(2)유리식과 무리식의 극한값의 계산

01 유리식의 극한값의 계산

$\frac{\infty}{\infty}$ 꼴의 극한 ⇨ 분모의 최고차항으로 분모, 분자를 나누어 극한값을 구한다.
$\infty-\infty$ 꼴의 극한 ⇨ 최고차항으로 묶어 극한값을 구한다.

◪ 다음 극한을 조사하고, 극한이 존재하면 그 극한값을 구하여라.

1. $\lim_{n\to\infty}\frac{n+1}{2n-3}$

2. $\lim_{n\to\infty}\frac{3n^2-3n+4}{n^2+5n+2}$

3. $\lim_{n\to\infty}\frac{3n-2}{n^2+n}$

4. $\lim_{n\to\infty}\frac{2n^2+n-5}{6n-1}$

5. $\lim_{n\to\infty}(n^2-2n)$

6. $\lim_{n\to\infty}(2+4n-n^2)$

◪ 다음 극한값을 구하여라.

7. $\lim_{n\to\infty}\frac{3n+1}{n-1}$

8. $\lim_{n\to\infty}\frac{4n+1}{2n-3}$

9. $\lim_{n\to\infty}\frac{2n+1}{3n-5}$

10. $\lim_{n\to\infty}\frac{2n+3}{5n-1}$

11. $\lim_{n\to\infty}\frac{3n+1}{n^2-2n-1}$

12. $\lim_{n\to\infty}\frac{3n+2}{n^2+3n-1}$

13. $\lim_{n\to\infty}\frac{2n^2-3n+1}{n^2+1}$

14. $\lim_{n\to\infty}\frac{n^2+n-1}{2n^2+3n+1}$

I410-UCI(KEPA)-0199-2019-001-000244110

15. $\lim_{n\to\infty}\frac{n^2-3n+1}{3n^2+5n-7}$

16. $\lim_{n\to\infty}\frac{6n^2-2n+1}{2n^2+3}$

17. $\lim_{n\to\infty}\frac{9n^2+4n}{3n^2+4}$

18. $\lim_{n\to\infty}\frac{(n+1)(3n-1)}{2n^2+1}$

19. $\lim_{n\to\infty}\frac{n^2+n}{n(2n+1)}$

20. $\lim_{n\to\infty}\frac{n^2-n}{n(3n-1)}$

21. $\lim_{n\to\infty}\frac{2n^2-3n+2}{(n+1)(n+2)}$

22. $\lim_{n\to\infty}\frac{2n^2+3n-2}{(n-1)(n-2)}$

23. $\lim_{n\to\infty}\frac{n(2n-1)}{(n+1)(n+2)}$

24. $\lim_{n\to\infty}\frac{(2n-1)(3n+2)}{n(n+4)}$

25. $\lim_{n\to\infty}\frac{(2n-1)(3n+2)}{(2n+1)^2}$

26. $\lim_{n\to\infty}\frac{5n^2+2n}{n^3+3n^2+4n}$

27. $\lim_{n\to\infty}\frac{2n^2-1}{2n^3+3n+1}$

28. $\lim_{n\to\infty}\frac{n^3+2n-1}{2n^3-n}$

29. $\lim_{n\to\infty}\frac{n^3-4n+1}{4n^3-n}$

30. $\lim_{n\to\infty}\frac{n^3-2n^2+n+2}{n^2(3n+5)}$

31. $\lim_{n\to\infty}\frac{1+2+3+\cdots+n}{n^2+3}$

32. $\lim_{n\to\infty}\frac{1+2+3+\cdots+n}{n^2-2n}$

33. $\lim_{n\to\infty}\frac{1+3+5+\cdots+(2n-1)}{4n^2+1}$

34. $\lim_{n\to\infty}\frac{2+5+8+\cdots+(3n-1)}{2n^2+3}$

35. $\lim_{n\to\infty}\frac{1\times2+2\times3+\cdots+n(n+1)}{n^3}$

36. $\lim_{n\to\infty}\frac{1\cdot3+2\cdot4+\cdots+n(n+2)}{n^3}$

37. $\lim_{n\to\infty}\frac{1\times3+2\times4+3\times5+\cdots+n(n+2)}{n(n+1)(n+2)}$

38. $\lim_{n\to\infty}\frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$

39. $\lim_{n\to\infty}\frac{(n+1)(1^2+2^2+\cdots+n^2)}{(1+2+\cdots+n)^2}$

40. $\lim_{n\to\infty}\frac{(1+2+\cdots+n)^2}{(n+1)(1^2+2^2+\cdots+n^2)}$

41.

$$\lim_{n\to\infty}\left\{\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{n+1}\right)\right\}^2\cdot\frac{1}{1+2+\cdots+n}$$

02 무리식의 극한값의 계산

$\infty-\infty$ 꼴의 극한

⇨ 무리식을 포함한 경우 근호를 포함한 쪽을 유리화하여 구한다.

◪ **다음 극한값을 구하여라.**

42. $\lim_{n\to\infty}(\sqrt{n+1}-\sqrt{n})$

43. $\lim_{n\to\infty}(\sqrt{n^2+5n}-n)$

44. $\lim_{n\to\infty}(\sqrt{n^2+2n}-n)$

45. $\lim_{n\to\infty}(\sqrt{n^2+2n+3}-n)$

46. $\lim_{n\to\infty}(\sqrt{n^2+4n+1}-n)$

47. $\lim_{n\to\infty}(\sqrt{4n^2+2n-3}-2n)$

48. $\lim_{n\to\infty}(\sqrt{4n^2+6n}-2n)$

I410-UCI(KEPA)-0199-2019-001-000244110

49. $\lim_{n\to\infty}(\sqrt{4n^2+3n+2}-2n)$

50. $\lim_{n\to\infty}(\sqrt{9n^2+6n}-3n)$

51. $\lim_{n\to\infty}(\sqrt{n^2+1}-\sqrt{n^2-5})$

52. $\lim_{n\to\infty}(\sqrt{n^2+3}-\sqrt{n^2-1})$

53. $\lim_{n\to\infty}(\sqrt{n^2+2n}-\sqrt{n^2-n})$

54. $\lim_{n\to\infty}(\sqrt{n^2+3n}-\sqrt{n^2-n})$

55. $\lim_{n\to\infty}\frac{\sqrt{2}}{\sqrt{n^2+n}-n}$

56. $\lim_{n\to\infty}\frac{1}{\sqrt{n^2+3n}-n}$

57. $\lim_{n\to\infty}\frac{1}{\sqrt{n^2+4n}-n}$

58. $\lim_{n\to\infty}\frac{4}{n-\sqrt{n^2-3n}}$

59. $\lim_{n\to\infty}\frac{2}{n-\sqrt{n^2-4n}}$

60. $\lim_{n\to\infty}\frac{1}{n(\sqrt{4n^2+1}-2n)}$

61. $\lim_{n\to\infty}\frac{1}{\sqrt{n^2+n}-\sqrt{n^2-n}}$

62. $\lim_{n\to\infty}\frac{1}{\sqrt{n^2-n}-\sqrt{n^2+n}}$

63. $\lim_{n\to\infty}\frac{2}{\sqrt{n^2+2n}-\sqrt{n^2+1}}$

64. $\lim_{n\to\infty}\frac{6}{\sqrt{n^2-3n}-\sqrt{n^2-1}}$

65. $\lim_{n\to\infty}\frac{\sqrt{5n+1}\sqrt{5n-1}+\sqrt{n+2}\sqrt{n-2}}{4n}$

정답 및 해설

1) 수렴, $\frac{1}{2}$

⇨ $\lim_{n\to\infty}\frac{n+1}{2n-3}=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{2-\frac{3}{n}}=\frac{1+0}{2-0}=\frac{1}{2}$

2) 수렴, 3

⇨ $\lim_{n\to\infty}\frac{3n^2-3n+4}{n^2+5n+2}=\lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{4}{n^2}}{1+\frac{5}{n}+\frac{2}{n^2}}$
$=\frac{3-0+0}{1+0+0}=3$

3) 수렴, 0

⇨ $\lim_{n\to\infty}\frac{3n-2}{n^2+n}=\lim_{n\to\infty}\frac{\frac{3}{n}-\frac{1}{n^2}}{1+\frac{1}{n}}=\frac{0-0}{1+0}=0$

4) 발산

⇨ $\lim_{n\to\infty}\frac{2n^2+n-5}{6n-1}=\lim_{n\to\infty}\frac{2n+1-\frac{5}{n}}{6-\frac{1}{n}}=\infty$

5) 발산

⇨ $\lim_{n\to\infty}(n^2-2n)=\lim_{n\to\infty}n^2\left(1-\frac{2}{n}\right)=\infty$

6) 발산

⇨ $\lim_{n\to\infty}(2+4n-n^2)=\lim_{n\to\infty}n^2\left(\frac{2}{n^2}+\frac{4}{n}-1\right)=-\infty$

7) 3

⇨ $\lim_{n\to\infty}\frac{3n+1}{n-1}=\lim_{n\to\infty}\frac{3+\frac{1}{n}}{1-\frac{1}{n}}=\frac{3+0}{1-0}=3$

8) 2

⇨ $\lim_{n\to\infty}\frac{4n+1}{2n-3}=\lim_{n\to\infty}\frac{4+\frac{1}{n}}{2-\frac{3}{n}}=\frac{4+0}{2-0}=2$

9) $\frac{2}{3}$

⇨ 분모와 분자를 분모의 최고차항인 n으로 나누면

$\lim_{n\to\infty}\frac{2n+1}{3n-5}=\lim_{n\to\infty}\frac{2+\frac{1}{n}}{3-\frac{5}{n}}=\frac{2+0}{3-0}=\frac{2}{3}$

10) $\frac{2}{5}$

⇨ 분모와 분자를 분모의 최고차항인 n으로 나누면

$\lim_{n\to\infty}\frac{2n+3}{5n-2}=\lim_{n\to\infty}\frac{2+\frac{3}{n}}{5-\frac{2}{n}}=\frac{2+0}{5-0}=\frac{2}{5}$

11) 0

⇨ 분모와 분자를 분모의 최고차항인 n^2으로 나누면

$\lim_{n\to\infty}\frac{3n+1}{n^2-2n-1}=\lim_{n\to\infty}\frac{\frac{3}{n}+\frac{1}{n^2}}{1-\frac{2}{n}-\frac{1}{n^2}}=\frac{0}{1}=0$

12) 0

⇨ $\lim_{n\to\infty}\frac{3n+2}{n^2+3n-1}=\lim_{n\to\infty}\frac{\frac{3}{n}+\frac{2}{n^2}}{1+\frac{3}{n}-\frac{1}{n^2}}=\frac{0}{1}=0$

13) 2

⇨ 분모와 분자를 분모의 최고차항인 n^2으로 나누면

$\lim_{n\to\infty}\frac{2n^2-3n+1}{n^2+1}=\lim_{n\to\infty}\frac{2-\frac{3}{n}+\frac{1}{n^2}}{1+\frac{1}{n^2}}=\frac{2}{1}=2$

14) $\frac{1}{2}$

⇨ $\lim_{n\to\infty}\frac{n^2+n-1}{2n^2+3n+1}=\lim_{n\to\infty}\frac{1+\frac{1}{n}-\frac{1}{n^2}}{2+\frac{3}{n}+\frac{1}{n^2}}=\frac{1}{2}$

15) $\frac{1}{3}$

⇨ $\lim_{n\to\infty}\frac{n^2-3n+1}{3n^2+5n-7}=\lim_{n\to\infty}\frac{1-\frac{3}{n}+\frac{1}{n^2}}{3+\frac{5}{n}-\frac{7}{n^2}}=\frac{1}{3}$

16) 3

⇨ $\lim_{n\to\infty}\frac{6n^2-2n+1}{2n^2+3}=\lim_{n\to\infty}\frac{6-\frac{2}{n}+\frac{1}{n^2}}{2+\frac{3}{n^2}}=\frac{6}{2}=3$

17) 3

⇨ $\lim_{n\to\infty}\frac{9n^2+4n}{3n^2+4}=\frac{9}{3}=3$

18) $\frac{3}{2}$

⇨ $\lim_{n\to\infty}\frac{(n+1)(3n-1)}{2n^2+1}=\lim_{n\to\infty}\frac{3n^2+2n-1}{2n^2+1}$

$$=\lim_{n\to\infty}\frac{3+\frac{2}{n}-\frac{1}{n^2}}{2+\frac{1}{n^2}}=\frac{3}{2}$$

19) $\frac{1}{2}$

⇨ $\lim_{n\to\infty}\frac{n^2+n}{n(2n+1)}=\lim_{n\to\infty}\frac{n^2+n}{2n^2+n}$

$$=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{2+\frac{1}{n}}$$

$$=\frac{1}{2}$$

20) $\frac{1}{3}$

⇨ $\lim_{n\to\infty}\frac{n^2-n}{n(3n-1)}=\lim_{n\to\infty}\frac{n^2-n}{3n^2-n}$

$$=\lim_{n\to\infty}\frac{1-\frac{1}{n}}{3-\frac{1}{n}}=\frac{1}{3}$$

21) 2

⇨ $\lim_{n\to\infty}\frac{2n^2-3n+2}{(n+1)(n+2)}=\lim_{n\to\infty}\frac{2n^2-3n+2}{n^2+3n+2}$

$$=\lim_{n\to\infty}\frac{2-\frac{3}{n}+\frac{2}{n^2}}{1+\frac{3}{n}+\frac{2}{n^2}}$$

$$=\frac{2-0+0}{1+0+0}=2$$

22) 2

⇨ $\lim_{n\to\infty}\frac{2n^2+3n-2}{(n-1)(n-2)}=\lim_{n\to\infty}\frac{2n^2+3n-2}{n^2-3n+2}$

$$=\lim_{n\to\infty}\frac{2+\frac{3}{n}-\frac{2}{n^2}}{1-\frac{3}{n}+\frac{2}{n^2}}$$

$$=\frac{2+0-0}{1-0+0}=2$$

23) 2

⇨ $\lim_{n\to\infty}\frac{n(2n-1)}{(n+1)(n+2)}=\lim_{n\to\infty}\frac{1\times\left(2-\frac{1}{n}\right)}{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}=2$

24) 6

⇨ $\lim_{n\to\infty}\frac{(2n-1)(3n+2)}{n(n+4)}=\lim_{n\to\infty}\frac{6n^2+n-2}{n^2+4n}$

$$=\lim_{n\to\infty}\frac{6+\frac{1}{n}-\frac{2}{n^2}}{1+\frac{4}{n}}$$

$$=6$$

25) $\frac{3}{2}$

⇨ $\frac{\infty}{\infty}$ 꼴의 극한이므로 최고차항의 계수를 비교하자.

$\therefore\ \lim_{n\to\infty}\frac{(2n-1)(3n+2)}{(2n+1)^2}=\frac{2\cdot 3}{2^2}=\frac{6}{4}=\frac{3}{2}$

26) 0

⇨ 분모와 분자를 분모의 최고차항인 n^3으로 나누면

$$\lim_{n\to\infty}\frac{5n^2+2n}{n^3+3n^2+4n}=\lim_{n\to\infty}\frac{\frac{5}{n}-\frac{2}{n^2}}{1+\frac{3}{n}+\frac{4}{n^2}}=0$$

27) 0

⇨ 분모와 분자를 분모의 최고차항인 n^3으로 나누면

$$\lim_{n\to\infty}\frac{2n^2-1}{2n^3+3n+1}=\lim_{n\to\infty}\frac{\frac{2}{n}-\frac{1}{n^3}}{2+\frac{3}{n^2}+\frac{1}{n^3}}=0$$

28) $\frac{1}{2}$

⇨ 분모와 분자를 분모의 최고차항인 n^3으로 나누면

$$\lim_{n\to\infty}\frac{n^3+2n-1}{2n^3-n}=\lim_{n\to\infty}\frac{1+\frac{2}{n^2}-\frac{1}{n^3}}{2-\frac{1}{n^2}}=\frac{1}{2}$$

29) $\frac{1}{4}$

⇨ $\lim_{n\to\infty}\frac{n^3-4n+1}{4n^3-n}=\lim_{n\to\infty}\frac{1-\frac{4}{n^2}+\frac{1}{n^3}}{4-\frac{1}{n^2}}=\frac{1}{4}$

30) $\frac{1}{3}$

⇨ $\lim_{n\to\infty}\frac{n^3-2n^2+n+2}{n^2(3n+5)}=\lim_{n\to\infty}\frac{n^3-2n^2+n+2}{3n^3+5n^2}$

$$=\lim_{n\to\infty}\frac{1-\frac{2}{n}+\frac{1}{n^2}+\frac{2}{n^3}}{3+\frac{5}{n}}$$

$$=\frac{1}{3}$$

31) $\frac{1}{2}$

$\Rightarrow \lim_{n\to\infty}\frac{1+2+3+\cdots+n}{n^2+3}=\lim_{n\to\infty}\frac{\frac{1}{2}n(n+1)}{n^2+3}$
$=\lim_{n\to\infty}\frac{\frac{1}{2}\left(1+\frac{1}{n}\right)}{1+\frac{3}{n^2}}=\frac{1}{2}$

32) $\frac{1}{2}$

$\Rightarrow \lim_{n\to\infty}\frac{1+2+3+\cdots+n}{n^2-2n}$
$=\lim_{n\to\infty}\frac{\frac{1}{2}n(n+1)}{n^2-2n}$
$=\lim_{n\to\infty}\frac{\frac{1}{2}\cdot 1\cdot\left(1+\frac{1}{n}\right)}{1-\frac{2}{n}}=\frac{1}{2}$

33) $\frac{1}{4}$

$\Rightarrow \lim_{n\to\infty}\frac{1+3+5+\cdots+(2n-1)}{4n^2+1}$
$=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}(2k-1)}{4n^2+1}=\lim_{n\to\infty}\frac{2\sum_{k=1}^{n}k-n}{4n^2+1}$
$=\lim_{n\to\infty}\frac{2\cdot\frac{n(n+1)}{2}-n}{4n^2+1}=\lim_{n\to\infty}\frac{n^2}{4n^2+1}$
$=\lim_{n\to\infty}\frac{1}{4+\frac{1}{n^2}}=\frac{1}{4}$

34) $\frac{3}{4}$

$\Rightarrow \lim_{n\to\infty}\frac{2+5+8+\cdots+(3n-1)}{2n^2+3}$
$=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}(3k-1)}{2n^2+3}=\lim_{n\to\infty}\frac{3\sum_{k=1}^{n}k-n}{2n^2+3}$
$=\lim_{n\to\infty}\frac{\frac{3n(n+1)}{2}-n}{2n^2+3}=\lim_{n\to\infty}\frac{\frac{3}{2}n^2+\frac{1}{2}n}{2n^2+3}$
$=\lim_{n\to\infty}\frac{\frac{3}{2}+\frac{1}{2n}}{2+\frac{3}{n^2}}=\frac{3}{4}$

35) $\frac{1}{3}$

$\Rightarrow \lim_{n\to\infty}\frac{1\times2+2\times3+\cdots+n(n+1)}{n^3}$
$=\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}k(k+1)=\lim_{n\to\infty}\frac{1}{n^3}\left(\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}k\right)$
$=\lim_{n\to\infty}\frac{1}{n^3}\left\{\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)\right\}$
$=\lim_{n\to\infty}\left\{\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+\frac{1}{2n}\left(1+\frac{1}{n}\right)\right\}$
$=\frac{1}{6}\times1\times2+0\times1=\frac{1}{3}$

36) $\frac{1}{3}$

$\Rightarrow \lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}k(k+2)=\lim_{n\to\infty}\frac{1}{n^3}\left(\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k\right)$
$=\lim_{n\to\infty}\frac{1}{n^3}\left\{\frac{1}{6}n(n+1)(2n+1)+n(n+1)\right\}$
$=\lim_{n\to\infty}\frac{1}{6}\cdot 1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+1\cdot\left(1+\frac{1}{n}\right)\frac{1}{n}$
$=\frac{1}{6}\cdot1\cdot1\cdot2+1\cdot1\cdot0=\frac{1}{3}$

37) $\frac{1}{3}$

$\Rightarrow \lim_{n\to\infty}\frac{1\times3+2\times4+3\times5+\cdots+n(n+2)}{n(n+1)(n+2)}$
$=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}k(k+2)}{n^3+3n^2+2n}=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}(k^2+2k)}{n^3+3n^2+2n}$
$=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k}{n^3+3n^2+2n}$
$=\lim_{n\to\infty}\frac{\frac{1}{6}n(n+1)(2n+1)+2\times\frac{1}{2}n(n+1)}{n^3+3n^2+2n}$
$=\lim_{n\to\infty}\frac{\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+\frac{1}{n}\left(1+\frac{1}{n}\right)}{1+\frac{3}{n}+\frac{2}{n^2}}$
$=\frac{\frac{1}{6}\times1\times2+0\times1}{1+0+0}=\frac{1}{3}$

38) $\frac{1}{3}$

$\Rightarrow \lim_{n\to\infty}\frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$
$=\lim_{n\to\infty}\frac{\frac{n(n+1)(2n+1)}{6}}{n^3}$
$=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}$
$=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$
$=\frac{1}{3}$

I410-UCI(KEPA)-0199-2019-001-000244110

39) $\frac{4}{3}$

⇨ $\lim_{n\to\infty}\frac{(n+1)(1^2+2^2+\cdots+n^2)}{(1+2+\cdots+n)^2}$

$=\lim_{n\to\infty}\frac{(n+1)\times\frac{1}{6}n(n+1)(2n+1)}{\left\{\frac{1}{2}n(n+1)\right\}^2}$

$=\lim_{n\to\infty}\frac{\frac{1}{6}n(n+1)^2(2n+1)}{\frac{1}{4}n^2(n+1)^2}=\lim_{n\to\infty}\frac{\frac{1}{6}(2n+1)}{\frac{1}{4}n}$

$=\lim_{n\to\infty}\frac{\frac{1}{6}\left(2+\frac{1}{n}\right)}{\frac{1}{4}}=\frac{\frac{1}{3}}{\frac{1}{4}}=\frac{4}{3}$

40) $\frac{3}{4}$

⇨ $\lim_{n\to\infty}\frac{(1+2+\cdots+n)^2}{(n+1)(1^2+2^2+\cdots+n^2)}$

$=\lim_{n\to\infty}\frac{\left\{\frac{1}{2}n(n+1)\right\}^2}{(n+1)\frac{1}{6}n(n+1)(2n+1)}$

$=\lim_{n\to\infty}\frac{\frac{1}{4}n^2(n+1)^2}{(n+1)\frac{1}{6}n(n+1)(2n+1)}$

$=\lim_{n\to\infty}\frac{\frac{1}{4}n}{\frac{1}{6}(2n+1)}=\lim_{n\to\infty}\frac{\frac{1}{4}}{\frac{1}{6}\left(2+\frac{1}{n}\right)}=\frac{3}{4}$

41) $\frac{1}{2}$

⇨

$\lim_{n\to\infty}\left\{\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{n+1}\right)\right\}^2\cdot\frac{1}{1+2+\cdots+n}$

$=\lim_{n\to\infty}\left(\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\ \cdots\ \cdot\frac{n+2}{n+1}\right)^2\cdot\frac{2}{n(n+1)}$

$=\lim_{n\to\infty}\left(\frac{n+2}{2}\right)^2\cdot\frac{2}{n(n+1)}=\frac{1}{2}$

42) 0

⇨ $\lim_{n\to\infty}(\sqrt{n+1}-\sqrt{n})$

$=\lim_{n\to\infty}\frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}}$

$=\lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}=0$

43) $\frac{5}{2}$

⇨ $\sqrt{n^2+5n}-n=\frac{\sqrt{n^2+5n}-n}{1}$으로 보고 분자를 유리화하면

$\lim_{n\to\infty}(\sqrt{n^2+5n}-n)$

$=\lim_{n\to\infty}\frac{(\sqrt{n^2+5n}-n)(\sqrt{n^2+5n}+n)}{\sqrt{n^2+5n}+n}$

$=\lim_{n\to\infty}\frac{(n^2+5n)-n^2}{\sqrt{n^2+5n}+n}=\lim_{n\to\infty}\frac{5n}{\sqrt{n^2+5n}+n}$

$=\lim_{n\to\infty}\frac{5}{\sqrt{1+\frac{5}{n}}+1}=\frac{5}{1+1}=\frac{5}{2}$

44) 1

⇨ $\sqrt{n^2+2n}-n=\frac{\sqrt{n^2+2n}-n}{1}$으로 보고 분자를 유리화한다.

$\lim_{n\to\infty}(\sqrt{n^2+2n}-n)$

$=\lim_{n\to\infty}\frac{(\sqrt{n^2+2n}-n)(\sqrt{n^2+2n}+n)}{\sqrt{n^2+2n}+n}$

$=\lim_{n\to\infty}\frac{(n^2+2n)-n^2}{\sqrt{n^2+2n}+n}$

$=\lim_{n\to\infty}\frac{2n}{\sqrt{n^2+2n}+n}$

$=\lim_{n\to\infty}\frac{2}{\sqrt{1+\frac{2}{n}}+1}=\frac{2}{1+1}=1$

45) 1

⇨ $\lim_{n\to\infty}(\sqrt{n^2+2n+3}-n)$

$=\lim_{n\to\infty}\frac{(\sqrt{n^2+2n+3}-n)(\sqrt{n^2+2n+3}+n)}{\sqrt{n^2+2n+3}+n}$

$=\lim_{n\to\infty}\frac{2n+3}{\sqrt{n^2+2n+3}+n}$

$=\lim_{n\to\infty}\frac{2+\frac{3}{n}}{\sqrt{1+\frac{2}{n}+\frac{3}{n^2}}+1}=1$

46) 2

⇨ $\lim_{n\to\infty}(\sqrt{n^2+4n+1}-n)$

$=\lim_{n\to\infty}\frac{(\sqrt{n^2+4n+1}-n)(\sqrt{n^2+4n+1}+n)}{\sqrt{n^2+4n+1}+n}$

$=\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}$

$=\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}=\frac{4}{1+1}=2$

47) $\frac{1}{2}$

⇨ $\lim_{n\to\infty}(\sqrt{4n^2+2n-3}-2n)$

$=\lim_{n\to\infty}\frac{2n-3}{\sqrt{4n^2+2n-3}+2n}$

$=\lim_{n\to\infty}\frac{2-\frac{3}{n}}{\sqrt{4+\frac{2}{n}-\frac{3}{n^2}}+2}=\frac{1}{2}$

48) $\frac{3}{2}$

⇨ $\lim_{n\to\infty}(\sqrt{4n^2+6n}-2n)$

$=\lim_{n\to\infty}\frac{(\sqrt{4n^2+6n}-2n)(\sqrt{4n^2+6n}+2n)}{\sqrt{4n^2+6n}+2n}$

$=\lim_{n\to\infty}\frac{4n^2+6n-4n^2}{\sqrt{4n^2+6n}+2n}=\lim_{n\to\infty}\frac{6n}{\sqrt{4n^2+6n}+2n}$

$=\lim_{n\to\infty}\frac{6}{\sqrt{4+\frac{6}{n}}+2}=\frac{6}{2+2}=\frac{3}{2}$

49) $\frac{3}{4}$

⇨ $\lim_{n\to\infty}(\sqrt{4n^2+3n+2}-2n)$

$=\lim_{n\to\infty}\frac{(\sqrt{4n^2+3n+2}-2n)(\sqrt{4n^2+3n+2}+2n)}{\sqrt{4n^2+3n+2}+2n}$

$=\lim_{n\to\infty}\frac{4n^2+3n+2-4n^2}{\sqrt{4n^2+3n+2}+2n}$

$=\lim_{n\to\infty}\frac{3+\frac{2}{n}}{\sqrt{4+\frac{3}{n}+\frac{2}{n^2}}+2}$

$=\frac{3}{2+2}=\frac{3}{4}$

50) 1

⇨ $\lim_{n\to\infty}\sqrt{9n^2+6n}-3n$

$=\lim_{n\to\infty}\frac{(\sqrt{9n^2+6n}-3n)(\sqrt{9n^2+6n}+3n)}{\sqrt{9n^2+6n}+3n}$

$=\lim_{n\to\infty}\frac{6n}{\sqrt{9n^2+6n}+3n}=\lim_{n\to\infty}\frac{6}{\sqrt{9+\frac{6}{n}}+3}=\frac{6}{3+3}=1$

51) 0

⇨ $\lim_{n\to\infty}(\sqrt{n^2+1}-\sqrt{n^2-5})$

$=\lim_{n\to\infty}\frac{(\sqrt{n^2+1}-\sqrt{n^2-5})(\sqrt{n^2+1}+\sqrt{n^2-5})}{\sqrt{n^2+1}+\sqrt{n^2-5}}$

$=\lim_{n\to\infty}\frac{(n^2+1)-(n^2-5)}{\sqrt{n^2+1}+\sqrt{n^2-5}}=\lim_{n\to\infty}\frac{6}{\sqrt{n^2+1}+\sqrt{n^2-5}}$

$=\lim_{n\to\infty}\frac{\frac{6}{n}}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{5}{n^2}}}=0$

52) 0

⇨ $\lim_{n\to\infty}(\sqrt{n^2+3}-\sqrt{n^2-1})$

$=\lim_{n\to\infty}\frac{(\sqrt{n^2+3}-\sqrt{n^2-1})(\sqrt{n^2+3}+\sqrt{n^2-1})}{\sqrt{n^2+3}+\sqrt{n^2-1}}$

$=\lim_{n\to\infty}\frac{(n^2+3)-(n^2-1)}{\sqrt{n^2+3}+\sqrt{n^2-1}}$

$=\lim_{n\to\infty}\frac{4}{\sqrt{n^2+3}+\sqrt{n^2-1}}=0$

53) $\frac{3}{2}$

⇨ $\lim_{n\to\infty}(\sqrt{n^2+2n}-\sqrt{n^2-n})$

$=\lim_{n\to\infty}\frac{(\sqrt{n^2+2n}-\sqrt{n^2-n})(\sqrt{n^2+2n}+\sqrt{n^2-n})}{\sqrt{n^2+2n}+\sqrt{n^2-n}}$

$=\lim_{n\to\infty}\frac{(n^2+2n)-(n^2-n)}{\sqrt{n^2+2n}+\sqrt{n^2-n}}=\lim_{n\to\infty}\frac{3n}{\sqrt{n^2+2n}+\sqrt{n^2-n}}$

$=\lim_{n\to\infty}\frac{3}{\sqrt{1+\frac{2}{n}}+\sqrt{1-\frac{1}{n}}}=\frac{3}{1+1}=\frac{3}{2}$

54) 2

⇨ $\lim_{n\to\infty}(\sqrt{n^2+3n}-\sqrt{n^2-n})$

$=\lim_{n\to\infty}\frac{(n^2+3n)-(n^2-n)}{\sqrt{n^2+3n}+\sqrt{n^2-n}}$

$=\lim_{n\to\infty}\frac{4n}{\sqrt{n^2+3n}+\sqrt{n^2-n}}$

$=\lim_{n\to\infty}\frac{4}{\sqrt{1+\frac{3}{n}}+\sqrt{1-\frac{1}{n}}}=2$

55) $2\sqrt{2}$

⇨ $\lim_{n\to\infty}\frac{\sqrt{2}}{\sqrt{n^2+n}-n}$

$=\lim_{n\to\infty}\frac{\sqrt{2}(\sqrt{n^2+n}+n)}{(\sqrt{n^2+n}-n)(\sqrt{n^2+n}+n)}$

$=\lim_{n\to\infty}\frac{\sqrt{2}(\sqrt{n^2+n}+n)}{(n^2+n)-n^2}$

$=\lim_{n\to\infty}\sqrt{2}\left(\sqrt{1+\frac{1}{n}}+1\right)$

$=2\sqrt{2}$

56) $\frac{2}{3}$

$\Rightarrow \lim_{n\to\infty}\frac{1}{\sqrt{n^2+3n}-n}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2+3n}+n}{(\sqrt{n^2+3n}-n)(\sqrt{n^2+3n}+n)}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2+3n}+n}{3n}$

$=\lim_{n\to\infty}\frac{\sqrt{1+\frac{3}{n}}+1}{3}=\frac{2}{3}$

57) $\frac{1}{2}$

$\Rightarrow \lim_{n\to\infty}\frac{1}{\sqrt{n^2+4n}-n}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2+4n}+n}{(\sqrt{n^2+4n}-n)(\sqrt{n^2+4n}+n)}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2+4n}+n}{(n^2+4n)-n^2}=\lim_{n\to\infty}\frac{\sqrt{n^2+4n}+n}{4n}$

$=\lim_{n\to\infty}\frac{\sqrt{1+\frac{4}{n}}+1}{4}=\frac{1+1}{4}=\frac{1}{2}$

58) $\frac{8}{3}$

$\Rightarrow \lim_{n\to\infty}\frac{4}{n-\sqrt{n^2-3n}}$

$=\lim_{n\to\infty}\frac{4(n+\sqrt{n^2-3n})}{(n-\sqrt{n^2-3n})(n+\sqrt{n^2-3n})}$

$=\lim_{n\to\infty}\frac{4(n+\sqrt{n^2-3n})}{n^2-(n^2-3n)}=\lim_{n\to\infty}\frac{4(n+\sqrt{n^2-3n})}{3n}$

$=\lim_{n\to\infty}\frac{4\left(1+\sqrt{1-\frac{3}{n}}\right)}{3}=\frac{8}{3}$

59) 1

$\Rightarrow \lim_{n\to\infty}\frac{2}{n-\sqrt{n^2-4n}}$

$=\lim_{n\to\infty}\frac{2(n+\sqrt{n^2-4n})}{(n-\sqrt{n^2-4n})(n+\sqrt{n^2-4n})}$

$=\lim_{n\to\infty}\frac{2(n+\sqrt{n^2-4n})}{n^2-(n^2-4n)}$

$=\lim_{n\to\infty}\frac{n+\sqrt{n^2-4n}}{2n}$

$=\lim_{n\to\infty}\frac{1+\sqrt{1-\frac{4}{n}}}{2}=\frac{1+1}{2}=1$

60) 4

$\Rightarrow \lim_{n\to\infty}\frac{1}{n(\sqrt{4n^2+1}-2n)}$

$=\lim_{n\to\infty}\frac{\sqrt{4n^2+1}+2n}{n(4n^2+1-4n^2)}$

$=\lim_{n\to\infty}\frac{\sqrt{4+\frac{1}{n^2}}+2}{1}$

$=2+2=4$

61) 1

$\Rightarrow \lim_{n\to\infty}\frac{1}{\sqrt{n^2+n}-\sqrt{n^2-n}}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2+n}+\sqrt{n^2-n}}{(\sqrt{n^2+n}-\sqrt{n^2-n})(\sqrt{n^2+n}+\sqrt{n^2-n})}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2+n}+\sqrt{n^2-n}}{(n^2+n)-(n^2-n)}=\lim_{n\to\infty}\frac{\sqrt{n^2+n}+\sqrt{n^2-n}}{2n}$

$=\lim_{n\to\infty}\frac{\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}}{2}=\frac{2}{2}=1$

62) -1

$\Rightarrow \lim_{n\to\infty}\frac{1}{\sqrt{n^2-n}-\sqrt{n^2+n}}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2-n}+\sqrt{n^2+n}}{(\sqrt{n^2-n}-\sqrt{n^2+n})(\sqrt{n^2-n}+\sqrt{n^2+n})}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2-n}+\sqrt{n^2+n}}{(n^2-n)-(n^2+n)}$

$=\lim_{n\to\infty}\frac{\sqrt{n^2-n}+\sqrt{n^2+n}}{-2n}$

$=\lim_{n\to\infty}\frac{\sqrt{1-\frac{1}{n}}+\sqrt{1+\frac{1}{n}}}{-2}$

$=\frac{1+1}{-2}=-1$

63) 2

$\Rightarrow \lim_{n\to\infty}\frac{2}{\sqrt{n^2+2n}-\sqrt{n^2+1}}$

$=\lim_{n\to\infty}\frac{2(\sqrt{n^2+2n}+\sqrt{n^2+1})}{(\sqrt{n^2+2n}-\sqrt{n^2+1})(\sqrt{n^2+2n}+\sqrt{n^2+1})}$

$=\lim_{n\to\infty}\frac{2(\sqrt{n^2+2n}+\sqrt{n^2+1})}{(n^2+2n)-(n^2+1)}$

$=\lim_{n\to\infty}\frac{2(\sqrt{n^2+2n}+\sqrt{n^2+1})}{2n-1}$

$=\lim_{n\to\infty}\frac{2\left(\sqrt{1+\frac{2}{n}}+\sqrt{1+\frac{1}{n^2}}\right)}{2-\frac{1}{n}}=\frac{2(1+1)}{2}=2$

64) -4

$\Rightarrow \lim_{n\to\infty}\frac{6}{\sqrt{n^2-3n}-\sqrt{n^2-1}}$

$=\lim_{n\to\infty}\frac{6(\sqrt{n^2-3n}+\sqrt{n^2-1})}{(n^2-3n)-(n^2-1)}$

$=\lim_{n\to\infty}\frac{6(\sqrt{n^2-3n}+\sqrt{n^2-1})}{-3n+1}$

$$=\lim_{n\to\infty}\frac{6\left(\sqrt{1-\frac{3}{n}}+\sqrt{1-\frac{1}{n^2}}\right)}{-3+\frac{1}{n}}$$

$$=\frac{6(1+1)}{-3}=-4$$

65) $\frac{3}{2}$

⇨ $\lim_{n\to\infty}\frac{\sqrt{5n+1}\sqrt{5n-1}+\sqrt{n+2}\sqrt{n-2}}{4n}$

$$=\lim_{n\to\infty}\frac{\sqrt{25n^2-1}+\sqrt{n^2-4}}{4n}$$

$$=\lim_{n\to\infty}\frac{\sqrt{25-\frac{1}{n^2}}+\sqrt{1-\frac{4}{n^2}}}{4}$$

$$=\frac{5+1}{4}=\frac{3}{2}$$