01 정적분의 뜻

(1) 정적분의 정의

닫힌구간 $[a, b]$에서 연속인 함수 $f(x)$의 한 부정적분을 $F(x)$라 할 때,

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

이것을 $f(x)$의 a에서 b까지의 **정적분**이라 한다.

(2) 정적분의 기본 정의

① $\int_a^a f(x)dx = 0$

② $\int_a^b f(x)dx = -\int_b^a f(x)dx$

◪ 다음 정적분의 값을 구하여라.

1. $\int_0^2 3dx$

2. $\int_1^2 xdx$

3. $\int_{-1}^3 (2x+4)dx$

4. $\int_0^4 (5x+4)\,dx$

5. $\int_1^2 3x^2 dx$

6. $\int_1^0 (x^2+x)\,dx$

7. $\int_{-1}^2 (3x^2+1)dx$

8. $\int_{-2}^1 (3y^2-1)\,dy$

9. $\int_2^2 (6x^2-2)dx$

10. $\int_1^{-2} (3x^2-4x)\,dx$

11. $\int_3^1 (3x^2+6x)dx$

12. $\int_1^{-2} (2x-3x^2)dx$

13. $\int_2^2 (x^2+2x+3)\,dx$

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14. $\int_1^2 (2t^2+3t-1)\,dt$

15. $\int_1^{-2} (6x^2+2x-5)\,dx$

16. $\int_1^2 x^3 dx$

17. $\int_0^{-1} (x^3-4x)\,dx$

18. $\int_1^3 (4x^3+2x)dx$

19. $\int_3^1 (4x^3-3x^2-2x)\,dx$

20. $\int_1^2 (2x^3-3x^2+2x-4)\,dx$

21. $\int_0^1 3x^4 dx$

22. $\int_0^1 10x^4 dx$

23. $\int_0^{-1} 10x^4 dx$

24. $\int_{-1}^0 x(x-4)\,dx$

25. $\int_0^2 (x+1)(x-1)\,dx$

26. $\int_1^3 (x-1)(x+2)\,dx$

27. $\int_0^1 (x-1)(x^2+x+1)\,dx$

28. $\int_8^8 (x-1)^8 dx$

02 정적분과 미분의 관계

닫힌구간 $[a,b]$에서 연속인 함수 $f(x)$에 대하여

$$\frac{d}{dx}\int_a^x f(t)dt = f(x) \quad (\text{단, } a<x<b)$$

◪ 다음을 구하여라.

29. $\frac{d}{dx}\int_1^x (t^2-2t)\,dt$

30. $\frac{d}{dx}\int_1^x (-t^2+4t)\,dt$

31. $\frac{d}{dx}\int_0^x (2t^2+3t-1)\,dt$

32. $\frac{d}{dx}\int_{-1}^x (t^3-5t^2+6)\,dt$

33. $\frac{d}{dx}\int_{-1}^x (2t^3+4t^2-1)\,dt$

34. $\frac{d}{dx}\int_2^x (-t^3+4t+2)\,dt$

03 정적분의 성질

두 함수 $f(x)$, $f(x)$가 세 실수 a, b, c를 포함하는 구간에서 연속일 때,

(1) $\int_a^b kf(x)dx=k\int_a^b f(x)dx$ (단, k는 실수)

(2) $\int_a^b \{f(x)+g(x)\}dx=\int_a^b f(x)dx+\int_a^b g(x)dx$

(3) $\int_a^b \{f(x)-g(x)\}dx=\int_a^b f(x)dx-\int_a^b g(x)dx$

(4) $\int_a^c f(x)dx+\int_c^b f(x)dx=\int_a^b f(x)dx$ (a, b, c의 대소에 관계없이 성립한다.)

◪ **다음 정적분의 값을 구하여라.**

35. $\int_{-1}^2 (2x-1)\,dx-\int_{-1}^2 (2x+1)dx$

36. $\int_{-1}^3 (2x-3)dx-\int_{-1}^3 (-2x+3)dx$

37. $\int_{-1}^1 (x^2+1)\,dx-\int_{-1}^1 x^2dx$

38. $\int_0^2 (1+x^2)dx+\int_2^0 (1-x^2)dx$

39. $\int_0^1 (2x-x^2)\,dx+\int_0^1 (2x+x^2)dx$

40. $\int_1^3 (x-1)^2dx-\int_3^1 2x\,dx$

41. $\int_{-1}^3 (3x^2+x-2)\,dx-\int_{-1}^3 (x+3)\,dx$

42. $\int_0^1 (x+1)^2dx+\int_1^0 (x-1)^2dx$

43. $\int_1^3 (x+1)^2dx-\int_1^3 (x-1)^2dx$

44. $\int_0^1 (x+2)^2dx-\int_0^1 (x-2)^2dx$

45. $\int_{-2}^1 (2x^2+6)\,dx-2\int_{-2}^1 (x^2-x+3)\,dx$

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46. $\int_1^2 (x^2-2x)dx+\int_1^2 (2x^2+2x+1)dx$

47. $\int_0^3 x(x-2)\,dx+\int_0^3 (y^2+2y)\,dy$

48. $\int_0^1 (2x-x^2)dx+\int_0^1 (2x+x^2)dx$

49. $\int_{-1}^0 (x^3-3x^2+2x)dx+\int_0^{-1} (x^3-3x^2-2x)dx$

50. $\int_1^3 (2x+1)^3dx+\int_1^3 (1-2x)^3dx$

◪ **다음 정적분의 값을 구하여라.**

51. $\int_{-1}^0 (3x+2)\,dx+\int_0^2 (3x+2)\,dx$

52. $\int_0^2 (9x+4)dx+\int_2^4 (9x+4)dx$

53. $\int_0^1 (x^2+1)dx+\int_1^3 (x^2+1)dx$

54. $\int_{-2}^1 (3x^2+1)dx+\int_1^3 (3x^2+1)dx$

55. $\int_1^2 (x^2+x)dx-\int_3^2 (t^2+t)dt$

56. $\int_0^2 (x-1)^2dx-\int_3^2 (x-1)^2dx$

57. $\int_0^1 (x+1)^2dx-\int_3^1 (x+1)^2dx$

58. $\int_0^2 x(3x-2)dx-\int_3^2 x(3x-2)dx$

59. $\int_{-2}^{-1} (x^2-4x+5)\,dx+\int_{-1}^1 (y^2-4y+5)\,dy$

60. $\int_{-2}^1 (x^3-x)dx+\int_1^2 (x^3-x)dx$

61. $\int_{-2}^1 (4x^3+2)dx+\int_1^2 (4x^3+2)dx$

62. $\int_0^2 (x^3-2x^2)dx+\int_2^1 (x^3-2x^2)dx$

63. $\int_{-1}^2 (4x^3-6x-1)dx-\int_3^2 (4x^3-6x-1)dx$

64. $\int_{-1}^{0}(x^3-3x^2+2x)\,dx+\int_{0}^{-1}(x^3-3x^2+2x)\,dx$

65. $\int_{-1}^{2}(5x^4-6x-1)\,dx-\int_{3}^{2}(5x^4-6x-1)\,dx$

66. $\int_{1}^{2}(3x^2-2x)dx+\int_{2}^{3}(3x^2-2x)dx$
$+\int_{3}^{4}(3x^2-2x)dx$

정답 및 해설

1) 6

⇨ $\int_0^2 3dx = [3x]_0^2 = 6-0 = 6$

2) $\frac{3}{2}$

⇨ $\int_1^2 x dx = \left[\frac{1}{2}x^2\right]_1^2 = 2-\frac{1}{2} = \frac{3}{2}$

3) 24

⇨ $\int_{-1}^3 (2x+4)dx = [x^2+4x]_{-1}^3 = (9+12)-(1-4)$

$= 24$

4) 56

⇨ $\int_0^4 (5x+4)\,dx = \left[\frac{5}{2}x^2+4x\right]_0^4 = 40+16 = 56$

5) 7

⇨ $\int_1^2 3x^2 dx = [x^3]_1^2 = 8-1 = 7$

6) $-\frac{5}{6}$

⇨ $\int_1^0 (x^2+x)dx = \left[\frac{1}{3}x^3+\frac{1}{2}x^2\right]_1^0$

$= 0-\left(\frac{1}{3}+\frac{1}{2}\right) = -\frac{5}{6}$

7) 12

⇨ $\int_{-1}^2 (3x^2+1)dx = [x^3+x]_{-1}^2 = (8+2)-(-1-1)$

$= 12$

8) 6

⇨ $\int_{-2}^1 (3y^2-1)\,dy = [y^3-y]_{-2}^1$

$= (1-1)-(-8+2) = 6$

9) 0

⇨ $\int_2^2 (6x^2-2)dx = 0$

10) -15

⇨ $\int_1^{-2} (3x^2-4x)dx = [x^3-2x^2]_1^{-2}$

$= (-8-8)-(1-2) = -16+1 = -15$

11) -50

⇨ $\int_3^1 (3x^2+6x)dx = [x^3+3x^2]_3^1$

$= (1+3)-(27+27) = -50$

12) 12

⇨ $\int_1^{-2} (2x-3x^2)dx = [x^2-x^3]_1^{-2}$

$= (4+8)-(1-1) = 12$

13) 0

⇨ $\int_2^2 (x^2+2x+3)\,dx = 0$

14) $\frac{49}{6}$

⇨ $\int_1^2 (2t^2+3t-1)\,dt = \left[\frac{2}{3}t^3+\frac{3}{2}t^2-t\right]_1^2$

$= \left(\frac{16}{3}+6-2\right)-\left(\frac{2}{3}+\frac{3}{2}-1\right)$

$= \frac{49}{6}$

15) 0

⇨ $\int_1^{-2} (6x^2+2x-5)\,dx$

$= -\int_{-2}^1 (6x^2+2x-5)\,dx$

$= -[2x^3+x^2-5x]_{-2}^1$

$= -\{(2+1-5)-(-16+4+10)\}$

$= 0$

16) $\frac{15}{4}$

⇨ $\int_1^2 x^3 dx = \left[\frac{1}{4}x^4\right]_1^2 = 4-\frac{1}{4} = \frac{15}{4}$

17) $-\frac{7}{4}$

⇨ $\int_0^{-1} (x^3-4x)\,dx = -\int_{-1}^0 (x^3-4x)\,dx$

$= -\left[\frac{1}{4}x^4-2x^2\right]_{-1}^0 = -\left\{0-\left(\frac{1}{4}-2\right)\right\} = -\frac{7}{4}$

18) 88

⇨ $\int_1^3 (4x^3+2x)dx = [x^4+x^2]_1^3 = (81+9)-(1+1)$

$= 88$

19) -46

⇨ $\int_3^1 (4x^3-3x^2-2x)\,dx = -\int_1^3 (4x^3-3x^2-2x)\,dx$

$= -[x^4-x^3-x^2]_1^3$

$= -\{(81-27-9)-(1-1-1)\}$

$= -46$

20) $-\frac{1}{2}$

⇨ $\int_1^2 (2x^3-3x^2+2x-4)\,dx=\left[\frac{1}{2}x^4-x^3+x^2-4x\right]_1^2$

$=\left(\frac{1}{2}\cdot 2^4-2^3+2^2-4\cdot 2\right)-\left(\frac{1}{2}-1+1-4\right)=-\frac{1}{2}$

21) $\frac{3}{5}$

⇨ $\int_0^1 3x^4\,dx=\left[\frac{3}{5}x^5\right]_0^1=\frac{3}{5}$

22) 2

⇨ $\int_0^1 10x^4\,dx=\left[2x^5\right]_0^1=2-0=2$

23) -2

⇨ $\int_0^{-1} 10x^4\,dx=\left[2x^5\right]_0^{-1}=-2$

24) $\frac{7}{3}$

⇨ $\int_{-1}^0 x(x-4)\,dx=\int_{-1}^0 (x^2-4x)\,dx$

$=\left[\frac{1}{3}x^3-2x^2\right]_{-1}^0=0-\left(-\frac{1}{3}-2\right)=\frac{7}{3}$

25) $\frac{2}{3}$

⇨ $\int_0^2 (x+1)(x-1)\,dx=\int_0^2 (x^2-1)\,dx$

$=\left[\frac{1}{3}x^3-x\right]_0^2=\frac{8}{3}-2=\frac{2}{3}$

26) $\frac{26}{3}$

⇨ $\int_1^3 (x-1)(x+2)\,dx$

$=\int_1^3 (x^2+x-2)\,dx=\left[\frac{1}{3}x^3+\frac{1}{2}x^2-2x\right]_1^3$

$=\left(\frac{1}{3}\cdot 3^3+\frac{1}{2}\cdot 3^2-2\cdot 3\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)=\frac{26}{3}$

27) $-\frac{3}{4}$

⇨ $\int_0^1 (x-1)(x^2+x+1)\,dx=\int_0^1 (x^3-1)\,dx$

$=\left[\frac{1}{4}x^4-x\right]_0^1=\frac{1}{4}-1=-\frac{3}{4}$

28) 0

⇨ $\int_8^8 (x-1)^8\,dx=0$

29) x^2-2x

⇨ $\frac{d}{dx}\int_1^x (t^2-2t)\,dt=x^2-2x$

30) $-x^2+4x$

⇨ $\frac{d}{dx}\int_1^x (-t^2+4t)\,dt=-x^2+4x$

31) $2x^2+3x-1$

⇨ $\frac{d}{dx}\int_0^x (2t^2+3t-1)\,dt=2x^2+3x-1$

32) x^3-5x^2+6

⇨ $\frac{d}{dx}\int_{-1}^x (t^3-5t^2+6)\,dt=x^3-5x^2+6$

33) $2x^3+4x^2-1$

⇨ $\frac{d}{dx}\int_{-1}^x (2t^3+4t^2-1)\,dt=2x^3+4x^2-1$

34) $-x^3+4x+2$

⇨ $\frac{d}{dx}\int_2^x (-t^3+4t+2)\,dt=-x^3+4x+2$

35) -6

⇨ $\int_{-1}^2 (2x-1)\,dx-\int_{-1}^2 (2x+1)\,dx$

$=\int_{-1}^2 \{(2x-1)-(2x+1)\}\,dx$

$=\int_{-1}^2 -2\,dx=\left[-2x\right]_{-1}^2=-4-2=-6$

36) -8

⇨ $\int_{-1}^3 (2x-3)\,dx-\int_{-1}^3 (-2x+3)\,dx$

$=\int_{-1}^3 \{(2x-3)-(-2x+3)\}\,dx$

$=\int_{-1}^3 (4x-6)\,dx$

$=\left[2x^2-6x\right]_{-1}^3$

$=0-8=-8$

37) 2

⇨ $\int_{-1}^1 (x^2+1)\,dx-\int_{-1}^1 x^2\,dx=\int_{-1}^1 \{(x^2+1)-x^2\}\,dx$

$=\int_{-1}^1 1\,dx=\left[x\right]_{-1}^1=2$

38) $\frac{16}{3}$

⇨ $\int_0^2 (1+x^2)\,dx+\int_2^0 (1-x^2)\,dx$

$=\int_0^2 (1+x^2)\,dx-\int_0^2 (1-x^2)\,dx$

$=\int_0^2 \{(1+x^2)-(1-x^2)\}\,dx$

$= \int_0^2 2x^2 dx = \left[\frac{2}{3}x^3\right]_0^2 = \frac{16}{3}$

39) 2

$\Rightarrow \int_0^1 (2x-x^2)dx + \int_0^1 (2x+x^2)dx$

$= \int_0^1 \{(2x-x^2)+(2x+x^2)\}dx$

$= \int_0^1 4x dx = [2x^2]_0^1 = 2-0 = 2$

40) $\frac{32}{3}$

$\Rightarrow \int_1^3 (x-1)^2 dx - \int_3^1 2x dx$

$= \int_1^3 (x-1)^2 dx + \int_1^3 2x dx$

$= \int_1^3 \{(x^2-2x+1)+2x\}dx$

$= \int_1^3 (x^2+1)dx = \left[\frac{1}{3}x^3+x\right]_1^3$

$= (9+3) - \left(\frac{1}{3}+1\right) = \frac{32}{3}$

41) 8

$\Rightarrow \int_{-1}^3 (3x^2+x-2)\,dx - \int_{-1}^3 (x+3)\,dx$

$= \int_{-1}^3 (3x^2+x-2-x-3)\,dx$

$= \int_{-1}^3 (3x^2-5)\,dx = [x^3-5x]_{-1}^3 = 12-4 = 8$

42) 2

$\Rightarrow \int_0^1 (x+1)^2 dx + \int_1^0 (x-1)^2 dx$

$= \int_0^1 (x^2+2x+1)\,dx - \int_0^1 (x^2-2x+1)\,dx$

$= \int_0^1 (x^2+2x+1-x^2+2x-1)\,dx$

$= \int_0^1 4x\,dx = [2x^2]_0^1 = 2$

43) 16

$\Rightarrow \int_1^3 (x+1)^2 dx - \int_1^3 (x-1)^2 dx$

$= \int_1^3 \{(x+1)^2-(x-1)^2\}dx$

$= \int_1^3 4x dx = [2x^2]_1^3 = 18-2 = 16$

44) 4

$\Rightarrow \int_0^1 (x+2)^2 dx - \int_0^1 (x-2)^2 dx$

$= \int_0^1 \{(x+2)^2-(x-2)^2\}dx$

$= \int_0^1 8x dx = [4x^2]_0^1 = 4$

45) -3

$\Rightarrow \int_{-2}^1 (2x^2+6)\,dx - 2\int_{-2}^1 (x^2-x+3)\,dx$

$= \int_{-2}^1 (2x^2+6)\,dx - \int_{-2}^1 (2x^2-2x+6)\,dx$

$= \int_{-2}^1 (2x^2+6-2x^2+2x-6)\,dx$

$= \int_{-2}^1 2x\,dx = [x^2]_{-2}^1$

$= 1-4 = -3$

46) 8

$\Rightarrow \int_1^2 (x^2-2x)dx + \int_1^2 (2x^2+2x+1)dx$

$= \int_1^2 (3x^2+1)dx = [x^3+x]_1^2$

$= 10-2 = 8$

47) 18

$\Rightarrow \int_0^3 x(x-2)\,dx + \int_0^3 (y^2+2y)\,dy$

$= \int_0^3 (x^2-2x)\,dx + \int_0^3 (x^2+2x)\,dx$

$= \int_0^3 (x^2-2x+x^2+2x)\,dx$

$= \int_0^3 2x^2 dx = \left[\frac{2}{3}x^3\right]_0^3$

$= 18$

48) 2

$\Rightarrow \int_0^1 (2x-x^2)dx + \int_0^1 (2x+x^2)dx$

$= \int_0^1 \{(2x-x^2)+(2x+x^2)\}dx$

$= \int_0^1 4x dx = [2x^2]_0^1 = 2$

49) -2

$\Rightarrow \int_{-1}^0 (x^3-3x^2+2x)dx + \int_0^{-1} (x^3-3x^2-2x)dx$

$= \int_{-1}^0 (x^3-3x^2+2x)dx - \int_{-1}^0 (x^3-3x^2-2x)dx$

$= \int_{-1}^0 \{(x^3-3x^2+2x)-(x^3-3x^2-2x)\}dx$

$= \int_{-1}^0 4x dx = [2x^2]_{-1}^0 = 0-2 = -2$

50) 212

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$\Rightarrow \int_1^3 (2x+1)^3 dx + \int_1^3 (1-2x)^3 dx$

$= \int_1^3 (24x^2+2) dx = \left[8x^3+2x\right]_1^3$

$= 222-10 = 212$

51) $\frac{21}{2}$

$\Rightarrow \int_{-1}^0 (3x+2)\,dx + \int_0^2 (3x+2)\,dx$

$= \int_{-1}^2 (3x+2)\,dx = \left[\frac{3}{2}x^2+2x\right]_{-1}^2$

$= 10-\left(-\frac{1}{2}\right) = \frac{21}{2}$

52) 88

$\Rightarrow \int_0^2 (9x+4) dx + \int_2^4 (9x+4) dx$

$= \int_0^4 (9x+4) dx = \left[\frac{9}{2}x^2+4x\right]_0^4 = 72+16 = 88$

53) 12

$\Rightarrow \int_0^1 (x^2+1) dx + \int_1^3 (x^2+1) dx = \int_0^3 (x^2+1) dx$

$= \left[\frac{1}{3}x^3+x\right]_0^3 = (9+3)-0 = 12$

54) 40

$\Rightarrow \int_{-2}^1 (3x^2+1) dx + \int_1^3 (3x^2+1) dx$

$= \int_{-2}^3 (3x^2+1) dx = \left[x^3+x\right]_{-2}^3 = 30-(-10) = 40$

55) $\frac{38}{3}$

$\Rightarrow \int_1^2 (x^2+x) dx - \int_3^2 (t^2+t) dx$

$= \int_1^2 (x^2+x) dx - \int_3^2 (x^2+x) dx$

$= \int_1^2 (x^2+x) dx + \int_2^3 (x^2+x) dx = \int_1^3 (x^2+x) dx$

$= \left[\frac{1}{3}x^3+\frac{1}{2}x^2\right]_1^3 = \left(9+\frac{9}{2}\right)-\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{38}{3}$

56) 3

$\Rightarrow \int_0^2 (x-1)^2 dx - \int_3^2 (x-1)^2 dx$

$= \int_0^2 (x-1)^2 dx + \int_2^3 (x-1)^2 dx$

$= \int_0^3 (x-1)^2 dx = \int_0^3 (x^2-2x+1) dx$

$= \left[\frac{1}{3}x^3-x^2+x\right]_0^3 = (9-9+3)-0 = 3$

57) 21

$\Rightarrow \int_0^1 (x+1)^2 dx - \int_3^1 (x+1)^2 dx$

$= \int_0^1 (x+1)^2 dx + \int_1^3 (x+1)^2 dx$

$= \int_0^3 (x+1)^2 dx$

$= \int_0^3 (x^2+2x+1) dx$

$= \left[\frac{1}{3}x^3+x^2+x\right]_0^3$

$= 9+9+3 = 21$

58) 18

$\Rightarrow \int_0^2 x(3x-2) dx - \int_3^2 x(3x-2) dx$

$= \int_0^2 (3x^2-2x) dx - \int_3^2 (3x^2-2x) dx$

$= \int_0^2 (3x^2-2x) dx + \int_2^3 (3x^2-2x) dx$

$= \int_0^3 (3x^2-2x) dx = \left[x^3-x^2\right]_0^3$

$= 27-9 = 18$

59) 24

$\Rightarrow \int_{-2}^{-1} (x^2-4x+5)\,dx + \int_{-1}^1 (y^2-4y+5)\,dy$

$= \int_{-2}^{-1} (x^2-4x+5)\,dx + \int_{-1}^1 (x^2-4x+5)\,dx$

$= \int_{-2}^1 (x^2-4x+5)\,dx = \left[\frac{1}{3}x^3-2x^2+5x\right]_{-2}^1$

$= \frac{10}{3}-\left(-\frac{62}{3}\right) = 24$

60) 0

$\Rightarrow \int_{-2}^1 (x^3-x) dx + \int_1^2 (x^3-x) dx$

$= \int_{-2}^2 (x^3-x) dx = \left[\frac{1}{4}x^4-\frac{1}{2}x^2\right]_{-2}^2$

$= (4-2)-(4-2) = 0$

61) 8

$\Rightarrow \int_{-2}^1 (4x^3+2) dx + \int_1^2 (4x^3+2) dx$

$= \int_{-2}^2 (4x^3+2) dx$

$= \left[x^4+2x\right]_{-2}^2 = (16+4)-(16-4) = 8$

62) $-\frac{5}{12}$

$\Rightarrow \int_0^2 (x^3-2x^2) dx + \int_2^1 (x^3-2x^2) dx$

$= \int_0^1 (x^3 - 2x^2)dx$

$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3\right]_0^1$

$= \frac{1}{4} - \frac{2}{3} = -\frac{5}{12}$

63) 52

⇨ $\int_{-1}^{2} (4x^3 - 6x - 1)dx - \int_3^2 (4x^3 - 6x - 1)dx$

$= \int_{-1}^{2} (4x^3 - 6x - 1)dx + \int_2^3 (4x^3 - 6x - 1)dx$

$= \int_{-1}^{3} (4x^3 - 6x - 1)dx = \left[x^4 - 3x^2 - x\right]_{-1}^{3}$

$= (81 - 27 - 3) - (1 - 3 + 1) = 51 + 1 = 52$

64) 0

⇨ $\int_{-1}^{0} (x^3 - 3x^2 + 2x)\,dx + \int_0^{-1} (x^3 - 3x^2 + 2x)\,dx$

$= \int_{-1}^{-1} (x^3 - 3x^2 + 2x)\,dx = 0$

65) 216

⇨ $\int_{-1}^{2} (5x^4 - 6x - 1)\,dx - \int_3^2 (5x^4 - 6x - 1)\,dx$

$= \int_{-1}^{2} (5x^4 - 6x - 1)\,dx + \int_2^3 (5x^4 - 6x - 1)\,dx$

$= \int_{-1}^{3} (5x^4 - 6x - 1)\,dx$

$= \left[x^5 - 3x^2 - x\right]_{-1}^{3}$

$= 213 - (-3) = 216$

66) 48

⇨ $\int_1^2 (3x^2 - 2x)dx + \int_2^3 (3x^2 - 2x)dx$

$+ \int_3^4 (3x^2 - 2x)dx = \int_1^4 (3x^2 - 2x)dx$

$= \left[x^3 - x^2\right]_1^4$

$= (64 - 16) - (1 - 1)$

$= 48$