◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시
1) 제작연월일 : 2019-08-13
2) 제작자 : 교육지대㈜
3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

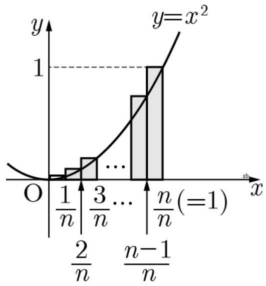
## 01 구분구적법

(1) 주어진 도형을 $n$개의 기본 도형으로 세분한다.

(2) $n$개의 기본 도형의 넓이의 합 $S_n$ 또는 부피의 합 $V_n$을 구한다.

(3) $\lim_{n\to\infty} S_n$ 또는 $\lim_{n\to\infty} V_n$을 구한다.

**1.** **다음은 곡선 $y=x^2$과 $x$축 및 직선 $x=1$로 둘러싸인 도형의 넓이 $S$를 구분구적법을 이용하여 구하는 과정이다.**




닫힌구간 $[0,1]$을 $n$등분 한 각 구간의 오른쪽 끝 점의 $x$좌표는 차례로

$\frac{1}{n}$, $\frac{2}{n}$, $\cdots$, $\frac{n-1}{n}$, $\frac{n}{n}(=1)$이고, 이에 대응하는 $y$의 값은 각각

$\left(\frac{1}{n}\right)^2$, $\left(\frac{2}{n}\right)^2$, $\cdots$, $\left(\frac{n-1}{n}\right)^2$, $\left(\frac{n}{n}\right)^2$

이므로 색칠한 직사각형의 넓이의 합을 $S_n$이라 하면

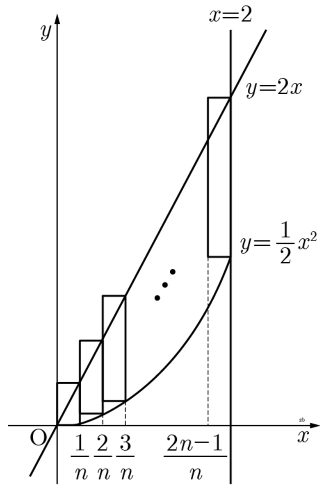
$$S_n = \boxed{(가)}\times\left(\frac{1}{n}\right)^2+\boxed{(나)}\times\left(\frac{2}{n}\right)^2+\cdots+\boxed{(다)}\times\left(\frac{n}{n}\right)^2$$

$$=\sum_{k=1}^{n}\boxed{(라)}$$

$$\therefore S=\lim_{n\to\infty}S_n=\boxed{(마)}$$

**(가), (나), (다), (라), (마)에 알맞은 식**

**2.** **다음은 곡선 $y=\frac{1}{2}x^2\ (0\le x\le 2)$과 두 직선 $y=2x$, $x=2$로 둘러싸인 도형의 넓이를 구하는 과정이다.**




두 함수 $f(x)$와 $g(x)$를 $f(x)=\frac{1}{2}x^2$, $g(x)=2x$라 하자.

그림과 같이 닫힌 구간 $[0,\ 2]$를 $2n$등분하여 구간 $\left[0,\ \frac{1}{n}\right]$, $\left[\frac{1}{n},\ \frac{2}{n}\right]$, $\left[\frac{2}{n},\ \frac{3}{n}\right]$, $\cdots\left[\frac{2n-1}{n},\ 2\right]$를 얻는다.

각 구간에서 가로의 길이가 $\frac{1}{n}$이고 구간의 오른쪽 끝점에서의 두 함숫값의 차를 세로의 길이로 하는 직사각형을 만든다. 왼쪽에서 $k$번째 직사각형의 넓이를 $S_k$라 하면

$S_k=\boxed{(가)}$

직사각형 $n$개의 넓이의 합은

$$\sum_{k=1}^{2n}S_k=\sum_{k=1}^{2n}\boxed{(가)}$$

구하는 도형의 넓이는

$$\lim_{n\to\infty}\sum_{k=1}^{2n}S_k=\boxed{(나)}$$

$$=\int_0^2\boxed{(다)}\,dx=\int_0^1\boxed{(라)}\,dx$$

**(가), (나), (다), (라)에 알맞은 식을 써라.**

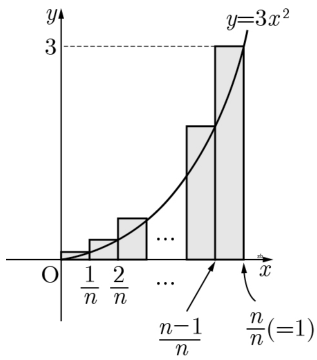

Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

**3.** 다음은 정적분의 정의를 이용하여 $\int_0^2 x^2 dx$의 값을 구하는 과정이다.

> $f(x)=x^2$이라고 하면 함수 $f(x)$는 닫힌구간 $[0,2]$에서 연속이다. 정적분의 정의에서 $a=0$, $b=2$라고 하면
>
> $\triangle x=\dfrac{b-a}{n}=\dfrac{2}{n}$, $x_k=a+k\triangle x=\dfrac{2k}{n}$
>
> $f(x_k)=\dfrac{(가)}{n^2}$이므로
>
> $\int_0^2 x^2dx=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}f(x_k)\triangle x$
>
> $=\lim\limits_{n\to\infty}\dfrac{(나)}{n^3}\sum\limits_{k=1}^{n}k^2$
>
> $=\lim\limits_{n\to\infty}\left\{(다)\cdot\dfrac{n(n+1)(2n+1)}{6}\right\}$
>
> $=\dfrac{4}{3}\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)\left(2+\dfrac{1}{n}\right)=(라)$

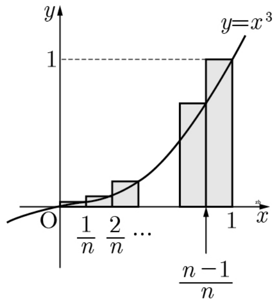
**(가), (나), (다), (라)에 알맞은 식을 써라.**

**4.** 다음은 곡선 $y=3x^2$과 $x$축 및 직선 $x=1$로 둘러싸인 도형의 넓이를 구분구적법을 이용하여 구하는 과정이다. $\boxed{\phantom{xx}}$안에 알맞은 식을 구하여라.

> <보기>
>
> 
> 
>
> 구간 $[0,1]$을 $n$등분하면 양 끝점과 각 분점의 $x$좌표는 앞에서부터 차례대로
>
> $0,\ \dfrac{1}{n},\ \dfrac{2}{n},\ \cdots,\ \dfrac{n-1}{n},\ 1$
>
> 이므로 위의 그림의 직사각형의 넓이의 합을 $S_n$이라고 하면 구하는 넓이 $S$는
>
> $S=\lim\limits_{n\to\infty}S_n=\lim\limits_{n\to\infty}\boxed{\phantom{xx}}=1$

◪ **다음 물음에 답하여라.**

**5.** 곡선 $y=x^3$과 $x$축 및 직선 $x=1$로 둘러싸인 부분의 넓이를 구분구적법으로 구하여라.




## 02 정적분과 급수

(1) 함수 $f(x)$가 닫힌구간 $[a,\ b]$에서 연속일 때,

$$\lim_{n\to\infty}\sum_{k=1}^{n}f(x_k)\Delta x=\int_a^b f(x)\,dx$$

(단, $\Delta x=\dfrac{b-a}{n}$, $x_k=a+k\Delta x$)

(2) 정적분과 급수의 합의 관계

① $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}f\left(a+\dfrac{(b-a)k}{n}\right)\cdot\dfrac{b-a}{n}=\int_a^b f(x)\,dx$

② $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}f\left(a+\dfrac{p}{n}k\right)\cdot\dfrac{p}{n}=\int_a^{a+p}f(x)\,dx=\int_0^p f(a+x)\,dx$

③ $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}f\left(\dfrac{p}{n}k\right)\cdot\dfrac{p}{n}=\int_0^p f(x)\,dx$

④ $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}f\left(\dfrac{k}{n}\right)\cdot\dfrac{1}{n}=\int_0^1 f(x)\,dx$

◪ **정적분을 이용하여 다음 극한값을 구하여라.**

**6.** $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\dfrac{2k}{n}\right)^2\times\dfrac{2}{n}$

**7.** $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(4+\dfrac{2k}{n}\right)\times\dfrac{2}{n}$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

**8.** $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+4k^2}$

**9.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^3\frac{4}{n}$

**10.** $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2(3n-k)}{n^4}$

**11.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left\{\left(\frac{2k}{n}\right)^2+1\right\}\frac{2}{n}$

**12.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{k}{n}\right)^3\frac{4}{n}$

**13.** $\lim_{n\to\infty}\sum_{k=1}^{n}(-n+3k)^5\cdot\frac{4}{n^6}$

**14.** $\lim_{n\to\infty}\sum_{k=1}^{4n}\left(1+\frac{k}{2n}\right)^4\frac{2}{n}$

**15.** $\lim_{n\to\infty}\sum_{k=1}^{n}(n+2k)^3\cdot\frac{3}{n^4}$

**16.** $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4k}{n^2}e^{\frac{k}{n}}$

**17.** $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k(k-2n)}{n^3}\left(\frac{n-k}{n}\right)$

**18.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^5\cdot\frac{3}{n}$

**19.** $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sin^2\frac{k\pi}{n}$

**20.** $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n+k}$

**21.** $\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{k^2(5k^2+7)}{n^3(n^2+3)}$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co,.Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

◪ 다음 물음에 답하여라.

**22.** $\int_{1}^{5} x^4 dx = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{a}{n}\left(1+\frac{ak}{n}\right)^4$일 때, 상수 $a$의 값을 구하여라.

**23.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{3k}{n}\right)^2\frac{6}{n}=a\int_{1}^{b}x^2dx$일 때, 두 자연수 $a,\ b$의 합을 구하여라.

**24.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^2\frac{4}{n}=\int_{1}^{b}ax^2dx$일 때, 자연수 $a,\ b$에 대한 곱 $ab$의 값을 구하여라.

**25.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{3k}{n}\right)^3\times\frac{1}{n}=\frac{q}{p}$일 때, $p+q$의 값을 구하여라. (단, $p,\ q$는 서로소인 자연수)

**26.** $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{8k}{n}\right)^2\cdot\frac{4}{n}=a\int_{0}^{b}(3+2x)^2dx$일 때, 상수 $a,\ b$에 대하여 $a+b$의 값을 구하여라.

**27.** 등식 $\lim_{n\to\infty}\sum_{k=1}^{n}\left\{\frac{n+2k}{n^2}\ln\left(1+\frac{2k}{n}\right)\right\}=a\ln3+b$를 만족하는 두 유리수 $a,\ b$에 대하여 $a+b$의 값을 구하여라.

◪ 다음 함수 $f(x)$에 대하여 알맞은 값을 구하여라.

**28.** 함수 $f(x)=x^4$에 대하여 $\lim_{n\to\infty}\sum_{k=1}^{n}f\left(1+\frac{2k}{n}\right)\frac{10}{n}$의 값을 구하여라.

**29.** 함수 $f(x)=x^2+2x$에 대하여 $\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}f\left(\frac{2k}{n}-1\right)$의 값을 구하여라.

**30.** 함수 $f(x)=e^x+2x$에 대하여 $\lim_{n\to\infty}\sum_{k=1}^{n}f\left(\frac{k}{n}\right)\frac{1}{n}+\lim_{n\to\infty}\sum_{k=1}^{2n}f\left(1+\frac{k}{n}\right)\frac{1}{n}$의 값을 구하여라.

**31.** 함수 $f(x)=\frac{1}{x^2+x}$에 대하여 $\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}f\left(1+\frac{2k}{n}\right)$의 값을 구하여라.

**32.** 함수 $f(x)=x^3+x+1$에 대하여 $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}f\left(1+\frac{2k}{n}\right)$의 값을 구하여라.

족보닷컴 zocbo.com Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331
I410-UCI(KEPA)-0199-2019-001-000244110

◪ 정적분을 이용하여 다음 극한값을 구하여라.

**33.** $\lim_{n\to\infty}\frac{\pi}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{n\pi}{n}\right)$

**34.** $\lim_{n\to\infty}\frac{3}{n}\left\{\left(\frac{n+1}{n}\right)^2+\left(\frac{n+2}{n}\right)^2+\cdots+\left(\frac{n+n}{n}\right)^2\right\}$

**35.** $\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{\frac{n}{1}}+\sqrt{\frac{n}{2}}+\sqrt{\frac{n}{3}}+\cdots+\sqrt{\frac{n}{n}}\right)$

**36.** $\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{n\sqrt{n}}$

**37.** $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$

**38.** $\lim_{n\to\infty}\left(\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+\frac{3^2}{n^3+3^3}+\cdots+\frac{n^2}{n^3+n^3}\right)$

**39.** $\lim_{n\to\infty}\frac{2}{n}\left(e^{1+\frac{4}{n}}+e^{1+\frac{8}{n}}+\cdots+e^{1+\frac{4n}{n}}\right)$

**40.** $\lim_{n\to\infty}\frac{1}{n}\ln\left(\frac{n+1}{n}\times\frac{n+2}{n}\times\frac{n+3}{n}\times\cdots\times\frac{2n}{n}\right)$

**41.** $\lim_{n\to\infty}\frac{\sqrt{n}}{n^2}(\sqrt{n+1}+\sqrt{n+2}+\sqrt{n+3}+\cdots+\sqrt{n+n})$

**42.** $\lim_{n\to\infty}\frac{\sqrt{3}+\sqrt{6}+\cdots+\sqrt{3n}}{n\sqrt{n}}$

**43.** $2\lim_{n\to\infty}\left(\frac{1}{n^2+1^2}+\frac{2}{n^2+2^2}+\cdots+\frac{n}{2n^2}\right)$

**44.** $\lim_{n\to\infty}\frac{(n+1)^5+(n+2)^5+(n+3)^5+\cdots+(n+n)^5}{1^5+2^5+3^5+\cdots+n^5}$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

**45.** $\lim_{n\to\infty}\frac{n(1^3+2^3+3^3+\cdots+n^3)}{(1+2+\cdots+n)(1^2+2^2+\cdots+n^2)}$

**46.** $\lim_{n\to\infty}\frac{1}{n}\left(e^{\frac{1}{n}-1}+e^{\frac{2}{n}-1}+\cdots+e^{\frac{n}{n}-1}\right)$

**47.** $\lim_{n\to\infty}\frac{e^{\frac{1}{n}}+2e^{\frac{2}{n}}+\cdots+(n-1)e^{\frac{n-1}{n}}+ne}{n^2}$

**48.** $\lim_{n\to\infty}\left\{\frac{2}{n}\cdot e^{\frac{2}{n}}+\frac{4}{n}\cdot e^{\frac{4}{n}}+\cdots\frac{2n}{n}\cdot e^{\frac{2n}{n}}\right\}\frac{2}{n}$

**49.** $\lim_{n\to\infty}\frac{1}{n^2}\left\{\frac{1}{\sqrt[n]{e}}+\frac{2}{\sqrt[n]{e^2}}+\frac{3}{\sqrt[n]{e^3}}+\cdots+\frac{n}{\sqrt[n]{e^n}}\right\}$

**50.** $\lim_{n\to\infty}\left\{\frac{6\cdot 2^2}{n^3+2^3}+\frac{6\cdot 4^2}{n^3+4^3}+\frac{6\cdot 6^2}{n^3+6^3}+\cdots+\frac{6(2n)^2}{n^3+(2n)^3}\right\}$

**51.** $\lim_{n\to\infty}\frac{1}{n}\left(e^{1+\frac{2}{n}}+e^{1+\frac{4}{n}}+e^{1+\frac{6}{n}}+\cdots+e^{1+\frac{2n}{n}}\right)$

**52.**
$\lim_{n\to\infty}\frac{27\pi^2}{n^2}\left(\sin\frac{3\pi}{n}+2\sin\frac{6\pi}{n}+3\sin\frac{9\pi}{n}+\cdots+n\sin3\pi\right)$

**53.** $\lim_{n\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{2n}+\sin\frac{2\pi}{2n}+\sin\frac{3\pi}{2n}+\cdots+\sin\frac{\pi}{2}\right)$

**54.** $\lim_{n\to\infty}\frac{2}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{n\pi}{n}\right)$

**55.** $\lim_{n\to\infty}\frac{\pi^2}{n^2}\left(\sin\frac{\pi}{n}+2\sin\frac{2\pi}{n}+3\sin\frac{3\pi}{n}+\cdots+n\sin\frac{n\pi}{n}\right)$

**56.** $\lim_{n\to\infty}\frac{1}{n^2}\left(\cos\frac{\pi}{n}+2\cos\frac{2}{n}\pi+3\cos\frac{3}{n}\pi+\cdots+n\cos\frac{n}{n}\pi\right)$


Copyright (c) since 2000 EDUZONE Co,.Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

**57.**

$$\lim_{n\to\infty}\frac{\pi^3}{8n^3}\left(\cos\frac{\pi}{2n}+4\cos\frac{2\pi}{2n}+9\cos\frac{3\pi}{2n}+\cdots+n^2\cos\frac{n\pi}{2n}\right)$$

**58.**

$$\lim_{n\to\infty}\left\{\frac{\pi}{n}\tan\left(\frac{\pi}{4}+\frac{\pi}{12n}\right)+\frac{\pi}{n}\tan\left(\frac{\pi}{4}+\frac{2\pi}{12n}\right)+\cdots+\frac{\pi}{n}\tan\left(\frac{\pi}{4}+\frac{n\pi}{12n}\right)\right\}$$

**59.** $\displaystyle\lim_{n\to\infty}\frac{6}{n}\ln\left(\frac{n+3}{n}\times\frac{n+6}{n}\times\frac{n+9}{n}\times\cdots\times\frac{4n}{n}\right)$

**60.** $\displaystyle\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right]^{\frac{1}{n}}$

**61.**

$$\lim_{n\to\infty}n\left\{\frac{\ln(n+1)-\ln n}{(2n+1)^2}+\frac{\ln(n+2)-\ln n}{(2n+2)^2}+\cdots+\frac{\ln 2n-\ln n}{(2n+n)^2}\right\}$$

◪ 다음 물음에 답하여라.

**62.** $\displaystyle\lim_{n\to\infty}\frac{\pi}{n}\left(\cos\frac{\pi}{n}+\cos\frac{2\pi}{n}+\cos\frac{3\pi}{n}+\cdots+\cos\frac{n\pi}{n}\right)$를 정적분으로 나타내면 $\displaystyle\int_0^a \cos x\,dx$일 때, 상수 $a$의 값을 구하여라.

**63.** 정적분을 이용하여 $\displaystyle a=\lim_{n\to\infty}\frac{(n+1)^4+(n+2)^4+(n+3)^4+\cdots+(n+n)^4}{n^5}$일 때, $5a$의 값을 구하여라.

◪ 다음 함수 $f(x)$에 대하여 알맞은 값을 구하여라.

**64.** 함수 $f(x)=x^4$에 대하여 $\displaystyle\lim_{n\to\infty}\frac{1}{n^5}\{f(n+1)+f(n+2)+\cdots+f(2n)\}$의 값을 구하여라.

**65.** 함수 $f(x)=e^{2x}$에 대하여 $\displaystyle\lim_{n\to\infty}\frac{1}{n}\left\{f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+f\left(\frac{3}{n}\right)+\cdots+f\left(\frac{n}{n}\right)\right\}$의 값을 구하여라.

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co,.Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

**66.** 함수 $f(x)=\tan x$에 대하여

$$\lim_{n\to\infty}\frac{1}{n}\left[\left\{f\left(\frac{\pi}{n}\right)\right\}^2+\left\{f\left(\frac{2\pi}{n}\right)\right\}^2+\left\{f\left(\frac{3\pi}{n}\right)\right\}^2+\cdots+\left\{f\left(\frac{n\pi}{n}\right)\right\}^2\right]$$

의 값을 구하여라.

**67.** 함수 $f(x)=2xe^{x^2}$에 대하여

$$\lim_{n\to\infty}\frac{1}{n}\left\{f\left(\frac{2}{n}\right)+f\left(\frac{4}{n}\right)+f\left(\frac{6}{n}\right)+\cdots+f\left(\frac{2n}{n}\right)\right\}$$

의 값을 구하여라.

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co,.Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331
I410-UCI(KEPA)-0199-2019-001-000244110

## 정답 및 해설

1) $\frac{1}{n}$, $\frac{1}{n}$, $\frac{1}{n}$, $\frac{k^2}{n^3}$, $\frac{1}{3}$

⇨ $S_n$은 밑변의 길이가 $\frac{1}{n}$이고 높이가 각각 $\left(\frac{1}{n}\right)^2$, $\left(\frac{2}{n}\right)^2$, $\cdots$, $\left(\frac{n+1}{n}\right)^2$, $\left(\frac{n}{n}\right)^2$인 직사각형의 넓이의 합이므로

$$S_n=\boxed{\frac{1}{n}}\times\left(\frac{1}{n}\right)^2+\boxed{\frac{1}{n}}\times\left(\frac{2}{n}\right)^2+\cdots+\boxed{\frac{1}{n}}\times\left(\frac{n}{n}\right)^2$$
$$=\sum_{k=1}^{n}\frac{1}{n}\times\left(\frac{k}{n}\right)^2$$
$$=\sum_{k=1}^{n}\boxed{\frac{k^2}{n^3}}$$

$$\therefore S=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2}{n^3}$$
$$=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}$$
$$=\boxed{\frac{1}{3}}$$

2) (가) $\frac{1}{n}\left(\frac{2k}{n}-\frac{1}{2}\left(\frac{k}{n}\right)^2\right)$

(나) $\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{1}{n}\left(\frac{2k}{n}-\frac{1}{2}\left(\frac{k}{n}\right)^2\right)$

(다) $2x-\frac{1}{2}x^2$

(라) $4x-2x^2$

⇨ $S_k=\frac{1}{n}\left(\frac{2k}{n}-\frac{1}{2}\left(\frac{k}{n}\right)^2\right)$

$$\lim_{n\to\infty}\sum_{k=1}^{2n}S_k=\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{1}{n}\left(\frac{2k}{n}-\frac{1}{2}\left(\frac{k}{n}\right)^2\right)$$
$$=\int_0^2 2x\,dx-\frac{1}{2}\int_0^2 x^2dx$$
$$=\int_0^2\left(2x-\frac{1}{2}x^2\right)dx=2\int_0^1(4x-2x^2)dx$$

3) (가) $4k^2$ (나) 8 (다) $\frac{8}{n^3}$ (라) $\frac{8}{3}$

⇨ $x_k=\frac{2k}{n}$이므로 $f(x_k)=\frac{4k^2}{n^2}$ $\therefore$ (가)$=4k^2$

$$\lim_{n\to\infty}\sum_{k=1}^{n}f(x_k)\triangle x=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4k^2}{n^2}\cdot\frac{2}{n}=\lim_{n\to\infty}\frac{8}{n^3}\sum_{k=1}^{n}k^2$$
$$=\lim_{n\to\infty}\left\{\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right\}$$
$$=\frac{4}{3}\lim_{n\to\infty}\left\{\frac{n(n+1)(2n+1)}{n^3}\right\}=\frac{4}{3}\lim_{n\to\infty}\frac{(n+1)(2n+1)}{n^2}$$
$$=\frac{4}{3}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)=\frac{8}{3}$$

이므로 (나)$=8$, (다)$=\frac{8}{n^3}$, (라)$=\frac{8}{3}$

4) $\frac{3}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2$

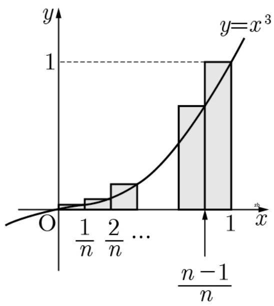
⇨ $k$번째 직사각형의 넓이는 $3\left(\frac{k}{n}\right)^2\frac{1}{n}$

$n$개의 직사각형의 넓이의 합은

$$S_n=\sum_{k=1}^{n}3\left(\frac{k}{n}\right)^2\frac{1}{n}$$
$$S=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\sum_{k=1}^{n}3\left(\frac{k}{n}\right)^2\frac{1}{n}=\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2$$

5) $\frac{1}{4}$

⇨




그림과 같이 곡선 위에 만든 직사각형의 넓이의 합을 $T_n$이라고 하면

$$T_n=\frac{1}{n}\left(\frac{1}{n}\right)^3+\frac{1}{n}\left(\frac{2}{n}\right)^3+\cdots+\frac{1}{n}\left(\frac{n}{n}\right)^3$$
$$=\frac{1}{n^4}(1^3+2^3+\cdots+n^3)$$
$$=\frac{1}{n^4}\left\{\frac{n(n+1)}{2}\right\}^2=\frac{1}{4}\left(1+\frac{1}{n}\right)^2$$

따라서 구하는 도형의 넓이는

$$\lim_{n\to\infty}T_n=\lim_{n\to\infty}\frac{1}{4}\left(1+\frac{1}{n}\right)^2=\frac{1}{4}$$

6) $\frac{8}{3}$

⇨ $f(x)=x^2$, $a=0$, $b=2$로 놓으면

$\varDelta x=\frac{2}{n}$, $x_k=\frac{2k}{n}$

$$\therefore\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{2k}{n}\right)^2\times\frac{2}{n}=\lim_{n\to\infty}\sum_{k=1}^{n}f(x_k)\varDelta x$$
$$=\int_0^2 x^2dx$$
$$=\left[\frac{1}{3}x^3\right]_0^2=\frac{8}{3}$$

7) 10

⇨ $f(x)=4+x$, $a=0$, $b=2$로 놓으면

$\varDelta x=\frac{2}{n}$, $x_k=\frac{2k}{n}$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

$$\therefore \lim_{n\to\infty}\sum_{k=1}^{n}\left(4+\frac{2k}{n}\right)\times\frac{2}{n}=\lim_{n\to\infty}\sum_{k=1}^{n}f(x_k)\varDelta x$$
$$=\int_0^2(4+x)dx$$
$$=\left[4x+\frac{1}{2}x^2\right]_0^2=10$$

8) $\frac{1}{8}\ln 5$

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+4k^2}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{\frac{k}{n}}{1+4\left(\frac{k}{n}\right)^2}\cdot\frac{1}{n}$$
$$=\int_0^1\frac{x}{1+4x^2}dx$$

이때, $1+4x^2=t$로 놓으면 $8x=\frac{dt}{dx}$

$x=0$일 때 $t=1$, $x=1$일 때 $t=5$

$$\int_0^1\frac{x}{1+4x^2}dx=\frac{1}{8}\int_1^5\frac{1}{t}dt$$
$$=\frac{1}{8}\Big[\ \ln|t|\ \Big]_1^5$$
$$=\frac{1}{8}\ln 5$$

9) 40

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^3\frac{4}{n}=2\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^3\frac{2}{n}$$
$$=2\int_1^3x^3dx=2\left[\frac{1}{4}x^4\right]_1^3=2\left(\frac{81}{4}-\frac{1}{4}\right)=40$$

10) $\frac{3}{4}$

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2\left(3-\frac{k}{n}\right)\frac{1}{n}=\int_0^1\{x^2(3-x)\}dx$$
$$=\int_0^1(3x^2-x^3)dx=\left[x^3-\frac{1}{4}x^4\right]_0^1=\frac{3}{4}$$

11) $\frac{14}{3}$

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left\{\left(\frac{2k}{n}\right)^2+1\right\}\frac{2}{n}=\int_0^2(x^2+1)dx$$
$$=\left[\frac{1}{3}x^3+x\right]_0^2=\frac{8}{3}+2=\frac{14}{3}$$

12) 65

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{k}{n}\right)^3\frac{4}{n}$$
$$=4\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{k}{n}\right)^3\frac{1}{n}$$
$$=4\int_2^3x^3dx=4\left[\frac{1}{4}x^4\right]_2^3=4\times\left(\frac{81}{4}-\frac{16}{4}\right)=65$$

13) 14

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}(-n+3k)^5\cdot\frac{4}{n^6}$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{-n+3k}{n}\right)^5\cdot\frac{4}{n}$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(-1+\frac{3k}{n}\right)^5\cdot\frac{3}{n}\cdot\frac{4}{3}$$
$$=\frac{4}{3}\int_{-1}^2x^5\,dx=\frac{4}{3}\left[\frac{1}{6}x^6\right]_{-1}^2=\frac{4}{3}\left(\frac{64}{6}-\frac{1}{6}\right)=14$$

14) $\frac{968}{5}$

$\Rightarrow$

$$\lim_{n\to\infty}\sum_{k=1}^{4n}\left(1+\frac{k}{2n}\right)^4\frac{2}{n}=4\lim_{n\to\infty}\sum_{k=1}^{4n}\left(1+\frac{k}{2n}\right)^4\frac{1}{2n}=4\int_1^3x^4dx$$
$$=4\left[\frac{1}{5}x^5\right]_1^3=4\left(\frac{243}{5}-\frac{1}{5}\right)=\frac{968}{5}$$

15) 30

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}(n+2k)^3\cdot\frac{3}{n^4}=\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^3\cdot\frac{3}{n}$$
$$=\frac{3}{2}\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^3\cdot\frac{2}{n}$$
$$=\frac{3}{2}\int_1^3x^3dx$$
$$=\frac{3}{2}\left[\frac{1}{4}x^4\right]_1^3=\frac{3}{2}\times\frac{1}{4}\times(81-1)=30$$

16) 4

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\frac{4k}{n^2}e^{\frac{k}{n}}=4\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n}e^{\frac{k}{n}}\cdot\frac{1}{n}$$
$$=4\int_0^1xe^xdx$$
$$=4\Big[\ xe^x-e^x\ \Big]_0^1=4$$

17) $-\frac{1}{4}$

$$\Rightarrow \lim_{n\to\infty}\sum_{k=1}^{n}\frac{k(k-2n)}{n^3}\left(\frac{n-k}{n}\right)$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\frac{k}{n}\cdot\left(\frac{k}{n}-2\right)\cdot\left(1-\frac{k}{n}\right)$$

($\frac{k}{n}=x$로 치환하면)

$$=\int_0^1x(x-2)(1-x)dx$$
$$=\int_0^1(-x^3+3x^2-2x)dx$$
$$=\left[-\frac{1}{4}x^4+x^3-x^2\right]_0^1$$
$$=-\frac{1}{4}$$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co,.Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

18) 182

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^5\cdot\frac{3}{n}$

$=\frac{3}{2}\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^5\cdot\frac{2}{n}$

$=\frac{3}{2}\int_1^3 x^5dx=\frac{3}{2}\left[\frac{1}{6}x^6\right]_1^3=\frac{1}{4}\times(729-1)=182$

19) $\frac{1}{2}$

⇨ $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sin^2\frac{k\pi}{n}$

$=\int_0^1\sin^2\pi x\,dx$

$=\int_0^1\left(\frac{1-\cos2\pi x}{2}\right)dx$

$=\frac{1}{2}\left[\ x-\frac{1}{2\pi}\sin2\pi x\ \right]_0^1$

$=\frac{1}{2}$

20) $\ln4$

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n+k}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{1+\frac{k}{n}}\frac{1}{n}=\int_0^1\frac{2}{1+x}dx$

$=[2\ln|1+x|]_0^1=2\ln2=\ln4$

21) 32

⇨

$\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{k^2(5k^2+7)}{n^3(n^2+3)}=\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{\left(\frac{k}{n}\right)^2\left(5\left(\frac{k}{n}\right)^2+\frac{7}{n^2}\right)}{n\left(1+\frac{3}{n^2}\right)}$

$=\int_0^2x^2(5x^2)dx=\int_0^2 5x^4dx=[x^5]_0^2=32$

22) 4

⇨ $\int_1^5x^4dx=\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{4k}{n}\right)^4\cdot\frac{4}{n}$ $\quad\therefore\ a=4$

23) 6

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{3k}{n}\right)^2\frac{6}{n}=2\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{3k}{n}\right)^2\frac{3}{n}$

$=2\int_1^4x^2dx$

따라서 $a=2$, $b=4$이므로 $a+b=2+4=6$이다.

24) 6

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^2\frac{4}{n}=\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^2\cdot\frac{2}{n}\cdot2$

$=\int_1^3 2x^2\,dx$

따라서 $a=2$, $b=3$이므로 $ab=6$이다.

25) 207

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{3k}{n}\right)^3\frac{1}{n}$

$=\int_0^1(2+3x)^3dx=\left[\frac{1}{3}\times\frac{1}{4}(2+3x)^4\right]_0^1$

$=\frac{5^4-2^4}{12}=\frac{625-16}{12}=\frac{203}{4}=\frac{q}{p}$

따라서 $p=4$, $q=203$이므로 $p+q=207$

26) 5

⇨ $x=\frac{4k}{n}$로 놓으면

$\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{8k}{n}\right)^2\cdot\frac{4}{n}=\int_0^4(3+2x)^2dx$

따라서 $a=1$, $b=4$이므로 $a+b=5$이다.

27) $\frac{5}{4}$

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\left\{\frac{1}{n}\left(1+2\frac{k}{n}\right)\ln\left(1+\frac{2k}{n}\right)\right\}=\frac{1}{2}\int_1^3x\ln x dx$

$=\frac{1}{2}\left[\frac{x^2}{2}\ln x-\frac{x^2}{4}\right]_1^3=\frac{1}{2}\left(\frac{9}{2}\ln3-\frac{9}{4}+\frac{1}{4}\right)=\frac{9}{4}\ln3-1$

$a=\frac{9}{4}$, $b=-1$ $\qquad\therefore\ a+b=\frac{5}{4}$

28) 242

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}f\left(1+\frac{2k}{n}\right)\frac{10}{n}=\lim_{n\to\infty}\sum_{k=1}^{n}f\left(1+\frac{2k}{n}\right)\frac{2}{n}\cdot5$

$=5\int_1^3f(x)dx=5\int_1^3x^4dx$

$=5\left[\frac{1}{5}x^5\right]_1^3=5\left(\frac{243}{5}-\frac{1}{5}\right)=5\times\frac{242}{5}=242$

29) 1

⇨ $\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}f\left(\frac{2k}{n}-1\right)=\frac{3}{2}\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}f\left(\frac{2k}{n}-1\right)$

$=\frac{3}{2}\int_{-1}^1f(x)dx=\frac{3}{2}\times2\int_0^1x^2dx$

$=3\left[\frac{1}{3}x^3\right]_0^1=3\times\frac{1}{3}\times1=1$

30) $e^3+8$

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}f\left(\frac{k}{n}\right)\frac{1}{n}=\int_0^1f(x)dx$ ,

$\lim_{n\to\infty}\sum_{k=1}^{2n}f\left(1+\frac{k}{n}\right)\frac{1}{n}=\lim_{n\to\infty}\sum_{k=1}^{2n}f\left(1+\frac{2k}{2n}\right)\frac{2}{2n}$

$=\int_1^3f(x)dx$

이므로

$\lim_{n\to\infty}\sum_{k=1}^{n}f\left(\frac{k}{n}\right)\frac{1}{n}+\lim_{n\to\infty}\sum_{k=1}^{2n}f\left(1+\frac{k}{n}\right)\frac{1}{n}$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

$=\int_0^1 f(x)dx+\int_1^3 f(x)dx$

$=\int_0^3 f(x)dx$

$=\int_0^3 (e^x+2x)dx$

$=\left[\ e^x+x^2\ \right]_0^3=e^3+8$

31) $\ln 3-\ln 2$

⇨ $f(x)=\dfrac{1}{x^2+x}=\dfrac{1}{x(x+1)}=\dfrac{1}{x}-\dfrac{1}{x+1}$ 이므로

$=\lim\limits_{n\to\infty}\dfrac{2}{n}\sum\limits_{k=1}^{n} f\left(1+\dfrac{2k}{n}\right)$

$=\int_1^3 f(x)dx$

$=\int_1^3\left(\dfrac{1}{x}-\dfrac{1}{x+1}\right)dx$

$=\left[\ \ln|x|-\ln|x+1|\ \right]_1^3$

$=\ln 3-\ln 2$

32) 13

⇨ $\lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{k=1}^{n} f\left(1+\dfrac{2k}{n}\right)=\dfrac{1}{2}\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n} f\left(1+\dfrac{2k}{n}\right)\cdot\dfrac{2}{n}$

$=\dfrac{1}{2}\int_1^3 f(x)dx$

$=\dfrac{1}{2}\int_1^3 (x^3+x+1)dx=\dfrac{1}{2}\left[\dfrac{1}{4}x^4+\dfrac{1}{2}x^2+x\right]_1^3$

$=\dfrac{1}{2}\left\{\left(\dfrac{81}{4}+\dfrac{9}{2}+3\right)-\left(\dfrac{1}{4}+\dfrac{1}{2}+1\right)\right\}$

$=\dfrac{1}{2}(20+4+2)=\dfrac{1}{2}\times 26=13$

33) 2

⇨ $\lim\limits_{n\to\infty}\dfrac{\pi}{n}\left(\sin\dfrac{\pi}{n}+\sin\dfrac{2\pi}{n}+\cdots+\sin\dfrac{n\pi}{n}\right)$

$=\lim\limits_{n\to\infty}\dfrac{\pi}{n}\sum\limits_{k=1}^{n}\sin\dfrac{k\pi}{n}$

$f(x)=\sin x,\ a=0,\ b=\pi$로 놓으면

$\Delta x=\dfrac{\pi}{n},\ x_k=\dfrac{k\pi}{n}$

$\therefore$ (주어진 식) $=\int_0^{\pi}\sin x dx=[-\cos x]_0^{\pi}=2$

34) 7

⇨ $\lim\limits_{n\to\infty}\dfrac{3}{n}\left\{\left(\dfrac{n+1}{n}\right)^2+\left(\dfrac{n+2}{n}\right)^2+\cdots+\left(\dfrac{n+n}{n}\right)^2\right\}$

$=\lim\limits_{n\to\infty}\dfrac{3}{n}\sum\limits_{k=1}^{n}\left(1+\dfrac{k}{n}\right)^2$

$=3\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(1+\dfrac{k}{n}\right)^2\times\dfrac{1}{n}$

$f(x)=x^2,\ a=1,\ b=2$로 놓으면

$\Delta x=\dfrac{1}{n},\ x_k=1+\dfrac{k}{n}$

$\therefore$ (주어진 식) $=3\int_1^2 x^2dx=3\left[\dfrac{1}{3}x^3\right]_1^2=7$

35) 2

⇨ $\lim\limits_{n\to\infty}\dfrac{1}{n}\left(\sqrt{\dfrac{n}{1}}+\sqrt{\dfrac{n}{2}}+\sqrt{\dfrac{n}{3}}+\cdots+\sqrt{\dfrac{n}{n}}\right)$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{1}{n}\sqrt{\dfrac{n}{k}}$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{1}{\sqrt{\dfrac{k}{n}}}\cdot\dfrac{1}{n}$

$=\int_0^1\dfrac{1}{\sqrt{x}}dx$

$=\left[\ 2\sqrt{x}\ \right]_0^1=2$

36) $\dfrac{2}{3}$

⇨ $\lim\limits_{n\to\infty}\dfrac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{n\sqrt{n}}$

$=\lim\limits_{n\to\infty}\dfrac{1}{n}\left(\sqrt{\dfrac{1}{n}}+\sqrt{\dfrac{2}{n}}+\cdots+\sqrt{\dfrac{n}{n}}\right)$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\sqrt{\dfrac{k}{n}}\cdot\dfrac{1}{n}$

$=\int_0^1\sqrt{x}\,dx$

$=\left[\ \dfrac{2}{3}x^{\frac{3}{2}}\ \right]_0^1=\dfrac{2}{3}$

37) $\ln 2$

⇨ $\lim\limits_{n\to\infty}\left(\dfrac{1}{n+1}+\dfrac{1}{n+2}+\cdots+\dfrac{1}{2n}\right)$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{1}{n+k}$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{1}{1+\dfrac{k}{n}}\cdot\dfrac{1}{n}$

$=\int_0^1\dfrac{1}{1+x}dx=\left[\ \ln|1+x|\ \right]_0^1$

$=\ln 2$

38) $\dfrac{1}{3}\ln 2$

⇨ $\lim\limits_{n\to\infty}\left(\dfrac{1^2}{n^3+1^3}+\dfrac{2^2}{n^3+2^3}+\dfrac{3^2}{n^3+3^3}+\cdots+\dfrac{n^2}{n^3+n^3}\right)$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{k^2}{n^3+k^3}$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{\left(\dfrac{k}{n}\right)^2}{1+\left(\dfrac{k}{n}\right)^3}\cdot\dfrac{1}{n}$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

$=\int_0^1 \frac{x^2}{1+x^3}dx$

이때, $1+x^3=t$로 놓으면 $3x^2=\frac{dt}{dx}$

$x=0$일 때 $t=1$, $x=1$일 때 $t=2$

$\int_0^1 \frac{x^2}{1+x^3}dx=\frac{1}{3}\int_1^2 \frac{1}{t}dt$

$=\frac{1}{3}\left[\ \ln|t|\ \right]_0^2$

$=\frac{1}{3}\ln 2$

39) $\frac{1}{2}(e^5-e)$

⇨ $\lim_{n\to\infty}\frac{2}{n}\left(e^{1+\frac{4}{n}}+e^{1+\frac{8}{n}}+\cdots+e^{1+\frac{4n}{n}}\right)$

$=\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}e^{1+\frac{4k}{n}}$

$=\frac{1}{2}\lim_{n\to\infty}\sum_{k=1}^{n}e^{1+\frac{4k}{n}}\cdot\frac{4}{n}$

$=\frac{1}{2}\int_1^5 e^x dx$

$=\frac{1}{2}\left[\ e^x\ \right]_1^5$

$=\frac{1}{2}(e^5-e)$

40) $2\ln 2-1$

⇨ $\lim_{n\to\infty}\frac{1}{n}\ln\left(\frac{n+1}{n}\times\frac{n+2}{n}\times\frac{n+3}{n}\times\cdots\times\frac{2n}{n}\right)$

$=\lim_{n\to\infty}\frac{1}{n}\left(\ln\frac{n+1}{n}+\ln\frac{n+2}{n}+\ln\frac{n+3}{n}+\cdots+\ln\frac{2n}{n}\right)$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\ln\frac{n+k}{n}$

$=\int_1^2 \ln x\,dx$

$=\left[\ x\ln x-x\ \right]_1^2=2\ln 2-1$

41) $\frac{2}{3}(2\sqrt{2}-1)$

⇨

$\lim_{n\to\infty}\frac{\sqrt{n}}{n^2}(\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{n+n})$

$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{1+\frac{k}{n}}\right)\frac{1}{n}=\int_1^2\sqrt{x}\,dx=\left[\frac{2}{3}x\sqrt{x}\right]_1^2=\frac{2}{3}(2\sqrt{2}-1)$

42) $\frac{2\sqrt{3}}{3}$

⇨ $\lim_{n\to\infty}\frac{\sqrt{3}+\sqrt{6}+\cdots+\sqrt{3n}}{n\sqrt{n}}$

$=\lim_{n\to\infty}\left(\sqrt{\frac{3}{n}}+\sqrt{\frac{6}{n}}+\cdots+\sqrt{\frac{3n}{n}}\right)\frac{1}{n}$

$=\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{\frac{3k}{n}}\times\frac{1}{n}=\int_0^1\sqrt{3x}\,dx$

$=\sqrt{3}\left[\frac{2}{3}x\sqrt{x}\right]_0^1=\frac{2\sqrt{3}}{3}$

43) $\ln 2$

⇨ $2\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n^2+k^2}\right)=2\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(\frac{\frac{k}{n}}{1+\left(\frac{k}{n}\right)^2}\right)$

$=2\int_0^1\frac{x}{1+x^2}dx$ ($1+x^2=t$로 치환하자. $2xdx=dt$)

$=\int_1^2\frac{1}{t}dt=[\ln|t|]_1^2=\ln 2$

44) 63

⇨ $\lim_{n\to\infty}\frac{(n+1)^5+(n+2)^5+(n+3)^5+\cdots+(n+n)^5}{1^5+2^5+3^5+\cdots+n^5}$

$=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}(n+k)^5}{\sum_{k=1}^{n}k^5}=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^5\frac{1}{n}}{\sum_{k=1}^{n}\left(\frac{k}{n}\right)^5\frac{1}{n}}$

$=\frac{\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^5\frac{1}{n}}{\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^5\frac{1}{n}}=\frac{\int_1^2 x^5dx}{\int_0^1 x^5dx}$

$=\frac{\left[\frac{1}{6}x^6\right]_1^2}{\left[\frac{1}{6}x^6\right]_0^1}=\frac{\frac{64}{6}-\frac{1}{6}}{\frac{1}{6}}=63$

45) $\frac{3}{2}$

⇨ $\lim_{n\to\infty}\frac{n(1^3+2^3+3^3+\cdots+n^3)}{(1+2+\cdots+n)(1^2+2^2+\cdots+n^2)}$

$=\lim_{n\to\infty}\frac{n\sum_{k=1}^{n}k^3}{\sum_{k=1}^{n}k\cdot\sum_{k=1}^{n}k^2}=\lim_{n\to\infty}\frac{\frac{1}{n^5}\cdot n\sum_{k=1}^{n}k^3}{\left(\frac{1}{n^2}\sum_{k=1}^{n}k\right)\cdot\left(\frac{1}{n^3}\sum_{k=1}^{n}k^2\right)}$

$=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}\left(\frac{k}{n}\right)^3\cdot\frac{1}{n}}{\left(\sum_{k=1}^{n}\frac{k}{n}\cdot\frac{1}{n}\right)\cdot\left\{\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2\cdot\frac{1}{n}\right\}}$

$=\frac{\int_0^1 x^3dx}{\int_0^1 xdx\cdot\int_0^1 x^2dx}=\frac{\left[\frac{1}{4}x^4\right]_0^1}{\left[\frac{1}{2}x^2\right]_0^1\cdot\left[\frac{1}{3}x^3\right]_0^1}$

$=\frac{\frac{1}{4}}{\frac{1}{2}\cdot\frac{1}{3}}=\frac{6}{4}=\frac{3}{2}$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331
I410-UCI(KEPA)-0199-2019-001-000244110

46) $1-\frac{1}{e}$

⇨ $\lim_{n\to\infty}\frac{1}{n}\left(e^{\frac{1}{n}-1}+e^{\frac{2}{n}-1}+\cdots+e^{\frac{n}{n}-1}\right)$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}e^{\frac{k}{n}-1}=\int_0^1 e^{x-1}dx$

$=\left[e^{x-1}\right]_0^1=1-\frac{1}{e}$

47) 1

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n}e^{\frac{k}{n}}\frac{1}{n}=\int_0^1 xe^x dx=\left[xe^x\right]_0^1-\int_0^1 e^x dx$

$=e-(e-1)=1$

48) $e^2+1$

⇨ 주어진 식을 정리하면

$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{2k}{n}e^{\frac{2k}{n}}\right)\frac{2}{n}=\int_0^2 xe^x dx$

$=\left[xe^x\right]_0^2-\int_0^2 e^x dx=2e^2-(e^2-1)=e^2+1$

49) $\frac{e-2}{e}$

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{\frac{k}{n}}{e^{\frac{k}{n}}}\cdot\frac{1}{n}=\int_0^1\frac{x}{e^x}dx=\int_0^1 xe^{-x}dx$

$=\left[-xe^{-x}\right]_0^1+\int_0^1 e^{-x}dx$

$=-\frac{1}{e}+\left[-e^{-x}\right]_0^1=-\frac{1}{e}-\frac{1}{e}+1=1-\frac{2}{e}=\frac{e-2}{e}$

50) 2ln3

⇨ $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{6(2k)^2}{n^3+(2k)^3}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{24\left(\frac{k}{n}\right)^2}{1+8\left(\frac{k}{n}\right)^3}\times\frac{1}{n}$

$=\int_0^1\frac{24x^2}{1+8x^3}dx=\left[\ln|1+8x^3|\right]_0^1=\ln 9=2\ln 3$

51) $\frac{1}{2}(e^3-e)$

⇨ $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}e^{1+\frac{2k}{n}}=\frac{1}{2}\int_1^3 e^x dx=\frac{1}{2}(e^3-e)$

52) $9\pi$

⇨ 주어진 극한식을 급수로 표현하면

$27\pi^2\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n}\sin\frac{3k\pi}{n}\frac{1}{n}$

$=27\pi^2\int_0^1 x\sin 3\pi x\,dx=27\pi^2\times\frac{1}{3\pi}=9\pi$

가 된다.

53) $\frac{2}{\pi}$

⇨ $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{\infty}\sin\frac{k\pi}{2n}=\int_0^1\sin\frac{\pi}{2}x dx$

$=\frac{2}{\pi}\left[-\cos\frac{\pi}{2}x\right]_0^1=\frac{2}{\pi}$

54) $\frac{4}{\pi}$

⇨ $\lim_{n\to\infty}\frac{2}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{n\pi}{n}\right)$

$=\frac{2}{\pi}\lim_{n\to\infty}\frac{\pi}{n}\sum_{k=1}^{n}\sin\frac{k\pi}{n}$

$=\frac{2}{\pi}\int_0^{\pi}\sin x\,dx$

$=\frac{2}{\pi}\left[-\cos x\right]_0^{\pi}=\frac{4}{\pi}$

55) $\pi$

⇨ $\sum_{k=1}^{n}\left(\frac{k\pi}{n}\sin\frac{k\pi}{n}\right)\frac{\pi}{n}=\int_0^{\pi}x\sin x dx$

$=\left[-x\cos x\right]_0^{\pi}+\int_0^{\pi}\cos x dx=\pi$

56) $-\frac{2}{\pi^2}$

⇨

$\lim_{n\to\infty}\frac{1}{n^2}\left(\cos\frac{\pi}{n}+2\cos\frac{2\pi}{n}+\cdots+n\cos\frac{n\pi}{n}\right)=\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}\cos\frac{k}{n}\pi\right)\frac{1}{n}$

$=\int_0^1 x\cos\pi x\,dx=\left[\frac{1}{\pi}x\sin\pi x\right]_0^1-\frac{1}{\pi}\int_0^1\sin\pi x dx$

$=\frac{1}{\pi^2}\left[\cos\pi x\right]_0^1=-\frac{2}{\pi^2}$

57) $\frac{\pi^2}{4}-2$

⇨ 주어진 식을 정리하면

$\lim_{n\to\infty}\frac{\pi^3}{8n^3}\sum_{k=1}^{n}k^2\cos\frac{k\pi}{2n}$

$=\lim_{n\to\infty}\frac{\pi^3}{8}\sum_{k=1}^{n}\frac{k^2}{n^2}\cos\frac{\pi}{2}\frac{k}{n}\frac{1}{n}$

$=\frac{\pi^3}{8}\int_0^1 x^2\cos\frac{\pi}{2}x\,dx$

$=\frac{\pi^3}{8}\left(\frac{2}{\pi}-\frac{2}{\pi}\int_0^1 2x\sin\frac{\pi}{2}x\,dx\right)$

$=\frac{\pi^3}{8}\left\{\frac{2}{\pi}+\left(\frac{2}{\pi}\right)^2\int_0^1 2\left(-\cos\frac{\pi}{2}x\right)dx\right\}$

$=\frac{\pi^2}{4}-2$

이다.

58) $6\ln 2$

Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

⇨ $\frac{\pi}{4}+\frac{\pi k}{12n}\to x$, $\frac{\pi}{12n}\to dx$로 놓으면,

$\frac{\pi}{4}\le x\le\frac{\pi}{3}$

$\therefore$ (주어진 극한)$=12\lim_{n\to\infty}\sum_{k=1}^{n}\tan\left(\frac{\pi}{4}+\frac{\pi k}{12n}\right)\cdot\frac{\pi}{12n}$

$=12\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\tan x dx=12\left[-\ln|\cos x|\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$=12\left(-\ln\frac{1}{2}+\ln\frac{\sqrt{2}}{2}\right)=6\ln 2$

59) $16\ln 2-6$

⇨ 주어진 극한식을 정리하면

$\lim_{n\to\infty}\frac{6}{n}\sum_{k=1}^{n}\ln\left(1+\frac{3k}{n}\right)=\lim_{n\to\infty}6\sum_{k=1}^{n}\ln\left(1+\frac{3k}{n}\right)\frac{1}{n}$

$=6\int_0^1\ln(1+3x)\,dx$ ,$1+3x=t$

$=16\ln 2-6$

60) $\frac{4}{e}$

⇨ 주어진 극한을 P라 놓으면

$\ln \mathrm{P}=\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(1+\frac{k}{n}\right)\cdot\frac{1}{n}=\int_1^2\ln x dx$

$=[x\ln x-x]_1^2=2\ln 2-1=\ln\frac{4}{e}$

$\therefore \mathrm{P}=\frac{4}{\mathrm{e}}$

61) $\frac{5}{3}\ln 2-\ln 3$

⇨ 주어진 식은

$n\sum_{k=1}^{n}\left(\frac{\ln\left(\frac{n+k}{n}\right)}{(2n+k)^2}\right)=\sum_{k=1}^{n}\ln\left(1+\frac{k}{n}\right)\frac{1}{\left(2+\frac{k}{n}\right)^2 n^2}\times n$

$=\int_0^1\frac{\ln(1+x)}{(2+x)^2}dx=\left[-\frac{\ln(1+x)}{2+x}\right]_0^1+\int_0^1\frac{1}{(1+x)(2+x)}dx$

$=-\frac{\ln 2}{3}+\int_0^1\left(\frac{1}{1+x}-\frac{1}{2+x}\right)dx$

$=-\frac{\ln 2}{3}+[\ln(x+1)-\ln(x+2)]_0^1=-\frac{\ln 2}{3}+\ln 2-\ln 3+\ln 2$

$=\frac{5}{3}\ln 2-\ln 3$

62) $\pi$

⇨ $\lim_{n\to\infty}\frac{\pi}{n}\left(\cos\frac{\pi}{n}+\cos\frac{2\pi}{n}+\cos\frac{3\pi}{n}+\cdots+\cos\frac{n\pi}{n}\right)$

$=\lim_{n\to\infty}\frac{\pi}{n}\sum_{k=1}^{n}\cos\frac{k\pi}{n}$

$=\lim_{n\to\infty}\sum_{k=1}^{n}\cos\frac{k\pi}{n}\cdot\frac{\pi}{n}$

$=\int_0^{\pi}\cos x dx$

$\therefore a=\pi$

63) 31

⇨ $a=\lim_{n\to\infty}\frac{1}{n}\left\{\left(\frac{n+1}{n}\right)^4+\left(\frac{n+2}{n}\right)^4+\cdots+\left(\frac{n+n}{n}\right)^4\right\}$

$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^4\frac{1}{n}=\int_1^2 x^4\,dx=\left[\frac{1}{5}x^5\right]_1^2=\frac{32}{5}-\frac{1}{5}$

$=\frac{31}{5}$

$\therefore\ 5a=5\times\frac{31}{5}=31$

64) $\frac{31}{5}$

⇨ $\lim_{n\to\infty}\frac{1}{n^5}\{f(n+1)+f(n+2)+\cdots+f(2n)\}$

$=\lim_{n\to\infty}\frac{1}{n^5}\sum_{k=1}^{n}f(n+k)=\lim_{n\to\infty}\frac{1}{n^5}\sum_{k=1}^{n}(n+k)^4$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^4$

$=\int_1^2 x^4dx=\left[\frac{1}{5}x^5\right]_1^2=\frac{32-1}{5}=\frac{31}{5}$

65) $\frac{1}{2}e^2-\frac{1}{2}$

⇨ $\lim_{n\to\infty}\frac{1}{n}\left\{f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+f\left(\frac{3}{n}\right)+\cdots+f\left(\frac{n}{n}\right)\right\}$

$=\lim_{n\to\infty}\sum_{k=1}^{n}f\left(\frac{k}{n}\right)\cdot\frac{1}{n}$

$=\int_0^1 f(x)dx=\int_0^1 e^{2x}dx$

$=\left[\ \frac{1}{2}e^{2x}\ \right]_0^1=\frac{1}{2}e^2-\frac{1}{2}$

66) $-1$

⇨

$\lim_{n\to\infty}\frac{1}{n}\left[\left\{f\left(\frac{\pi}{n}\right)^2\right\}+\left\{f\left(\frac{2\pi}{n}\right)^2\right\}+\left\{f\left(\frac{3\pi}{n}\right)^2\right\}+\cdots+\left\{f\left(\frac{n\pi}{n}\right)^2\right\}\right]$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left\{f\left(\frac{k\pi}{n}\right)\right\}^2$

$=\frac{1}{\pi}\lim_{n\to\infty}\sum_{k=1}^{n}\left\{f\left(\frac{k\pi}{n}\right)\right\}^2\cdot\frac{\pi}{n}$

$=\frac{1}{\pi}\int_0^{\pi}\{f(x)\}^2dx=\frac{1}{\pi}\int_0^{\pi}\tan^2x dx$

이때, $1+\tan^2x=\sec^2x$에서 $\tan^2x=\sec^2x-1$이므로

$\frac{1}{\pi}\int_0^{\pi}\tan^2xdx=\frac{1}{\pi}\int_0^{\pi}(\sec^2x-1)dx$

$=\frac{1}{\pi}\left[\ \tan x-x\ \right]_0^{\pi}$

$=\frac{1}{\pi}\cdot(-\pi)$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co.,Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331

$=-1$

67) $\frac{1}{2}(e^4-1)$

⇨ $\lim_{n\to\infty}\frac{1}{n}\left\{f\left(\frac{2}{n}\right)+f\left(\frac{4}{n}\right)+f\left(\frac{6}{n}\right)+\cdots f\left(\frac{2n}{n}\right)\right\}$

$=\lim_{n\to\infty}\sum_{k=1}^{n}f\left(\frac{2k}{n}\right)\frac{1}{n}$

$=\frac{1}{2}\lim_{n\to\infty}\sum_{k=1}^{n}f\left(\frac{2k}{n}\right)\frac{2}{n}$

$=\frac{1}{2}\int_0^2 f(x)dx$

$=\frac{1}{2}\int_0^2 2xe^{x^2}dx$

$=\frac{1}{2}\left[\ e^{x^2}\ \right]_0^2$

$=\frac{1}{2}(e^4-1)$

족보닷컴 zocbo.com
Copyright (c) since 2000 EDUZONE Co,.Ltd all rights reserved
교육지대(주) 서울특별시 구로구 구로동 디지털로33길 55 이앤씨벤처드림타워2차 805호 대표전화 : 1544-5633 FAX : 02-521-0331
I410-UCI(KEPA)-0199-2019-001-000244110