01 지수의 확장

(1) 0 또는 음의 정수인 지수의 정의 : $a \neq 0$이고 n이 양의 정수일 때

① $a^0 = 1$ ② $a^{-n} = \frac{1}{a^n}$

(2) 유리수인 지수의 정의 : $a > 0$이고, m, $n(n \geq 2)$이 정수일 때

① $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ ② $a^{\frac{1}{n}} = \sqrt[n]{a}$

◪ 다음 수를 근호를 사용하여 나타내어라.

1. $4^{-\frac{3}{4}}$

2. $\left(\frac{1}{9}\right)^{-\frac{3}{4}}$

3. $2^{\frac{2}{3}}$

4. $11^{\frac{2}{3}}$

5. $5^{-\frac{3}{2}}$

6. $\left(\frac{125}{64}\right)^{\frac{1}{6}}$

7. $\left(\frac{1}{32}\right)^{\frac{3}{10}}$

◪ 다음 값을 구하여라.

8. $\left(-\frac{1}{2}\right)^0$

9. $4^0 + (-5)^0$

10. $\left(\frac{1}{3}\right)^{-2}$

11. $(-5)^{-2}$

12. $(-2)^{-3} + (-3)^{-2}$

13. $\left(\frac{5}{2}\right)^0 + \left(\frac{2}{3}\right)^{-2}$

14. $(-5)^{-2} + \left(\frac{5}{3}\right)^{-3}$

15. $(-3)^0 - 2^{-2}$

02 지수법칙

1. **지수가 정수일 때**의 지수법칙
 : $a \neq 0$, $b \neq 0$이고 m, n이 정수일 때
 ① $a^m a^n = a^{m+n}$ ② $a^m \div a^n = a^{m-n}$
 ③ $(a^m)^n = a^{mn}$ ④ $(ab)^n = a^n b^n$

2. **지수가 유리수일 때**의 지수법칙
 : $a > 0$, $b > 0$이고 p, q가 유리수일 때
 ① $a^p a^q = a^{p+q}$ ② $a^p \div a^q = a^{p-q}$
 ③ $(a^p)^q = a^{pq}$ ④ $(ab)^p = a^p b^p$

3. **지수가 실수일 때**의 지수법칙
 : $a > 0$, $b > 0$이고 x, y가 실수일 때
 ① $a^x a^y = a^{x+y}$ ② $a^x \div a^y = a^{x-y}$
 ③ $(a^x)^y = a^{xy}$ ④ $(ab)^x = a^x b^x$

◪ $a > 0$**일 때, 다음을 간단히 하여라.**

16. $a^3 \times a^4 \div a^9$

17. $(a^{-3})^{-4} \div (a^5)^{-3}$

18. $(a^{-2})^5 \times (a^3)^{-4}$

19. $a^5 \times a^2 \div a^8$

20. $\dfrac{(a^{-5})^2 \times (a^2)^5}{a^2 \times a^{-5}}$

21. $\dfrac{(a^4)^{-3} \times (a^{-2})^{-8}}{(a^{-7})^3}$

22. $(a^5)^{-2} \times (a^{-3})^{-4} \div (a^{-2})^2$

23. $(a^{-4})^2 \times (a^{-5})^{-3} \div a^{-5}$

24. $(a^3)^{-2} \times (a^{-5})^{-4} \div (a^{-2})^7$

25. $\sqrt{\sqrt[3]{a}}$

26. $\sqrt[3]{a\sqrt{a\sqrt{a}}}$

27. $\sqrt[3]{\sqrt{a} \times \sqrt[5]{a}}$

28. $\sqrt[4]{a\ \sqrt[3]{a}}$

29. $\sqrt[4]{\sqrt[3]{a\ \sqrt{a}}}$

30. $\sqrt{a\sqrt{a\sqrt{a}}}$

31. $\sqrt{a\sqrt[3]{a\sqrt[4]{a^3}}}$

32. $\sqrt{9a\sqrt{a\sqrt{a}}}$

33. $\sqrt{a\sqrt{a^2\sqrt{a^3}}}$

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◪ $a>0$일 때, 다음을 간단히 하여라.

34. $(a^{\frac{1}{3}})^{-\frac{3}{5}}\times(a^{\frac{3}{2}})^{\frac{4}{9}}$

35. $a^{\frac{1}{2}}\times a^{-\frac{2}{3}}\div a^{\frac{3}{4}}$

36. $a^{\frac{2}{5}}\div a^{-\frac{1}{2}}$

37. $a^{\frac{1}{3}}\times a^{\frac{1}{2}}$

38. $a^{\sqrt{2}}\div a^{2\sqrt{2}}\times a^{\sqrt{3}}$

39. $(a^{-\frac{3}{4}})^2\times\sqrt{a}\div a^{\frac{3}{4}}$

40. $(\sqrt{a^3}\times\sqrt[5]{a}\times a^{-\frac{1}{2}})^{\frac{1}{3}}$

41. $\sqrt[4]{a^3}\times\sqrt[3]{a^2}\div\sqrt{a^5}$

42. $\sqrt[4]{a^5}\times\sqrt{a^3}\div\sqrt[3]{a^5}$

43. $a^{-\frac{1}{2}}\div a^{\frac{1}{4}}\times a^{\frac{3}{4}}$

44. $(\sqrt{a^3}\div\sqrt[5]{a})^{\frac{1}{3}}$

45. $(\sqrt[3]{a^2}\times\sqrt[4]{a^3})^{\frac{2}{3}}$

46. $\sqrt[3]{\sqrt[4]{\sqrt{a^7}}}\times\sqrt[4]{\sqrt[3]{a^5}}$

47. $\dfrac{\sqrt[10]{a^6}}{(\sqrt[5]{a^4})^2}$

48. $\sqrt[7]{a^{11}}\times\sqrt[14]{a^6}$

49. $a^{\sqrt{3}}\times a^{\sqrt{12}}$

50. $a^{\sqrt{32}}\times a^{\sqrt{8}}\div a^{\sqrt{18}}$

51. $(a^{\frac{\sqrt{3}}{2}})^4\div a^{\sqrt{3}}$

52. $a^{-\frac{\sqrt{2}}{3}}\times a^{\frac{-2\sqrt{2}}{3}}\div a^{-3\sqrt{2}}$

53. $(a^6)^{-\frac{1}{3}}\div(a^{-3\sqrt{2}})^{\frac{1}{\sqrt{2}}}$

◪ $a>0,\ b>0$일 때, 다음 식을 간단히 하여라.

54. $(a^3b^2)^{\frac{1}{12}}\times(a^{\frac{1}{3}}b^{\frac{1}{4}})^4$

55. $\sqrt[3]{ab^2}\times\sqrt[6]{ab^5}\div\sqrt{ab}$

56. $\sqrt[3]{\sqrt[5]{a^{34}b^6}}\div\sqrt[5]{\sqrt[3]{a^4b^{21}}}$

57. $\sqrt[3]{a^2b^5}\div\sqrt[4]{a^5b^2}\times\sqrt{a^3b}$

58. $(a^{2\sqrt{2}}\times b^{3\sqrt{2}})^{\frac{3}{\sqrt{2}}}$

59. $\dfrac{\sqrt{\sqrt[3]{a^4b}}\times\sqrt[3]{a^2b^4}}{\sqrt[4]{\sqrt[3]{a^4b^6}}}$

60. $\left(\dfrac{a^{\sqrt{3}}}{b^{\sqrt{18}}}\right)^{\frac{2}{\sqrt{6}}}$

◪ 다음을 간단히 하여라.

61. $(81^{-\frac{1}{3}})^{\frac{9}{4}}$

62. $\left\{\left(\dfrac{3}{5}\right)^{-\frac{5}{2}}\right\}^{\frac{4}{5}}$

63. $25^{-\frac{3}{2}}\times100^{\frac{3}{2}}$

64. $9^{-\frac{3}{2}}\times36^{\frac{1}{2}}$

65. $81^{-\frac{3}{4}}\div125^{-\frac{2}{3}}$

66. $7^{\frac{5}{4}}\times7^{-\frac{3}{2}}\div7^{-2}$

67. $32^{-\frac{3}{5}}\div27^{-\frac{2}{3}}$

68. $16^{\frac{1}{4}}\div16^{\frac{1}{8}}$

69. $\left\{\left(\dfrac{3}{2}\right)^{-\frac{4}{3}}\right\}^{\frac{9}{4}}$

70. $5^{\frac{3}{4}} \times 625^{\frac{1}{4}}$

71. $16^{-\frac{3}{2}} \times 64^{\frac{3}{2}} \div 27^{-\frac{1}{3}}$

72. $\sqrt{\dfrac{\sqrt[5]{5^{30}}}{\sqrt[3]{7^{12}}}}$

73. $(5^{\frac{3}{2}})^2 \div \sqrt{5}$

74. $(\sqrt[12]{3^5})^2 \times (\sqrt[12]{3^2})^7$

75. $\sqrt[4]{\sqrt[3]{16}} \times \sqrt[6]{\sqrt{256}}$

76. $\left\{\left(\dfrac{27}{64}\right)^{-\frac{1}{3}}\right\}^{\frac{3}{2}} \times \left(\dfrac{3}{4}\right)^{\frac{1}{2}}$

77. $(3^{-\frac{2}{3}})^6 \times \left\{\left(\dfrac{2}{3}\right)^{-\frac{3}{2}}\right\}^4$

78. $\sqrt{\dfrac{\sqrt[3]{1024}}{\sqrt[6]{27}}} \div \sqrt[3]{\dfrac{\sqrt{16}}{\sqrt[4]{27}}}$

79. $(4^{\sqrt{3}})^{\frac{\sqrt{3}}{2}}$

80. $3^{\sqrt{5}} \times 4^{\sqrt{5}}$

81. $(4^{\frac{1}{\sqrt{6}}} \times 3^{\sqrt{\frac{2}{3}}})^{\sqrt{3}}$

82. $(2^{\sqrt{8}} \times 3^{\sqrt{2}})^{\sqrt{2}}$

83. $3^{\frac{\sqrt{2}}{3}} \times 3^{\frac{\sqrt{8}}{3}}$

84. $8^{-\frac{\sqrt{3}}{6}} \times 2^{\frac{\sqrt{3}}{2}}$

85. $5^{\sqrt{3}+1} \div 5^{\sqrt{3}-2}$

86. $3^{\frac{\sqrt{5}}{2}} \times 3^{\frac{3\sqrt{5}}{2}}$

87. $(2^{5\sqrt{3}})^{\frac{\sqrt{6}}{3}} \times 2^{2-3\sqrt{2}} \div 2^{2\sqrt{2}-1}$

88. $4^{\sqrt{2}} \times 4^{\sqrt{18}} \div 4^{\sqrt{8}}$

89. $(3^{\sqrt{8}} \times 2^{\sqrt{2}})^{\sqrt{2}}$

90. $2^{\sqrt{2}+1} \div 2^{\sqrt{2}-1}$

정답 및 해설

1) $\frac{\sqrt{2}}{4}$

⇨ $4^{-\frac{3}{4}}=(2^2)^{-\frac{3}{4}}=2^{-\frac{3}{2}}=\frac{1}{2^{\frac{3}{2}}}=\frac{1}{\sqrt{2^3}}=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}$

2) $3\sqrt{3}$

⇨ $\left(\frac{1}{9}\right)^{-\frac{3}{4}}=(3^{-2})^{-\frac{3}{4}}=3^{\frac{3}{2}}=\sqrt{3^3}=3\sqrt{3}$

3) $\sqrt[3]{4}$

⇨ $2^{\frac{2}{3}}=\sqrt[3]{2^2}=\sqrt[3]{4}$

4) $\sqrt[3]{121}$

⇨ $11^{\frac{2}{3}}=\sqrt[3]{11^2}=\sqrt[3]{121}$

5) $\frac{\sqrt{5}}{25}$

⇨ $5^{-\frac{3}{2}}=5^{\frac{-3}{2}}=\sqrt{5^{-3}}=\sqrt{\frac{1}{5^3}}$

$=\frac{1}{\sqrt{5^3}}=\frac{1}{\sqrt{125}}=\frac{\sqrt{5}}{25}$

6) $\frac{\sqrt{5}}{2}$

⇨ $\left(\frac{125}{64}\right)^{\frac{1}{6}}=\left\{\left(\frac{5}{4}\right)^3\right\}^{\frac{1}{6}}=\left(\frac{5}{4}\right)^{3\times\frac{1}{6}}=\left(\frac{5}{4}\right)^{\frac{1}{2}}=\frac{\sqrt{5}}{2}$

7) $\frac{\sqrt{2}}{4}$

⇨ $\left(\frac{1}{32}\right)^{\frac{3}{10}}=\left\{\left(\frac{1}{2}\right)^5\right\}^{\frac{3}{10}}=\left(\frac{1}{2}\right)^{5\times\frac{3}{10}}=\left(\frac{1}{2}\right)^{\frac{3}{2}}$

$=\sqrt{\frac{1}{8}}=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}$

8) 1

9) 2

⇨ $4^0+(-5)^0=1+1=2$

10) 9

11) $\frac{1}{25}$

12) $-\frac{1}{72}$

⇨ $(-2)^{-3}+(-3)^{-2}=\frac{1}{(-2)^3}+\frac{1}{(-3)^2}$

$=-\frac{1}{8}+\frac{1}{9}=-\frac{1}{72}$

13) $\frac{13}{4}$

⇨ $\left(\frac{5}{2}\right)^0+\left(\frac{2}{3}\right)^{-2}=1+\frac{1}{\left(\frac{2}{3}\right)^2}=1+\frac{1}{\frac{4}{9}}=\frac{13}{4}$

14) $\frac{32}{125}$

⇨ $(-5)^{-2}+\left(\frac{5}{3}\right)^{-3}=\frac{1}{(-5)^2}+\frac{1}{\left(\frac{5}{3}\right)^3}=\frac{1}{25}+\frac{1}{\frac{125}{27}}$

$=\frac{5}{125}+\frac{27}{125}=\frac{32}{125}$

15) $\frac{3}{4}$

⇨ $(-3)^0-2^{-2}=1-\frac{1}{2^2}=1-\frac{1}{4}=\frac{3}{4}$

16) $\frac{1}{a^2}$

⇨ $a^3\times a^4\div a^9=a^3\times a^4\times a^{-9}$

$=a^{3+4-9}=a^{-2}=\frac{1}{a^2}$

17) a^{27}

18) $\frac{1}{a^{22}}$

⇨ $(a^{-2})^5\times(a^3)^{-4}=a^{-10}\times a^{-12}=a^{-22}=\frac{1}{a^{22}}$

19) $\frac{1}{a}$

⇨ $a^5\times a^2\div a^8=a^{5+2-8}=a^{-1}=\frac{1}{a}$

20) a^3

⇨ $\frac{(a^{-5})^2\times(a^2)^5}{a^2\times a^{-5}}=\frac{a^{-10}\times a^{10}}{a^{2+(-5)}}=\frac{a^{(-10)+10}}{a^{-3}}$

$=a^{0-(-3)}=a^3$

21) a^{25}

⇨ $\frac{(a^4)^{-3}\times(a^{-2})^{-8}}{(a^{-7})^3}=\frac{a^{-12}\times a^{16}}{a^{-21}}$

$=a^{-12+16-(-21)}=a^{25}$

22) a^6

⇨ $(a^5)^{-2}\times(a^{-3})^{-4}\div(a^{-2})^2=a^{-10}\times a^{12}\div a^{-4}$

$=a^{-10+12-(-4)}=a^6$

23) a^{12}

⇨ $(a^{-4})^2\times(a^{-5})^{-3}\div a^{-5}=a^{-8}\times a^{15}\times a^{5}$
$=a^{(-8)+15+5}=a^{12}$

24) a^{28}
⇨ $(a^{3})^{-2}\times(a^{-5})^{-4}\div(a^{-2})^{7}=a^{-6}\times a^{20}\div a^{-14}$
$=a^{-6+20-(-14)}=a^{28}$

25) $a^{\frac{1}{6}}$
⇨ $\sqrt{\sqrt[3]{a}}=(a^{\frac{1}{3}})^{\frac{1}{2}}=a^{\frac{1}{3}\times\frac{1}{2}}=a^{\frac{1}{6}}$

26) $a^{\frac{7}{12}}$
⇨ $\sqrt[3]{a\sqrt{a\sqrt{a}}}=\left\{a(a\times a^{\frac{1}{2}})^{\frac{1}{2}}\right\}^{\frac{1}{3}}=(a\times a^{\frac{3}{2}\times\frac{1}{2}})^{\frac{1}{3}}$
$=(a^{\frac{7}{4}})^{\frac{1}{3}}=a^{\frac{7}{12}}$

27) $a^{\frac{7}{30}}$
⇨ $\sqrt[3]{\sqrt{a}\times\sqrt[5]{a}}=(a^{\frac{1}{2}}\times a^{\frac{1}{5}})^{\frac{1}{3}}$
$=(a^{\frac{1}{2}+\frac{1}{5}})^{\frac{1}{3}}=(a^{\frac{7}{10}})^{\frac{1}{3}}=a^{\frac{7}{30}}$

28) $a^{\frac{1}{3}}$
⇨ $\sqrt[4]{a\sqrt[3]{a}}=\sqrt[4]{a\times a^{\frac{1}{3}}}=(a^{\frac{4}{3}})^{\frac{1}{4}}=a^{\frac{1}{3}}$

29) $a^{\frac{1}{8}}$
⇨ $\sqrt[4]{\sqrt[3]{a\sqrt{a}}}=\sqrt[4]{\sqrt[3]{a\times a^{\frac{1}{2}}}}=\sqrt[4]{(a^{\frac{3}{2}})^{\frac{1}{3}}}=\sqrt[4]{a^{\frac{1}{2}}}=(a^{\frac{1}{2}})^{\frac{1}{4}}$
$=a^{\frac{1}{8}}$

30) $a^{\frac{7}{8}}$
⇨ $\sqrt{a\sqrt{a\sqrt{a}}}=\sqrt{a\sqrt{a\times a^{\frac{1}{2}}}}=\sqrt{a\sqrt{a^{\frac{3}{2}}}}$
$=\sqrt{a\times a^{\frac{3}{4}}}=\sqrt{a^{\frac{7}{4}}}=a^{\frac{7}{8}}$

31) $a^{\frac{19}{24}}$
⇨ $\sqrt{a\sqrt[3]{a\sqrt[4]{a^3}}}$
$=\sqrt{a\sqrt[3]{a\times a^{\frac{3}{4}}}}=\sqrt{a\times(a^{\frac{7}{4}})^{\frac{1}{3}}}=\sqrt{a\times a^{\frac{7}{12}}}$
$=(a^{\frac{19}{12}})^{\frac{1}{2}}=a^{\frac{19}{24}}$

32) $3a^{\frac{7}{8}}$
⇨ $\sqrt{9a\sqrt{a\sqrt{a}}}=\left\{9a(a\times a^{\frac{1}{2}})^{\frac{1}{2}}\right\}^{\frac{1}{2}}=(9a\times a^{\frac{3}{2}\times\frac{1}{2}})^{\frac{1}{2}}$
$=(9a^{\frac{7}{4}})^{\frac{1}{2}}=9^{\frac{1}{2}}a^{\frac{7}{8}}=3a^{\frac{7}{8}}$

33) $a^{\frac{11}{8}}$
⇨ $\sqrt{a\sqrt{a^2\sqrt{a^3}}}=\left\{a(a^2\times a^{\frac{3}{2}})^{\frac{1}{2}}\right\}^{\frac{1}{2}}=\left\{a(a^{\frac{7}{2}})^{\frac{1}{2}}\right\}^{\frac{1}{2}}$
$=(a\times a^{\frac{7}{4}})^{\frac{1}{2}}$
$=(a^{\frac{11}{4}})^{\frac{1}{2}}=a^{\frac{11}{8}}$

34) $a^{\frac{7}{15}}$
⇨ $(a^{\frac{1}{3}})^{-\frac{3}{5}}\times(a^{\frac{3}{2}})^{\frac{4}{9}}=a^{-\frac{1}{5}}\times a^{\frac{2}{3}}=a^{-\frac{1}{5}+\frac{2}{3}}=a^{\frac{7}{15}}$

35) $a^{-\frac{11}{12}}$
⇨ $a^{\frac{1}{2}}\times a^{-\frac{2}{3}}\div a^{\frac{3}{4}}=a^{\frac{1}{2}+\left(-\frac{2}{3}\right)-\frac{3}{4}}=a^{-\frac{11}{12}}$

36) $a^{\frac{9}{10}}$
⇨ $a^{\frac{2}{5}}\div a^{-\frac{1}{2}}=a^{\frac{2}{5}-\left(-\frac{1}{2}\right)}=a^{\frac{9}{10}}$

37) $a^{\frac{5}{6}}$
⇨ $a^{\frac{1}{3}}\times a^{\frac{1}{2}}=a^{\frac{1}{3}+\frac{1}{2}}=a^{\frac{5}{6}}$

38) $a^{\sqrt{3}-\sqrt{2}}$
⇨ $a^{\sqrt{2}}\div a^{2\sqrt{2}}\times a^{\sqrt{3}}=a^{\sqrt{2}-2\sqrt{2}+\sqrt{3}}=a^{\sqrt{3}-\sqrt{2}}$

39) $a^{-\frac{7}{4}}$
⇨ $(a^{-\frac{3}{4}})^2\times\sqrt{a}\div a^{\frac{3}{4}}=a^{-\frac{3}{2}}\times a^{\frac{1}{2}}\div a^{\frac{3}{4}}=a^{-\frac{3}{2}+\frac{1}{2}-\frac{3}{4}}$
$=a^{-\frac{7}{4}}$

40) $a^{\frac{2}{5}}$
⇨ $(\sqrt{a^3}\times\sqrt[5]{a}\times a^{-\frac{1}{2}})^{\frac{1}{3}}=(a^{\frac{3}{2}}\times a^{\frac{1}{5}}\times a^{-\frac{1}{2}})^{\frac{1}{3}}$
$=(a^{\frac{3}{2}+\frac{1}{5}-\frac{1}{2}})^{\frac{1}{3}}=(a^{\frac{6}{5}})^{\frac{1}{3}}=a^{\frac{2}{5}}$

41) $a^{-\frac{13}{12}}$
⇨ $\sqrt[4]{a^3}\times\sqrt[3]{a^2}\div\sqrt{a^5}=a^{\frac{3}{4}}\times a^{\frac{2}{3}}\div a^{\frac{5}{2}}$
$=a^{\frac{3}{4}+\frac{2}{3}-\frac{5}{2}}=a^{-\frac{13}{12}}$

42) $a^{\frac{13}{12}}$
⇨ $\sqrt[4]{a^5}\times\sqrt{a^3}\div\sqrt[3]{a^5}=a^{\frac{5}{4}}\times a^{\frac{3}{2}}\div a^{\frac{5}{3}}$
$=a^{\frac{5}{4}+\frac{3}{2}-\frac{5}{3}}=a^{\frac{13}{12}}$

43) 1

⇨ $a^{-\frac{1}{2}} \div a^{\frac{1}{4}} \times a^{\frac{3}{4}} = a^{\left(-\frac{1}{2}\right)-\frac{1}{4}+\frac{3}{4}} = a^0 = 1$

44) $a^{\frac{13}{30}}$

⇨ $(\sqrt{a^3} \div \sqrt[5]{a})^{\frac{1}{3}} = (a^{\frac{3}{2}} \div a^{\frac{1}{5}})^{\frac{1}{3}}$

$= (a^{\frac{3}{2}-\frac{1}{5}})^{\frac{1}{3}}$

$= (a^{\frac{13}{10}})^{\frac{1}{3}} = a^{\frac{13}{30}}$

45) $a^{\frac{17}{18}}$

⇨ $(\sqrt[3]{a^2} \times \sqrt[4]{a^3})^{\frac{2}{3}} = (a^{\frac{2}{3}} \times a^{\frac{3}{4}})^{\frac{2}{3}} = (a^{\frac{17}{12}})^{\frac{2}{3}} = a^{\frac{17}{18}}$

46) a

⇨ $\sqrt[3]{\sqrt[4]{a^7}} \times \sqrt[4]{\sqrt[3]{a^5}} = \sqrt[12]{a^7} \times \sqrt[12]{a^5} = \sqrt[12]{a^7 \times a^5}$

$= \sqrt[12]{a^{12}} = a$

47) $\frac{1}{a}$

⇨ $\frac{\sqrt[10]{a^6}}{(\sqrt[5]{a^4})^2} = \frac{\sqrt[5]{a^3}}{\sqrt[5]{a^8}} = \sqrt[5]{\frac{a^3}{a^8}} = \sqrt[5]{\frac{1}{a^5}}$

$= \sqrt[5]{\left(\frac{1}{a}\right)^5} = \frac{1}{a}$

48) a^2

⇨ $\sqrt[7]{a^{11}} \times \sqrt[14]{a^6} = \sqrt[7]{a^{11}} \times \sqrt[7]{a^3} = \sqrt[7]{a^{11} \times a^3} = \sqrt[7]{a^{14}}$

$= \sqrt[7]{(a^2)^7} = a^2$

49) $a^{3\sqrt{3}}$

⇨ $a^{\sqrt{3}} \times a^{\sqrt{12}} = a^{\sqrt{3}+2\sqrt{3}} = a^{3\sqrt{3}}$

50) $a^{3\sqrt{2}}$

⇨ $a^{\sqrt{32}} \times a^{\sqrt{8}} \div a^{\sqrt{18}} = a^{4\sqrt{2}+2\sqrt{2}-3\sqrt{2}} = a^{3\sqrt{2}}$

51) $a^{\sqrt{3}}$

⇨ $(a^{\frac{\sqrt{3}}{2}})^4 \div a^{\sqrt{3}} = a^{2\sqrt{3}-\sqrt{3}} = a^{\sqrt{3}}$

52) $a^{2\sqrt{2}}$

⇨ $a^{-\frac{\sqrt{2}}{3}} \times a^{-\frac{2\sqrt{2}}{3}} \div a^{-3\sqrt{2}} = a^{\left(-\frac{\sqrt{2}}{3}\right)+\left(-\frac{2\sqrt{2}}{3}\right)-(-3\sqrt{2})}$

$= a^{2\sqrt{2}}$

53) a

⇨ $(a^6)^{-\frac{1}{3}} \div (a^{-3\sqrt{2}})^{\frac{1}{\sqrt{2}}} = a^{-2} \div a^{-3}$

$= \frac{1}{a^2} \div \frac{1}{a^3} = \frac{a^3}{a^2} = a$

54) $a^{\frac{19}{12}} b^{\frac{7}{6}}$

⇨ $(a^3b^2)^{\frac{1}{12}} \times (a^{\frac{1}{3}} b^{\frac{1}{4}})^4 = a^{\frac{1}{4}} b^{\frac{1}{6}} \times a^{\frac{4}{3}} b$

$= a^{\frac{1}{4}+\frac{4}{3}} b^{\frac{1}{6}+1} = a^{\frac{19}{12}} b^{\frac{7}{6}}$

55) b

⇨ $\sqrt[3]{ab^2} \times \sqrt[6]{ab^5} \div \sqrt{ab} = a^{\frac{1}{3}} b^{\frac{2}{3}} \times a^{\frac{1}{6}} b^{\frac{5}{6}} \div a^{\frac{1}{2}} b^{\frac{1}{2}}$

$= a^{\frac{1}{3}+\frac{1}{6}-\frac{1}{2}} b^{\frac{2}{3}+\frac{5}{6}-\frac{1}{2}} = b$

56) $\frac{a^2}{b}$

⇨ $\sqrt[3]{\sqrt[5]{a^{34}b^6}} \div \sqrt[5]{\sqrt[3]{a^4b^{21}}} = \sqrt[15]{a^{34}b^6} \div \sqrt[15]{a^4b^{21}}$

$= \frac{\sqrt[15]{a^{34}b^6}}{\sqrt[15]{a^4b^{21}}} = \sqrt[15]{\frac{a^{34}b^6}{a^4b^{21}}}$

$= \sqrt[15]{\frac{a^{30}}{b^{15}}} = \sqrt[15]{\left(\frac{a^2}{b}\right)^{15}} = \frac{a^2}{b}$

57) $a^{\frac{11}{12}} b^{\frac{5}{3}}$

⇨ $\sqrt[3]{a^2b^5} \div \sqrt[4]{a^5b^2} \times \sqrt{a^3b}$

$= a^{\frac{2}{3}} b^{\frac{5}{3}} \div a^{\frac{5}{4}} b^{\frac{2}{4}} \times a^{\frac{3}{2}} b^{\frac{1}{2}}$

$= a^{\frac{2}{3}-\frac{5}{4}+\frac{3}{2}} b^{\frac{5}{3}-\frac{2}{4}+\frac{1}{2}} = a^{\frac{11}{12}} b^{\frac{5}{3}}$

58) a^6b^9

⇨ $(a^{2\sqrt{2}} \times b^{3\sqrt{2}})^{\frac{3}{\sqrt{2}}} = (a^{2\sqrt{2}})^{\frac{3}{\sqrt{2}}} \times (b^{3\sqrt{2}})^{\frac{3}{\sqrt{2}}} = a^6b^9$

59) ab

⇨ $\frac{\sqrt{\sqrt[3]{a^4b} \times \sqrt[3]{a^2b^4}}}{\sqrt[4]{\sqrt[3]{a^4b^6}}} = \frac{\sqrt[6]{a^4b} \times \sqrt[3]{a^2b^4}}{\sqrt[12]{a^4b^6}}$

$= \frac{\sqrt[6]{a^4b} \times \sqrt[6]{a^4b^8}}{\sqrt[6]{a^2b^3}} = \sqrt[6]{\frac{a^4b \times a^4b^8}{a^2b^3}}$

$= \sqrt[6]{a^6b^6} = \sqrt[6]{(ab)^6} = ab$

60) $a^{\sqrt{2}} b^{-2\sqrt{3}}$

⇨ $\left(\frac{a^{\sqrt{3}}}{b^{\sqrt{18}}}\right)^{\frac{2}{\sqrt{6}}} = \frac{(a^{\sqrt{3}})^{\frac{2}{\sqrt{6}}}}{(b^{3\sqrt{2}})^{\frac{2}{\sqrt{6}}}} = \frac{a^{\frac{2}{\sqrt{2}}}}{b^{\frac{6}{\sqrt{3}}}} = a^{\sqrt{2}} b^{-2\sqrt{3}}$

61) $\frac{1}{27}$

⇨ $(81^{-\frac{1}{3}})^{\frac{9}{4}} = 81^{-\frac{3}{4}} = (3^4)^{-\frac{3}{4}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

62) $\frac{25}{9}$

⇨ $\left\{\left(\frac{3}{5}\right)^{-\frac{5}{2}}\right\}^{\frac{4}{5}} = \left(\frac{3}{5}\right)^{-\frac{5}{2} \times \frac{4}{5}} = \left(\frac{3}{5}\right)^{-2}$

$= \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

63) 8

⇨ $25^{-\frac{3}{2}} \times 100^{\frac{3}{2}} = (5^2)^{-\frac{3}{2}} \times (2^2 \times 5^2)^{\frac{3}{2}}$

$= 2^3 \times 5^{-3+3} = 8$

64) $\frac{2}{9}$

⇨ $9^{-\frac{3}{2}} \times 36^{\frac{1}{2}} = (3^2)^{-\frac{3}{2}} \times (6^2)^{\frac{1}{2}} = 3^{-3} \times 6 = \frac{6}{3^3} = \frac{2}{9}$

65) $\frac{25}{27}$

⇨ $81^{-\frac{3}{4}} \div 125^{-\frac{2}{3}} = (3^4)^{-\frac{3}{4}} \div (5^3)^{-\frac{2}{3}} = 3^{-3} \div 5^{-2}$

$= \frac{1}{3^3} \div \frac{1}{5^2} = \frac{1}{3^3} \times 5^2 = \frac{25}{27}$

66) $7^{\frac{7}{4}}$

⇨ $7^{\frac{5}{4}} \times 7^{-\frac{3}{2}} \div 7^{-2} = 7^{\frac{5}{4}+\left(-\frac{3}{2}\right)-(-2)} = 7^{\frac{7}{4}}$

67) $\frac{9}{8}$

⇨ $32^{-\frac{3}{5}} \div 27^{-\frac{2}{3}} = (2^5)^{-\frac{3}{5}} \div (3^3)^{-\frac{2}{3}} = 2^{-3} \div 3^{-2}$

$= \frac{1}{2^3} \div \frac{1}{3^2} = \frac{3^2}{2^3} = \frac{9}{8}$

68) $\sqrt{2}$

⇨ $16^{\frac{1}{4}} \div 16^{\frac{1}{8}} = 16^{\frac{1}{4}-\frac{1}{8}} = 16^{\frac{1}{8}} = (2^4)^{\frac{1}{8}} = 2^{\frac{1}{2}} = \sqrt{2}$

69) $\frac{8}{27}$

⇨ $\left\{\left(\frac{3}{2}\right)^{-\frac{4}{3}}\right\}^{\frac{9}{4}} = \left(\frac{3}{2}\right)^{-3} = \frac{1}{\left(\frac{3}{2}\right)^3} = \frac{8}{27}$

70) $5^{\frac{7}{4}}$

⇨ $5^{\frac{3}{4}} \times 625^{\frac{1}{4}} = 5^{\frac{3}{4}} \times (5^4)^{\frac{1}{4}} = 5^{\frac{3}{4}+1} = 5^{\frac{7}{4}}$

71) 24

⇨ $16^{-\frac{3}{2}} \times 64^{\frac{3}{2}} \div 27^{-\frac{1}{3}} = (2^4)^{-\frac{3}{2}} \times (2^6)^{\frac{3}{2}} \div (3^3)^{-\frac{1}{3}}$

$= 2^{-6} \times 2^9 \div 3^{-1} = 2^3 \div \frac{1}{3}$

$= 8 \times 3 = 24$

72) $\frac{125}{49}$

⇨ $\sqrt{\frac{\sqrt[5]{5^{30}}}{\sqrt[3]{7^{12}}}} = \frac{\sqrt{\sqrt[5]{5^{30}}}}{\sqrt{\sqrt[3]{7^{12}}}} = \frac{\sqrt[10]{5^{30}}}{\sqrt[6]{7^{12}}}$

$= \frac{\sqrt[10]{(5^3)^{10}}}{\sqrt[6]{(7^2)^6}} = \frac{5^3}{7^2} = \frac{125}{49}$

73) $5^{\frac{5}{2}}$

⇨ $(5^{\frac{3}{2}})^2 \div \sqrt{5} = 5^{\frac{3}{2}\times 2} \div 5^{\frac{1}{2}} = 5^{3-\frac{1}{2}} = 5^{\frac{5}{2}}$

74) 9

⇨ $(\sqrt[12]{3^5})^2 \times (\sqrt[12]{3^2})^7$

$= \sqrt[12]{(3^5)^2} \times \sqrt[12]{(3^2)^7} = \sqrt[12]{3^{10}} \times \sqrt[12]{3^{14}}$

$= \sqrt[12]{3^{10} \times 3^{14}} = \sqrt[12]{3^{24}} = \sqrt[12]{(3^2)^{12}}$

$= 3^2 = 9$

75) 2

⇨ $\sqrt[4]{\sqrt[3]{16}} \times \sqrt[6]{\sqrt{256}}$

$= \sqrt[12]{16} \times \sqrt[12]{256} = \sqrt[12]{2^4} \times \sqrt[12]{2^8}$

$= \sqrt[12]{2^4 \times 2^8} = \sqrt[12]{2^{12}} = 2$

76) $\frac{4}{3}$

⇨ $\left\{\left(\frac{27}{64}\right)^{-\frac{1}{3}}\right\}^{\frac{3}{2}} \times \left(\frac{3}{4}\right)^{\frac{1}{2}} = \left(\frac{27}{64}\right)^{-\frac{1}{2}} \times \left(\frac{3}{4}\right)^{\frac{1}{2}}$

$= \left(\frac{3^3}{4^3}\right)^{-\frac{1}{2}} \times \left(\frac{3}{4}\right)^{\frac{1}{2}}$

$= \left(\frac{3}{4}\right)^{-\frac{3}{2}+\frac{1}{2}} = \left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

77) $\frac{9}{64}$

⇨ $(3^{-\frac{2}{3}})^6 \times \left\{\left(\frac{2}{3}\right)^{-\frac{3}{2}}\right\}^4 = 3^{-4} \times \left(\frac{2}{3}\right)^{-6} = \frac{1}{3^4} \times \frac{1}{\left(\frac{2}{3}\right)^6}$

$= \frac{1}{3^4} \times \frac{3^6}{2^6} = \frac{3^2}{2^6} = \frac{9}{64}$

78) 2

⇨ $\sqrt{\frac{\sqrt[3]{1024}}{\sqrt[6]{27}}} \div \sqrt[3]{\frac{\sqrt{16}}{\sqrt[4]{27}}} = \frac{\sqrt{\sqrt[3]{1024}}}{\sqrt{\sqrt[6]{27}}} \div \frac{\sqrt[3]{\sqrt{16}}}{\sqrt[3]{\sqrt[4]{27}}}$

$= \frac{\sqrt[6]{1024}}{\sqrt[12]{27}} \div \frac{\sqrt[6]{16}}{\sqrt[12]{27}} = \frac{\sqrt[6]{1024}}{\sqrt[12]{27}} \times \frac{\sqrt[12]{27}}{\sqrt[6]{16}}$

$= \frac{\sqrt[6]{1024}}{\sqrt[6]{16}} = \sqrt[6]{\frac{2^{10}}{2^4}} = \sqrt[6]{2^6} = 2$

79) 8

⇨ $(4^{\sqrt{3}})^{\frac{\sqrt{3}}{2}} = 4^{\sqrt{3}\times\frac{\sqrt{3}}{2}} = 4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^3 = 8$

80) $12^{\sqrt{5}}$

⇨ $3^{\sqrt{5}} \times 4^{\sqrt{5}} = (3\times 4)^{\sqrt{5}} = 12^{\sqrt{5}}$

81) $6^{\sqrt{2}}$

⇨ $(4^{\frac{1}{\sqrt{6}}}\times 3^{\sqrt{\frac{2}{3}}})^{\sqrt{3}}=(2^{\frac{2}{\sqrt{6}}}\times 3^{\frac{\sqrt{2}}{\sqrt{3}}})^{\sqrt{3}}$
$=2^{\sqrt{2}}\times 3^{\sqrt{2}}=6^{\sqrt{2}}$

82) 144
⇨ $(2^{\sqrt{8}}\times 3^{\sqrt{2}})^{\sqrt{2}}=2^{\sqrt{16}}\times 3^2=2^4\times 3^2=144$

83) $3^{\sqrt{2}}$
⇨ $3^{\frac{\sqrt{2}}{3}}\times 3^{\frac{\sqrt{8}}{3}}=3^{\frac{\sqrt{2}}{3}+\frac{2\sqrt{2}}{3}}=3^{\frac{3\sqrt{2}}{3}}=3^{\sqrt{2}}$

84) 1
⇨ $8^{-\frac{\sqrt{3}}{6}}\times 2^{\frac{\sqrt{3}}{2}}=(2^3)^{-\frac{\sqrt{3}}{6}}\times 2^{\frac{\sqrt{3}}{2}}=2^{-\frac{\sqrt{3}}{2}}\times 2^{\frac{\sqrt{3}}{2}}$
$=2^{-\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}=1$

85) 125
⇨ $5^{\sqrt{3}+1}\div 5^{\sqrt{3}-2}=5^{\sqrt{3}+1-(\sqrt{3}-2)}=5^3=125$

86) $3^{2\sqrt{5}}$
⇨ $3^{\frac{\sqrt{5}}{2}}\times 3^{\frac{3\sqrt{5}}{2}}=3^{\frac{\sqrt{5}}{2}+\frac{3\sqrt{5}}{2}}=3^{2\sqrt{5}}$

87) 8
⇨ $(2^{5\sqrt{3}})^{\frac{\sqrt{6}}{3}}\times 2^{2-3\sqrt{2}}\div 2^{2\sqrt{2}-1}$
$=2^{5\sqrt{2}}\times 2^{2-3\sqrt{2}}\div 2^{2\sqrt{2}-1}$
$=2^{5\sqrt{2}+2-3\sqrt{2}-(2\sqrt{2}-1)}$
$=2^3=8$

88) $2^{4\sqrt{2}}$
⇨ $4^{\sqrt{2}}\times 4^{\sqrt{18}}\div 4^{\sqrt{8}}=4^{\sqrt{2}+\sqrt{18}-\sqrt{8}}=4^{2\sqrt{2}}=2^{4\sqrt{2}}$

89) 324
⇨$(3^{\sqrt{8}}\times 2^{\sqrt{2}})^{\sqrt{2}}=(3^{\sqrt{8}})^{\sqrt{2}}\times(2^{\sqrt{2}})^{\sqrt{2}}$
$=3^4\times 2^2=324$

90) 4
⇨ $2^{\sqrt{2}+1}\div 2^{\sqrt{2}-1}=2^{\sqrt{2}+1-(\sqrt{2}-1)}=2^2=4$