실력완성 | 고1





수학 계산력 강화

(5)음수의 제곱근





◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

- 1) 제작연월일 : 2018-03-05
- 2) 제작자 : 교육지대㈜
- 3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호 되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무 단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 / 음수의 제곱근

a>0일 때

- (1) $\sqrt{-a} = \sqrt{a}i$
- (2) -a의 제곱근은 $\pm \sqrt{a}i$
- ☑ 허수단위 i를 사용하여 다음 수의 제곱근을 구하여 라.
- **1.** -1
- **2.** -3
- **3.** −4
- **4.** -5
- 5. -8
- 6. -18
- 7. -25
- 8. $-\frac{1}{16}$

- 9. $-\frac{1}{2}$
- **10.** $-\frac{2}{3}$
- **11.** $-\frac{3}{4}$
- 12. $-\frac{1}{9}$
- **13.** $-\frac{1}{4}$
- ☑ 다음 수를 허수단위 i를 사용하여 나타내어라.
- **14.** $\sqrt{-9}$
- **15.** $\sqrt{-12}$
- **16.** $-\sqrt{-8}$
- **17.** $-\sqrt{-\frac{9}{4}}$

ightharpoonup 다음을 계산하여 a+bi(a,b는 실수)의 꼴로 나타내어

18.
$$\sqrt{-2} + \sqrt{-8}$$

19.
$$\sqrt{-3} + \sqrt{-27}$$

20.
$$\sqrt{-9} + \sqrt{-16}$$

21.
$$\sqrt{-4} + \sqrt{-8}$$

22.
$$\sqrt{-32} - \sqrt{-8}$$

23.
$$4\sqrt{-12}-2\sqrt{-27}$$

24.
$$3\sqrt{-2} - \sqrt{-8}$$

25.
$$\sqrt{-25} - \sqrt{-1}$$

26.
$$\sqrt{-16} + \sqrt{-4}$$

27.
$$\sqrt{-2} + \sqrt{-8} + \sqrt{-32}$$

28.
$$\sqrt{-7} + \sqrt{-49}$$

29.
$$\sqrt{-1} - \sqrt{-16}$$

30.
$$-5\sqrt{-16}+2\sqrt{-9}$$

31.
$$3\sqrt{-2}-4\sqrt{-8}$$

32.
$$4\sqrt{-1}-6\sqrt{-36}+\sqrt{-12}$$

33.
$$\sqrt{-4} - \sqrt{-25}$$

☑ 다음 이차방정식의 해를 복소수 범위에서 구하여라.

34.
$$x^2 = 2$$

35.
$$x^2 = -2$$

36.
$$x^2 = -3$$

37.
$$x^2 = -18$$

38.
$$x^2 = \frac{1}{2}$$

39.
$$x^2 = -\frac{1}{2}$$

40.
$$3x^2 = -9$$

41.
$$\frac{1}{2}x^2 = -2$$

42.
$$-\frac{1}{3}x^2 = 9$$

43.
$$x^2 = -9$$

02 / 음수의 제곱근의 성질

(1)
$$a < 0$$
, $b < 0$ 이면 $\sqrt{a}\sqrt{b} = -\sqrt{ab}$

(2)
$$a>0$$
, $b<0$ 일 때, $\frac{\sqrt{a}}{\sqrt{b}}=-\sqrt{\frac{a}{b}}$

a>0, b<0일 때를 제외하면 a, b의 부호에 관계없이

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \ (b \neq 0)$$

참고 위 성질의 역으로 다음이 성립한다.

(1)
$$\sqrt{a}\sqrt{b}$$
= $-\sqrt{ab}$ 이면 $a<0,\ b<0$ 또는 $a=0$ 또는 $b=0$

(2)
$$\frac{\sqrt{a}}{\sqrt{b}} = -\sqrt{\frac{a}{b}}$$
 이면 $a>0$, $b<0$ 또는 $a=0$, $b\neq 0$

☑ 다음을 a+bi(a,b는 실수) 꼴로 나타내어라.

44.
$$\sqrt{-2}\sqrt{-8}$$

45.
$$\sqrt{3} \sqrt{-27}$$

46.
$$\sqrt{-2}\sqrt{8}$$

47.
$$\sqrt{-2}\sqrt{-3}$$

48.
$$\sqrt{-2}\sqrt{3}$$

49.
$$\sqrt{-4} \sqrt{-6}$$

50.
$$\sqrt{-5} \sqrt{-3}$$

51.
$$\sqrt{-8}\sqrt{-9}$$

52.
$$\frac{\sqrt{16}}{\sqrt{-4}}$$

53.
$$\frac{\sqrt{-30}}{\sqrt{-6}}$$

54.
$$\frac{\sqrt{-6}}{\sqrt{2}}$$

55.
$$\frac{\sqrt{5}}{\sqrt{-4}}$$

56.
$$\frac{\sqrt{-12}}{\sqrt{-3}}$$

57.
$$\frac{\sqrt{8}}{\sqrt{-2}}$$

58.
$$\frac{\sqrt{-10}}{\sqrt{2}}$$

59.
$$\frac{\sqrt{18}}{\sqrt{-3}}$$

60.
$$\frac{\sqrt{-8}}{\sqrt{-2}}$$

61.
$$\frac{1 - \sqrt{-12}}{2 + \sqrt{-3}}$$

62.
$$\frac{1 - \sqrt{-8}}{2 + \sqrt{-2}}$$

63.
$$\frac{2-2\sqrt{-1}}{1+\sqrt{-1}}$$

ightharpoonup 다음을 계산하여 a+bi꼴로 나타내어라.(단, a,b는 실 수, $i = \sqrt{-1}$)

64.
$$2\sqrt{-8} - \sqrt{-18} + 2\sqrt{-50}$$

65.
$$\sqrt{-12} - \sqrt{-8} \sqrt{-2} + \frac{\sqrt{15}}{\sqrt{-5}}$$

66.
$$\frac{1-\sqrt{-2}}{2+\sqrt{-2}} + \frac{3+\sqrt{-2}}{2-\sqrt{-2}}$$

67.
$$3\sqrt{-12} - \sqrt{-48} - 6\sqrt{-3}$$

68.
$$\sqrt{-2} - \sqrt{-8} - \sqrt{-3} \sqrt{6} - \frac{\sqrt{16}}{\sqrt{-2}}$$

69.
$$\sqrt{4}\sqrt{-9} + \sqrt{-4}\sqrt{-9} + \frac{\sqrt{9}}{\sqrt{-4}} + \frac{\sqrt{-9}}{\sqrt{-4}}$$

70.
$$\sqrt{-9} \sqrt{-25} + \frac{\sqrt{9}}{\sqrt{-25}}$$

71.
$$\sqrt{-2} + \sqrt{-32} - \sqrt{-8}$$

72.
$$2\sqrt{-25} - 3\sqrt{-9} + 5\sqrt{-36}$$

73.
$$\sqrt{8}\sqrt{-2} + \sqrt{-8}\sqrt{-2} + \frac{\sqrt{8}}{\sqrt{-2}} + \frac{\sqrt{-8}}{\sqrt{-2}}$$

74.
$$\frac{\sqrt{32}}{\sqrt{-2}} + \frac{\sqrt{-48}}{\sqrt{-4}} + \sqrt{-2}\sqrt{-6}$$

75.
$$\frac{1-\sqrt{-2}}{1+\sqrt{-2}} + \frac{1+\sqrt{-2}}{1-\sqrt{-2}}$$

76.
$$\sqrt{-4}\sqrt{-9} + \frac{\sqrt{-8}}{\sqrt{-2}} - \sqrt{(-4)^2} - \sqrt{-2^2}$$

77.
$$\left(\sqrt{-15}\sqrt{5} + \frac{\sqrt{-27}}{\sqrt{-3}}\right)\left(\sqrt{-3}\sqrt{-1} + \sqrt{-9}\right)$$

☑ 다음 물음에 답하여라.

- **78.** 0이 아닌 두 실수 a,b에 대하여 $\sqrt{a}\sqrt{b}=-\sqrt{ab}$ 일 때, $|a| - |b| + \sqrt{(a+b)^2}$ 를 간단히 나타내어라.
- **79.** 0이 아닌 두 실수 a,b에 대하여 $\frac{\sqrt{a}}{\sqrt{b}} = -\sqrt{\frac{a}{b}}$ 일 때, |a-b|-|a|+|b|를 간단히 나타내어라.
- 80. 0이 아닌 두 실수 a, b에 대하여 $\frac{\sqrt{a}}{\sqrt{b}} = -\sqrt{\frac{a}{b}}$ 일 때, $\sqrt{a^2} \sqrt{b^2}$ 을 간단히 나타내어라.
- 81. 실수 a, b에 대하여 $\frac{\sqrt{a}}{\sqrt{b}} = -\sqrt{\frac{a}{b}}$ 일 때, $\sqrt{(a-b)^2} - \sqrt{b^2} + (\sqrt{b})^2 + (\sqrt{a})^2$ 을 간단히 나타내
- 82. 0이 아닌 실수 a, b, c, d에 대하여 $\sqrt{a}\,\sqrt{b} = -\,\sqrt{ab}\,, \qquad \sqrt{rac{d}{c}} = -\,rac{\sqrt{d}}{\sqrt{c}}$ 가 성립할 $\sqrt{(a+b+c)^2}-2|a-d|$ 를 간단히 나타내어라.
- 83. 0이 아닌 세 실수 a, b, c가 $\sqrt{a} \sqrt{b} = \sqrt{ab}$ 와 $\frac{\sqrt{c}}{\sqrt{h}} = -\sqrt{\frac{c}{h}}$ 를 동시에 만족할 $|a-b| - \sqrt{(b-c)^2} - |a+c|$ 를 간단히 나타내어라.

- **84.** 0이 아닌 실수 a, b, c, d에 대하여 $\sqrt{ab} = -\sqrt{a}\,\sqrt{b}$, $\sqrt{\frac{c}{d}} = -\frac{\sqrt{c}}{\sqrt{d}}$ 일 때, $|a| + |b+d| - \sqrt{c^2} - \sqrt{(a-c)^2}$ 을 간단히 하여라.
- 85. 0이 아닌 네 실수 a, b, c, d에 대하여 $\frac{\sqrt{a}}{\sqrt{b}}$ = $-\sqrt{\frac{a}{b}}$, $\sqrt{c}\sqrt{d}$ = $-\sqrt{cd}$ 일 때, $\sqrt{a^2}+\sqrt{b^2}-\sqrt{(c-a)^2}-|b+d|+|d|$ 를 간단히 나타

정답 및 해설

$$1) \pm i$$

2)
$$\pm \sqrt{3}i$$

$$\Rightarrow \pm \sqrt{-3} = \pm \sqrt{3}i$$

3)
$$\pm 2i$$

$$\Rightarrow \pm \sqrt{4} i = \pm 2i$$

4)
$$\pm \sqrt{5}i$$

5)
$$\pm 2\sqrt{2}i$$

$$\Rightarrow \pm \sqrt{8} i = \pm 2\sqrt{2} i$$

6)
$$\pm 3\sqrt{2}i$$

$$\Rightarrow \pm \sqrt{-18} = \pm \sqrt{18} i = \pm 3\sqrt{2} i$$

7)
$$\pm 5i$$

$$\Rightarrow \pm \sqrt{-25} = \pm \sqrt{25} i = \pm 5i$$

8)
$$\pm \frac{1}{4}i$$

$$\Rightarrow \pm \sqrt{-\frac{1}{16}} = \pm \sqrt{\frac{1}{16}} i = \pm \frac{1}{4}i$$

9)
$$\pm \frac{\sqrt{2}}{2}i$$

10)
$$\pm \frac{\sqrt{6}}{2}i$$

$$\Rightarrow \pm \sqrt{\frac{2}{3}} i = \pm \frac{\sqrt{6}}{3} i$$

11)
$$\pm \frac{\sqrt{3}}{2}i$$

12)
$$\pm \frac{1}{3}i$$

13)
$$\pm \frac{1}{2}i$$

$$\Rightarrow \pm \sqrt{\frac{1}{4}}i = \pm \frac{1}{2}i$$

$$\Rightarrow \sqrt{-9} = \sqrt{9}i = 3i$$

15)
$$2\sqrt{3}i$$

$$\Rightarrow \sqrt{-12} = \sqrt{12} i = 2\sqrt{3} i$$

16)
$$-2\sqrt{2}i$$

$$\Rightarrow -\sqrt{-8} = -\sqrt{8}i = -2\sqrt{2}i$$

17)
$$-\frac{3}{2}i$$

$$\Rightarrow -\sqrt{-\frac{9}{4}} = -\sqrt{\frac{9}{4}}\,i = -\frac{3}{2}i$$

18)
$$3\sqrt{2}i$$

$$\Box$$

$$\sqrt{-2} + \sqrt{-8} = \sqrt{2}i + \sqrt{2^3}i = \sqrt{2}i + 2\sqrt{2}i$$
$$= (\sqrt{2} + 2\sqrt{2})i = 3\sqrt{2}i$$

19)
$$4\sqrt{3}i$$

$$\Rightarrow$$

$$\sqrt{-3} + \sqrt{-27} = \sqrt{3}i + 3\sqrt{3}i = 4\sqrt{3}i$$

$$\Rightarrow$$

$$\sqrt{-9} + \sqrt{-16} = 3i + 4i = 7i$$

21)
$$2(1+\sqrt{2})i$$

$$\Rightarrow \sqrt{-4} + \sqrt{-8} = 2i + 2\sqrt{2}i = 2(1 + \sqrt{2})i$$

22)
$$2\sqrt{2}i$$

$$\Rightarrow$$

$$\sqrt{-32} - \sqrt{-8} = 4\sqrt{2}i - 2\sqrt{2}i = 2\sqrt{2}i$$

23)
$$2\sqrt{3}i$$

$$\Rightarrow$$

$$\begin{array}{l} 4\sqrt{-12} - 2\sqrt{-27} = 4 \cdot 2\sqrt{3} \ i - 2 \cdot 3\sqrt{3} \ i \\ = 8\sqrt{3} \ i - 6\sqrt{3} \ i = 2\sqrt{3} \ i \end{array}$$

24)
$$\sqrt{2}i$$

$$\Rightarrow$$

$$3\sqrt{-2} - \sqrt{-8} = 3\sqrt{2}i - 2\sqrt{2}i = \sqrt{2}i$$

25) 4i

$$\Rightarrow$$

$$\sqrt{-25} - \sqrt{-1} = 5i - i = 4i$$

$$\Rightarrow \sqrt{-16} + \sqrt{-4} = 4i + 2i = 6i$$

27)
$$7\sqrt{2}i$$

$$\Rightarrow$$

$$\sqrt{-2} + \sqrt{-8} + \sqrt{-32} = \sqrt{2}i + 2\sqrt{2}i + 4\sqrt{2}i = 7\sqrt{2}i$$

28)
$$(7+\sqrt{7})i$$

$$\Rightarrow \sqrt{-7} + \sqrt{-49} = \sqrt{7}i + 7i = (7 + \sqrt{7})i$$

29)
$$-3i$$

$$\Rightarrow \sqrt{-1} - \sqrt{-16} = i - 4i = -3i$$

30)
$$-14i$$

$$\Rightarrow$$

$$-5\sqrt{-16} + 2\sqrt{-9} = -5\cdot4i + 2\cdot3i = -20i + 6i = -14i$$

31)
$$-5\sqrt{2}i$$

$$\Rightarrow 3\sqrt{-2} - 4\sqrt{-8} = 3\sqrt{2}i - 8\sqrt{2}i = -5\sqrt{2}i$$

32)
$$-2(16-\sqrt{3})i$$

$$\Rightarrow 4\sqrt{-1} - 6\sqrt{-36} + \sqrt{-12}$$

$$=4\cdot i-6\cdot 6i+2\sqrt{3}i$$

$$=4i-36i+2\sqrt{3}i=-32i+2\sqrt{3}i$$

$$=-2(16-\sqrt{3})i$$

33)
$$-3i$$

$$\Rightarrow \sqrt{-4} - \sqrt{-25} = 2i - 5i = -3i$$

34)
$$x = \pm \sqrt{2}$$

35)
$$x = \pm \sqrt{2}i$$

36)
$$x = \pm \sqrt{3}i$$

37)
$$x = \pm 3\sqrt{2}i$$

$$\Rightarrow x = \pm \sqrt{18}i$$
 $\therefore x = \pm 3\sqrt{2}i$

38)
$$x = \pm \frac{\sqrt{2}}{2}$$

39)
$$x = \pm \frac{\sqrt{2}}{2}i$$

40)
$$x = \pm \sqrt{3}i$$

$$\Rightarrow x^2 = -3 \qquad \therefore x = \pm \sqrt{3} i$$

41)
$$x = \pm 2i$$

$$\Rightarrow x^2 = -4$$
 $\therefore x = \pm 2i$

42)
$$x = \pm 3\sqrt{3}i$$

$$\Rightarrow x^2 = -27, x = \pm \sqrt{27}i$$
 $\therefore x = \pm 3\sqrt{3}i$

43)
$$x = \pm 3i$$

$$\Rightarrow$$
 -9의 제곱근은 $\pm \sqrt{9}i = \pm 3i$ 이므로

$$x^2 = -9$$
의 해는 $x = \pm 3i$

44)
$$-4$$

$$\Rightarrow \sqrt{-2} \sqrt{-8} = \sqrt{2} i \cdot \sqrt{8} i = \sqrt{16} i^2 = -4$$

$$\Rightarrow \sqrt{3} \sqrt{-27} = \sqrt{3} \sqrt{27} i = \sqrt{81} i = 9i$$

$$\Rightarrow \sqrt{-2} \sqrt{8} = \sqrt{2} i \times 2 \sqrt{2} = 4i$$

47)
$$-\sqrt{6}$$

$$\Rightarrow \sqrt{-2}\sqrt{-3} = \sqrt{2}i \times \sqrt{3}i = -\sqrt{6}$$

48)
$$\sqrt{6}i$$

$$\Rightarrow \sqrt{-2}\sqrt{3} = \sqrt{2}i \times \sqrt{3} = \sqrt{6}i$$

49)
$$-2\sqrt{6}$$

$$\Rightarrow \sqrt{-4} \sqrt{-6} = 2i \times \sqrt{6} i = -2\sqrt{6}$$

50)
$$-\sqrt{15}$$

$$\Rightarrow \sqrt{-5}\sqrt{-3} = \sqrt{5}i \times \sqrt{3}i = -\sqrt{15}$$

51)
$$-6\sqrt{2}$$

$$\Rightarrow$$

$$\sqrt{-8} \sqrt{-9} = 2\sqrt{2} i \cdot 3i = -6\sqrt{2}$$

52)
$$-2i$$

$$\Rightarrow \frac{\sqrt{16}}{\sqrt{-4}} = \frac{4}{\sqrt{4}i} = \frac{4i}{2i^2} = \frac{4i}{-2} = -2i$$

$$\Rightarrow \frac{\sqrt{-30}}{\sqrt{-6}} = \frac{\sqrt{30}i}{\sqrt{6}i} = \sqrt{5}$$

54)
$$\sqrt{3}i$$

$$\Rightarrow \frac{\sqrt{-6}}{\sqrt{2}} = \frac{\sqrt{6}i}{\sqrt{2}} = \sqrt{3}i$$

55)
$$-\frac{\sqrt{5}}{2}i$$

$$\Rightarrow \frac{\sqrt{5}}{\sqrt{-4}} = \frac{\sqrt{5}}{2i} = \frac{\sqrt{5}i}{2i^2} = -\frac{\sqrt{5}}{2}i$$

$$\Rightarrow \frac{\sqrt{-12}}{\sqrt{-3}} = \frac{2\sqrt{3}i}{\sqrt{3}i} = 2$$

$$57) - 2$$

$$\Rightarrow \frac{\sqrt{8}}{\sqrt{-2}} = \frac{2\sqrt{2}}{\sqrt{2}i} = \frac{2\sqrt{2}i}{\sqrt{2}i^2} = -2i$$

58)
$$\sqrt{5}i$$

$$\Rightarrow$$

$$\frac{\sqrt{-10}}{\sqrt{2}} = \frac{\sqrt{10}\,i}{\sqrt{2}} = \sqrt{5}\,i$$

59)
$$-\sqrt{6}i$$

$$\frac{\sqrt{18}}{\sqrt{-3}} = \frac{3\sqrt{2}}{\sqrt{3}i} = \frac{3\sqrt{2}\cdot\sqrt{3}i}{3i^2} = -\sqrt{6}i$$

60) 2

$$\Rightarrow$$

$$\frac{\sqrt{-8}}{\sqrt{-2}} = \frac{2\sqrt{2}i}{\sqrt{2}i} = 2$$

61)
$$-\frac{4}{7} - \frac{5\sqrt{3}}{7}i$$

$$\begin{split} \frac{1-\sqrt{-12}}{2+\sqrt{-3}} &= \frac{1-2\sqrt{3}\,i}{2+\sqrt{3}\,i} = \frac{(1-2\sqrt{3}\,i)(2-\sqrt{3}\,i)}{(2+\sqrt{3}\,i)(2-\sqrt{3}\,i)} \\ &= \frac{2-\sqrt{3}\,i-4\sqrt{3}\,i+6i^2}{4-3i^2} \\ &= \frac{-4-5\sqrt{3}\,i}{7} = -\frac{4}{7} - \frac{5\sqrt{3}}{7}i \end{split}$$

62)
$$-\frac{1}{3} - \frac{5\sqrt{2}}{6}i$$

$$\begin{split} \frac{1-\sqrt{-8}}{2+\sqrt{-2}} &= \frac{1-2\sqrt{2}i}{2+\sqrt{2}i} \\ &= \frac{(1-2\sqrt{2}i)(2-\sqrt{2}i)}{(2+\sqrt{2}i)(2-\sqrt{2}i)} \\ &= \frac{2-\sqrt{2}i-4\sqrt{2}i+4i^2}{4-2i^2} \\ &= \frac{-2-5\sqrt{2}i}{6} = -\frac{1}{3} - \frac{5\sqrt{2}}{6}i \end{split}$$

63)
$$-2i$$

 \Rightarrow

$$\frac{2-2\sqrt{-1}}{1+\sqrt{-1}} = \frac{2-2i}{1+i} = \frac{2(1-i)^2}{(1+i)(1-i)} = \frac{2(1-i)^2}{2} = -2i$$

64)
$$11\sqrt{2}i$$

$$\begin{array}{c} 2\sqrt{-8} - \sqrt{-18} + 2\sqrt{-50} = 2 \cdot 2\sqrt{2} \, i - 3\sqrt{2} \, i + 2 \cdot 5\sqrt{2} \, i \\ = 4\sqrt{2} \, i - 3\sqrt{2} \, i + 10\sqrt{2} \, i \\ = 11\sqrt{2} \, i \end{array}$$

65)
$$4 + \sqrt{3}i$$

$$\sqrt{-12} - \sqrt{-8} \sqrt{-2} + \frac{\sqrt{15}}{\sqrt{-5}} = 2\sqrt{3} i - 2\sqrt{2} i \cdot \sqrt{2} i + \frac{\sqrt{15}}{\sqrt{5} i}$$

$$= 2\sqrt{3} i + 4 + \frac{\sqrt{3} i}{i^2}$$

$$= 2\sqrt{3} i + 4 - \sqrt{3} i$$

$$= 4 + \sqrt{3} i$$

66)
$$\frac{2}{3} + \frac{\sqrt{2}}{3}i$$

$$\begin{array}{l} \Longrightarrow \\ \frac{1-\sqrt{-2}}{2+\sqrt{-2}} + \frac{3+\sqrt{-2}}{2-\sqrt{-2}} \\ = \frac{1-\sqrt{2}\,i}{2+\sqrt{2}\,i} + \frac{3+\sqrt{2}\,i}{2-\sqrt{2}\,i} \\ = \frac{(1-\sqrt{2}\,i)(2-\sqrt{2}\,i) + (3+\sqrt{2}\,i)(2+\sqrt{2}\,i)}{(2+\sqrt{2}\,i)(2-\sqrt{2}\,i)} \\ = \frac{-3\sqrt{2}\,i + (4+5\sqrt{2}\,i)}{4-2i^2} \\ = \frac{4+2\sqrt{2}\,i}{6} = \frac{2}{3} + \frac{\sqrt{2}}{3}\,i \end{array}$$

67)
$$-4\sqrt{3}i$$

$$3\sqrt{-12} - \sqrt{-48} - 6\sqrt{-3} = 3 \cdot 2\sqrt{3} i - 4\sqrt{3} i - 6\sqrt{3} i$$

$$= -4\sqrt{3} i$$

68)
$$-2\sqrt{2}i$$

$$\begin{aligned} &\sqrt{-2} - \sqrt{-8} - \sqrt{-3} \sqrt{6} - \frac{\sqrt{16}}{\sqrt{-2}} \\ &= \sqrt{2} i - 2\sqrt{2} i - \sqrt{3} i \cdot \sqrt{6} - \frac{4}{\sqrt{2} i} \\ &= \sqrt{2} i - 2\sqrt{2} i - 3\sqrt{2} i + 2\sqrt{2} i \\ &= -2\sqrt{2} i \end{aligned}$$

69)
$$-\frac{9}{2} + \frac{9}{2}i$$

$$\begin{split} &\sqrt{4}\,\sqrt{-9} + \sqrt{-4}\,\sqrt{-9} + \frac{\sqrt{9}}{\sqrt{-4}} + \frac{\sqrt{-9}}{\sqrt{-4}} \\ &= 2\cdot 3i + 2i\cdot 3i + \frac{3}{2i} + \frac{3i}{2i} \\ &= 6i - 6 - \frac{3}{2}i + \frac{3}{2} = -\frac{9}{2} + \frac{9}{2}i \end{split}$$

70)
$$-15 - \frac{3}{5}i$$

$$\Rightarrow \sqrt{-9} \sqrt{-25} + \frac{\sqrt{9}}{\sqrt{-25}} = 3i \cdot 5i + \frac{3}{5i} = -15 - \frac{3}{5}i$$

71)
$$3\sqrt{2}i$$

$$\sqrt{-2} + \sqrt{-32} - \sqrt{-8} = \sqrt{2}i + 4\sqrt{2}i - 2\sqrt{2}i$$
$$= (1+4-2)\sqrt{2}i = 3\sqrt{2}i$$

$$2\sqrt{-25} - 3\sqrt{-9} + 5\sqrt{-36} = 2\cdot 5i - 3\cdot 3i + 5\cdot 6i$$

= $10i - 9i + 30i = 31i$

73)
$$-2+2i$$

$$\sqrt{8}\sqrt{-2} + \sqrt{-8}\sqrt{-2} + \frac{\sqrt{8}}{\sqrt{-2}} + \frac{\sqrt{-8}}{\sqrt{-2}}$$

$$= 2\sqrt{2} \cdot \sqrt{2}i + 2\sqrt{2}i \cdot \sqrt{2}i + \frac{2\sqrt{2}}{\sqrt{2}i} + \frac{2\sqrt{2}i}{\sqrt{2}i}$$

$$= 4i - 4 - 2i + 2 = -2 + 2i$$

74)
$$-4i$$

$$\frac{\sqrt{32}}{\sqrt{-2}} + \frac{\sqrt{-48}}{\sqrt{-4}} + \sqrt{-2}\sqrt{-6} = \frac{\sqrt{32}}{\sqrt{2}i} + \frac{\sqrt{48}i}{\sqrt{4}i} + \sqrt{2}i\sqrt{6}i$$

$$= \frac{4}{i} + \sqrt{12} - \sqrt{12} = \frac{4i}{i^2} = -4i$$

75)
$$-\frac{2}{3}$$

$$\begin{split} &\frac{1-\sqrt{-2}}{1+\sqrt{-2}} + \frac{1+\sqrt{-2}}{1-\sqrt{-2}} = \frac{1-\sqrt{2}\,i}{1+\sqrt{2}\,i} + \frac{1+\sqrt{2}\,i}{1-\sqrt{2}\,i} \\ &= \frac{(1-\sqrt{2}\,i)^2 + (1+\sqrt{2}\,i)^2}{(1+\sqrt{2}\,i)(1-\sqrt{2}\,i)} \end{split}$$

$$=\frac{1-2\sqrt{2}\,i-2+1+2\sqrt{2}\,i-2}{3}=-\frac{2}{3}$$

76)
$$-8-2i$$

$$\Rightarrow \sqrt{-4} \sqrt{-9} + \frac{\sqrt{-8}}{\sqrt{-2}} - \sqrt{(-4)^2} - \sqrt{-2^2}$$

$$=2i\cdot 3i+\frac{2\sqrt{2}\,i}{\sqrt{2}\,i}-4-2i=\!\!-6+2-4-2i=\!\!-8-2i$$

77)
$$-18\sqrt{3}-6i$$

$$\sqrt{-15} \sqrt{5} + \frac{\sqrt{-27}}{\sqrt{-3}} = \sqrt{15} i \sqrt{5} + \frac{\sqrt{27} i}{\sqrt{3} i}$$

$$=5\sqrt{3}i+3$$

$$\therefore$$
 (준식)= $(3+5\sqrt{3}i)(-\sqrt{3}+3i)$
= $-3\sqrt{3}+9i-15i-15\sqrt{3}$
= $-18\sqrt{3}-6i$

78)
$$-2a$$

$$\sqrt{a}\sqrt{b} = -\sqrt{ab}$$
일 때, $a < 0, b < 0$ 이므로
 $|a| - |b| + \sqrt{(a+b)^2} = -a - (-b) - (a+b) = -2a$

79)
$$-2b$$

$$\frac{\sqrt{a}}{\sqrt{b}} = -\sqrt{\frac{a}{b}}$$
일 때, $a > 0, b < 0$ 이므로
$$|a-b|-|a|+|b|=a-b-a+(-b)=-2b$$

80)
$$-ab$$

$$\frac{\sqrt{a}}{\sqrt{b}}$$
 = $-\sqrt{\frac{a}{b}}$ 일 때, $a > 0, b < 0$ 이므로 $\sqrt{a^2}\sqrt{b^2} = a \cdot (-b) = -ab$

81)
$$2a+b$$

$$\sqrt{(a-b)^2} - \sqrt{b^2} + (\sqrt{b})^2 + (\sqrt{a})^2$$

= $(a-b) - (-b) + b + a = 2a + b$

82)
$$a-b-c-2d$$

$$\sqrt{a}\sqrt{b} = -\sqrt{ab}$$
에서 $a < 0, b < 0$ 이고

$$\sqrt{\frac{d}{c}} = -\frac{\sqrt{d}}{\sqrt{c}}$$
에서 $c < 0, d > 0$ 이다.

$$\sqrt{(a+b+c)^2} - 2|a-d|
= -(a+b+c) + 2(a-d)
= a-b-c-2d$$

83)
$$-2a$$

당
$$\frac{\sqrt{c}}{\sqrt{b}} = -\sqrt{\frac{c}{b}}$$
 이므로 $c > 0$, $b < 0$ 이고,
이 때 $\sqrt{a}\sqrt{b} = \sqrt{ab}$ 이므로 $b < 0$ 이면 $a > 0$ 이다.
 $|a-b| - \sqrt{(b-c)^2} - |a+c|$
 $= (a-b) - (-b+c) - (a+c)$
 $= -2c$

84)
$$-b-2c-d$$

$$\sqrt{ab} = -\sqrt{a}\sqrt{b}$$
이므로 $a < 0$, $b < 0$
 $\sqrt{\frac{c}{d}} = -\frac{\sqrt{c}}{\sqrt{d}}$ 이므로 $c > 0$, $d < 0$

$$\therefore |a| + |b+d| - \sqrt{c^2} - \sqrt{(a-c)^2}$$

$$= |a| + |b+d| - |c| - |a-c|$$

$$= -a - b - d - c + a - c$$

$$= -b - 2c - d$$