7}{6}\pi\right)=\tan\left(-\frac{\pi}{6}\right)=-\tan\frac{\pi}{6}=-\left(\frac{1}{\sqrt{3}}\right)=-\frac{\sqrt{3}}{3}$

(주어진 값)$=\frac{2\sqrt{3}}{3}+\frac{\sqrt{2}}{2}-\frac{2\sqrt{3}}{3}=\frac{\sqrt{2}}{2}$

58) $-\frac{5}{12}$

59) $\frac{1}{\tan\theta}$

⇨ $\sin\left(\frac{3}{2}\pi-\theta\right)=-\cos\theta$, $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

$\sin(\pi+\theta)=-\sin\theta$, $\tan(\pi-\theta)=-\tan\theta$

$\cos\left(\frac{3}{2}\pi+\theta\right)=\sin\theta$

따라서 주어진 값은

$\frac{-\cos\theta}{-\sin\theta\cos^2\theta}-\frac{-\sin\theta(-\tan\theta)}{\sin\theta}=\frac{\cos\theta}{\sin\theta\cos^2\theta}-\frac{\sin\theta\tan\theta}{\sin\theta}$

$=\frac{1}{\sin\theta\cos\theta}-\tan\theta=\frac{1}{\sin\theta\cos\theta}-\frac{\sin\theta}{\cos\theta}=\frac{1-\sin^2\theta}{\sin\theta\cos\theta}$

$=\frac{\cos^2\theta}{\sin\theta\cos\theta}=\frac{\cos\theta}{\sin\theta}=\frac{1}{\tan\theta}$

60) $-\tan^2\theta$

⇨ $\cos(\pi+\theta)=-\cos\theta$, $\sin\left(\frac{3}{2}\pi+\theta\right)=-\cos\theta$

$\tan\frac{19}{4}\pi=\tan\left(4\pi+\frac{3}{4}\pi\right)=\tan\frac{3}{4}\pi=-1$

$\sin\frac{5}{2}\pi=\sin\left(2\pi+\frac{\pi}{2}\right)=\sin\frac{\pi}{2}=1$

$\cos\left(\frac{5}{2}\pi-\theta\right)=\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$

(주어진 식)$=\frac{-\cos\theta}{-\cos\theta}+\frac{(-1)}{1-\sin^2\theta}=1-\frac{1}{\cos^2\theta}$

$=\frac{\cos^2\theta-1}{\cos^2\theta}=-\frac{\sin^2\theta}{\cos^2\theta}=-\tan^2\theta$

61) 1

⇨ $\frac{\cos(\pi-\theta)\tan(\pi-\theta)}{\cos\left(\frac{\pi}{2}-\theta\right)}=\frac{-\cos\theta\times(-\tan\theta)}{\sin\theta}$

$=\cos\theta\times\frac{\sin\theta}{\cos\theta}\times\frac{1}{\sin\theta}=1$

62) 0

⇨ $\sin\left(\frac{5\pi}{2}-\theta\right)+\cos(-\pi+\theta)+\sin\left(\frac{3}{2}\pi+\theta\right)+\cos(-\theta)$

$=\cos\theta-\cos\theta-\cos\theta+\cos\theta=0$

63) 0

⇨ $\sin\left(\frac{\pi}{2}-\theta\right)-\sin(\pi-\theta)+\cos(\pi+\theta)+\cos\left(\frac{3}{2}\pi+\theta\right)$

$=\cos\theta-\sin\theta-\cos\theta+\sin\theta=0$

64) 1

⇨ $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$

$\sin(\pi+\theta)=-\sin\theta$

$\therefore\ \sin^2\left(\frac{\pi}{2}+\theta\right)+\sin^2(\pi+\theta)=\cos^2\theta+\sin^2\theta=1$

65) 1

⇨ $\frac{\sin(\pi+\theta)\tan^2(\pi-\theta)}{\cos\left(\frac{3}{2}\pi+\theta\right)}-\frac{\sin\left(\frac{3}{2}\pi-\theta\right)}{\sin\left(\frac{\pi}{2}+\theta\right)\cos^2\theta}$

$=\frac{-\sin\theta\tan^2\theta}{\sin\theta}-\frac{-\cos\theta}{\cos^3\theta}=-\tan^2\theta+\frac{1}{\cos^2\theta}=1$

66) -1

⇨ $\dfrac{\sin\theta\sin\left(\dfrac{\pi}{2}+\theta\right)}{\tan\left(\dfrac{\pi}{2}+\theta\right)}+\cos\theta\tan\left(\dfrac{\pi}{2}-\theta\right)\cos\left(\dfrac{\pi}{2}+\theta\right)$

$=\sin\theta\times\cos\theta\times(-\tan\theta)+\cos\theta\times\dfrac{1}{\tan\theta}\times(-\sin\theta)$

$=\sin\theta\times\cos\theta\times\left(-\dfrac{\sin\theta}{\cos\theta}\right)+\cos\theta\times\dfrac{\cos\theta}{\sin\theta}\times(-\sin\theta)$

$=-\sin^2\theta-\cos^2\theta=-(\sin^2\theta+\cos^2\theta)=-1$

67) $\dfrac{\sqrt{3}}{2}$

⇨ $\sin\left(\dfrac{\pi}{2}-\dfrac{\pi}{3}\right)+\sin\left(\pi+\dfrac{\pi}{6}\right)-\cos\left(\dfrac{\pi}{2}+\dfrac{\pi}{3}\right)$

$=\cos\dfrac{\pi}{3}-\sin\dfrac{\pi}{6}+\sin\dfrac{\pi}{3}$

$=\dfrac{1}{2}-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}=\dfrac{\sqrt{3}}{2}$

68) 1

⇨ $1^\circ+89^\circ=3^\circ+87^\circ=\cdots=43^\circ+47^\circ=90^\circ$ 에서

$\tan 1^\circ=\tan(90^\circ-89^\circ)=\dfrac{1}{\tan 89^\circ}$,

$\tan 3^\circ=\tan(90^\circ-87^\circ)=\dfrac{1}{\tan 87^\circ}$,

$\vdots$

$\tan 43^\circ=\tan(90^\circ-47^\circ)=\dfrac{1}{\tan 47^\circ}$ 이므로

$\tan 1^\circ\tan 89^\circ=\dfrac{1}{\tan 89^\circ}\cdot\tan 89^\circ=1$,

$\tan 3^\circ\tan 87^\circ=\dfrac{1}{\tan 87^\circ}\cdot\tan 87^\circ=1$,

$\vdots$

$\tan 43^\circ\tan 47^\circ=\dfrac{1}{\tan 47^\circ}\cdot\tan 47^\circ=1$

$\therefore\ \tan 1^\circ\tan 3^\circ\cdots\tan 87^\circ\tan 89^\circ$

$=1\cdot 1\cdot\cdots\cdot 1\cdot\tan 45^\circ=1$

69) $\dfrac{19}{2}$

⇨ $\sin(90^\circ-\theta)=\cos\theta$이므로

$\sin^2 5^\circ+\sin^2 10^\circ+\sin^2 15^\circ+\cdots+\sin^2 90^\circ$

$=(\sin^2 5^\circ+\sin^2 85^\circ)+(\sin^2 10^\circ+\sin^2 80^\circ)+$

$\cdots+(\sin^2 40^\circ+\sin^2 50^\circ)+\sin^2 45^\circ+\sin^2 90^\circ$

$=(\sin^2 5^\circ+\cos^2 5^\circ)+(\sin^2 10^\circ+\cos^2 10^\circ)+$

$\cdots+(\sin^2 40^\circ+\cos^2 40^\circ)+\sin^2 45^\circ+\sin^2 90^\circ$

$=1\times 8+\left(\dfrac{\sqrt{2}}{2}\right)^2+1=\dfrac{19}{2}$

70) 1

⇨ $\tan(90^\circ-\theta)=\dfrac{1}{\tan\theta}$이므로

$\tan\theta\times\tan(90^\circ-\theta)=1$

$\therefore\ \tan 2^\circ\times\tan 4^\circ\times\cdots\times\tan 86^\circ\times\tan 88^\circ$

$=(\tan 2^\circ\times\tan 88^\circ)\times(\tan 4^\circ\times\tan 86^\circ)\times$

$\cdots\times(\tan 44^\circ\times\tan 46^\circ)$

$=\left(\tan 2^\circ\times\dfrac{1}{\tan 2^\circ}\right)\times\left(\tan 4^\circ\times\dfrac{1}{\tan 4^\circ}\right)\times$

$\cdots\times\left(\tan 44^\circ\times\dfrac{1}{\tan 44^\circ}\right)$

$=1$

71) 5

⇨ $\cos(90^\circ-\theta)=\sin\theta$ 이므로

$\cos^2\theta+\cos^2(90^\circ-\theta)=\cos^2\theta+\sin^2\theta=1$

$\therefore\ \cos^2 0^\circ+\cos^2 10^\circ+\cos^2 20^\circ+\cdots+\cos^2 90^\circ$

$=(\cos^2 0^\circ+\cos^2 90^\circ)+(\cos^2 10^\circ+\cos^2 80^\circ)+$

$\cdots+(\cos^2 40^\circ+\cos^2 50^\circ)$

$=(\cos^2 0^\circ+\sin^2 0^\circ)+(\cos^2 10^\circ+\sin^2 10^\circ)+$

$\cdots+(\cos^2 40^\circ+\sin^2 40^\circ)$

$=1+1+1+1+1=5$

72) 181

⇨ $\cos^2(2\pi-\theta)=\cos^2\theta$, $\cos^2(\pi-\theta)=\cos^2\theta$

, $\cos^2\left(\dfrac{\pi}{2}-\theta\right)=\sin^2\theta$

를 이용하여 주어진 식을 정리하면

$2(\cos^2 1^\circ+\cdots+\cos^2 179^\circ)+2\cos^2 0^\circ+\cos^2 180^\circ$

$=4(\cos^2 1^\circ+\cos^2 2^\circ+\cdots+\cos^2 88^\circ+\cos^2 89^\circ)+3$

$=4\times 44+4\cos^2 45^\circ+3=181$

$(\because\ \cos^2\alpha+\sin^2\alpha=1)$

73) 1

⇨ $\tan 1^\circ\tan 2^\circ\cdots\tan 88^\circ\tan 89^\circ$ 에서

$1^\circ+89^\circ=2^\circ+88^\circ=\cdots=44^\circ+46^\circ=90^\circ$ 이 므로

$\tan 1^\circ=\tan(90^\circ-89^\circ)=\dfrac{1}{\tan 89^\circ}$,

$\tan 2^\circ=\tan(90^\circ-88^\circ)=\dfrac{1}{\tan 88^\circ}$,

$\vdots$

$\tan 44^\circ=\tan(90^\circ-46^\circ)=\dfrac{1}{\tan 46^\circ}$

이때, $\tan 1^\circ\tan 89^\circ=\dfrac{1}{\tan 89^\circ}\tan 89^\circ=1$,

$\tan 2^\circ\tan 88^\circ=\dfrac{1}{\tan 88^\circ}\tan 88^\circ=1$,

$\vdots$

$\tan 44^\circ\tan 46^\circ=\dfrac{1}{\tan 46^\circ}\tan 46^\circ=1$

이므로 주어진 식의 값은

$\tan 1^\circ\tan 2^\circ\cdots\tan 88^\circ\tan 89^\circ$

$=1\cdot 1\cdot\cdots\cdot 1\cdot\tan 45^\circ=1$

74) 4

⇨ $\cos^2 10^\circ+\cos^2 20^\circ+\cdots+\cos^2 70^\circ+\cos^2 80^\circ$ 에서

$10^\circ+80^\circ=20^\circ+70^\circ=\cdots=40^\circ+50^\circ=90^\circ$ 이

므로

$\cos 10^\circ = \cos(90^\circ - 80^\circ) = \sin 80^\circ$,

$\cos 20^\circ = \cos(90^\circ - 70^\circ) = \sin 70^\circ$,

$\cos 30^\circ = \cos(90^\circ - 60^\circ) = \sin 60^\circ$

$\cos 40^\circ = \cos(90^\circ - 50^\circ) = \sin 50^\circ$

이때,

$\cos^2 10^\circ + \cos^2 80^\circ = \sin^2 80^\circ + \cos^2 80^\circ = 1$,

$\cos^2 20^\circ + \cos^2 70^\circ = \sin^2 70^\circ + \cos^2 70^\circ = 1$,

$\cos^2 30^\circ + \cos^2 60^\circ = \sin^2 60^\circ + \cos^2 60^\circ = 1$,

$\cos^2 40^\circ + \cos^2 50^\circ = \sin^2 50^\circ + \cos^2 50^\circ = 1$

이므로 주어진 식의 값은

$\cos^2 10^\circ + \cos^2 20^\circ + \cdots + \cos^2 70^\circ + \cos^2 80^\circ$

$= 1+1+1+1 = 4$

75) $\frac{91}{2}$

⇨ $\sin^2 1^\circ + \sin^2 2^\circ + \cdots + \sin^2 89^\circ + \sin^2 90^\circ$ 에서

$1^\circ + 89^\circ = 2^\circ + 88^\circ = \cdots = 44^\circ + 46^\circ = 90^\circ$ 이 므로

$\sin 1^\circ = \sin(90^\circ - 89^\circ) = \cos 89^\circ$

$\sin 2^\circ = \sin(90^\circ - 88^\circ) = \cos 88^\circ$,

$\vdots$

$\sin 44^\circ = \sin(90^\circ - 46^\circ) = \cos 46^\circ$

이때,

$\sin^2 1^\circ + \sin^2 89^\circ = \cos^2 89^\circ + \sin^2 89^\circ = 1$,

$\sin^2 2^\circ + \sin^2 88^\circ = \cos^2 88^\circ + \sin^2 88^\circ = 1$,

$\vdots$

$\sin^2 44^\circ + \sin^2 46^\circ = \cos^2 46^\circ + \sin^2 46^\circ = 1$

이므로 주어진 식의 값은

$\sin^2 1^\circ + \sin^2 2^\circ + \cdots + \sin^2 89^\circ + \sin^2 90^\circ$

$= 44 + \sin^2 45^\circ + \sin^2 90^\circ$

$= 44 + \frac{1}{2} + 1 = \frac{91}{2}$

76) 4

⇨ $\sin(90^\circ - x^\circ) = \cos(x^\circ)$

주어진 값은

$\sin^2 10^\circ + \sin^2 20^\circ + \cdots + \cos^2 10^\circ$

$(\sin^2 10^\circ + \cos^2 10^\circ) + \cdots + (\sin^2 40^\circ + \cos^2 40^\circ) = 4$

77) $\frac{89}{2}$

⇨ $1^\circ + 89^\circ = 2^\circ + 88^\circ = \cdots = 44^\circ + 46^\circ = 90^\circ$ 에서

$\cos 1^\circ = \cos(90^\circ - 89^\circ) = \sin 89^\circ$,

$\cos 2^\circ = \cos(90^\circ - 88^\circ) = \sin 88^\circ$,

$\vdots$

$\cos 44^\circ = \cos(90^\circ - 46^\circ) = \sin 46^\circ$ 이므로

$\cos^2 1^\circ + \cos^2 89^\circ = \sin^2 89^\circ + \cos^2 89^\circ = 1$,

$\cos^2 2^\circ + \cos^2 88^\circ = \sin^2 88^\circ + \cos^2 88^\circ = 1$,

$\vdots$

$\cos^2 44 + \cos^2 46^\circ = \sin^2 46^\circ + \cos^2 46^\circ = 1$

$\therefore \cos^2 1^\circ + \cos^2 2^\circ + \cdots + \cos^2 88^\circ + \cos^2 89^\circ$

$= 44 + \cos^2 45^\circ = 44 + \frac{1}{2} = \frac{89}{2}$

78) 5

⇨ $10^\circ + 80^\circ = 20^\circ + 70^\circ = \cdots = 40^\circ + 50^\circ$ 에서

$\sin 10^\circ = \sin(90^\circ - 80^\circ) = \cos 80^\circ$,

$\sin 20^\circ = \sin(90^\circ - 70^\circ) = \cos 70^\circ$,

$\sin 30^\circ = \sin(90^\circ - 60^\circ) = \cos 60^\circ$

$\sin 40^\circ = \sin(90^\circ - 50^\circ) = \cos 50^\circ$ 이므로

$\sin^2 10^\circ + \sin^2 80^\circ = \cos^2 80^\circ + \sin^2 80^\circ = 1$,

$\sin^2 20^\circ + \sin^2 70^\circ = \cos^2 70^\circ + \sin^2 70^\circ = 1$,

$\sin^2 30^\circ + \sin^2 60^\circ = \cos^2 60^\circ + \sin^2 60^\circ = 1$

$\sin^2 40^\circ + \sin^2 50^\circ = \cos^2 50^\circ + \sin^2 50^\circ = 1$

$\therefore \sin^2 10^\circ + \sin^2 20^\circ + \cdots + \sin^2 80^\circ + \sin^2 90^\circ$

$= 1+1+1+1+\sin^2 90^\circ = 4+1 = 5$