



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

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3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 삼각함수 사이의 관계

삼각함수 사이에는 다음과 같은 관계가 성립한다.

(1) $\tan \theta = \frac{\sin \theta}{\cos \theta}$

(2) $\sin^2 \theta + \cos^2 \theta = 1$

(참고) $(\sin x)^2$, $(\cos x)^2$ 은 $\sin^2 x$, $\cos^2 x$ 로 간단히 나타낸다.■ $\sin \theta - \cos \theta = \frac{1}{2}$ 일 때, 다음 식의 값을 구하여라.

1. $\sin \theta \cos \theta$

2. $\tan \theta + \frac{1}{\tan \theta}$

3. $\sin^3 \theta - \cos^3 \theta$

■ $\sin \theta + \cos \theta = \frac{1}{2}$ 일 때, 다음 식의 값을 구하여라.

4. $\sin \theta \cos \theta$

5. $\tan \theta + \frac{1}{\tan \theta}$

6. $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$

7. $\sin^4 \theta + \cos^4 \theta$

■ $\sin \theta + \cos \theta = \frac{1}{3}$ 일 때, 다음 식의 값을 구하여라.

8. $\sin \theta \cos \theta$

9. $(\sin \theta - \cos \theta)^2$

10. $\frac{1}{\sin \theta} + \frac{1}{\cos \theta}$

11. $\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta}$

12. $\tan \theta + \frac{1}{\tan \theta}$

13. $\sin^3 \theta - \cos^3 \theta$

14. $\sin^3 \theta + \cos^3 \theta$

15. $\frac{1}{\cos \theta} \left(\frac{1}{\tan \theta} + 1 \right)$

■ 다음 식의 값을 구하여라.

16. $\cos \theta = \frac{1}{2}$ 일 때, $\frac{\sin \theta}{\tan \theta}$

17. $\sin \theta = \frac{3}{7}$ 일 때, $\cos \theta \tan \theta$

18. $\cos \theta = \frac{1}{4}$ 일 때, $\frac{\tan \theta}{\sin \theta}$

19. $\tan \theta = 3$ 일 때, $(\sin \theta - \cos \theta)^2$

20. $\cos \theta = \frac{1}{3}$ 일 때, $1 + \tan^2 \theta$

21. $\sin \theta = -\frac{1}{\sqrt{3}}$ 일 때, $1 + \frac{1}{\tan^2 \theta}$

22. $\tan \theta = \frac{4}{3}$ 일 때, $\sin \theta \cos \theta$

23. $\sin \theta + \cos \theta = \frac{4}{3}$ 일 때, $\tan \theta + \frac{1}{\tan \theta}$

24. $\sin \theta + \cos \theta = \frac{7}{5}$ 일 때, $\tan \theta + \frac{1}{\tan \theta}$

25. $\sin \theta - \cos \theta = \frac{\sqrt{2}}{4}$ 일 때, $\sin \theta \cos \theta$ 의 값

26. $\sin \theta - \cos \theta = \frac{3}{2}$ 일 때, $\frac{1}{\cos \theta} \left(\tan \theta - \frac{1}{\tan^2 \theta} \right)$

27. $\sin \theta - \cos \theta = \sqrt{2}$ 일 때, $\sin^3 \theta - \cos^3 \theta$ 의 값

28. $\sin \theta \cos \theta = \frac{1}{4}$ 일 때, $\tan \theta + \frac{1}{\tan \theta}$ 의 값

29. $\sin \theta - \cos \theta = \frac{1}{3}$ 일 때, $(\sin \theta + \cos \theta)^2$

▣ θ 는 제2사분면의 각이고 $\sin \theta \cos \theta = -\frac{1}{2}$ 일 때, 다음 식의 값을 구하여라.

30. $\sin \theta + \cos \theta$

31. $\cos \theta - \sin \theta$

32. $\sin^3 \theta - \cos^3 \theta$

33. $\frac{1}{\sin \theta} - \frac{1}{\cos \theta}$

34. $\tan \theta + \frac{1}{\tan \theta}$

▣ 다음 식의 값을 구하여라.

35. $0 < \theta < \frac{\pi}{2}$ 이고 $\sin \theta \cos \theta = \frac{1}{2}$ 일 때,
 $\frac{1}{\sin \theta} + \frac{1}{\cos \theta}$ 의 값

36. $\frac{\pi}{2} \leq \theta \leq \pi$ 이고, $\sin \theta + \cos \theta = \frac{1}{5}$ 일 때,
 $\frac{1}{\sin \theta} + \frac{1}{\cos \theta}$ 의 값

37. 각 θ 가 제4사분면의 각이고 $\sin \theta \cos \theta = -\frac{1}{4}$ 일 때,
 $\cos \theta - \sin \theta$ 의 값

38. 각 θ 가 제4사분면의 각이고 $\sin \theta \cos \theta = -\frac{1}{8}$ 일 때,
 $\cos \theta - \sin \theta$ 의 값

39. 각 θ 가 제2사분면의 각이고 $\sin \theta + \cos \theta = \frac{1}{2}$ 일 때,
 $\sin^2 \theta - \cos^2 \theta$ 의 값

40. 각 θ 가 제3사분면의 각이고 $\sin \theta - \cos \theta = \frac{\sqrt{2}}{2}$ 일 때,
 $\sin \theta \cos \theta$ 의 값

41. 각 θ 가 제2사분면의 각이고 $\sin \theta \cos \theta = -\frac{1}{4}$ 일 때,
 $\cos \theta - \sin \theta$ 의 값

42. 각 θ 가 제3사분면의 각이고 $\sin \theta - \cos \theta = \frac{\sqrt{2}}{2}$ 일 때,
 $\sin \theta + \cos \theta$ 의 값

▣ 다음 식을 간단히 하여라.

$$43. \frac{\sin \theta}{1 + \cos \theta} + \frac{\sin \theta}{1 - \cos \theta}$$

$$44. \tan \theta \times \cos \theta - \frac{1 + 2 \sin \theta \cos \theta}{\sin \theta + \cos \theta}$$

$$45. \frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}$$

$$46. \tan \theta - \frac{\cos \theta}{1 - \sin \theta}$$

$$47. \frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta}$$

$$48. \frac{\cos \theta}{1 + \sin \theta} + \tan \theta$$

$$49. (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$$

$$50. (\sin \theta + \cos \theta)^2 + \frac{(1 - \tan \theta)^2}{1 + \tan^2 \theta}$$

$$51. \left(\sin \theta - \frac{1}{\sin \theta} \right)^2 - \left(\tan \theta - \frac{1}{\tan \theta} \right)^2 + \left(\cos \theta - \frac{1}{\cos \theta} \right)^2$$

$$52. \left(1 + \frac{1}{\tan \theta} - \frac{1}{\sin \theta} \right) \left(1 + \frac{1}{\cos \theta} + \tan \theta \right)$$

$$53. \sqrt{1 - \cos^2 \theta} \quad \left(\text{단, } \frac{\pi}{2} < \theta < \pi \right)$$

$$54. \frac{1}{\sqrt{\tan^2 \theta + 1}} \quad \left(\text{단, } 0 < \theta < \frac{\pi}{2} \right)$$

$$55. \sqrt{1 - \cos^2 \theta \tan^2 \theta} \quad \left(\text{단, } \frac{\pi}{2} < \theta < \pi \right)$$

$$56. \sqrt{1 + 2 \sin \theta \cos \theta} \quad \left(\text{단, } 0 < \theta < \frac{\pi}{2} \right)$$

$$57. \sqrt{1 - 2 \sin \theta \cos \theta} \quad \left(\text{단, } \frac{\pi}{2} < \theta < \pi \right)$$

■ 다음 물음에 답하여라.

58. $\sin x + \cos x = -1$ 일 때, $\sin^{2016} x + \cos^{2016} x$ 의 값을 구하여라.

59. $\log_2 \sin^2 \theta + \log_4 \cos^4 \theta = \log_2 \frac{1}{16}$ 이 성립할 때,
 $\sin \theta - \cos \theta$ 의 값을 구하시오. (단, $0 < \theta < \frac{\pi}{2}$)

02 삼각함수 사이의 관계의 응용

(1) 삼각함수와 이차방정식

이차방정식의 두 근이 삼각함수로 주어진 경우에는
 이차방정식의 근과 계수와의 관계를 이용하여 삼각함수
 사이에 관계를 이용할 수 있는 식을 세운다.

■ 다음 이차방정식의 두 근이 $\sin \theta$, $\cos \theta$ 일 때, 상수 k
 의 값을 구하여라.

60. $x^2 - \sqrt{2}x + k = 0$

61. $3x^2 + 2x + k = 0$

62. $x^2 - \frac{\sqrt{2}}{2}x + k = 0$

63. $x^2 + kx + \frac{1}{4} = 0$ ($k > 0$)

64. $4x^2 + kx - 1 = 0$ ($k > 0$)

65. $10x^2 - kx + 3 = 0$ ($k > 0$)

66. $3x^2 - kx + 1 = 0$ ($k > 0$)

67. $x^2 + kx + \frac{1}{3} = 0$ ($k > 0$)

■ 다음 물음에 답하여라.

68. x 에 대한 이차방정식 $2x^2 - 2\sqrt{2}x + 1 = 0$ 의 두
 근이 $\sin \theta$, $\cos \theta$ 일 때, $|\sin \theta - \cos \theta|$ 의 값을 구하여
 라.

69. 이차방정식 $x^2 - 2\sqrt{2}x + k = 0$ 의 두 근이
 $\frac{1}{\sin \theta}$, $\frac{1}{\cos \theta}$ 일 때, 상수 k 의 값을 구하여라.
 (단, $0 < \theta < \frac{\pi}{2}$)

70. 이차방정식 $5x^2 + \sqrt{5}x + a = 0$ 의 두 근이 $\sin\theta$, $\cos\theta$ 이고 θ 는 제2사분면의 각일 때, $a + \tan\theta$ 의 값을 구하여라. (단, a 는 상수이다.)

71. 이차방정식 $x^2 - (2a-1)x + 1 - 2a = 0$ 의 두 근이 $\sin\theta$, $\cos\theta$ 일 때, $\sin^3\theta + \cos^3\theta$ 의 값을 구하여라. (단, $a > 0$)

72. 이차방정식 $2x^2 - x + k = 0$ 의 두 근을 $\sin\theta$, $\cos\theta$ 라 할 때, $\tan\theta$, $\frac{1}{\tan\theta}$ 를 두 근으로 하고 x^2 의 계수가 3인 이차방정식을 구하여라.



정답 및 해설

1) $\frac{3}{8}$

$$\Rightarrow \sin \theta - \cos \theta = \frac{1}{2} \text{ 양변을 제곱하자.}$$

$$\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = \frac{1}{4}$$

$$1 - 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$2 \sin \theta \cos \theta = \frac{3}{4}$$

$$\therefore \sin \theta \cos \theta = \frac{3}{8}$$

2) $\frac{8}{3}$

$$\begin{aligned} \Rightarrow \tan \theta + \frac{1}{\tan \theta} &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \\ &= \frac{1}{\cos \theta \sin \theta} = \frac{1}{\frac{3}{8}} = \frac{8}{3} \end{aligned}$$

3) $\frac{11}{16}$

$$\begin{aligned} \Rightarrow \sin^3 \theta - \cos^3 \theta &= (\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta) \\ &= \frac{1}{2} \times \left(1 + \frac{3}{8}\right) = \frac{1}{2} \times \frac{11}{8} = \frac{11}{16} \end{aligned}$$

4) $-\frac{3}{8}$

$$\Rightarrow \sin \theta + \cos \theta = \frac{1}{2} \text{ 의 양변을 제곱하면}$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$1 + 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$\therefore \sin \theta \cos \theta = -\frac{3}{8}$$

5) $-\frac{8}{3}$

$$\begin{aligned} \Rightarrow \tan \theta + \frac{1}{\tan \theta} &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} \end{aligned}$$

$$\text{이때, } \sin \theta + \cos \theta = \frac{1}{2} \text{ 의 양변을 제곱하면}$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = \frac{1}{4} \text{ 에서}$$

$$1 + 2 \sin \theta \cos \theta = \frac{1}{4} \text{ 이므로}$$

$$\sin \theta \cos \theta = -\frac{3}{8}$$

$$\therefore (\text{주어진 식}) = \frac{1}{\sin \theta \cos \theta} = \frac{1}{-\frac{3}{8}} = -\frac{8}{3}$$

6) $-\frac{8}{3}$

$$\begin{aligned} \Rightarrow \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} = -\frac{8}{3} \end{aligned}$$

7) $\frac{23}{32}$

$$\Rightarrow \sin \theta + \cos \theta = \frac{1}{2} \text{ 의 양변을 제곱하면}$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$2 \sin \theta \cos \theta = -\frac{3}{4} \quad \therefore \sin \theta \cos \theta = -\frac{3}{8}$$

$$\therefore \sin^4 \theta + \cos^4 \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$= 1^2 - 2(\sin \theta \cos \theta)^2$$

$$= 1 - 2 \cdot \left(-\frac{3}{8}\right)^2$$

$$= 1 - \frac{9}{32} = \frac{23}{32}$$

8) $-\frac{4}{9}$

$$\Rightarrow \sin \theta + \cos \theta = \frac{1}{3} \text{ 의 양변을 제곱하면}$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = \frac{1}{9}$$

$$2 \sin \theta \cos \theta = -\frac{8}{9} \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$\therefore \sin \theta \cos \theta = -\frac{4}{9}$$

9) $\frac{17}{9}$

$$\begin{aligned} \Rightarrow (\sin \theta - \cos \theta)^2 &= \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta \\ &= 1 - 2 \cdot \left(-\frac{4}{9}\right) = \frac{17}{9} \end{aligned}$$

10) $-\frac{3}{4}$

$$\Rightarrow \frac{1}{\sin \theta} + \frac{1}{\cos \theta} = \frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} = \frac{\frac{1}{3}}{-\frac{4}{9}} = -\frac{3}{4}$$

11) $\frac{81}{16}$

$$\Rightarrow \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$= \frac{1}{(\sin \theta \cos \theta)^2} = \frac{1}{\left(-\frac{4}{9}\right)^2} = \frac{81}{16}$$

$$12) -\frac{9}{4}$$

$$\begin{aligned} \Rightarrow \tan \theta + \frac{1}{\tan \theta} &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{-\frac{4}{9}} = -\frac{9}{4} \end{aligned}$$

$$13) \pm \frac{5\sqrt{17}}{27}$$

$$\begin{aligned} \Rightarrow \sin \theta + \cos \theta &= \frac{1}{3} \\ \sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta &= \frac{1}{9} \\ 2\sin \theta \cos \theta &= \frac{1}{9} - 1 = -\frac{8}{9} \\ \sin \theta \cos \theta &= -\frac{4}{9} \\ (\sin \theta - \cos \theta)^2 &= \sin^2 \theta - 2\sin \theta \cos \theta + \cos^2 \theta \\ &= 1 + \frac{8}{9} = \frac{17}{9} \\ \sin \theta - \cos \theta &= \pm \frac{\sqrt{17}}{3} \\ \sin \theta \cos \theta &= -\frac{4}{9} \\ \sin^3 \theta - \cos^3 \theta &= (\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta) \\ &= \pm \frac{\sqrt{17}}{3} \times \left(1 - \frac{4}{9}\right) = \left(\pm \frac{\sqrt{17}}{3}\right) \times \frac{5}{9} = \pm \frac{5\sqrt{17}}{27} \end{aligned}$$

$$14) \frac{13}{27}$$

$$\begin{aligned} \Rightarrow \sin^3 \theta + \cos^3 \theta &= (\sin \theta + \cos \theta)^3 - 3\sin \theta \cos \theta (\sin \theta + \cos \theta) \\ &= \left(\frac{1}{3}\right)^3 - 3 \cdot \left(-\frac{4}{9}\right) \cdot \left(\frac{1}{3}\right) \\ &= \frac{1}{27} + \frac{4}{9} = \frac{13}{27} \end{aligned}$$

$$15) -\frac{3}{4}$$

$$\begin{aligned} \Rightarrow \sin \theta + \cos \theta &= \frac{1}{3} \text{의 양변을 제곱하면} \\ 1 + 2\sin \theta \cos \theta &= \frac{1}{9} \text{이므로 } \sin \theta \cos \theta = -\frac{4}{9} \\ \therefore \frac{1}{\cos \theta} \left(\frac{1}{\tan \theta} + 1\right) &= \frac{1}{\cos \theta} \left(\frac{\cos \theta}{\sin \theta} + 1\right) = \frac{1}{\sin \theta} + \frac{1}{\cos \theta} \\ &= \frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} = \frac{\frac{1}{3}}{-\frac{4}{9}} = -\frac{3}{4} \end{aligned}$$

$$16) \frac{1}{2}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \text{ 일 때,}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}, \tan \theta = \pm \sqrt{3} \text{ (복호동순)이므로}$$

$$\frac{\sin \theta}{\tan \theta} = \sin \theta \cdot \frac{1}{\tan \theta} = \left(\pm \frac{\sqrt{3}}{2}\right) \left(\pm \frac{1}{\sqrt{3}}\right) = \frac{1}{2}$$

$$17) \frac{3}{7}$$

$$\Rightarrow \sin \theta = \frac{3}{7} \text{ 이므로}$$

$$\cos \theta \tan \theta = \cos \theta \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta = \frac{3}{7}$$

$$18) 4$$

$$\Rightarrow \frac{\tan \theta}{\sin \theta} = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{1}{\cos \theta} = \frac{1}{\frac{1}{4}} = 4$$

$$19) \frac{2}{5}$$

$$\Rightarrow \tan \theta = 3 \text{ 일 때,}$$

$$\sin \theta = \pm \frac{3}{\sqrt{10}}, \cos \theta = \pm \frac{1}{\sqrt{10}} \text{ (복호동순)이므로}$$

$$\begin{aligned} (\sin \theta - \cos \theta)^2 &= \sin^2 \theta + \cos^2 \theta - 2\sin \theta \cos \theta \\ &= 1 - 2\sin \theta \cos \theta \\ &= 1 - 2 \cdot \frac{3}{10} = \frac{2}{5} \end{aligned}$$

$$20) 9$$

$$\Rightarrow \cos \theta = \frac{1}{3} \text{ 이므로}$$

$$\begin{aligned} 1 + \tan^2 \theta &= 1 + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} \\ &= \frac{1}{\cos^2 \theta} = \left(\frac{1}{\cos \theta}\right)^2 \\ &= 3^2 = 9 \end{aligned}$$

$$21) 3$$

$$\Rightarrow \tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} \text{ 에서 } \frac{1}{\tan^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} \text{ 이므로}$$

$$\begin{aligned} 1 + \frac{1}{\tan^2 \theta} &= 1 + \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} \\ &= \frac{1}{\left(-\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{\frac{1}{3}} = 3 \end{aligned}$$

$$22) \frac{12}{25}$$

$$\Rightarrow \tan \theta = \frac{4}{3} \text{ 일 때,}$$

$$\sin \theta = \pm \frac{4}{5}, \cos \theta = \pm \frac{3}{5} \text{ (복호동순)이므로}$$

$$\sin \theta \cos \theta = \left(\pm \frac{4}{5} \right) \left(\pm \frac{3}{5} \right) = \frac{12}{25}$$

$$23) \frac{18}{7}$$

$$\Rightarrow \sin \theta + \cos \theta = \frac{4}{3}$$

$$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = \frac{16}{9}$$

$$2 \sin \theta \cos \theta = \frac{7}{9}$$

$$\sin \theta \cos \theta = \frac{7}{18}$$

$$\begin{aligned} \tan \theta + \frac{1}{\tan \theta} &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} = \frac{18}{7} \end{aligned}$$

$$24) \frac{25}{12}$$

$$\Rightarrow \sin \theta + \cos \theta = \frac{7}{5}$$

$$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = \frac{49}{25}$$

$$2 \sin \theta \cos \theta = \frac{24}{25}$$

$$\sin \theta \cos \theta = \frac{12}{25}$$

$$\begin{aligned} \tan \theta + \frac{1}{\tan \theta} &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} = \frac{25}{12} \end{aligned}$$

$$25) \frac{7}{16}$$

$$\Rightarrow \sin \theta - \cos \theta = \frac{\sqrt{2}}{4} \text{ 의 양변을 제곱하면}$$

$$\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = \frac{1}{8}$$

$$1 - 2 \sin \theta \cos \theta = \frac{1}{8}$$

$$\therefore \sin \theta \cos \theta = \frac{7}{16}$$

$$26) \frac{36}{25}$$

$$\Rightarrow (\sin \theta - \cos \theta)^2 = \frac{9}{4}$$

$$\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = \frac{9}{4}$$

$$1 - 2 \sin \theta \cos \theta = \frac{9}{4}$$

$$\therefore \sin \theta \cos \theta = -\frac{5}{8}$$

$$\begin{aligned} \frac{1}{\cos \theta} \left(\tan \theta - \frac{1}{\tan^2 \theta} \right) &= \frac{\sin \theta}{\cos^2 \theta} - \frac{\cos \theta}{\sin^2 \theta} \\ &= \frac{\sin^3 \theta - \cos^3 \theta}{\cos^2 \theta \sin^2 \theta} \\ &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{\cos^2 \theta \sin^2 \theta} \\ &= \frac{\frac{3}{2} \times \left(1 - \frac{5}{8} \right)}{\frac{25}{64}} = \frac{36}{25} \end{aligned}$$

$$27) \frac{\sqrt{2}}{2}$$

$$\Rightarrow \sin \theta - \cos \theta = \sqrt{2}$$

양변을 제곱하자.

$$\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = 2$$

$$1 - 2 \sin \theta \cos \theta = 2$$

$$\sin \theta \cos \theta = -\frac{1}{2}$$

$$\begin{aligned} \sin^3 \theta - \cos^3 \theta &= (\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta) \\ &= \sqrt{2} \left(1 - \frac{1}{2} \right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \end{aligned}$$

$$28) 4$$

$$\Rightarrow \tan \theta + \frac{1}{\tan \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{1}{\cos \theta \sin \theta} = \frac{1}{\frac{1}{4}} = 4$$

$$29) \frac{17}{9}$$

$$\Rightarrow \sin \theta - \cos \theta = \frac{1}{3} \text{ 의 양변을 제곱하면}$$

$$\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = \frac{1}{9}$$

$$1 - 2 \sin \theta \cos \theta = \frac{1}{9}$$

$$\therefore \sin \theta \cos \theta = \frac{4}{9}$$

$$\begin{aligned} (\sin \theta + \cos \theta)^2 &= (\sin \theta - \cos \theta)^2 + 4 \sin \theta \cos \theta \\ &= \left(\frac{1}{3} \right)^2 + \frac{16}{9} = \frac{17}{9} \end{aligned}$$

$$30) 0$$

$$\begin{aligned} \Rightarrow (\sin \theta + \cos \theta)^2 &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\ &= 1 + 2 \cdot \left(-\frac{1}{2} \right) = 0 \end{aligned}$$

$$31) -\sqrt{2}$$

$$\begin{aligned} \Rightarrow (\cos \theta - \sin \theta)^2 &= \cos^2 \theta + \sin^2 \theta - 2 \cos \theta \sin \theta \\ &= 1 - 2 \cdot \left(-\frac{1}{2} \right) = 2 \end{aligned}$$

$$\sin \theta > 0, \cos \theta < 0 \text{ 이므로 } \cos \theta - \sin \theta = -\sqrt{2}$$

$$32) \frac{\sqrt{2}}{2}$$

$$\begin{aligned} \Rightarrow \sin^3 \theta - \cos^3 \theta &= (\sin \theta - \cos \theta)^3 + 3 \sin \theta \cos \theta (\sin \theta - \cos \theta) \\ &= 2\sqrt{2} + 3 \cdot \left(-\frac{1}{2}\right) \cdot \sqrt{2} = \frac{\sqrt{2}}{2} \end{aligned}$$

$$33) 2\sqrt{2}$$

$$\Rightarrow \frac{1}{\sin \theta} - \frac{1}{\cos \theta} = \frac{\cos \theta - \sin \theta}{\sin \theta \cos \theta} = \frac{-\sqrt{2}}{-\frac{1}{2}} = 2\sqrt{2}$$

$$34) -2$$

$$\begin{aligned} \Rightarrow \tan \theta + \frac{1}{\tan \theta} &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{-\frac{1}{2}} = -2 \end{aligned}$$

$$35) 2\sqrt{2}$$

$$\begin{aligned} \Rightarrow \sin \theta + \cos \theta &= \sqrt{1 + 2\sin \theta \cos \theta} = \sqrt{2} \\ \frac{1}{\sin \theta} + \frac{1}{\cos \theta} &= \frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta} = \frac{\sqrt{2}}{\frac{1}{2}} = 2\sqrt{2} \end{aligned}$$

$$36) -\frac{5}{12}$$

$$37) \frac{\sqrt{6}}{2}$$

$$\begin{aligned} \Rightarrow (\cos \theta - \sin \theta)^2 &= \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta \\ &= 1 - 2 \cdot \left(-\frac{1}{4}\right) = \frac{3}{2} \end{aligned}$$

이때, θ 는 제4사분면의 각이므로

$\sin \theta < 0$, $\cos \theta > 0$ 에서 $\cos \theta - \sin \theta > 0$

$$\therefore \cos \theta - \sin \theta = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$$

$$38) \frac{\sqrt{5}}{2}$$

$$\begin{aligned} \Rightarrow (\cos \theta - \sin \theta)^2 &= 1 - 2\cos \theta \sin \theta = 1 + \frac{1}{4} = \frac{5}{4} \\ \theta &\text{가 제4사분면의 각이므로 } \cos \theta > 0, \sin \theta < 0 \\ \text{따라서 } \cos \theta - \sin \theta &> 0 \\ \therefore \cos \theta - \sin \theta &= \frac{\sqrt{5}}{2} \end{aligned}$$

$$39) \frac{\sqrt{7}}{4}$$

$$\begin{aligned} \Rightarrow (\sin \theta + \cos \theta)^2 &= \frac{1}{4}, \quad 1 + 2\sin \theta \cos \theta = \frac{1}{4} \\ \therefore 2\sin \theta \cos \theta &= -\frac{3}{4} \end{aligned}$$

$$(\sin \theta - \cos \theta)^2 = 1 + \frac{3}{4} = \frac{7}{4}$$

θ 가 제2사분면 각이므로

$$\sin \theta - \cos \theta > 0 \quad \therefore \sin \theta - \cos \theta = \frac{\sqrt{7}}{2}$$

$$\therefore \sin^2 \theta - \cos^2 \theta = (\sin \theta + \cos \theta)(\sin \theta - \cos \theta) = \frac{\sqrt{7}}{4}$$

$$40) \frac{1}{4}$$

$$41) -\frac{\sqrt{6}}{2}$$

$$\begin{aligned} \Rightarrow (\cos \theta - \sin \theta)^2 &= \cos^2 \theta - 2 \cos \theta \sin \theta + \sin^2 \theta \\ &= 1 - 2 \times \left(-\frac{1}{4}\right) = \frac{3}{2} \end{aligned}$$

이때, θ 가 제2사분면의 각이므로

$\sin \theta > 0$, $\cos \theta < 0$ $\therefore \cos \theta - \sin \theta < 0$

$$\therefore \cos \theta - \sin \theta = -\sqrt{\frac{3}{2}} = -\frac{\sqrt{6}}{2}$$

$$42) -\frac{\sqrt{6}}{2}$$

$$43) \frac{2}{\sin \theta}$$

$$\Rightarrow \frac{\sin \theta}{1 + \cos \theta} + \frac{\sin \theta}{1 - \cos \theta} = \sin \theta \left(\frac{2}{1 - \cos^2 \theta} \right) = \frac{2}{\sin \theta}$$

$$44) -\cos \theta$$

$$\begin{aligned} \Rightarrow \tan \theta \times \cos \theta - \frac{1 + 2\sin \theta \cos \theta}{\sin \theta + \cos \theta} \\ &= \sin \theta - \frac{(\sin \theta + \cos \theta)^2}{\sin \theta + \cos \theta} \\ &= \sin \theta - (\sin \theta + \cos \theta) = -\cos \theta \end{aligned}$$

$$45) \frac{2}{\cos \theta}$$

$$\begin{aligned} \Rightarrow \frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} &= \frac{(1 + \sin \theta)^2 + \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{1 + 2\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta (1 + \sin \theta)} = \frac{2(1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} = \frac{2}{\cos \theta} \end{aligned}$$

$$46) -\frac{1}{\cos \theta}$$

$$\begin{aligned} \Rightarrow \tan \theta - \frac{\cos \theta}{1 - \sin \theta} &= \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{1 - \sin \theta} \\ &= \frac{\sin \theta (1 - \sin \theta) - \cos^2 \theta}{\cos \theta (1 - \sin \theta)} \\ &= \frac{\sin \theta - (\sin^2 \theta + \cos^2 \theta)}{\cos \theta (1 - \sin \theta)} \\ &= \frac{\sin \theta - 1}{\cos \theta (1 - \sin \theta)} \\ &= -\frac{1}{\cos \theta} \end{aligned}$$

$$47) \frac{2}{\sin \theta \cos \theta}$$

$$\begin{aligned} \Rightarrow & \frac{\tan \theta}{1+\cos \theta} + \frac{\tan \theta}{1-\cos \theta} \\ &= \tan \theta \times \frac{(1-\cos \theta)+(1+\cos \theta)}{1-\cos^2 \theta} \\ &= \frac{2 \tan \theta}{\sin^2 \theta} = 2 \times \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin^2 \theta} \\ &= \frac{2}{\sin \theta \cos \theta} \end{aligned}$$

$$\begin{aligned} 48) & \frac{1}{\cos \theta} \\ \Rightarrow & \frac{\cos \theta}{1+\sin \theta} + \tan \theta = \frac{\cos \theta}{1+\sin \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta + \sin^2 \theta + \sin \theta}{(1+\sin \theta) \cos \theta} \\ &= \frac{1+\sin \theta}{(1+\sin \theta) \cos \theta} \\ &= \frac{1}{\cos \theta} \end{aligned}$$

$$\begin{aligned} 49) & 2 \\ \Rightarrow & (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 \\ &= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta \\ &\quad + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta \\ &= 2(\sin^2 \theta + \cos^2 \theta) = 2 \end{aligned}$$

$$\begin{aligned} 50) & 2 \\ \Rightarrow & 1 + \tan^2 \theta = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \\ & \text{주어진 값을 정리하자.} \\ & \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta + \cos^2 \theta (1 - \tan^2 \theta)^2 \\ &= 1 + 2 \sin \theta \cos \theta + \cos^2 \theta \left(1 - \frac{\sin^2 \theta}{\cos^2 \theta}\right)^2 \\ &= 1 + 2 \sin \theta \cos \theta + (\cos \theta - \sin \theta)^2 \\ &= 1 + 2 \sin \theta \cos \theta + \cos^2 \theta - 2 \sin \theta \cos \theta + \sin^2 \theta \\ &= 1 + 1 = 2 \end{aligned}$$

$$\begin{aligned} 51) & 1 \\ \Rightarrow & \left(\sin \theta - \frac{1}{\sin \theta}\right)^2 - \left(\tan \theta - \frac{1}{\tan \theta}\right)^2 + \left(\cos \theta - \frac{1}{\cos \theta}\right)^2 \\ &= \left(\sin^2 \theta - 2 + \frac{1}{\sin^2 \theta}\right) - \left(\tan^2 \theta - 2 + \frac{1}{\tan^2 \theta}\right) \\ &\quad + \left(\cos^2 \theta - 2 + \frac{1}{\cos^2 \theta}\right) \\ &= \sin^2 \theta - 2 + \frac{1}{\sin^2 \theta} - \tan^2 \theta + 2 - \frac{1}{\tan^2 \theta} + \cos^2 \theta - 2 + \frac{1}{\cos^2 \theta} \\ &= -1 + \frac{1}{\sin^2 \theta} - \tan^2 \theta - \frac{1}{\tan^2 \theta} + \frac{1}{\cos^2 \theta} \\ &= -1 + \frac{1}{\sin^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \\ &= -1 + \frac{1 - \cos^2 \theta}{\sin^2 \theta} + \frac{1 - \sin^2 \theta}{\cos^2 \theta} \\ &= -1 + \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = -1 + 1 + 1 = 1 \end{aligned}$$

$$\begin{aligned} 52) & 2 \\ \Rightarrow & \left(1 + \frac{1}{\tan \theta} - \frac{1}{\sin \theta}\right) \left(1 + \frac{1}{\cos \theta} + \tan \theta\right) \\ &= \frac{\sin \theta + \cos \theta - 1}{\sin \theta} \cdot \frac{\cos \theta + 1 + \sin \theta}{\cos \theta} \\ &= \frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta \cos \theta} = \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2 \end{aligned}$$

$$\begin{aligned} 53) & \sin \theta \\ \Rightarrow & \frac{\pi}{2} < \theta < \pi \text{에서 } \sin \theta > 0, \cos \theta < 0 \\ & \sqrt{1 - \cos^2 \theta} = \sqrt{\sin^2 \theta} = |\sin \theta| \\ &= \sin \theta \quad (\because \sin \theta > 0) \end{aligned}$$

$$\begin{aligned} 54) & \cos \theta \\ \Rightarrow & 0 < \theta < \frac{\pi}{2} \text{에서 } \cos \theta > 0 \text{ 이므로} \\ & \frac{1}{\sqrt{\tan^2 \theta + 1}} \\ &= \frac{1}{\sqrt{\frac{\sin^2 \theta}{\cos^2 \theta} + 1}} = \frac{1}{\sqrt{\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}}} \\ &= \frac{1}{\sqrt{\frac{1}{\cos^2 \theta}}} = \frac{1}{\left|\frac{1}{\cos \theta}\right|} = \frac{1}{\cos \theta} = \cos \theta \end{aligned}$$

$$\begin{aligned} 55) & -\cos \theta \\ \Rightarrow & \sqrt{1 - \cos^2 \theta \tan^2 \theta} \\ &= \sqrt{\sin^2 \theta + \cos^2 \theta - \cos^2 \theta \cdot \frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \sqrt{\cos^2 \theta} = |\cos \theta| = -\cos \theta \quad (\because \cos \theta < 0) \end{aligned}$$

$$\begin{aligned} 56) & \sin \theta + \cos \theta \\ \Rightarrow & 0 < \theta < \frac{\pi}{2} \text{에서 } \sin \theta > 0, \cos \theta > 0 \text{ 이므로} \\ & \sqrt{1 + 2 \sin \theta \cos \theta} \\ &= \sqrt{(\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cos \theta} \\ &= \sqrt{(\sin \theta + \cos \theta)^2} \\ &= |\sin \theta + \cos \theta| \\ &= \sin \theta + \cos \theta \end{aligned}$$

$$\begin{aligned} 57) & \sin \theta - \cos \theta \\ \Rightarrow & \frac{\pi}{2} < \theta < \pi \text{에서 } \sin \theta > 0, \cos \theta < 0 \\ & \sqrt{1 - 2 \sin \theta \cos \theta} \\ &= \sqrt{\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta} \\ &= \sqrt{(\sin \theta - \cos \theta)^2} \\ &= |\sin \theta - \cos \theta| \\ &= \sin \theta - \cos \theta \quad (\because \sin \theta > 0, \cos \theta < 0) \end{aligned}$$

$$58) 1$$

⇒ 양변을 제곱하면

$$1 + 2\sin x \cos x = 1 \quad \therefore \sin x \cos x = 0$$

$\sin x = -1, \cos x = 0$ 또는 $\sin x = 0, \cos x = -1$ 이므로

$$\sin^{2016} x + \cos^{2016} x = 1$$

59) $\pm \frac{\sqrt{2}}{2}$

⇒ $\log_4(\sin^4 \theta \cos^4 \theta) = \log_4 \frac{1}{16^2}$

$$\sin^4 \theta \cos^4 \theta = \frac{1}{16^2}, \quad \sin^2 \theta \cos^2 \theta = \frac{1}{16}$$

$$\therefore \sin \theta \cos \theta = \pm \frac{1}{4}$$

$$\begin{aligned} (\sin \theta - \cos \theta)^2 &= \sin^2 \theta - 2\sin \theta \cos \theta + \cos^2 \theta \\ &= 1 - 2\sin \theta \cos \theta = 1 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2} \end{aligned}$$

$$\therefore \sin \theta - \cos \theta = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

60) $\frac{1}{2}$

⇒ $x^2 - \sqrt{2}x + k = 0$ 의 두 근이 $\sin \theta, \cos \theta$ 이므로

$$\sin \theta + \cos \theta = \sqrt{2} \quad \dots\dots \textcircled{7}$$

$$\sin \theta \cos \theta = k \quad \dots\dots \textcircled{8}$$

⑦의 양변을 제곱한 후 ⑧을 대입하면

$$\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = 2$$

$$1 + 2k = 2 \quad \therefore k = \frac{1}{2}$$

61) $-\frac{5}{6}$

⇒ 이차방정식의 근과 계수의 관계에 의하여

$$\sin \theta + \cos \theta = -\frac{2}{3}, \quad \sin \theta \cos \theta = \frac{k}{3}$$

$\sin \theta + \cos \theta = -\frac{2}{3}$ 의 양변을 제곱하면

$$\sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta = \frac{4}{9}$$

$$1 + 2\sin \theta \cos \theta = \frac{4}{9}$$

$$\therefore \sin \theta \cos \theta = -\frac{5}{18}$$

$$\frac{k}{3} = -\frac{5}{18} \quad \therefore k = -\frac{5}{6}$$

62) $-\frac{1}{4}$

⇒ $x^2 - \frac{\sqrt{2}}{2}x + k = 0$ 의 두 근이 $\sin \theta, \cos \theta$ 이므로

$$\sin \theta + \cos \theta = \frac{\sqrt{2}}{2} \quad \dots\dots \textcircled{7}$$

$$\sin \theta \cos \theta = k \quad \dots\dots \textcircled{8}$$

⑦의 양변을 제곱한 후 ⑧을 대입하면

$$\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = \frac{1}{2}$$

$$1 + 2k = \frac{1}{2} \quad \therefore k = -\frac{1}{4}$$

63) $\frac{\sqrt{6}}{2}$

⇒ $x^2 + kx + \frac{1}{4} = 0$ 의 두 근이 $\sin \theta, \cos \theta$ 이므로

$$\sin \theta + \cos \theta = -k \quad \dots\dots \textcircled{7}$$

$$\sin \theta \cos \theta = \frac{1}{4} \quad \dots\dots \textcircled{8}$$

⑦의 양변을 제곱한 후 ⑧을 대입하면

$$\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = k^2$$

$$1 + 2 \cdot \frac{1}{4} = k^2 \quad \therefore k = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2} \quad (\because k > 0)$$

64) $2\sqrt{2}$

⇒ 이차방정식의 근과 계수의 관계에 의하여

$$\sin \theta + \cos \theta = -\frac{k}{4}, \quad \sin \theta \cos \theta = -\frac{1}{4}$$

$\sin \theta + \cos \theta = -\frac{k}{4}$ 의 양변을 제곱하면

$$\sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta = \frac{k^2}{16}$$

$$1 + 2 \times \left(-\frac{1}{4}\right) = \frac{k^2}{16}$$

$$k^2 = 8 \quad \therefore k = 2\sqrt{2} \quad (\because k > 0)$$

65) $4\sqrt{10}$

66) $\sqrt{15}$

⇒ $\sin \theta + \cos \theta = \frac{k}{3}$

$$\sin \theta \cos \theta = \frac{1}{3}$$

$$\sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta = \frac{k^2}{9}$$

$$1 + \frac{2}{3} = \frac{k^2}{9}, \quad \frac{5}{3} = \frac{k^2}{9}$$

$$k^2 = 15$$

$$\therefore k = \sqrt{15}$$

67) $\frac{\sqrt{15}}{3}$

⇒ $x^2 + kx + \frac{1}{3} = 0$ 의 두 근이 $\sin \theta, \cos \theta$ 이므로

$$\sin \theta + \cos \theta = -k \quad \dots\dots \textcircled{7}$$

$$\sin \theta \cos \theta = \frac{1}{3} \quad \dots\dots \textcircled{8}$$

⑦의 양변을 제곱한 후 ⑧을 대입하면

$$\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = k^2$$

$$1 + 2 \cdot \frac{1}{3} = k^2 \quad \therefore k = \sqrt{\frac{5}{3}} = \frac{\sqrt{15}}{3} \quad (\because k > 0)$$

68) 0

$$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}, \sin\theta\cos\theta = \frac{1}{2}$$

$$(\sin\theta - \cos\theta)^2 = 0 \quad \therefore |\sin\theta - \cos\theta| = 0$$

69) 2

$$\Rightarrow x^2 - 2\sqrt{2}x + k = 0 \text{의 두 근이 } \frac{1}{\sin\theta}, \frac{1}{\cos\theta} \text{ 이므로}$$

로

$$\frac{1}{\sin\theta} + \frac{1}{\cos\theta} = 2\sqrt{2} \quad \dots\dots \textcircled{A}$$

$$\frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} = k \quad \dots\dots \textcircled{B}$$

①의 양변에 $\sin\theta\cos\theta$ 를 곱하면

$$\sin\theta + \cos\theta = 2\sqrt{2} \sin\theta\cos\theta$$

$$\textcircled{B} \text{에서 } \sin\theta\cos\theta = \frac{1}{k} \text{ 이므로}$$

$$\sin\theta + \cos\theta = \frac{2\sqrt{2}}{k}$$

이 식의 양변을 제곱하면

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{8}{k^2}$$

$$\therefore 1 + \frac{2}{k} = \frac{8}{k^2}$$

양변에 k^2 을 곱하면

$$k^2 + 2k - 8 = 0, (k+4)(k-2) = 0$$

$$\therefore k = 2 \left(\because 0 < \theta < \frac{\pi}{2} \text{에서 } k > 0 \right)$$

70) $-\frac{5}{2}$

$$\Rightarrow \sin\theta + \cos\theta = -\frac{\sqrt{5}}{5} \quad \dots \textcircled{1}$$

$$\sin\theta\cos\theta = \frac{a}{5} \quad \dots \textcircled{2}$$

①식의 양변을 제곱하고 ②식을 대입하면

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \frac{1}{5}$$

$$1 + 2 \times \left(\frac{a}{5}\right) = \frac{1}{5}, \frac{2a}{5} = -\frac{4}{5} \quad \therefore a = -2$$

따라서 주어진 이차방정식은

$$5x^2 + \sqrt{5}x - 2 = 0, (\sqrt{5}x + 2)(\sqrt{5}x - 1) = 0$$

$$\frac{\pi}{2} < \theta < \pi \text{이므로 두 근은}$$

$$\cos\theta = -\frac{2}{\sqrt{5}}, \sin\theta = \frac{1}{\sqrt{5}}$$

$$\therefore \tan\theta = -\frac{1}{2}$$

$$\therefore a + \tan\theta = -2 - \frac{1}{2} = -\frac{5}{2}$$

71) $2 - \sqrt{2}$

$$\Rightarrow \text{근과 계수의 관계에 의해 } \sin\theta + \cos\theta = 2a - 1, \sin\theta\cos\theta = 1 - 2a \text{ 이다.}$$

$$(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = \sin^2\theta + \cos^2\theta = 1 \text{ 임을 이용하면 } (2a-1)^2 - 2(1-2a) = 1 \text{ 이고, 여기서 } 2a-1 = \sqrt{2}-1 \text{ 임을 알 수 있다.}$$

따라서 구하는 값을 정리하면 다음과 같다.

$$\begin{aligned} \sin^3\theta + \cos^3\theta &= (\sin\theta + \cos\theta)^3 - 3\sin\theta\cos\theta(\sin\theta + \cos\theta) \\ &= (2a-1)^3 + 3(2a-1)^2 = (\sqrt{2}-1)^3 + 3(\sqrt{2}-1)^2 \\ &= 2 - \sqrt{2} \end{aligned}$$

72) $3x^2 + 8x + 3 = 0$

$$\Rightarrow \sin\theta + \cos\theta = \frac{1}{2} \quad \dots \textcircled{A}, \sin\theta\cos\theta = \frac{k}{2} \quad \dots \textcircled{B}$$

①을 제곱하여 ②을 대입하면

$$(\sin\theta + \cos\theta)^2 = \frac{1}{2^2}, 1 + k = \frac{1}{4} \quad \therefore k = -\frac{3}{4}$$

$$\begin{aligned} \tan\theta + \frac{1}{\tan\theta} &= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} \\ &= \frac{2}{k} = -\frac{8}{3}, \end{aligned}$$

$$\tan\theta \times \frac{1}{\tan\theta} = 1 \text{ 이므로 } \tan\theta, \frac{1}{\tan\theta} \text{를 두 근으로}$$

하고 최고차항 계수를 3으로 하는 이차방정식은

$$3x^2 + 8x + 3 = 0$$