01 로그의 밑의 변환 공식

$a>0,\ a\neq 1,\ b>0$일 때

(1) $\log_a b=\dfrac{\log_c b}{\log_c a}$ (단, $c>0,\ c\neq 1$)

(2) $\log_a b=\dfrac{1}{\log_b a}$ (단, $b\neq 1$)

◪ 다음 식을 간단히 하여라.

1. $\log_2 9\times\log_3 8$

2. $\log_2 3\times\log_3 2$

3. $\log_2\sqrt{5}\times\log_{25} 8$

4. $\log_{25} 9\times\log_{27} 5$

5. $\log_5 3\times\log_3\sqrt{5}$

6. $\log_{16} 25\times\log_9 8\times\log_5 81$

7. $\log_3 5\times\log_5 7\times\log_7 9$

8. $\log_5 3\times\log_2 5\times\log_3 2$

9. $\log_{25} 8\times\log_8 81\times\log_9 125$

10. $\log_3 49\times\log_5 27\times\log_7\sqrt{5}$

11. $\log_4 27\times\log_9 25\times\log_5 16$

12. $\log_9 25+\log_{27} 625$

13. $\log_{12} 16+\dfrac{1}{\log_9 12}$

14. $\log_3 5\times\log_5 7\times\log_7 9\times\log_9 3$

15. $\log_8 81\times\log_9 125\times\log_5 64$

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16. $\log_{25} 3 \times \log_{16} 5 \times \log_9 32$

17. $(\log_6 3)^2 + \dfrac{\log_6 18}{\log_2 6}$

18. $\log_2 20 - \dfrac{1}{\log_5 2}$

19. $\log_3 63 - \dfrac{1}{\log_7 3}$

20. $\log_6 24 + \dfrac{1}{\log_9 6}$

21. $\log_3 \sqrt{10} - \dfrac{1}{2}\log_3 \dfrac{1}{6} - \dfrac{3}{2}\log_3 \sqrt[3]{20}$

22. $\log_6 27 \times \log_9 6 + \log_2 4 + \log_{\frac{1}{3}} 27$

23. $\log_8 27 \times \log_3 25 \times \log_5 32$

24. $\log_2 \dfrac{1}{3} + 2\log_2 \sqrt{3} - \log_2 3 \cdot \log_3 4$

25. $\log_2 (\log_2 3) + \log_2 (\log_3 4)$

26. $(\log_2 5 + \log_4 125)(\log_5 4 - \log_{25} 8)$

27. $(\log_2 3 + \log_8 27)(\log_3 4 + \log_{\sqrt{3}} 2)$

28. $(\log_3 5 - \log_9 125)(\log_{25} 27 + \log_5 9)$

29. $(\log_2 15 - \log_2 3)(\log_5 24 - \log_5 3)$

30. $(\log_2 3 + \log_4 9)(\log_3 4 - \log_9 8)$

31. $\log_2 3 \times \log_3 4 \times \log_4 5 \times \cdots \times \log_{31} 32$

32. $\log_6(\log_3 2) + \log_6(\log_4 3) + \log_6(\log_5 4) + \cdots + \log_6(\log_{64} 63)$

02 로그의 정의를 이용하여 식을 문자로 나타내기

$x=\log_{10}a \Leftrightarrow 10^x=a$임을 이용하여 주어진 식을 문자로 나타낼 수 있다.

◪ $10^x=a,\ 10^y=b$**일 때, 다음을** $x,\ y$**로 나타내어라.**

33. $\log_b \sqrt{a}$

34. $\log_a b^2$

35. $\log_{10} \dfrac{b^2}{a}$

36. $\log_{10} ab$

◪ $10^a=2,\ 10^b=3$**일 때, 다음을** $a,\ b$**로 나타내어라.**

37. $\log_{12} \sqrt{6}$

38. $\log_4 81$

39. $\log 54$

◪ $10^x=a,\ 10^y=b,\ 10^z=c$**일 때, 다음을** $x,\ y,\ z$**로 나타내어라.**

40. $\log_{a^2bc} \sqrt[3]{abc^2}$

41. $\log_{b^2c} \sqrt{a^3b^5}$

42. $\log_{ab} \sqrt[3]{bc^2}$

◪ $\log_{10}2=a,\ \log_{10}3=b$**일 때, 다음을** $a,\ b$**로 나타내어라.**

43. $\log_2 3$

44. $\log_5 24$

45. $\log_2 \sqrt{27}$

46. $\log_6 18$

47. $\log_6 12$

48. $\log_3 12$

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49. $\log_6 5$

50. $\log_3 \sqrt{18}$

51. $\log_4 \sqrt{54}$

52. $\log_{12} 72$

◪ 다음 물음에 답하여라.

53. $\log_2 3 = a, \log_3 5 = b$일 때, $\log_{10} 5$를 a, b로 나타내어라.

54. $7^a = 2,\ 7^b = 3$일 때, $\log_6 24$을 $a,\ b$로 나타내어라.

55. $8^x = 3,\ 9^y = 2$일 때, xy의 값을 구하여라.

56. $\log_a x = 3,\ \log_a y = 5$일 때, $\log_{xy} a$의 값을 구하여라.

57. $7^a = 2,\ 7^b = 3$일 때, $\log_7 \frac{16}{3}$을 $a,\ b$로 나타내어라.

58. $\log_{10} 2 = a,\ \log_{10} 3 = b$일 때, $\log_{12} \sqrt{24}$를 $a,\ b$로 나타내어라.

59. $3^a = 2,\ 3^b = 5$일 때, $\log_{18} 15$를 $a,\ b$로 나타내어라.

60. $\log_2 3 = a, \log_2 5 = b$ 일 때, $\log_{45} 100$을 a, b로 나타내면?

61. $\log_2 3 = a, \log_5 2 = b$일 때, $\log_{12} 270$을 a, b로 나타내어라.

62. $\log_{10} 15 = a,\ \log_{10} 45 = b$일 때, $\log_{10} \frac{5}{3}$를 $a,\ b$로 나타내어라.

63. $\log_{10} 36 = a,\ \log_{10} 24 = b$일 때, $\log_{10} 45$를 $a,\ b$로 나타내어라.

64. $2^a = 10,\ 10^b = 3$일 때, $\log_6 45$를 $a,\ b$로 나타내어라.

65. $2^a=10$, $3^b=10$**일 때,** $\log_3 5$**을** a, b**로 나타내어라.**

66. $3^a=2, 3^b=5$**일 때,** $\log_{12}50$**을** a, b**로 나타내어라.**

67. $2^a=3$, $3^b=5$**일 때,** $\log_{18}30$**을** a, b**로 나타내어라.**

68. $10^x=a$, $10^y=b$, $10^z=c$**일 때,** $\log_{10}\frac{b^5c}{a^3}$**를** x, y, z**로 나타내어라.**

정답 및 해설

1) 6

$\Rightarrow \log_2 9 \cdot \log_3 8 = \frac{\log_{10} 9}{\log_{10} 2} \cdot \frac{\log_{10} 8}{\log_{10} 3}$

$= \frac{\log_{10} 3^2}{\log_{10} 2} \cdot \frac{\log_{10} 2^3}{\log_{10} 3}$

$= \frac{2\log_{10} 3}{\log_{10} 2} \cdot \frac{3\log_{10} 2}{\log_{10} 3} = 6$

2) 1

$\Rightarrow \log_2 3 \times \log_3 2 = \log_2 3 \times \frac{1}{\log_2 3} = 1$

3) $\frac{3}{4}$

$\Rightarrow \log_2 \sqrt{5} \cdot \log_{25} 8 = \frac{\log_{10} \sqrt{5}}{\log_{10} 2} \cdot \frac{\log_{10} 8}{\log_{10} 25}$

$= \frac{\log_{10} 5^{\frac{1}{2}}}{\log_{10} 2} \cdot \frac{\log_{10} 2^3}{\log_{10} 5^2}$

$= \frac{\frac{1}{2}\log_{10} 5}{\log_{10} 2} \cdot \frac{3\log_{10} 2}{2\log_{10} 5} = \frac{3}{4}$

4) $\frac{1}{3}$

$\Rightarrow \log_{25} 9 \times \log_{27} 5 = \frac{\log_{10} 9}{\log_{10} 25} \times \frac{\log_{10} 5}{\log_{10} 27}$

$= \frac{\log_{10} 3^2}{\log_{10} 5^2} \times \frac{\log_{10} 5}{\log_{10} 3^3}$

$= \frac{2\log_{10} 3}{2\log_{10} 5} \times \frac{\log_{10} 5}{3\log_{10} 3}$

$= \frac{1}{3}$

5) $\frac{1}{2}$

$\Rightarrow \log_5 3 \times \log_3 \sqrt{5} = \log_5 3 \times \frac{1}{2}\log_3 5$

$= \frac{1}{2}\log_5 3 \times \frac{1}{\log_5 3} = \frac{1}{2}$

6) 3

$\Rightarrow \log_{16} 25 \cdot \log_9 8 \cdot \log_5 81$

$= \frac{\log_{10} 25}{\log_{10} 16} \cdot \frac{\log_{10} 8}{\log_{10} 9} \cdot \frac{\log_{10} 81}{\log_{10} 5}$

$= \frac{\log_{10} 5^2}{\log_{10} 2^4} \cdot \frac{\log_{10} 2^3}{\log_{10} 3^2} \cdot \frac{\log_{10} 3^4}{\log_{10} 5}$

$= \frac{2\log_{10} 5}{4\log_{10} 2} \cdot \frac{3\log_{10} 2}{2\log_{10} 3} \cdot \frac{4\log_{10} 3}{\log_{10} 5} = 3$

7) 2

$\Rightarrow \log_3 5 \times \log_5 7 \times \log_7 9 = \frac{\log_{10} 5}{\log_{10} 3} \times \frac{\log_{10} 7}{\log_{10} 5} \times \frac{\log_{10} 9}{\log_{10} 7}$

$= \frac{\log_{10} 9}{\log_{10} 3} = \frac{2\log_{10} 3}{\log_{10} 3} = 2$

8) 1

$\Rightarrow \log_5 3 \cdot \log_2 5 \cdot \log_3 2$

$= \frac{\log_{10} 3}{\log_{10} 5} \cdot \frac{\log_{10} 5}{\log_{10} 2} \cdot \frac{\log_{10} 2}{\log_{10} 3} = 1$

9) 3

$\Rightarrow \log_{25} 8 \times \log_8 81 \times \log_9 125$

$= \log_{25} 81 \cdot \log_9 125$

$= 2\log_{25} 9 \cdot \log_9 125$

$= 2\log_{25} 125$

$= 2\log_{5^2} 5^3$

$= 2 \times \frac{3}{2}$

$= 3$

10) 3

$\Rightarrow \log_3 49 \cdot \log_5 27 \cdot \log_7 \sqrt{5}$

$= \frac{\log_{10} 49}{\log_{10} 3} \cdot \frac{\log_{10} 27}{\log_{10} 5} \cdot \frac{\log_{10} \sqrt{5}}{\log_{10} 7}$

$= \frac{\log_{10} 7^2}{\log_{10} 3} \cdot \frac{\log_{10} 3^3}{\log_{10} 5} \cdot \frac{\log_{10} 5^{\frac{1}{2}}}{\log_{10} 7}$

$= \frac{2\log_{10} 7}{\log_{10} 3} \cdot \frac{3\log_{10} 3}{\log_{10} 5} \cdot \frac{\frac{1}{2}\log_{10} 5}{\log_{10} 7} = 3$

11) 6

12) $\frac{7}{3}\log_3 5$

$\Rightarrow \log_9 25 + \log_{27} 625 = \log_{3^2} 5^2 + \log_{3^3} 5^4$

$= \frac{2}{2}\log_3 5 + \frac{4}{3}\log_3 5$

$= \frac{7}{3}\log_3 5$

13) 2

$\Rightarrow \log_{12} 16 + \frac{1}{\log_9 12}$

$= \log_{12} 16 + \log_{12} 9 = \log_{12} (16 \times 9)$

$= \log_{12} 144 = \log_{12} 12^2 = 2$

14) 1

15) 12

$\Rightarrow \log_8 81 \times \log_9 125 \times \log_5 64$

$=\frac{4}{3}\log_2 3\times\frac{3}{2}\log_3 5\times 6\log_5 2=12$

16) $\frac{5}{16}$

⇨ $\log_{25}3\cdot\log_{16}5\cdot\log_9 32$

$=\frac{\log_{10}3}{\log_{10}25}\cdot\frac{\log_{10}5}{\log_{10}16}\cdot\frac{\log_{10}32}{\log_{10}9}$

$=\frac{\log_{10}3}{\log_{10}5^2}\cdot\frac{\log_{10}5}{\log_{10}2^4}\cdot\frac{\log_{10}2^5}{\log_{10}3^2}$

$=\frac{\log_{10}3}{2\log_{10}5}\cdot\frac{\log_{10}5}{4\log_{10}2}\cdot\frac{5\log_{10}2}{2\log_{10}3}=\frac{5}{16}$

17) 1

18) 2

⇨ $\log_2 20-\frac{1}{\log_5 2}=\log_2 20-\log_2 5$

$=\log_2\frac{20}{5}=\log_2 4=2$

19) 2

⇨ $\log_3 63-\frac{1}{\log_7 3}=\log_3 63-\log_3 7$

$=\log_3\frac{63}{7}=\log_3 3^2=2$

20) 3

⇨ $\log_6 24+\frac{1}{\log_9 6}=\log_6 24+\log_6 9$

$=\log_6(24\times 9)=\log_6 6^3=3$

21) $\frac{1}{2}$

⇨ $\log_3\sqrt{10}-\frac{1}{2}\log_3\frac{1}{6}-\frac{3}{2}\log_3\sqrt[3]{20}$

$=\log_3\sqrt{10}-\log_3\frac{1}{\sqrt{6}}-\log_3\sqrt{20}$

$=\log_3\frac{\sqrt{10}\times\sqrt{6}}{\sqrt{20}}=\log_3\sqrt{3}=\frac{1}{2}$

22) $\frac{1}{2}$

⇨ $\log_6 27\times\log_9 6+\log_2 4+\log_{\frac{1}{3}}27$

$=3\log_6 3\times\frac{1}{2}\log_3 6+2-3=\frac{3}{2}-1=\frac{1}{2}$

23) 10

⇨ $\log_8 27\times\log_3 25\times\log_5 32=\log_2 3\times 2\log_3 5\times 5\log_5 2$

$=10$

24) -2

⇨ $\log_2\frac{1}{3}+2\log_2\sqrt{3}-\log_2 3\cdot\log_3 4$

$=-\log_2 3+\log_2 3-\log_2 3\cdot\frac{\log_2 4}{\log_2 3}=-2$

25) 1

⇨ $\log_2(\log_2 3)+\log_2(\log_3 4)$

$=\log_2(\log_2 3\times\log_3 4)$

$=\log_2(\log_2 3\times 2\log_3 2)$

$=\log_2\left(\log_2 3\times\frac{2}{\log_2 3}\right)$

$=\log_2 2=1$

26) $\frac{5}{4}$

⇨ $(\log_2 5+\log_4 125)(\log_5 4-\log_{25}8)$

$=(\log_2 5+\log_{2^2}5^3)(\log_5 2^2-\log_{5^2}2^3)$

$=\left(\log_2 5+\frac{3}{2}\log_2 5\right)\left(2\log_5 2-\frac{3}{2}\log_5 2\right)$

$=\left(\frac{5}{2}\log_2 5\right)\left(\frac{1}{2}\log_5 2\right)$

$=\frac{5}{2}\log_2 5\cdot\frac{1}{2\log_2 5}=\frac{5}{4}$

27) 8

⇨ $(\log_2 3+\log_8 27)(\log_3 4+\log_{\sqrt{3}}2)$

$=(\log_2 3+\frac{3}{3}\log_2 3)(2\log_3 2+2\log_3 2)$

$=2\log_2 3\cdot 4\log_3 2=8$

28) $-\frac{7}{4}$

⇨ $(\log_3 5-\log_9 125)(\log_{25}27+\log_5 9)$

$=\left(\frac{\log_{10}5}{\log_{10}3}-\frac{\log_{10}125}{\log_{10}9}\right)\left(\frac{\log_{10}27}{\log_{10}25}+\frac{\log_{10}9}{\log_{10}5}\right)$

$=\left(\frac{\log_{10}5}{\log_{10}3}-\frac{3\log_{10}5}{2\log_{10}3}\right)\left(\frac{3\log_{10}3}{2\log_{10}5}+\frac{2\log_{10}3}{\log_{10}5}\right)$

$=\frac{2\log_{10}5-3\log_{10}5}{2\log_{10}3}\cdot\frac{3\log_{10}3+4\log_{10}3}{2\log_{10}5}$

$=\frac{-\log_{10}5}{2\log_{10}3}\cdot\frac{7\log_{10}3}{2\log_{10}5}=-\frac{7}{4}$

29) 3

⇨ $(\log_2 15-\log_2 3)(\log_5 24-\log_5 3)$

$=\log_2\frac{15}{3}\cdot\log_5\frac{24}{3}=\log_2 5\cdot\log_5 8$

$=\frac{\log_{10}5}{\log_{10}2}\cdot\frac{\log_{10}8}{\log_{10}5}=\frac{\log_{10}5}{\log_{10}2}\cdot\frac{3\log_{10}2}{\log_{10}5}=3$

30) 1

⇨ $(\log_2 3+\log_4 9)(\log_3 4-\log_9 8)$

$=\left(\frac{\log_{10}3}{\log_{10}2}+\frac{\log_{10}9}{\log_{10}4}\right)\left(\frac{\log_{10}4}{\log_{10}3}-\frac{\log_{10}8}{\log_{10}9}\right)$

$$=\left(\frac{\log_{10}3}{\log_{10}2}+\frac{2\log_{10}3}{2\log_{10}2}\right)\left(\frac{2\log_{10}2}{\log_{10}3}-\frac{3\log_{10}2}{2\log_{10}3}\right)$$
$$=\frac{2\log_{10}3+2\log_{10}3}{2\log_{10}2}\cdot\frac{4\log_{10}2-3\log_{10}2}{2\log_{10}3}$$
$$=\frac{2\log_{10}3}{\log_{10}2}\cdot\frac{\log_{10}2}{2\log_{10}3}=1$$

31) 5

⇨ $\log_2 3\times\log_3 4\times\log_4 5\times\cdots\times\log_{31}32$
$$=\log_2 3\times\frac{\log_2 4}{\log_2 3}\times\frac{\log_2 5}{\log_2 4}\times\cdots\times\frac{\log_2 32}{\log_2 31}$$
$$=\log_2 32=\log_2 2^5=5$$

32) -1

⇨ (준식)$=\log_6(\log_3 2\cdot\log_4 3\cdot\log_5 4\cdot\cdots\cdot\log_{64}63)$
$$=\log_6\frac{\log_6 2}{\log_6 64}=\log_6\frac{1}{6}=-1$$

33) $\frac{x}{2y}$

⇨ $10^x=a,\ 10^y=b$에서 $x=\log_{10}a,\ y=\log_{10}b$
$$\log_b\sqrt{a}=\log_b a^{\frac{1}{2}}=\frac{1}{2}\log_b a=\frac{1}{2}\cdot\frac{\log_{10}a}{\log_{10}b}=\frac{x}{2y}$$

34) $\frac{2y}{x}$

⇨ $10^x=a,\ 10^y=b$에서 $x=\log_{10}a,\ y=\log_{10}b$
$$\log_a b^2=\frac{\log_{10}b^2}{\log_{10}a}=\frac{2\log_{10}b}{\log_{10}a}=\frac{2y}{x}$$

35) $2y-x$

⇨ $10^x=a,\ 10^y=b$에서 $x=\log_{10}a,\ y=\log_{10}b$
$$\log_{10}\frac{b^2}{a}=\log_{10}b^2-\log_{10}a=2\log_{10}b-\log_{10}a=2y-x$$

36) $x+y$

⇨ $10^x=a,\ 10^y=b$에서 $x=\log_{10}a,\ y=\log_{10}b$
$$\log_{10}ab=\log_{10}a+\log_a b=x+y$$

37) $\frac{a+b}{4a+2b}$

⇨ $\log_{12}\sqrt{6}=\frac{\log\sqrt{6}}{\log 12}=\frac{\log(2\times3)^{\frac{1}{2}}}{\log(2^2\times3)}$
$$=\frac{\frac{1}{2}(\log2+\log3)}{2\log2+\log3}=\frac{a+b}{4a+2b}$$

38) $\frac{2b}{a}$

⇨ $\log_4 81=\frac{\log81}{\log4}=\frac{\log3^4}{\log2^2}=\frac{4\log3}{2\log2}=\frac{2b}{a}$

39) $a+3b$

⇨ $\log54=\log(2\times3^3)=\log2+\log3^3$
$=\log2+3\log3=a+3b$

40) $\frac{x+y+2z}{6x+3y+3z}$

⇨ $10^x=a,\ 10^y=b,\ 10^z=c$에서
$x=\log_{10}a,\ y=\log_{10}b,\ z=\log_{10}c$이므로
$$\log_{a^2bc}\sqrt[3]{abc^2}=\frac{\log_{10}\sqrt[3]{abc^2}}{\log_{10}a^2bc}$$
$$=\frac{\frac{1}{3}(\log_{10}a+\log_{10}b+2\log_{10}c)}{2\log_{10}a+\log_{10}b+\log_{10}c}$$
$$=\frac{x+y+2z}{3(2x+y+z)}=\frac{x+y+2z}{6x+3y+3z}$$

41) $\frac{3x+5y}{4y+2z}$

⇨ $10^x=a,\ 10^y=b,\ 10^z=c$에서
$x=\log_{10}a,\ y=\log_{10}b,\ z=\log_{10}c$이므로
$$\log_{b^2c}\sqrt{a^3b^5}=\frac{\log_{10}\sqrt{a^3b^5}}{\log_{10}b^2c}$$
$$=\frac{\frac{1}{2}(3\log_{10}a+5\log_{10}b)}{2\log_{10}b+\log_{10}c}=\frac{3x+5y}{2(2y+z)}=\frac{3x+5y}{4y+2z}$$

42) $\frac{y+2z}{3x+3y}$

⇨ $10^x=a,\ 10^y=b,\ 10^z=c$에서
$x=\log_{10}a,\ y=\log_{10}b,\ z=\log_{10}c$이므로
$$\log_{ab}\sqrt[3]{bc^2}=\frac{\log_{10}\sqrt[3]{bc^2}}{\log_{10}ab}$$
$$=\frac{\frac{1}{3}(\log_{10}b+2\log_{10}c)}{\log_{10}a+\log_{10}b}=\frac{y+2z}{3(x+y)}=\frac{y+2z}{3x+3y}$$

43) $\frac{b}{a}$

⇨ $\log_2 3=\frac{\log_{10}3}{\log_{10}2}=\frac{b}{a}$

44) $\frac{3a+b}{1-a}$

⇨ $\log_5 24=\frac{\log_{10}24}{\log_{10}5}=\frac{\log_{10}(2^3\times3)}{\log_{10}\frac{10}{2}}$
$$=\frac{3\log_{10}2+\log_{10}3}{\log_{10}10-\log_{10}2}=\frac{3a+b}{1-a}$$

45) $\frac{3b}{2a}$

⇨ $\log_2\sqrt{27}=\log_2 3^{\frac{3}{2}}=\frac{3}{2}\log_2 3=\frac{3}{2}\times\frac{\log_{10}3}{\log_{10}2}=\frac{3b}{2a}$

46) $\frac{a+2b}{a+b}$

⇨ $\log_6 18=\frac{\log_{10}18}{\log_{10}6}=\frac{\log_{10}(2\times3^2)}{\log_{10}(2\times3)}$

$=\frac{\log_{10}2+2\log_{10}3}{\log_{10}2+\log_{10}3}=\frac{a+2b}{a+b}$

47) $\frac{2a+b}{a+b}$

⇨ $\log_6 12=\frac{\log_{10}12}{\log_{10}6}=\frac{\log_{10}(2^2\times3)}{\log_{10}(2\times3)}$

$=\frac{\log_{10}2^2+\log_{10}3}{\log_{10}2+\log_{10}3}=\frac{2\log_{10}2+\log_{10}3}{\log_{10}2+\log_{10}3}=\frac{2a+b}{a+b}$

48) $\frac{2a+b}{b}$

⇨ $\log_3 12=\frac{\log_{10}12}{\log_{10}3}=\frac{\log_{10}(2^2\times3)}{\log_{10}3}$

$=\frac{2\log_{10}2+\log_{10}3}{\log_{10}3}=\frac{2a+b}{b}$

49) $\frac{1-a}{a+b}$

⇨ $\log_6 5=\frac{\log_{10}5}{\log_{10}6}=\frac{\log_{10}\frac{10}{2}}{\log_{10}(2\times3)}$

$=\frac{\log_{10}10-\log_{10}2}{\log_{10}2+\log_{10}3}=\frac{1-a}{a+b}$

50) $\frac{a+2b}{2b}$

⇨ $\log_3\sqrt{18}=\frac{1}{2}\log_3(2\times3^2)=\frac{1}{2}(\log_3 2+2\log_3 3)$

$=\frac{1}{2}\left(\frac{\log_{10}2}{\log_{10}3}+2\right)=\frac{1}{2}\left(\frac{a}{b}+2\right)=\frac{a+2b}{2b}$

51) $\frac{a+3b}{4a}$

⇨ $\log_4\sqrt{54}=\log_4 54^{\frac{1}{2}}=\frac{1}{2}\log_4 54$

$=\frac{1}{2}\cdot\frac{\log_{10}54}{\log_{10}4}=\frac{\log_{10}(2\times3^3)}{2\log_{10}2^2}$

$=\frac{\log_{10}2+3\log_{10}3}{4\log_{10}2}=\frac{a+3b}{4a}$

52) $\frac{3a+2b}{2a+b}$

⇨ $\log_{12}72=\frac{\log_{10}72}{\log_{10}12}=\frac{\log_{10}(2^3\times3^2)}{\log_{10}(2^2\times3)}$

$=\frac{3\log_{10}2+2\log_{10}3}{2\log_{10}2+\log_{10}3}=\frac{3a+2b}{2a+b}$

53) $\frac{ab}{1+ab}$

⇨ $\log_2 3=a$ 이므로 $\log_3 2=\frac{1}{a}$

$\log_3 5=b$

$\therefore \log_{10}5=\frac{\log_3 5}{\log_3 10}=\frac{\log_3 5}{\log_3 2+\log_3 5}$

$=\frac{b}{\frac{1}{a}+b}=\frac{b}{\frac{1+ab}{a}}=\frac{ab}{1+ab}$

54) $\frac{3a+b}{a+b}$

⇨ $7^a=2,\ 7^b=3$에서 $a=\log_7 2,\ b=\log_7 3$

$\log_6 24=\frac{\log_7 24}{\log_7 6}=\frac{\log_7(2^3\times3)}{\log_7(2\times3)}$

$=\frac{3\log_7 2+\log_7 3}{\log_7 2+\log_7 3}=\frac{3a+b}{a+b}$

55) $\frac{1}{6}$

⇨ $8^x=3$에서 $x=\log_8 3$

$9^y=2$에서 $y=\log_9 2$

$\therefore\ xy=\log_8 3\times\log_9 2=\frac{\log_2 3}{\log_2 8}\times\frac{\log_2 2}{\log_2 9}$

$=\frac{\log_2 3}{3}\times\frac{1}{2\log_2 3}=\frac{1}{6}$

56) $\frac{1}{8}$

⇨ $\log_{xy}a=\frac{1}{\log_a xy}=\frac{1}{\log_a x+\log_a y}=\frac{1}{3+5}=\frac{1}{8}$

57) $4a-b$

⇨ $7^a=2,\ 7^b=3$에서 $a=\log_7 2,\ b=\log_7 3$

$\log_7\frac{16}{3}=\log_7 16-\log_7 3=\log_7 2^4-\log_7 3$

$=4\log_7 2-\log_7 3=4a-b$

58) $\frac{3a+b}{4a+2b}$

⇨ $\log_{12}\sqrt{24}=\log_{12}24^{\frac{1}{2}}=\frac{1}{2}\log_{12}24$

$=\frac{1}{2}\cdot\frac{\log_{10}24}{\log_{10}12}=\frac{\log_{10}(2^3\times3)}{2\log_{10}(2^2\times3)}$

$=\frac{3\log_{10}2+\log_{10}3}{2(2\log_{10}2+\log_{10}3)}=\frac{3a+b}{4a+2b}$

59) $\frac{b+1}{a+2}$

⇨ $3^a=2,\ 3^b=5$에서 $a=\log_3 2,\ b=\log_3 5$

$\therefore\ \log_{18}15=\frac{\log_3 15}{\log_3 18}=\frac{\log_3(3\times5)}{\log_3(2\times3^2)}$

$= \dfrac{\log_3 3+\log_3 5}{\log_3 2+2\log_3 3} = \dfrac{b+1}{a+2}$

60) $\dfrac{2+2b}{2a+b}$

61) $\dfrac{3ab+b+1}{ab+2b}$

62) $3a-2b$

⇨ $\log_{10} 15 = \log_{10}(3\times 5) = \log_{10} 3+\log_{10} 5$

$\therefore\ a = \log_{10} 3+\log_{10} 5$ …… ㉠

$\log_{10} 45 = \log_{10}(3^2\times 5) = 2\log_{10} 3+\log_{10} 5$

$\therefore\ b = 2\log_{10} 3+\log_{10} 5$ …… ㉡

㉠, ㉡을 연립하여 풀면

$\log_{10} 3 = -a+b,\ \log_{10} 5 = 2a-b$

$\therefore\ \log_{10}\dfrac{5}{3} = \log_{10} 5-\log_{10} 3 = (2a-b)-(-a+b)$

$= 3a-2b$

63) $\dfrac{7}{4}a-\dfrac{3}{2}b+1$

⇨ $\log_{10} 36 = \log_{10}(2^2\times 3^2) = 2\log_{10} 2+2\log_{10} 3$

$\therefore\ a = 2\log_{10} 2+2\log_{10} 3$ …… ㉠

$\log_{10} 24 = \log_{10}(2^3\times 3) = 3\log_{10} 2+\log_{10} 3$

$\therefore\ b = 3\log_{10} 2+\log_{10} 3$ …… ㉡

㉠, ㉡을 연립하여 풀면

$\log_{10} 2 = -\dfrac{1}{4}a+\dfrac{1}{2}b,\ \log_{10} 3 = \dfrac{3}{4}a-\dfrac{1}{2}b$

$\therefore\ \log_{10} 45 = \log_{10}(5\times 3^2)$

$= \log_{10} 5+2\log_{10} 3$

$= 1-\log_{10} 2+2\log_{10} 3$

$= 1-\left(-\dfrac{1}{4}a+\dfrac{1}{2}b\right)+2\left(\dfrac{3}{4}a-\dfrac{1}{2}b\right)$

$= \dfrac{7}{4}a-\dfrac{3}{2}b+1$

64) $\dfrac{2ab+a-1}{ab+1}$

⇨ $\log_2 10 = a$이므로 $\log_{10} 2 = \dfrac{1}{a}$

$\log_{10} 3 = b$

$\therefore \log_6 45 = \dfrac{\log_{10} 45}{\log_{10} 6} = \dfrac{2\log_{10} 3+\log_{10} 5}{\log_{10} 2+\log_{10} 3}$

$= \dfrac{2\log_{10} 3+\log_{10} 10-\log_{10} 2}{\log_{10} 2+\log_{10} 3}$

$= \dfrac{2b+1-\dfrac{1}{a}}{\dfrac{1}{a}+b} = \dfrac{\dfrac{2ab+a-1}{a}}{\dfrac{1+ab}{a}} = \dfrac{2ab+a-1}{1+ab}$

65) $\dfrac{ab-b}{a}$

⇨ $2^a = 10 \Rightarrow a = \log_2 10$ 이므로 $\dfrac{1}{a} = \log 2$

$3^b = 10 \Rightarrow b = \log_3 10$ 이므로 $\dfrac{1}{b} = \log 3$

$\therefore \log_3 5 = \dfrac{\log 5}{\log 3} = \dfrac{\log\dfrac{10}{2}}{\log 3} = \dfrac{\log 10-\log 2}{\log 3} = \dfrac{1-\dfrac{1}{a}}{\dfrac{1}{b}}$

$= \dfrac{\dfrac{a-1}{a}}{\dfrac{1}{b}} = \dfrac{ab-b}{a}$

66) $\dfrac{a+2b}{2a+1}$

⇨ $3^a = 2$이므로 $\log_3 2 = a$ 이다.

$3^b = 5$ 이므로 $\log_3 5 = b$ 이다.

$\therefore \log_{12} 50 = \dfrac{\log_3 50}{\log_3 12} = \dfrac{\log_3(5^2\cdot 2)}{\log_3(2^2\cdot 3)}$

$= \dfrac{2\log_3 5+\log_3 2}{2\log_3 2+\log_3 3} = \dfrac{2b+a}{2a+1}$

67) $\dfrac{1+a+ab}{1+2a}$

68) $-3x+5y+z$

⇨ $10^x = a,\ 10^y = b,\ 10^z = c$에서

$x = \log_{10} a,\ y = \log_{10} b,\ z = \log_{10} c$

$\log_{10}\dfrac{b^5c}{a^3} = 5\log_{10} b+\log_{10} c-3\log_{10} a$

$= -3x+5y+z$