



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

1) 제작연월일 : 2019-02-13

2) 제작자 : 교육지대(주)

3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 지수의 확장(1) 0 또는 음의 정수인 지수의 정의 : $a \neq 0$ 이고 n 이 양의 정수일 때

① $a^0 = 1$

② $a^{-n} = \frac{1}{a^n}$

(2) 유리수인 지수의 정의 : $a > 0$ 이고, $m, n (n \geq 2)$ 이 정수일 때

① $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

② $a^{\frac{1}{n}} = \sqrt[n]{a}$

■ 다음 수를 근호를 사용하여 나타내어라.

1. $4^{-\frac{3}{4}}$

2. $\left(\frac{1}{9}\right)^{-\frac{3}{4}}$

3. $2^{\frac{2}{3}}$

4. $11^{\frac{2}{3}}$

5. $5^{-\frac{3}{2}}$

6. $\left(\frac{125}{64}\right)^{\frac{1}{6}}$

7. $\left(\frac{1}{32}\right)^{\frac{3}{10}}$

■ 다음 값을 구하여라.

8. $\left(-\frac{1}{2}\right)^0$

9. $4^0 + (-5)^0$

10. $\left(\frac{1}{3}\right)^{-2}$

11. $(-5)^{-2}$

12. $(-2)^{-3} + (-3)^{-2}$

13. $\left(\frac{5}{2}\right)^0 + \left(\frac{2}{3}\right)^{-2}$

14. $(-5)^{-2} + \left(\frac{5}{3}\right)^{-3}$

15. $(-3)^0 - 2^{-2}$

02 지수법칙

1. 지수가 정수일 때의 지수법칙

: $a \neq 0, b \neq 0$ 이고 m, n 이 정수일 때

- ① $a^m a^n = a^{m+n}$ ② $a^m \div a^n = a^{m-n}$
 ③ $(a^m)^n = a^{mn}$ ④ $(ab)^n = a^n b^n$

2. 지수가 유리수일 때의 지수법칙

: $a > 0, b > 0$ 이고 p, q 가 유리수일 때

- ① $a^p a^q = a^{p+q}$ ② $a^p \div a^q = a^{p-q}$
 ③ $(a^p)^q = a^{pq}$ ④ $(ab)^p = a^p b^p$

3. 지수가 실수일 때의 지수법칙

: $a > 0, b > 0$ 이고 x, y 가 실수일 때

- ① $a^x a^y = a^{x+y}$ ② $a^x \div a^y = a^{x-y}$
 ③ $(a^x)^y = a^{xy}$ ④ $(ab)^x = a^x b^x$

■ $a > 0$ 일 때, 다음을 간단히 하여라.

16. $a^3 \times a^4 \div a^9$

17. $(a^{-3})^{-4} \div (a^5)^{-3}$

18. $(a^{-2})^5 \times (a^3)^{-4}$

19. $a^5 \times a^2 \div a^8$

20. $\frac{(a^{-5})^2 \times (a^2)^5}{a^2 \times a^{-5}}$

21. $\frac{(a^4)^{-3} \times (a^{-2})^{-8}}{(a^{-7})^3}$

22. $(a^5)^{-2} \times (a^{-3})^{-4} \div (a^{-2})^2$

23. $(a^{-4})^2 \times (a^{-5})^{-3} \div a^{-5}$

24. $(a^3)^{-2} \times (a^{-5})^{-4} \div (a^{-2})^7$

25. $\sqrt[3]{\sqrt{a}}$

26. $\sqrt[3]{a \sqrt{a \sqrt{a}}}$

27. $\sqrt[3]{\sqrt{a} \times \sqrt[5]{a}}$

28. $\sqrt[4]{a \sqrt[3]{a}}$

29. $\sqrt[4]{\sqrt[3]{a} \sqrt{a}}$

30. $\sqrt{a \sqrt{a \sqrt{a}}}$

31. $\sqrt{a \sqrt[3]{a \sqrt[4]{a^3}}}$

32. $\sqrt{9a \sqrt{a \sqrt{a}}}$

33. $\sqrt{a \sqrt{a^2 \sqrt{a^3}}}$

■ $a > 0$ 일 때, 다음을 간단히 하여라.

34. $(a^{\frac{1}{3}})^{-\frac{3}{5}} \times (a^{\frac{3}{2}})^{\frac{4}{9}}$

35. $a^{\frac{1}{2}} \times a^{-\frac{2}{3}} \div a^{\frac{3}{4}}$

36. $a^{\frac{2}{5}} \div a^{-\frac{1}{2}}$

37. $a^{\frac{1}{3}} \times a^{\frac{1}{2}}$

38. $a^{\sqrt{2}} \div a^{2\sqrt{2}} \times a^{\sqrt{3}}$

39. $(a^{-\frac{3}{4}})^2 \times \sqrt{a} \div a^{\frac{3}{4}}$

40. $(\sqrt{a^3} \times \sqrt[5]{a} \times a^{-\frac{1}{2}})^{\frac{1}{3}}$

41. $\sqrt[4]{a^3} \times \sqrt[3]{a^2} \div \sqrt{a^5}$

42. $\sqrt[4]{a^5} \times \sqrt{a^3} \div \sqrt[3]{a^5}$

43. $a^{-\frac{1}{2}} \div a^{\frac{1}{4}} \times a^{\frac{3}{4}}$

44. $(\sqrt{a^3} \div \sqrt[5]{a})^{\frac{1}{3}}$

45. $(\sqrt[3]{a^2} \times \sqrt[4]{a^3})^{\frac{2}{3}}$

46. $\sqrt[3]{\sqrt[4]{a^7}} \times \sqrt[4]{\sqrt[3]{a^5}}$

47. $\frac{\sqrt[10]{a^6}}{(\sqrt[5]{a^4})^2}$

48. $\sqrt[7]{a^{11}} \times \sqrt[14]{a^6}$

49. $a^{\sqrt{3}} \times a^{\sqrt{12}}$

50. $a^{\sqrt{32}} \times a^{\sqrt{8}} \div a^{\sqrt{18}}$

51. $(a^{\frac{\sqrt{3}}{2}})^4 \div a^{\sqrt{3}}$

52. $a^{-\frac{\sqrt{2}}{3}} \times a^{-\frac{2\sqrt{2}}{3}} \div a^{-3\sqrt{2}}$

53. $(a^6)^{-\frac{1}{3}} \div (a^{-3\sqrt{2}})^{\frac{1}{\sqrt{2}}}$

■ $a > 0, b > 0$ 일 때, 다음 식을 간단히 하여라.

54. $(a^3b^2)^{\frac{1}{12}} \times (a^{\frac{1}{3}}b^{\frac{1}{4}})^4$

55. $\sqrt[3]{ab^2} \times \sqrt[6]{ab^5} \div \sqrt{ab}$

56. $\sqrt[3]{\sqrt[5]{a^{34}b^6}} \div \sqrt[5]{\sqrt[3]{a^4b^{21}}}$

57. $\sqrt[3]{a^2b^5} \div \sqrt[4]{a^5b^2} \times \sqrt{a^3b}$

58. $(a^{2\sqrt{2}} \times b^{3\sqrt{2}})^{\frac{3}{\sqrt{2}}}$

59. $\frac{\sqrt[3]{\sqrt[3]{a^4b}} \times \sqrt[3]{a^2b^4}}{\sqrt[4]{\sqrt[3]{a^4b^6}}}$

60. $\left(\frac{a^{\sqrt{3}}}{b^{\sqrt{18}}}\right)^{\frac{2}{\sqrt{6}}}$

■ 다음을 간단히 하여라.

61. $(81^{-\frac{1}{3}})^{\frac{9}{4}}$

62. $\left\{\left(\frac{3}{5}\right)^{-\frac{5}{2}}\right\}^{\frac{4}{5}}$

63. $25^{-\frac{3}{2}} \times 100^{\frac{3}{2}}$

64. $9^{-\frac{3}{2}} \times 36^{\frac{1}{2}}$

65. $81^{-\frac{3}{4}} \div 125^{-\frac{2}{3}}$

66. $7^{\frac{5}{4}} \times 7^{-\frac{3}{2}} \div 7^{-2}$

67. $32^{-\frac{3}{5}} \div 27^{-\frac{2}{3}}$

68. $16^{\frac{1}{4}} \div 16^{\frac{1}{8}}$

69. $\left\{\left(\frac{3}{2}\right)^{-\frac{4}{3}}\right\}^{\frac{9}{4}}$

$$70. \quad 5^{\frac{3}{4}} \times 625^{\frac{1}{4}}$$

$$71. \quad 16^{-\frac{3}{2}} \times 64^{\frac{3}{2}} \div 27^{-\frac{1}{3}}$$

$$72. \quad \sqrt{\frac{\sqrt[5]{5^{30}}}{\sqrt[3]{7^{12}}}}$$

$$73. \quad (5^{\frac{3}{2}})^2 \div \sqrt{5}$$

$$74. \quad (\sqrt[12]{3^5})^2 \times (\sqrt[12]{3^2})^7$$

$$75. \quad \sqrt[4]{\sqrt[3]{16}} \times \sqrt[6]{\sqrt{256}}$$

$$76. \quad \left\{ \left(\frac{27}{64} \right)^{-\frac{1}{3}} \right\}^{\frac{3}{2}} \times \left(\frac{3}{4} \right)^{\frac{1}{2}}$$

$$77. \quad (3^{-\frac{2}{3}})^6 \times \left\{ \left(\frac{2}{3} \right)^{-\frac{3}{2}} \right\}^4$$

$$78. \quad \sqrt{\frac{\sqrt[3]{1024}}{\sqrt[6]{27}}} \div \sqrt[3]{\frac{\sqrt{16}}{\sqrt[4]{27}}}$$

$$79. \quad (4^{\sqrt{3}})^{\frac{\sqrt{3}}{2}}$$

$$80. \quad 3^{\sqrt{5}} \times 4^{\sqrt{5}}$$

$$81. \quad (4^{\frac{1}{\sqrt{6}}} \times 3^{\sqrt{\frac{2}{3}}})^{\sqrt{3}}$$

$$82. \quad (2^{\sqrt{8}} \times 3^{\sqrt{2}})^{\sqrt{2}}$$

$$83. \quad 3^{\frac{\sqrt{2}}{3}} \times 3^{\frac{\sqrt{8}}{3}}$$

$$84. \quad 8^{-\frac{\sqrt{3}}{6}} \times 2^{\frac{\sqrt{3}}{2}}$$

$$85. \quad 5^{\sqrt{3}+1} \div 5^{\sqrt{3}-2}$$

$$86. \quad 3^{\frac{\sqrt{5}}{2}} \times 3^{\frac{3\sqrt{5}}{2}}$$

$$87. \quad (2^5\sqrt{3})^{\frac{\sqrt{6}}{3}} \times 2^{2-3\sqrt{2}} \div 2^{2\sqrt{2}-1}$$

88. $4^{\sqrt{2}} \times 4^{\sqrt{18}} \div 4^{\sqrt{8}}$

89. $(3^{\sqrt{8}} \times 2^{\sqrt{2}})^{\sqrt{2}}$

90. $2^{\sqrt{2}+1} \div 2^{\sqrt{2}-1}$



정답 및 해설

1) $\frac{\sqrt{2}}{4}$

$$\Rightarrow 4^{-\frac{3}{4}} = (2^2)^{-\frac{3}{4}} = 2^{-\frac{3}{2}} = \frac{1}{2^{\frac{3}{2}}} = \frac{1}{\sqrt{2^3}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

2) $3\sqrt{3}$

$$\Rightarrow \left(\frac{1}{9}\right)^{-\frac{3}{4}} = (3^{-2})^{-\frac{3}{4}} = 3^{\frac{3}{2}} = \sqrt{3^3} = 3\sqrt{3}$$

3) $\sqrt[3]{4}$

$$\Rightarrow 2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4}$$

4) $\sqrt[3]{121}$

$$\Rightarrow 11^{\frac{2}{3}} = \sqrt[3]{11^2} = \sqrt[3]{121}$$

5) $\frac{\sqrt{5}}{25}$

$$\begin{aligned} \Rightarrow 5^{-\frac{3}{2}} &= 5^{\frac{-3}{2}} = \sqrt{5^{-3}} = \sqrt{\frac{1}{5^3}} \\ &= \frac{1}{\sqrt{5^3}} = \frac{1}{\sqrt{125}} = \frac{\sqrt{5}}{25} \end{aligned}$$

6) $\frac{\sqrt{5}}{2}$

$$\Rightarrow \left(\frac{125}{64}\right)^{\frac{1}{6}} = \left\{\left(\frac{5}{4}\right)^3\right\}^{\frac{1}{6}} = \left(\frac{5}{4}\right)^{3 \times \frac{1}{6}} = \left(\frac{5}{4}\right)^{\frac{1}{2}} = \frac{\sqrt{5}}{2}$$

7) $\frac{\sqrt{2}}{4}$

$$\begin{aligned} \Rightarrow \left(\frac{1}{32}\right)^{\frac{3}{10}} &= \left\{\left(\frac{1}{2}\right)^5\right\}^{\frac{3}{10}} = \left(\frac{1}{2}\right)^{5 \times \frac{3}{10}} = \left(\frac{1}{2}\right)^{\frac{3}{2}} \\ &= \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \end{aligned}$$

8) 1

9) 2

$$\Rightarrow 4^0 + (-5)^0 = 1 + 1 = 2$$

10) 9

11) $\frac{1}{25}$

12) $-\frac{1}{72}$

$$\Rightarrow (-2)^{-3} + (-3)^{-2} = \frac{1}{(-2)^3} + \frac{1}{(-3)^2}$$

$$= -\frac{1}{8} + \frac{1}{9} = -\frac{1}{72}$$

13) $\frac{13}{4}$

$$\Rightarrow \left(\frac{5}{2}\right)^0 + \left(\frac{2}{3}\right)^{-2} = 1 + \frac{1}{\left(\frac{2}{3}\right)^2} = 1 + \frac{1}{\frac{4}{9}} = \frac{13}{4}$$

14) $\frac{32}{125}$

$$\begin{aligned} \Rightarrow (-5)^{-2} + \left(\frac{5}{3}\right)^{-3} &= \frac{1}{(-5)^2} + \frac{1}{\left(\frac{5}{3}\right)^3} = \frac{1}{25} + \frac{1}{\frac{125}{27}} \\ &= \frac{5}{125} + \frac{27}{125} = \frac{32}{125} \end{aligned}$$

15) $\frac{3}{4}$

$$\Rightarrow (-3)^0 - 2^{-2} = 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

16) $\frac{1}{a^2}$

$$\begin{aligned} \Rightarrow a^3 \times a^4 \div a^9 &= a^3 \times a^4 \times a^{-9} \\ &= a^{3+4-9} = a^{-2} = \frac{1}{a^2} \end{aligned}$$

17) a^{27}

18) $\frac{1}{a^{22}}$

$$\Rightarrow (a^{-2})^5 \times (a^3)^{-4} = a^{-10} \times a^{-12} = a^{-22} = \frac{1}{a^{22}}$$

19) $\frac{1}{a}$

$$\Rightarrow a^5 \times a^2 \div a^8 = a^{5+2-8} = a^{-1} = \frac{1}{a}$$

20) a^3

$$\begin{aligned} \Rightarrow \frac{(a^{-5})^2 \times (a^2)^5}{a^2 \times a^{-5}} &= \frac{a^{-10} \times a^{10}}{a^{2+(-5)}} = \frac{a^{(-10)+10}}{a^{-3}} \\ &= a^{0-(-3)} = a^3 \end{aligned}$$

21) a^{25}

$$\begin{aligned} \Rightarrow \frac{(a^4)^{-3} \times (a^{-2})^{-8}}{(a^{-7})^3} &= \frac{a^{-12} \times a^{16}}{a^{-21}} \\ &= a^{-12+16-(-21)} = a^{25} \end{aligned}$$

22) a^6

$$\begin{aligned} \Rightarrow (a^5)^{-2} \times (a^{-3})^{-4} \div (a^{-2})^2 &= a^{-10} \times a^{12} \div a^{-4} \\ &= a^{-10+12-(-4)} = a^6 \end{aligned}$$

23) a^{12}

$$\Rightarrow (a^{-4})^2 \times (a^{-5})^{-3} \div a^{-5} = a^{-8} \times a^{15} \times a^5 \\ = a^{(-8)+15+5} = a^{12}$$

$$24) a^{28}$$

$$\Rightarrow (a^3)^{-2} \times (a^{-5})^{-4} \div (a^{-2})^7 = a^{-6} \times a^{20} \div a^{-14} \\ = a^{-6+20-(-14)} = a^{28}$$

$$25) a^{\frac{1}{6}}$$

$$\Rightarrow \sqrt[3]{\sqrt{a}} = (a^{\frac{1}{3}})^{\frac{1}{2}} = a^{\frac{1}{3} \times \frac{1}{2}} = a^{\frac{1}{6}}$$

$$26) a^{\frac{7}{12}}$$

$$\Rightarrow \sqrt[3]{\sqrt{a} \sqrt[4]{a}} = \left\{ a(a \times a^{\frac{1}{2}})^{\frac{1}{2}} \right\}^{\frac{1}{3}} = (a \times a^{\frac{3}{2} \times \frac{1}{2}})^{\frac{1}{3}} \\ = (a^{\frac{7}{4}})^{\frac{1}{3}} = a^{\frac{7}{12}}$$

$$27) a^{\frac{7}{30}}$$

$$\Rightarrow \sqrt[3]{\sqrt{a} \times \sqrt[5]{a}} = (a^{\frac{1}{2}} \times a^{\frac{1}{5}})^{\frac{1}{3}} \\ = (a^{\frac{1}{2} + \frac{1}{5}})^{\frac{1}{3}} = (a^{\frac{7}{10}})^{\frac{1}{3}} = a^{\frac{7}{30}}$$

$$28) a^{\frac{1}{3}}$$

$$\Rightarrow \sqrt[4]{a \sqrt[3]{a}} = \sqrt[4]{a \times a^{\frac{1}{3}}} = (a^{\frac{4}{3}})^{\frac{1}{4}} = a^{\frac{1}{3}}$$

$$29) a^{\frac{1}{8}}$$

$$\Rightarrow \sqrt[4]{\sqrt[3]{a} \sqrt{a}} = \sqrt[4]{\sqrt[3]{a \times a^{\frac{1}{2}}}} = \sqrt[4]{(a^{\frac{3}{2}})^{\frac{1}{3}}} = \sqrt[4]{a^{\frac{1}{2}}} = (a^{\frac{1}{2}})^{\frac{1}{4}} \\ = a^{\frac{1}{8}}$$

$$30) a^{\frac{7}{8}}$$

$$\Rightarrow \sqrt{a \sqrt[3]{a} \sqrt[4]{a}} = \sqrt{a \sqrt{a \times a^{\frac{1}{2}}}} = \sqrt{a \sqrt{a^{\frac{3}{2}}}} \\ = \sqrt{a \times a^{\frac{3}{4}}} = \sqrt{a^{\frac{7}{4}}} = a^{\frac{7}{8}}$$

$$31) a^{\frac{19}{24}}$$

$$\Rightarrow \sqrt{a \sqrt[3]{a} \sqrt[4]{a^3}} \\ = \sqrt{a \sqrt[3]{a \times a^{\frac{3}{4}}}} = \sqrt{a \times (a^{\frac{7}{4}})^{\frac{1}{3}}} = \sqrt{a \times a^{\frac{7}{12}}} \\ = (a^{\frac{19}{12}})^{\frac{1}{2}} = a^{\frac{19}{24}}$$

$$32) 3a^{\frac{7}{8}}$$

$$\Rightarrow \sqrt{9a \sqrt{a} \sqrt[4]{a}} = \left\{ 9a(a \times a^{\frac{1}{2}})^{\frac{1}{2}} \right\}^{\frac{1}{2}} = (9a \times a^{\frac{3}{2} \times \frac{1}{2}})^{\frac{1}{2}}$$

$$= (9a^{\frac{7}{4}})^{\frac{1}{2}} = 9^{\frac{1}{2}} a^{\frac{7}{8}} = 3a^{\frac{7}{8}}$$

$$33) a^{\frac{11}{8}}$$

$$\Rightarrow \sqrt{a \sqrt[2]{a^2} \sqrt[4]{a^3}} = \left\{ a(a^2 \times a^{\frac{3}{2}})^{\frac{1}{2}} \right\}^{\frac{1}{2}} = \left\{ a(a^{\frac{7}{2}})^{\frac{1}{2}} \right\}^{\frac{1}{2}} \\ = (a \times a^{\frac{7}{4}})^{\frac{1}{2}} \\ = (a^{\frac{11}{4}})^{\frac{1}{2}} = a^{\frac{11}{8}}$$

$$34) a^{\frac{7}{15}}$$

$$\Rightarrow (a^{\frac{1}{3}})^{-\frac{3}{5}} \times (a^{\frac{3}{2}})^{\frac{4}{9}} = a^{-\frac{1}{5}} \times a^{\frac{2}{3}} = a^{-\frac{1}{5} + \frac{2}{3}} = a^{\frac{7}{15}}$$

$$35) a^{-\frac{11}{12}}$$

$$\Rightarrow a^{\frac{1}{2}} \times a^{-\frac{2}{3}} \div a^{\frac{3}{4}} = a^{\frac{1}{2} + (-\frac{2}{3}) - \frac{3}{4}} = a^{-\frac{11}{12}}$$

$$36) a^{\frac{9}{10}}$$

$$\Rightarrow a^{\frac{2}{5}} \div a^{-\frac{1}{2}} = a^{\frac{2}{5} - (-\frac{1}{2})} = a^{\frac{9}{10}}$$

$$37) a^{\frac{5}{6}}$$

$$\Rightarrow a^{\frac{1}{3}} \times a^{\frac{1}{2}} = a^{\frac{1}{3} + \frac{1}{2}} = a^{\frac{5}{6}}$$

$$38) a^{\sqrt{3}-\sqrt{2}}$$

$$\Rightarrow a^{\sqrt{2}} \div a^{2\sqrt{2}} \times a^{\sqrt{3}} = a^{\sqrt{2}-2\sqrt{2}+\sqrt{3}} = a^{\sqrt{3}-\sqrt{2}}$$

$$39) a^{-\frac{7}{4}}$$

$$\Rightarrow (a^{-\frac{3}{4}})^2 \times \sqrt{a} \div a^{\frac{3}{4}} = a^{-\frac{3}{2}} \times a^{\frac{1}{2}} \div a^{\frac{3}{4}} = a^{-\frac{3}{2} + \frac{1}{2} - \frac{3}{4}} \\ = a^{-\frac{7}{4}}$$

$$40) a^{\frac{2}{5}}$$

$$\Rightarrow (\sqrt{a^3} \times \sqrt[5]{a} \times a^{-\frac{1}{2}})^{\frac{1}{3}} = (a^{\frac{3}{2}} \times a^{\frac{1}{5}} \times a^{-\frac{1}{2}})^{\frac{1}{3}} \\ = (a^{\frac{3}{2} + \frac{1}{5} - \frac{1}{2}})^{\frac{1}{3}} = (a^{\frac{6}{5}})^{\frac{1}{3}} = a^{\frac{2}{5}}$$

$$41) a^{-\frac{13}{12}}$$

$$\Rightarrow \sqrt[4]{a^3} \times \sqrt[3]{a^2} \div \sqrt{a^5} = a^{\frac{3}{4}} \times a^{\frac{2}{3}} \div a^{\frac{5}{2}} \\ = a^{\frac{3}{4} + \frac{2}{3} - \frac{5}{2}} = a^{-\frac{13}{12}}$$

$$42) a^{\frac{13}{12}}$$

$$\Rightarrow \sqrt[4]{a^5} \times \sqrt{a^3} \div \sqrt[3]{a^5} = a^{\frac{5}{4}} \times a^{\frac{3}{2}} \div a^{\frac{5}{3}} \\ = a^{\frac{5}{4} + \frac{3}{2} - \frac{5}{3}} = a^{\frac{13}{12}}$$

43) 1

$$\Rightarrow a^{-\frac{1}{2}} \div a^{\frac{1}{4}} \times a^{\frac{3}{4}} = a^{\left(-\frac{1}{2}\right) - \frac{1}{4} + \frac{3}{4}} = a^0 = 1$$

44) $a^{\frac{13}{30}}$

$$\begin{aligned}\Rightarrow (\sqrt{a^3} \div \sqrt[5]{a})^{\frac{1}{3}} &= (a^{\frac{3}{2}} \div a^{\frac{1}{5}})^{\frac{1}{3}} \\ &= (a^{\frac{3}{2} - \frac{1}{5}})^{\frac{1}{3}} \\ &= (a^{\frac{13}{10}})^{\frac{1}{3}} = a^{\frac{13}{30}}\end{aligned}$$

45) $a^{\frac{17}{18}}$

$$\Rightarrow (\sqrt[3]{a^2} \times \sqrt[4]{a^3})^{\frac{2}{3}} = (a^{\frac{2}{3}} \times a^{\frac{3}{4}})^{\frac{2}{3}} = (a^{\frac{17}{12}})^{\frac{2}{3}} = a^{\frac{17}{18}}$$

46) a

$$\begin{aligned}\Rightarrow \sqrt[3]{\sqrt[4]{a^7}} \times \sqrt[4]{\sqrt[3]{a^5}} &= \sqrt[12]{a^7} \times \sqrt[12]{a^5} = \sqrt[12]{a^7 \times a^5} \\ &= \sqrt[12]{a^{12}} = a\end{aligned}$$

47) $\frac{1}{a}$

$$\begin{aligned}\Rightarrow \frac{\sqrt[10]{a^6}}{(\sqrt[5]{a^4})^2} &= \frac{\sqrt[5]{a^3}}{\sqrt[5]{a^8}} = \sqrt[5]{\frac{a^3}{a^8}} = \sqrt[5]{\frac{1}{a^5}} \\ &= \sqrt[5]{\left(\frac{1}{a}\right)^5} = \frac{1}{a}\end{aligned}$$

48) a^2

$$\begin{aligned}\Rightarrow \sqrt[7]{a^{11}} \times \sqrt[14]{a^6} &= \sqrt[7]{a^{11}} \times \sqrt[7]{a^3} = \sqrt[7]{a^{11} \times a^3} = \sqrt[7]{a^{14}} \\ &= \sqrt[7]{(a^2)^7} = a^2\end{aligned}$$

49) $a^3\sqrt{3}$

$$\Rightarrow a^{\sqrt{3}} \times a^{\sqrt{12}} = a^{\sqrt{3} + 2\sqrt{3}} = a^{3\sqrt{3}}$$

50) $a^3\sqrt{2}$

$$\Rightarrow a^{\sqrt{32}} \times a^{\sqrt{8}} \div a^{\sqrt{18}} = a^{4\sqrt{2} + 2\sqrt{2} - 3\sqrt{2}} = a^{3\sqrt{2}}$$

51) $a^{\sqrt{3}}$

$$\Rightarrow (a^{\frac{\sqrt{3}}{2}})^4 \div a^{\sqrt{3}} = a^{2\sqrt{3} - \sqrt{3}} = a^{\sqrt{3}}$$

52) $a^{2\sqrt{2}}$

$$\begin{aligned}\Rightarrow a^{-\frac{\sqrt{2}}{3}} \times a^{-\frac{2\sqrt{2}}{3}} \div a^{-3\sqrt{2}} &= a^{\left(-\frac{\sqrt{2}}{3}\right) + \left(-\frac{2\sqrt{2}}{3}\right) - (-3\sqrt{2})} \\ &= a^{2\sqrt{2}}\end{aligned}$$

53) a

$$\begin{aligned}\Rightarrow (a^6)^{-\frac{1}{3}} \div (a^{-3\sqrt{2}})^{\frac{1}{\sqrt{2}}} &= a^{-2} \div a^{-3} \\ &= \frac{1}{a^2} \div \frac{1}{a^3} = \frac{a^3}{a^2} = a\end{aligned}$$

54) $a^{\frac{19}{12}}b^{\frac{7}{6}}$

$$\begin{aligned}\Rightarrow (a^3b^2)^{\frac{1}{12}} \times (a^{\frac{1}{3}}b^{\frac{1}{4}})^4 &= a^{\frac{1}{4}}b^{\frac{1}{6}} \times a^{\frac{4}{3}}b \\ &= a^{\frac{1}{4} + \frac{4}{3}}b^{\frac{1}{6} + 1} = a^{\frac{19}{12}}b^{\frac{7}{6}}\end{aligned}$$

55) b

$$\begin{aligned}\Rightarrow \sqrt[3]{ab^2} \times \sqrt[6]{ab^5} \div \sqrt{ab} &= a^{\frac{1}{3}}b^{\frac{2}{3}} \times a^{\frac{1}{6}}b^{\frac{5}{6}} \div a^{\frac{1}{2}}b^{\frac{1}{2}} \\ &= a^{\frac{1}{3} + \frac{1}{6} - \frac{1}{2}}b^{\frac{2}{3} + \frac{5}{6} - \frac{1}{2}} = b\end{aligned}$$

56) $\frac{a^2}{b}$

$$\begin{aligned}\Rightarrow \sqrt[3]{\sqrt[5]{a^{34}b^6}} \div \sqrt[5]{\sqrt[3]{a^4b^{21}}} &= \sqrt[15]{a^{34}b^6} \div \sqrt[15]{a^4b^{21}} \\ &= \frac{\sqrt[15]{a^{34}b^6}}{\sqrt[15]{a^4b^{21}}} = \sqrt[15]{\frac{a^{34}b^6}{a^4b^{21}}} \\ &= \sqrt[15]{\frac{a^{30}}{b^{15}}} = \sqrt[15]{\left(\frac{a^2}{b}\right)^{15}} = \frac{a^2}{b}\end{aligned}$$

57) $a^{\frac{11}{12}}b^{\frac{5}{3}}$

$$\begin{aligned}\Rightarrow \sqrt[3]{a^2b^5} \div \sqrt[4]{a^5b^2} \times \sqrt{a^3b} \\ &= a^{\frac{2}{3}}b^{\frac{5}{3}} \div a^{\frac{5}{4}}b^{\frac{2}{4}} \times a^{\frac{3}{2}}b^{\frac{1}{2}} \\ &= a^{\frac{2}{3} - \frac{5}{4} + \frac{3}{2}}b^{\frac{5}{3} - \frac{2}{4} + \frac{1}{2}} = a^{\frac{11}{12}}b^{\frac{5}{3}}\end{aligned}$$

58) a^6b^9

$$\Rightarrow (a^2\sqrt{2} \times b^3\sqrt{2})^{\frac{3}{\sqrt{2}}} = (a^2\sqrt{2})^{\frac{3}{\sqrt{2}}} \times (b^3\sqrt{2})^{\frac{3}{\sqrt{2}}} = a^6b^9$$

59) ab

$$\begin{aligned}\Rightarrow \frac{\sqrt[3]{a^4b} \times \sqrt[3]{a^2b^4}}{\sqrt[4]{\sqrt[3]{a^4b^6}}} &= \frac{\sqrt[6]{a^4b} \times \sqrt[6]{a^2b^4}}{\sqrt[12]{a^4b^6}} \\ &= \frac{\sqrt[6]{a^4b} \times \sqrt[6]{a^2b^4}}{\sqrt[6]{a^4b^6}} = \sqrt[6]{\frac{a^4b \times a^2b^4}{a^4b^6}} \\ &= \sqrt[6]{a^6b^6} = \sqrt[6]{(ab)^6} = ab\end{aligned}$$

60) $a^{\sqrt{2}}b^{-2\sqrt{3}}$

$$\Rightarrow \left(\frac{a^{\sqrt{3}}}{b^{\sqrt{18}}}\right)^{\frac{2}{\sqrt{6}}} = \frac{(a^{\sqrt{3}})^{\frac{2}{\sqrt{6}}}}{(b^{\sqrt{18}})^{\frac{2}{\sqrt{6}}}} = \frac{a^{\frac{\sqrt{2}}{2}}}{b^{\frac{6}{\sqrt{3}}}} = a^{\sqrt{2}}b^{-2\sqrt{3}}$$

61) $\frac{1}{27}$

$$\Rightarrow (81^{-\frac{1}{3}})^{\frac{9}{4}} = 81^{-\frac{3}{4}} = (3^4)^{-\frac{3}{4}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

62) $\frac{25}{9}$

$$\begin{aligned}\Rightarrow \left\{\left(\frac{3}{5}\right)^{-\frac{5}{2}}\right\}^{\frac{4}{5}} &= \left(\frac{3}{5}\right)^{-\frac{5}{2} \times \frac{4}{5}} = \left(\frac{3}{5}\right)^{-2} \\ &= \left(\frac{5}{3}\right)^2 = \frac{25}{9}\end{aligned}$$

63) 8

$$\Rightarrow 25^{-\frac{3}{2}} \times 100^{\frac{3}{2}} = (5^2)^{-\frac{3}{2}} \times (2^2 \times 5^2)^{\frac{3}{2}} \\ = 2^3 \times 5^{-3+3} = 8$$

64) $\frac{2}{9}$

$$\Rightarrow 9^{-\frac{3}{2}} \times 36^{\frac{1}{2}} = (3^2)^{-\frac{3}{2}} \times (6^2)^{\frac{1}{2}} = 3^{-3} \times 6 = \frac{6}{3^3} = \frac{2}{9}$$

65) $\frac{25}{27}$

$$\Rightarrow 81^{-\frac{3}{4}} \div 125^{-\frac{2}{3}} = (3^4)^{-\frac{3}{4}} \div (5^3)^{-\frac{2}{3}} = 3^{-3} \div 5^{-2} \\ = \frac{1}{3^3} \div \frac{1}{5^2} = \frac{1}{3^3} \times 5^2 = \frac{25}{27}$$

66) $7^{\frac{7}{4}}$

$$\Rightarrow 7^{\frac{5}{4}} \times 7^{-\frac{3}{2}} \div 7^{-2} = 7^{\frac{5}{4} + (-\frac{3}{2}) - (-2)} = 7^{\frac{7}{4}}$$

67) $\frac{9}{8}$

$$\Rightarrow 32^{-\frac{3}{5}} \div 27^{-\frac{2}{3}} = (2^5)^{-\frac{3}{5}} \div (3^3)^{-\frac{2}{3}} = 2^{-3} \div 3^{-2} \\ = \frac{1}{2^3} \div \frac{1}{3^2} = \frac{3^2}{2^3} = \frac{9}{8}$$

68) $\sqrt{2}$

$$\Rightarrow 16^{\frac{1}{4}} \div 16^{\frac{1}{8}} = 16^{\frac{1}{4} - \frac{1}{8}} = 16^{\frac{1}{8}} = (2^4)^{\frac{1}{8}} = 2^{\frac{1}{2}} = \sqrt{2}$$

69) $\frac{8}{27}$

$$\Rightarrow \left\{ \left(\frac{3}{2} \right)^{-\frac{4}{3}} \right\}^{\frac{9}{4}} = \left(\frac{3}{2} \right)^{-3} = \frac{1}{\left(\frac{3}{2} \right)^3} = \frac{8}{27}$$

70) $5^{\frac{7}{4}}$

$$\Rightarrow 5^{\frac{3}{4}} \times 625^{\frac{1}{4}} = 5^{\frac{3}{4}} \times (5^4)^{\frac{1}{4}} = 5^{\frac{3}{4} + 1} = 5^{\frac{7}{4}}$$

71) 24

$$\Rightarrow 16^{-\frac{3}{2}} \times 64^{\frac{3}{2}} \div 27^{-\frac{1}{3}} = (2^4)^{-\frac{3}{2}} \times (2^6)^{\frac{3}{2}} \div (3^3)^{-\frac{1}{3}} \\ = 2^{-6} \times 2^9 \div 3^{-1} = 2^3 \div \frac{1}{3} \\ = 8 \times 3 = 24$$

72) $\frac{125}{49}$

$$\Rightarrow \sqrt{\frac{5\sqrt{5^{30}}}{\sqrt[3]{7^{12}}}} = \frac{\sqrt{5\sqrt{5^{30}}}}{\sqrt[3]{7^{12}}} = \frac{10\sqrt{5^{30}}}{\sqrt[6]{7^{12}}}$$

$$= \frac{10\sqrt{(5^3)^{10}}}{\sqrt[6]{(7^2)^6}} = \frac{5^3}{7^2} = \frac{125}{49}$$

73) $5^{\frac{5}{2}}$

$$\Rightarrow (5^{\frac{3}{2}})^2 \div \sqrt{5} = 5^{\frac{3}{2} \times 2} \div 5^{\frac{1}{2}} = 5^{3 - \frac{1}{2}} = 5^{\frac{5}{2}}$$

74) 9

$$\Rightarrow (\sqrt[12]{3^5})^2 \times (\sqrt[12]{3^2})^7 \\ = \sqrt[12]{(3^5)^2} \times \sqrt[12]{(3^2)^7} = \sqrt[12]{3^{10}} \times \sqrt[12]{3^{14}} \\ = \sqrt[12]{3^{10} \times 3^{14}} = \sqrt[12]{3^{24}} = \sqrt[12]{(3^2)^{12}} \\ = 3^2 = 9$$

75) 2

$$\Rightarrow \sqrt[4]{\sqrt[3]{16}} \times \sqrt[6]{\sqrt{256}} \\ = \sqrt[12]{16} \times \sqrt[12]{256} = \sqrt[12]{2^4} \times \sqrt[12]{2^8} \\ = \sqrt[12]{2^4 \times 2^8} = \sqrt[12]{2^{12}} = 2$$

76) $\frac{4}{3}$

$$\Rightarrow \left\{ \left(\frac{27}{64} \right)^{-\frac{1}{3}} \right\}^{\frac{3}{2}} \times \left(\frac{3}{4} \right)^{\frac{1}{2}} = \left(\frac{27}{64} \right)^{-\frac{1}{2}} \times \left(\frac{3}{4} \right)^{\frac{1}{2}} \\ = \left(\frac{3^3}{4^3} \right)^{-\frac{1}{2}} \times \left(\frac{3}{4} \right)^{\frac{1}{2}} \\ = \left(\frac{3}{4} \right)^{-\frac{3}{2} + \frac{1}{2}} = \left(\frac{3}{4} \right)^{-1} = \frac{4}{3}$$

77) $\frac{9}{64}$

$$\Rightarrow (3^{-\frac{2}{3}})^6 \times \left\{ \left(\frac{2}{3} \right)^{-\frac{3}{2}} \right\}^4 = 3^{-4} \times \left(\frac{2}{3} \right)^{-6} = \frac{1}{3^4} \times \left(\frac{1}{\left(\frac{2}{3} \right)^6} \right) \\ = \frac{1}{3^4} \times \frac{3^6}{2^6} = \frac{3^2}{2^6} = \frac{9}{64}$$

78) 2

$$\Rightarrow \sqrt{\frac{\sqrt[3]{1024}}{\sqrt[6]{27}}} \div \sqrt[3]{\frac{\sqrt[4]{16}}{\sqrt[4]{27}}} = \frac{\sqrt{\sqrt[3]{1024}}}{\sqrt{\sqrt[6]{27}}} \div \frac{\sqrt[3]{\sqrt[4]{16}}}{\sqrt[3]{\sqrt[4]{27}}} \\ = \frac{\sqrt[6]{1024}}{\sqrt[12]{27}} \div \frac{\sqrt[6]{16}}{\sqrt[12]{27}} = \frac{\sqrt[6]{1024}}{\sqrt[12]{27}} \times \frac{\sqrt[12]{27}}{\sqrt[6]{16}} \\ = \frac{\sqrt[6]{1024}}{\sqrt[6]{16}} = \sqrt[6]{\frac{2^{10}}{2^4}} = \sqrt[6]{2^6} = 2$$

79) 8

$$\Rightarrow (4^{\sqrt{3}})^{\frac{\sqrt{3}}{2}} = 4^{\sqrt{3} \times \frac{\sqrt{3}}{2}} = 4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^3 = 8$$

80) $12^{\sqrt{5}}$

$$\Rightarrow 3^{\sqrt{5}} \times 4^{\sqrt{5}} = (3 \times 4)^{\sqrt{5}} = 12^{\sqrt{5}}$$

81) $6^{\sqrt{2}}$

$$\begin{aligned}\Rightarrow (4^{\frac{1}{\sqrt{6}}} \times 3^{\sqrt{\frac{2}{3}}})^{\sqrt{3}} &= (2^{\frac{2}{\sqrt{6}}} \times 3^{\frac{\sqrt{2}}{\sqrt{3}}})^{\sqrt{3}} \\ &= 2^{\sqrt{2}} \times 3^{\sqrt{2}} = 6^{\sqrt{2}}\end{aligned}$$

82) 144

$$\Rightarrow (2^{\sqrt{8}} \times 3^{\sqrt{2}})^{\sqrt{2}} = 2^{\sqrt{16}} \times 3^2 = 2^4 \times 3^2 = 144$$

83) $3^{\sqrt{2}}$

$$\Rightarrow 3^{\frac{\sqrt{2}}{3}} \times 3^{\frac{\sqrt{8}}{3}} = 3^{\frac{\sqrt{2}}{3} + \frac{2\sqrt{2}}{3}} = 3^{\frac{3\sqrt{2}}{3}} = 3^{\sqrt{2}}$$

84) 1

$$\begin{aligned}\Rightarrow 8^{-\frac{\sqrt{3}}{6}} \times 2^{\frac{\sqrt{3}}{2}} &= (2^3)^{-\frac{\sqrt{3}}{6}} \times 2^{\frac{\sqrt{3}}{2}} = 2^{-\frac{\sqrt{3}}{2}} \times 2^{\frac{\sqrt{3}}{2}} \\ &= 2^{-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}} = 1\end{aligned}$$

85) 125

$$\Rightarrow 5^{\sqrt{3}+1} \div 5^{\sqrt{3}-2} = 5^{\sqrt{3}+1-(\sqrt{3}-2)} = 5^3 = 125$$

86) $3^{2\sqrt{5}}$

$$\Rightarrow 3^{\frac{\sqrt{5}}{2}} \times 3^{\frac{3\sqrt{5}}{2}} = 3^{\frac{\sqrt{5}}{2} + \frac{3\sqrt{5}}{2}} = 3^{2\sqrt{5}}$$

87) 8

$$\begin{aligned}\Rightarrow (2^5 \sqrt{3})^{\frac{\sqrt{6}}{3}} \times 2^{2-3\sqrt{2}} \div 2^{2\sqrt{2}-1} \\ &= 2^{5\sqrt{2}} \times 2^{2-3\sqrt{2}} \div 2^{2\sqrt{2}-1} \\ &= 2^{5\sqrt{2}+2-3\sqrt{2}-(2\sqrt{2}-1)} \\ &= 2^3 = 8\end{aligned}$$

88) $2^{4\sqrt{2}}$

$$\Rightarrow 4^{\sqrt{2}} \times 4^{\sqrt{18}} \div 4^{\sqrt{8}} = 4^{\sqrt{2} + \sqrt{18} - \sqrt{8}} = 4^{2\sqrt{2}} = 2^{4\sqrt{2}}$$

89) 324

$$\begin{aligned}\Rightarrow (3^{\sqrt{8}} \times 2^{\sqrt{2}})^{\sqrt{2}} &= (3^{\sqrt{8}})^{\sqrt{2}} \times (2^{\sqrt{2}})^{\sqrt{2}} \\ &= 3^4 \times 2^2 = 324\end{aligned}$$

90) 4

$$\Rightarrow 2^{\sqrt{2}+1} \div 2^{\sqrt{2}-1} = 2^{\sqrt{2}+1-(\sqrt{2}-1)} = 2^2 = 4$$