01 치환적분법을 이용한 정적분의 계산

(1) 치환적분을 이용한 정적분: 구간 $[a,\ b]$에서 연속인 함수 $f(x)$에 대하여 미분가능한 함수 $x=g(t)$의 도함수 $g'(t)$가 구간 $[\alpha,\ \beta]$에서 연속이고 $a=g(\alpha)$, $b=g(\beta)$이면

⇨ $\int_a^b f(x)dx=\int_\alpha^\beta f(g(t))g'(t)dt$

(2) 삼각치환을 이용한 정적분

① 피적분함수가 $\sqrt{a^2-x^2}$, $\dfrac{1}{\sqrt{a^2-x^2}}$ $(a>0)$의 꼴인 경우

⇨ $x=a\sin\theta\left(-\dfrac{\pi}{2}\le\theta\le\dfrac{\pi}{2}\right)$로 치환한다.

② 피적분함수가 $\dfrac{1}{a^2+x^2}(a>0)$의 꼴인 경우

⇨ $x=a\tan\theta\left(-\dfrac{\pi}{2}\le\theta\le\dfrac{\pi}{2}\right)$로 치환한다.

◪ 다음 정적분의 값을 구하여라.

1. $\int_0^1\sqrt{x+2}\,dx$

2. $\int_{-2}^1\sqrt{2-x}\,dx$

3. $\int_{\sqrt{3}}^2 x\sqrt{x^2-3}\,dx$

4. $\int_2^{\sqrt{7}} x\sqrt{x^2-3}\,dx$

5. $\int_0^4\dfrac{1}{\sqrt{x}(\sqrt{x}+2)}dx$

6. $\int_0^4\dfrac{1}{\sqrt{1+2x}}dx$

7. $\int_0^3(2x-1)^2dx$

8. $\int_0^{\sqrt{3}}\dfrac{2x}{x^2+1}dx$

9. $\int_0^2 x(x^2-1)^2dx$

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10. $\int_{\frac{4}{5}}^{1} 5(5x-4)^4\, dx$

11. $\int_{0}^{1} 4x(x^2+1)^3 dx$

12. $\int_{1}^{2} \frac{x}{3x^2+1} dx$

13. $\int_{2}^{5} 2x\sqrt{x^2+5}\, dx$

14. $\int_{\sqrt{e}}^{e^2} \frac{1}{x\ln x} dx$

15. $\int_{e}^{e^2} \frac{1}{x(\ln x)^2} dx$

16. $\int_{1}^{e} \frac{(\ln x)^2}{x} dx$

17. $\int_{0}^{\frac{\pi}{4}} \tan x dx$

18. $\int_{1}^{3} \frac{x+1}{x^2+2x-1} dx$

19. $\int_{0}^{1} \frac{e^x+1}{e^x+x} dx$

20. $\int_{0}^{1} 2xe^{x^2} dx$

21. $\int_{1}^{3} e^{2x-1} dx$

22. $\int_{0}^{1} x^2 e^{x^3+1} dx$

23. $\int_{0}^{\ln 2} \frac{1}{e^x+1} dx$

24. $\displaystyle\int_0^{\ln(2e-1)} \frac{e^x}{e^x+1}dx$

25. $\displaystyle\int_0^{\frac{\pi}{2}} 2\cos\left(2x-\frac{\pi}{6}\right)dx$

26. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2(\tan x-1)\sec^2 x dx$

27. $\displaystyle\int_{-\frac{\pi}{4}}^{0} (\tan^2 x+1)dx$

28. $\displaystyle\int_{-\frac{\pi}{2}}^{0} (2+\sin x)^3 \cos x dx$

29. $\displaystyle\int_0^{\pi} \cos\left(\frac{x}{2}+\pi\right)dx$

30. $\displaystyle\int_0^{\frac{\pi}{4}} (1-\tan^2 x)\sec^2 x dx$

31. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$

32. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx$

33. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{1+\sin x}dx$

34. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x dx$

35. $\displaystyle\int_1^{e^2} \frac{(\ln x)^2}{2x}dx$

36. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot e^{\cos x} dx$

◪ 다음 정적분의 값을 구하여라.

37. $\int_{1}^{\sqrt{3}} \frac{1}{x^2+1} dx$

38. $\int_{0}^{\sqrt{3}} \frac{1}{x^2+9} dx$

39. $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{x^2+9} dx$

40. $\int_{-2}^{\sqrt{3}-1} \frac{1}{x^2+2x+2} dx$

41. $\int_{0}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}} dx$

42. $\int_{-1}^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}} dx$

◪ 다음 물음에 답하여라.

43. 함수 $f(x)=e^x$에 대하여 $\int_{0}^{1} \{f(x)+f(1-x)\} dx$의 값을 구하여라.

44. $k\int_{e}^{2e} \frac{1}{x(\ln x)^2} dx = \ln 2$를 만족하는 k값을 구하여라.

45. $\int_{0}^{a} \sqrt{a^2-x^2}\, dx = \frac{\pi}{2}$일 때, 상수 a^2의 값을 구하여라.

46. $\int_{-a}^{a} \frac{1}{a^2+x^2} dx = \frac{\pi}{6}$일 때, 상수 a의 값을 구하여라.

47. 정적분 $\int_{0}^{1} 16x(x^2+1)^3 dx$의 값을 a, 정적분 $\int_{1}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx$의 값을 b라고 할 때, ab의 값을 구하여라.

02 부분적분법을 이용한 정적분의 계산

두 함수 $f(x)$, $g(x)$가 미분가능하고 $f'(x)$, $g'(x)$가 닫힌구간 $[a,\ b]$에서 연속이면

⇨ $\int_a^b f(x)g'(x)dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$

◪ 다음 정적분의 값을 구하여라.

48. $\int_1^e \ln x dx$

49. $\int_0^1 2xe^x dx$

50. $\int_0^1 xe^x dx$

51. $\int_0^1 xe^{-x} dx$

52. $\int_1^e \ln x^2 dx$

53. $\int_1^{\sqrt{e}} x\ln x dx$

54. $\int_1^e (\ln x)^2 dx$

55. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2x-1)\sin 2x dx$

56. $\int_0^{\ln 2} xe^x dx$

57. $\int_0^{\frac{\pi}{4}} 8x\sin 2x dx$

58. $\int_0^{\frac{\pi}{2}} e^x \cos x dx$

59. $\int_0^{\frac{\pi}{2}} x\cos 2x dx$

60. $\int_0^{\frac{\pi}{2}} x\sin x dx$

61. $2\int_{0}^{\frac{\pi}{2}} x\cos x dx$

62. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\cos x dx$

63. $\int_{0}^{\frac{\pi}{4}} \sin x \cos x dx$

64. $\int_{1}^{e^2} \frac{\ln x}{x^2} dx$

65. $\int_{0}^{1} (1+x)e^x dx$

66. $\int_{1}^{e} 4x\ln x dx$

67. $\int_{0}^{\frac{\pi}{2}} 5e^{2x}\sin x dx$

68. $\int_{0}^{\frac{1}{2}\pi} e^{2x}\sin 2x dx$

69. $\int_{-1}^{1} |x|e^{x+1} dx$

◪ 다음 물음에 답하여라.

70. $\int_{1}^{2} \ln x dx = -q + \ln p$일 때, $p+q$의 값을 구하여라. (단, p, q는 자연수)

71. $\int_{e}^{e^2} \frac{\ln x}{x^2} dx = \frac{a}{e} + \frac{b}{e^2}$일 때, $a+b$의 값을 구하여라. (단, a,b는 정수)

72. 함수 $f(x) = \begin{cases} x-1 & (x \le 1) \\ \ln x & (x > 1) \end{cases}$에 대하여 $\int_{0}^{2} f(x) dx$의 값을 구하여라.

73. $\int_{0}^{a} xe^x dx = e^2+1$일 때, 상수 a의 값을 구하여라.

74. $a=\int_{1}^{e}\frac{\ln x}{x^2}dx$, $b=\int_{-1}^{1}\frac{x^3+2x-2}{x^2}dx$**라 할 때, $b-a$의 값을 구하여라.**

75. **함수** $f(x)=xe^{-x}$**에 대하여**
$\int_{0}^{2}f(x)dx-\int_{-2}^{2}f(x)dx+\int_{-2}^{1}f(x)dx$**의 값을 구하여라.**

76. **함수** $f(x)=2x\ln x$ **에 대하여**
$\int_{2}^{4}f(x)dx-\int_{3}^{4}f(x)dx+\int_{1}^{2}f(x)dx$**의 값을 구하여라.**

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정답 및 해설

1) $2\sqrt{3}-\dfrac{4\sqrt{2}}{3}$

⇨ $x+2=t$로 놓으면 $1=\dfrac{dt}{dx}$

$x=0$일 때 $t=2$, $x=1$일 때 $t=3$이므로

$$\int_0^1 \sqrt{x+2}\,dx=\int_2^3 \sqrt{t}\,dt=\int_2^3 t^{\frac{1}{2}}dt$$
$$=\left[\frac{2}{3}t^{\frac{3}{2}}\right]_2^3=2\sqrt{3}-\frac{4\sqrt{2}}{3}$$

2) $\dfrac{14}{3}$

⇨ $2-x=t$로 놓으면

$-1=\dfrac{dt}{dx}\quad \therefore dx=-dt$

$x=1$일 때 $t=1$, $x=-2$일 때 $t=4$

$$\therefore \int_{-2}^1 \sqrt{2-x}\,dx=-\int_4^1 \sqrt{t}\,dt=\int_1^4 \sqrt{t}\,dt$$
$$=\left[\frac{2}{3}t\sqrt{t}\right]_1^4$$
$$=\frac{2}{3}(4\sqrt{4}-1)$$
$$=\frac{14}{3}$$

3) $\dfrac{1}{3}$

⇨ $x^2-3=t$로 놓으면

$2x=\dfrac{dt}{dx}\quad \therefore xdx=\dfrac{1}{2}dt$

$x=2$일 때 $t=1$, $x=\sqrt{3}$일 때 $t=0$

$$\therefore \int_{\sqrt{3}}^2 x\sqrt{x^2-3}\,dx=\frac{1}{2}\int_0^1 \sqrt{t}\,dt$$
$$=\frac{1}{2}\left[\frac{2}{3}t\sqrt{t}\right]_0^1$$
$$=\frac{1}{3}(1-0)=\frac{1}{3}$$

4) $\dfrac{7}{3}$

⇨ $x^2-3=t$, $2xdx=dt$

$(x=2\to t=1,\ x=\sqrt{7}\to t=4)$

$$\int_2^{\sqrt{7}} x\sqrt{x^2-3}\,dx=\int_1^4 \frac{1}{2}\sqrt{t}\,dt=\frac{1}{2}\cdot\frac{2}{3}\left[t\sqrt{t}\right]_1^4$$
$$=\frac{1}{3}(4\sqrt{4}-1)=\frac{7}{3}$$

5) $\ln 4$

⇨ $\sqrt{x}=t$로 치환하면 $\dfrac{1}{2\sqrt{x}}dx=dt$

$$\int_0^2 \frac{2}{(t+2)}dt=2\left[\ln|t+2|\right]_0^2=2\ln4-2\ln2=2\ln2=\ln4$$

6) 2

⇨ $1+2x=t$로 놓고 양변을 x에 대하여 미분하면

$2=\dfrac{dt}{dx}\quad \therefore dx=\dfrac{1}{2}dt$

$x=0$일 때 $t=1$, $x=4$일 때 $t=9$

$$\int_0^4 \frac{1}{\sqrt{1+2x}}dx=\frac{1}{2}\int_1^9 \frac{1}{\sqrt{t}}dt=\frac{1}{2}\int_1^9 t^{-\frac{1}{2}}dt$$
$$=\frac{1}{2}\left[2\sqrt{t}\right]_1^9=3-1=2$$

7) 21

⇨ $2x-1=t$로 놓으면 $2=\dfrac{dt}{dx}$

$x=0$일 때 $t=-1$, $x=3$일 때 $t=5$이므로

$$\int_0^3 (2x-1)^2dx=\frac{1}{2}\int_{-1}^5 t^2dt=\frac{1}{2}\left[\frac{1}{3}t^3\right]_{-1}^5$$
$$=\frac{1}{2}\left\{\frac{125}{3}-\left(-\frac{1}{3}\right)\right\}=21$$

8) $\ln 4$

⇨ $x^2+1=t$로 치환하자. $2xdx=dt$

$$\int_1^4 \frac{1}{t}dt=\left[\ln|t|\right]_1^4=\ln4$$

9) $\dfrac{14}{3}$

⇨ $x^2-1=t$로 놓으면 $2x=\dfrac{dt}{dx}$

$x=0$일 때 $t=-1$, $x=2$일 때 $t=3$이므로

$$\int_0^2 x(x^2-1)^2dx=\frac{1}{2}\int_{-1}^3 t^2dt$$
$$=\left[\frac{1}{6}t^3\right]_{-1}^3=\frac{14}{3}$$

10) $\dfrac{1}{5}$

⇨ $$\int_{\frac{4}{5}}^1 5(5x-4)^4dx=\left[\frac{1}{5}(5x-4)^5\right]_{\frac{4}{5}}^1=\frac{1}{5}(1-0)=\frac{1}{5}$$

11) $\dfrac{15}{2}$

⇨ $x^2+1=t$라 하면 $2xdx=dt$

$$\int_1^2 2t^3dt=\left[\frac{2}{4}t^4\right]_1^2=\frac{1}{2}2^4-\frac{1}{2}=\frac{15}{2}$$

12) $\dfrac{1}{6}\ln\dfrac{13}{4}$

⇨ $3x^2+1=t$로 놓으면 $6x=\dfrac{dt}{dx}$

$x=1$일 때 $t=4$, $x=2$일 때 $t=13$이므로

$$\int_1^2 \frac{x}{3x^2+1}dx = \int_4^{13}\frac{1}{t}\times\frac{1}{6}dt = \left[\frac{1}{6}\ln|t|\right]_4^{13}$$
$$= \frac{1}{6}\ln 13 - \frac{1}{6}\ln 4$$
$$= \frac{1}{6}\ln\frac{13}{4}$$

13) $20\sqrt{30}-18$

⇨ $x^2+5=t$로 놓으면 $2x=\frac{dt}{dx}$

$x=2$일 때 $t=9$, $x=5$일 때 $t=30$

$$\int_2^5 2x\sqrt{x^2+5}\,dx = \int_9^{30}\sqrt{t}\,dt$$
$$=\left[\ \frac{2}{3}t\sqrt{t}\ \right]_9^{30}$$
$$=20\sqrt{30}-18$$

14) $2\ln 2$

⇨ $$\int_{\sqrt{e}}^{e^2}\frac{1}{x\ln x}dx = \int_{\sqrt{e}}^{e^2}\frac{\frac{1}{x}}{\ln x}dx$$
$$=\int_{\sqrt{e}}^{e^2}\frac{(\ln x)'}{\ln x}dx$$
$$=\left[\ \ln|\ln x|\ \right]_{\sqrt{e}}^{e^2}$$
$$=\ln 2-\ln\frac{1}{2}$$
$$=2\ln 2$$

15) $\frac{1}{2}$

⇨ $\ln x=t$로 놓으면

$\frac{1}{x}=\frac{dt}{dx}$ $\quad\therefore \frac{1}{x}dx=dt$

$x=e^2$일 때 $t=2$, $x=e$일 때 $t=1$

$$\therefore \int_e^{e^2}\frac{1}{x(\ln x)^2}dx = \int_1^2\frac{1}{t^2}dt$$
$$=\left[-\frac{1}{t}\right]_1^2$$
$$=-\frac{1}{2}-(-1)=\frac{1}{2}$$

16) $\frac{1}{3}$

⇨

$\ln x=t$라 하면 $\frac{1}{x}=\frac{dt}{dx}$ $\qquad\therefore\ dx=xdt$

$x=1$일 때, $t=0$이고, $x=e$일 때 $t=1$이므로

$$\int_1^e\frac{(\ln x)^2}{x}dx=\int_0^1 t^2dt=\left[\frac{1}{3}t^3\right]_0^1=\frac{1}{3}$$

17) $\frac{1}{2}\ln 2$

⇨ $$\int_0^{\frac{\pi}{4}}\tan x\,dx=\int_0^{\frac{\pi}{4}}\frac{\sin x}{\cos x}dx$$
$$=-\int_0^{\frac{\pi}{4}}\frac{(-\sin x)}{\cos x}dx$$
$$=-\int_0^{\frac{\pi}{4}}\frac{(\cos x)'}{\cos x}dx$$
$$=\left[\ -\ln|\cos x|\ \right]_0^{\frac{\pi}{4}}$$
$$=-\ln\left|\cos\frac{\pi}{4}\right|-(-\ln|\cos 0|)$$
$$=-\ln\frac{\sqrt{2}}{2}=\frac{1}{2}\ln 2$$

18) $\frac{1}{2}\ln 7$

⇨ $$\int_1^3\frac{x+1}{x^2+2x-1}dx=\frac{1}{2}\int_1^3\frac{(x^2+2x-1)'}{x^2+2x-1}dx$$
$$=\frac{1}{2}\left[\ \ln|x^2+2x-1|\ \right]_1^3$$
$$=\frac{1}{2}(\ln 14-\ln 2)$$
$$=\frac{1}{2}\ln 7$$

19) $\ln(e+1)$

⇨ $$\int_0^1\frac{e^x+1}{e^x+x}dx=\int_0^1\frac{(e^x+x)'}{e^x+x}dx$$
$$=\left[\ \ln|e^x+x|\ \right]_0^1$$
$$=\ln|e^1+1|-\ln|e^0+0|$$
$$=\ln(e+1)$$

20) $e-1$

⇨ $x^2=t$로 놓으면

$2x=\frac{dt}{dx}$ $\quad\therefore 2xdx=dt$

$x=1$일 때 $t=1$, $x=0$일 때 $t=0$

$$\therefore\int_0^1 2xe^{x^2}dx=\int_0^1 e^t dt$$
$$=[e^t]_0^1=e-1$$

21) $\frac{1}{2}(e^5-e)$

⇨ $2x-1=t$로 놓으면 $2=\frac{dt}{dx}$

$x=1$일 때 $t=1$, $x=3$일 때 $t=5$

$$\int_1^3 e^{2x-1}dx=\frac{1}{2}\int_1^5 e^t dt$$
$$=\frac{1}{2}\left[\ e^t\ \right]_1^5$$
$$=\frac{1}{2}(e^5-e)$$

22) $\frac{e}{3}(e-1)$

⇨ $x^3+1=t$로 치환하자. $3x^2dx=dt$

$\int_1^2 \frac{1}{3}e^t dt=\frac{1}{3}[e^t]_1^2=\frac{1}{3}(e^2-e)=\frac{e}{3}(e-1)$

23) $\ln\frac{4}{3}$

⇨ $e^x+1=t$로 치환하자.

$e^x dx=dt$, $dx=\frac{1}{t-1}dt$

$\int_2^3 \frac{1}{t}\left(\frac{1}{t-1}\right)dt=\int_2^3\left(\frac{1}{t-1}-\frac{1}{t}\right)dt$

$=[\ln|t-1|-\ln|t|]_2^3=\ln 2-\ln 3+\ln 2=\ln\frac{4}{3}$

24) 1

⇨ $e^x+1=t$로 놓고 양변을 x에 대하여 미분하면

$e^x=\frac{dt}{dx}$ $\therefore e^x dx=dt$

$x=0$일 때 $t=2$, $x=\ln(2e-1)$일 때 $t=2e$

$\therefore \int_0^{\ln(2e-1)}\frac{e^x}{e^x+1}dx=\int_2^{2e}\frac{1}{t}dt=[\ln t]_2^{2e}$

$=\ln 2e-\ln 2=1$

25) 1

⇨ $2x-\frac{\pi}{6}=t$로 놓으면

$2=\frac{dt}{dx}$ $\therefore 2dx=dt$

$x=\frac{\pi}{2}$일 때 $t=\frac{5}{6}\pi$, $x=0$일 때 $t=-\frac{\pi}{6}$

$\therefore \int_0^{\frac{\pi}{2}}2\cos\left(2x-\frac{\pi}{6}\right)dx=\int_{-\frac{\pi}{6}}^{\frac{5}{6}\pi}\cos t\,dt$

$=[\sin t]_{-\frac{\pi}{6}}^{\frac{5}{6}\pi}$

$=\frac{1}{2}-\left(-\frac{1}{2}\right)=1$

26) $4-2\sqrt{3}$

⇨ $\tan x-1=t$로 놓으면

$\sec^2 x=\frac{dt}{dx}$ $\therefore \sec^2 x dx=dt$

$x=\frac{\pi}{3}$일 때 $t=\sqrt{3}-1$, $x=\frac{\pi}{4}$일 때 $t=0$

$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{3}}2(\tan x-1)\sec^2 x dx$

$=\int_0^{\sqrt{3}-1}2t\,dt=[t^2]_0^{\sqrt{3}-1}=(\sqrt{3}-1)^2=4-2\sqrt{3}$

27) 1

⇨ $\tan x=t$라 하면 $\sec^2 x dx=dt$

$x=-\frac{\pi}{4}$일 때, $t=-1$이고, $x=0$일 때 $t=0$이다.

$\int_{-\frac{\pi}{4}}^0(\tan^2 x+1)dx=\int_{-\frac{\pi}{4}}^0\sec^2 x dx=\int_{-1}^0 1dt=1$

28) $\frac{15}{4}$

⇨ $2+\sin x=t$로 놓고 양변을 x에 대하여 미분하면

$\cos x=\frac{dt}{dx}$ $\therefore \cos x dx=dt$

$x=-\frac{\pi}{2}$일 때 $t=1$, $x=0$일 때 $t=2$

$\int_{-\frac{\pi}{2}}^0(2+\sin x)^3\cos x\,dx=\int_1^2 t^3dt=\left[\frac{1}{4}t^4\right]_1^2=\frac{15}{4}$

29) -2

⇨ $\frac{x}{2}+\pi=t$라 하면 $\frac{1}{2}dx=dt$이고

$0\le x\le\pi$는 $\pi\le t\le\frac{3}{2}\pi$이므로

$\int_0^\pi\cos\left(\frac{x}{2}+\pi\right)dx=2\int_\pi^{\frac{3\pi}{2}}\cos t dt=2[\sin t]_\pi^{\frac{3\pi}{2}}=-2$

30) $\frac{2}{3}$

⇨ $\tan x=t$ 라 하면,

$\tan x=t$, $\sec^2 x\ dx=dt$, $\int_0^{\frac{\pi}{4}}\to\int_0^1$이므로

주어진 정적분은 $\int_0^1 1-t^2dt=\frac{2}{3}$이다.

31) $\frac{1}{3}$

⇨ $\sin x=t$로 놓으면 $\cos x=\frac{dt}{dx}$

$x=0$일 때 $t=0$, $x=\frac{\pi}{2}$일 때 $t=1$

$\int_0^{\frac{\pi}{2}}\sin^2 x\cos x dx=\int_0^1 t^2dt$

$=\left[\frac{1}{3}t^3\right]_0^1$

$=\frac{1}{3}$

32) $\frac{1}{3}$

⇨

$\cos x=t$라 하면 $-\sin x=\frac{dt}{dx}$ $\therefore -\sin x dx=dt$

$x=0$일 때, $t=1$, $x=\frac{\pi}{2}$일 때, $t=0$이므로

$\int_0^{\frac{\pi}{2}}\cos^2 x\sin x dx=-\int_1^0 t^2dt=-\left[\frac{1}{3}t^3\right]_1^0=\frac{1}{3}$

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33) $\frac{1}{2}$

⇨ $1+\sin x=t$로 놓고 양변을 x에 대하여 미분하면

$\cos x=\frac{dt}{dx}$ $\therefore \cos x dx=dt$

$x=0$일 때 $t=1$, $x=\frac{\pi}{2}$일 때 $t=2$

$$\int_0^{\frac{\pi}{2}}\frac{\cos^3 x}{1+\sin x}dx=\int_0^{\frac{\pi}{2}}\frac{(1-\sin^2 x)\cos x}{1+\sin x}dx$$
$$=\int_0^{\frac{\pi}{2}}(1-\sin x)\cos x\, dx$$
$$=\int_1^2(2-t)dt$$
$$=\left[2t-\frac{1}{2}t^2\right]_1^2=\frac{1}{2}$$

34) $\frac{2}{3}$

⇨ $\int_0^{\frac{\pi}{2}}\cos^3 x\, dx=\int_0^{\frac{\pi}{2}}(1-\sin^2 x)\cos x\, dx$이므로

$\sin x=t$로 놓고 양변을 x에 대하여 미분하면

$\cos x=\frac{dt}{dx}$ $\therefore \cos x dx=dt$

$x=0$일 때 $t=0$, $x=\frac{\pi}{2}$일 때 $t=1$

$$\therefore \int_0^{\frac{\pi}{2}}\cos^3 x dx=\int_0^{\frac{\pi}{2}}(1-\sin^2 x)\cos x\, dx$$
$$=\int_0^1(1-t^2)dt$$
$$=\left[t-\frac{1}{3}t^3\right]_0^1=\frac{2}{3}$$

35) $\frac{4}{3}$

⇨ $\ln x=t$로 놓으면 $\frac{1}{x}=\frac{dt}{dx}$

$x=1$일 때 $t=0$, $x=e^2$일 때 $t=2$

$$\int_1^{e^2}\frac{(\ln x)^2}{2x}dx=\frac{1}{2}\int_0^2 t^2 dt$$
$$=\frac{1}{2}\left[\ \frac{1}{3}t^3\ \right]_0^2$$
$$=\frac{4}{3}$$

36) $e^{\frac{\sqrt{3}}{2}}-1$

⇨ $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin x\cdot e^{\cos x}dx$에서

$\cos x=t$라 하면 $-\sin x dx=dt$

$x=\frac{\pi}{6}$일 때, $t=\frac{\sqrt{3}}{2}$, $x=\frac{\pi}{2}$일 때, $t=0$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin x\cdot e^{\cos x}dx=\int_{\frac{\sqrt{3}}{2}}^0\sin t\cdot e^t\cdot\frac{dt}{-\sin x}$$
$$=\int_{\frac{\sqrt{3}}{2}}^0-e^t dt=\left[-e^t\right]_{\frac{\sqrt{3}}{2}}^0=-1+e^{\frac{\sqrt{3}}{2}}$$

37) $\frac{\pi}{12}$

⇨ $x=\tan\theta\ \left(-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right)$로 놓으면

$\frac{dx}{d\theta}=\sec^2\theta$

$x=1$일 때 $\theta=\frac{\pi}{4}$, $x=\sqrt{3}$일 때 $\theta=\frac{\pi}{3}$

또, $x^2+1=\tan^2\theta+1=\sec^2\theta$이므로

$$\int_1^{\sqrt{3}}\frac{1}{x^2+1}dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sec^2\theta}\cdot\sec^2\theta\, d\theta$$
$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}1d\theta$$
$$=\left[\ \theta\ \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$
$$=\frac{\pi}{12}$$

38) $\frac{\pi}{18}$

⇨ $x=3\tan\theta\ \left(-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right)$로 놓으면

$\frac{dx}{d\theta}=3\sec^2\theta$

$x=0$일 때 $\theta=0$, $x=\sqrt{3}$일 때 $\theta=\frac{\pi}{6}$

또, $x^2+9=9(\tan^2\theta+1)=9\sec^2\theta$이므로

$$\int_0^{\sqrt{3}}\frac{1}{x^2+9}dx=\int_0^{\frac{\pi}{6}}\frac{1}{9\sec^2\theta}\cdot 3\sec^2\theta\, d\theta$$
$$=\int_0^{\frac{\pi}{6}}\frac{1}{3}d\theta$$
$$=\left[\ \frac{1}{3}\theta\ \right]_0^{\frac{\pi}{6}}$$
$$=\frac{\pi}{18}$$

39) $\frac{\pi}{9}$

⇨

$x=3\tan\theta$라 하면 $1=3\sec^2\theta\cdot\frac{d\theta}{dx}$ $\therefore$ $dx=3\sec^2\theta d\theta$

$x=-\sqrt{3}$일 때, $\theta=-\frac{\pi}{6}$, $x=\sqrt{3}$일 때, $\theta=\frac{\pi}{6}$

$$\int_{-\sqrt{3}}^{\sqrt{3}}\frac{1}{x^2+9}dx=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{1}{9(1+\tan^2\theta)}3\sec^2\theta\, d\theta$$

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$=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{1}{9\sec^2\theta}3\sec^2\theta\,d\theta=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{1}{3}d\theta=2\int_0^{\frac{\pi}{6}}\frac{1}{3}d\theta$

$=2\left[\frac{1}{3}\theta\right]_0^{\frac{\pi}{6}}=2\times\frac{1}{3}\times\frac{\pi}{6}=\frac{\pi}{9}$

40) $\frac{7}{12}\pi$

⇨ $x+1=\tan\theta\ \left(-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right)$로 놓으면

$\frac{dx}{d\theta}=\sec^2\theta$

$x=-2$일 때 $\theta=-\frac{\pi}{4}$, $x=\sqrt{3}-1$일 때 $\theta=\frac{\pi}{3}$

또, $x^2+2x+2=(x+1)^2+1=\tan^2\theta+1=\sec^2\theta$이므로

$\int_{-2}^{\sqrt{3}-1}\frac{1}{x^2+2x+2}dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sec^2\theta}\cdot\sec^2\theta\,d\theta$

$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}}1\,d\theta$

$=\left[\ \theta\ \right]_{-\frac{\pi}{4}}^{\frac{\pi}{3}}$

$=\frac{7}{12}\pi$

41) $\frac{\pi}{3}$

⇨ $x=\sin\theta$, $dx=\cos\theta\,d\theta$이고,

$x=0$일 때, $\theta=0$이고, $x=\frac{\sqrt{3}}{2}$일 때, $\theta=\frac{\pi}{3}$이므로

$\int_0^{\frac{\sqrt{3}}{2}}\frac{1}{\sqrt{1-x^2}}dx$

$=\int_0^{\frac{\pi}{3}}\frac{\cos\theta}{\sqrt{1-\sin^2\theta}}d\theta=\int_0^{\frac{\pi}{3}}1d\theta=\frac{\pi}{3}$

42) $\frac{5}{12}\pi$

⇨ $x=2\sin t$라 하면 $dx=2\cos t dt$

$\int_{-1}^{\sqrt{2}}\frac{1}{\sqrt{4-x^2}}dx=\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{2\cos t}{\sqrt{4-4\sin^2 t}}dt$

$=\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}}1\,dt=\frac{5}{12}\pi$

43) $2(e-1)$

⇨ $1-x=t$로 놓으면 $-1=\frac{dt}{dx}$

$x=0$일 때 $t=1$, $x=1$일 때 $t=0$

$\int_0^1 f(1-x)dx=-\int_1^0 f(t)dt=\int_0^1 f(t)dt$이므로

$\int_0^1\{f(x)+f(1-x)\}dx=\int_0^1 f(x)dx+\int_0^1 f(1-x)dx$

$=\int_0^1 f(x)dx+\int_0^1 f(t)dt$

$=2\int_0^1 f(x)dx$

$=2\int_0^1 e^x dx$

$=2\left[\ e^x\ \right]_0^1$

$=2(e^1-e^0)$

$=2(e-1)$

44) $\ln 2e$

⇨ $\int_e^{2e}\frac{1}{x(\ln x)^2}dx$

$\ln x=t$라 하면 $\frac{1}{x}dx=dt$

$x=e$일 때 $t=1$, $x=2e$일 때 $t=\ln 2e$

$\int_1^{\ln 2e}\frac{1}{t^2}dt=\left[-\frac{1}{t}\right]_1^{\ln 2e}=-\frac{1}{\ln 2e}+1$

$\left(-\frac{1}{\ln 2e}+1\right)k=\ln 2$

$\therefore\ k=\dfrac{\ln 2}{-\dfrac{1}{\ln 2e}+1}=\dfrac{\ln 2}{-\dfrac{1}{\ln 2+1}+1}=\dfrac{\ln 2}{\dfrac{\ln 2}{\ln 2+1}}$

$=\ln 2+1=\ln 2e$

45) 2

⇨ $x=a\sin\theta\ \left(-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}\right)$로 놓으면

$\frac{dx}{d\theta}=a\cos\theta$

$x=0$일 때 $\theta=0$, $x=a$일 때 $\theta=\frac{\pi}{2}$

$\int_0^a\sqrt{a^2-x^2}\,dx=\int_0^{\frac{\pi}{2}}\sqrt{a^2-a^2\sin^2\theta}\,a\cos\theta\,d\theta$

$=a^2\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta$

$=a^2\int_0^{\frac{\pi}{2}}\frac{1+\cos 2\theta}{2}d\theta$

$=a^2\left[\ \frac{1}{2}\theta+\frac{1}{4}\sin 2\theta\ \right]_0^{\frac{\pi}{2}}$

$=\frac{\pi}{4}a^2$

$\frac{\pi}{4}a^2=\frac{\pi}{2}$이므로 $a^2=2$

46) 3

⇨ $x=a\tan\theta\ \left(-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right)$로 놓으면

$\frac{dx}{d\theta}=a\sec^2\theta$

$x=-a$일 때 $\theta=-\frac{\pi}{4}$, $x=a$일 때 $\theta=\frac{\pi}{4}$

$$\int_{-a}^{a}\frac{1}{a^2+x^2}dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{a^2+a^2\tan^2\theta}\cdot a\sec^2\theta\, d\theta$$

$$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{a^2\sec^2\theta}\cdot a\sec^2\theta\, d\theta$$

$$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{a}d\theta$$

$$=\frac{\pi}{2a}$$

$\frac{\pi}{2a}=\frac{\pi}{6}$이므로 $a=3$

47) 5π

⇨ $\int_0^1 16x(x^2+1)^3dx$

$x^2+1=t$로 치환하자. $2xdx=dt$

$\int_1^2 8t^3dt=[2t^4]_1^2=2(16-1)=30$ $\quad\therefore a=30$

$\int_1^{\sqrt{3}}\frac{1}{\sqrt{4-x^2}}dx$

$x=2\sin t$로 치환하자. $dx=2\cos tdt$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}dt=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$ $\quad\therefore b=\frac{\pi}{6}$

$\therefore\ ab=5\pi$

48) 1

⇨ $f(x)=\ln x$, $g'(x)=1$로 놓으면

$f'(x)=\frac{1}{x}$, $g(x)=x$

$$\therefore\int_1^e \ln xdx=[x\ln x]_1^e-\int_1^e\frac{1}{x}\times xdx$$
$$=e-[x]_1^e=e-(e-1)$$
$$=1$$

49) 2

⇨ $f(x)=2x$, $g'(x)=e^x$으로 놓으면

$f'(x)=2$, $g(x)=e^x$

$$\therefore\int_0^1 2xe^xdx=[2xe^x]_0^1-\int_0^1 2e^xdx$$
$$=2e-[2e^x]_0^1$$
$$=2e-(2e-2)$$
$$=2$$

50) 1

⇨ $f(x)=x$, $g'(x)=e^x$으로 놓으면

$f'(x)=1$, $g(x)=e^x$이므로

$$\int_0^1 xe^xdx=[xe^x]_0^1-\int_0^1 e^xdx$$
$$=e-[e^x]_0^1=e-(e-1)=1$$

51) $1-\frac{2}{e}$

⇨

$$\int_0^1 xe^{-x}dx=[-xe^{-x}]_0^1+\int_0^1 e^{-x}dx=-\frac{1}{e}+[-e^{-x}]_0^1$$
$$=-\frac{1}{e}-\frac{1}{e}+1=1-\frac{2}{e}$$

52) 2

⇨ $\int_1^e 2\ln xdx=[2x\ln x]_1^e-\int_1^e 2dx$

$=2e-2(e-1)=2$

53) $\frac{1}{4}$

⇨ $f(x)=\ln x$, $g'(x)=x$로 놓으면

$f'(x)=\frac{1}{x}$, $g(x)=\frac{1}{2}x^2$

$$\int_1^{\sqrt{e}}x\ln xdx=\left[\ln x\cdot\frac{1}{2}x^2\right]_1^{\sqrt{e}}-\int_1^{\sqrt{e}}\frac{1}{x}\cdot\frac{1}{2}x^2dx$$
$$=\left[\frac{1}{2}x^2\ln x\right]_1^{\sqrt{e}}-\int_1^{\sqrt{e}}\frac{1}{2}x\,dx$$
$$=\frac{e}{4}-\left[\frac{1}{4}x^2\right]_1^{\sqrt{e}}$$
$$=\frac{e}{4}-\left(\frac{e}{4}-\frac{1}{4}\right)$$
$$=\frac{1}{4}$$

54) $e-2$

⇨

$t=\ln x$라 하면 $x=e^t$, $1=e^t\cdot\frac{dt}{dx}$ $\quad\therefore dx=e^t\cdot dt$

$x=1$일 때, $t=0$, $x=e$일 때, $t=1$이므로

$$\int_1^e(\ln x)^2dx=\int_0^1 t^2e^tdt=[t^2e^t]_0^1-\int_0^1 2te^tdt$$
$$=e-[2te^t]_0^1+\int_0^1 2e^tdt=e-2e+2e-2=e-2$$

55) $\frac{\pi}{2}-1$

⇨ $f(x)=2x-1$, $g'(x)=\sin 2x$로 놓으면

$f'(x)=2$, $g(x)=-\frac{1}{2}\cos 2x$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2x-1)\sin 2xdx$$

$$=\left[-\frac{1}{2}(2x-1)\cos 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos 2xdx$$

$$=\frac{\pi}{2}-\frac{1}{2}+\left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=\frac{\pi}{2}-\frac{1}{2}-\frac{1}{2}$$

$$=\frac{\pi}{2}-1$$

56) $2\ln 2-1$

⇨ $f(x)=x$, $g'(x)=e^x$로 놓으면

$f'(x)=1$, $g(x)=e^x$

$$\int_0^{\ln 2} xe^x dx=\left[\ xe^x\ \right]_0^{\ln 2}-\int_0^{\ln 2} e^x dx$$

$$=2\ln 2-\left[\ e^x\ \right]_0^{\ln 2}$$

$$=2\ln 2-1$$

57) 2

58) $\frac{1}{2}\left(e^{\frac{\pi}{2}}-1\right)$

⇨ $f(x)=e^x$, $g'(x)=\cos x$ 로 놓으면

$f'(x)=e^x$, $g(x)=\sin x$

$$\int_0^{\frac{\pi}{2}} e^x \cos x dx$$

$$=\left[\ e^x \sin x\ \right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}} e^x \sin x dx$$

$$=\left[\ e^x \sin x\ \right]_0^{\frac{\pi}{2}}-\left\{\left[\ -e^x \cos x\ \right]_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}} e^x \cos x dx\right\}$$

에서

$$2\int_0^{\frac{\pi}{2}} e^x \cos x dx=\left[\ e^x \sin x\ \right]_0^{\frac{\pi}{2}}+\left[\ e^x \cos x\ \right]_0^{\frac{\pi}{2}}$$

$$=e^{\frac{\pi}{2}}-1$$

$$\therefore \int_0^{\frac{\pi}{2}} e^x \cos x dx=\frac{1}{2}\left(e^{\frac{\pi}{2}}-1\right)$$

59) $-\frac{1}{2}$

⇨ $$\int_0^{\frac{\pi}{2}} x\cos 2x dx=\left[\frac{1}{2}x\sin 2x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\frac{1}{2}\sin 2x dx$$

$$=\left[\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}}=-\frac{1}{4}-\frac{1}{4}=-\frac{1}{2}$$

60) 1

⇨ $f(x)=x$, $g'(x)=\sin x$로 놓으면

$f'(x)=1$, $g(x)=-\cos x$ 이므로

$$\int_0^{\frac{\pi}{2}} x\sin x dx=[-x\cos x]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(-\cos x)dx$$

$$=0+[\sin x]_0^{\frac{\pi}{2}}=1$$

61) $\pi-2$

⇨ $$2\int_0^{\frac{\pi}{2}} x\cos x dx=2[x\sin x]_0^{\frac{\pi}{2}}-2\int_0^{\frac{\pi}{2}}\sin x dx$$

$$=2\left(\frac{\pi}{2}\right)-2[-\cos x]_0^{\frac{\pi}{2}}=\pi-2$$

62) $\frac{\pi^2}{2}-4$

⇨ $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\cos x dx=[x^2\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\sin x dx$$

$$=[x^2\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-[-2x\cos x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos x dx$$

$$=[x^2\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-[-2x\cos x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-[2\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{\pi^2}{2}-4$$

63) $\frac{1}{4}$

⇨ $f(x)=\sin x$, $g'(x)=\cos x$로 놓으면

$f'(x)=\cos x$, $g(x)=\sin x$이므로

$$\int_0^{\frac{\pi}{4}}\sin x\cos x\,dx=[\sin^2 x]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}\sin x\cos x\,dx$$

$$2\int_0^{\frac{\pi}{4}}\sin x\cos x\,dx=[\sin^2 x]_0^{\frac{\pi}{4}}=\frac{1}{2}$$

$$\therefore \int_0^{\frac{\pi}{4}}\sin x\cos x\,dx=\frac{1}{4}$$

64) $1-\frac{3}{e^2}$

⇨ $f(x)=\ln x$, $g'(x)=\frac{1}{x^2}$로 놓으면

$f'(x)=\frac{1}{x}$, $g(x)=-\frac{1}{x}$이므로

$$\int_1^{e^2}\frac{\ln x}{x^2}dx=\left[-\frac{1}{x}\ln x\right]_1^{e^2}+\int_1^{e^2}\frac{1}{x^2}dx$$

$$=-\frac{2}{e^2}+\left[-\frac{1}{x}\right]_1^{e^2}$$

$$=1-\frac{3}{e^2}$$

65) e

⇨ $f(x)=1+x$, $g'(x)=e^x$으로 놓으면

$f'(x)=1$, $g(x)=e^x$이므로

$$\int_0^1(1+x)e^x dx=[(1+x)e^x]_0^1-\int_0^1 e^x dx$$

$$=2e-1-[e^x]_0^1=e$$

66) e^2+1

⇨ $f(x)=\ln x$, $g'(x)=4x$로 놓으면

$f'(x)=\frac{1}{x}$, $g(x)=2x^2$이므로

$$\int_1^e 4x\ln x\,dx=[2x^2\ln x]_1^e-\int_1^e 2x\,dx$$

$$=2e^2-[x^2]_1^e=e^2+1$$

67) $2e^{\pi}+1$

⇨

$$\int_0^{\frac{\pi}{2}} 5e^{2x}\sin x dx = 5\left[\frac{1}{2}e^{2x}\sin x\right]_0^{\frac{\pi}{2}} - 5\int_0^{\frac{\pi}{2}}\frac{1}{2}e^{2x}\cos x dx$$

$$=\frac{5}{2}e^{\pi}-\frac{5}{2}\left[\frac{1}{2}e^{2x}\cos x\right]_0^{\frac{\pi}{2}}-\frac{5}{4}\int_0^{\frac{\pi}{2}}e^{2x}\sin x dx$$

$$\frac{25}{4}\int_0^{\frac{\pi}{2}}e^{2x}\sin x dx=\frac{5}{2}e^{\pi}+\frac{5}{4}$$

$$\therefore \int_0^{\frac{\pi}{2}}5e^{2x}\sin x dx=\frac{4}{5}\left(\frac{5}{2}e^{\pi}+\frac{5}{4}\right)=2e^{\pi}+1$$

68) $\frac{1}{2}e^{\pi}+\frac{1}{2}$

⇨ $2x=t$라 하면 $2dx=dt$이고, $0\le x\le \frac{\pi}{2}$일 때,

$0\le t\le \pi$이므로

$$\int_0^{\frac{\pi}{2}}e^{2x}\sin 2x dx=\frac{1}{2}\int_0^{\pi}e^t\sin t dt=\frac{1}{2}\left[e^t\sin t\right]_0^{\pi}-\frac{1}{2}\int_0^{\pi}e^t\cos t dt$$

$=-\frac{1}{2}\left[e^t\cos t\right]_0^{\pi}-\frac{1}{2}\int_0^{\pi}e^t\sin t\, dt$이므로

$$\int_0^{\pi}e^t\sin t dt=-\frac{1}{2}\left[e^t\cos t\right]_0^{\pi}=\frac{1}{2}(e^{\pi}+1)$$

69) $2e$

70) 5

⇨ 부분적분에 의해서

$$[x\ln x]_1^2-\int_1^2 1\,dx=\ln 4-1$$

$\therefore\ p+q=4+1=5$

71) -1

⇨

$$\int_e^{e^2}\frac{\ln x}{x^2}dx=\left[-\frac{\ln x}{x}\right]_e^{e^2}-\int_e^{e^2}-\frac{1}{x^2}dx=\left(-\frac{2}{e^2}+\frac{1}{e}\right)-\left[\frac{1}{x}\right]_e^{e^2}$$

$$=\left(-\frac{2}{e^2}+\frac{1}{e}\right)-\left(\frac{1}{e^2}-\frac{1}{e}\right)=-\frac{3}{e^2}+\frac{2}{e}$$

$\therefore\ a+b=2+(-3)=-1$

72) $2\ln 2-\frac{3}{2}$

⇨ $\int_0^2 f(x)dx=\int_0^1(x-1)dx+\int_1^2\ln x dx$

이때,

$$\int_0^1(x-1)dx=\left[\frac{1}{2}x^2-x\right]_0^1=\frac{1}{2}-1=-\frac{1}{2}\cdots ㉠$$

한편, $\int_1^2\ln x\, dx$에서

$u(x)=\ln x,\ v'(x)=1$로 놓으면

$u'(x)=\frac{1}{x}, v(x)=x$이므로

$$\int_1^2\ln x\, dx=[x\ln x]_1^2-\int_1^2 1dx$$

$$=2\ln 2-[x]_1^2=2\ln 2-1\cdots ㉡$$

따라서 ㉠,㉡에서 구하는 값은 $2\ln 2-\frac{3}{2}$

73) 2

⇨ $f(x)=x,\ g'(x)=e^x$으로 놓으면

$f'(x)=1,\ g(x)=e^x$이므로

$$\int_0^a xe^x\, dx=[xe^x]_0^a-\int_0^a e^x\, dx$$

$$=ae^a-[e^x]_0^a=(a-1)e^a+1$$

$\int_0^a xe^x\, dx=e^2+1$이므로

$(a-1)e^a+1=e^2+1,\ (a-1)e^a=e^2$

$\therefore a=2$

74) $3+\frac{2}{e}$

⇨ $a=\int_1^e\frac{\ln x}{x^2}dx$

$\ln x=t$로 치환하자. $\frac{1}{x}dx=dt$

$$=\int_0^1\frac{t}{e^t}dt=\int_0^1 te^{-t}dt=\left[(-1)e^{-t}t\right]_0^1+\int_0^1 e^{-t}dt$$

$$=(-1)e^{-1}+\left[-e^{-t}\right]_0^1=(-1)e^{-1}-e^{-1}+1$$

$$=1-\frac{2}{e}$$

$$b=\int_{-1}^1\left(x+\frac{2}{x}-\frac{2}{x^2}\right)dx=\int_{-1}^1\frac{-2}{x^2}dx$$

$$=\left[\frac{2}{x}\right]_{-1}^1=2-(-2)=4$$

$b-a=3+\frac{2}{e}$

75) $1-\frac{2}{e}$

⇨ $\int_0^2 f(x)dx-\int_{-2}^2 f(x)dx+\int_{-2}^1 f(x)dx$

$$=\int_0^2 f(x)dx+\int_2^{-2}f(x)dx+\int_{-2}^1 f(x)dx$$

$$=\int_0^1 f(x)dx=\int_0^1 xe^{-x}dx$$

$u(x)=x,\ v'(x)=e^{-x}$으로 놓으면

$u'(x)=1,\ v(x)=-e^{-x}$이므로

$$\int_0^1 xe^{-x}dx=\left[-xe^{-x}\right]_0^1+\int_0^1 e^{-x}dx$$

$$=-\frac{1}{e}+\left[-e^{-x}\right]_0^1=1-\frac{2}{e}$$

76) $9\ln 3-4$

⇨

$$\int_{2}^{4} f(x)dx - \int_{3}^{4} f(x)dx + \int_{1}^{2} f(x)dx = \int_{1}^{4} f(x)dx - \int_{3}^{4} f(x)dx$$

$$= \int_{1}^{3} f(x)dx = \int_{1}^{3} 2x\ln x dx = \left[x^2 \ln x\right]_1^3 - \int_{1}^{3} x dx = 9\ln 3 - 4$$

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