



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시
 1) 제작연월일 : 2019-08-13
 2) 제작자 : 교육지대(주)
 3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 / 치환적분법을 이용한 정적분의 계산

(1) 치환적분을 이용한 정적분: 구간 $[a, b]$ 에서 연속인 함수 $f(x)$ 에 대하여 미분가능한 함수 $x=g(t)$ 의 도함수 $g'(t)$ 가 구간 $[\alpha, \beta]$ 에서 연속이고 $a=g(\alpha)$, $b=g(\beta)$ 이면

$$\Rightarrow \int_a^b f(x)dx = \int_{\alpha}^{\beta} f(g(t))g'(t)dt$$

(2) 삼각치환을 이용한 정적분

① 피적분함수가 $\sqrt{a^2-x^2}$, $\frac{1}{\sqrt{a^2-x^2}}$ ($a>0$)의 꼴인 경우

$$\Rightarrow x = a \sin \theta \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right) \text{로 치환한다.}$$

② 피적분함수가 $\frac{1}{a^2+x^2}$ ($a>0$)의 꼴인 경우

$$\Rightarrow x = a \tan \theta \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right) \text{로 치환한다.}$$

■ 다음 정적분의 값을 구하여라.

1. $\int_0^1 \sqrt{x+2} dx$

2. $\int_{-2}^1 \sqrt{2-x} dx$

3. $\int_{\sqrt{3}}^2 x \sqrt{x^2-3} dx$

4. $\int_2^{\sqrt{7}} x \sqrt{x^2-3} dx$

5. $\int_0^4 \frac{1}{\sqrt{x}(\sqrt{x}+2)} dx$

6. $\int_0^4 \frac{1}{\sqrt{1+2x}} dx$

7. $\int_0^3 (2x-1)^2 dx$

8. $\int_0^{\sqrt{3}} \frac{2x}{x^2+1} dx$

9. $\int_0^2 x(x^2-1)^2 dx$

$$10. \int_{\frac{4}{5}}^1 5(5x-4)^4 dx$$

$$17. \int_0^{\frac{\pi}{4}} \tan x dx$$

$$11. \int_0^1 4x(x^2+1)^3 dx$$

$$18. \int_1^3 \frac{x+1}{x^2+2x-1} dx$$

$$12. \int_1^2 \frac{x}{3x^2+1} dx$$

$$19. \int_0^1 \frac{e^x+1}{e^x+x} dx$$

$$13. \int_2^5 2x\sqrt{x^2+5} dx$$

$$20. \int_0^1 2xe^{x^2} dx$$

$$14. \int_{\sqrt{e}}^{e^2} \frac{1}{x \ln x} dx$$

$$21. \int_1^3 e^{2x-1} dx$$

$$15. \int_e^{e^2} \frac{1}{x(\ln x)^2} dx$$

$$22. \int_0^1 x^2 e^{x^3+1} dx$$

$$16. \int_1^e \frac{(\ln x)^2}{x} dx$$

$$23. \int_0^{\ln 2} \frac{1}{e^x+1} dx$$

$$24. \int_0^{\ln(2e-1)} \frac{e^x}{e^x+1} dx$$

$$25. \int_0^{\frac{\pi}{2}} 2\cos\left(2x - \frac{\pi}{6}\right) dx$$

$$26. \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2(\tan x - 1)\sec^2 x dx$$

$$27. \int_{-\frac{\pi}{4}}^0 (\tan^2 x + 1) dx$$

$$28. \int_{-\frac{\pi}{2}}^0 (2 + \sin x)^3 \cos x dx$$

$$29. \int_0^{\pi} \cos\left(\frac{x}{2} + \pi\right) dx$$

$$30. \int_0^{\frac{\pi}{4}} (1 - \tan^2 x) \sec^2 x dx$$

$$31. \int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$$

$$32. \int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx$$

$$33. \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{1 + \sin x} dx$$

$$34. \int_0^{\frac{\pi}{2}} \cos^3 x dx$$

$$35. \int_1^{e^2} \frac{(\ln x)^2}{2x} dx$$

$$36. \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot e^{\cos x} dx$$

■ 다음 정적분의 값을 구하여라.

37. $\int_1^{\sqrt{3}} \frac{1}{x^2+1} dx$

38. $\int_0^{\sqrt{3}} \frac{1}{x^2+9} dx$

39. $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{x^2+9} dx$

40. $\int_{-2}^{\sqrt{3}-1} \frac{1}{x^2+2x+2} dx$

41. $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}} dx$

42. $\int_{-1}^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}} dx$

■ 다음 물음에 답하여라.

43. 함수 $f(x) = e^x$ 에 대하여

$\int_0^1 \{f(x) + f(1-x)\} dx$ 의 값을 구하여라.

44. $k \int_e^{2e} \frac{1}{x(\ln x)^2} dx = \ln 2$ 를 만족하는 k 값을 구하여라.

45. $\int_0^a \sqrt{a^2-x^2} dx = \frac{\pi}{2}$ 일 때, 상수 a^2 의 값을 구하여라.

46. $\int_{-a}^a \frac{1}{a^2+x^2} dx = \frac{\pi}{6}$ 일 때, 상수 a 의 값을 구하여라.

47. 정적분 $\int_0^1 16x(x^2+1)^3 dx$ 의 값을 a , 정적분 $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx$ 의 값을 b 라고 할 때, ab 의 값을 구하여라.

02 / 부분적분법을 이용한 정적분의 계산

두 함수 $f(x)$, $g(x)$ 가 미분가능하고 $f'(x)$, $g'(x)$ 가
 닫힌구간 $[a, b]$ 에서 연속이면

$$\Rightarrow \int_a^b f(x)g'(x)dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$$

■ 다음 정적분의 값을 구하여라.

48. $\int_1^e \ln x dx$

49. $\int_0^1 2xe^x dx$

50. $\int_0^1 xe^x dx$

51. $\int_0^1 xe^{-x} dx$

52. $\int_1^e \ln x^2 dx$

53. $\int_1^{\sqrt{e}} x \ln x dx$

54. $\int_1^e (\ln x)^2 dx$

55. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2x-1) \sin 2x dx$

56. $\int_0^{\ln 2} xe^x dx$

57. $\int_0^{\frac{\pi}{4}} 8x \sin 2x dx$

58. $\int_0^{\frac{\pi}{2}} e^x \cos x dx$

59. $\int_0^{\frac{\pi}{2}} x \cos 2x dx$

60. $\int_0^{\frac{\pi}{2}} x \sin x dx$

61. $2 \int_0^{\frac{\pi}{2}} x \cos x dx$

62. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x dx$

63. $\int_0^{\frac{\pi}{4}} \sin x \cos x dx$

64. $\int_1^{e^2} \frac{\ln x}{x^2} dx$

65. $\int_0^1 (1+x)e^x dx$

66. $\int_1^e 4x \ln x dx$

67. $\int_0^{\frac{\pi}{2}} 5e^{2x} \sin x dx$

68. $\int_0^{\frac{1}{2}\pi} e^{2x} \sin 2x dx$

69. $\int_{-1}^1 |x| e^{x+1} dx$

■ 다음 물음에 답하여라.

70. $\int_1^2 \ln x dx = -q + \ln p$ 일 때, $p+q$ 의 값을 구하여라.
(단, p, q 는 자연수)

71. $\int_e^{e^2} \frac{\ln x}{x^2} dx = \frac{a}{e} + \frac{b}{e^2}$ 일 때, $a+b$ 의 값을 구하여라.
(단, a, b 는 정수)

72. 함수 $f(x) = \begin{cases} x-1 & (x \leq 1) \\ \ln x & (x > 1) \end{cases}$ 에 대하여 $\int_0^2 f(x) dx$ 의 값을 구하여라.

73. $\int_0^a x e^x dx = e^2 + 1$ 일 때, 상수 a 의 값을 구하여라.

74. $a = \int_1^e \frac{\ln x}{x^2} dx$, $b = \int_{-1}^1 \frac{x^3 + 2x - 2}{x^2} dx$ 라 할 때,
 $b - a$ 의 값을 구하여라.

75. 함수 $f(x) = xe^{-x}$ 에 대하여
 $\int_0^2 f(x) dx - \int_{-2}^2 f(x) dx + \int_{-2}^1 f(x) dx$ 의 값을 구하
 여라.

76. 함수 $f(x) = 2x \ln x$ 에 대하여
 $\int_2^4 f(x) dx - \int_3^4 f(x) dx + \int_1^2 f(x) dx$ 의 값을
 구하여라.



정답 및 해설

1) $2\sqrt{3} - \frac{4\sqrt{2}}{3}$

$$\Rightarrow x+2=t \text{로 놓으면 } 1 = \frac{dt}{dx}$$

$$x=0 \text{일 때 } t=2, x=1 \text{일 때 } t=3 \text{이므로}$$

$$\begin{aligned} \int_0^1 \sqrt{x+2} dx &= \int_2^3 \sqrt{t} dt = \int_2^3 t^{\frac{1}{2}} dt \\ &= \left[\frac{2}{3} t^{\frac{3}{2}} \right]_2^3 = 2\sqrt{3} - \frac{4\sqrt{2}}{3} \end{aligned}$$

2) $\frac{14}{3}$

$$\Rightarrow 2-x=t \text{로 놓으면}$$

$$-1 = \frac{dt}{dx} \therefore dx = -dt$$

$$x=1 \text{일 때 } t=1, x=-2 \text{일 때 } t=4$$

$$\begin{aligned} \therefore \int_{-2}^1 \sqrt{2-x} dx &= -\int_4^1 \sqrt{t} dt = \int_1^4 \sqrt{t} dt \\ &= \left[\frac{2}{3} t\sqrt{t} \right]_1^4 \\ &= \frac{2}{3} (4\sqrt{4} - 1) \\ &= \frac{14}{3} \end{aligned}$$

3) $\frac{1}{3}$

$$\Rightarrow x^2-3=t \text{로 놓으면}$$

$$2x = \frac{dt}{dx} \therefore xdx = \frac{1}{2} dt$$

$$x=2 \text{일 때 } t=1, x=\sqrt{3} \text{일 때 } t=0$$

$$\begin{aligned} \therefore \int_{\sqrt{3}}^2 x\sqrt{x^2-3} dx &= \frac{1}{2} \int_0^1 \sqrt{t} dt \\ &= \frac{1}{2} \left[\frac{2}{3} t\sqrt{t} \right]_0^1 \\ &= \frac{1}{3} (1-0) = \frac{1}{3} \end{aligned}$$

4) $\frac{7}{3}$

$$\Rightarrow x^2-3=t, 2xdx=dt$$

$$(x=2 \rightarrow t=1, x=\sqrt{7} \rightarrow t=4)$$

$$\begin{aligned} \int_2^{\sqrt{7}} x\sqrt{x^2-3} dx &= \int_1^4 \frac{1}{2} \sqrt{t} dt = \frac{1}{2} \cdot \frac{2}{3} [t\sqrt{t}]_1^4 \\ &= \frac{1}{3} (4\sqrt{4} - 1) = \frac{7}{3} \end{aligned}$$

5) $\ln 4$

$$\Rightarrow \sqrt{x}=t \text{로 치환하면 } \frac{1}{2\sqrt{x}} dx = dt$$

$$\int_0^2 \frac{2}{(t+2)} dt = 2[\ln|t+2|]_0^2 = 2\ln 4 - 2\ln 2 = 2\ln 2 = \ln 4$$

6) 2

$$\Rightarrow 1+2x=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$2 = \frac{dt}{dx} \therefore dx = \frac{1}{2} dt$$

$$x=0 \text{일 때 } t=1, x=4 \text{일 때 } t=9$$

$$\begin{aligned} \int_0^4 \frac{1}{\sqrt{1+2x}} dx &= \frac{1}{2} \int_1^9 \frac{1}{\sqrt{t}} dt = \frac{1}{2} \int_1^9 t^{-\frac{1}{2}} dt \\ &= \frac{1}{2} [2\sqrt{t}]_1^9 = 3-1 = 2 \end{aligned}$$

7) 21

$$\Rightarrow 2x-1=t \text{로 놓으면 } 2 = \frac{dt}{dx}$$

$$x=0 \text{일 때 } t=-1, x=3 \text{일 때 } t=5 \text{이므로}$$

$$\begin{aligned} \int_0^3 (2x-1)^2 dx &= \frac{1}{2} \int_{-1}^5 t^2 dt = \frac{1}{2} \left[\frac{1}{3} t^3 \right]_{-1}^5 \\ &= \frac{1}{2} \left(\frac{125}{3} - \left(-\frac{1}{3} \right) \right) = 21 \end{aligned}$$

8) $\ln 4$

$$\Rightarrow x^2+1=t \text{로 치환하자. } 2xdx=dt$$

$$\int_1^4 \frac{1}{t} dt = [\ln|t|]_1^4 = \ln 4$$

9) $\frac{14}{3}$

$$\Rightarrow x^2-1=t \text{로 놓으면 } 2x = \frac{dt}{dx}$$

$$x=0 \text{일 때 } t=-1, x=2 \text{일 때 } t=3 \text{이므로}$$

$$\begin{aligned} \int_0^2 x(x^2-1)^2 dx &= \frac{1}{2} \int_{-1}^3 t^2 dt \\ &= \left[\frac{1}{6} t^3 \right]_{-1}^3 = \frac{14}{3} \end{aligned}$$

10) $\frac{1}{5}$

$$\Rightarrow \int_{\frac{4}{5}}^1 5(5x-4)^4 dx = \left[\frac{1}{5} (5x-4)^5 \right]_{\frac{4}{5}}^1 = \frac{1}{5} (1-0) = \frac{1}{5}$$

11) $\frac{15}{2}$

$$\Rightarrow x^2+1=t \text{라 하면 } 2xdx=dt$$

$$\int_1^2 2t^3 dt = \left[\frac{2}{4} t^4 \right]_1^2 = \frac{1}{2} 2^4 - \frac{1}{2} = \frac{15}{2}$$

12) $\frac{1}{6} \ln \frac{13}{4}$

$$\Rightarrow 3x^2+1=t \text{로 놓으면 } 6x = \frac{dt}{dx}$$

$$x=1 \text{일 때 } t=4, x=2 \text{일 때 } t=13 \text{이므로}$$

$$\begin{aligned}\int_1^2 \frac{x}{3x^2+1} dx &= \int_4^{13} \frac{1}{t} \times \frac{1}{6} dt = \left[\frac{1}{6} \ln |t| \right]_4^{13} \\ &= \frac{1}{6} \ln 13 - \frac{1}{6} \ln 4 \\ &= \frac{1}{6} \ln \frac{13}{4}\end{aligned}$$

13) $20\sqrt{30}-18$

$$\begin{aligned}\Rightarrow x^2+5=t \text{로 놓으면 } 2x &= \frac{dt}{dx} \\ x=2 \text{일 때 } t=9, \quad x=5 \text{일 때 } t=30 \\ \int_2^5 2x\sqrt{x^2+5} dx &= \int_9^{30} \sqrt{t} dt \\ &= \left[\frac{2}{3} t\sqrt{t} \right]_9^{30} \\ &= 20\sqrt{30}-18\end{aligned}$$

14) $2\ln 2$

$$\begin{aligned}\Rightarrow \int_{\sqrt{e}}^{e^2} \frac{1}{x \ln x} dx &= \int_{\sqrt{e}}^{e^2} \frac{\frac{1}{x}}{\ln x} dx \\ &= \int_{\sqrt{e}}^{e^2} \frac{(\ln x)'}{\ln x} dx \\ &= \left[\ln |\ln x| \right]_{\sqrt{e}}^{e^2} \\ &= \ln 2 - \ln \frac{1}{2} \\ &= 2\ln 2\end{aligned}$$

15) $\frac{1}{2}$

$$\begin{aligned}\Rightarrow \ln x = t \text{로 놓으면} \\ \frac{1}{x} = \frac{dt}{dx} \quad \therefore \frac{1}{x} dx = dt \\ x=e^2 \text{일 때 } t=2, \quad x=e \text{일 때 } t=1 \\ \therefore \int_e^{e^2} \frac{1}{x(\ln x)^2} dx &= \int_1^2 \frac{1}{t^2} dt \\ &= \left[-\frac{1}{t} \right]_1^2 \\ &= -\frac{1}{2} - (-1) = \frac{1}{2}\end{aligned}$$

16) $\frac{1}{3}$

$$\begin{aligned}\Rightarrow \ln x = t \text{라 하면 } \frac{1}{x} &= \frac{dt}{dx} \quad \therefore dx = x dt \\ x=1 \text{일 때, } t=0 \text{이고, } x=e \text{일 때 } t=1 \text{이므로} \\ \int_1^e \frac{(\ln x)^2}{x} dx &= \int_0^1 t^2 dt = \left[\frac{1}{3} t^3 \right]_0^1 = \frac{1}{3}\end{aligned}$$

17) $\frac{1}{2} \ln 2$

$$\begin{aligned}\Rightarrow \int_0^{\frac{\pi}{4}} \tan x dx &= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx \\ &= - \int_0^{\frac{\pi}{4}} \frac{(-\sin x)}{\cos x} dx \\ &= - \int_0^{\frac{\pi}{4}} \frac{(\cos x)'}{\cos x} dx \\ &= \left[-\ln |\cos x| \right]_0^{\frac{\pi}{4}} \\ &= -\ln \left| \cos \frac{\pi}{4} \right| - (-\ln |\cos 0|) \\ &= -\ln \frac{\sqrt{2}}{2} = \frac{1}{2} \ln 2\end{aligned}$$

18) $\frac{1}{2} \ln 7$

$$\begin{aligned}\Rightarrow \int_1^3 \frac{x+1}{x^2+2x-1} dx &= \frac{1}{2} \int_1^3 \frac{(x^2+2x-1)'}{x^2+2x-1} dx \\ &= \frac{1}{2} \left[\ln |x^2+2x-1| \right]_1^3 \\ &= \frac{1}{2} (\ln 14 - \ln 2) \\ &= \frac{1}{2} \ln 7\end{aligned}$$

19) $\ln(e+1)$

$$\begin{aligned}\Rightarrow \int_0^1 \frac{e^x+1}{e^x+x} dx &= \int_0^1 \frac{(e^x+x)'}{e^x+x} dx \\ &= \left[\ln |e^x+x| \right]_0^1 \\ &= \ln |e^1+1| - \ln |e^0+0| \\ &= \ln(e+1)\end{aligned}$$

20) $e-1$

$$\begin{aligned}\Rightarrow x^2=t \text{로 놓으면} \\ 2x = \frac{dt}{dx} \quad \therefore 2x dx = dt \\ x=1 \text{일 때 } t=1, \quad x=0 \text{일 때 } t=0 \\ \therefore \int_0^1 2xe^{x^2} dx &= \int_0^1 e^t dt \\ &= [e^t]_0^1 = e-1\end{aligned}$$

21) $\frac{1}{2}(e^5-e)$

$$\begin{aligned}\Rightarrow 2x-1=t \text{로 놓으면 } 2 &= \frac{dt}{dx} \\ x=1 \text{일 때 } t=1, \quad x=3 \text{일 때 } t=5 \\ \int_1^3 e^{2x-1} dx &= \frac{1}{2} \int_1^5 e^t dt \\ &= \frac{1}{2} \left[e^t \right]_1^5 \\ &= \frac{1}{2} (e^5 - e)\end{aligned}$$

$$22) \frac{e}{3}(e-1)$$

$\Rightarrow x^3+1=t$ 로 치환하자. $3x^2dx=dt$

$$\int_1^2 \frac{1}{3}e^t dt = \frac{1}{3}[e^t]_1^2 = \frac{1}{3}(e^2-e) = \frac{e}{3}(e-1)$$

$$23) \ln \frac{4}{3}$$

$\Rightarrow e^x+1=t$ 로 치환하자.

$$e^x dx = dt, \quad dx = \frac{1}{t-1} dt$$

$$\begin{aligned} \int_2^3 \frac{1}{t-1} dt &= \int_2^3 \left(\frac{1}{t-1} - \frac{1}{t} \right) dt \\ &= [\ln|t-1| - \ln|t|]_2^3 = \ln 2 - \ln 3 + \ln 2 = \ln \frac{4}{3} \end{aligned}$$

$$24) 1$$

$\Rightarrow e^x+1=t$ 로 놓고 양변을 x 에 대하여 미분하면

$$e^x = \frac{dt}{dx} \quad \therefore e^x dx = dt$$

$x=0$ 일 때 $t=2$, $x=\ln(2e-1)$ 일 때 $t=2e$

$$\begin{aligned} \therefore \int_0^{\ln(2e-1)} \frac{e^x}{e^x+1} dx &= \int_2^{2e} \frac{1}{t} dt = [\ln t]_2^{2e} \\ &= \ln 2e - \ln 2 = 1 \end{aligned}$$

$$25) 1$$

$\Rightarrow 2x - \frac{\pi}{6} = t$ 로 놓으면

$$2 = \frac{dt}{dx} \quad \therefore 2dx = dt$$

$x = \frac{\pi}{2}$ 일 때 $t = \frac{5}{6}\pi$, $x=0$ 일 때 $t = -\frac{\pi}{6}$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{2}} 2 \cos\left(2x - \frac{\pi}{6}\right) dx &= \int_{-\frac{\pi}{6}}^{\frac{5}{6}\pi} \cos t dt \\ &= [\sin t]_{-\frac{\pi}{6}}^{\frac{5}{6}\pi} \\ &= \frac{1}{2} - \left(-\frac{1}{2}\right) = 1 \end{aligned}$$

$$26) 4-2\sqrt{3}$$

$\Rightarrow \tan x - 1 = t$ 로 놓으면

$$\sec^2 x = \frac{dt}{dx} \quad \therefore \sec^2 x dx = dt$$

$x = \frac{\pi}{3}$ 일 때 $t = \sqrt{3}-1$, $x = \frac{\pi}{4}$ 일 때 $t=0$

$$\begin{aligned} \therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2(\tan x - 1) \sec^2 x dx &= \int_0^{\sqrt{3}-1} 2t dt = [t^2]_0^{\sqrt{3}-1} = (\sqrt{3}-1)^2 = 4-2\sqrt{3} \end{aligned}$$

$$27) 1$$

$\Rightarrow \tan x = t$ 라 하면 $\sec^2 x dx = dt$

$x = -\frac{\pi}{4}$ 일 때, $t = -1$ 이고, $x=0$ 일 때 $t=0$ 이다.

$$\int_{-\frac{\pi}{4}}^0 (\tan^2 x + 1) dx = \int_{-1}^0 \sec^2 x dx = \int_{-1}^0 1 dt = 1$$

$$28) \frac{15}{4}$$

$\Rightarrow 2 + \sin x = t$ 로 놓고 양변을 x 에 대하여 미분하면

$$\cos x = \frac{dt}{dx} \quad \therefore \cos x dx = dt$$

$x = -\frac{\pi}{2}$ 일 때 $t=1$, $x=0$ 일 때 $t=2$

$$\int_{-\frac{\pi}{2}}^0 (2 + \sin x)^3 \cos x dx = \int_1^2 t^3 dt = \left[\frac{1}{4} t^4 \right]_1^2 = \frac{15}{4}$$

$$29) -2$$

$\Rightarrow \frac{x}{2} + \pi = t$ 라 하면 $\frac{1}{2} dx = dt$ 이고

$0 \leq x \leq \pi$ 는 $\pi \leq t \leq \frac{3}{2}\pi$ 이므로

$$\int_0^\pi \cos\left(\frac{x}{2} + \pi\right) dx = 2 \int_\pi^{\frac{3\pi}{2}} \cos t dt = 2[\sin t]_\pi^{\frac{3\pi}{2}} = -2$$

$$30) \frac{2}{3}$$

$\Rightarrow \tan x = t$ 라 하면,

$\tan x = t$, $\sec^2 x dx = dt$, $\int_0^{\frac{\pi}{4}} \rightarrow \int_0^1$ 이므로

주어진 정적분은 $\int_0^1 1-t^2 dt = \frac{2}{3}$ 이다.

$$31) \frac{1}{3}$$

$\Rightarrow \sin x = t$ 로 놓으면 $\cos x = \frac{dt}{dx}$

$x=0$ 일 때 $t=0$, $x=\frac{\pi}{2}$ 일 때 $t=1$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx &= \int_0^1 t^2 dt \\ &= \left[\frac{1}{3} t^3 \right]_0^1 \\ &= \frac{1}{3} \end{aligned}$$

$$32) \frac{1}{3}$$

\Rightarrow

$\cos x = t$ 라 하면 $-\sin x = \frac{dt}{dx} \quad \therefore -\sin x dx = dt$

$x=0$ 일 때, $t=1$, $x=\frac{\pi}{2}$ 일 때, $t=0$ 이므로

$$\int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx = - \int_1^0 t^2 dt = - \left[\frac{1}{3} t^3 \right]_1^0 = \frac{1}{3}$$

$$33) \frac{1}{2}$$

$\Rightarrow 1 + \sin x = t$ 로 놓고 양변을 x 에 대하여 미분하면

$$\cos x = \frac{dt}{dx} \quad \therefore \cos x dx = dt$$

$$x=0 \text{일 때 } t=1, \quad x=\frac{\pi}{2} \text{일 때 } t=2$$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{1 + \sin x} dx &= \int_1^2 \frac{(1 - \sin^2 x) \cos x}{1 + \sin x} dx \\ &= \int_1^2 (1 - \sin x) \cos x dx \\ &= \int_1^2 (2 - t) dt \\ &= \left[2t - \frac{1}{2} t^2 \right]_1^2 = \frac{1}{2} \end{aligned}$$

$$34) \frac{2}{3}$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \cos^3 x dx = \int_0^{\frac{\pi}{2}} (1 - \sin^2 x) \cos x dx \text{이므로}$$

$\sin x = t$ 로 놓고 양변을 x 에 대하여 미분하면

$$\cos x = \frac{dt}{dx} \quad \therefore \cos x dx = dt$$

$$x=0 \text{일 때 } t=0, \quad x=\frac{\pi}{2} \text{일 때 } t=1$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{2}} \cos^3 x dx &= \int_0^1 (1 - \sin^2 x) \cos x dx \\ &= \int_0^1 (1 - t^2) dt \\ &= \left[t - \frac{1}{3} t^3 \right]_0^1 = \frac{2}{3} \end{aligned}$$

$$35) \frac{4}{3}$$

$$\Rightarrow \ln x = t \text{로 놓으면 } \frac{1}{x} = \frac{dt}{dx}$$

$$x=1 \text{일 때 } t=0, \quad x=e^2 \text{일 때 } t=2$$

$$\begin{aligned} \int_1^{e^2} \frac{(\ln x)^2}{2x} dx &= \frac{1}{2} \int_0^2 t^2 dt \\ &= \frac{1}{2} \left[\frac{1}{3} t^3 \right]_0^2 \\ &= \frac{4}{3} \end{aligned}$$

$$36) e^{\frac{\sqrt{3}}{2}} - 1$$

$$\Rightarrow \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot e^{\cos x} dx \text{에서}$$

$$\cos x = t \text{라 하면 } -\sin x dx = dt$$

$$x=\frac{\pi}{6} \text{일 때, } t=\frac{\sqrt{3}}{2}, \quad x=\frac{\pi}{2} \text{일 때, } t=0$$

$$\begin{aligned} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot e^{\cos x} dx &= \int_{\frac{\sqrt{3}}{2}}^0 \sin t \cdot e^t \cdot \frac{dt}{-\sin x} \\ &= \int_{\frac{\sqrt{3}}{2}}^0 -e^t dt = [-e^t]_{\frac{\sqrt{3}}{2}}^0 = -1 + e^{\frac{\sqrt{3}}{2}} \end{aligned}$$

$$37) \frac{\pi}{12}$$

$$\Rightarrow x = \tan \theta \quad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right) \text{로 놓으면}$$

$$\frac{dx}{d\theta} = \sec^2 \theta$$

$$x=1 \text{일 때 } \theta = \frac{\pi}{4}, \quad x=\sqrt{3} \text{일 때 } \theta = \frac{\pi}{3}$$

$$\text{또, } x^2 + 1 = \tan^2 \theta + 1 = \sec^2 \theta \text{이므로}$$

$$\begin{aligned} \int_1^{\sqrt{3}} \frac{1}{x^2 + 1} dx &= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sec^2 \theta} \cdot \sec^2 \theta d\theta \\ &= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1 d\theta \\ &= \left[\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} \\ &= \frac{\pi}{12} \end{aligned}$$

$$38) \frac{\pi}{18}$$

$$\Rightarrow x = 3 \tan \theta \quad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right) \text{로 놓으면}$$

$$\frac{dx}{d\theta} = 3 \sec^2 \theta$$

$$x=0 \text{일 때 } \theta=0, \quad x=\sqrt{3} \text{일 때 } \theta=\frac{\pi}{6}$$

$$\text{또, } x^2 + 9 = 9(\tan^2 \theta + 1) = 9 \sec^2 \theta \text{이므로}$$

$$\begin{aligned} \int_0^{\sqrt{3}} \frac{1}{x^2 + 9} dx &= \int_0^{\frac{\pi}{6}} \frac{1}{9 \sec^2 \theta} \cdot 3 \sec^2 \theta d\theta \\ &= \int_0^{\frac{\pi}{6}} \frac{1}{3} d\theta \\ &= \left[\frac{1}{3} \theta \right]_0^{\frac{\pi}{6}} \\ &= \frac{\pi}{18} \end{aligned}$$

$$39) \frac{\pi}{9}$$

\Rightarrow

$$x = 3 \tan \theta \text{라 하면 } 1 = 3 \sec^2 \theta \cdot \frac{d\theta}{dx} \quad \therefore dx = 3 \sec^2 \theta d\theta$$

$$x = -\sqrt{3} \text{일 때, } \theta = -\frac{\pi}{6}, \quad x = \sqrt{3} \text{일 때, } \theta = \frac{\pi}{6}$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{x^2 + 9} dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{9(1 + \tan^2 \theta)} 3 \sec^2 \theta d\theta$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{9\sec^2\theta} 3\sec^2\theta d\theta = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{3} d\theta = 2 \int_0^{\frac{\pi}{6}} \frac{1}{3} d\theta$$

$$= 2 \left[\frac{1}{3} \theta \right]_0^{\frac{\pi}{6}} = 2 \times \frac{1}{3} \times \frac{\pi}{6} = \frac{\pi}{9}$$

$$40) \frac{7}{12}\pi$$

$$\Rightarrow x+1 = \tan\theta \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right) \text{로 놓으면}$$

$$\frac{dx}{d\theta} = \sec^2\theta$$

$$x = -2 \text{ 일 때 } \theta = -\frac{\pi}{4}, \quad x = \sqrt{3}-1 \text{ 일 때 } \theta = \frac{\pi}{3}$$

$$\text{또, } x^2+2x+2 = (x+1)^2+1 = \tan^2\theta+1 = \sec^2\theta \text{ 이므로}$$

$$\int_{-2}^{\sqrt{3}-1} \frac{1}{x^2+2x+2} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sec^2\theta} \cdot \sec^2\theta d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} 1 d\theta$$

$$= \left[\theta \right]_{-\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{7}{12}\pi$$

$$41) \frac{\pi}{3}$$

$$\Rightarrow x = \sin\theta, \quad dx = \cos\theta d\theta \text{ 이고,}$$

$$x=0 \text{ 일 때, } \theta=0 \text{ 이고, } x=\frac{\sqrt{3}}{2} \text{ 일 때, } \theta=\frac{\pi}{3} \text{ 이므로}$$

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}} dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{\cos\theta}{\sqrt{1-\sin^2\theta}} d\theta = \int_0^{\frac{\pi}{3}} 1 d\theta = \frac{\pi}{3}$$

$$42) \frac{5}{12}\pi$$

$$\Rightarrow x = 2\sin t \text{ 라 하면 } dx = 2\cos t dt$$

$$\int_{-1}^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}} dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{2\cos t}{\sqrt{4-4\sin^2 t}} dt$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} 1 dt = \frac{5}{12}\pi$$

$$43) 2(e-1)$$

$$\Rightarrow 1-x=t \text{ 로 놓으면 } -1 = \frac{dt}{dx}$$

$$x=0 \text{ 일 때 } t=1, \quad x=1 \text{ 일 때 } t=0$$

$$\int_0^1 f(1-x) dx = - \int_1^0 f(t) dt = \int_0^1 f(t) dt \text{ 이므로}$$

$$\int_0^1 \{f(x) + f(1-x)\} dx = \int_0^1 f(x) dx + \int_0^1 f(1-x) dx$$

$$= \int_0^1 f(x) dx + \int_0^1 f(t) dt$$

$$= 2 \int_0^1 f(x) dx$$

$$= 2 \int_0^1 e^x dx$$

$$= 2 \left[e^x \right]_0^1$$

$$= 2(e^1 - e^0)$$

$$= 2(e-1)$$

$$44) \ln 2e$$

$$\Rightarrow \int_e^{2e} \frac{1}{x(\ln x)^2} dx$$

$$\ln x = t \text{ 라 하면 } \frac{1}{x} dx = dt$$

$$x=e \text{ 일 때 } t=1, \quad x=2e \text{ 일 때 } t=\ln 2e$$

$$\int_1^{\ln 2e} \frac{1}{t^2} dt = \left[-\frac{1}{t} \right]_1^{\ln 2e} = -\frac{1}{\ln 2e} + 1$$

$$\left(-\frac{1}{\ln 2e} + 1 \right) k = \ln 2$$

$$\therefore k = \frac{\ln 2}{-\frac{1}{\ln 2e} + 1} = \frac{\ln 2}{-\frac{1}{\ln 2+1} + 1} = \frac{\ln 2}{\frac{\ln 2}{\ln 2+1}}$$

$$= \ln 2 + 1 = \ln 2e$$

$$45) 2$$

$$\Rightarrow x = a \sin\theta \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right) \text{로 놓으면}$$

$$\frac{dx}{d\theta} = a \cos\theta$$

$$x=0 \text{ 일 때 } \theta=0, \quad x=a \text{ 일 때 } \theta=\frac{\pi}{2}$$

$$\int_0^a \sqrt{a^2-x^2} dx = \int_0^{\frac{\pi}{2}} \sqrt{a^2-a^2\sin^2\theta} a \cos\theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2} d\theta$$

$$= a^2 \left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4} a^2$$

$$\frac{\pi}{4} a^2 = \frac{\pi}{2} \text{ 이므로 } a^2 = 2$$

$$46) 3$$

$$\Rightarrow x = a \tan\theta \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right) \text{로 놓으면}$$

$$\frac{dx}{d\theta} = a \sec^2\theta$$

$$x=-a \text{ 일 때 } \theta=-\frac{\pi}{4}, \quad x=a \text{ 일 때 } \theta=\frac{\pi}{4}$$

$$\begin{aligned}
 \int_{-a}^a \frac{1}{a^2+x^2} dx &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{a^2+a^2 \tan^2 \theta} \cdot a \sec^2 \theta d\theta \\
 &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{a^2 \sec^2 \theta} \cdot a \sec^2 \theta d\theta \\
 &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{a} d\theta \\
 &= \frac{\pi}{2a}
 \end{aligned}$$

$$\frac{\pi}{2a} = \frac{\pi}{6} \text{ 이므로 } a=3$$

47) 5π

$$\Rightarrow \int_0^1 16x(x^2+1)^3 dx$$

 $x^2+1=t$ 로 치환하자. $2xdx=dt$

$$\int_1^2 8t^3 dt = [2t^4]_1^2 = 2(16-1) = 30 \quad \therefore a=30$$

$$\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx$$

 $x=2\sin t$ 로 치환하자. $dx=2\cos t dt$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dt = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \quad \therefore b = \frac{\pi}{6}$$

$$\therefore ab = 5\pi$$

48) 1

 $\Rightarrow f(x)=\ln x, g'(x)=1$ 로 놓으면

$$f'(x) = \frac{1}{x}, g(x) = x$$

$$\begin{aligned}
 \therefore \int_1^e \ln x dx &= [x \ln x]_1^e - \int_1^e \frac{1}{x} \times x dx \\
 &= e - [x]_1^e = e - (e-1) \\
 &= 1
 \end{aligned}$$

49) 2

 $\Rightarrow f(x)=2x, g'(x)=e^x$ 로 놓으면

$$f'(x)=2, g(x)=e^x$$

$$\begin{aligned}
 \therefore \int_0^1 2xe^x dx &= [2xe^x]_0^1 - \int_0^1 2e^x dx \\
 &= 2e - [2e^x]_0^1 \\
 &= 2e - (2e-2) \\
 &= 2
 \end{aligned}$$

50) 1

 $\Rightarrow f(x)=x, g'(x)=e^x$ 로 놓으면

$$f'(x)=1, g(x)=e^x \text{ 이므로}$$

$$\begin{aligned}
 \int_0^1 xe^x dx &= [xe^x]_0^1 - \int_0^1 e^x dx \\
 &= e - [e^x]_0^1 = e - (e-1) = 1
 \end{aligned}$$

$$51) 1 - \frac{2}{e}$$

 \Rightarrow

$$\begin{aligned}
 \int_0^1 xe^{-x} dx &= [-xe^{-x}]_0^1 + \int_0^1 e^{-x} dx = -\frac{1}{e} + [-e^{-x}]_0^1 \\
 &= -\frac{1}{e} - \frac{1}{e} + 1 = 1 - \frac{2}{e}
 \end{aligned}$$

52) 2

$$\begin{aligned}
 \Rightarrow \int_1^e 2 \ln x dx &= [2x \ln x]_1^e - \int_1^e 2 dx \\
 &= 2e - 2(e-1) = 2
 \end{aligned}$$

$$53) \frac{1}{4}$$

 $\Rightarrow f(x)=\ln x, g'(x)=x$ 로 놓으면

$$f'(x) = \frac{1}{x}, g(x) = \frac{1}{2}x^2$$

$$\begin{aligned}
 \int_1^{\sqrt{e}} x \ln x dx &= \left[\ln x \cdot \frac{1}{2}x^2 \right]_1^{\sqrt{e}} - \int_1^{\sqrt{e}} \frac{1}{x} \cdot \frac{1}{2}x^2 dx \\
 &= \left[\frac{1}{2}x^2 \ln x \right]_1^{\sqrt{e}} - \int_1^{\sqrt{e}} \frac{1}{2}x dx \\
 &= \frac{e}{4} - \left[\frac{1}{4}x^2 \right]_1^{\sqrt{e}} \\
 &= \frac{e}{4} - \left(\frac{e}{4} - \frac{1}{4} \right) \\
 &= \frac{1}{4}
 \end{aligned}$$

54) $e-2$ \Rightarrow

$$t=\ln x \text{ 라 하면 } x=e^t, 1=e^t \cdot \frac{dt}{dx} \quad \therefore dx=e^t \cdot dt$$

 $x=1$ 일 때, $t=0$, $x=e$ 일 때, $t=1$ 이므로

$$\begin{aligned}
 \int_1^e (\ln x)^2 dx &= \int_0^1 t^2 e^t dt = [t^2 e^t]_0^1 - \int_0^1 2te^t dt \\
 &= e - [2te^t]_0^1 + \int_0^1 2e^t dt = e - 2e + 2e - 2 = e - 2
 \end{aligned}$$

$$55) \frac{\pi}{2} - 1$$

 $\Rightarrow f(x)=2x-1, g'(x)=\sin 2x$ 로 놓으면

$$f'(x)=2, g(x)=-\frac{1}{2}\cos 2x$$

$$\begin{aligned}
 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2x-1) \sin 2x dx &= \left[-\frac{1}{2}(2x-1) \cos 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x dx \\
 &= \frac{\pi}{2} - \frac{1}{2} + \left[\frac{1}{2} \sin 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
 &= \frac{\pi}{2} - \frac{1}{2} - \frac{1}{2} \\
 &= \frac{\pi}{2} - 1
 \end{aligned}$$

56) $2\ln 2 - 1$ $\Rightarrow f(x) = x, g'(x) = e^x$ 로 놓으면 $f'(x) = 1, g(x) = e^x$

$$\begin{aligned}\int_0^{\ln 2} x e^x dx &= \left[x e^x \right]_0^{\ln 2} - \int_0^{\ln 2} e^x dx \\ &= 2\ln 2 - \left[e^x \right]_0^{\ln 2} \\ &= 2\ln 2 - 1\end{aligned}$$

57) 2

58) $\frac{1}{2} \left(e^{\frac{\pi}{2}} - 1 \right)$ $\Rightarrow f(x) = e^x, g'(x) = \cos x$ 로 놓으면 $f'(x) = e^x, g(x) = \sin x$

$$\begin{aligned}\int_0^{\frac{\pi}{2}} e^x \cos x dx &= \left[e^x \sin x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \sin x dx \\ &= \left[e^x \sin x \right]_0^{\frac{\pi}{2}} - \left\{ \left[-e^x \cos x \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} e^x \cos x dx \right\}\end{aligned}$$

에서

$$\begin{aligned}2 \int_0^{\frac{\pi}{2}} e^x \cos x dx &= \left[e^x \sin x \right]_0^{\frac{\pi}{2}} + \left[e^x \cos x \right]_0^{\frac{\pi}{2}} \\ &= e^{\frac{\pi}{2}} - 1 \\ \therefore \int_0^{\frac{\pi}{2}} e^x \cos x dx &= \frac{1}{2} \left(e^{\frac{\pi}{2}} - 1 \right)\end{aligned}$$

59) $-\frac{1}{2}$

$$\begin{aligned}\Rightarrow \int_0^{\frac{\pi}{2}} x \cos 2x dx &= \left[\frac{1}{2} x \sin 2x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin 2x dx \\ &= \left[\frac{1}{4} \cos 2x \right]_0^{\frac{\pi}{2}} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}\end{aligned}$$

60) 1

 $\Rightarrow f(x) = x, g'(x) = \sin x$ 로 놓으면 $f'(x) = 1, g(x) = -\cos x$ 이므로

$$\begin{aligned}\int_0^{\frac{\pi}{2}} x \sin x dx &= \left[-x \cos x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (-\cos x) dx \\ &= 0 + \left[\sin x \right]_0^{\frac{\pi}{2}} = 1\end{aligned}$$

61) $\pi - 2$

$$\begin{aligned}\Rightarrow 2 \int_0^{\frac{\pi}{2}} x \cos x dx &= 2 \left[x \sin x \right]_0^{\frac{\pi}{2}} - 2 \int_0^{\frac{\pi}{2}} \sin x dx \\ &= 2 \left(\frac{\pi}{2} \right) - 2 \left[-\cos x \right]_0^{\frac{\pi}{2}} = \pi - 2\end{aligned}$$

62) $\frac{\pi^2}{2} - 4$

$$\begin{aligned}\Rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x dx &= \left[x^2 \sin x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x \sin x dx \\ &= \left[x^2 \sin x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \left[-2x \cos x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2 \cos x dx \\ &= \left[x^2 \sin x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \left[-2x \cos x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \left[2 \sin x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{\pi^2}{2} - 4\end{aligned}$$

63) $\frac{1}{4}$ $\Rightarrow f(x) = \sin x, g'(x) = \cos x$ 로 놓으면 $f'(x) = \cos x, g(x) = \sin x$ 이므로

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \sin x \cos x dx &= \left[\sin^2 x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin x \cos x dx \\ 2 \int_0^{\frac{\pi}{4}} \sin x \cos x dx &= \left[\sin^2 x \right]_0^{\frac{\pi}{4}} = \frac{1}{2} \\ \therefore \int_0^{\frac{\pi}{4}} \sin x \cos x dx &= \frac{1}{4}\end{aligned}$$

64) $1 - \frac{3}{e^2}$ $\Rightarrow f(x) = \ln x, g'(x) = \frac{1}{x^2}$ 로 놓으면 $f'(x) = \frac{1}{x}, g(x) = -\frac{1}{x}$ 이므로

$$\begin{aligned}\int_1^{e^2} \frac{\ln x}{x^2} dx &= \left[-\frac{1}{x} \ln x \right]_1^{e^2} + \int_1^{e^2} \frac{1}{x^2} dx \\ &= -\frac{2}{e^2} + \left[-\frac{1}{x} \right]_1^{e^2} \\ &= 1 - \frac{3}{e^2}\end{aligned}$$

65) e $\Rightarrow f(x) = 1+x, g'(x) = e^x$ 으로 놓으면 $f'(x) = 1, g(x) = e^x$ 이므로

$$\begin{aligned}\int_0^1 (1+x) e^x dx &= \left[(1+x) e^x \right]_0^1 - \int_0^1 e^x dx \\ &= 2e - 1 - \left[e^x \right]_0^1 = e\end{aligned}$$

66) $e^2 + 1$ $\Rightarrow f(x) = \ln x, g'(x) = 4x$ 로 놓으면 $f'(x) = \frac{1}{x}, g(x) = 2x^2$ 이므로

$$\begin{aligned}\int_1^e 4x \ln x dx &= \left[2x^2 \ln x \right]_1^e - \int_1^e 2x dx \\ &= 2e^2 - \left[x^2 \right]_1^e = e^2 + 1\end{aligned}$$

67) $2e^\pi + 1$ \Rightarrow

$$\begin{aligned} \int_0^{\frac{\pi}{2}} 5e^{2x} \sin x dx &= 5 \left[\frac{1}{2} e^{2x} \sin x \right]_0^{\frac{\pi}{2}} - 5 \int_0^{\frac{\pi}{2}} \frac{1}{2} e^{2x} \cos x dx \\ &= \frac{5}{2} e^\pi - \frac{5}{2} \left[\frac{1}{2} e^{2x} \cos x \right]_0^{\frac{\pi}{2}} - \frac{5}{4} \int_0^{\frac{\pi}{2}} e^{2x} \sin x dx \\ \frac{25}{4} \int_0^{\frac{\pi}{2}} e^{2x} \sin x dx &= \frac{5}{2} e^\pi + \frac{5}{4} \\ \therefore \int_0^{\frac{\pi}{2}} 5e^{2x} \sin x dx &= \frac{4}{5} \left(\frac{5}{2} e^\pi + \frac{5}{4} \right) = 2e^\pi + 1 \end{aligned}$$

68) $\frac{1}{2}e^\pi + \frac{1}{2}$ $\Rightarrow 2x = t$ 라 하면 $2dx = dt$ 이고, $0 \leq x \leq \frac{\pi}{2}$ 일 때, $0 \leq t \leq \pi$ 이므로

$$\begin{aligned} \int_0^{\frac{\pi}{2}} e^{2x} \sin 2x dx &= \frac{1}{2} \int_0^\pi e^t \sin t dt = \frac{1}{2} [e^t \sin t]_0^\pi - \frac{1}{2} \int_0^\pi e^t \cos t dt \\ &= -\frac{1}{2} [e^t \cos t]_0^\pi - \frac{1}{2} \int_0^\pi e^t \sin t dt \text{ 이므로} \\ \int_0^\pi e^t \sin t dt &= -\frac{1}{2} [e^t \cos t]_0^\pi = \frac{1}{2} (e^\pi + 1) \end{aligned}$$

69) $2e$

70) 5

 \Rightarrow 부분적분에 의해서

$$[x \ln x]_1^2 - \int_1^2 1 dx = \ln 4 - 1$$

$$\therefore p + q = 4 + 1 = 5$$

71) -1 \Rightarrow

$$\begin{aligned} \int_e^{e^2} \frac{\ln x}{x^2} dx &= \left[-\frac{\ln x}{x} \right]_e^{e^2} - \int_e^{e^2} -\frac{1}{x^2} dx = \left(-\frac{2}{e^2} + \frac{1}{e} \right) - \left[\frac{1}{x} \right]_e^{e^2} \\ &= \left(-\frac{2}{e^2} + \frac{1}{e} \right) - \left(\frac{1}{e^2} - \frac{1}{e} \right) = -\frac{3}{e^2} + \frac{2}{e} \\ \therefore a + b &= 2 + (-3) = -1 \end{aligned}$$

72) $2\ln 2 - \frac{3}{2}$

$$\Rightarrow \int_0^2 f(x) dx = \int_0^1 (x-1) dx + \int_1^2 \ln x dx$$

이때,

$$\int_0^1 (x-1) dx = \left[\frac{1}{2} x^2 - x \right]_0^1 = \frac{1}{2} - 1 = -\frac{1}{2} \dots \textcircled{1}$$

한편, $\int_1^2 \ln x dx$ 에서 $u(x) = \ln x$, $v'(x) = 1$ 로 놓으면 $u'(x) = \frac{1}{x}$, $v(x) = x$ 이므로

$$\begin{aligned} \int_1^2 \ln x dx &= [x \ln x]_1^2 - \int_1^2 1 dx \\ &= 2\ln 2 - [x]_1^2 = 2\ln 2 - 1 \dots \textcircled{2} \end{aligned}$$

따라서 $\textcircled{1}, \textcircled{2}$ 에서 구하는 값은 $2\ln 2 - \frac{3}{2}$

73) 2

 $\Rightarrow f(x) = x$, $g'(x) = e^x$ 으로 놓으면 $f'(x) = 1$, $g(x) = e^x$ 이므로

$$\begin{aligned} \int_0^a x e^x dx &= [x e^x]_0^a - \int_0^a e^x dx \\ &= a e^a - [e^x]_0^a = (a-1)e^a + 1 \end{aligned}$$

$$\int_0^a x e^x dx = e^2 + 1 \text{ 이므로}$$

$$(a-1)e^a + 1 = e^2 + 1, (a-1)e^a = e^2$$

$$\therefore a = 2$$

74) $3 + \frac{2}{e}$

$$\Rightarrow a = \int_1^e \frac{\ln x}{x^2} dx$$

 $\ln x = t$ 로 치환하자. $\frac{1}{x} dx = dt$

$$= \int_0^1 \frac{t}{e^t} dt = \int_0^1 t e^{-t} dt = [(-1)e^{-t}t]_0^1 + \int_0^1 e^{-t} dt$$

$$= (-1)e^{-1} + [-e^{-t}]_0^1 = (-1)e^{-1} - e^{-1} + 1$$

$$= 1 - \frac{2}{e}$$

$$b = \int_{-1}^1 \left(x + \frac{2}{x} - \frac{2}{x^2} \right) dx = \int_{-1}^1 \frac{-2}{x^2} dx$$

$$= \left[\frac{2}{x} \right]_{-1}^1 = 2 - (-2) = 4$$

$$b - a = 3 + \frac{2}{e}$$

75) $1 - \frac{2}{e}$

$$\Rightarrow \int_0^2 f(x) dx - \int_{-2}^2 f(x) dx + \int_{-2}^1 f(x) dx$$

$$= \int_0^2 f(x) dx + \int_2^{-2} f(x) dx + \int_{-2}^1 f(x) dx$$

$$= \int_0^1 f(x) dx = \int_0^1 x e^{-x} dx$$

 $u(x) = x$, $v'(x) = e^{-x}$ 으로 놓으면 $u'(x) = 1$, $v(x) = -e^{-x}$ 이므로

$$\int_0^1 x e^{-x} dx = [-x e^{-x}]_0^1 + \int_0^1 e^{-x} dx$$

$$= -\frac{1}{e} + [-e^{-x}]_0^1 = 1 - \frac{2}{e}$$

76) $9\ln 3 - 4$ \Rightarrow

$$\begin{aligned}
 \int_2^4 f(x)dx - \int_3^4 f(x)dx + \int_1^2 f(x)dx &= \int_1^4 f(x)dx - \int_3^4 f(x)dx \\
 &= \int_1^3 f(x)dx = \int_1^3 2x \ln x dx = [x^2 \ln x]_1^3 - \int_1^3 x dx = 9 \ln 3 - 4
 \end{aligned}$$