01 자연수의 거듭제곱의 합

(1) $\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

(2) $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

(3) $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+\cdots+n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

◪ 다음을 계산하여라.

1. $\sum_{k=1}^{10} k^3$

2. $\sum_{k=1}^{10} k^2$

3. $\sum_{k=2}^{10} (3k+1)$

4. $\sum_{k=3}^{10} (2k-5)$

5. $\sum_{k=1}^{10} (k-2)^3$

6. $\sum_{k=1}^{10} (4k+2)$

7. $\sum_{k=1}^{10} (k+1)(k-3)$

8. $\sum_{k=4}^{10} k(k+1)$

9. $\sum_{k=2}^{10} (3k-5)$

10. $\sum_{k=3}^{10} k(k+2)$

11. $\sum_{k=1}^{10} (k^3-k^2+k-1)$

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12. $\sum_{k=1}^{10}(k-1)(k+2)$

13. $\sum_{k=1}^{5}k(k-1)(k+1)$

14. $\sum_{k=1}^{10}(k^2-3k+1)$

15. $\sum_{k=1}^{8}k(k+1)(k-1)$

16. $\sum_{k=6}^{10}(2k+1)^2$

17. $\sum_{k=1}^{10}(2k^2-3k+1)$

18. $\sum_{k=4}^{10}(k^2-k)$

19. $\sum_{k=1}^{8}k(k-1)(k-2)$

20. $\sum_{k=1}^{10}(k^3-2k^2+4k-3)$

21. $\sum_{k=1}^{10}(k^2+2k+4)-\sum_{k=1}^{10}(k^2-1)$

22. $\sum_{k=1}^{10}(k+5)(k-2)-\sum_{k=1}^{10}(k-5)(k+2)$

23. $\sum_{k=1}^{10}k(k+1)$

24. $\sum_{k=1}^{8}(k+1)(k+2)$

25. $\sum_{k=1}^{10}(k+1)^3-\sum_{k=1}^{11}(k-1)^3$

26. $\sum_{k=1}^{10}(k^2+1)-\sum_{k=1}^{10}(k-1)$

27. $\sum_{k=1}^{20}k^2$

28. $\sum_{k=1}^{8}2k(k-1)-\sum_{i=1}^{8}(i^2-1)$

29. $\sum_{k=1}^{10}(2k-3)^2$

30. $\sum_{k=1}^{10}(k^2+2k-3)$

31. $\sum_{k=1}^{10}(2k-1)$

◪ **다음을 계산하여라.**

32. $\sum_{k=1}^{10}\left\{\left(\frac{1}{2}\right)^k+\left(\frac{1}{3}\right)^k\right\}$

33. $\sum_{k=1}^{10}(4^k-k^2)$

34. $\sum_{k=1}^{10}(k+3^k)$

35. $\sum_{k=2}^{7}3\cdot 2^k$

36. $\sum_{k=1}^{10}\left(\frac{2}{3}\right)^k$

37. $\sum_{k=1}^{10}2^k$

38. $\sum_{k=3}^{12}(2^{k-2}-1)$

◪ 다음 합을 구하여라.

39. $\sum_{k=1}^{n}(3k+1)$

40. $\sum_{k=1}^{n}(2k-1)^2$

41. $\sum_{k=1}^{n}(k+1)$

42. $\sum_{k=1}^{n}(k+1)^2$

43. $\sum_{k=1}^{n}k^2(k+1)$

44. $\sum_{k=1}^{n}2^k$

45. $\sum_{k=2}^{n-1}(3^k+3k)$

46. $\sum_{k=1}^{n}(2k+3\times 2^{k-1})$

47. $\sum_{k=1}^{n}(k^2+3k+2)$

48. $\sum_{k=1}^{n}k(k^2+1)$

◪ 다음 수열의 첫째항부터 제10항까지의 합을 구하여라.

49. $1,\ 1+3,\ 1+3+3^2,\ 1+3+3^2+3^3,\ \cdots$

50. $1,\ 1+2,\ 1+2+3,\ 1+2+3+4,\ \cdots$

51. $1\cdot 14,\ 2\cdot 13,\ 3\cdot 12,\ \cdots$

52. $1\cdot 11,\ 2\cdot 10,\ 3\cdot 9,\ 4\cdot 8,\ \cdots$

53. $2\cdot4,\ 4\cdot5,\ 6\cdot6,\ 8\cdot7,\ \cdots$

54. $2\cdot3,\ 3\cdot4,\ 4\cdot5,\ 5\cdot6,\ \cdots$

55. $1,\ 1+2,\ 1+2+2^2,\ 1+2+2^2+2^3,\ \cdots$

56. $2,\ 22,\ 222,\ 2222,\ \cdots$

57. $9,\ 99,\ 999,\ 9999,\ \cdots$

◪ **다음 수열의 합을 구하여라.**

58. $1+2+3+\cdots+20$

59. $1^2+2^2+3^2+\ \cdots+10^2$

60. $5^2+6^2+7^2+\cdots+15^2$

61. $3^3+4^3+5^3+\cdots+15^3$

62. $1^3+2^3+3^3+\cdots+9^3$

63. $1\cdot3+3\cdot4+5\cdot5+\cdots+19\cdot12$

64. $1\cdot2+3\cdot4+5\cdot6+7\cdot8+\cdots+19\cdot20$

65. $1\cdot3+2\cdot5+3\cdot7+\cdots+10\cdot21$

◪ 다음 수열의 첫째항부터 제n항까지의 합 S_n을 자연수의 거듭제곱의 합을 이용하여 구하여라.

66. $2\times1,\ 4\times4,\ 6\times7,\ 8\times10,\ \cdots$

67. $1^2\cdot2,\ 2^2\cdot3,\ 3^2\cdot4,\ \cdots$

68. $1\times2,\ 2\times3,\ 3\times4,\ \cdots$

69. $1,\ 1+2,\ 1+2+3,\ 1+2+3+4,\ \cdots$

◪ 다음 물음에 답하여라.

70. 등식 $1^2+2^2+3^2+\cdots+n^2=140$을 만족시키는 자연수 n의 값을 구하여라.

71. 수열 $2-1,\ 2^2-3,\ 2^3-5,\ 2^4-7,\ \cdots$의 첫째항부터 제$9$항까지의 합을 구하여라.

72. 다음 수열의 첫째항부터 제15항까지의 합이 $\frac{1}{4}\left\{p+\left(\frac{1}{3}\right)^q\right\}$일 때, $p+q$의 값을 구하여라.

$1,\ 1+\frac{1}{3},\ 1+\frac{1}{3}+\frac{1}{9},\ 1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27},\ \cdots$

73. $\sum_{k=1}^{n}a_k=n^2-2n+2$일 때, $\sum_{k=1}^{10}a_{2k-1}$의 값을 구하여라.

02 $\sum$를 여러 개 포함한 식

$\sum$를 여러개 포함한 식은 상수인 것과 상수가 아닌 것을 구별하여 계산한다.

74. $\sum_{p=1}^{5}\left\{\sum_{q=1}^{9}(p+q)\right\}$

75. $\sum_{i=1}^{10}\left\{\sum_{j=1}^{10}(i+j^2+1)\right\}$

76. $\sum_{j=1}^{10}\left\{\sum_{i=1}^{10}(2i+j)\right\}$

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77. $\sum_{i=1}^{10}\left\{\sum_{k=1}^{i}(2k+4)\right\}$

78. $\sum_{i=1}^{10}\left\{\sum_{j=1}^{6}(i^2-2ij+2)\right\}$

79. $\sum_{l=1}^{10}\left\{\sum_{k=1}^{l}(2k+1)\right\}$

80. $\sum_{k=1}^{7}(1+2+3+\cdots+k)$

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정답 및 해설

1) 3025

⇨ $\sum_{k=1}^{10} k^3 = \left\{\frac{10(10+1)}{2}\right\}^2 = 3025$

2) 385

⇨ $\sum_{k=1}^{10} k^2 = \frac{10(10+1)(20+1)}{6} = 385$

3) 171

⇨ $\sum_{k=2}^{10}(3k+1) = \sum_{k=1}^{10}(3k+1) - \sum_{k=1}^{1}(3k+1)$

$= \left(3\times\frac{10\times 11}{2} + 1\times 10\right) - 4$

$= 175 - 4 = 171$

4) 64

⇨ $\sum_{k=3}^{10}(2k-5)$

$= \sum_{k=1}^{10}(2k-5) - \sum_{k=1}^{2}(2k-5)$

$= \left(2\times\frac{10\times 11}{2} - 5\times 10\right) - \left(2\times\frac{2\times 3}{2} - 5\times 2\right)$

$= 60 - (-4) = 64$

5) 1295

⇨ $\sum_{k=1}^{10}(k-2)^3$

$= \sum_{k=1}^{10}(k^3 - 6k^2 + 12k - 8)$

$= \sum_{k=1}^{10}k^3 - 6\sum_{k=1}^{10}k^2 + 12\sum_{k=1}^{10}k - \sum_{k=1}^{10}8$

$= \left(\frac{10\times 11}{2}\right)^2 - 6\times\frac{10\times 11\times 21}{6} + 12\times\frac{10\times 11}{2} - 80$

$= 3025 - 2310 + 660 - 80 = 1295$

6) 240

⇨ $\sum_{k=1}^{10}(4k+2) = 4\sum_{k=1}^{10}k + \sum_{k=1}^{10}2$

$= 4\cdot\frac{10\cdot 11}{2} + 2\cdot 10 = 240$

7) 245

⇨ $\sum_{k=1}^{10}(k+1)(k-3)$

$= \sum_{k=1}^{10}(k^2 - 2k - 3) = \sum_{k=1}^{10}k^2 - 2\sum_{k=1}^{10}k - \sum_{k=1}^{10}3$

$= \frac{10\times 11\times 21}{6} - 2\times\frac{10\times 11}{2} - 30$

$= 385 - 110 - 30 = 245$

8) 420

⇨ $\sum_{k=4}^{10}k(k+1)$

$= \sum_{k=1}^{10}k(k+1) - \sum_{k=1}^{3}k(k+1)$

$= \sum_{k=1}^{10}(k^2+k) - \sum_{k=1}^{3}(k^2+k)$

$= \sum_{k=1}^{10}k^2 + \sum_{k=1}^{10}k - \sum_{k=1}^{3}k^2 - \sum_{k=1}^{3}k$

$= \frac{10\times 11\times 21}{6} + \frac{10\times 11}{2} - \frac{3\times 4\times 7}{6} - \frac{3\times 4}{2}$

$= 385 + 55 - 14 - 6$

$= 420$

9) 117

⇨ $\sum_{k=2}^{10}(3k-5) = \sum_{k=1}^{10}(3k-5) - \sum_{k=1}^{1}(3k-5)$

$= \sum_{k=1}^{10}(3k-5) - (3\times 1 - 5)$

$= 3\sum_{k=1}^{10}k - \sum_{k=1}^{10}5 - (-2)$

$= 3\times\frac{10\times 11}{2} - 50 + 2$

$= 117$

10) 484

⇨ $\sum_{k=3}^{10}k(k+2) = \sum_{k=3}^{10}(k^2+2k)$

$= \sum_{k=1}^{10}(k^2+2k) - \sum_{k=1}^{2}(k^2+2k)$

$= \left(\frac{10\times 11\times 21}{6} + 2\times\frac{10\times 11}{2}\right) - \left(\frac{2\times 3\times 5}{6} + 2\times\frac{2\times 3}{2}\right)$

$= 495 - 11 = 484$

11) 2685

⇨ $\sum_{k=1}^{10}(k^3 - k^2 + k - 1)$

$= \sum_{k=1}^{10}k^3 - \sum_{k=1}^{10}k^2 + \sum_{k=1}^{10}k - \sum_{k=1}^{10}1$

$= \left(\frac{10\times 11}{2}\right)^2 - \frac{10\times 11\times 21}{6} + \frac{10\times 11}{2} - 10$

$= 3025 - 385 + 55 - 10 = 2685$

12) 420

⇨ $\sum_{k=1}^{10}(k-1)(k+2) = \sum_{k=1}^{10}(k^2+k-2)$

$= \sum_{k=1}^{10}k^2 + \sum_{k=1}^{10}k - \sum_{k=1}^{10}2$

$= \frac{10\times 11\times 21}{6} + \frac{10\times 11}{2} - 20$

$= 385 + 55 - 20 = 420$

13) 210

$\Rightarrow \sum_{k=1}^{5} k(k-1)(k+1)$

$= \sum_{k=1}^{5}(k^3-k)$

$= \left(\frac{5 \cdot 6}{2}\right)^2 - \frac{5 \cdot 6}{2} = 225-15=210$

14) 230

$\Rightarrow \sum_{k=1}^{10}(k^2-3k+1) = \sum_{k=1}^{10}k^2 - 3\sum_{k=1}^{10}k + \sum_{k=1}^{10}1$

$= \frac{10\times11\times21}{6} - 3\times\frac{10\times11}{2} + 10$

$= 385-165+10=230$

15) 1260

$\Rightarrow \sum_{k=1}^{8}k(k+1)(k-1) = \sum_{k=1}^{8}(k^3-k)$

$= \sum_{k=1}^{8}k^3 - \sum_{k=1}^{8}k$

$= \left(\frac{8\cdot9}{2}\right)^2 - \frac{8\cdot9}{2} = 1260$

16) 1485

$\Rightarrow \sum_{k=6}^{10}(2k+1)^2 = \sum_{k=6}^{10}(4k^2+4k+1)$

$= \sum_{k=1}^{10}(4k^2+4k+1) - \sum_{k=1}^{5}(4k^2+4k+1)$

$= \left(4\times\frac{10\times11\times21}{6} + 4\times\frac{10\times11}{2} + 10\right)$

$-\left(4\times\frac{5\times6\times11}{6} + 4\times\frac{5\times6}{2} + 5\right)$

$= 1770-285=1485$

17) 615

$\Rightarrow \sum_{k=1}^{10}(2k^2-3k+1) = 2\sum_{k=1}^{10}k^2 - 3\sum_{k=1}^{10}k + \sum_{k=1}^{10}1$

$= 2\cdot\frac{10\cdot11\cdot21}{6} - 3\cdot\frac{10\cdot11}{2} + 1\cdot10$

$= 770-165+10$

$= 615$

18) 322

$\Rightarrow \sum_{k=4}^{10}(k^2-k)$

$= \sum_{k=1}^{10}(k^2-k) - \sum_{k=1}^{3}(k^2-k)$

$= \left(\frac{10\times11\times21}{6} - \frac{10\times11}{2}\right) - \left(\frac{3\times4\times7}{6} - \frac{3\times4}{2}\right)$

$= 330-8=322$

19) 756

$\Rightarrow \sum_{k=1}^{8}k(k-1)(k-2)$

$= \sum_{k=1}^{8}(k^3-3k^2+2k) = \sum_{k=1}^{8}k^3 - 3\sum_{k=1}^{8}k^2 + 2\sum_{k=1}^{8}k$

$= \left\{\frac{8(8+1)}{2}\right\}^2 - 3\times\frac{8\times(8+1)\times(2\times8+1)}{6} + 2\times\frac{8(8+1)}{2}$

$= 36^2 - 3\times204+72$

$= 1296-612+72=756$

20) 2445

$\Rightarrow \sum_{k=1}^{10}(k^3-2k^2+4k-3)$

$= \sum_{k=1}^{10}k^3 - 2\sum_{k=1}^{10}k^2 + 4\sum_{k=1}^{10}k - \sum_{k=1}^{10}3$

$= \left\{\frac{10(10+1)}{2}\right\}^2 - 2\times\frac{10(10+1)(20+1)}{6}$

$+4\times\frac{10(10+1)}{2} - 3\cdot10$

$= 3025-770+220-30$

$= 2445$

21) 160

$\Rightarrow$ (준식) $= \sum_{k=1}^{10}(k^2+2k+4-k^2+1) = \sum_{k=1}^{10}(2k+5)$

$= 2\cdot\frac{10\cdot11}{2} + 5\cdot10 = 160$

22) 330

$\Rightarrow \sum_{k=1}^{10}(k+5)(k-2) - \sum_{k=1}^{10}(k-5)(k+2)$

$= \sum_{k=1}^{10}\{(k+5)(k-2)-(k-5)(k+2)\}$

$= \sum_{k=1}^{10}(k^2+3k-10-k^2+3k+10)$

$= \sum_{k=1}^{10}6k$

$= 6\times\frac{10\cdot11}{2}$

$= 330$

23) 440

$\Rightarrow \sum_{k=1}^{10}k(k+1) = \sum_{k=1}^{10}k^2 + \sum_{k=1}^{10}k$

$= \frac{10\times11\times21}{6} + \frac{10\times11}{2}$

$= 385+55=440$

24) 328

$\Rightarrow$ (준식) $= \sum_{k=1}^{8}(k^2+3k+2)$

$= \frac{8\cdot9\cdot17}{6} + 3\cdot\frac{8\cdot9}{2} + 2\cdot8 = 204+108+16=328$

25) 1330

$\Rightarrow \displaystyle\sum_{k=1}^{10}(k+1)^3-\sum_{k=1}^{11}(k-1)^3$

$=\displaystyle\sum_{k=1}^{10}(k+1)^3-\left\{\sum_{k=1}^{10}(k-1)^3+(11-1)^3\right\}$

$=\displaystyle\sum_{k=1}^{10}\{(k+1)^3-(k-1)^3\}-10^3$

$=\displaystyle\sum_{k=1}^{10}(6k^2+2)-10^3$

$=6\displaystyle\sum_{k=1}^{10}k^2+\sum_{k=1}^{10}2-10^3$

$=6\times\dfrac{10\times11\times21}{6}+2\times10-1000$

$=1330$

26) 350

$\Rightarrow \displaystyle\sum_{k=1}^{10}(k^2+1)-\sum_{k=1}^{10}(k-1)=\sum_{k=1}^{10}(k^2-k+2)$

$=\dfrac{10\cdot11\cdot21}{6}-\dfrac{10\cdot11}{2}+2\cdot10=350$

27) 2870

$\Rightarrow \displaystyle\sum_{k=1}^{20}k^2=\dfrac{20\cdot21\cdot41}{6}=2870$

28) 140

$\Rightarrow \displaystyle\sum_{k=1}^{8}2k(k-1)-\sum_{i=1}^{8}(i^2-1)$

$=\displaystyle\sum_{k=1}^{8}(2k^2-2k)-\sum_{k=1}^{8}(k^2-1)$

$=\displaystyle\sum_{k=1}^{8}(2k^2-2k-k^2+1)$

$=\displaystyle\sum_{k=1}^{8}(k^2-2k+1)$

$=\dfrac{8\cdot9\cdot17}{6}-2\times\dfrac{8\cdot9}{2}+8$

$=140$

29) 970

$\Rightarrow \displaystyle\sum_{k=1}^{10}(2k-3)^2=\sum_{k=1}^{10}(4k^2-12k+9)$

$=4\displaystyle\sum_{k=1}^{10}k^2-12\sum_{k=1}^{10}k+\sum_{k=1}^{10}9$

$=4\times\dfrac{10\times11\times21}{6}-12\times\dfrac{10\times11}{2}+9\times10$

$=970$

30) 465

$\Rightarrow \displaystyle\sum_{k=1}^{10}(k^2+2k-3)$

$=\displaystyle\sum_{k=1}^{10}k^2+2\sum_{k=1}^{10}k-\sum_{k=1}^{10}3$

$=\dfrac{10(10+1)(20+1)}{6}+2\times\dfrac{10(10+1)}{2}-3\cdot10$

$=385+110-30=465$

31) 100

$\Rightarrow \displaystyle\sum_{k=1}^{10}(2k-1)=2\sum_{k=1}^{10}k-\sum_{k=1}^{10}1$

$=2\times\dfrac{10(10+1)}{2}-10=100$

32) $\dfrac{3}{2}-\dfrac{1}{2^{10}}-\dfrac{1}{2\cdot3^{10}}$

$\Rightarrow \displaystyle\sum_{k=1}^{10}\left\{\left(\frac{1}{2}\right)^k+\left(\frac{1}{3}\right)^k\right\}$

$=\dfrac{\frac{1}{2}\left\{1-\left(\frac{1}{2}\right)^{10}\right\}}{1-\frac{1}{2}}+\dfrac{\frac{1}{3}\left\{1-\left(\frac{1}{3}\right)^{10}\right\}}{1-\frac{1}{3}}$

$=1-\dfrac{1}{2^{10}}+\dfrac{1}{2}\left(1-\dfrac{1}{3^{10}}\right)$

$=\dfrac{3}{2}-\dfrac{1}{2^{10}}-\dfrac{1}{2\cdot3^{10}}$

33) $\dfrac{4^{11}-1159}{3}$

$\Rightarrow \displaystyle\sum_{k=1}^{10}(4^k-k^2)=\sum_{k=1}^{10}4^k-\sum_{k=1}^{10}k^2$

$=\dfrac{4(4^{10}-1)}{4-1}-\dfrac{10\times11\times21}{6}$

$=\dfrac{4^{11}-1159}{3}$

34) $\dfrac{3^{11}+107}{2}$

$\Rightarrow \displaystyle\sum_{k=1}^{10}(k+3^k)=\sum_{k=1}^{10}k+\sum_{k=1}^{10}3^k$

$=\dfrac{10\times11}{2}+\dfrac{3(3^{10}-1)}{3-1}$

$=\dfrac{3^{11}+107}{2}$

35) 756

$\Rightarrow \displaystyle\sum_{k=2}^{7}3\cdot2^k=\dfrac{12(2^6-1)}{2-1}=756$

36) $2\left\{1-\left(\dfrac{2}{3}\right)^{10}\right\}$

$\Rightarrow \displaystyle\sum_{k=1}^{10}\left(\frac{2}{3}\right)^k=\dfrac{\frac{2}{3}\left\{1-\left(\frac{2}{3}\right)^{10}\right\}}{1-\frac{2}{3}}=2\left\{1-\left(\dfrac{2}{3}\right)^{10}\right\}$

37) 2046

$\Rightarrow \displaystyle\sum_{k=1}^{10}2^k=2+2^2+2^3+\cdots+2^{10}=\dfrac{2(2^{10}-1)}{2-1}=2046$

38) 2036

⇨ $\sum_{k=3}^{12}(2^{k-2}-1)=\sum_{n=1}^{10}(2^n-1)=\frac{2(2^{10}-1)}{2-1}-10$

$=2^{11}-12=2036$

39) $\frac{3n^2+5n}{2}$

⇨ $\sum_{k=1}^{n}(3k+1)=3\sum_{k=1}^{n}k+\sum_{k=1}^{n}1$

$=3\times\frac{n(n+1)}{2}+n=\frac{3n^2+5n}{2}$

40) $\frac{n(2n+1)(2n-1)}{3}$

⇨ $\sum_{k=1}^{n}(2k-1)^2$

$=\sum_{k=1}^{n}(4k^2-4k+1)=4\sum_{k=1}^{n}k^2-4\sum_{k=1}^{n}k+\sum_{k=1}^{n}1$

$=4\times\frac{n(n+1)(2n+1)}{6}-4\times\frac{n(n+1)}{2}+n$

$=\frac{n}{6}\{4(n+1)(2n+1)-12(n+1)+6\}$

$=\frac{n}{6}(8n^2-2)$

$=\frac{n(4n^2-1)}{3}$

$=\frac{n(2n+1)(2n-1)}{3}$

41) $\frac{1}{2}n(n+3)$

⇨ $\sum_{k=1}^{n}(k+1)=\sum_{k=1}^{n}k+\sum_{k=1}^{n}1$

$=\frac{n(n+1)}{2}+n$

$=\frac{n}{2}(n+1+2)$

$=\frac{1}{2}n(n+3)$

42) $\frac{n(2n^2+9n+13)}{6}$

⇨ $\sum_{k=1}^{n}(k+1)^2=\sum_{k=1}^{n}(k^2+2k+1)$

$=\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k+\sum_{k=1}^{n}1$

$=\frac{n(n+1)(2n+1)}{6}+n(n+1)+n$

$=\frac{1}{6}n\{(n+1)(2n+1)+6(n+1)+6\}$

$=\frac{n(2n^2+9n+13)}{6}$

43) $\frac{n(n+1)(n+2)(3n+1)}{12}$

⇨ $\sum_{k=1}^{n}k^2(k+1)$

$=\sum_{k=1}^{n}(k^3+k^2)=\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}k^2$

$=\left\{\frac{n(n+1)}{2}\right\}^2+\frac{n(n+1)(2n+1)}{6}$

$=\frac{n(n+1)}{12}\{3n(n+1)+2(2n+1)\}$

$=\frac{n(n+1)(3n^2+7n+2)}{12}$

$=\frac{n(n+1)(n+2)(3n+1)}{12}$

44) $2^{n+1}-2$

⇨ 첫째항이 2, 공비가 2인 등비수열의 합이므로

$\sum_{k=1}^{n}2^k=\frac{2(2^n-1)}{2-1}=2^{n+1}-2$

45) $\frac{3}{2}(3^{n-1}+n^2-n-5)$

⇨ $\sum_{k=2}^{n-1}(3^k+3k)=\sum_{k=1}^{n-1}(3^k+3k)-\sum_{k=1}^{1}(3^k+3k)$

$=\sum_{k=1}^{n-1}3^k+3\sum_{k=1}^{n-1}k-(3^1+3\times1)$

$=\frac{3(3^{n-1}-1)}{3-1}+\frac{3(n-1)n}{2}-6$

$=\frac{3}{2}(3^{n-1}+n^2-n-5)$

46) $n^2+n+3\times2^n-3$

⇨ $\sum_{k=1}^{n}(2k+3\times2^{k-1})=2\sum_{k=1}^{n}k+\sum_{k=1}^{n}3\times2^{k-1}$

$=2\times\frac{n(n+1)}{2}+\frac{3(2^n-1)}{2-1}$

$=n^2+n+3\times2^n-3$

47) $\frac{n(n^2+6n+11)}{3}$

⇨ $\sum_{k=1}^{n}(k^2+3k+2)=\sum_{k=1}^{n}k^2+3\sum_{k=1}^{n}k+\sum_{k=1}^{n}2$

$=\frac{n(n+1)(2n+1)}{6}+3\times\frac{n(n+1)}{2}+2n$

$=\frac{n}{6}\{(n+1)(2n+1)+9(n+1)+12\}$

$=\frac{n}{6}(2n^2+12n+22)$

$=\frac{n(n^2+6n+11)}{3}$

48) $\frac{n(n+1)(n^2+n+2)}{4}$

⇨ $\sum_{k=1}^{n}k(k^2+1)=\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}k$

$=\left\{\frac{n(n+1)}{2}\right\}^2+\frac{n(n+1)}{2}$

$=\frac{n(n+1)}{2}\left\{\frac{n(n+1)}{2}+1\right\}$

$=\frac{n(n+1)}{2}\times\frac{n^2+n+2}{2}$

$=\frac{n(n+1)(n^2+n+2)}{4}$

49) $\frac{3^{11}-23}{4}$

⇨ $a_n=1+3+3^2+3^3+\cdots+3^{n-1}$

$=\frac{1\cdot(3^n-1)}{3-1}=\frac{1}{2}(3^n-1)$

$\therefore\ \sum_{k=1}^{10}a_k=\sum_{k=1}^{10}\frac{1}{2}(3^k-1)$

$=\frac{1}{2}\left\{\frac{3(3^{10}-1)}{3-1}-10\right\}=\frac{3^{11}-23}{4}$

50) 220

⇨ 일반항이 $1+2+3+\cdots+n=\frac{n(n+1)}{2}$이므로

$\sum_{k=1}^{10}\frac{k(k+1)}{2}=\frac{1}{2}\sum_{k=1}^{10}(k^2+k)$

$=\frac{1}{2}\left(\frac{10\times11\times21}{6}+\frac{10\times11}{2}\right)$

$=\frac{1}{2}(385+55)=220$

51) 440

⇨ $1\cdot14+2\cdot13+3\cdot12+\cdots+14\cdot1=\sum_{k=1}^{14}k(15-k)$

$=\sum_{k=1}^{15}(15k-k^2)=15\cdot\frac{14\cdot15}{2}-\frac{14\cdot15\cdot29}{6}=560$

52) 275

⇨ 일반항이 $n(12-n)$이므로

$\sum_{k=1}^{10}k(12-k)=\sum_{k=1}^{10}(12k-k^2)$

$=12\times\frac{10\times11}{2}-\frac{10\times11\times21}{6}$

$=660-385=275$

53) 1100

⇨ 일반항이 $2n(n+3)$이므로

$\sum_{k=1}^{10}2k(k+3)=\sum_{k=1}^{10}(2k^2+6k)$

$=2\times\frac{10\times11\times21}{6}+6\times\frac{10\times11}{2}$

$=770+330=1100$

54) 570

⇨ 일반항이 $(n+1)(n+2)$이므로

$\sum_{k=1}^{10}(k+1)(k+2)=\sum_{k=1}^{10}(k^2+3k+2)$

$=\frac{10\times11\times21}{6}+3\times\frac{10\times11}{2}+2\times10$

$=385+165+20=570$

55) $2^{11}-12$

⇨ $a_n=1+2+2^2+2^3+\cdots+2^{n-1}$

$=\frac{1\cdot(2^n-1)}{2-1}=2^n-1$

$\sum_{k=1}^{10}a_k=\sum_{k=1}^{10}(2^k-1)$

$=\frac{2(2^{10}-1)}{2-1}-10=2^{11}-12$

56) $\frac{2(10^{11}-100)}{81}$

⇨ $2=\frac{2}{9}\times9=\frac{2}{9}(10-1)$,

$22=\frac{2}{9}\times99=\frac{2}{9}(10^2-1)$,

$222=\frac{2}{9}\times999=\frac{2}{9}(10^3-1)$, $\cdots$이므로

$a_n=\frac{2}{9}(10^n-1)$

$\sum_{k=1}^{10}a_k=\sum_{k=1}^{10}\frac{2}{9}(10^k-1)$

$=\frac{2}{9}\left\{\frac{10(10^{10}-1)}{10-1}-10\right\}$

$=\frac{2}{9}\times\frac{10^{11}-100}{9}=\frac{2(10^{11}-100)}{81}$

57) $\frac{10^{11}-100}{9}$

⇨ $9=10-1$, $99=10^2-1$, $999=10^3-1$, $\cdots$이므로

$a_n=10^n-1$

$\sum_{k=1}^{10}(10^k-1)=\frac{10(10^{10}-1)}{10-1}-10=\frac{10^{11}-100}{9}$

58) 210

⇨ $1+2+3+\cdots+20=\sum_{k=1}^{20}k=\frac{20\cdot21}{2}=210$

59) 385

⇨ (준식)$=\sum_{k=1}^{10}k^2=\frac{10\cdot11\cdot21}{6}=385$

60) 1210

⇨ 주어진 수열의 일반항은 a_n은 $a_n=(n+4)^2$이므로

$(n+4)^2=15^2$에서 $n+4=15$ $\quad\therefore\ n=11$

$\therefore$ (주어진 식)$=\sum_{k=1}^{11}(k+4)^2$

$=\sum_{k=1}^{11}(k^2+8k+16)$

$=\sum_{k=1}^{11}k^2+8\sum_{k=1}^{11}k+\sum_{k=1}^{11}16$

$=\frac{11\cdot12\cdot23}{6}+8\cdot\frac{11\cdot12}{2}+16\cdot11$

$=506+528+176=1210$

61) 14391

⇨ $3^3+4^3+5^3+\cdots+15^3=\sum_{k=3}^{15}k^3=\sum_{k=1}^{15}k^3-\sum_{k=1}^{2}k^3$

$=\left(\frac{15\times16}{2}\right)^2-\left(\frac{2\times3}{2}\right)^2=14391$

62) 2025

⇨ 주어진 수열의 일반항은 a_n은 n^3

$\therefore$ (주어진 식)$=\sum_{k=1}^{9}k^3=\left(\frac{9\cdot10}{2}\right)^2=45^2=2025$

63) 915

⇨ $1\cdot3+3\cdot4+5\cdot5+\cdots+19\cdot12$

$=\sum_{k=1}^{10}(2k-1)(k+2)$

$=\sum_{k=1}^{10}(2k^2+3k-2)$

$=2\cdot\frac{10\cdot11\cdot21}{6}+3\cdot\frac{10\cdot11}{2}-2\cdot10$

$=770+165-20=915$

64) 1430

⇨ $1\cdot2+3\cdot4+5\cdot6+7\cdot8+\cdots+19\cdot20$

$=\sum_{k=1}^{10}(2k-1)\times2k=\sum_{k=1}^{10}(4k^2-2k)$

$=4\times\frac{10\times11\times21}{6}-2\times\frac{10\times11}{2}$

$=1540-110=1430$

65) 825

⇨ $1\cdot3+2\cdot5+3\cdot7+\cdots+10\cdot21$

$=\sum_{k=1}^{10}k(2k+1)$

$=\sum_{k=1}^{10}(2k^2+k)=2\cdot\frac{10\cdot11\cdot21}{6}+\frac{10\cdot11}{2}=825$

66) $n(n+1)(2n-1)$

⇨ 2, 4, 6, 8, $\cdots$, $2n$이라고 하면

1, 4, 7, 10, $\cdots$, $1+3(n-1)=3n-2$이므로

$a_n=2n(3n-2)=6n^2-4n$

$\therefore\ S_n=\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}(6k^2-4k)=6\sum_{k=1}^{n}k^2-4\sum_{k=1}^{n}k$

$=6\times\frac{n(n+1)(2n+1)}{6}-4\times\frac{n(n+1)}{2}$

$=n(n+1)(2n+1-2)=n(n+1)(2n-1)$

67) $\frac{n(n+1)(n+2)(3n+1)}{12}$

⇨ $\sum_{k=1}^{n}k^2(k+1)=\sum_{k=1}^{n}(k^3+k^2)$

$=\left\{\frac{n(n+1)}{2}\right\}^2+\frac{n(n+1)(2n+1)}{6}$

$=\frac{n(n+1)}{12}\{3n(n+1)+2(2n+1)\}$

$=\frac{n(n+1)(3n^2+7n+2)}{12}$

$=\frac{n(n+1)(n+2)(3n+1)}{12}$

68) $\frac{1}{3}n(n+1)(n+2)$

⇨ 1, 2, 3, $\cdots$, n이라고 하면 2, 3, 4, $\cdots$, $n+1$이므로 $a_n=n(n+1)$

$\therefore\ S_n=\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}k(k+1)=\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}k$

$=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

$=\frac{n(n+1)(2n+1+3)}{6}$

$=\frac{1}{3}n(n+1)(n+2)$

69) $\frac{1}{6}n(n+1)(n+2)$

⇨ $a_1=1$, $a_2=1+2$, $a_3=1+2+3$, $\cdots$이라고 하면

$a_n=1+2+3+\cdots+n=\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$

$\therefore\ S_n=\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}\frac{k(k+1)}{2}=\frac{1}{2}\left(\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}k\right)$

$=\frac{1}{2}\left\{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right\}$

$=\frac{1}{2}\times\frac{1}{6}n(n+1)(2n+1+3)=\frac{1}{6}n(n+1)(n+2)$

70) 7

⇨ $1^2+2^2+3^2+\cdots+n^2=\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$이므로

$\frac{n(n+1)(2n+1)}{6}=140$, $n(n+1)(2n+1)=840$

이때, $840=7\cdot8\cdot15$이므로 $n=7$

71) 941

⇨ 수열 $2-1$, 2^2-3, 2^3-5, 2^4-7, $\cdots$ 의 일반항을 a_n이라 하면 수열 2, 2^2, 2^3, 2^4, $\cdots$ 은 첫째항이 2, 공비가 2인 등비수열이므로 일반항은 2^n이다.

수열 1, 3, 5, 7, $\cdots$ 은 첫째항이 1, 공차가 2인 등차수열이므로 일반항은 $2n-1$ 이다.

$\therefore a_n = 2^n - (2k-1)$

$\therefore \sum_{k=1}^{9}(2^k - 2k + 1) = \sum_{k=1}^{9}2^k - 2\sum_{k=1}^{9}k + \sum_{k=1}^{9}1$

$= \dfrac{2(2^9-1)}{2-1} - 2\times\dfrac{9 \cdot 10}{2} + 9$

$= 1024 - 2 - 90 + 9$

$= 941$

72) 101

⇨ 일반항을 a_n이라 하면

$a_n = 1 + \dfrac{1}{3} + \dfrac{1}{9} + \cdots + (\dfrac{1}{3})^{n-1}$

$= \dfrac{1-(\frac{1}{3})^n}{1-\frac{1}{3}} = \dfrac{3}{2}(1-(\dfrac{1}{3})^n)$

$\therefore \sum_{n=1}^{15}\dfrac{3}{2}(1-(\dfrac{1}{3})^n) = \dfrac{3}{2}\left\{15 - \dfrac{\frac{1}{3}(1-(\frac{1}{3})^{15})}{1-\frac{1}{3}}\right\}$

$= \dfrac{3}{2}\left\{15 - \dfrac{1}{2}(1-(\dfrac{1}{3})^{15})\right\} = \dfrac{3}{4}\left\{30 - 1 + (\dfrac{1}{3})^{15}\right\}$

$= \dfrac{1}{4}\left\{87 + (\dfrac{1}{3})^{14}\right\}$

$\therefore p + q = 87 + 14 = 101$

73) 172

⇨ $S_n = \sum_{k=1}^{n}a_k = n^2 - 2n + 2$ 이므로

$a_1 = S_1 = 1 - 2 + 2 = 1$

$a_n = S_n - S_{n-1} \quad (n \geq 2)$

$= n^2 - 2n + 2 - \{(n-1)^2 - 2(n-1) + 2\}$

$= 2n - 3$

$\therefore a_1 = 1 \quad , \quad a_{2k-1} = 4k - 5 \ (k \geq 2)$

$\therefore \sum_{k=1}^{10}a_{2k-1} = a_1 + \sum_{k=2}^{10}a_{2k-1}$

$= 1 + \sum_{k=2}^{10}(4k-5) = 1 + \sum_{k=1}^{10}(4k-5) + 1$

$= 2 + 4\times\dfrac{10 \cdot 11}{2} - 50$

$= 2 + 220 - 50$

$= 172$

74) 360

⇨ $\sum_{p=1}^{5}\left\{\sum_{q=1}^{9}(p+q)\right\} = \sum_{p=1}^{5}(9p+45)$

$= 9\times\left(\dfrac{5\times 6}{2} + 25\right) = 360$

75) 4500

⇨ $\sum_{i=1}^{10}\left\{\sum_{j=1}^{10}(i + j^2 + 1)\right\} = \sum_{i=1}^{10}\left\{10(i+1) + \dfrac{10 \cdot 11 \cdot 21}{6}\right\}$

$= \sum_{i=1}^{10}(10i + 395) = 10 \cdot \dfrac{10 \cdot 11}{2} + 395 \cdot 10 = 4500$

76) 1650

⇨ $\sum_{j=1}^{10}\left\{\sum_{i=1}^{10}(2i+j)\right\}$

$= \sum_{j=1}^{10}\left(2\times\dfrac{10 \cdot 11}{2} + 10j\right)$

$= \sum_{j=1}^{10}(110 + 10j)$

$= 1100 + 10\times\dfrac{10 \cdot 11}{2}$

$= 1650$

77) 660

⇨ $\sum_{i=1}^{10}\left\{\sum_{k=1}^{i}(2k+4)\right\} = \sum_{i=1}^{10}\{i(i+1) + 4i\} = \sum_{i=1}^{10}(i^2 + 5i)$

$= \dfrac{10\times 11\times 21}{6} + 5\times\dfrac{10\times 11}{2} = 60\times 11 = 660$

78) 120

⇨ $\sum_{i=1}^{10}\left\{\sum_{j=1}^{6}(i^2 - 2ij + 2)\right\} = \sum_{i=1}^{10}\left\{6(i^2+2) - 2i \cdot \dfrac{6 \cdot 7}{2}\right\}$

$= \sum_{i=1}^{10}(6i^2 - 42i + 12)$

$= 6 \cdot \dfrac{10 \cdot 11 \cdot 21}{6} - 42 \cdot \dfrac{10 \cdot 11}{2} + 12 \cdot 10$

$= 2310 - 2310 + 120 = 120$

79) 495

⇨ $\sum_{l=1}^{10}\left\{\sum_{k=1}^{l}(2k+1)\right\} = \sum_{l=1}^{10}\left\{2\times\dfrac{l(l+1)}{2} + l\right\}$

$= \sum_{l=1}^{10}(l^2 + 2l)$

$= \dfrac{10 \cdot 11 \cdot 21}{6} + 2\times\dfrac{10 \cdot 11}{2} = 495$

80) 84

⇨ (준식) $= \sum_{k=1}^{7}(1 + 2 + 3 + \cdots + k) = \sum_{k=1}^{7}\dfrac{k(k+1)}{2}$

$= \dfrac{1}{2}\sum_{k=1}^{7}(k^2 + k) = \dfrac{1}{2}(\dfrac{7 \cdot 8 \cdot 15}{6} + \dfrac{7 \cdot 8}{2}) = 84$