01 유리식의 뜻과 사칙연산

(1) 유리식의 뜻과 성질

① 유리식 : 두 다항식 A, $B(B\neq 0)$에 대하여 $\frac{A}{B}$꼴로 나타낼 수 있는 식을 유리식이라 한다. 이때 B가 0이 아닌 상수이면 $\frac{A}{B}$는 다항식이므로 다항식도 유리식이다.

② 유리식의 성질

다항식 A, B, $C(B\neq 0,\ C\neq 0)$에 대하여

$\frac{A}{B}=\frac{A\times C}{B\times C}$ $\qquad$ $\frac{A}{B}=\frac{A\div C}{B\div C}$

(2) 유리식의 사칙연산

다항식 A, B, C, $D(C\neq 0,\ D\neq 0)$에 대하여

① $\frac{A}{C}\pm\frac{B}{C}=\frac{A\pm B}{C}$, $\frac{A}{C}+\frac{B}{D}=\frac{AD+BC}{CD}$

⇨ 분모를 통분한 후, 분자끼리 계산한다.

② $\frac{A}{C}\times\frac{B}{D}=\frac{AB}{CD}$

⇨ 분모는 분모끼리, 분자는 분자끼리 곱하여 계산한다.

③ $\frac{A}{C}\div\frac{B}{D}=\frac{A}{C}\times\frac{D}{B}=\frac{AD}{BC}$

⇨ 나누는 식의 분자와 분모를 바꾸어 곱하여 계산한다.

◪ 다음 유리식을 계산하여라.

1. $\frac{3}{x+1}-2$

2. $\frac{1}{x+1}+\frac{2}{x-1}$

3. $\frac{2}{x+2}+\frac{1}{x-2}$

4. $\frac{1}{x-1}+\frac{3}{x+2}$

5. $\frac{2x+3}{x-1}+\frac{5}{1-x}$

6. $\frac{x}{x-2}+\frac{x-1}{2-x}$

7. $\frac{x+2}{x-1}-\frac{x-1}{x+2}$

8. $\frac{x+2}{x+3}-\frac{x-2}{x-1}$

9. $\frac{x+2}{x-3}-\frac{x-3}{x-2}$

10. $\frac{2}{x+1}+\frac{x}{x^2-x+1}$

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11. $\frac{x+1}{x^2-4}+\frac{3}{x+2}$

12. $\frac{3x-1}{x^2-1}-\frac{x}{x^2-x}$

13. $\frac{x+3}{x^2-1}-\frac{x+4}{x^2-x-2}$

14. $\frac{x}{x-1}-\frac{4}{x^2+2x-3}$

15. $\frac{x}{x^2-2x-3}-\frac{x-1}{x^2+3x+2}$

◪ 다음 중 다항식인 것에는 '다항식' 분수식인 것에는 '분수식'을 써넣어라.

16. $\frac{1}{x}$ ( )

17. $\frac{1}{x+1}$ ( )

18. $1-\frac{x}{2}$ ( )

19. $\frac{2x+1}{3}$ ( )

20. $\frac{1}{1-4x}$ ( )

21. $\frac{3}{2x+1}$ ( )

22. $3x+\frac{1}{4}$ ( )

23. $\frac{x+1}{x}$ ( )

24. $\frac{3x-4}{2}$ ( )

25. $2x^2-\frac{x}{2}$ ( )

26. $\frac{4}{x^2-1}$ ( )

27. $y=\frac{3x-1}{x+1}$ ( )

28. $y=4-\frac{3x}{2}$ ( )

29. $y=\frac{2x^2-1}{3}$ ( )

30. $\dfrac{2x}{3x^2-1}$ ( )

31. $y=\dfrac{2x^3+2x-1}{x^2-1}$ ( )

◪ 분모가 0이 되지 않도록 하는 모든 실수 x에 대하여 다음 등식이 성립할 때, 상수 a, b의 값을 구하여라.

32. $\dfrac{1}{x(x+1)}=\dfrac{1}{x}+\dfrac{b}{x+a}$

33. $\dfrac{1}{(x+1)(x-2)}=\dfrac{a}{x+1}+\dfrac{b}{x-2}$

34. $\dfrac{a}{2x-1}+\dfrac{b}{x-2}=\dfrac{5x-4}{2x^2-5x+2}$

35. $\dfrac{1}{x-1}-\dfrac{2}{x+1}=\dfrac{ax+b}{x^2-1}$

36. $\dfrac{-6x}{x^3+1}=\dfrac{a}{x+1}+\dfrac{bx-2}{x^2-x+1}$

37. $\dfrac{6}{(x-1)(x-2)(x-3)}=\dfrac{a}{x-1}+\dfrac{b}{x-2}+\dfrac{3}{x-3}$

38. $\dfrac{a}{x}+\dfrac{bx+2}{x^2+1}=\dfrac{2x-1}{x^3+x}$

◪ 다음 유리식을 약분하여라.

39. $\dfrac{x^2-4x-5}{x+1}$

40. $\dfrac{x^3-1}{x^2+x+1}$

41. $\dfrac{x^2+x-6}{2x^2-3x-2}$

42. $\dfrac{12a^5b^2}{8a^3b^3}$

43. $\dfrac{6a^3x^3y}{4a^2xy^2}$

◪ 다음 유리식을 통분하여라.

44. $\dfrac{x-1}{x^2-2x}$, $\dfrac{2}{x-2}$

45. $\dfrac{1}{3a^3b}$, $\dfrac{1}{4ab^2}$

46. $\dfrac{c}{3ab^2x}$, $\dfrac{a}{2bcx^2}$

47. $\frac{2}{x^2-4x+3}, \frac{x+1}{x^2-x-6}$

48. $\frac{2x+3y}{x^2+xy-2y^2}, \frac{x-y}{2x^2+7xy+6y^2}$

◪ 다음 유리식을 계산하여라.

49. $\frac{2}{x+1}\times\frac{x+1}{x+2}$

50. $\frac{2}{x+1}\times\frac{x^2+x}{x+2}$

51. $\frac{x-1}{2x}\times\frac{x}{x^2-1}$

52. $\frac{x+1}{x}\times\frac{3x}{x^2-1}$

53. $\frac{x}{x^2-4}\times\frac{x+2}{x-2}$

54. $\frac{x+1}{x^2+2x}\times\frac{x+2}{x^2-1}$

55. $\frac{x^2-1}{x^2+2x}\times\frac{x+2}{x+1}$

56. $\frac{2x-4}{x^2-3x}\times\frac{x^2-9}{(x-2)^2}$

57. $\frac{x^3+3x}{x^2-16}\times\frac{x-4}{x^2+3}$

58. $\frac{3x+3}{x^2-3x+2}\times\frac{x^2-4}{(x+1)^2}$

59. $\frac{x^2+x-6}{x^2-4x-5}\times\frac{x^2-3x-10}{x^2+2x-3}$

60. $\frac{x^2-4}{(x+2)^2}\times\frac{x^2+3x+2}{x-1}+1$

◪ 다음 유리식을 계산하여라.

61. $\frac{x}{x-1}\div\frac{x+1}{x-1}$

62. $\frac{x-2}{x+1}\div\frac{x-2}{x+3}$

63. $\frac{x}{x^2-1}\div\frac{x+1}{x-1}$

64. $\frac{6a^3b}{x^3y^3}\div\frac{3a^2b}{x^2y}$

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65. $\dfrac{x^2-2x}{x+1} \div \dfrac{x^2-4}{x^2-1}$

66. $\dfrac{x^2-3x+2}{x-3} \div \dfrac{x-1}{x-3}$

67. $\dfrac{x^2-4x+3}{x^2-4} \div \dfrac{x-3}{x-2}$

68. $\dfrac{x^2-2x+1}{x-2} \div \dfrac{x-1}{x-2}$

02 특수한 형태의 유리식의 계산

(1) 부분분수로의 변형

분모가 두 개 이상의 인수의 곱으로 되어 있으면 한 개의 유리식을 두 개 이상의 유리식으로 나누어 계산한다.

⇨ $\dfrac{C}{AB}=\dfrac{C}{B-A}\left(\dfrac{1}{A}-\dfrac{1}{B}\right)$ (단, $A \neq B$)

(2) 번분수 꼴

분모($\neq 0$) 또는 분자가 유리식으로 되어 있으면 주어진 식의 형태에 따라 다음과 같이 계산한다.

⇨ $\dfrac{A}{\dfrac{B}{C}}=\dfrac{AC}{B}$, $\dfrac{\dfrac{A}{B}}{C}=\dfrac{A}{BC}$,

$\dfrac{\dfrac{A}{B}}{\dfrac{C}{D}}=\dfrac{A}{B} \div \dfrac{C}{D}=\dfrac{A}{B} \times \dfrac{D}{C}=\dfrac{AD}{BC}$

(3) 비례식의 성질

0이 아닌 실수 k에 대하여

① $a:b=c:d \Leftrightarrow \dfrac{a}{b}=\dfrac{c}{d}=k \Leftrightarrow a=bk,\ c=dk$

② $a:b:c=d:e:f \Leftrightarrow \dfrac{a}{d}=\dfrac{b}{e}=\dfrac{c}{f}=k$

$\Leftrightarrow a=dk,\ b=ek,\ c=fk$

◪ **분모를 0으로 만들지 않는 모든 실수 x에 대하여 다음 등식이 성립하도록 하는 상수 a, b의 값을 구하여라.**

69. $\dfrac{1}{x-3}+\dfrac{2}{2x-1}=\dfrac{ax+b}{2x^2-7x+3}$

70. $\dfrac{a}{x-1}+\dfrac{b}{2x+1}=\dfrac{4x-1}{2x^2-x-1}$

71. $\dfrac{a}{x}+\dfrac{bx+1}{x^2-1}=\dfrac{x+2}{x^3-x}$

72. $\dfrac{3x}{x^3-1}=\dfrac{a}{x-1}+\dfrac{bx+1}{x^2+x+1}$

73. $\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+2)(x+3)}=\dfrac{b}{(x+1)(x+a)}$

74. $\dfrac{2}{(x+1)(x+3)}+\dfrac{2}{(x+3)(x+5)}=\dfrac{b}{(x+a)(x+5)}$

75. $\dfrac{1}{x(x+2)}+\dfrac{1}{(x+2)(x+4)}=\dfrac{b}{(x+a)(x+4)}$

76. $\dfrac{1}{(x-2)(x-1)}+\dfrac{2}{(x-1)(x+1)}+\dfrac{1}{(x+1)(x+2)}$
$=\dfrac{b}{(x+a)(x+2)}$

77. $\dfrac{2}{(x-1)(x+1)}+\dfrac{4}{(x+1)(x+5)}+\dfrac{2}{(x+5)(x+7)}$
$=\dfrac{b}{(x-1)(x+a)}$

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78. $\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}=\frac{b}{x(x+a)}$

79. $\frac{1}{x^2-x}+\frac{1}{x^2+x}+\frac{1}{x^2+3x+2}=\frac{b}{(x+a)(x+2)}$

◪ **다음 식의 값을 구하여라. (단, $xy\neq0$)**

80. $x:y=1:2$**일 때,** $\frac{2x}{x+y}$

81. $x:y=1:2$**일 때,** $\frac{xy}{x^2+y^2}$

82. $\frac{x}{3}=\frac{y}{2}$**일 때,** $\frac{y}{x}-\frac{x}{y}$

83. $\frac{x}{3}=\frac{y}{2}$**일 때,** $\frac{2xy}{x^2+y^2}$

84. $\frac{x-y}{2}=\frac{x-2y}{3}$**일 때,** $\frac{x+xy+y}{x^2+y^2}$

◪ **다음을 계산하여라.**

85. $x:y:z=2:3:1$**일 때,** $\frac{x+z}{x-y}$

86. $x:y:z=2:3:1$**일 때,** $\frac{y}{x}+\frac{z}{y}+\frac{x}{z}$

87. $x:y:z=2:3:5$**일 때,** $\frac{2yz}{x^2+y^2}$

88. $x:y:z=1:2:3$**일 때,** $\frac{x+2y+3z}{x+y+z}$

89. $x:y:z=2:3:1$**일 때,** $\frac{xy+yz+zx}{x^2+y^2+z^2}$

90. $x:y=3:2,\ y:z=3:1$ **일 때,** $\frac{x^2-y^2+z^2}{xy-yz+zx}$

◪ **다음을 계산하여라. (단, $xyz\neq0$)**

91. $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$**일 때,** $\frac{y}{x}+\frac{z}{y}-\frac{x}{z}$

92. $\frac{x}{2}=\frac{y}{4}=\frac{z}{3}$**일 때,** $\frac{x-3z}{x-y+2z}$

93. $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$**일 때,** $\frac{x-y-z}{x+y+z}$

94. $\frac{x}{2}=\frac{y}{3}=\frac{z}{2}$**일 때,** $\frac{x^2-y^2+z^2}{xy+yz-zx}$

95. $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$**일 때,** $\frac{x^2-y^2+z^2}{xy+3yz+zx}$

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정답 및 해설

1) $\dfrac{-2x+1}{x+1}$

⇨ $\dfrac{3}{x+1}-2=\dfrac{3-2(x+1)}{x+1}=\dfrac{-2x+1}{x+1}$

2) $\dfrac{3x+1}{(x+1)(x-1)}$

⇨ $\dfrac{1}{x+1}+\dfrac{2}{x-1}=\dfrac{x-1+2(x+1)}{(x+1)(x-1)}=\dfrac{3x+1}{(x+1)(x-1)}$

3) $\dfrac{3x-2}{(x+2)(x-2)}$

⇨ $\dfrac{2}{x+2}+\dfrac{1}{x-2}=\dfrac{2(x-2)+x+2}{(x+2)(x-2)}$

$=\dfrac{3x-2}{(x+2)(x-2)}$

4) $\dfrac{4x-1}{(x-1)(x+2)}$

⇨ $\dfrac{1}{x-1}+\dfrac{3}{x+2}=\dfrac{x+2+3(x-1)}{(x-1)(x+2)}=\dfrac{4x-1}{(x-1)(x+2)}$

5) 2

⇨ $\dfrac{2x+3}{x-1}+\dfrac{5}{1-x}=\dfrac{2x+3}{x-1}-\dfrac{5}{x-1}=\dfrac{2x+3-5}{x-1}$

$=\dfrac{2x-2}{x-1}=\dfrac{2(x-1)}{x-1}=2$

6) $\dfrac{1}{x-2}$

⇨ $\dfrac{x}{x-2}+\dfrac{x-1}{2-x}=\dfrac{x-(x-1)}{x-2}=\dfrac{1}{x-2}$

7) $\dfrac{6x+3}{(x-1)(x+2)}$

⇨ $\dfrac{x+2}{x-1}-\dfrac{x-1}{x+2}=\dfrac{(x+2)^2-(x-1)^2}{(x-1)(x+2)}$

$=\dfrac{x^2+4x+4-(x^2-2x+1)}{(x-1)(x+2)}$

$=\dfrac{6x+3}{(x-1)(x+2)}$

8) $\dfrac{4}{(x+3)(x-1)}$

⇨ $\dfrac{x+2}{x+3}-\dfrac{x-2}{x-1}=\dfrac{(x+2)(x-1)-(x-2)(x+3)}{(x+3)(x-1)}$

$=\dfrac{(x^2+x-2)-(x^2+x-6)}{(x+3)(x-1)}$

$=\dfrac{4}{(x+3)(x-1)}$

9) $\dfrac{6x-13}{(x-3)(x-2)}$

⇨ $\dfrac{x+2}{x-3}-\dfrac{x-3}{x-2}=\dfrac{x-3+5}{x-3}-\dfrac{x-2-1}{x-2}$

$=1+\dfrac{5}{x-3}-\left(1-\dfrac{1}{x-2}\right)$

$=\dfrac{5}{x-3}+\dfrac{1}{x-2}$

$=\dfrac{5(x-2)+x-3}{(x-3)(x-2)}$

$=\dfrac{6x-13}{(x-3)(x-2)}$

10) $\dfrac{3x^2-x+2}{x^3+1}$

⇨ $\dfrac{2}{x+1}+\dfrac{x}{x^2-x+1}=\dfrac{2(x^2-x+1)+x(x+1)}{(x+1)(x^2-x+1)}$

$=\dfrac{3x^2-x+2}{x^3+1}$

11) $\dfrac{4x-5}{(x+2)(x-2)}$

⇨ $\dfrac{x+1}{x^2-4}+\dfrac{3}{x+2}=\dfrac{x+1}{(x+2)(x-2)}+\dfrac{3}{x+2}$

$=\dfrac{x+1+3(x-2)}{(x+2)(x-2)}$

$=\dfrac{4x-5}{(x+2)(x-2)}$

12) $\dfrac{2}{x+1}$

⇨ $\dfrac{3x-1}{x^2-1}-\dfrac{x}{x^2-x}=\dfrac{3x-1}{(x+1)(x-1)}-\dfrac{x}{x(x-1)}$

$=\dfrac{x(3x-1)-x(x+1)}{x(x+1)(x-1)}$

$=\dfrac{2x^2-2x}{x(x+1)(x-1)}$

$=\dfrac{2x(x-1)}{x(x+1)(x-1)}=\dfrac{2}{x+1}$

13) $-\dfrac{2}{(x-1)(x-2)}$

⇨ $\dfrac{x+3}{x^2-1}-\dfrac{x+4}{x^2-x-2}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{x+4}{(x+1)(x-2)}$

$=\dfrac{(x+3)(x-2)-(x+4)(x-1)}{(x+1)(x-1)(x-2)}$

$=\dfrac{-2x-2}{(x+1)(x-1)(x-2)}$

$=\dfrac{-2(x+1)}{(x+1)(x-1)(x-2)}$

$=-\dfrac{2}{(x-1)(x-2)}$

14) $\dfrac{x+4}{x+3}$

⇨ $\dfrac{x}{x-1}-\dfrac{4}{x^2+2x-3}=\dfrac{x}{x-1}-\dfrac{4}{(x-1)(x+3)}$

$= \dfrac{x(x+3)-4}{(x-1)(x+3)}$

$= \dfrac{x^2+3x-4}{(x-1)(x+3)}$

$= \dfrac{(x-1)(x+4)}{(x-1)(x+3)}$

$= \dfrac{x+4}{x+3}$

15) $\dfrac{6x-3}{(x+1)(x+2)(x-3)}$

⇨ $\dfrac{x}{x^2-2x-3} - \dfrac{x-1}{x^2+3x+2}$

$= \dfrac{x}{(x+1)(x-3)} - \dfrac{x-1}{(x+1)(x+2)}$

$= \dfrac{x(x+2)-(x-1)(x-3)}{(x+1)(x+2)(x-3)}$

$= \dfrac{6x-3}{(x+1)(x+2)(x-3)}$

16) 분수식

17) 분수

18) 다항

19) 다항식

20) 분수식

21) 분수식

22) 다항식

23) 분수식

24) 다항식

25) 다항

26) 분수식

27) 분수

28) 다항

29) 다항

30) 분수

31) 분수

32) $a=1,\ b=-1$

⇨ $\dfrac{1}{x(x+1)} = \dfrac{1}{(x+1)-x}\left(\dfrac{1}{x} - \dfrac{1}{x+1}\right) = \dfrac{1}{x} - \dfrac{1}{x+1}$

$\therefore\ a=1,\ b=-1$

33) $a=-\dfrac{1}{3},\ b=\dfrac{1}{3}$

⇨ $\dfrac{1}{(x+1)(x-2)} = \dfrac{1}{(x-2)-(x+1)}\left(\dfrac{1}{x+1} - \dfrac{1}{x-2}\right)$

$= -\dfrac{1}{3(x+1)} + \dfrac{1}{3(x-2)}$

$\therefore\ a=-\dfrac{1}{3},\ b=\dfrac{1}{3}$

34) $a=1,\ b=2$

⇨ $\dfrac{a}{2x-1} + \dfrac{b}{x-2} = \dfrac{5x-4}{2x^2-5x+2}$의 양변에

$2x^2-5x+2$, 즉 $(x-2)(2x-1)$을 곱하면

$a(x-2)+b(2x-1)=5x-4$

$\therefore\ (a+2b)x-(2a+b)=5x-4$

이 식이 x에 대한 항등식이므로

$a+2b=5,\ 2a+b=4$ $\quad\therefore\ a=1,\ b=2$

35) $a=-1,\ b=3$

⇨ $\dfrac{1}{x-1} - \dfrac{2}{x+1} = \dfrac{ax+b}{x^2-1}$의 양변에 x^2-1, 즉

$(x-1)(x+1)$을 곱하면 $x+1-2(x-1)=ax+b$

$\therefore\ -x+3=ax+b$

이 식이 x에 대한 항등식이므로 $a=-1,\ b=3$

36) $a=2,\ b=-2$

⇨ $\dfrac{-6x}{x^3+1} = \dfrac{a}{x+1} + \dfrac{bx-2}{x^2-x+1}$의 양변에 x^3+1,

즉 $(x+1)(x^2-x+1)$을 곱하면

$-6x=a(x^2-x+1)+(bx-2)(x+1)$

$\therefore\ -6x=(a+b)x^2+(-a+b-2)x+a-2$

이 식이 x에 대한 항등식이므로

$a+b=0,\ -a+b-2=-6,\ a-2=0$

$\therefore\ a=2,\ b=-2$

37) $a=3,\ b=-6$

⇨ $\dfrac{6}{(x-1)(x-2)(x-3)} = \dfrac{a}{x-1} + \dfrac{b}{x-2} + \dfrac{3}{x-3}$의

양변에 $(x-1)(x-2)(x-3)$을 곱하면

$6=a(x-2)(x-3)+b(x-3)(x-1)+3(x-1)(x-2)$

$\therefore\ 6=(a+b+3)x^2+(-5a-4b-9)x+6a+3b+6$

이 식이 x에 대한 항등식이므로

$a+b+3=0,\ -5a-4b-9=0,\ 6a+3b+6=6$

$\therefore\ a=3,\ b=-6$

38) $a=-1,\ b=1$

⇨ $\dfrac{a}{x} + \dfrac{bx+2}{x^2+1} = \dfrac{2x-1}{x^3+x}$의 양변에 x^3+x,

즉 $x(x^2+1)$을 곱하면

$a(x^2+1)+x(bx+2)=2x-1$

$\therefore\ (a+b)x^2+2x+a=2x-1$

이 식이 x에 대한 항등식이므로

$a+b=0,\ a=-1$ $\quad\therefore\ a=-1,\ b=1$

39) $x-5$

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⇨ $\frac{x^2-4x-5}{x+1}=\frac{(x-5)(x+1)}{x+1}=x-5$

40) $x-1$

⇨ $\frac{x^3-1}{x^2+x+1}=\frac{(x-1)(x^2+x+1)}{x^2+x+1}=x-1$

41) $\frac{x+3}{2x+1}$

⇨ $\frac{x^2+x-6}{2x^2-3x-2}=\frac{(x+3)(x-2)}{(2x+1)(x-2)}=\frac{x+3}{2x+1}$

42) $\frac{3a^2}{2b}$

43) $\frac{3ax^2}{2y}$

⇨ $\frac{6a^3x^3y}{4a^2xy^2}=\frac{3ax^2}{2y}$

44) $\frac{x-1}{x(x-2)}$, $\frac{2x}{x(x-2)}$

⇨ $\frac{x-1}{x^2-2x}=\frac{x-1}{x(x-2)}$이므로 분모를 $x(x-2)$로 통분하면 $\frac{x-1}{x(x-2)}$, $\frac{2x}{x(x-2)}$

45) $\frac{4b}{12a^3b^2}$, $\frac{3a^2}{12a^3b^2}$

46) $\frac{2c^2x}{6ab^2cx^2}$, $\frac{3a^2b}{6ab^2cx^2}$

⇨ $\frac{c}{3ab^2x}$, $\frac{a}{2bcx^2}$의 분모를 $6ab^2cx^2$으로 통분하면 $\frac{2c^2x}{6ab^2cx^2}$, $\frac{3a^2b}{6ab^2cx^2}$

47) $\frac{2(x+2)}{(x-1)(x+2)(x-3)}$, $\frac{(x+1)(x-1)}{(x-1)(x+2)(x-3)}$

⇨ $x^2-4x+3=(x-1)(x-3)$,
$x^2-x-6=(x+2)(x-3)$이므로
주어진 두 식을 통분하면
$\frac{2(x+2)}{(x-1)(x+2)(x-3)}$, $\frac{(x+1)(x-1)}{(x-1)(x+2)(x-3)}$

48) $\frac{(2x+3y)^2}{(x-y)(x+2y)(2x+3y)}$,
$\frac{(x-y)^2}{(x-y)(x+2y)(2x+3y)}$

⇨ 주어진 두 분수식의 분모를 각각 인수분해하면
$x^2+xy-2y^2=(x-y)(x+2y)$
$2x^2+7xy+6y^2=(x+2y)(2x+3y)$
따라서 분모를 $(x-y)(x+2y)(2x+3y)$로 통분하면
$\frac{(2x+3y)^2}{(x-y)(x+2y)(2x+3y)}$, $\frac{(x-y)^2}{(x-y)(x+2y)(2x+3y)}$

49) $\frac{2}{x+2}$

⇨ $\frac{2}{x+1}\times\frac{x+1}{x+2}=\frac{2}{x+2}$

50) $\frac{2x}{x+2}$

⇨ $\frac{2}{x+1}\times\frac{x^2+x}{x+2}=\frac{2}{x+1}\times\frac{x(x+1)}{x+2}=\frac{2x}{x+2}$

51) $\frac{1}{2(x+1)}$

⇨ $\frac{x-1}{2x}\times\frac{x}{x^2-1}=\frac{x-1}{2x}\times\frac{x}{(x+1)(x-1)}$
$=\frac{1}{2(x+1)}$

52) $\frac{3}{x-1}$

⇨ $\frac{x+1}{x}\times\frac{3x}{x^2-1}=\frac{x+1}{x}\times\frac{3x}{(x+1)(x-1)}=\frac{3}{x-1}$

53) $\frac{x}{(x-2)^2}$

⇨ $\frac{x}{x^2-4}\times\frac{x+2}{x-2}=\frac{x}{(x+2)(x-2)}\times\frac{x+2}{x-2}=\frac{x}{(x-2)^2}$

54) $\frac{1}{x(x-1)}$

⇨ $\frac{x+1}{x^2+2x}\times\frac{x+2}{x^2-1}=\frac{x+1}{x(x+2)}\times\frac{x+2}{(x-1)(x+1)}$
$=\frac{1}{x(x-1)}$

55) $\frac{x-1}{x}$

⇨ $\frac{x^2-1}{x^2+2x}\times\frac{x+2}{x+1}=\frac{(x-1)(x+1)}{x(x+2)}\times\frac{x+2}{x+1}=\frac{x-1}{x}$

56) $\frac{2(x+3)}{x(x-2)}$

⇨ $\frac{2x-4}{x^2-3x}\times\frac{x^2-9}{(x-2)^2}=\frac{2(x-2)}{x(x-3)}\times\frac{(x+3)(x-3)}{(x-2)^2}$
$=\frac{2(x+3)}{x(x-2)}$

57) $\frac{x}{x+4}$

⇨ $\frac{x^2+3x}{x^2-16}\times\frac{x-4}{x^2+3}=\frac{x(x^2+3)}{(x+4)(x-4)}\times\frac{x-4}{x^2+3}$
$=\frac{x}{x+4}$

58) $\frac{3(x+2)}{x^2-1}$

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⇨ $\frac{3x+3}{x^2-3x+2}\times\frac{x^2-4}{(x+1)^2}$
$=\frac{3(x+1)}{(x-1)(x-2)}\times\frac{(x+2)(x-2)}{(x+1)^2}$
$=\frac{3(x+2)}{(x-1)(x+1)}$
$=\frac{3(x+2)}{x^2-1}$

59) $\frac{(x-2)(x+2)}{(x+1)(x-1)}$

⇨ $\frac{x^2+x-6}{x^2-4x-5}\times\frac{x^2-3x-10}{x^2+2x-3}=\frac{(x-2)(x+2)}{(x+1)(x-1)}$

60) $\frac{x^2-3}{x-1}$

⇨ $\frac{x^2-4}{(x+2)^2}\times\frac{x^2+3x+2}{x-1}+1$
$=\frac{(x+2)(x-2)}{(x+2)^2}\times\frac{(x+2)(x+1)}{x-1}+1$
$=\frac{(x-2)(x+1)}{x-1}+1$
$=\frac{x^2-x-2+(x-1)}{x-1}=\frac{x^2-3}{x-1}$

61) $\frac{x}{x+1}$

⇨ $\frac{x}{x-1}\div\frac{x+1}{x-1}=\frac{x}{x-1}\times\frac{x-1}{x+1}=\frac{x}{x+1}$

62) $\frac{x+3}{x+1}$

⇨ $\frac{x-2}{x+1}\div\frac{x-2}{x+3}=\frac{x-2}{x+1}\times\frac{x+3}{x-2}=\frac{x+3}{x+1}$

63) $\frac{x}{(x+1)^2}$

⇨ $\frac{x}{x^2-1}\div\frac{x+1}{x-1}=\frac{x}{(x+1)(x-1)}\times\frac{x-1}{x+1}=\frac{x}{(x+1)^2}$

64) $\frac{2a}{xy^2}$

⇨ $\frac{6a^3b}{x^3y^3}\div\frac{3a^2b}{x^2y}=\frac{6a^3b}{x^3y^3}\times\frac{x^2y}{3a^2b}=\frac{2a}{xy^2}$

65) $\frac{x(x-1)}{x+2}$

⇨ $\frac{x^2-2x}{x+1}\div\frac{x^2-4}{x^2-1}=\frac{x(x-2)}{x+1}\times\frac{(x-1)(x+1)}{(x-2)(x+2)}$
$=\frac{x(x-1)}{x+2}$

66) $x-2$

⇨ $\frac{x^2-3x+2}{x-3}\div\frac{x-1}{x-3}=\frac{x^2-3x+2}{x-3}\times\frac{x-3}{x-1}$
$=\frac{(x-2)(x-1)}{x-3}\times\frac{x-3}{x-1}$
$=x-2$

67) $\frac{x-1}{x+2}$

⇨ $\frac{x^2-4x+3}{x^2-4}\div\frac{x-3}{x-2}=\frac{x^2-4x+3}{x^2-4}\times\frac{x-2}{x-3}$
$=\frac{(x-3)(x-1)}{(x+2)(x-2)}\times\frac{x-2}{x-3}$
$=\frac{x-1}{x+2}$

68) $x-1$

⇨ $\frac{x^2-2x+1}{x-2}\div\frac{x-1}{x-2}=\frac{x^2-2x+1}{x-2}\times\frac{x-2}{x-1}$
$=\frac{(x-1)^2}{x-2}\times\frac{x-2}{x-1}$
$=x-1$

69) $a=4,\ b=-7$

⇨ $\frac{1}{x-3}+\frac{2}{2x-1}=\frac{ax+b}{2x^2-7x+3}$의 양변에
$2x^2-7x+3$, 즉 $(x-3)(2x-1)$을 곱하면
$2x-1+2(x-3)=ax+b$
$\therefore\ 4x-7=ax+b$
이 식이 x에 대한 항등식이므로 $a=4,\ b=-7$

70) $a=1,\ b=2$

⇨ $\frac{a}{x-1}+\frac{b}{2x+1}=\frac{4x-1}{2x^2-x-1}$의 양변에
$2x^2-x-1$, 즉 $(x-1)(2x+1)$을 곱하면
$a(2x+1)+b(x-1)=4x-1$
$\therefore\ (2a+b)x+a-b=4x-1$
이 식이 x에 대한 항등식이므로
$2a+b=4,\ a-b=-1$ $\therefore\ a=1,\ b=2$

71) $a=-2,\ b=2$

⇨ $\frac{a}{x}+\frac{bx+1}{x^2-1}=\frac{x+2}{x^3-x}$의 양변에
x^3-x, 즉 $x(x^2-1)$을 곱하면
$a(x^2-1)+x(bx+1)=x+2$
$\therefore\ (a+b)x^2+x-a=x+2$
이 식이 x에 대한 항등식이므로
$a+b=0,\ -a=2$ $\therefore\ a=-2,\ b=2$

72) $a=1,\ b=-1$

⇨ $\frac{3x}{x^3-1}=\frac{a}{x-1}+\frac{bx+1}{x^2+x+1}$의 양변에 x^3-1,
즉 $(x-1)(x^2+x+1)$을 곱하면
$3x=a(x^2+x+1)+(bx+a)(x-1)$
$\therefore\ 3x=(a+b)x^2+(a-b+1)x+a-1$
이 식이 x에 대한 항등식이므로
$a+b=0,\ a-b+1=3,\ a-1=0$

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$\therefore\ a=1,\ b=-1$

73) $a=3,\ b=2$

$\Rightarrow \dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+2)(x+3)}$

$=\left(\dfrac{1}{x+1}-\dfrac{1}{x+2}\right)+\left(\dfrac{1}{x+2}-\dfrac{1}{x+3}\right)$

$=\dfrac{1}{x+1}-\dfrac{1}{x+3}=\dfrac{2}{(x+1)(x+3)}$

$\therefore\ a=3,\ b=2$

74) $a=1,\ b=4$

$\Rightarrow \dfrac{2}{(x+1)(x+3)}+\dfrac{2}{(x+3)(x+5)}$

$=\left(\dfrac{1}{x+1}-\dfrac{1}{x+3}\right)+\left(\dfrac{1}{x+3}-\dfrac{1}{x+5}\right)$

$=\dfrac{1}{x+1}-\dfrac{1}{x+5}=\dfrac{4}{(x+1)(x+5)}$

$\therefore\ a=1,\ b=4$

75) $a=0,\ b=2$

$\Rightarrow \dfrac{1}{x(x+2)}+\dfrac{1}{(x+2)(x+4)}$

$=\dfrac{1}{2}\left(\dfrac{1}{x}-\dfrac{1}{x+2}\right)+\dfrac{1}{2}\left(\dfrac{1}{x+2}-\dfrac{1}{x+4}\right)$

$=\dfrac{1}{2}\left(\dfrac{1}{x}-\dfrac{1}{x+4}\right)$

$=\dfrac{1}{2}\cdot\dfrac{4}{x(x+4)}=\dfrac{2}{x(x+4)}$

$\therefore\ a=0\ b=2$

76) $a=-2,\ b=4$

$\Rightarrow \dfrac{1}{(x-2)(x-1)}+\dfrac{2}{(x-1)(x+1)}+\dfrac{1}{(x+1)(x+2)}$

$=\left(\dfrac{1}{x-2}-\dfrac{1}{x-1}\right)+\left(\dfrac{1}{x-1}-\dfrac{1}{x+1}\right)+\left(\dfrac{1}{x+1}-\dfrac{1}{x+2}\right)$

$=\dfrac{1}{x-2}-\dfrac{1}{x+2}=\dfrac{4}{(x-2)(x+2)}$

$\therefore a=-2,\ b=4$

77) $a=7,\ b=8$

$\Rightarrow \dfrac{2}{(x-1)(x+1)}+\dfrac{4}{(x+1)(x+5)}+\dfrac{2}{(x+5)(x+7)}$

$=\left(\dfrac{1}{x-1}-\dfrac{1}{x+1}\right)+\left(\dfrac{1}{x+1}-\dfrac{1}{x+5}\right)+\left(\dfrac{1}{x+5}-\dfrac{1}{x+7}\right)$

$=\dfrac{1}{x-1}-\dfrac{1}{x+7}=\dfrac{8}{(x-1)(x+7)}$

$\therefore\ a=7,\ b=8$

78) $a=3,\ b=3$

$\Rightarrow \dfrac{1}{x(x+1)}+\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+2)(x+3)}$

$=\left(\dfrac{1}{x}-\dfrac{1}{x+1}\right)+\left(\dfrac{1}{x+1}-\dfrac{1}{x+2}\right)+\left(\dfrac{1}{x+2}-\dfrac{1}{x+3}\right)$

$=\dfrac{1}{x}-\dfrac{1}{x+3}=\dfrac{3}{x(x+3)}$

$\therefore\ a=3,\ b=3$

79) $a=-1,\ b=3$

$\Rightarrow \dfrac{1}{x^2-x}+\dfrac{1}{x^2+x}+\dfrac{1}{x^2+3x+2}$

$=\dfrac{1}{(x-1)x}+\dfrac{1}{x(x+1)}+\dfrac{1}{(x+1)(x+2)}$

$=\left(\dfrac{1}{x-1}-\dfrac{1}{x}\right)+\left(\dfrac{1}{x}-\dfrac{1}{x+1}\right)+\left(\dfrac{1}{x+1}-\dfrac{1}{x+2}\right)$

$=\dfrac{1}{x-1}-\dfrac{1}{x+2}=\dfrac{3}{(x-1)(x+2)}$

$\therefore\ a=-1,\ b=3$

80) $\dfrac{2}{3}$

$\Rightarrow x:y=1:2$에서 $x=k,\ y=2k\ (k\neq0)$라 하면

$\dfrac{2x}{x+y}=\dfrac{2k}{k+2k}=\dfrac{2k}{3k}=\dfrac{2}{3}$

81) $\dfrac{2}{5}$

$\Rightarrow x:y=1:2$에서 $x=k,\ y=2k\ (k\neq0)$라 하면

$\dfrac{xy}{x^2+y^2}=\dfrac{2k^2}{k^2+4k^2}=\dfrac{2k^2}{5k^2}=\dfrac{2}{5}$

82) $-\dfrac{5}{6}$

$\Rightarrow \dfrac{x}{3}=\dfrac{y}{2}$에서 $x=3k,\ y=2k\ (k\neq0)$라 하면

$\dfrac{y}{x}-\dfrac{x}{y}=\dfrac{2k}{3k}-\dfrac{3k}{2k}=\dfrac{2}{3}-\dfrac{3}{2}=-\dfrac{5}{6}$

83) $\dfrac{12}{13}$

$\Rightarrow \dfrac{x}{3}=\dfrac{y}{2}$에서 $x=3k,\ y=2k\ (k\neq0)$라 하면

$\dfrac{2xy}{x^2+y^2}=\dfrac{12k^2}{9k^2+4k^2}=\dfrac{12k^2}{13k^2}=\dfrac{12}{13}$

84) $-\dfrac{1}{2}$

$\Rightarrow \dfrac{x-y}{2}=\dfrac{x-2y}{3}=k\ (k\neq0)$에서

$x-y=2k,\ x-2y=3k,$

즉 $x=k,\ y=-k$라 하면

$\dfrac{x+xy+y}{x^2+y^2}=\dfrac{k-k^2-k}{k^2+k^2}=\dfrac{-k^2}{2k^2}=-\dfrac{1}{2}$

85) -3

$\Rightarrow x:y:z=2:3:1$에서

$x=2k,\ y=3k,\ z=k\ (k\neq0)$라 하면

$\dfrac{x+z}{x-y}=\dfrac{2k+k}{2k-3k}=\dfrac{3k}{-k}=-3$

86) $\dfrac{23}{6}$

$\Rightarrow x:y:z=2:3:1$에서

$x=2k,\ y=3k,\ z=k\ (k\neq0)$라 하면

$\dfrac{y}{x}+\dfrac{z}{y}+\dfrac{x}{z}=\dfrac{3k}{2k}+\dfrac{k}{3k}+\dfrac{2k}{k}=\dfrac{3}{2}+\dfrac{1}{3}+2=\dfrac{23}{6}$

87) $\dfrac{30}{13}$

⇨ $x:y:z=2:3:5$에서 $x=2k,\ y=3k,\ z=5k$ $(k\neq0)$라면

$\dfrac{2yz}{x^2+y^2}=\dfrac{30k^2}{4k^4+9k^2}=\dfrac{30k^2}{13k^2}=\dfrac{30}{13}$

88) $\dfrac{7}{3}$

⇨ $x:y:z=1:2:3$에서

$x=k,\ y=2k,\ z=3k\ (k\neq0)$라 하면

$\dfrac{x+2y+3z}{x+y+z}=\dfrac{k+4k+9k}{k+2k+3k}=\dfrac{14k}{6k}=\dfrac{7}{3}$

89) $\dfrac{11}{14}$

⇨ $x:y:z=2:3:1$에서

$x=2k,\ y=3k,\ z=k\ (k\neq0)$라 하면

$\dfrac{xy+yz+zx}{x^2+y^2+z^2}=\dfrac{6k^2+3k^2+2k^2}{4k^2+9k^2+k^2}=\dfrac{11k^2}{14k^2}=\dfrac{11}{14}$

90) $\dfrac{49}{60}$

⇨ $x:y=3:2,\ y:z=3:1$에서

$x=9k,\ y=6k,\ z=2k\ (k\neq0)$라 하면

$\dfrac{x^2-y^2+z^2}{xy-yz+ax}=\dfrac{81k^2-36k^2+4k^2}{54k^2-12k^2+18k^2}=\dfrac{49k^2}{60k^2}=\dfrac{49}{60}$

91) $\dfrac{7}{3}$

⇨ $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{4}$에서 $x=2k,\ y=3k,\ z=4k\ (k\neq0)$라 하면 $\dfrac{y}{x}+\dfrac{z}{y}-\dfrac{x}{z}=\dfrac{3k}{2k}+\dfrac{4k}{3k}-\dfrac{2k}{4k}=\dfrac{3}{2}+\dfrac{4}{3}-\dfrac{1}{2}=\dfrac{7}{3}$

92) $-\dfrac{7}{4}$

⇨ $\dfrac{x}{3}=\dfrac{y}{1}=\dfrac{z}{3}$에서 $x=2k,\ y=4k,\ z=3k\ (k\neq0)$라면 $\dfrac{x-3z}{x-y+2z}=\dfrac{2k-9k}{2k-4k+6k}=\dfrac{-7k}{4k}=-\dfrac{7}{4}$

93) $-\dfrac{5}{9}$

⇨ $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{4}$에서 $x=2k,\ y=3k,\ z=4k\ (k\neq0)$라 하면 $\dfrac{x-y-z}{x+y+z}=\dfrac{2k-3k-4k}{2k+3k+4k}=\dfrac{-5k}{9k}=-\dfrac{5}{9}$

94) $-\dfrac{1}{8}$

⇨ $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{2}$에서 $x=2k,\ y=3k,\ z=2k\ (k\neq0)$라 하면 $\dfrac{x^2-y^2+z^2}{xy+yz-zx}=\dfrac{4k^2-9k^2+4k^2}{6k^2+6k^2-4k^2}=\dfrac{-k^2}{8k^2}=-\dfrac{1}{8}$

95) $\dfrac{11}{50}$

⇨ $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{4}$에서

$x=2k,\ y=3k,\ z=4k\ (k\neq0)$라 하면

$\dfrac{x^2-y^2+z^2}{xy+3yz+zx}=\dfrac{4k^2-9k^2+16k^2}{6k^2+36k^2+8k^2}=\dfrac{11k^2}{50k^2}=\dfrac{11}{50}$

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