01 분수꼴로 주어진 수열의 합

주어진 분수를 부분분수로 변형한 후 전개하여 계산한다.

(1) $\sum_{k=1}^{n}\frac{1}{k(k+1)}=\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

(2) $\sum_{k=1}^{n}\frac{1}{k(k+a)}=\frac{1}{a}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+a}\right)$

(3) $\sum_{k=1}^{n}\frac{1}{(k+a)(k+b)}=\frac{1}{b-a}\sum_{k=1}^{n}\left(\frac{1}{k+a}-\frac{1}{k+b}\right)$

(4) $\sum_{k=1}^{n}\frac{1}{k(k+1)(k+2)}=\frac{1}{2}\sum_{k=1}^{n}\left\{\frac{1}{k(k+1)}-\frac{1}{(k+1)(k+2)}\right\}$

◪ **다음 분수를 부분분수로 변형하여라.**

1. $\frac{1}{n(n+1)}$

2. $\frac{1}{n(n+2)}$

3. $\frac{1}{(n+1)(n+3)}$

4. $\frac{1}{(2n-1)(2n+1)}$

5. $\frac{2}{(n+1)(n+3)}$

6. $\frac{1}{(n+1)(n+2)}$

◪ **다음을 계산하여라.**

7. $\sum_{k=1}^{10}\frac{2}{(2k-1)(2k+1)}$

8. $\sum_{k=1}^{10}\frac{2}{k(k+2)}$

9. $\sum_{k=1}^{9}\frac{1}{(k+1)(k+3)}$

10. $\sum_{k=1}^{7}\frac{1}{(k+2)(k+3)}$

11. $\sum_{k=2}^{20}\frac{1}{(k-1)k}$

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12. $\displaystyle\sum_{k=1}^{10}\frac{1}{k(k+1)}$

13. $\displaystyle\sum_{k=1}^{14}\left\{\frac{k(k+1)}{5}+\frac{15}{k(k+1)}\right\}$

◪ 다음을 구하여라.

14. $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots+\frac{1}{49\times51}$

15. $\frac{1}{2\times4}+\frac{1}{3\times5}+\frac{1}{4\times6}+\cdots+\frac{1}{8\times10}$

16. $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\cdots+\frac{1}{19\cdot21}$

17. $\frac{1}{1\cdot3}+\frac{1}{2\cdot4}+\frac{1}{3\cdot5}+\cdots+\frac{1}{8\cdot10}$

18. $\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\cdots+\frac{1}{11\cdot12}$

19. $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\cdots+\frac{1}{10\times12}$

20. $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots+\frac{1}{100\times101}$

21. $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{5\cdot6}+\cdots+\frac{1}{101\cdot102}$

◪ 다음 수열의 합을 구하여라.

22. $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots+\frac{1}{(2n-1)(2n+1)}$

23. $\frac{1}{2\times4}+\frac{1}{3\times5}+\frac{1}{4\times6}+\cdots+\frac{1}{(n+1)(n+3)}$

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24. $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\cdots+\frac{1}{n(n+2)}$

25. $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{4\times5}+\cdots+\frac{1}{n(n+1)}$

26. $\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\cdots+\frac{1}{(n+1)(n+2)}$

27. $\frac{1}{2^2-1},\ \frac{1}{4^2-1},\ \frac{1}{6^2-1},\ \frac{1}{8^2-1},\ \cdots,\ \frac{1}{(2n)^2-1}$

◪ 다음 물음에 답하여라.

28. 수열 $1,\ \frac{1}{1+2},\ \frac{1}{1+2+3},\ \frac{1}{1+2+3+4},\ \cdots$의 첫째항부터 제$10$항까지의 합을 구하여라.

29. 수열 $\frac{1}{3^2-1},\ \frac{1}{5^2-1},\ \frac{1}{7^2-1},\ \frac{1}{9^2-1},\ \cdots$의 첫째항부터 제$10$항까지의 합을 구하여라.

30. 수열 $\{a_n\}$의 첫째항부터 제 n항까지의 합을 S_n이라 하자. $S_n=n^2+2n$일 때, $\sum_{k=1}^{15}\frac{1}{a_ka_{k+1}}$의 값을 구하여라.

31. 수열 $\{a_n\}$이 $\frac{1}{2^2+2},\ \frac{1}{3^2+3},\ \frac{1}{4^2+4},\ \cdots$일 때, $\sum_{k=1}^{11}a_k$의 값을 구하여라.

02 분모에 근호가 있는 수열의 합

일반항이 분수식이고, a_k의 분모에 근호가 있는 두 식의 합으로 나타내어 질 경우, 분모를 유리화한 후, a_k의 k에 $1,\ 2,\ 3,\ \cdots,\ n$을 차례로 대입하여 합의 꼴로 나타내어 계산한다.

◪ 다음을 계산하여라.

32. $\sum_{k=1}^{80}\frac{1}{\sqrt{k+1}+\sqrt{k}}$

33. $\sum_{k=1}^{99}\frac{1}{\sqrt{k}+\sqrt{k+1}}$

34. $\sum_{k=1}^{10} \frac{2}{\sqrt{2k+1}+\sqrt{2k-1}}$

35. $\sum_{k=1}^{11} \frac{1}{\sqrt{2k+3}+\sqrt{2k+1}}$

36. $\sum_{k=1}^{10} \frac{1}{\sqrt{k+3}+\sqrt{k+2}}$

37. $\sum_{k=1}^{8} \frac{1}{\sqrt{k+1}+\sqrt{k}}$

38. $\sum_{k=1}^{10} \frac{3}{\sqrt{3k+2}+\sqrt{3k-1}}$

39. $\sum_{k=1}^{8} (\sqrt{k}-\sqrt{k+1})$

40. $\sum_{k=1}^{48} \frac{1}{\sqrt{k+2}+\sqrt{k}}$

41. $\sum_{k=1}^{10} \frac{1}{\sqrt{k+3}+\sqrt{k+1}}$

◪ **다음 식을 계산하여라.**

42. $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{15}+\sqrt{16}}$

43. $\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\cdots+\frac{1}{\sqrt{49}+\sqrt{47}}$

44. $\frac{2}{\sqrt{3}+1}+\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{2}{\sqrt{7}+\sqrt{5}}+\cdots+\frac{2}{\sqrt{81}+\sqrt{79}}$

45. $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}$

46. $\frac{1}{\sqrt{5}+\sqrt{2}}+\frac{1}{\sqrt{8}+\sqrt{5}}+\frac{1}{\sqrt{11}+\sqrt{8}}+\cdots+\frac{1}{\sqrt{50}+\sqrt{47}}$

47. $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{25}+\sqrt{24}}$

48. $\frac{3}{\sqrt{4}+1}+\frac{3}{\sqrt{7}+\sqrt{4}}+\frac{3}{\sqrt{10}+\sqrt{7}}+\cdots+\frac{3}{\sqrt{121}+\sqrt{118}}$

◪ 다음 물음에 답하여라.

49. $f(x)=\frac{1}{\sqrt{x}+\sqrt{x+1}}$일 때, $f(1)+f(2)+f(3)+\cdots+f(99)$를 구하여라.

50. $\sum_{k=1}^{n}\frac{1}{\sqrt{k+1}+\sqrt{k}}=4$일 때, 자연수 n의 값을 구하여라.

51. 수열의 합
$\frac{1}{\sqrt{2}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{6}}+\cdots+\frac{1}{\sqrt{62}+\sqrt{64}}=a+b\sqrt{2}$
일 때, $a+b$의 값을 구하여라. (단, a, b는 유리수이다.)

52. 첫째항이 1, 공차가 2인 등차수열 $\{a_n\}$에 대하여
$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\frac{1}{\sqrt{a_3}+\sqrt{a_4}}+\cdots+\frac{1}{\sqrt{a_{24}}+\sqrt{a_{25}}}$의 값을 구하여라.

03 (등차)$\times$(등비)꼴의 수열의 합

① 수열의 합 S에 등비수열의 공비 r을 곱한다.
② $S-rS$또는 $rS-S$를 계산하여 S를 구한다. (단, $r \neq 1$)

◪ 다음에서 S의 값을 구하여라.

53. $S=\sum_{k=1}^{10} k \cdot \left(\frac{1}{4}\right)^k$

54. $S=\sum_{k=1}^{10} (3k-1)2^k$

55. $S=\sum_{k=1}^{10} k \cdot 3^k$

56. $S=1 \cdot \frac{1}{2}+2 \cdot \left(\frac{1}{2}\right)^2+3 \cdot \left(\frac{1}{2}\right)^3+\cdots+9 \cdot \left(\frac{1}{2}\right)^9$

57. $S=1 \cdot \frac{1}{4}+4 \cdot \left(\frac{1}{4}\right)^2+\cdots+22 \cdot \left(\frac{1}{4}\right)^8$

58. $S=1 \cdot 2+3 \cdot 2^2+5 \cdot 2^3+\cdots+15 \cdot 2^8$

59. $S=1 \cdot 2+2 \cdot 2^2+3 \cdot 2^3+4 \cdot 2^4+5 \cdot 2^5$

60. $S=6 \cdot 4^0+5 \cdot 4^1+\cdots+1 \cdot 4^5$

61. $S=1+2 \cdot 2+3 \cdot 2^2+\cdots+10 \cdot 2^9$

62. $S=2 \cdot 3+4 \cdot 3^2+\cdots+14 \cdot 3^7$

63. $1 \cdot 2+2 \cdot 2^2+3 \cdot 2^3+ \cdots +9 \cdot 2^9$

04 규칙성을 가지는 수열의 합

① 주어진 수열을 규칙성을 갖도록 묶는다.
② 구하고자 하는 항이 몇 번째 묶음의 몇 번째 항인지 파악한다.
③ 각 묶음의 항의 개수와 첫째항이 갖는 규칙성을 조사한다.

◪ 다음 물음에 답하여라.

64. 수열 $1,\ 3,\ 3,\ 5,\ 5,\ 5,\ 7,\ 7,\ 7,\ 7,\ \cdots$에서 제 40항을 구하시오.

65. 수열 $\frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{4},\ \frac{2}{4},\ \frac{3}{4},\ \frac{1}{5},\ \frac{2}{5},\ \frac{3}{5},\ \frac{4}{5},\ \cdots$ $\frac{3}{8}$은 제 몇 항인지 구하여라.

66. 수열 $1,\ 2,\ 1,\ 3,\ 2,\ 1,\ 4,\ 3,\ 2,\ 1,\ \cdots$에 대하여 처음으로 나타나는 7은 제 몇 항인지 구하여라.

67. 수열 $1,\ 1,\ 2,\ 1,\ 2,\ 3,\ 1,\ 2,\ 3,\ 4,\ \cdots$에 대하여 처음으로 나타나는 6은 제 몇 항인지 구하여라.

68. 수열 $1,\ 1,\ 2,\ 1,\ 1,\ 2,\ 3,\ 2,\ 1,\ \cdots$에 대하여 처음으로 나타나는 5는 제 몇 항인지 구하여라.

69. 수열 $1,\ 2,\ 2,\ 3,\ 3,\ 3,\ 4,\ 4,\ 4,\ 4,\ \cdots$에 대하여 처음으로 나타나는 7은 제 몇 항인지 구하여라.

70. 수열 $\frac{1}{1},\ \frac{2}{1},\ \frac{1}{2},\ \frac{3}{1},\ \frac{2}{2},\ \frac{1}{3},\ \frac{4}{1},\ \frac{3}{2},\ \frac{2}{3},\ \frac{1}{4},\ \cdots$ 애 대하여 처음으로 나타나는 $\frac{3}{5}$은 제 몇 항인지 구하여라.

71. 수열 $\frac{2}{3},\ \frac{2}{9},\ \frac{4}{9},\ \frac{2}{27},\ \frac{4}{27},\ \frac{6}{27},\ \frac{8}{27},\ \frac{2}{81},\ \cdots$에서 제 100항을 구하여라.

72. 수열 $\frac{1}{1},\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{1}{3},\ \frac{3}{3},\ \frac{5}{3},\ \frac{7}{3},\ \frac{9}{3},\ \frac{1}{4},\ \frac{3}{4},\ \frac{5}{4},\ \frac{7}{4},\ \frac{9}{4},\ \frac{11}{4},\ \frac{13}{4},\ \cdots$에서 $\frac{19}{12}$는 제 몇 항인지 구하여라.

73. 수열 $\frac{1}{2},\ \frac{2}{3},\ \frac{1}{3},\ \frac{3}{4},\ \frac{2}{4},\ \frac{1}{4},\ \frac{4}{5},\ \frac{3}{5},\ \frac{2}{5},\ \frac{1}{5},\ \cdots$ 에 대하여 처음으로 나타나는 $\frac{1}{7}$은 제 몇 항인지 구하여라.

정답 및 해설

1) $\frac{1}{n}-\frac{1}{n+1}$

$\Rightarrow \frac{1}{n(n+1)}=\frac{1}{(n+1)-n}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$=\frac{1}{n}-\frac{1}{n+1}$

2) $\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

$\Rightarrow \frac{1}{n(n+2)}=\frac{1}{(n+2)-n}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

$=\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

3) $\frac{1}{2}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$

$\Rightarrow \frac{1}{(n+1)(n+3)}=\frac{1}{(n+3)-(n+1)}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$

$=\frac{1}{2}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$

4) $\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$

$\Rightarrow \frac{1}{(2n-1)(2n+1)}$

$=\frac{1}{(2n+1)-(2n-1)}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$

$=\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$

5) $\frac{1}{n+1}-\frac{1}{n+3}$

$\Rightarrow \frac{2}{(n+1)(n+3)}=\frac{2}{2}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$

$=\frac{1}{n+1}-\frac{1}{n+3}$

6) $\frac{1}{n+1}-\frac{1}{n+2}$

$\Rightarrow \frac{1}{(n+1)(n+2)}=\frac{1}{n+1}-\frac{1}{n+2}$

7) $\frac{20}{21}$

$\Rightarrow \sum_{k=1}^{10}\frac{2}{(2k-1)(2k+1)}$

$=\sum_{k=1}^{10}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$

$=\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\cdots+\left(\frac{1}{19}-\frac{1}{21}\right)$

$=1-\frac{1}{21}=\frac{20}{21}$

8) $\frac{175}{132}$

$\Rightarrow \sum_{k=1}^{10}\frac{2}{k(k+2)}=\sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+2}\right)$

$=\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots$

$+\left(\frac{1}{9}-\frac{1}{11}\right)+\left(\frac{1}{10}-\frac{1}{12}\right)$

$=1+\frac{1}{2}-\frac{1}{11}-\frac{1}{12}=\frac{175}{132}$

9) $\frac{29}{88}$

$\Rightarrow \sum_{k=1}^{9}\frac{1}{(k+1)(k+3)}=\sum_{k=1}^{9}\frac{1}{2}\left(\frac{1}{k+1}-\frac{1}{k+3}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\cdots\right.$

$\left.+\left(\frac{1}{9}-\frac{1}{11}\right)+\left(\frac{1}{10}-\frac{1}{12}\right)\right\}$

$=\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{11}-\frac{1}{12}\right)=\frac{29}{88}$

10) $\frac{7}{30}$

$\Rightarrow \sum_{k=1}^{7}\frac{1}{(k+2)(k+3)}$

$=\sum_{k=1}^{7}\left(\frac{1}{k+2}-\frac{1}{k+3}\right)$

$=\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\cdots+\left(\frac{1}{9}-\frac{1}{10}\right)$

$=\frac{1}{3}-\frac{1}{10}=\frac{7}{30}$

11) $\frac{19}{20}$

$\Rightarrow \sum_{k=2}^{20}\frac{1}{(k-1)k}=\sum_{k=2}^{20}\left(\frac{1}{k-1}-\frac{1}{k}\right)$

$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{19}-\frac{1}{20}\right)$

$=1-\frac{1}{20}=\frac{19}{20}$

12) $\frac{10}{11}$

$\Rightarrow \sum_{k=1}^{10}\frac{1}{k(k+1)}=\sum_{k=1}^{1}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{10}-\frac{1}{11}\right)$

$=1-\frac{1}{11}=\frac{10}{11}$

13) 238

$\Rightarrow \sum_{k=1}^{14}\frac{k(k+1)}{5}=\sum_{k=1}^{14}\frac{1}{5}(k^2+k)$

$=\frac{1}{5}\left(\frac{14\cdot15\cdot29}{6}+\frac{14\cdot15}{2}\right)=\frac{1}{5}(1015+105)=224$

$$\sum_{k=1}^{14}\frac{15}{k(k+1)}=\sum_{k=1}^{14}15\left(\frac{1}{k}-\frac{1}{k+1}\right)$$
$$=15\left\{\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{14}-\frac{1}{15}\right)\right\}$$
$$=15\left(1-\frac{1}{15}\right)=14$$
$$\therefore \sum_{k=1}^{14}\frac{k(k+1)}{5}=224+14=238$$

14) $\frac{25}{51}$

⇨ $a_n=\frac{1}{(2n-1)(2n+1)}=\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$이라고 하면 $2n-1=49$에서 $n=25$로 항수는 25이므로

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots+\frac{1}{49\times51}$$
$$=\sum_{k=1}^{25}a_k=\frac{1}{2}\sum_{k=1}^{25}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$$
$$=\frac{1}{2}\times\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{49}-\frac{1}{51}\right)\right\}$$
$$=\frac{1}{2}\times\left(1-\frac{1}{51}\right)=\frac{25}{51}$$

15) $\frac{14}{45}$

⇨ $a_n=\frac{1}{(n+1)(n+3)}=\frac{1}{2}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$이라고 하면

$n+1=8$에서 $n=7$로 항수는 7이므로

$$\frac{1}{2\times4}+\frac{1}{3\times5}+\frac{1}{4\times6}+\cdots+\frac{1}{8\times10}$$
$$=\sum_{k=1}^{7}a_k=\frac{1}{2}\sum_{k=1}^{7}\left(\frac{1}{k+1}-\frac{1}{k+3}\right)$$
$$=\frac{1}{2}\times\left\{\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\cdots+\left(\frac{1}{7}-\frac{1}{9}\right)+\left(\frac{1}{8}-\frac{1}{10}\right)\right\}$$
$$=\frac{1}{2}\times\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{9}-\frac{1}{10}\right)$$
$$=\frac{1}{2}\times\frac{56}{90}=\frac{14}{45}$$

16) $\frac{10}{21}$

⇨ $$\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\cdots+\frac{1}{19\cdot21}$$
$$=\sum_{k=1}^{10}\frac{1}{(2k-1)(2k+1)}$$
$$=\sum_{k=1}^{10}\frac{1}{2}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$$
$$=\frac{1}{2}\left\{\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\cdots+\left(\frac{1}{19}-\frac{1}{21}\right)\right\}$$
$$=\frac{1}{2}\left(1-\frac{1}{21}\right)=\frac{10}{21}$$

17) $\frac{29}{45}$

⇨ $$\frac{1}{1\cdot3}+\frac{1}{2\cdot4}+\frac{1}{3\cdot5}+\cdots+\frac{1}{8\cdot10}$$
$$=\sum_{k=1}^{8}\frac{1}{k(k+2)}=\sum_{k=1}^{8}\frac{1}{2}\left(\frac{1}{k}-\frac{1}{k+2}\right)$$
$$=\frac{1}{2}\left\{\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{7}-\frac{1}{9}\right)+\left(\frac{1}{8}-\frac{1}{10}\right)\right\}$$
$$=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{9}-\frac{1}{10}\right)=\frac{29}{45}$$

18) $\frac{5}{12}$

⇨ $$\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\cdots+\frac{1}{11\cdot12}$$
$$=\sum_{k=1}^{10}\frac{1}{(k+1)(k+2)}$$
$$=\sum_{k=1}^{10}\left(\frac{1}{k+1}-\frac{1}{k+2}\right)$$
$$=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{11}-\frac{1}{12}\right)$$
$$=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$$

19) $\frac{175}{264}$

⇨ $a_n=\frac{1}{n(n+2)}=\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right)$이라고 하면

항수는 10이므로

$$\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\cdots+\frac{1}{10\times12}$$
$$=\sum_{k=1}^{10}a_k=\frac{1}{2}\sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+2}\right)$$
$$=\frac{1}{2}\times\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\cdots+\left(\frac{1}{9}-\frac{1}{11}\right)+\left(\frac{1}{10}-\frac{1}{12}\right)\right\}$$
$$=\frac{1}{2}\times\left(1+\frac{1}{2}-\frac{1}{11}-\frac{1}{12}\right)$$
$$=\frac{1}{2}\times\frac{175}{132}=\frac{175}{264}$$

20) $\frac{100}{101}$

⇨ $a_n=\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$이라고 하면 항수는 100이므로

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots+\frac{1}{100\times101}$$
$$=\sum_{k=1}^{100}a_k=\sum_{k=1}^{100}\left(\frac{1}{k}-\frac{1}{k+1}\right)$$

$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{100}-\frac{1}{101}\right)$

$=1-\frac{1}{101}=\frac{100}{101}$

21) $\frac{101}{102}$

⇨ $\sum_{k=1}^{101}\frac{1}{k(k+1)}=\sum_{k=1}^{101}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{101}-\frac{1}{102}=1-\frac{1}{102}=\frac{101}{102}$

22) $\frac{n}{2n+1}$

⇨ $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots+\frac{1}{(2n-1)(2n+1)}$

$=\sum_{k=1}^{n}\frac{1}{(2k-1)(2k+1)}$

$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)\right\}$

$=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)=\frac{n}{2n+1}$

23) $\frac{1}{2}\left(\frac{5}{6}-\frac{1}{n+2}-\frac{1}{n+3}\right)$

⇨ $\frac{1}{2\times4}+\frac{1}{3\times5}+\frac{1}{4\times6}+\cdots+\frac{1}{(n+1)(n+3)}$

$=\sum_{k=1}^{n}\frac{1}{(k+1)(k+3)}$

$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k+1}-\frac{1}{k+3}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\right.$

$\left.\cdots+\left(\frac{1}{n}-\frac{1}{n+2}\right)+\left(\frac{1}{n+1}-\frac{1}{n+3}\right)\right\}$

$=\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}\right)$

$=\frac{1}{2}\left(\frac{5}{6}-\frac{1}{n+2}-\frac{1}{n+3}\right)$

24) $\frac{1}{2}\left(\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$

⇨ $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\cdots+\frac{1}{n(n+2)}$

$=\sum_{k=1}^{n}\frac{1}{k(k+2)}=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\right.$

$\left.\cdots+\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right\}$

$=\frac{1}{2}\left(\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$

25) $\frac{n}{n+1}$

⇨ $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots+\frac{1}{n(n+1)}$

$=\sum_{k=1}^{n}\frac{1}{k(k+1)}=\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$=1-\frac{1}{n+1}=\frac{n}{n+1}$

26) $\frac{n}{2(n+2)}$

⇨ $\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\cdots+\frac{1}{(n+1)(n+2)}$

$=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$=\frac{1}{2}-\frac{1}{n+2}=\frac{n}{2(n+2)}$

27) $\frac{n}{2n+1}$

⇨ $a_n=\frac{1}{(2n)^2-1}=\frac{1}{(2n-1)(2n+1)}$

$=\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$이라고 하면

$\frac{1}{2^2-1}+\frac{1}{4^2-1}+\frac{1}{6^2-1}+\frac{1}{8^2-1}+\cdots+\frac{1}{(2n)^2-1}$

$=\sum_{k=1}^{n}a_k=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\right.$

$\left.\cdots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)\right.$

$=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$

$=\frac{n}{2n+1}$

28) $\frac{20}{11}$

⇨ $1+2+3+\cdots+n=\frac{n(n+1)}{2}$이므로

$a_n=\frac{1}{1+2+3+\cdots+n}=\frac{2}{n(n+1)}$

$\sum_{k=1}^{10}a_k=\sum_{k=1}^{10}\frac{2}{k(k+1)}$

$=2\sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=2\left\{\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{10}-\frac{1}{11}\right)\right\}$

$=2\left(1-\frac{1}{11}\right)=\frac{20}{11}$

29) $\frac{5}{22}$

⇨ $a_n=\dfrac{1}{(2n+1)^2-1}=\dfrac{1}{(2n+1-1)(2n+1+1)}$

$=\dfrac{1}{4n(n+1)}$

$\therefore \displaystyle\sum_{k=11}^{10} a_k=\sum_{k=1}^{10}\frac{1}{4k(k+1)}$

$=\dfrac{1}{4}\displaystyle\sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=\dfrac{1}{4}\left\{\left(1-\dfrac{1}{2}\right)+\left(\dfrac{1}{2}-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{4}\right)+\cdots+\left(\dfrac{1}{10}-\dfrac{1}{11}\right)\right\}$

$=\dfrac{1}{4}\left(1-\dfrac{1}{11}\right)=\dfrac{5}{22}$

30) $\dfrac{5}{33}$

⇨ $S_n=n^2+2n$이면 $S_{n-1}=(n-1)^2+2(n-1)=n^2-1$

이므로 $a_n=\mathrm{S}_n-\mathrm{S}_{n-1}=2n+1$이다.

$\dfrac{1}{a_ka_{k+1}}=\dfrac{1}{2}\left(\dfrac{1}{2k+1}-\dfrac{1}{2k+3}\right)$에서

$\displaystyle\sum_{k=1}^{15}\frac{1}{a_ka_{k+1}}=\frac{1}{2}\sum_{k=1}^{15}\left(\frac{1}{2k+1}-\frac{1}{2k+3}\right)$

$=\dfrac{1}{2}\left(\dfrac{1}{3}-\dfrac{1}{33}\right)=\dfrac{5}{33}$

31) $\dfrac{11}{26}$

⇨ $a_n=\dfrac{1}{(n+1)^2+(n+1)}=\dfrac{1}{(n+1)(n+2)}$

$=\dfrac{1}{n+1}-\dfrac{1}{n+2}$

$\therefore \displaystyle\sum_{k=1}^{11}a_k=\sum_{k=1}^{11}\left(\frac{1}{k+1}-\frac{1}{k+2}\right)=\frac{1}{2}-\frac{1}{13}=\frac{11}{26}$

32) 8

⇨ $\displaystyle\sum_{k=1}^{80}\frac{1}{\sqrt{k+1}+\sqrt{k}}=\sum_{k=1}^{80}(\sqrt{k+1}-\sqrt{k})$

$=(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+$
$\cdots+(\sqrt{81}-\sqrt{80})$

$=-1+\sqrt{81}=8$

33) 9

⇨ $\dfrac{1}{\sqrt{k}+\sqrt{k+1}}=\dfrac{\sqrt{k}-\sqrt{k+1}}{k-(k+1)}=\sqrt{k+1}-\sqrt{k}$

$\displaystyle\sum_{n=1}^{99}\frac{1}{\sqrt{k}+\sqrt{k+1}}=\sum_{k=1}^{99}(\sqrt{k+1}-\sqrt{k})$

$=\sqrt{100}-\sqrt{1}=9$

34) $\sqrt{21}-1$

⇨ $\displaystyle\sum_{k=1}^{10}\frac{2}{\sqrt{2k+1}+\sqrt{2k-1}}=\sum_{k=1}^{10}(\sqrt{2k+1}-\sqrt{2k-1})$

$=(\sqrt{3}-1)+(\sqrt{5}-\sqrt{3})+(\sqrt{7}-\sqrt{5})+$
$\cdots+(\sqrt{21}-\sqrt{19})$

$=\sqrt{21}-1$

35) $\dfrac{1}{2}(5-\sqrt{3})$

⇨ $\displaystyle\sum_{k=1}^{11}\frac{1}{\sqrt{2k+3}+\sqrt{2k+1}}$

$=\displaystyle\sum_{k=1}^{11}\frac{\sqrt{2k+3}-\sqrt{2k+1}}{2}$

$=\dfrac{1}{2}\{(\sqrt{5}-\sqrt{3})+(\sqrt{7}-\sqrt{5})+(\sqrt{9}-\sqrt{7})+$
$\cdots+(\sqrt{25}-\sqrt{23})\}$

$=\dfrac{1}{2}(\sqrt{25}-\sqrt{3})=\dfrac{1}{2}(5-\sqrt{3})$

36) $\sqrt{13}-\sqrt{3}$

⇨ $\displaystyle\sum_{k=1}^{10}\frac{1}{\sqrt{k+3}+\sqrt{k+2}}=\sum_{k=1}^{10}(\sqrt{k+3}-\sqrt{k+2})$

$=(\sqrt{4}-\sqrt{3})+(\sqrt{5}-\sqrt{4})+\cdots+(\sqrt{13}-\sqrt{12})$

$=\sqrt{13}-\sqrt{3}$

37) 2

⇨ $\displaystyle\sum_{k=1}^{8}\frac{1}{\sqrt{k+1}+\sqrt{k}}$

$=\displaystyle\sum_{k=1}^{8}\frac{\sqrt{k+1}-\sqrt{k}}{(\sqrt{k+1}+\sqrt{k})(\sqrt{k+1}-\sqrt{k})}$

$=\displaystyle\sum_{k=1}^{8}(\sqrt{k+1}-\sqrt{k})$

$=(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+\cdots+(\sqrt{9}-\sqrt{8})$

$=\sqrt{9}-1=2$

38) $3\sqrt{2}$

⇨ $\displaystyle\sum_{k=1}^{10}\frac{3}{\sqrt{3k+2}+\sqrt{3k-1}}$

$=\displaystyle\sum_{k=1}^{10}(\sqrt{3k+2}-\sqrt{3k-1})$

$=(\sqrt{5}-\sqrt{2})+(\sqrt{8}-\sqrt{5})+((\sqrt{11}-\sqrt{8})+$
$\cdots+(\sqrt{32}-\sqrt{29})$

$=\sqrt{32}-\sqrt{2}=3\sqrt{2}$

39) -2

⇨ $\displaystyle\sum_{k=1}^{8}(\sqrt{k}-\sqrt{k+1})$

$=(1-\sqrt{2})+(\sqrt{2}-\sqrt{3})+(\sqrt{3}-\sqrt{4})+\cdots+(\sqrt{8}-\sqrt{9})$

$=1-\sqrt{9}=-2$

40) $3+2\sqrt{2}$

⇨ $\dfrac{1}{\sqrt{k+2}+\sqrt{k}}=\dfrac{\sqrt{k+2}-\sqrt{k}}{(\sqrt{k+2}+\sqrt{k})(\sqrt{k+2}-\sqrt{k})}$

$=\dfrac{1}{2}(\sqrt{k+2}-\sqrt{k})$

이므로 주어진 수열의 합은

$\therefore \displaystyle\sum_{k=1}^{48}\frac{1}{\sqrt{k+2}+\sqrt{k}}=\sum_{k=1}^{48}\left(\frac{1}{2}\right)(\sqrt{k+2}-\sqrt{k})$

$=\left(\frac{1}{2}\right)\{(\sqrt{3}-\sqrt{1})+(\sqrt{4}-\sqrt{2})+\cdots+(\sqrt{50}-\sqrt{48})\}$

$=\frac{1}{2}(\sqrt{50}+\sqrt{49}-\sqrt{2}-\sqrt{1})$

$=\frac{1}{2}(5\sqrt{2}+7-\sqrt{2}-1)$

$=3+2\sqrt{2}$

41) $\frac{1}{2}(\sqrt{13}+\sqrt{3}-\sqrt{2})$

⇨ $\sum_{k=1}^{10}\frac{1}{\sqrt{k+3}+\sqrt{k+1}}$

$=\sum_{k=1}^{10}\frac{\sqrt{k+3}-\sqrt{k+1}}{2}$

$=\frac{1}{2}\{(\sqrt{4}-\sqrt{2})+(\sqrt{5}-\sqrt{3})+(\sqrt{6}-\sqrt{4})+$
$\cdots+(\sqrt{12}-\sqrt{10})+(\sqrt{13}-\sqrt{11})$

$=\frac{1}{2}(\sqrt{13}+\sqrt{12}-\sqrt{2}-\sqrt{3})$

$=\frac{1}{2}(\sqrt{13}+\sqrt{3}-\sqrt{2})$

42) 3

⇨ 주어진 수열의 일반항을 a_n이라 하면

$a_n=\frac{1}{\sqrt{n}+\sqrt{n+1}}=\frac{\sqrt{n+1}-\sqrt{n}}{(\sqrt{n+1}+\sqrt{n})(\sqrt{n+1}-\sqrt{n})}$

$=\sqrt{n+1}-\sqrt{n}$

$\therefore \sum_{k=1}^{15}a_k=\sum_{k=1}^{15}(\sqrt{k+1}-\sqrt{k})$

$=(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+$
$\cdots+(\sqrt{16}-\sqrt{15})$

$=\sqrt{16}-1=3$

43) 3

⇨ $\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{7}+\sqrt{5}}+$
$\cdots+\frac{1}{\sqrt{49}+\sqrt{47}}$

$=\frac{1}{2}\times\{(\sqrt{3}-1)+(\sqrt{5}-\sqrt{3})+(\sqrt{7}-\sqrt{5})+$
$\cdots+(\sqrt{49}-\sqrt{47})\}$

$=\frac{1}{2}\times(\sqrt{49}-1)=3$

44) 8

⇨ $\frac{2}{\sqrt{3}+1}+\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{2}{\sqrt{7}+\sqrt{5}}+$
$\cdots+\frac{2}{\sqrt{81}+\sqrt{79}}$

$=\sum_{k=1}^{40}\frac{2}{\sqrt{2k+1}+\sqrt{2k-1}}$

$=\sum_{k=1}^{40}(\sqrt{2k+1}-\sqrt{2k-1})$

$=(\sqrt{3}-1)+(\sqrt{5}-\sqrt{3})+(\sqrt{7}-\sqrt{5})+$
$\cdots+(\sqrt{81}-\sqrt{79})$

$=\sqrt{81}-1=8$

45) 9

⇨ $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+$
$\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}$

$=(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots+(\sqrt{100}-\sqrt{99})$

$=\sqrt{100}-1=9$

46) $\frac{4}{3}\sqrt{2}$

⇨ $\frac{1}{\sqrt{5}+\sqrt{2}}+\frac{1}{\sqrt{8}+\sqrt{5}}+\frac{1}{\sqrt{11}+\sqrt{8}}+$
$\cdots+\frac{1}{\sqrt{50}+\sqrt{47}}$

$=\sum_{k=1}^{16}\frac{1}{\sqrt{3k+2}+\sqrt{3k-1}}$

$=\sum_{k=1}^{16}\frac{\sqrt{3k+2}-\sqrt{3k-1}}{3}$

$=\frac{1}{3}\{(\sqrt{5}-\sqrt{2})+(\sqrt{8}-\sqrt{5})+(\sqrt{11}-\sqrt{8})+$
$\cdots+(\sqrt{50}-\sqrt{47})\}$

$=\frac{1}{3}(\sqrt{50}-\sqrt{2})$

$=\frac{1}{3}(5\sqrt{2}-\sqrt{2})=\frac{4}{3}\sqrt{2}$

47) 4

⇨ $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+$
$\cdots+\frac{1}{\sqrt{25}+\sqrt{24}}$

$=\sum_{k=1}^{24}\frac{1}{\sqrt{k+1}+\sqrt{k}}$

$=\sum_{k=1}^{24}(\sqrt{k+1}-\sqrt{k})$

$=(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+$
$\cdots+(\sqrt{25}-\sqrt{24})$

$=\sqrt{25}-1=4$

48) 10

⇨ $\frac{3}{\sqrt{4}+1}+\frac{3}{\sqrt{7}+\sqrt{4}}+\frac{3}{\sqrt{10}+\sqrt{7}}+$
$\cdots+\frac{3}{\sqrt{121}+\sqrt{118}}$

$=\sum_{k=1}^{40}\frac{3}{\sqrt{3k+1}+\sqrt{3k-2}}$

$=\sum_{k=1}^{40}(\sqrt{3k+1}-\sqrt{3k-2})$

$=(\sqrt{4}-1)+(\sqrt{7}-\sqrt{4})+(\sqrt{10}-\sqrt{7})+$
$\cdots+(\sqrt{121}-\sqrt{118})$

$= \sqrt{121} - 1 = 10$

49) 9

⇨ $f(x) = \dfrac{1}{\sqrt{x} + \sqrt{x+1}}$

$= \dfrac{(\sqrt{x} - \sqrt{x+1})}{(\sqrt{x} + \sqrt{x+1})(\sqrt{x} - \sqrt{x+1})}$

$= -(\sqrt{x} - \sqrt{x+1})$

$\therefore f(1) + f(2) + f(3) + \cdots + f(99)$

$= -\{(\sqrt{1} - \sqrt{2}) + (\sqrt{2} - \sqrt{3}) + \cdots + (\sqrt{99} - \sqrt{100})\}$

$= -(\sqrt{1} - \sqrt{100}) = 9$

50) 24

⇨ $\displaystyle\sum_{k=1}^{n}\left(\frac{1}{\sqrt{k+1} + \sqrt{k}}\right)$

$= \displaystyle\sum_{k=1}^{n}\frac{\sqrt{k+1} - \sqrt{k}}{(\sqrt{k+1} + \sqrt{k})(\sqrt{k+1} - \sqrt{k})}$

$= \displaystyle\sum_{k=1}^{n}(\sqrt{k+1} - \sqrt{k})$

$= (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + \cdots + (\sqrt{n+1} - \sqrt{n})$

$= \sqrt{n+1} - 1 = 4$

$\therefore n = 24$

51) $\dfrac{7}{2}$

⇨ $\dfrac{1}{\sqrt{2} + \sqrt{4}} + \dfrac{1}{\sqrt{4} + \sqrt{6}} + \cdots + \dfrac{1}{\sqrt{62} + \sqrt{64}}$

$= \dfrac{\sqrt{2} - \sqrt{4}}{(\sqrt{2} + \sqrt{4})(\sqrt{2} - \sqrt{4})} + \cdots + \dfrac{(\sqrt{62} - \sqrt{64})}{(\sqrt{62} + \sqrt{64})(\sqrt{62} - \sqrt{64})}$

$= \dfrac{\sqrt{2} - \sqrt{4}}{-2} + \dfrac{\sqrt{2} - \sqrt{4}}{-2} + \cdots + \dfrac{\sqrt{62} - \sqrt{64}}{-2}$

$= \dfrac{\sqrt{2} - \sqrt{64}}{-2} = 4 - \dfrac{1}{2}\sqrt{2}$

$\therefore a + b = 4 + \left(-\dfrac{1}{2}\right) = \dfrac{7}{2}$

52) 3

⇨ 주어진 조건에 의하여 $a_n = 2n - 1$이다.

$\displaystyle\sum_{k=1}^{24}\frac{1}{\sqrt{a_k} + \sqrt{a_{k+1}}} = \sum_{k=1}^{24}\frac{1}{2}(\sqrt{a_{k+1}} - \sqrt{a_k})$

$= \dfrac{1}{2}(-\sqrt{a_1} + \sqrt{a_{25}}) = \dfrac{1}{2}(-1 + 7) = 3$이다.

53) $\dfrac{4}{9} - \dfrac{34}{9} \cdot \left(\dfrac{1}{4}\right)^{10}$

⇨

$$\begin{array}{rl} S = & 1 \cdot \frac{1}{4} + 2 \cdot \left(\frac{1}{4}\right)^2 + 3 \cdot \left(\frac{1}{4}\right)^3 + \cdots + 10 \cdot \left(\frac{1}{4}\right)^{10} \\ -\big)\ \frac{1}{4}S = & \quad 1 \cdot \left(\frac{1}{4}\right)^2 + 2 \cdot \left(\frac{1}{4}\right)^3 + \cdots + 9 \cdot \left(\frac{1}{4}\right)^{10} + 10 \cdot \left(\frac{1}{4}\right)^{11} \\ \hline \frac{3}{4}S = & \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots + \left(\frac{1}{4}\right)^{10} - 10 \cdot \left(\frac{1}{4}\right)^{11} \end{array}$$

$= \dfrac{\frac{1}{4}\left\{1 - \left(\frac{1}{4}\right)^{10}\right\}}{1 - \frac{1}{4}} - 10 \cdot \left(\dfrac{1}{4}\right)^{11}$

$= \dfrac{1}{3}\left\{1 - \left(\dfrac{1}{4}\right)^{10}\right\} - 10 \cdot \left(\dfrac{1}{4}\right)^{11}$

$\therefore S = \dfrac{4}{9}\left\{1 - \left(\dfrac{1}{4}\right)^{10}\right\} - \dfrac{40}{3} \cdot \left(\dfrac{1}{4}\right)^{11} = \dfrac{4}{9} - \dfrac{34}{9} \cdot \left(\dfrac{1}{4}\right)^{10}$

54) $26 \cdot 2^{11} + 8$

⇨

$$\begin{array}{rl} S = & 2 \cdot 2 + 5 \cdot 2^2 + 8 \cdot 2^3 + \cdots + 29 \cdot 2^{10} \\ -\big)\ 2S = & \qquad 2 \cdot 2^2 + 5 \cdot 2^3 + \cdots + 26 \cdot 2^{10} + 29 \cdot 2^{11} \\ \hline -S = & 2 \cdot 2 + 3 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 3 \cdot 2^{10} - 29 \cdot 2^{11} \end{array}$$

이때,

$3 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 3 \cdot 2^{10} = 3 \times \dfrac{4(2^9 - 1)}{2 - 1} = 12(2^9 - 1)$

이므로

$-S = 4 + 12(2^9 - 1) - 29 \cdot 2^{11} = -8 + 3 \cdot 2^{11} - 29 \cdot 2^{11}$

$\therefore S = 26 \cdot 2^{11} + 8$

55) $\dfrac{19}{4} \cdot 3^{11} + \dfrac{3}{4}$

⇨ $S = 1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \cdots + 10 \cdot 3^{10}$이고,

$$\begin{array}{rl} S = & 1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \cdots + 10 \cdot 3^{10} \\ -\big)\ 3S = & \qquad 1 \cdot 3^2 + 2 \cdot 3^3 + \cdots + 9 \cdot 3^{10} + 10 \cdot 3^{11} \\ \hline -2S = & 3 + 3^2 + 3^3 + \cdots + 3^{10} - 10 \cdot 3^{11} \end{array}$$

이때, $3 + 3^2 + 3^3 + \cdots + 3^{10} = \dfrac{3(3^{10} - 1)}{3 - 1} = \dfrac{3^{11} - 3}{2}$이므로 $-2S = -\dfrac{19}{2} \cdot 3^{11} - \dfrac{3}{2}$

$\therefore S = \dfrac{19}{4} \cdot 3^{11} + \dfrac{3}{4}$

56) $2 - \dfrac{11}{2^9}$

⇨

$$\begin{array}{rl} S = & 1 \cdot \frac{1}{2} + 2 \cdot \left(\frac{1}{2}\right)^2 + 3 \cdot \left(\frac{1}{2}\right)^3 + \cdots + 9 \cdot \left(\frac{1}{2}\right)^9 \\ -\big)\ \frac{1}{2}S = & \quad 1 \cdot \left(\frac{1}{2}\right)^2 + 2 \cdot \left(\frac{1}{2}\right)^3 + \cdots + 8 \cdot \left(\frac{1}{2}\right)^9 + 9 \cdot \left(\frac{1}{2}\right)^{10} \\ \hline \frac{1}{2}S = & \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + \left(\frac{1}{2}\right)^9 - 9 \cdot \left(\frac{1}{2}\right)^{10} \end{array}$$

$= \dfrac{\frac{1}{2}\left\{1 - \left(\frac{1}{2}\right)^9\right\}}{1 - \frac{1}{2}} - 9 \cdot \left(\dfrac{1}{2}\right)^{10}$

$= 1 - \dfrac{11}{2} \cdot \left(\dfrac{1}{2}\right)^9$

$\therefore S = 2 - \dfrac{11}{2^9}$

57) $\dfrac{2}{3} - \dfrac{26}{3} \cdot \left(\dfrac{1}{4}\right)^8$

⇨

$$S=1\cdot\frac{1}{4}^{1}+4\cdot\left(\frac{1}{4}\right)^2+\cdots+22\cdot\left(\frac{1}{4}\right)^8$$

$$-)\ \frac{1}{4}S=\qquad 1\cdot\left(\frac{1}{4}\right)^2+\cdots+19\cdot\left(\frac{1}{4}\right)^8+22\cdot\left(\frac{1}{4}\right)^9$$

$$\frac{3}{4}S=\quad\frac{1}{4}+3\cdot\left(\frac{1}{4}\right)^2+\cdots+\ 3\cdot\left(\frac{1}{4}\right)^8-22\cdot\left(\frac{1}{4}\right)^9$$

이때,

$$3\cdot\left(\frac{1}{4}\right)^2+\cdots+3\cdot\left(\frac{1}{4}\right)^8=3\times\frac{\left(\frac{1}{4}\right)^2\left\{1-\left(\frac{1}{4}\right)^7\right\}}{1-\frac{1}{4}}$$

$$=\frac{1}{4}-\left(\frac{1}{4}\right)^8$$

이므로

$$\frac{3}{4}S=\frac{1}{4}+\left\{\frac{1}{4}-\left(\frac{1}{4}\right)^8\right\}-22\cdot\left(\frac{1}{4}\right)^9=\frac{1}{2}-\frac{13}{2}\cdot\left(\frac{1}{4}\right)^8$$

$$\therefore\ S=\frac{2}{3}-\frac{26}{3}\cdot\left(\frac{1}{4}\right)^8$$

58) $13\cdot2^9+6$

⇨ $$S=1\cdot2+3\cdot2^2+5\cdot2^3+\cdots+15\cdot2^8$$

$$-)\ 2S=\qquad 1\cdot2^2+3\cdot2^3+\cdots+13\cdot2^8+15\cdot2^9$$

$$-S=\quad 2+2\cdot2^2+2\cdot2^3+\cdots+2\cdot2^8-15\cdot2^9$$

$$=2+2^3+2^4+\cdots+2^9-15\cdot2^9$$

이때, $2^3+2^4+\cdots+2^9=\dfrac{8(2^7-1)}{2-1}=2^{10}-8$이므로

$-S=2+(2^{10}-8)-15\cdot2^9$

$\therefore\ S=15\cdot2^9-2^{10}+6=13\cdot2^9+6$

59) 258

⇨

$$S=1\cdot2+2\cdot2^2+3\cdot2^3+4\cdot2^4+5\cdot2^5$$

$$-)\ 2S=\qquad 1\cdot2^2+2\cdot2^3+3\cdot2^4+4\cdot2^5+5\cdot2^6$$

$$-S=\quad 2+\quad 2^2+\quad 2^3+\quad 2^4+\quad 2^5-5\cdot2^6$$

이때, $2+2^2+2^3+2^4+2^5=\dfrac{2(2^5-1)}{2-1}=2^6-2$이므로

$-S=(2^6-2)-5\cdot2^6$

$\therefore\ S=4\cdot2^6+2=258$

60) $\dfrac{1}{9}\cdot4^7-\dfrac{22}{9}$

⇨ $$S=6\cdot4^0+5\cdot4^1+\cdots+1\cdot4^5$$

$$-)\ 4S=\qquad 6\cdot4^1+\cdots+2\cdot4^5+1\cdot4^6$$

$$-3S=\quad 6-\quad 4-\cdots-\quad 4^4-\quad 4^6$$

이때,

$$-4-\cdots-4^5-4^6=--1\times\frac{4(4^6-1)}{4-1}$$

$$=-\frac{1}{3}\cdot4^7+\frac{4}{3}$$

이므로

$$-3S=6+\left(-\frac{1}{3}\cdot4^7+\frac{4}{3}\right)=-\frac{1}{3}\cdot4^7+\frac{22}{3}$$

$$\therefore\ S=\frac{1}{9}\cdot4^7-\frac{22}{9}$$

61) $9\cdot2^{10}+1$

⇨ $$S=1+2\cdot2+3\cdot2^2+\cdots+9\cdot2^8+10\cdot2^9$$

$$-)\ 2S=\qquad 2+2\cdot2^2+\cdots+8\cdot2^8+9\cdot2^9+10\cdot2^{10}$$

$$-S=1+\quad 2+\quad 2^2+\cdots+\quad 2^8+\quad 2^9-10\cdot2^{10}$$

$$=\frac{2^{10}-1}{2-1}-10\cdot2^{10}$$

$$=2^{10}-1-10\cdot2^{10}$$

$$=-9\cdot2^{10}-1$$

$$\therefore\ S=9\cdot2^{10}+1$$

62) $\dfrac{13}{2}\cdot3^8+\dfrac{3}{2}$

⇨ $$S=2\cdot3+4\cdot3^2+\cdots+14\cdot3^7$$

$$-)\ 3S=\qquad 2\cdot3^2+\cdots+12\cdot3^7+14\cdot3^8$$

$$-2S=\ 2\cdot3+2\cdot3^2+\cdots+\ 2\cdot3^7-14\cdot3^8$$

이때,

$$2\cdot3+2\cdot3^2+\cdots+2\cdot3^7=2\times\frac{3(3^7-1)}{3-1}=3^8-3$$

$$-2S=3^8-3-14\cdot3^8$$

$$\therefore\ S=\frac{13}{2}\cdot3^8+\frac{3}{2}$$

63) 8194

⇨ $S=1\cdot2+2\cdot2^2+3\cdot2^3+\cdots+9\cdot2^9$ $\cdots$ ㉠

㉠의 양변에 2를 곱하면

$2S=1\cdot2^2+2\cdot2^3+3\cdot2^4+\cdots+9\cdot2^{10}$ $\cdots$ ㉡

㉠−㉡을 하면

$$-S=2+2^2+2^3+\cdots+2^9-9\cdot2^{10}$$

$$=\frac{2(2^9-1)}{2-1}-9\cdot2^{10}$$

$$=2^{10}-2-9\cdot2^{10}$$

$$=-2-8\cdot2^{10}$$

$$\therefore S=2+8\cdot2^{10}=2+2^{13}=8194$$

64) 17

⇨ 주어진 수열을

$(1),\ (3,\ 3),\ (5,\ 5,\ 5),\ (7,\ 7,\ 7,\ 7),\ \cdots$

과 같이 묶으면 n번째 묶음에 있는 각 항의 개수는 $2n-1$이고, n번째 묶음의 항의 개수는 n이므로 첫 번째 묶음 부터 n번째 묶음까지의 항의 개수는 $\displaystyle\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$

8번째 묶음까지의 전체 항의 개수는 $\dfrac{8\cdot9}{2}=36$이므로 제 40항은 9번째 묶음의 4번째 항이다.

따라서 제40항은 $2\cdot9-1=17$이다.

65) 제24항

⇨ 6번째 묶음 까지의 항의 개수는

$1+2+3+4+5+6=21$이고

$\dfrac{3}{8}$은 7번째 묶음의 3번째 항이므로

$\frac{3}{8}$이 처음으로 나타나는 항은 제$(21+3)$항, 즉 제 24항이다.

66) 제22항

⇨ 주어진 수열을
$(1),\ (2,\ 1),\ (3,\ 2,\ 1),\ (4,\ 3,\ 2,\ 1),\ \cdots$
과 같이 묶으면 7은 7번째 묶음의 번째 항에서 처음 나타난다.
각 묶음의 항의 개수는 1, 2, 3, 4, …이므로 6번째 묶음까지의 항의 개수는
$1+2+3+4+5+6=21$
따라서 처음으로 나타나는 7은 제$(21+1)$항, 즉 제22항이다.

67) 제21항

⇨ 주어진 수열을
$(1),\ (1,\ 2),\ (1,\ 2,\ 3),\ (1,\ 2,\ 3,\ 4),\ \cdots$
과 같이 묶으면 6은 6번째 묶음의 6번째 항에서 처음으로 나타난다.
이때, 각 묶음의 항의 개수는 1, 2, 3, 4, …이므로 5번째 묶음까지의 항의 개수는
$1+2+3+4+5=15$
따라서 처음으로 나타나는 6은 제$(15+6)$항, 즉 제21항이다.

68) 제21항

⇨ 주어진 수열을
$(1),\ (1,\ 2,\ 1),\ (1,\ 2,\ 3,\ 2,\ 1),\ \cdots$
과 같이 묶으면 5는 5번째 묶음의 5번째 항에서 처음으로 나타난다.
이때, 각 묶음의 항의 개수는 1, 3, 5, 7, …이므로 4번째 묶음까지의 항의 개수는
$1+3+5+7=16$
따라서 처음으로 나타나는 5는 제$(16+5)$항,
즉 제21항이다.

69) 제22항

⇨ 주어진 수열을
$(1),\ (2,\ 2),\ (3,\ 3,\ 3),\ (4,\ 4,\ 4,\ 4),\ \cdots$
과 같이 묶으면 7은 7번째 묶음의 첫 번째 항에서 처음으로 나타난다.
이때, 각 묶음의 항의 개수는 1, 2, 3, 4, …이므로 6번째 묶음까지의 항의 개수는
$1+2+3+4+5+6=21$
따라서 처음으로 나타나는 7은 제$(21+1)$항, 즉 제22항이다.

70) 제26항

⇨ 주어진 수열을
$\left(\frac{1}{1}\right),\ \left(\frac{2}{1},\ \frac{1}{2}\right),\ \left(\frac{3}{1},\ \frac{2}{2},\ \frac{1}{3}\right),\ \left(\frac{4}{1},\ \frac{3}{2},\ \frac{2}{3},\ \frac{1}{4}\right),\ \cdots$
과 같이 묶어 살펴보면 첫 번째 묶음의 첫 번째 항은 분자, 분모의 합은 2, 두 번째 묶음의 첫 번째 항의 분자, 분모의 합은 3, 세 번째 묶음의 첫 번째 항의 분자, 분모의 합은 4, …이다.
따라서 $\frac{3}{5}$은 분자, 분모의 합이 8인 7번째 묶음에서 5번째 항이다.
각 묶음의 항의 개수는 1, 2, 3, 4, …이므로 6번째 묶음 까지의 항의 개수는
$1+2+3+4+5+6=21$
따라서 $\frac{3}{5}$은 제$(21+5)$항, 즉 제26항이다.

71) $\frac{74}{2187}$

⇨ 주어진 수열을
$\left(\frac{2}{3}\right),\ \left(\frac{2}{9},\frac{4}{9}\right),\ \left(\frac{2}{27},\frac{4}{27},\frac{6}{27},\frac{8}{27}\right),\ \cdots$
과 같이 묶으면 n번째 묶음에 포함되는 항의 개수는 2^{n-1}개다.
따라서 n번째 묶음의 마지막 수는 2^n-1번째 항이다.
이때, 7번째 묶음의 첫 번째 수는 제64항이고
7번째 묶음의 수를 나열하면
$\left(\frac{2}{3^7},\ \frac{4}{3^7},\ \cdots,\ \frac{2k}{3^7},\ \cdots\right)$이므로
제100항은 7번째 묶음의 37번째 수이므로 $\frac{74}{2187}$이다.

72) 131

⇨ 주어진 수열을 규칙성 있게 묶음으로 나타내면
$\frac{1}{1},\ \left(\frac{1}{2},\frac{3}{2},\frac{5}{2}\right),\ \left(\frac{1}{3},\frac{3}{3},\frac{5}{3},\frac{7}{3},\frac{9}{3}\right),\ \cdots$이므로
$\frac{19}{12}$는 $\left(\frac{1}{12},\ \frac{3}{12},\ \frac{5}{12},\ \cdots\ ,\frac{19}{12},\ \cdots\right)$인 묶음에 속하므로 12번째 묶음의 10번째 수이다.
이 때 n번째 묶음에 포함되는 수의 개수는 $2n-1$개 이므로 11번째 묶음의 마지막 수는
$\sum_{k=1}^{11}(2k-1)=121$에서 제 121항이다.
따라서 $\frac{19}{12}$는 제131항이다.

73) 제21항이다.

⇨ 주어진 수열을 규칙에 맞게 묶어보면
$\left(\frac{1}{2}\right),\ \left(\frac{2}{3},\ \frac{1}{3}\right),\ \left(\frac{3}{4},\ \frac{2}{4},\ \frac{1}{4}\right),\ \left(\frac{4}{5},\ \frac{3}{5},\ \frac{2}{5},\ \frac{1}{5}\right),\ \cdots$
이므로 $\frac{1}{7}$은 6번째 묶음의 6번째 항에서 처음으로 나타난다.
이때, 각 묶음의 항의 개수는 1, 2, 3, 4, …이므로 5번째 묶음까지의 항의 개수는
$1+2+3+4+5=15$
따라서 처음으로 나타나는 $\frac{1}{7}$은 제$(15+6)$항,

즉 제21항이다.