



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

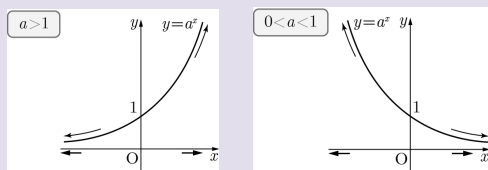
1) 제작연월일 : 2019-08-12

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3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 지수함수의 극한

지수함수 $y = a^x (a > 0, a \neq 1)$ 에서(1) $a > 1$ 일 때 : $\lim_{x \rightarrow \infty} a^x = \infty, \lim_{x \rightarrow -\infty} a^x = 0$ (2) $0 < a < 1$ 일 때 : $\lim_{x \rightarrow \infty} a^x = 0, \lim_{x \rightarrow -\infty} a^x = \infty$ 

■ 다음 극한을 조사하고, 극한이 존재하면 그 극한값을 구하여라.

1. $\lim_{x \rightarrow -2} 3^x$

2. $\lim_{x \rightarrow 0} \left(\frac{3}{4}\right)^x$

3. $\lim_{x \rightarrow 0^+} \frac{x}{3^{\frac{1}{x}}}$

4. $\lim_{x \rightarrow 1} \frac{6^{x-1} - 1}{x^3 - 1}$

5. $\lim_{x \rightarrow \infty} 3^x$

6. $\lim_{x \rightarrow \infty} \left(\frac{5}{4}\right)^x$

7. $\lim_{x \rightarrow \infty} \left(\frac{1}{2}\right)^x$

8. $\lim_{x \rightarrow \infty} \frac{3^x}{2^{2x}}$

9. $\lim_{x \rightarrow \infty} \frac{3^x}{2 + 3^{x+1}}$

10. $\lim_{x \rightarrow \infty} \frac{2^x}{1 + 2^x}$

11. $\lim_{x \rightarrow \infty} (3^x - 5^x)$

12. $\lim_{x \rightarrow \infty} \left\{ \left(\frac{2}{3}\right)^x - 5 \right\}$

13. $\lim_{x \rightarrow \infty} \frac{7^x}{3^{2x}}$

$$14. \lim_{x \rightarrow \infty} (2^x - 5^x)$$

$$15. \lim_{x \rightarrow \infty} (3^x - 2^x)$$

$$16. \lim_{x \rightarrow \infty} \frac{5^{x+1} - 2^x}{5^x + 3^x}$$

$$17. \lim_{x \rightarrow \infty} \frac{2^x - 7^x}{3^x + 7^x}$$

$$18. \lim_{x \rightarrow \infty} \left\{ \frac{5^{x-1} + 3^{x+2}}{5^x - 3^x} \right\}$$

$$19. \lim_{x \rightarrow \infty} \frac{7^{x+2} - 5^{x+3}}{5^x - 7^{x+1}}$$

$$20. \lim_{x \rightarrow -\infty} \frac{3^x + 1}{3^x - 1}$$

$$21. \lim_{x \rightarrow \infty} \frac{4^x - 2^x}{4^x + 2^x}$$

$$22. \lim_{x \rightarrow -\infty} \frac{4^x + 3^{-x}}{5^x - 3^{-x}}$$

$$23. \lim_{x \rightarrow \infty} \frac{3^x + 2^x}{3^x - 2^x}$$

$$24. \lim_{x \rightarrow \infty} (3^x + 4^x)^{\frac{1}{x}}$$

$$25. \lim_{x \rightarrow \infty} \frac{3^{x+1}(2^{x+1} + 3^{-x})}{6^{x-1} - 2}$$

$$26. \lim_{x \rightarrow \infty} \frac{5^{x+1} - 2^x}{5^x + 2^x}$$

$$27. \lim_{x \rightarrow -\infty} \left(\frac{2}{3} \right)^x$$

$$28. \lim_{x \rightarrow -\infty} 5^x$$

$$29. \lim_{x \rightarrow -\infty} \frac{5^x + 5^{-x}}{5^x - 5^{-x}}$$

$$30. \lim_{x \rightarrow -\infty} \frac{4^x + 3^{-x}}{5^x - 3^{-x}}$$

$$31. \lim_{x \rightarrow -\infty} \frac{3^x - 3x^3 - 1}{1 + 3x^3}$$

$$32. \lim_{x \rightarrow \infty} \frac{4^{x+1} + 2^x}{4^x - 2^x}$$

■ 주어진 극한값을 만족하는 상수 a 의 값을 구하여라.

$$33. \lim_{x \rightarrow \infty} \frac{a \cdot 4^x + 3^x}{4^{x+1} - 2^x} = 8$$

$$34. \lim_{x \rightarrow \infty} \frac{a \cdot 5^x + 3}{5^{x-1} - 4} = 25$$

$$35. \lim_{x \rightarrow \infty} \frac{a \cdot 3^{x+1} - 4}{3^{x-2} + 2} = 81$$

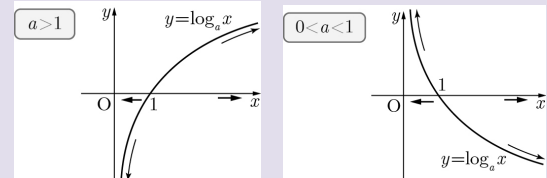
$$36. \lim_{x \rightarrow \infty} \frac{4^{x+a} + 3^{x+2} + 2^x}{4^x + 3^{x+1}} = \frac{1}{64}$$

02 로그함수의 극한

로그함수 $y = \log_a x (a > 0, a \neq 1)$ 에서

(1) $a > 1$ 일 때 : $\lim_{x \rightarrow 0+} \log_a x = -\infty, \lim_{x \rightarrow \infty} \log_a x = \infty$

(2) $0 < a < 1$ 일 때 : $\lim_{x \rightarrow 0+} \log_a x = \infty, \lim_{x \rightarrow \infty} \log_a x = -\infty$



■ 다음 극한을 조사하고, 극한이 존재하면 그 극한값을 구하여라.

$$37. \lim_{x \rightarrow 4} \log_2 x$$

$$38. \lim_{x \rightarrow 0+} \log_{\frac{1}{2}} x$$

$$39. \lim_{x \rightarrow 0+} \log_3 x$$

$$40. \lim_{x \rightarrow 0+} \log x$$

$$41. \lim_{x \rightarrow 1} \log x$$

$$42. \lim_{x \rightarrow 9} \log_{\frac{1}{3}} x$$

$$43. \lim_{x \rightarrow 1} \frac{\log_2 x}{1-x}$$

$$44. \lim_{x \rightarrow 1} (\log_2 |x^2 - 1| - \log_2 |x^3 - 1|)$$

$$45. \lim_{x \rightarrow -2} \log_3 \frac{x+1}{x^3-1}$$

$$46. \lim_{x \rightarrow 4} \frac{\log_4 (x-3)}{x-4}$$

$$47. \lim_{x \rightarrow 2} \{\log_3 (5x+2) - \log_3 (x+2)\}$$

$$48. \lim_{x \rightarrow 0+} \left\{ \frac{\log_3 \frac{3}{x}}{\log_3 \left(\frac{6}{x} + 1 \right)} \right\}$$

$$49. \lim_{x \rightarrow \infty} \log_3 2x$$

$$50. \lim_{x \rightarrow \infty} \log_{\frac{1}{5}} x$$

$$51. \lim_{x \rightarrow \infty} \log_2 x$$

$$52. \lim_{x \rightarrow \infty} \log_4 (x^2 + 1)$$

$$53. \lim_{x \rightarrow 0+} \log_{\frac{1}{3}} x$$

$$54. \lim_{x \rightarrow 4+} \log_{\frac{1}{2}} (x-4)$$

$$55. \lim_{x \rightarrow \infty} \log_3 \frac{1}{x}$$

$$56. \lim_{x \rightarrow \infty} \log_5 \frac{x^2+1}{x^2-1}$$

$$57. \lim_{x \rightarrow \infty} \{\log_2 (12x+3) - \log_2 3x\}$$

$$58. \lim_{x \rightarrow \infty} \{\log (20x+1) - \log 2x\}$$

$$59. \lim_{x \rightarrow \infty} \{\log_3 (9x+1) - \log_3 x\}$$

$$60. \lim_{x \rightarrow \infty} \{\log_2(3x+2) - \log_2 3x\}$$

$$61. \lim_{x \rightarrow \infty} \{\log_2 3^x - \log_2(3^x - 1)\}$$

$$62. \lim_{x \rightarrow \infty} \log_2 \frac{4x+1}{x+2}$$

$$63. \lim_{x \rightarrow \infty} \{\log_2(4x+1) - \log_2 x\}$$

$$64. \lim_{x \rightarrow \infty} \frac{1}{x} \log_3(2^x + 3^x)$$

$$65. \lim_{x \rightarrow \infty} \{\log_2 5^x - \log_2(5^x - 1)\}$$

$$66. \lim_{x \rightarrow \infty} \{\log_2(4x+1) - \log_2(x-1)\}$$

$$67. \lim_{x \rightarrow \infty} \{\log_4(x^3 + 3x + 1) - \log_4(4x^3 + 3)\}$$

$$68. \lim_{x \rightarrow \infty} \left\{ \frac{\log_3(x^4 + 2x^3 - 1)}{\log_3(x^2 - x - 4)} + \frac{\log_3(6^x + 9^x)}{\log_3 3^x} \right\}$$

■ 주어진 극한값을 만족하는 상수 a 의 값을 구하여라.

$$69. \lim_{x \rightarrow \infty} \{\log_3(ax-1) - \log_3(2x+1)\} = 3$$

$$70. \lim_{x \rightarrow \infty} \{\log ax - \log(2x+5)\} = 1$$

$$71. \lim_{x \rightarrow \infty} \{\log_2(ax+1) - \log_2 x\} = 2$$



정답 및 해설

1) $\frac{1}{9}$

$$\Rightarrow \lim_{x \rightarrow -2} 3^x = 3^{-2} = \frac{1}{9}$$

2) 1

$$\Rightarrow \lim_{x \rightarrow 0} \left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^0 = 1$$

3) 0

$$\Rightarrow \lim_{x \rightarrow +0} \frac{x}{3^{\frac{1}{x}}} = \lim_{x \rightarrow +0} \frac{x}{\sqrt[3]{3}} = 0$$

4) $\frac{1}{3} \ln 6$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \frac{6^{x-1} - 1}{x^3 - 1} &= \lim_{x \rightarrow 1} \frac{6^{x-1} - 1}{x - 1} \cdot \frac{1}{x^2 + x + 1} \\ &= \ln 6 \cdot \frac{1}{3} = \frac{1}{3} \ln 6 \end{aligned}$$

5) ∞

$$\Rightarrow a > 1 \text{ 일 때 } \lim_{x \rightarrow \infty} a^x = \infty \text{ 이므로 } \lim_{x \rightarrow \infty} 3^x = \infty$$

6) ∞

$$\Rightarrow \lim_{x \rightarrow \infty} \left(\frac{5}{4}\right)^x = \infty$$

7) 0

$$\Rightarrow 0 < a < 1 \text{ 일 때 } \lim_{x \rightarrow \infty} a^x = 0 \text{ 이므로 } \lim_{x \rightarrow \infty} \left(\frac{1}{2}\right)^x = 0$$

8) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{3^x}{2^{2x}} = \lim_{x \rightarrow \infty} \left(\frac{3}{4}\right)^x = 0$$

9) $\frac{1}{3}$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{3^x}{2 + 3^{x+1}} = \lim_{x \rightarrow \infty} \frac{1}{\frac{2}{3^x} + 3} = \frac{1}{3}$$

10) 1

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{2^x}{1 + 2^x} = \lim_{x \rightarrow \infty} \frac{1}{\left(\frac{1}{2}\right)^x + 1} = \frac{1}{0 + 1} = 1$$

11) $-\infty$

$$\Rightarrow \lim_{x \rightarrow \infty} (3^x - 5^x) = \lim_{x \rightarrow \infty} 5^x \left\{ \left(\frac{3}{5}\right)^x - 1 \right\}$$

$$\text{이때 } \lim_{x \rightarrow \infty} \left\{ \left(\frac{3}{5}\right)^x - 1 \right\} = -1 \text{ 이므로}$$

$$\lim_{x \rightarrow \infty} (3^x - 5^x) = \lim_{x \rightarrow \infty} 5^x \left\{ \left(\frac{3}{5}\right)^x - 1 \right\} = -\infty$$

12) -5

$$\Rightarrow \lim_{x \rightarrow \infty} \left\{ \left(\frac{2}{3}\right)^x - 5 \right\} = -5$$

13) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{7^x}{3^{2x}} = \lim_{x \rightarrow \infty} \left(\frac{7}{9}\right)^x = 0$$

14) $-\infty$

$$\Rightarrow \text{괄호의 식을 } 5^x \text{ 으로 묶으면}$$

$$\lim_{x \rightarrow \infty} (2^x - 5^x) = \lim_{x \rightarrow \infty} 5^x \left\{ \left(\frac{2}{5}\right)^x - 1 \right\} = -\infty$$

15) ∞

$$\Rightarrow \text{괄호의 식을 } 3^x \text{ 으로 묶으면}$$

$$\lim_{x \rightarrow \infty} (3^x - 2^x) = \lim_{x \rightarrow \infty} 3^x \left\{ 1 - \left(\frac{2}{3}\right)^x \right\} = \infty$$

16) 5

$$\Rightarrow \text{분모, 분자를 } 5^x \text{ 으로 각각 나누어 } 0 < a < 1 \text{ 일 때, } \lim_{x \rightarrow \infty} a^x = 0 \text{ 임을 이용한다.}$$

$$\lim_{x \rightarrow \infty} \frac{5^{x+1} - 2^x}{5^x + 3^x} = \lim_{x \rightarrow \infty} \frac{5 - \left(\frac{2}{5}\right)^x}{1 + \left(\frac{3}{5}\right)^x} = \frac{5 - 0}{1 + 0} = 5$$

17) -1

$$\Rightarrow \text{분모, 분자를 } 7^x \text{ 으로 나누면}$$

$$\lim_{x \rightarrow \infty} \frac{2^x - 7^x}{3^x + 7^x} = \lim_{x \rightarrow \infty} \frac{\left(\frac{2}{7}\right)^x - 1}{\left(\frac{3}{7}\right)^x + 1} = -1$$

18) $\frac{1}{5}$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{5^{x-1} + 3^{x+2}}{5^x - 3^x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{5} + 9\left(\frac{3}{5}\right)^x}{1 - \left(\frac{3}{5}\right)^x} = \frac{1}{5}$$

19) -7

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{7^{x+2} - 5^{x+3}}{5^x - 7^{x+1}} = \lim_{x \rightarrow \infty} \frac{7^2 - 5^3 \left(\frac{5}{7}\right)^x}{\left(\frac{5}{7}\right)^x - 7} = \frac{7^2}{-7} = -7$$

20) -1

$$\Rightarrow \lim_{x \rightarrow -\infty} 3^x = 0 \text{ 이므로 } \lim_{x \rightarrow -\infty} \frac{3^x + 1}{3^x - 1} = \frac{0 + 1}{0 - 1} = -1$$

21) 1

⇒ $0 < a < 1$ 일 때, $\lim_{x \rightarrow \infty} a^x = 0$ 이므로 분모, 분자를 각각 4^x 으로 나누어 구한다.

$$\text{즉, } \lim_{x \rightarrow \infty} \frac{4^x - 2^x}{4^x + 2^x} = \lim_{x \rightarrow \infty} \frac{1 - \left(\frac{1}{2}\right)^x}{1 + \left(\frac{1}{2}\right)^x} = \frac{1-0}{1+0} = 1$$

22) -1

23) 1

24) 4

⇒

$$\lim_{x \rightarrow \infty} (3^x + 4^x)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} \left\{ 4^x \left(\left(\frac{3}{4} \right)^x + 1 \right) \right\}^{\frac{1}{x}} = 4(0+1) = 4$$

25) 36

26) 5

⇒ 분모, 분자를 5^x 으로 나누면

$$\lim_{x \rightarrow \infty} \frac{5^{x+1} - 2^x}{5^x + 2^x} = \lim_{x \rightarrow \infty} \frac{5 - \left(\frac{2}{5}\right)^x}{1 + \left(\frac{2}{5}\right)^x} = 5$$

27) ∞

$$\Rightarrow \lim_{x \rightarrow -\infty} \left(\frac{2}{3} \right)^x = \infty$$

28) 0

⇒ $a > 1$ 일 때 $\lim_{x \rightarrow -\infty} a^x = 0$ 이므로 $\lim_{x \rightarrow -\infty} 5^x = 0$

29) -1

$$\Rightarrow \lim_{x \rightarrow -\infty} \frac{5^x + 5^{-x}}{5^x - 5^{-x}} = \lim_{x \rightarrow -\infty} \frac{5^{2x} + 1}{5^{2x} - 1} = \lim_{x \rightarrow -\infty} \frac{25^x + 1}{25^x - 1}$$

이때 $\lim_{x \rightarrow -\infty} 25^x = 0$ 이므로

$$\lim_{x \rightarrow -\infty} \frac{5^x + 5^{-x}}{5^x - 5^{-x}} = \lim_{x \rightarrow -\infty} \frac{25^x + 1}{25^x - 1} = -1$$

30) -1

31) -1

32) 4

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{4^{x+1} + 2^x}{4^x - 2^x} = \lim_{x \rightarrow \infty} \frac{4 + \left(\frac{1}{2}\right)^x}{1 - \left(\frac{1}{2}\right)^x} = 4$$

33) 32

⇒ 분자, 분모를 4^x 으로 각각 나누면

$$\lim_{x \rightarrow \infty} \frac{a \cdot 4^x + 3^x}{4^{x+1} - 2^x} = \lim_{x \rightarrow \infty} \frac{a \cdot 1 + \left(\frac{3}{4}\right)^x}{4 \cdot 1 - \left(\frac{1}{2}\right)^x} = \frac{a}{4} = 8$$

$$\therefore a = 32$$

34) 5

35) 3

[해설]

$$\lim_{x \rightarrow \infty} \frac{a \times 3^{x+1} - 4}{3^{x-2} + 2} = \lim_{x \rightarrow \infty} \frac{3a \times 3^x - 4}{\frac{1}{9} \times 3^x + 2} = \lim_{x \rightarrow \infty} \frac{3a - \frac{4}{3^x}}{\frac{1}{9} + \frac{2}{3^x}} = 3^3 a = 81 \text{ 이므로 } a = 3 \text{ 이다.}$$

36) -3

⇒ 분자, 분모를 4^x 으로 나누면

$$\lim_{x \rightarrow \infty} \frac{4^x + 9 \times \left(\frac{3}{4}\right)^x + \left(\frac{1}{2}\right)^x}{1 + 3 \times \left(\frac{3}{4}\right)^x} = 4^a$$

$$4^a = \frac{1}{64} = 4^{-3} \text{ 에서 } a = -3$$

37) 2

$$\Rightarrow \lim_{x \rightarrow 4} \log_2 x = \log_2 4 = 2$$

38) ∞

$$\Rightarrow \lim_{x \rightarrow 0+} \log_{\frac{1}{2}} x = \infty$$

39) $-\infty$

⇒ $a > 1$ 일 때 $\lim_{x \rightarrow 0+} \log_a x = -\infty$ 이므로

$$\lim_{x \rightarrow 0+} \log_3 x = -\infty$$

40) $-\infty$

$$\Rightarrow \lim_{x \rightarrow 0+} \log x = -\infty$$

41) 0

$$\Rightarrow \lim_{x \rightarrow 1} \log x = \log 1 = 0$$

42) -2

$$\Rightarrow \lim_{x \rightarrow 9} \log_{\frac{1}{3}} x = \log_{\frac{1}{3}} 9 = -2$$

$$43) -\frac{1}{\ln 2}$$

⇒ $x - 1 = t$ 로 놓자.

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\log_2 x}{1-x} &= \lim_{t \rightarrow 0} \frac{\log_2 (1+t)}{-t} \\ &= \lim_{t \rightarrow 0} \frac{\log_2 (1+t)}{t} \cdot (-1) = -\frac{1}{\ln 2} \end{aligned}$$

44) $1 - \log_2 3$

45) -2

$$\Rightarrow \lim_{x \rightarrow -2} \log_3 \frac{x+1}{x^3-1} = \log_3 \frac{-2+1}{-8-1} = \log_3 \frac{1}{9} = -2$$

46) $\frac{1}{2\ln 2}$

$$\Rightarrow x-4=t \text{라 놓으면 } x \rightarrow 4 \text{일 때, } t \rightarrow 0$$

$$\lim_{x \rightarrow 4} \frac{\log_4(x-3)}{x-4} = \lim_{t \rightarrow 0} \frac{\log_4(1+t)}{t} \\ = \frac{1}{\ln 4} = \frac{1}{2\ln 2}$$

47) 1

$$\Rightarrow \lim_{x \rightarrow 2} \{\log_3(5x+2) - \log_3(x+2)\} \\ = \lim_{x \rightarrow 2} \log_3 \frac{5x+2}{x+2} = \log_3 \left(\lim_{x \rightarrow 2} \frac{5x+2}{x+2} \right) = \log_3 3 = 1$$

48) 1

$$\Rightarrow \frac{1}{x} = t \text{라 놓으면 } x \rightarrow 0+ \text{일 때, } t \rightarrow \infty \text{이므로}$$

$$\lim_{x \rightarrow 0+} \left\{ \frac{\log_3 \frac{3}{x}}{\log_3 \left(\frac{6}{x} + 1 \right)} \right\} = \lim_{t \rightarrow \infty} \frac{\log_3 3t}{\log_3(6t+1)} \\ = \lim_{t \rightarrow \infty} \frac{\log_3 t + 1}{\log_3 t + \log_3 \left(6 + \frac{1}{t} \right)}$$

분자와 분모를 $\log_3 t$ 로 나누면

$$= \lim_{t \rightarrow \infty} \frac{1 + \frac{1}{\log_3 t}}{\log_3 \left(6 + \frac{1}{t} \right) + 1} = \frac{1}{1} = 1$$

49) ∞

$$\Rightarrow \lim_{x \rightarrow \infty} \log_3 2x = \infty$$

50) $-\infty$

$$\Rightarrow 0 < a < 1 \text{일 때 } \lim_{x \rightarrow \infty} \log_a x = -\infty \text{이므로}$$

$$\lim_{x \rightarrow \infty} \log_{\frac{1}{5}} x = -\infty$$

51) ∞

$$\Rightarrow a > 1 \text{일 때 } \lim_{x \rightarrow \infty} \log_a x = \infty \text{이므로}$$

$$\lim_{x \rightarrow \infty} \log_2 x = \infty$$

52) ∞

$$\Rightarrow \lim_{x \rightarrow \infty} \log_4(x^2+1) = \infty$$

53) ∞

$$\Rightarrow \lim_{x \rightarrow 0+} \log_{\frac{1}{3}} x = \infty$$

54) ∞

$$\Rightarrow x-4=t \text{로 놓으면 } x \rightarrow 4+ \text{일 때 } t \rightarrow 0+ \text{이므로}$$

$$\lim_{x \rightarrow 4+} \log_{\frac{1}{2}}(x-4) = \lim_{t \rightarrow 0+} \log_{\frac{1}{2}} t = \infty$$

55) $-\infty$

$$\Rightarrow \lim_{x \rightarrow \infty} \log_3 \frac{1}{x} = \lim_{x \rightarrow \infty} (-\log_3 x) = -\infty$$

56) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \log_5 \frac{x^2+1}{x^2-1} = \log_5 \left(\lim_{x \rightarrow \infty} \frac{1 + \frac{1}{x^2}}{1 - \frac{1}{x^2}} \right) \\ = \log_5 1 = 0$$

57) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \{\log_2(12x+3) - \log_2 3x\} \\ = \lim_{x \rightarrow \infty} \log_2 \frac{12x+3}{3x} = \log_2 \left(\lim_{x \rightarrow \infty} \frac{12x+3}{3x} \right) = \log_2 4 = 2$$

58) 1

$$\Rightarrow \lim_{x \rightarrow \infty} \{\log(20x+1) - \log 2x\} \\ = \lim_{x \rightarrow \infty} \log \frac{20x+1}{2x} \\ = \lim_{x \rightarrow \infty} \log \left(10 + \frac{1}{2x} \right) = \log 10 = 1$$

59) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \log_3 \frac{9x+1}{x} = \lim_{x \rightarrow \infty} \log_3 \left(9 + \frac{1}{x} \right) = \log_3 9 = 2$$

60) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \{\log_2(3x+2) - \log_2 3x\} = \lim_{x \rightarrow \infty} \log_2 \frac{3x+2}{3x} = \log_2 1 = 0$$

61) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \log_2 \left(\frac{3^x}{3^x-1} \right) = \log_2 1 = 0$$

62) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \log_2 \frac{4x+1}{x+2} = \log_2 \left(\lim_{x \rightarrow \infty} \frac{4x+1}{x+2} \right) = \log_2 4 \\ = \log_2 2^2 = 2$$

63) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \{\log_2(4x+1) - \log_2 x\} \\ = \lim_{x \rightarrow \infty} \log_2 \frac{4x+1}{x} \\ = \lim_{x \rightarrow \infty} \log_2 \left(4 + \frac{1}{x} \right) = \log_2 4 = 2$$

64) 1

$$\Rightarrow 2^x + 3^x = 3^x \left\{ \left(\frac{2}{3} \right)^x + 1 \right\} \text{이므로}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} \log_3(2^x + 3^x) = \lim_{x \rightarrow \infty} \log_3 \left[3^x \left\{ \left(\frac{2}{3} \right)^x + 1 \right\} \right]^{\frac{1}{x}} \\ = \log_3 3 = 1$$

65) 0

$$\Rightarrow \lim_{x \rightarrow \infty} \log_2 \left(\frac{5^x}{5^x - 1} \right) = \log_2 1 = 0$$

66) 2

$$\Rightarrow \lim_{x \rightarrow \infty} \{ \log_2(4x+1) - \log_2(x-1) \} \\ = \lim_{x \rightarrow \infty} \log_2 \frac{4x+1}{x-1} = \log_2 \left(\lim_{x \rightarrow \infty} \frac{4x+1}{x-1} \right) = \log_2 4 = 2$$

67) -1

$$\Rightarrow \lim_{x \rightarrow \infty} \{ \log_4(x^3 + 3x + 1) - \log_4(4x^3 + 3) \} \\ = \lim_{x \rightarrow \infty} \log_4 \frac{x^3 + 3x + 1}{4x^3 + 3} \\ = \lim_{x \rightarrow \infty} \log_4 \frac{1 + \frac{3}{x^2} + \frac{1}{x^3}}{4 + \frac{3}{x^3}} = \log_4 \frac{1}{4} = -1$$

68) 4

$$\Rightarrow \lim_{x \rightarrow \infty} \left\{ \frac{\log_3 x^4 \left(1 + \frac{2}{x} - \frac{1}{x^4} \right) + \log_3 9^x \left(\left(\frac{2}{3} \right)^x + 1 \right)}{\log_3 x^2 \left(1 - \frac{1}{x} - \frac{4}{x^2} \right)} \right\} \\ = \lim_{x \rightarrow \infty} \left\{ \frac{4 \log_3 x + \log_3 \left(1 + \frac{2}{x} - \frac{1}{x^4} \right) + 2x + \log_3 \left(\left(\frac{2}{3} \right)^x + 1 \right)}{2 \log_3 x + \log_3 \left(1 - \frac{1}{x} - \frac{4}{x^2} \right)} \right\} \\ = 2 + 2 = 4$$

69) 54

$$\Rightarrow \lim_{x \rightarrow \infty} \{ \log_3(ax-1) - \log_3(2x+1) \} \\ = \log_3 \left(\lim_{x \rightarrow \infty} \frac{ax-1}{2x+1} \right) \\ = \log_3 \left(\lim_{x \rightarrow \infty} \frac{a - \frac{1}{x}}{2 + \frac{1}{x}} \right) = \log_3 \frac{a}{2} \\ \log_3 \frac{a}{2} = 3 \text{이므로 } a = 54$$

70) 20

$$\Rightarrow \lim_{x \rightarrow \infty} \{ \log ax - \log(2x+5) \} = \lim_{x \rightarrow \infty} \log \frac{ax}{2x+5} \\ = \log \frac{a}{2}$$

$$\text{따라서 } \log \frac{a}{2} = 1 \text{이므로 } \frac{a}{2} = 10, a = 20$$

71) 4

$$\Rightarrow \lim_{x \rightarrow \infty} \{ \log_2(ax+1) - \log_2 x \} = \lim_{x \rightarrow \infty} \log_2 \left(a + \frac{1}{x} \right) = \log_2 a \\ \log_2 a = 2 \text{이므로 } a = 4$$