



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

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3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 / 등비급수의 성질

두 등비급수 $\sum_{n=1}^{\infty} p^n$, $\sum_{n=1}^{\infty} q^n$ 이 수렴하면

(1) $\sum_{n=1}^{\infty} cp^n = c \sum_{n=1}^{\infty} p^n$

(2) $\sum_{n=1}^{\infty} (p^n + q^n) = \sum_{n=1}^{\infty} p^n + \sum_{n=1}^{\infty} q^n$

(3) $\sum_{n=1}^{\infty} (p^n - q^n) = \sum_{n=1}^{\infty} p^n - \sum_{n=1}^{\infty} q^n$

■ 다음 등비급수의 합을 구하여라.

1. $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

2. $\sum_{n=1}^{\infty} 5 \left(\frac{3}{4}\right)^{n-1}$

3. $\sum_{n=1}^{\infty} 3 \left(-\frac{1}{2}\right)^{n-1}$

4. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4}$

5. $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$

6. $\sum_{n=1}^{\infty} \left(\frac{3}{2^n} + \frac{2}{3^n}\right)$

7. $\sum_{n=1}^{\infty} \left(\frac{2^{n+1}}{3^n} + \frac{4}{2^n}\right)$

8. $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{3^n}{4^n}\right)$

9. $\sum_{n=1}^{\infty} \left(\frac{3^n}{4^n} - \frac{3}{2^n}\right)$

10. $\sum_{n=1}^{\infty} \left(\frac{4}{3^n} - \frac{3}{4^n}\right)$

11. $\sum_{n=1}^{\infty} \left(\frac{2^{n+1}}{4^n} + \frac{6}{3^n}\right)$

12. $\sum_{n=1}^{\infty} \left(\frac{5}{4^n} + \frac{4}{5^n}\right)$

13. $\sum_{n=1}^{\infty} \left(\frac{3}{4^n} + \frac{2}{(-5)^{n-2}}\right)$

$$14. \sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{3^n} \right)$$

$$15. \sum_{n=1}^{\infty} \left(\frac{3^n}{5^n} + \frac{2}{(-3)^n} \right)$$

$$16. \sum_{n=1}^{\infty} \left(\frac{1}{2^{n-1}} + \frac{3}{5^{n-1}} \right)$$

$$17. \sum_{n=1}^{\infty} \left(\frac{1}{5^{n-1}} + \frac{5}{6^{n-1}} \right)$$

$$18. \sum_{n=1}^{\infty} \frac{1 + (-1)^n}{3^n}$$

$$19. \sum_{n=1}^{\infty} 8 \left(\frac{1 + 2^n}{3^n} \right)$$

$$20. \sum_{n=1}^{\infty} \frac{1 + (-2)^n}{3^n}$$

$$21. \sum_{n=1}^{\infty} \frac{2^n + (-2)^n}{3^n}$$

$$22. \sum_{n=1}^{\infty} \frac{2^n + 1}{4^n}$$

$$23. \sum_{n=1}^{\infty} \frac{2^n - 1}{4^n}$$

$$24. \sum_{n=1}^{\infty} \frac{1 + 2^{n+1}}{4^n}$$

$$25. \sum_{n=1}^{\infty} \frac{3^n + 5}{4^n}$$

$$26. \sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$$

$$27. \sum_{n=1}^{\infty} \frac{3^n - 2^n}{4^n}$$

$$28. \sum_{n=1}^{\infty} \frac{2^{n-1} + 3^{n+1}}{4^n}$$

$$29. \sum_{n=1}^{\infty} \frac{3^{n+1} - 3^{n-1}}{4^n}$$

$$30. \sum_{n=1}^{\infty} \frac{2^{2n} - 1}{5^n}$$

$$31. \sum_{n=1}^{\infty} \frac{1 + (-3)^n}{5^n}$$

$$32. \sum_{n=1}^{\infty} \frac{3^n + 1}{5^n}$$

$$33. \sum_{n=1}^{\infty} \frac{3^n - 1}{5^n}$$

$$34. \sum_{n=1}^{\infty} \frac{2^n - 3^n}{5^n}$$

$$35. \sum_{n=1}^{\infty} \frac{4^n + (-2)^n}{5^n}$$

$$36. \sum_{n=1}^{\infty} \frac{2^n + 4^n}{5^n}$$

$$37. \sum_{n=1}^{\infty} \frac{3^n + 4^n}{5^n}$$

$$38. \sum_{n=1}^{\infty} \frac{2^{n-1} + 3^n}{5^n}$$

$$39. \sum_{n=1}^{\infty} \frac{2 \cdot 3^n + (-2)^{2n}}{5^n}$$

$$40. \sum_{n=1}^{\infty} \frac{1 + 2^n}{6^n}$$

$$41. \sum_{n=1}^{\infty} \frac{3^n - 2^n}{6^n}$$

$$42. \sum_{n=1}^{\infty} \frac{3^n + 4^n}{6^n}$$

$$43. \sum_{n=1}^{\infty} \frac{2^{n+1} + 4^n}{6^n}$$

$$44. \sum_{n=1}^{\infty} \frac{2^{n-1} + 3^n}{6^n}$$

$$45. \sum_{n=1}^{\infty} \frac{2^{n+1} + (-3)^n}{6^n}$$

$$46. \sum_{n=1}^{\infty} \frac{1 + 5^n}{6^n}$$

$$47. \sum_{n=1}^{\infty} \frac{5^n + (-1)^{n-1}}{6^n}$$

$$48. \sum_{n=1}^{\infty} \frac{2^{n-1} + 3^{n-1}}{6^{n-1}}$$

$$49. \sum_{n=1}^{\infty} \frac{2^{n+1} - 3^{n-1}}{6^{n+1}}$$

50. $\sum_{n=1}^{\infty} \frac{3^{n+1} + (-5)^{n-1}}{6^n}$

51. $\sum_{n=1}^{\infty} \frac{3^n - 2^n}{7^n}$

52. $\sum_{n=1}^{\infty} \frac{3^n + 2^{2n}}{9^n}$

53. $\sum_{n=1}^{\infty} \frac{(-2)^n - 5^{n+1}}{10^n}$

54. $\sum_{n=1}^{\infty} \frac{3^{n+1} - 4^n + 6^{n-1}}{12^{n+1}}$

55. $\sum_{n=1}^{\infty} \left(\frac{1}{1 + \sqrt{3}} \right)^{n-1}$

▣ 다음 값을 구하여라.

56. 등비수열 $\{a_n\}$ 에 대하여 $a_1 = 3$, $a_2 = 1$ 일 때,
 $\sum_{n=1}^{\infty} (a_n)^2$ 의 값

57. 첫째항이 a 이고 공비가 $\frac{1}{4}$ 인 등비수열 $\{a_n\}$ 에 대
 하여 $\sum_{n=1}^{\infty} a_n = 16$ 일 때, a^2 의 값

58. 첫째항이 2인 등비수열 $\{a_n\}$ 에 대하여 $\sum_{n=1}^{\infty} a_n = 4$
 일 때, $\sum_{n=1}^{\infty} a_n^2$ 의 값

59. 공비가 $\frac{1}{5}$ 인 등비수열 $\{a_n\}$ 에 대하여 $\sum_{n=1}^{\infty} a_n = 15$
 일 때, 첫째항 a_1 의 값

60. 급수 $\sum_{n=1}^{\infty} r^{2n}$ 의 합이 4일 때, 급수 $\sum_{n=1}^{\infty} r^{4n-2}$ 의 합



정답 및 해설

1) 3

$$\Rightarrow \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n = \frac{\frac{3}{4}}{1 - \frac{3}{4}} = 3$$

2) 20

\Rightarrow 주어진 등비급수는 첫째항이 5, 공비가 $\frac{3}{4}$ 이므로

$$\sum_{n=1}^{\infty} 5 \left(\frac{3}{4}\right)^{n-1} = \frac{5}{1 - \frac{3}{4}} = 20$$

3) 2

\Rightarrow 주어진 등비급수는 첫째항이 3, 공비가 $-\frac{1}{2}$ 이므로

$$\sum_{n=1}^{\infty} 3 \left(-\frac{1}{2}\right)^{n-1} = \frac{3}{1 - \left(-\frac{1}{2}\right)} = 2$$

4) 4

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{4}\right)^n + \left(\frac{3}{4}\right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n \\ &= \frac{\frac{1}{2}}{1 - \frac{1}{2}} + \frac{\frac{3}{4}}{1 - \frac{3}{4}} \\ &= 1 + 3 = 4 \end{aligned}$$

5) $\frac{1}{2}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{2^n}$ 은 첫째항이 $\frac{1}{2}$ 이고, 공비가 $\frac{1}{2}$ 인 등비급

수이고, $\sum_{n=1}^{\infty} \frac{1}{3^n}$ 은 첫째항이 $\frac{1}{3}$, 공비가 $\frac{1}{3}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right) &= \sum_{n=1}^{\infty} \frac{1}{2^n} - \sum_{n=1}^{\infty} \frac{1}{3^n} \\ &= \frac{\frac{1}{2}}{1 - \frac{1}{2}} - \frac{\frac{1}{3}}{1 - \frac{1}{3}} = 1 - \frac{1}{2} = \frac{1}{2} \end{aligned}$$

6) 4

$\Rightarrow \sum_{n=1}^{\infty} \frac{3}{2^n}$ 은 첫째항이 $\frac{3}{2}$ 이고, 공비가 $\frac{1}{2}$ 인 등비급

수이고, $\sum_{n=1}^{\infty} \frac{2}{3^n}$ 은 첫째항이 $\frac{2}{3}$, 공비가 $\frac{1}{3}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{3}{2^n} + \frac{2}{3^n}\right) &= \sum_{n=1}^{\infty} \frac{3}{2^n} + \sum_{n=1}^{\infty} \frac{2}{3^n} \\ &= \frac{\frac{3}{2}}{1 - \frac{1}{2}} + \frac{\frac{2}{3}}{1 - \frac{1}{3}} = 3 + 1 = 4 \end{aligned}$$

7) 8

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \left(\frac{2^{n+1}}{3^n} + \frac{4}{2^n}\right) &= 2 \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n + 4 \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \\ &= \frac{\frac{4}{3}}{1 - \frac{2}{3}} + \frac{2}{1 - \frac{1}{2}} = 4 + 4 = 8 \end{aligned}$$

8) 4

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{2^n}$ 은 첫째항이 $\frac{1}{2}$ 이고, 공비가 $\frac{1}{2}$ 인 등비급수이고, $\sum_{n=1}^{\infty} \frac{3^n}{4^n}$ 은 첫째항이 $\frac{3}{4}$, 공비가 $\frac{3}{4}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{3^n}{4^n}\right) &= \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=1}^{\infty} \frac{3^n}{4^n} \\ &= \frac{\frac{1}{2}}{1 - \frac{1}{2}} + \frac{\frac{3}{4}}{1 - \frac{3}{4}} = 1 + 3 = 4 \end{aligned}$$

9) 0

$\Rightarrow \sum_{n=1}^{\infty} \frac{3^n}{4^n}$ 은 첫째항이 $\frac{3}{4}$ 이고, 공비가 $\frac{3}{4}$ 인 등비급

수이고, $\sum_{n=1}^{\infty} \frac{3}{2^n}$ 은 첫째항이 $\frac{3}{2}$, 공비가 $\frac{1}{2}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{3^n}{4^n} - \frac{3}{2^n}\right) &= \sum_{n=1}^{\infty} \frac{3^n}{4^n} - \sum_{n=1}^{\infty} \frac{3}{2^n} \\ &= \frac{\frac{3}{4}}{1 - \frac{3}{4}} - \frac{\frac{3}{2}}{1 - \frac{1}{2}} = 3 - 3 = 0 \end{aligned}$$

10) 1

$\Rightarrow \sum_{n=1}^{\infty} \frac{4}{3^n}$ 은 첫째항이 $\frac{4}{3}$ 이고, 공비가 $\frac{1}{3}$ 인 등비급

수이고, $\sum_{n=1}^{\infty} \frac{3}{4^n}$ 은 첫째항이 $\frac{3}{4}$, 공비가 $\frac{1}{4}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{4}{3^n} - \frac{3}{4^n}\right) &= \sum_{n=1}^{\infty} \frac{4}{3^n} - \sum_{n=1}^{\infty} \frac{3}{4^n} \\ &= \frac{\frac{4}{3}}{1 - \frac{1}{3}} - \frac{\frac{3}{4}}{1 - \frac{1}{4}} = 2 - 1 = 1 \end{aligned}$$

11) 5

$$\begin{aligned} &\Rightarrow \sum_{n=1}^{\infty} \left(\frac{2^{n+1}}{4^n} + \frac{6}{3^n} \right) \\ &= \sum_{n=1}^{\infty} 2 \times \left(\frac{1}{2} \right)^n + \sum_{n=1}^{\infty} 6 \times \left(\frac{1}{3} \right)^n \\ &= \frac{1}{1-\frac{1}{2}} + \frac{2}{1-\frac{1}{3}} = 2 + 3 = 5 \end{aligned}$$

12) $\frac{8}{3}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{5}{4^n}$ 은 첫째항이 $\frac{5}{4}$ 이고, 공비가 $\frac{1}{4}$ 인

등비급수이고, $\sum_{n=1}^{\infty} \frac{4}{5^n}$ 은 첫째항이 $\frac{4}{5}$, 공비가 $\frac{1}{5}$ 인

등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{5}{4^n} + \frac{4}{5^n} \right) &= \sum_{n=1}^{\infty} \frac{5}{4^n} + \sum_{n=1}^{\infty} \frac{4}{5^n} \\ &= \frac{\frac{5}{4}}{1-\frac{1}{4}} + \frac{\frac{4}{5}}{1-\frac{1}{5}} = \frac{5}{3} + 1 = \frac{8}{3} \end{aligned}$$

13) $-\frac{22}{3}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{3}{4^n}$ 은 첫째항이 $\frac{3}{4}$ 이고, 공비가 $\frac{1}{4}$ 인 등비급

수이고, $\sum_{n=1}^{\infty} \frac{2}{(-5)^{n-2}}$ 은 첫째항이 -10 , 공비가 $-\frac{1}{5}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{3}{4^n} + \frac{2}{(-5)^{n-2}} \right) &= \sum_{n=1}^{\infty} \frac{3}{4^n} + \sum_{n=1}^{\infty} \frac{2}{(-5)^{n-2}} \\ &= \frac{\frac{3}{4}}{1-\frac{1}{4}} + \frac{-10}{1+\frac{1}{5}} = 1 - \frac{25}{3} = -\frac{22}{3} \end{aligned}$$

14) $\frac{7}{4}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{3}{5^n}$ 은 첫째항이 $\frac{3}{5}$, 공비가 $\frac{1}{5}$ 인 등비급수이

고, $\sum_{n=1}^{\infty} \frac{2}{3^n}$ 은 첫째항이 $\frac{2}{3}$, 공비가 $\frac{1}{3}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{3^n} \right) &= \sum_{n=1}^{\infty} \frac{3}{5^n} + \sum_{n=1}^{\infty} \frac{2}{3^n} \\ &= \frac{\frac{3}{5}}{1-\frac{1}{5}} + \frac{\frac{2}{3}}{1-\frac{1}{3}} \\ &= \frac{3}{4} + 1 = \frac{7}{4} \end{aligned}$$

15) 1

$\Rightarrow \sum_{n=1}^{\infty} \frac{3^n}{5^n}$ 은 첫째항이 $\frac{3}{5}$, 공비가 $\frac{3}{5}$ 인 등비급수이

고, $\sum_{n=1}^{\infty} \frac{2}{(-3)^n}$ 은 첫째항이 $-\frac{2}{3}$, 공비가 $-\frac{1}{3}$ 인 등비급수이다.

$$\begin{aligned} \therefore \sum_{n=1}^{\infty} \left(\frac{3^n}{5^n} + \frac{2}{(-3)^n} \right) &= \sum_{n=1}^{\infty} \frac{3^n}{5^n} + \sum_{n=1}^{\infty} \frac{2}{(-3)^n} \\ &= \frac{\frac{3}{5}}{1-\frac{3}{5}} + \frac{-\frac{2}{3}}{1+\frac{1}{3}} = \frac{3}{2} - \frac{1}{2} = 1 \end{aligned}$$

16) $\frac{23}{4}$

$$\begin{aligned} &\Rightarrow \sum_{n=1}^{\infty} \left(\frac{1}{2^{n-1}} + \frac{3}{5^{n-1}} \right) \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^{n-1} + \sum_{n=1}^{\infty} 3 \times \left(\frac{1}{5} \right)^{n-1} \\ &= \frac{1}{1-\frac{1}{2}} + \frac{3}{1-\frac{1}{5}} \\ &= 2 + \frac{15}{4} = \frac{23}{4} \end{aligned}$$

17) $\frac{29}{4}$

$$\begin{aligned} &\Rightarrow \sum_{n=1}^{\infty} \left(\frac{1}{5^{n-1}} + \frac{5}{6^{n-1}} \right) = \sum_{n=1}^{\infty} \left(\frac{1}{5} \right)^{n-1} + 5 \sum_{n=1}^{\infty} \left(\frac{1}{6} \right)^{n-1} \\ &= \frac{1}{1-\frac{1}{5}} + \frac{5}{1-\frac{1}{6}} = \frac{5}{4} + 6 = \frac{29}{4} \end{aligned}$$

18) $\frac{1}{4}$

$$\begin{aligned} &\Rightarrow \sum_{n=1}^{\infty} \frac{1+(-1)^n}{3^n} = \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{3} \right)^n + \left(-\frac{1}{3} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^n + \sum_{n=1}^{\infty} \left(-\frac{1}{3} \right)^n \\ &= \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{-\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \end{aligned}$$

19) 20

$$\begin{aligned} &\Rightarrow \sum_{n=1}^{\infty} 8 \left(\frac{1+2^n}{3^n} \right) = 8 \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{3} \right)^n + \left(\frac{2}{3} \right)^n \right\} \\ &= 8 \sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^n + 8 \sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^n \\ &= 8 \times \frac{\frac{1}{3}}{1-\frac{1}{3}} + 8 \times \frac{\frac{2}{3}}{1-\frac{2}{3}} = 8 \times \frac{1}{2} + 8 \times 2 = 4 + 16 = 20 \end{aligned}$$

$$20) \frac{1}{10}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{1+(-2)^n}{3^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{3} \right)^n + \left(-\frac{2}{3} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^n + \sum_{n=1}^{\infty} \left(-\frac{2}{3} \right)^n \\ &= \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{-\frac{2}{3}}{1+\frac{2}{3}} = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} \end{aligned}$$

$$21) \frac{8}{5}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n+(-2)^n}{3^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{3} \right)^n + \left(-\frac{2}{3} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^n + \sum_{n=1}^{\infty} \left(-\frac{2}{3} \right)^n \\ &= \frac{\frac{2}{3}}{1-\frac{2}{3}} + \frac{-\frac{2}{3}}{1+\frac{2}{3}} = 2 - \frac{2}{5} = \frac{8}{5} \end{aligned}$$

$$22) \frac{4}{3}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n+1}{4^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{4} \right)^n + \left(\frac{1}{4} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n + \sum_{n=1}^{\infty} \left(\frac{1}{4} \right)^n \\ &= \frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{\frac{1}{4}}{1-\frac{1}{4}} = 1 + \frac{1}{3} = \frac{4}{3} \end{aligned}$$

$$23) \frac{2}{3}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n-1}{4^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{4} \right)^n - \left(\frac{1}{4} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{4} \right)^n \\ &= \frac{\frac{1}{2}}{1-\frac{1}{2}} - \frac{\frac{1}{4}}{1-\frac{1}{4}} = 1 - \frac{1}{3} = \frac{2}{3} \end{aligned}$$

$$24) \frac{7}{3}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{1+2^{n+1}}{4^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{4} \right)^n + 2 \left(\frac{2}{4} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{4} \right)^n + 2 \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n \\ &= \frac{\frac{1}{4}}{1-\frac{1}{4}} + 2 \times \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1}{3} + 2 \times 1 = \frac{7}{3} \end{aligned}$$

$$25) \frac{14}{3}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n+5}{4^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{4} \right)^n + 5 \left(\frac{1}{4} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{3}{4} \right)^n + 5 \sum_{n=1}^{\infty} \left(\frac{1}{4} \right)^n \\ &= \frac{\frac{3}{4}}{1-\frac{3}{4}} + 5 \times \frac{\frac{1}{4}}{1-\frac{1}{4}} = 3 + 5 \times \frac{1}{3} = \frac{14}{3} \end{aligned}$$

$$26) 4$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{2} \right)^n + \left(\frac{3}{4} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n + \sum_{n=1}^{\infty} \left(\frac{3}{4} \right)^n \\ &= \frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{\frac{3}{4}}{1-\frac{3}{4}} = 1 + 3 = 4 \end{aligned}$$

$$27) 2$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n-2^n}{4^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{4} \right)^n - \left(\frac{2}{4} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{3}{4} \right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n \\ &= \frac{\frac{3}{4}}{1-\frac{3}{4}} - \frac{\frac{1}{2}}{1-\frac{1}{2}} = 3 - 1 = 2 \end{aligned}$$

$$28) \frac{19}{2}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{n-1}+3^{n+1}}{4^n} &= \sum_{n=1}^{\infty} \left\{ \frac{1}{4} \left(\frac{2}{4} \right)^{n-1} + 3 \left(\frac{3}{4} \right)^n \right\} \\ &= \frac{1}{4} \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^{n-1} + 3 \sum_{n=1}^{\infty} \left(\frac{3}{4} \right)^n \\ &= \frac{1}{4} \times \frac{1}{1-\frac{1}{2}} + 3 \times \frac{\frac{3}{4}}{1-\frac{3}{4}} = \frac{1}{4} \times 2 + 3 \times 3 = \frac{1}{2} + 9 = \frac{19}{2} \end{aligned}$$

$$29) 8$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^{n+1}-3^{n-1}}{4^n} &= \sum_{n=1}^{\infty} \left\{ 3 \left(\frac{3}{4} \right)^n - \frac{1}{4} \left(\frac{3}{4} \right)^{n-1} \right\} \\ &= 3 \sum_{n=1}^{\infty} \left(\frac{3}{4} \right)^n - \frac{1}{4} \sum_{n=1}^{\infty} \left(\frac{3}{4} \right)^{n-1} \\ &= 3 \times \frac{\frac{3}{4}}{1-\frac{3}{4}} - \frac{1}{4} \times \frac{1}{1-\frac{3}{4}} \\ &= 3 \times 3 - \frac{1}{4} \times 4 = 9 - 1 = 8 \end{aligned}$$

$$30) \frac{15}{4}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{2n}-1}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{4}{5} \right)^n - \left(\frac{1}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{4}{5} \right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{5} \right)^n \\ &= \frac{\frac{4}{5}}{1-\frac{4}{5}} - \frac{\frac{1}{5}}{1-\frac{1}{5}} \\ &= 4 - \frac{1}{4} = \frac{15}{4} \end{aligned}$$

$$31) -\frac{1}{8}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{1+(-3)^n}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{5} \right)^n + \left(-\frac{3}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{5} \right)^n + \sum_{n=1}^{\infty} \left(-\frac{3}{5} \right)^n \\ &= \frac{\frac{1}{5}}{1-\frac{1}{5}} + \frac{-\frac{3}{5}}{1+\frac{3}{5}} = \frac{1}{4} - \frac{3}{8} = -\frac{1}{8} \end{aligned}$$

$$32) \frac{7}{4}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n+1}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{5} \right)^n + \left(\frac{1}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{3}{5} \right)^n + \sum_{n=1}^{\infty} \left(\frac{1}{5} \right)^n \\ &= \frac{\frac{3}{5}}{1-\frac{3}{5}} + \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{3}{2} + \frac{1}{4} = \frac{7}{4} \end{aligned}$$

$$33) \frac{5}{4}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n-1}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{5} \right)^n - \left(\frac{1}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{3}{5} \right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{5} \right)^n \\ &= \frac{\frac{3}{5}}{1-\frac{3}{5}} - \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{3}{2} - \frac{1}{4} = \frac{5}{4} \end{aligned}$$

$$34) -\frac{5}{6}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n-3^n}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{5} \right)^n - \left(\frac{3}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{2}{5} \right)^n - \sum_{n=1}^{\infty} \left(\frac{3}{5} \right)^n \end{aligned}$$

$$= \frac{\frac{2}{5}}{1-\frac{2}{5}} - \frac{\frac{3}{5}}{1-\frac{3}{5}} = \frac{2}{3} - \frac{3}{2} = -\frac{5}{6}$$

$$35) \frac{26}{7}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{4^n+(-2)^n}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{4}{5} \right)^n + \left(-\frac{2}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{4}{5} \right)^n + \sum_{n=1}^{\infty} \left(-\frac{2}{5} \right)^n \\ &= \frac{\frac{4}{5}}{1-\frac{4}{5}} + \frac{-\frac{2}{5}}{1+\frac{2}{5}} \\ &= 4 - \frac{2}{7} = \frac{26}{7} \end{aligned}$$

$$36) \frac{14}{3}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n+4^n}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{5} \right)^n + \left(\frac{4}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{2}{5} \right)^n + \sum_{n=1}^{\infty} \left(\frac{4}{5} \right)^n \\ &= \frac{\frac{2}{5}}{1-\frac{2}{5}} + \frac{\frac{4}{5}}{1-\frac{4}{5}} \\ &= \frac{2}{3} + 4 = \frac{14}{3} \end{aligned}$$

$$37) \frac{11}{2}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n+4^n}{5^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{5} \right)^n + \left(\frac{4}{5} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{3}{5} \right)^n + \sum_{n=1}^{\infty} \left(\frac{4}{5} \right)^n \\ &= \frac{\frac{3}{5}}{1-\frac{3}{5}} + \frac{\frac{4}{5}}{1-\frac{4}{5}} \\ &= \frac{3}{2} + 4 = \frac{11}{2} \end{aligned}$$

$$38) \frac{11}{6}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{n-1}+3^n}{5^n} &= \sum_{n=1}^{\infty} \left\{ \frac{1}{5} \left(\frac{2}{5} \right)^{n-1} + \left(\frac{3}{5} \right)^n \right\} \\ &= \frac{1}{5} \sum_{n=1}^{\infty} \left(\frac{2}{5} \right)^{n-1} + \sum_{n=1}^{\infty} \left(\frac{3}{5} \right)^n \\ &= \frac{1}{5} \times \frac{1}{1-\frac{2}{5}} + \frac{\frac{3}{5}}{1-\frac{3}{5}} \end{aligned}$$

$$= \frac{1}{5} \times \frac{5}{3} + \frac{3}{2} = \frac{11}{6}$$

39) 7

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2 \cdot 3^n + (-2)^{2n}}{5^n} &= \sum_{n=1}^{\infty} \left\{ 2 \left(\frac{3}{5} \right)^n + \left(\frac{4}{5} \right)^n \right\} \\ &= 2 \sum_{n=1}^{\infty} \left(\frac{3}{5} \right)^n + \sum_{n=1}^{\infty} \left(\frac{4}{5} \right)^n \\ &= 2 \times \frac{\frac{3}{5}}{1 - \frac{3}{5}} + \frac{\frac{4}{5}}{1 - \frac{4}{5}} \\ &= 2 \times \frac{3}{2} + 4 = 7 \end{aligned}$$

40) $\frac{7}{10}$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{1+2^n}{6^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{6} \right)^n + \left(\frac{2}{6} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{6} \right)^n + \sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^n \\ &= \frac{\frac{1}{6}}{1 - \frac{1}{6}} + \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{5} + \frac{1}{2} = \frac{7}{10} \end{aligned}$$

41) $\frac{1}{2}$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n - 2^n}{6^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{6} \right)^n - \left(\frac{2}{6} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^n \\ &= \frac{\frac{1}{2}}{1 - \frac{1}{2}} - \frac{\frac{1}{3}}{1 - \frac{1}{3}} = 1 - \frac{1}{2} = \frac{1}{2} \end{aligned}$$

42) 3

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n + 4^n}{6^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{6} \right)^n + \left(\frac{4}{6} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n + \sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^n \\ &= \frac{\frac{1}{2}}{1 - \frac{1}{2}} + \frac{\frac{2}{3}}{1 - \frac{2}{3}} = 1 + 2 = 3 \end{aligned}$$

43) 3

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{n+1} + 4^n}{6^n} &= \sum_{n=1}^{\infty} \left\{ 2 \left(\frac{2}{6} \right)^n + \left(\frac{4}{6} \right)^n \right\} \end{aligned}$$

$$\begin{aligned} &= \sum_{n=1}^{\infty} 2 \left(\frac{1}{3} \right)^n + \sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^n \\ &= 2 \times \frac{\frac{1}{3}}{1 - \frac{1}{3}} + \frac{\frac{2}{3}}{1 - \frac{2}{3}} = 2 \times \frac{1}{2} + 2 = 3 \end{aligned}$$

44) $\frac{5}{4}$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{n-1} + 3^n}{6^n} &= \sum_{n=1}^{\infty} \left\{ \frac{1}{6} \left(\frac{2}{6} \right)^{n-1} + \left(\frac{3}{6} \right)^n \right\} \\ &= \frac{1}{6} \sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^{n-1} + \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n \\ &= \frac{1}{6} \times \frac{1}{1 - \frac{1}{3}} + \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{1}{6} \times \frac{3}{2} + 1 = \frac{5}{4} \end{aligned}$$

45) $\frac{2}{3}$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{n+1} + (-3)^n}{6^n} &= \sum_{n=1}^{\infty} \left\{ 2 \left(\frac{2}{6} \right)^n + \left(-\frac{3}{6} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} 2 \left(\frac{1}{3} \right)^n + \sum_{n=1}^{\infty} \left(-\frac{1}{2} \right)^n \\ &= 2 \times \frac{\frac{1}{3}}{1 - \frac{1}{3}} + \frac{-\frac{1}{2}}{1 + \frac{1}{2}} = 2 \times \frac{1}{2} - \frac{1}{3} = \frac{2}{3} \end{aligned}$$

46) $\frac{26}{5}$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{1+5^n}{6^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{6} \right)^n + \left(\frac{5}{6} \right)^n \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{6} \right)^n + \sum_{n=1}^{\infty} \left(\frac{5}{6} \right)^n \\ &= \frac{\frac{1}{6}}{1 - \frac{1}{6}} + \frac{\frac{5}{6}}{1 - \frac{5}{6}} = \frac{1}{5} + 5 = \frac{26}{5} \end{aligned}$$

47) $\frac{36}{7}$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{5^n + (-1)^{n-1}}{6^n} &= \sum_{n=1}^{\infty} \left\{ \left(\frac{5}{6} \right)^n + \frac{1}{6} \left(-\frac{1}{6} \right)^{n-1} \right\} \\ &= \sum_{n=1}^{\infty} \left(\frac{5}{6} \right)^n + \sum_{n=1}^{\infty} \frac{1}{6} \left(-\frac{1}{6} \right)^{n-1} \end{aligned}$$

$$= \frac{\frac{5}{6}}{1 - \frac{5}{6}} + \frac{\frac{1}{6}}{1 - \left(-\frac{1}{6}\right)} = 5 + \frac{1}{7} = \frac{36}{7}$$

$$48) \frac{7}{2}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{n-1} + 3^{n-1}}{6^{n-1}} \\ = \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{6}\right)^{n-1} + \left(\frac{3}{6}\right)^{n-1} \right\} \\ = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n-1} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} \\ = \frac{1}{1 - \frac{1}{3}} + \frac{1}{1 - \frac{1}{2}} = \frac{3}{2} + 2 = \frac{7}{2} \end{aligned}$$

$$49) \frac{1}{9}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{2^{n+1} - 3^{n-1}}{6^{n+1}} \\ = \sum_{n=1}^{\infty} \left\{ \left(\frac{2}{6}\right)^{n+1} - \frac{1}{36} \times \left(\frac{3}{6}\right)^{n-1} \right\} \\ = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n+1} - \frac{1}{36} \times \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} \\ = \frac{\frac{1}{9}}{1 - \frac{1}{3}} - \frac{1}{36} \times \frac{1}{1 - \frac{1}{2}} = \frac{1}{6} - \frac{1}{36} \times 2 = \frac{2}{18} = \frac{1}{9} \end{aligned}$$

$$50) \frac{34}{11}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^{n+1} + (-5)^{n-1}}{6^n} \\ = \sum_{n=1}^{\infty} \left\{ 3 \times \left(\frac{3}{6}\right)^n + \frac{1}{6} \times \left(\frac{-5}{6}\right)^{n-1} \right\} \\ = \sum_{n=1}^{\infty} 3 \times \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \frac{1}{6} \times \left(\frac{-5}{6}\right)^{n-1} \\ = \frac{\frac{3}{2}}{1 - \frac{1}{2}} + \frac{\frac{1}{6}}{1 + \frac{5}{6}} = 3 + \frac{1}{11} = \frac{34}{11} \end{aligned}$$

$$51) \frac{7}{20}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n - 2^n}{7^n} = \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{7}\right)^n - \left(\frac{2}{7}\right)^n \right\} \\ = \sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n - \sum_{n=1}^{\infty} \left(\frac{2}{7}\right)^n \\ = \frac{\frac{3}{7}}{1 - \frac{3}{7}} - \frac{\frac{2}{7}}{1 - \frac{2}{7}} = \frac{3}{4} - \frac{2}{5} = \frac{7}{20} \end{aligned}$$

$$52) \frac{13}{10}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^n + 2^{2n}}{9^n} = \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{9}\right)^n + \left(\frac{4}{9}\right)^n \right\} \\ = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^n \\ = \frac{\frac{1}{3}}{1 - \frac{1}{3}} + \frac{\frac{4}{9}}{1 - \frac{4}{9}} = \frac{1}{2} + \frac{4}{5} = \frac{13}{10} \end{aligned}$$

$$53) -\frac{31}{6}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{(-2)^n - 5^{n+1}}{10^n} \\ = \sum_{n=1}^{\infty} \left\{ \left(-\frac{2}{10}\right)^n - 5 \times \left(\frac{5}{10}\right)^n \right\} \\ = \sum_{n=1}^{\infty} \left(-\frac{1}{5}\right)^n - 5 \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \\ = \frac{-\frac{1}{5}}{1 + \frac{1}{5}} - 5 \times \frac{\frac{1}{2}}{1 - \frac{1}{2}} = -\frac{1}{6} - 5 \times 1 = -\frac{31}{6} \end{aligned}$$

$$54) \frac{1}{18}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \frac{3^{n+1} - 4^n + 6^{n-1}}{12^{n+1}} \\ = \sum_{n=1}^{\infty} \left\{ \left(\frac{3}{12}\right)^{n+1} - \frac{1}{12} \times \left(\frac{4}{12}\right)^n + \frac{1}{144} \times \left(\frac{6}{12}\right)^{n-1} \right\} \\ = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^{n+1} - \frac{1}{12} \times \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \frac{1}{144} \times \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} \\ = \frac{\frac{1}{16}}{1 - \frac{1}{4}} - \frac{1}{12} \times \frac{\frac{1}{3}}{1 - \frac{1}{3}} + \frac{1}{144} \times \frac{1}{1 - \frac{1}{2}} \\ = \frac{1}{12} - \frac{1}{12} \times \frac{1}{2} + \frac{1}{144} \times 2 = \frac{1}{12} - \frac{1}{24} + \frac{1}{72} = \frac{4}{72} = \frac{1}{18} \end{aligned}$$

$$55) \frac{3 + \sqrt{3}}{3}$$

$$\begin{aligned} \Rightarrow \sum_{n=1}^{\infty} \left(\frac{1}{1 + \sqrt{3}} \right)^{n-1} &= \frac{1}{1 - \frac{1}{1 + \sqrt{3}}} = \frac{1}{\frac{\sqrt{3}}{1 + \sqrt{3}}} \\ &= \frac{1 + \sqrt{3}}{\sqrt{3}} = \frac{3 + \sqrt{3}}{3} \end{aligned}$$

$$56) \frac{81}{8}$$

\Rightarrow 등비수열 $\{a_n\}$ 의 공비를 r 라고 하면

$$r = \frac{a_2}{a_1} = \frac{1}{3} \quad \therefore a_n = 3 \cdot \left(\frac{1}{3}\right)^{n-1}$$

$$(a_n)^2 = \left\{ 3 \cdot \left(\frac{1}{3}\right)^{n-1} \right\}^2 = 9 \cdot \left(\frac{1}{9}\right)^{n-1}$$

$$\therefore \sum_{n=1}^{\infty} (a_n)^2 = \frac{9}{1 - \frac{1}{9}} = \frac{81}{8}$$

57) 144

$$\Rightarrow \sum_{n=1}^{\infty} a_n = \frac{a}{1-r} = \frac{a}{1-\frac{1}{4}} = 16 \text{에서}$$

$$a = 16 \times \frac{3}{4} = 12 \quad \therefore a^2 = 144$$

58) $\frac{16}{3}$ \Rightarrow 등비수열 $\{a_n\}$ 의 공비를 r 라 하면

$$\sum_{n=1}^{\infty} a_n = \frac{2}{1-r} = 4 \quad \therefore r = \frac{1}{2}$$

이때, $\sum_{n=1}^{\infty} a_n^2$ 은 첫째항이 $2^2 = 4$ 이고공비는 $r^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ 인 등비급수이므로

$$\sum_{n=1}^{\infty} a_n^2 = \frac{4}{1 - \frac{1}{4}} = \frac{16}{3}$$

59) 12

 \Rightarrow 등비수열 $\{a_n\}$ 의 첫째항이 a_1 ,

$$\text{공비가 } \frac{1}{5} \text{이므로 } \sum_{n=1}^{\infty} a_n = \frac{a_1}{1 - \frac{1}{5}} = \frac{5}{4} a_1$$

$$\text{즉, } \frac{5}{4} a_1 = 15 \text{이므로 } a_1 = 12$$

60) $\frac{20}{9}$ \Rightarrow 급수 $\sum_{n=1}^{\infty} r^{2n}$ 의 합이 4이므로

$$0 < r^2 < 1, \quad r \neq 0 \text{이고, } \sum_{n=1}^{\infty} r^{2n} = \frac{r^2}{1-r^2} = 4 \text{에서}$$

$$r^2 = 4 - 4r^2 \quad \therefore r^2 = \frac{4}{5}$$

급수 $\sum_{n=1}^{\infty} r^{4n-2} = r^2 + r^6 + r^{10} + \dots$ 은 첫째항이 $r^2 = \frac{4}{5}$,공비가 $r^4 = \frac{16}{25}$ 인 등비급수이므로 구하는 합은

$$\frac{\frac{4}{5}}{1 - \frac{16}{25}} = \frac{20}{9}$$