



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

1) 제작연월일 : 2019-08-13

2) 제작자 : 교육지대㈜

3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 치환적분법

미분가능한 함수 $g(t)$ 에 대하여 $x = g(t)$ 로 놓으면

$$\Leftrightarrow \int f(x)dx = \int f(g(t))g'(t)dt$$

(1) $f(ax+b)$ 의 꼴 : $\int f(x)dx = F(x) + C$ 이면

$$\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C$$

(단, a, b 는 상수, $a \neq 0$, C 는 적분상수)(2) $f(g(x))g'(x)$ 의 꼴 : $g(x) = t$ 로 놓으면

$$\int f(g(x))g'(x)dx = \int f(t)dt$$

(3) $\frac{f'(x)}{f(x)}$ 의 꼴 : $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$ (단, C 는 적분상수)

■ 다음 부정적분을 구하여라.

1. $\int (2x+1)^4 dx$

2. $\int (x+1)^3 dx$

3. $\int (4x+1)^3 dx$

4. $\int (3x-1)^5 dx$

5. $\int \sqrt{2x+3} dx$

6. $\int 6x(x^2-3)^5 dx$

7. $\int 3x^2(x^3-1)^2 dx$

8. $\int (2x+1)(x^2+x-1)^2 dx$

9. $\int 2(x-1)(x^2-2x-4)dx$

10. $\int (x^3+x)(x^4+2x^2)^2 dx$

11. $\int \cot x dx$

12. $\int \tan x dx$

13. $\int \sin 2x dx$

20. $\int \tan x \sec^2 x dx$

14. $\int \frac{1 - \sin x}{x + \cos x} dx$

21. $\int \cos^3 x \sin x dx$

15. $\int \sin^2 x \cos x dx$

22. $\int \frac{\sin x}{2 + \cos x} dx$

16. $\int \sin x (1 + \cos x)^2 dx$

23. $\int (\cos 3x + \sin 2x) dx$

17. $\int \cos^3 x \sin x dx$

24. $\int \sec^2(3x + 1) dx$

18. $\int \frac{\sec^2 x}{\tan x - 1} dx$

25. $\int x \sqrt{2x - 8} dx$

19. $\int \sin x \cos^2 x dx$

26. $\int \frac{2x}{\sqrt{x^2 + 3}} dx$

27. $\int x \sqrt{x+1} dx$

28. $\int \frac{x}{\sqrt{x+2}} dx$

29. $\int \frac{3e^x}{\sqrt{e^x+1}} dx$

30. $\int \frac{2x}{x^2+1} dx$

31. $\int \frac{x+2}{x^2+4x+3} dx$

32. $\int 2xe^{x^2} dx$

33. $\int 3x^2 e^{x^3} dx$

34. $\int \frac{e^x}{1+e^x} dx$

35. $\int 4x^3 e^{x^4} dx$

36. $\int x e^{-x^2} dx$

37. $\int e^x \sqrt{e^x+1} dx$

38. $\int \frac{e^x+1}{e^x+x} dx$

39. $\int \frac{1}{x \ln 2x} dx$

40. $\int (e^x-1)^3 e^x dx$

41. $\int \frac{\sin(\ln x)}{x} dx$

42. $\int \frac{\sqrt{\ln x+1}}{x} dx$

43. $\int \frac{(\ln x)^4}{x} dx$

44. $\int \frac{\ln 3x}{x} dx$

45. $\int \frac{2x}{(x^2+5)^4} dx$

46. $\int \frac{4x-6}{(x^2-3x+5)^2} dx$

47. $\int \frac{3x^2+x}{(2x^3+x^2+1)^2} dx$

48. $\int \frac{1}{(3x+1)^2} dx$

49. $\int \frac{2x-1}{x^2-x+1} dx$

50. $\int \frac{x+1}{(x^2+2x+2)^3} dx$

■ 다음 물음에 답하여라.

51. 함수 $f(x) = \int (1 - \sin x)^2 \cos x dx$ 에 대하여 $f(0) = 0$ 일 때, $f\left(\frac{\pi}{2}\right)$ 의 값을 구하여라.

52. 함수 $f(x) = \int e^{\frac{1}{2}x} dx$ 에 대하여 $f(0) = 1$ 일 때, 방정식 $f(x) = 0$ 의 해를 구하여라.

53. 함수 $f(x) = \int e^{\sin x} \cos x dx$ 에 대하여 $f(0) = 2$ 일 때, $f\left(\frac{\pi}{2}\right)$ 의 값을 구하여라.

54. $f(x) = \int \frac{(\ln x)^3 + 2(\ln x)^2 + 1}{x} dx$ 에 대하여 $f(e) = 1$ 일 때, $24f(e^2)$ 의 값을 구하여라.

55. 함수 $f(x) = \int 4xe^{x+2} dx$ 에 대하여 $f(0) = 2e^2$ 일 때, $f(1)$ 의 값을 구하여라.

56. 함수 $f(x)$ 가 $f(x) = \int \sin \sqrt{x} dx$, $f(0) = 0$ 일 때,
 $f\left(\frac{\pi^2}{9}\right)$ 의 값을 구하여라.

57. 함수 $f(x) = \int \frac{e^{2x}}{e^{2x} + 3} dx$ 에 대하여 $f(0) = \ln 2$ 일
 때, $f\left(\frac{1}{2}\right)$ 의 값을 구하여라.

■ 함수 $f(x)$ 의 도함수 $f'(x)$ 에 대하여 다음 물음에 알맞은 값을 구하여라.

58. 함수 $f(x)$ 의 도함수 $f'(x)$ 가 $f'(x) = \frac{3^x}{3^x + 1}$,
 $f(0) = \log_3 2$ 일 때, $f(1)$ 의 값을 구하여라.

59. 함수 $f(x)$ 의 도함수 $f'(x)$ 가 $f'(x) = \frac{2^x \ln 2}{2^x + 1}$,
 $f(0) = \ln 2$ 일 때, $f(3)$ 의 값을 구하여라.

60. 함수 $f(x)$ 의 도함수 $f'(x)$ 가 $f'(x) = \frac{2x+1}{x^2+x-4}$,
 $f(2) = \ln 2$ 일 때, $f(1)$ 의 값을 구하여라.

02 / 부분적분법

두 함수 $f(x)$, $g(x)$ 가 미분가능할 때

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

적분
↓
↑
미분
↓
↑
그대로

■ 다음 부정적분을 구하여라.

61. $\int x \sin x dx$

62. $\int x^2 \cos x dx$

63. $\int x e^x dx$

64. $\int x \cos x dx$

65. $\int (2x-1) \sin 2x dx$

66. $\int (3x+2) \sin x dx$

67. $\int x^2 \sin x dx$

68. $\int e^x \sin x dx$

69. $\int (x^2 - x) e^x dx$

70. $\int (x - 1) e^{2x} dx$

71. $\int (x^2 - 3x) \cos x dx$

72. $\int e^{2x} \cos x dx$

73. $\int e^{-x} \cos x dx$

74. $\int x^2 e^{-x} dx$

75. $\int x e^{x+1} dx$

76. $\int (x+2) e^x dx$

77. $\int \ln x dx$

78. $\int x (\ln x)^2 dx$

79. $\int x \ln x dx$

80. $\int (\ln x)^2 dx$

81. $\int x \ln 2x dx$

82. $\int 9x^2 \ln x dx$

▣ 다음 물음에 알맞은 값을 구하여라.

83. 함수 $f(x) = \int (x^2 + 4x + 1) \ln x dx$ 에 대하여 $f(1) = 0$ 일 때, $f(e)$ 의 값을 구하여라.

84. 함수 $f(x) = \int x e^x dx$ 에 대하여 $f(1) = 0$ 일 때, $f(3)$ 의 값을 구하여라.

85. 함수 $f(x) = \int x \sin 3x dx$ 에 대하여 $f(0) = 1$ 일 때, $f(\pi)$ 의 값을 구하여라.

86. 함수 $f(x) = \int e^x \cos x dx$ 이고 $f\left(\frac{\pi}{2}\right) = \frac{e^{\frac{\pi}{2}}}{2}$ 일 때, $f(0)$ 의 값을 구하여라.

87. 함수 $h(x) = \int e^x \cos x dx$ 에 대하여 $h(0) = \frac{1}{2}$ 일 때, $h(\pi)$ 의 값을 구하여라.

88. 함수 $f(x) = \int \ln x dx$ 에 대하여 $f(e) = 0$ 일 때, $f(1)$ 의 값을 구하여라.

89. 함수 $f(x)$ 의 한 부정적분을 $F(x)$ 라 할 때, $F(x) = xf(x) - x^2 \sin x$, $f(0) = 0$ 이 성립할 때, $f(\pi)$ 의 값을 구하여라.

90. $x > 0$ 인 모든 실수 x 에 대하여 함수 $f(x)$ 의 부정적분 중의 하나를 $F(x)$ 라고 할 때, $F(x) = xf(x) - x \ln x$, $f(1) = 0$ 이 성립할 때, $f(e^2)$ 의 값을 구하여라.



정답 및 해설

$$1) \frac{1}{10}(2x+1)^5 + C$$

$$\Rightarrow 2x+1=t \text{로 놓으면 } 2 = \frac{dt}{dx} \text{이므로}$$

$$\begin{aligned} \int (2x+1)^4 dx &= \int t^4 \times \frac{1}{2} dt = \frac{1}{2} \int t^4 dt \\ &= \frac{1}{2} \times \frac{1}{5} t^5 + C = \frac{1}{10} t^5 + C \\ &= \frac{1}{10} (2x+1)^5 + C \end{aligned}$$

$$2) \frac{1}{4}(x+1)^4 + C$$

$$\Rightarrow x+1=t \text{로 놓으면 } 1 = \frac{dt}{dx}$$

$$\begin{aligned} \int (x+1)^3 dx &= \int t^3 dt = \frac{1}{4} t^4 + C \\ &= \frac{1}{4} (x+1)^4 + C \end{aligned}$$

$$3) \frac{1}{16}(4x+1)^4 + C$$

$$\Rightarrow 4x+1=t \text{로 놓으면 } 4 = \frac{dt}{dx}$$

$$\begin{aligned} \int (4x+1)^3 dx &= \int t^3 \cdot \frac{1}{4} dt \\ &= \frac{1}{4} \int t^3 dt = \frac{1}{16} t^4 + C \\ &= \frac{1}{16} (4x+1)^4 + C \end{aligned}$$

$$4) \frac{1}{18}(3x-1)^6 + C$$

$$\Rightarrow (3x-1)' = 3 \text{이므로}$$

$$\begin{aligned} \int (3x-1)^5 dx &= \frac{1}{3} \times \frac{1}{6} (3x-1)^6 + C \\ &= \frac{1}{18} (3x-1)^6 + C \end{aligned}$$

$$5) \frac{1}{3}(\sqrt{2x+3})^3 + C$$

$$\Rightarrow (2x+3)' = 2 \text{이므로}$$

$$\begin{aligned} \int \sqrt{2x+3} dx &= \frac{1}{2} \times \frac{2}{3} (2x+3)^{\frac{3}{2}} + C \\ &= \frac{1}{3} (\sqrt{2x+3})^3 + C \end{aligned}$$

$$6) \frac{1}{2}(x^2-3)^6 + C$$

$$\Rightarrow x^2-3=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$2x = \frac{dt}{dx} \therefore 2x dx = dt$$

$$\begin{aligned} \therefore \int 6x(x^2-3)^5 dx &= \int 3t^5 dt = 3 \times \frac{1}{6} t^6 + C \\ &= \frac{1}{2} (x^2-3)^6 + C \end{aligned}$$

$$7) \frac{1}{3}(x^3-1)^3 + C$$

$$\Rightarrow x^3-1=t \text{로 놓으면 } 3x^2 = \frac{dt}{dx}$$

$$\begin{aligned} \int 3x^2(x^3-1)^2 dx &= \int t^2 dt \\ &= \frac{1}{3} t^3 + C = \frac{1}{3} (x^3-1)^3 + C \end{aligned}$$

$$8) \frac{1}{3}(x^2+x-1)^3 + C$$

$$\Rightarrow x^2+x-1=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$2x+1 = \frac{dt}{dx} \therefore (2x+1)dx = dt$$

$$\begin{aligned} \therefore \int (2x+1)(x^2+x-1)^2 dx &= \int t^2 dt = \frac{1}{3} t^3 + C \\ &= \frac{1}{3} (x^2+x-1)^3 + C \end{aligned}$$

$$9) \frac{1}{2}(x^2-2x-4)^2 + C$$

$$\Rightarrow x^2-2x-4=t \text{로 놓으면}$$

$$2x-2 = 2(x-1) = \frac{dt}{dx}$$

$$\begin{aligned} \int 2(x-1)(x^2-2x-4) dx &= \int t dt = \frac{1}{2} t^2 + C \\ &= \frac{1}{2} (x^2-2x-4)^2 + C \end{aligned}$$

$$10) \frac{1}{12}(x^4+2x^2)^3 + C$$

$$\Rightarrow x^4+2x^2=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$4x^3+4x = \frac{dt}{dx}$$

$$\therefore (x^3+x)dx = \frac{1}{4} dt$$

$$\begin{aligned} \therefore \int (x^3+x)(x^4+2x^2)^2 dx &= \frac{1}{4} \int t^2 dt \\ &= \frac{1}{4} \times \frac{1}{3} t^3 + C \\ &= \frac{1}{12} (x^4+2x^2)^3 + C \end{aligned}$$

$$11) \ln|\sin x| + C$$

$$\Rightarrow \cot x = \frac{\cos x}{\sin x} \text{이고 } (\sin x)' = \cos x \text{이므로}$$

$$\begin{aligned} \int \cot x dx &= \int \frac{\cos x}{\sin x} dx = \int \frac{(\sin x)'}{\sin x} dx \\ &= \ln|\sin x| + C \end{aligned}$$

$$12) -\ln|\cos x| + C$$

$$\begin{aligned}\Rightarrow \int \tan x dx &= \int \frac{\sin x}{\cos x} dx \\ &= - \int \frac{(-\sin x)}{\cos x} dx \\ &= - \int \frac{(\cos x)'}{\cos x} dx \\ &= - \ln |\cos x| + C\end{aligned}$$

$$13) -\frac{1}{2} \cos 2x + C$$

$$\Rightarrow 2x = t \text{로 놓으면 } 2 = \frac{dt}{dx}$$

$$\begin{aligned}\int \sin 2x dx &= \frac{1}{2} \int \sin t dt \\ &= -\frac{1}{2} \cos t + C \\ &= -\frac{1}{2} \cos 2x + C\end{aligned}$$

$$14) \ln |x + \cos x| + C$$

$$\Rightarrow (x + \cos x)' = 1 - \sin x \text{이므로}$$

$$\int \frac{1 - \sin x}{x + \cos x} dx = \int \frac{(x + \cos x)'}{x + \cos x} dx = \ln |x + \cos x| + C$$

$$15) \frac{1}{3} \sin^3 x + C$$

$$\Rightarrow \sin x = t \text{로 놓으면 } \cos x = \frac{dt}{dx} \text{이므로}$$

$$\begin{aligned}\int \sin^2 x \cos x dx &= \int t^2 dt = \frac{1}{3} t^3 + C \\ &= \frac{1}{3} \sin^3 x + C\end{aligned}$$

$$16) -\frac{1}{3} (1 + \cos x)^3 + C$$

$$\Rightarrow 1 + \cos x = t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$-\sin x = \frac{dt}{dx} \therefore \sin x dx = -dt$$

$$\begin{aligned}\therefore \int \sin x (1 + \cos x)^2 dx &= \int t^2 \times (-dt) = - \int t^2 dt \\ &= -\frac{1}{3} t^3 + C \\ &= -\frac{1}{3} (1 + \cos x)^3 + C\end{aligned}$$

$$17) -\frac{1}{4} \cos^4 x + C$$

$$\Rightarrow \cos x = t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$-\sin x = \frac{dt}{dx} \therefore \sin x dx = -dt$$

$$\begin{aligned}\therefore \int \cos^3 x \sin x dx &= \int t^3 \times (-dt) = - \int t^3 dt \\ &= -\frac{1}{4} t^4 + C = -\frac{1}{4} \cos^4 x + C\end{aligned}$$

$$18) \ln |\tan x - 1| + C$$

$$\Rightarrow \tan x - 1 = t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$\sec^2 x = \frac{dt}{dx} \therefore \sec^2 x dx = dt$$

$$\begin{aligned}\therefore \int \frac{\sec^2 x}{\tan x - 1} dx &= \int \frac{1}{t} dt = \ln |t| + C \\ &= \ln |\tan x - 1| + C\end{aligned}$$

$$19) -\frac{1}{3} \cos^3 x + C$$

$$\Rightarrow \cos x = t \text{로 놓으면 } -\sin x = \frac{dt}{dx}$$

$$\begin{aligned}\int \sin x \cos^2 x dx &= \int (-t^2) dt = -\frac{1}{3} t^3 + C \\ &= -\frac{1}{3} \cos^3 x + C\end{aligned}$$

$$20) \frac{1}{2} \tan^2 x + C$$

$$\Rightarrow \tan x = t \text{로 놓으면 } \sec^2 x = \frac{dt}{dx}$$

$$\begin{aligned}\int \tan x \sec^2 x dx &= \int t dt = \frac{1}{2} t^2 + C \\ &= \frac{1}{2} \tan^2 x + C\end{aligned}$$

$$21) -\frac{1}{4} \cos^4 x + C$$

$$\Rightarrow$$

$$\cos x = t \text{라 하면 } -\sin x = \frac{dt}{dx} \therefore -\sin x dx = dt$$

$$\therefore (\text{주어진 적분})$$

$$= \int -t^3 dt = -\frac{1}{4} t^4 + C = -\frac{1}{4} \cos^4 x + C$$

$$22) -\ln(2 + \cos x) + C$$

$$\Rightarrow (2 + \cos x)' = -\sin x \text{이고 } 2 + \cos x > 0 \text{이므로}$$

$$\begin{aligned}\int \frac{\sin x}{2 + \cos x} dx &= - \int \frac{-\sin x}{2 + \cos x} dx \\ &= - \int \frac{(2 + \cos x)'}{2 + \cos x} dx \\ &= -\ln(2 + \cos x) + C\end{aligned}$$

$$23) \frac{1}{3} \sin 3x - \frac{1}{2} \cos 2x + C$$

$$\Rightarrow (3x)' = 3, (2x)' = 2 \text{이므로}$$

$$\int (\cos 3x + \sin 2x) dx = \frac{1}{3} \sin 3x - \frac{1}{2} \cos 2x + C$$

$$24) \frac{1}{3} \tan(3x + 1) + C$$

$$\Rightarrow (3x + 1)' = 3 \text{이므로}$$

$$\int \sec^2(3x + 1) dx = \frac{1}{3} \tan(3x + 1) + C$$

$$25) \frac{1}{10} (\sqrt{2x-8})^5 + \frac{4}{3} (\sqrt{2x-8})^3 + C$$

$$\Rightarrow \sqrt{2x-8} = t, \text{ 즉 } x = \frac{1}{2}(t^2 + 8) \text{로 놓으면 } \frac{dx}{dt} = t$$

$$\begin{aligned}
 \int x \sqrt{2x-8} dx &= \int \frac{1}{2}(t^2+8) \cdot t^2 dt \\
 &= \int \left(\frac{1}{2}t^4 + 4t^2 \right) dt \\
 &= \frac{1}{10}t^5 + \frac{4}{3}t^3 + C \\
 &= \frac{1}{10}(\sqrt{2x-8})^5 + \frac{4}{3}(\sqrt{2x-8})^3 + C
 \end{aligned}$$

$$26) 2\sqrt{x^2+3} + C$$

$$\Rightarrow \sqrt{x^2+3}=t \text{로 놓으면 } x^2+3=t^2$$

$$\text{양변을 } x \text{에 대하여 미분하면 } 2x = 2t \frac{dt}{dx}$$

$$\therefore 2x dx = 2t dt$$

$$\begin{aligned}
 \therefore \int \frac{2x}{\sqrt{x^2+3}} dx &= \int \frac{1}{t} \times 2t dt = \int 2 dt \\
 &= 2t + C = 2\sqrt{x^2+3} + C
 \end{aligned}$$

$$27) \frac{2}{5}(\sqrt{x+1})^5 - \frac{2}{3}(\sqrt{x+1})^3 + C$$

$$\Rightarrow \sqrt{x+1}=t \text{로 놓으면 } x+1=t^2$$

$$\text{양변을 } x \text{에 대하여 미분하면 } 1 = 2t \frac{dt}{dx}$$

$$\therefore dx = 2t dt$$

$$\begin{aligned}
 \therefore \int x \sqrt{x+1} dx &= \int (t^2-1)t \times 2t dt \\
 &= \int (2t^4 - 2t^2) dt = \frac{2}{5}t^5 - \frac{2}{3}t^3 + C \\
 &= \frac{2}{5}(\sqrt{x+1})^5 - \frac{2}{3}(\sqrt{x+1})^3 + C
 \end{aligned}$$

$$28) \frac{2}{3}(\sqrt{x+2})^3 - 4\sqrt{x+2} + C$$

$$\Rightarrow \sqrt{x+2}=t \text{로 놓으면 } x+2=t^2$$

$$\text{양변을 } x \text{에 대하여 미분하면 } 1 = 2t \frac{dt}{dx}$$

$$\therefore dx = 2t dt$$

$$\begin{aligned}
 \therefore \int \frac{x}{\sqrt{x+2}} dx &= \int \frac{t^2-2}{t} \times 2t dt \\
 &= \int (2t^2 - 4) dt = \frac{2}{3}t^3 - 4t + C \\
 &= \frac{2}{3}(\sqrt{x+2})^3 - 4\sqrt{x+2} + C
 \end{aligned}$$

$$29) 6\sqrt{e^x+1} + C$$

$$\Rightarrow \sqrt{e^x+1}=t \text{로 놓으면 } e^x+1=t^2$$

$$\text{양변을 } x \text{에 대하여 미분하면 } e^x = 2t \frac{dt}{dx}$$

$$\therefore e^x dx = 2t dt$$

$$\begin{aligned}
 \therefore \int \frac{3e^x}{\sqrt{e^x+1}} dx &= \int \frac{3}{t} \times 2t dt = \int 6 dt \\
 &= 6t + C = 6\sqrt{e^x+1} + C
 \end{aligned}$$

$$30) \ln(x^2+1) + C$$

$$\Rightarrow (x^2+1)' = 2x \text{이고 } x^2+1 > 0 \text{이므로}$$

$$\begin{aligned}
 \int \frac{2x}{x^2+1} dx &= \int \frac{(x^2+1)'}{x^2+1} dx \\
 &= \ln(x^2+1) + C
 \end{aligned}$$

$$31) \frac{1}{2} \ln|x^2+4x+3| + C$$

$$\Rightarrow (x^2+4x+3)' = 2x+4 \text{이므로}$$

$$\begin{aligned}
 \int \frac{x+2}{x^2+4x+3} dx &= \frac{1}{2} \int \frac{2x+4}{x^2+4x+3} dx \\
 &= \frac{1}{2} \int \frac{(x^2+4x+3)'}{x^2+4x+3} dx \\
 &= \frac{1}{2} \ln|x^2+4x+3| + C
 \end{aligned}$$

$$32) e^{x^2} + C$$

$$\Rightarrow x^2=t \text{로 치환하자. } 2x dx = dt$$

$$\int e^t dt = e^t + C = e^{x^2} + C$$

$$33) e^{x^3} + C$$

$$\Rightarrow x^3=t \text{로 놓으면 } 3x^2 = \frac{dt}{dx} \text{이므로}$$

$$\int 3x^2 e^{x^3} dx = \int e^t dt = e^t + C = e^{x^3} + C$$

$$34) \ln(1+e^x) + C$$

$$\Rightarrow (1+e^x)' = e^x \text{이고 } 1+e^x > 0 \text{이므로}$$

$$\int \frac{e^x}{1+e^x} dx = \int \frac{(1+e^x)'}{1+e^x} dx = \ln(1+e^x) + C$$

$$35) e^{x^4} + C$$

$$\Rightarrow x^4=t \text{로 놓으면 } 4x^3 = \frac{dt}{dx}$$

$$\begin{aligned}
 \int 4x^3 e^{x^4} dx &= \int e^t dt \\
 &= e^t + C = e^{x^4} + C
 \end{aligned}$$

$$36) -\frac{1}{2}e^{-x^2} + C$$

$$\Rightarrow -x^2=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$-2x = \frac{dt}{dx} \therefore x dx = -\frac{1}{2} dt$$

$$\begin{aligned}
 \therefore \int x e^{-x^2} dx &= -\frac{1}{2} \int e^t dt = -\frac{1}{2} e^t + C \\
 &= -\frac{1}{2} e^{-x^2} + C
 \end{aligned}$$

$$37) \frac{2}{3}(\sqrt{e^x+1})^3 + C$$

$$\Rightarrow e^x+1=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$e^x = \frac{dt}{dx} \therefore e^x dx = dt$$

$$\therefore \int e^x \sqrt{e^x+1} dx = \int \sqrt{t} dt = \frac{2}{3} t^{\frac{3}{2}} + C$$

$$= \frac{2}{3}(\sqrt{e^x+1})^3 + C$$

$$38) \ln|e^x+x|+C$$

$$\begin{aligned} \Rightarrow \int \frac{e^x+1}{e^x+x} dx &= \int \frac{(e^x+x)'}{e^x+x} dx \\ &= \ln|e^x+x|+C \end{aligned}$$

$$39) \ln|\ln 2x|+C$$

$$\begin{aligned} \Rightarrow \int \frac{1}{x \ln 2x} dx &= \int \frac{\frac{1}{x}}{\ln 2x} dx \\ &= \int \frac{(\ln 2x)'}{\ln 2x} dx \\ &= \ln|\ln 2x|+C \end{aligned}$$

$$40) \frac{1}{4}(e^x-1)^4+C$$

$$\Rightarrow e^x-1=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$e^x = \frac{dt}{dx} \quad \therefore e^x dx = dt$$

$$\begin{aligned} \therefore \int (e^x-1)^3 e^x dx &= \int t^3 dt = \frac{1}{4}t^4 + C \\ &= \frac{1}{4}(e^x-1)^4 + C \end{aligned}$$

$$41) -\cos(\ln x)+C$$

$$\Rightarrow \ln x = t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$\frac{1}{x} = \frac{dt}{dx} \quad \therefore \frac{1}{x} dx = dt$$

$$\begin{aligned} \therefore \int \frac{\sin(\ln x)}{x} dx &= \int \sin t dt \\ &= -\cos t + C \\ &= -\cos(\ln x) + C \end{aligned}$$

$$42) \frac{2}{3}(\sqrt{\ln x+1})^3+C$$

$$\Rightarrow \ln x+1=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$\frac{1}{x} = \frac{dt}{dx} \quad \therefore \frac{1}{x} dx = dt$$

$$\begin{aligned} \therefore \int \frac{\sqrt{\ln x+1}}{x} dx &= \int \sqrt{t} dt \\ &= \frac{2}{3}t^{\frac{3}{2}} + C \\ &= \frac{2}{3}(\sqrt{\ln x+1})^3 + C \end{aligned}$$

$$43) \frac{1}{5}(\ln x)^5+C$$

$$\Rightarrow \ln x = t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$\frac{1}{x} = \frac{dt}{dx} \quad \therefore \frac{1}{x} dx = dt$$

$$\begin{aligned} \therefore \int \frac{(\ln x)^4}{x} dx &= \int t^4 dt = \frac{1}{5}t^5 + C \\ &= \frac{1}{5}(\ln x)^5 + C \end{aligned}$$

$$44) \frac{1}{2}(\ln 3x)^2+C$$

$$\Rightarrow \ln 3x = t \text{로 놓으면 } \frac{1}{x} = \frac{dt}{dx}$$

$$\begin{aligned} \int \frac{\ln 3x}{x} dx &= \int t dt = \frac{1}{2}t^2 + C \\ &= \frac{1}{2}(\ln 3x)^2 + C \end{aligned}$$

$$45) -\frac{1}{3(x^2+5)^3}+C$$

$$\Rightarrow x^2+5=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$2x = \frac{dt}{dx} \quad \therefore 2x dx = dt$$

$$\begin{aligned} \therefore \int \frac{2x}{(x^2+5)^4} dx &= \int \frac{1}{t^4} dt = \int t^{-4} dt \\ &= -\frac{1}{3}t^{-3} + C = -\frac{1}{3(x^2+5)^3} + C \end{aligned}$$

$$46) -\frac{2}{x^2-3x+5}+C$$

$$\Rightarrow x^2-3x+5=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$2x-3 = \frac{dt}{dx} \quad \therefore (2x-3)dx = dt$$

$$\begin{aligned} \therefore \int \frac{4x-6}{(x^2-3x+5)^2} dx &= 2 \int \frac{1}{t^2} dx = -\frac{2}{t} + C \\ &= -\frac{2}{x^2-3x+5} + C \end{aligned}$$

$$47) -\frac{1}{2(2x^3+x^2+1)}+C$$

$$\Rightarrow 2x^3+x^2+1=t \text{로 놓고 양변을 } x \text{에 대하여 미분하면}$$

$$6x^2+2x = \frac{dt}{dx} \quad \therefore (3x^2+x)dx = \frac{1}{2}dt$$

$$\begin{aligned} \therefore \int \frac{3x^2+x}{(2x^3+x^2+1)^2} dx &= \frac{1}{2} \int \frac{1}{t^2} dt = \frac{1}{2} \left(-\frac{1}{t} \right) + C \\ &= -\frac{1}{2(2x^3+x^2+1)} + C \end{aligned}$$

$$48) -\frac{1}{3(3x+1)}+C$$

$$\Rightarrow 3x+1=t \text{로 놓으면 } 3 = \frac{dt}{dx} \text{이므로}$$

$$\begin{aligned} \int \frac{1}{(3x+1)^2} dx &= \int \frac{1}{t^2} \times \frac{1}{3} dt = \frac{1}{3} \int t^{-2} dt \\ &= \frac{1}{3} \times (-t^{-1}) + C = -\frac{1}{3t} + C \\ &= -\frac{1}{3(3x+1)} + C \end{aligned}$$

$$49) \ln(x^2-x+1)+C$$

$$\Rightarrow (x^2-x+1)' = 2x-1 \text{이므로}$$

$$\int \frac{2x-1}{x^2-x+1} dx = \int \frac{(x^2-x+1)'}{x^2-x+1} dx \\ = \ln(x^2-x+1) + C (\because x^2-x+1 > 0)$$

$$50) -\frac{1}{4(x^2+2x+2)^2} + C$$

$\Rightarrow x^2+2x+2=t$ 로 놓고 양변을 x 에 대하여 미분하면

$$2x+2 = \frac{dt}{dx} \therefore (x+1)dx = \frac{1}{2}dt$$

$$\therefore \int \frac{x+1}{(x^2+2x+2)^3} dx = \frac{1}{2} \int \frac{1}{t^3} dt = \frac{1}{2} \int t^{-3} dt \\ = \frac{1}{2} \left(-\frac{1}{2} t^{-2} \right) + C \\ = -\frac{1}{4(x^2+2x+2)^2} + C$$

$$51) \frac{1}{3}$$

\Rightarrow 함수 $f(x) = \int (1-\sin x)^2 \cos x dx$ 에서

$$1-\sin x = t \text{로 놓으면 } -\cos x = \frac{dt}{dx}$$

$$\int (1-\sin x)^2 \cos x dx \\ = \int (-t^2) dt = -\frac{1}{3} t^3 + C \\ = -\frac{1}{3} (1-\sin x)^3 + C$$

$$\text{이때, } f(0) = 0 \text{이므로 } f(0) = -\frac{1}{3} + C = 0$$

$$\therefore C = \frac{1}{3}, f(x) = -\frac{1}{3} (1-\sin x)^3 + \frac{1}{3}$$

따라서 구하는 값은

$$f\left(\frac{\pi}{2}\right) = -\frac{1}{3} \left(1 - \sin \frac{\pi}{2}\right)^3 + \frac{1}{3} = \frac{1}{3}$$

$$52) -2\ln 2$$

$$\Rightarrow f(x) = \int e^{\frac{1}{2}x} dx = 2e^{\frac{1}{2}x} + C$$

$$f(0) = 1 \text{이므로}$$

$$2e^0 + C = 1 \therefore C = -1$$

$$\therefore f(x) = 2e^{\frac{1}{2}x} - 1$$

$$f(x) = 0 \text{에서 } 2e^{\frac{1}{2}x} = 1$$

$$e^{\frac{1}{2}x} = \frac{1}{2}, \frac{1}{2}x = -\ln 2$$

$$\therefore x = -2\ln 2$$

$$53) e+1$$

$\Rightarrow \sin x = t$ 라 하면 $\cos x dx = dt$ 이므로

$$f(x) = \int e^{\sin x} \cos x dx = \int e^t dt = e^t + C = e^{\sin x} + C$$

$$\text{이때 } f(0) = 1 + C = 2 \text{이므로 } C = 1$$

$$\therefore f\left(\frac{\pi}{2}\right) = e^1 + 1$$

$$54) 250$$

$$\Rightarrow f(x) = \int \frac{(\ln x)^3 + 2(\ln x)^2 + 1}{x} dx \text{에서}$$

$$\ln x = t \text{로 놓으면 } \frac{1}{x} = \frac{dt}{dx}$$

$$\int \frac{(\ln x)^3 + 2(\ln x)^2 + 1}{x} dx$$

$$= \int (t^3 + 2t^2 + 1) dt = \frac{1}{4} t^4 + \frac{2}{3} t^3 + t + C$$

$$= \frac{1}{4} (\ln x)^4 + \frac{2}{3} (\ln x)^3 + \ln x + C$$

$$f(e) = 1 \text{이므로}$$

$$\frac{1}{4} (\ln e)^4 + \frac{2}{3} (\ln e)^3 + \ln e + C = 1$$

$$\therefore C = -\frac{11}{12}, f(x) = \frac{1}{4} (\ln x)^4 + \frac{2}{3} (\ln x)^3 + \ln x - \frac{11}{12}$$

$x = e^2$ 일 때, 함수 $f(x)$ 의 값은

$$f(e^2) = \frac{1}{4} (\ln e^2)^4 + \frac{2}{3} (\ln e^2)^3 + \ln e^2 - \frac{11}{12} = \frac{125}{12}$$

따라서 구하는 값은

$$24f(e^2) = 24 \cdot \frac{125}{12} = 250$$

$$55) 2e^3$$

$$\Rightarrow f(x) = \int 4xe^{x^2+2} dx \text{에서}$$

$$x^2+2 = t \text{로 놓으면 } 2x = \frac{dt}{dx}$$

$$\int 4xe^{x^2+2} dx = 2 \int e^t dt \\ = 2e^t + C = 2e^{x^2+2} + C$$

$$\text{이때, } f(0) = 2e^2 \text{이므로}$$

$$2e^2 + C = 2e^2$$

$$\therefore C = 0, f(x) = 2e^{x^2+2}$$

따라서 구하는 값은

$$f(1) = 2e^{1+2} = 2e^3$$

$$56) \sqrt{3} - \frac{\pi}{3}$$

$$\Rightarrow \sqrt{x} = t, \frac{1}{2\sqrt{x}} dx = dt \text{이므로}$$

$$f(x) = \int \sin \sqrt{x} dx \\ = \int 2t \sin t dt = [-2t \cos t] + \int 2 \cos t dt$$

$$= -2\sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x} + C$$

$$f(0) = 0 \text{이므로 } C = 0$$

$$\therefore f(x) = -2\sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x}$$

$$\therefore f\left(\frac{\pi^2}{9}\right) = \sqrt{3} - \frac{\pi}{3}$$

$$57) \frac{1}{2} \ln(e+3)$$

$\Rightarrow (e^{2x}+3)' = 2e^{2x}$ 이고 $e^{2x}+3 > 0$ 이므로

$$\begin{aligned} f(x) &= \int \frac{e^{2x}}{e^{2x}+3} dx = \frac{1}{2} \int \frac{2e^{2x}}{e^{2x}+3} dx \\ &= \frac{1}{2} \int \frac{(e^{2x}+3)'}{e^{2x}+3} dx \\ &= \frac{1}{2} \ln(e^{2x}+3) + C \end{aligned}$$

$f(0) = \ln 2$ 이므로

$$\frac{1}{2} \ln 4 + C = \ln 2 + C = \ln 2$$

$$\therefore C = 0$$

따라서 $f(x) = \frac{1}{2} \ln(e^{2x}+3)$ 이므로

$$f\left(\frac{1}{2}\right) = \frac{1}{2} \ln(e+3)$$

$$58) 2\log_3 2$$

$$\Rightarrow f(x) = \int f'(x) dx = \int \frac{3^x}{3^x+1} dx \text{에서}$$

$(3^x+1)' = 3^x \ln 3$ 이므로

$$\begin{aligned} \int \frac{3^x}{3^x+1} dx &= \frac{1}{\ln 3} \int \frac{3^x \ln 3}{3^x+1} dx \\ &= \frac{1}{\ln 3} \int \frac{(3^x+1)'}{3^x+1} dx \\ &= \frac{1}{\ln 3} \cdot \ln |3^x+1| + C \\ &= \frac{\ln |3^x+1|}{\ln 3} + C \end{aligned}$$

이때, $f(0) = \log_3 2$ 이므로 $C = 0$

$$\therefore f(x) = \frac{\ln |3^x+1|}{\ln 3}$$

따라서 구하는 값은

$$f(1) = \frac{\ln |3^1+1|}{\ln 3} = \frac{\ln 4}{\ln 3} = 2\log_3 2$$

$$59) 2\ln 3$$

$$\Rightarrow f(x) = \int f'(x) dx \text{에서}$$

$$\begin{aligned} \int \frac{2^x \ln 2}{2^x+1} dx &= \int \frac{(2^x+1)'}{2^x+1} dx \\ &= \ln |2^x+1| + C \end{aligned}$$

이때, $f(0) = \ln |2^0+1| + C = \ln 2$ 이므로 $C = 0$

$$\therefore f(x) = \ln |2^x+1|$$

따라서 구하는 값은

$$f(3) = \ln |2^3+1| = 2\ln 3$$

$$60) \ln 2$$

$$\Rightarrow f(x) = \int f'(x) dx = \int \frac{2x+1}{x^2+x-4} dx \text{에서}$$

$$(x^2+x-4)' = 2x+1 \text{이므로}$$

$$\begin{aligned} \int \frac{2x+1}{x^2+x-4} dx &= \int \frac{(x^2+x-4)'}{x^2+x-4} dx \\ &= \ln |x^2+x-4| + C \end{aligned}$$

이때, $f(2) = \ln 2$ 이므로

$$\ln |2^2+2-4| + C = \ln 2 \quad \therefore C = 0$$

$$\therefore f(x) = \ln |x^2+x-4|$$

따라서 구하는 값은

$$f(1) = \ln |1^2+1-4| = \ln 2$$

$$61) -x \cos x + \sin x + C$$

$\Rightarrow f(x) = x, g'(x) = \sin x$ 로 놓으면

$$f'(x) = 1, g(x) = -\cos x$$

$$\begin{aligned} \therefore \int x \sin x dx &= x(-\cos x) - \int 1 \times (-\cos x) dx \\ &= -x \cos x + \int \cos x dx \\ &= -x \cos x + \sin x + C \end{aligned}$$

$$62) (x^2-2)\sin x + 2x \cos x + C$$

$\Rightarrow f(x) = x^2, g'(x) = \cos x$ 로 놓으면

$$f'(x) = 2x, g(x) = \sin x \text{이므로}$$

$$\int x^2 \cos x dx = x^2 \sin x - 2 \int x \sin x dx \cdots \textcircled{1}$$

한편, $\int x \sin x dx$ 에서

$u(x) = x, v'(x) = \sin x$ 로 놓으면

$$u'(x) = 1, v(x) = -\cos x \text{이므로}$$

$$\begin{aligned} \int x \sin x dx &= -x \cos x + \int \cos x dx \\ &= -x \cos x + \sin x + C_1 \cdots \textcircled{2} \end{aligned}$$

$\textcircled{2}$ 을 $\textcircled{1}$ 에 대입하면

$$\begin{aligned} \int x^2 \cos x dx &= x^2 \sin x - 2(-x \cos x + \sin x + C_1) \\ &= (x^2-2)\sin x + 2x \cos x + C \end{aligned}$$

$$63) xe^x - e^x + C$$

$\Rightarrow f(x) = x, g'(x) = e^x$ 으로 놓으면

$$f'(x) = 1, g(x) = e^x$$

$$\begin{aligned} \therefore \int xe^x dx &= xe^x - \int 1 \times e^x dx \\ &= xe^x - e^x + C \end{aligned}$$

$$64) x \sin x + \cos x + C$$

$\Rightarrow f(x) = x, g'(x) = \cos x$ 로 놓으면

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx \text{이므로}$$

$$\begin{aligned} \int x \cos x dx &= x \sin x - \int 1 \cdot \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$

$$65) -\frac{1}{2}(2x-1)\cos 2x + \frac{1}{2}\sin 2x + C$$

$\Rightarrow f(x) = 2x-1, g'(x) = \sin 2x$ 로 놓으면

$$f'(x) = 2, g(x) = -\frac{1}{2}\cos 2x$$

$$\begin{aligned} & \int (2x-1)\sin 2x dx \\ &= (2x-1) \cdot \left(-\frac{1}{2}\cos 2x\right) - \int 2 \cdot \left(-\frac{1}{2}\cos 2x\right) dx \\ &= -\frac{1}{2}(2x-1)\cos 2x + \frac{1}{2}\sin 2x + C \end{aligned}$$

$$\begin{aligned} 66) & -(3x+2)\cos x + 3\sin x + C \\ \Rightarrow & f(x) = 3x+2, \quad g'(x) = \sin x \text{로 놓으면} \\ & \int (3x+2)\sin x dx \\ &= (3x+2) \cdot (-\cos x) - \int 3 \cdot (-\cos x) dx \\ &= -(3x+2)\cos x + 3\sin x + C \end{aligned}$$

$$\begin{aligned} 67) & (-x^2+2)\cos x + 2x\sin x + C \\ \Rightarrow & f(x) = x^2, \quad g'(x) = \sin x \text{로 놓으면} \\ & f'(x) = 2x, \quad g(x) = -\cos x \\ & \int x^2\sin x dx = -x^2\cos x + \int 2x\cos x dx \\ &= -x^2\cos x + \left\{2x\sin x - \int 2\sin x dx\right\} \\ &= -x^2\cos x + 2x\sin x + 2\cos x + C \\ &= (-x^2+2)\cos x + 2x\sin x + C \end{aligned}$$

$$\begin{aligned} 68) & \frac{1}{2}e^x(\sin x - \cos x) + C \\ \Rightarrow & f(x) = \sin x, \quad g'(x) = e^x \text{으로 놓으면} \\ & f'(x) = \cos x, \quad g(x) = e^x \text{이므로} \\ & \int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx \cdots \textcircled{7} \\ & \int e^x \cos x dx \text{에서} \\ & u(x) = \cos x, \quad v'(x) = e^x \text{으로 놓으면} \\ & u'(x) = -\sin x, \quad v(x) = e^x \text{이므로} \\ & \int e^x \cos x dx = e^x \cos x + \int e^x \sin x dx \cdots \textcircled{8} \\ & \textcircled{8}을 \textcircled{7}에 대입하면 \\ & \int e^x \sin x dx = e^x \sin x - \left(e^x \cos x + \int e^x \sin x dx\right) \\ &= e^x \sin x - e^x \cos x - \int e^x \sin x dx \\ \therefore & \int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C \end{aligned}$$

$$\begin{aligned} 69) & (x^2-3x+3)e^x + C \\ \Rightarrow & f(x) = x^2-x, \quad g'(x) = e^x \text{으로 놓으면} \\ & f'(x) = 2x-1, \quad g(x) = e^x \text{이므로} \\ & \int (x^2-x)e^x dx = (x^2-x)e^x - \int (2x-1)e^x dx \cdots \textcircled{9} \\ & \int (2x-1)e^x dx \text{에서} \\ & u(x) = 2x-1, \quad v'(x) = e^x \text{으로 놓으면} \\ & u'(x) = 2, \quad v(x) = e^x \text{이므로} \\ & \int (2x-1)e^x dx = (2x-1)e^x - \int 2e^x dx \\ &= (2x-3)e^x + C_1 \cdots \textcircled{10} \\ & \textcircled{10}을 \textcircled{9}에 대입하면 \end{aligned}$$

$$\begin{aligned} & \int (x^2-x)e^x dx = (x^2-x)e^x - \{(2x-3)e^x + C_1\} \\ &= (x^2-3x+3)e^x + C \end{aligned}$$

$$\begin{aligned} 70) & \frac{e^{2x}(2x-3)}{4} + C \\ \Rightarrow & f(x) = x-1, \quad g'(x) = e^{2x} \text{로 놓으면} \\ & \int (x-1)e^{2x} dx = (x-1) \cdot \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} dx \\ &= \frac{e^{2x}(x-1)}{2} - \frac{e^{2x}}{4} + C \\ &= \frac{e^{2x}(2x-3)}{4} + C \end{aligned}$$

$$\begin{aligned} 71) & (x^2-3x-2)\sin x + (2x-3)\cos x + C \\ \Rightarrow & f(x) = x^2-3x, \quad g'(x) = \cos x \text{로 놓으면} \\ & f'(x) = 2x-3, \quad g(x) = \sin x \\ & \int (x^2-3x)\cos x dx \\ &= (x^2-3x)\sin x - \int (2x-3)\sin x dx \\ &= (x^2-3x)\sin x - \{(2x-3)(-\cos x) - \int 2(-\cos x) dx\} \\ &= (x^2-3x)\sin x + (2x-3)\cos x - 2\sin x + C \\ &= (x^2-3x-2)\sin x + (2x-3)\cos x + C \end{aligned}$$

$$\begin{aligned} 72) & \frac{e^{2x}}{5}(\sin x + 2\cos x) + C \\ \Rightarrow & f(x) = e^{2x}, \quad g'(x) = \cos x \text{로 놓으면} \\ & f'(x) = 2e^{2x}, \quad g(x) = \sin x \\ & \int e^{2x} \cos x dx = e^{2x} \sin x - 2 \int e^{2x} \sin x dx \cdots \textcircled{11} \\ & \text{한편 } \int e^{2x} \sin x dx \text{에서} \\ & u = e^{2x}, \quad v' = \sin x \text{라고 하면} \\ & u' = 2e^{2x}, \quad v = -\cos x \text{이므로} \\ & \int e^{2x} \sin x dx = -e^{2x} \cos x + 2 \int e^{2x} \cos x dx \cdots \textcircled{12} \\ & \textcircled{11}, \textcircled{12}에서 \\ & \int e^{2x} \cos x dx \\ &= e^{2x} \sin x - 2(-e^{2x} \cos x + 2 \int e^{2x} \cos x dx) \\ &= e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x dx \\ 5 \int e^{2x} \cos x dx &= e^{2x} \sin x + 2e^{2x} \cos x \\ \therefore \int e^{2x} \cos x dx &= \frac{e^{2x}}{5}(\sin x + 2\cos x) + C \end{aligned}$$

$$\begin{aligned} 73) & \frac{e^{-x}}{2}(\sin x - \cos x) + C \\ \Rightarrow & f(x) = e^{-x}, \quad g'(x) = \cos x \text{로 놓으면} \\ & f'(x) = -e^{-x}, \quad g(x) = \sin x \\ & \int e^{-x} \cos x dx \\ &= e^{-x} \sin x + \int e^{-x} \sin x dx \end{aligned}$$

$$\begin{aligned}
&= e^{-x} \sin x + \left(-e^{-x} \cos x - \int e^{-x} \cos x dx \right) \\
2 \int e^{-x} \cos x dx &= e^{-x} \sin x - e^{-x} \cos x \\
\int e^{-x} \cos x dx &= \frac{e^{-x}}{2} (\sin x - \cos x) + C
\end{aligned}$$

$$\begin{aligned}
74) & -e^{-x}(x^2+2x+2)+C \\
\Rightarrow f(x) &= x^2, g'(x) = e^{-x} \text{으로 놓으면} \\
f'(x) &= 2x, g(x) = -e^{-x} \text{이므로} \\
\int x^2 e^{-x} dx &= -x^2 e^{-x} + 2 \int x e^{-x} dx \cdots \textcircled{1} \\
\text{한편, } \int x e^{-x} dx &\text{에서}
\end{aligned}$$

$$\begin{aligned}
u(x) &= x, v'(x) = e^{-x} \text{으로 놓으면} \\
u'(x) &= 1, v(x) = -e^{-x} \text{이므로} \\
\int x e^{-x} dx &= -x e^{-x} + \int e^{-x} dx \\
&= -x e^{-x} - e^{-x} + C_1 \cdots \textcircled{2}
\end{aligned}$$

$$\begin{aligned}
\textcircled{1} \text{을 } \textcircled{2} \text{에 대입하면} \\
\int x^2 e^{-x} dx &= -x^2 e^{-x} + 2(-x e^{-x} - e^{-x} + C_1) \\
&= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C \\
&= -e^{-x}(x^2+2x+2) + C
\end{aligned}$$

$$\begin{aligned}
75) & x e^{x+1} - e^{x+1} + C \\
\Rightarrow f(x) &= x, g'(x) = e^{x+1} \text{으로 놓으면} \\
f'(x) &= 1, g(x) = e^{x+1} \text{이므로} \\
\int x e^{x+1} dx &= x e^{x+1} - \int e^{x+1} dx = x e^{x+1} - e^{x+1} + C
\end{aligned}$$

$$\begin{aligned}
76) & (x+1)e^x + C \\
\Rightarrow f(x) &= x+2, g'(x) = e^x \text{으로 놓으면} \\
f'(x) &= 1, g(x) = e^x \text{이므로} \\
\int (x+2)e^x dx &= (x+2)e^x - \int e^x dx \\
&= (x+2)e^x - e^x + C \\
&= (x+1)e^x + C
\end{aligned}$$

$$\begin{aligned}
77) & x \ln x - x + C \\
\Rightarrow f(x) &= \ln x, g'(x) = 1 \text{로 놓으면} \\
f'(x) &= \frac{1}{x}, g(x) = x \\
\therefore \int \ln x dx &= \ln x \times x - \int \frac{1}{x} \times x dx = x \ln x - \int 1 dx \\
&= x \ln x - x + C
\end{aligned}$$

$$\begin{aligned}
78) & \frac{1}{2} x^2 (\ln x)^2 - \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2 + C \\
\Rightarrow f(x) &= (\ln x)^2, g'(x) = x \text{로 놓으면} \\
f'(x) &= \frac{2 \ln x}{x}, g(x) = \frac{1}{2} x^2 \text{이므로} \\
\int x (\ln x)^2 dx &= \frac{1}{2} x^2 (\ln x)^2 - \int x \ln x dx \cdots \textcircled{1} \\
\text{한편, } \int x \ln x dx &\text{에서} \\
u(x) &= \ln x, v'(x) = x \text{로 놓으면}
\end{aligned}$$

$$\begin{aligned}
u'(x) &= \frac{1}{x}, v(x) = \frac{1}{2} x^2 \text{이므로} \\
\int x \ln x dx &= \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x dx \\
&= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C_1 \cdots \textcircled{2}
\end{aligned}$$

$$\begin{aligned}
\textcircled{1} \text{을 } \textcircled{2} \text{에 대입하면} \\
\int x (\ln x)^2 dx &= \frac{1}{2} x^2 (\ln x)^2 - \left(\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C_1 \right) \\
&= \frac{1}{2} x^2 (\ln x)^2 - \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2 + C
\end{aligned}$$

$$\begin{aligned}
79) & \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C \\
\Rightarrow f(x) &= \ln x, g'(x) = x \text{로 놓으면} \\
f'(x) &= \frac{1}{x}, g(x) = \frac{1}{2} x^2 \\
\int x \ln x dx &= \frac{1}{2} x^2 \ln x - \int \frac{1}{x} \cdot \frac{1}{2} x^2 dx \\
&= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx \\
&= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C
\end{aligned}$$

$$\begin{aligned}
80) & x (\ln x)^2 - 2x \ln x + 2x + C \\
\Rightarrow f(x) &= (\ln x)^2, g'(x) = 1 \text{로 놓으면} \\
f'(x) &= \frac{2 \ln x}{x}, g(x) = x \\
\int (\ln x)^2 dx &= x (\ln x)^2 - \int \frac{2 \ln x}{x} \cdot x dx \\
&= x (\ln x)^2 - 2 \int \ln x dx \\
&= x (\ln x)^2 - 2 \left\{ x \ln x - \int 1 dx \right\} \\
&= x (\ln x)^2 - 2(x \ln x - x) + C \\
&= x (\ln x)^2 - 2x \ln x + 2x + C
\end{aligned}$$

$$\begin{aligned}
81) & \frac{1}{2} x^2 \ln 2x - \frac{1}{4} x^2 + C \\
\Rightarrow f(x) &= \ln 2x, g'(x) = x \text{로 놓으면} \\
f'(x) &= \frac{1}{x}, g(x) = \frac{1}{2} x^2 \text{이므로} \\
\int x \ln 2x dx &= \frac{1}{2} x^2 \ln 2x - \int \frac{1}{2} x^2 \times \frac{1}{x} dx \\
&= \frac{1}{2} x^2 \ln 2x - \frac{1}{4} x^2 + C
\end{aligned}$$

$$\begin{aligned}
82) & 3x^3 \ln x - x^3 + C \\
\Rightarrow & \\
u: \ln x, u': \frac{1}{x}, v': 9x^2, v: 3x^3 \text{라 하면} \\
(\text{주어진 적분}) &= 3x^3 \cdot \ln x - \int 3x^2 dx \\
&= 3x^3 \cdot \ln x - x^3 + C
\end{aligned}$$

$$83) \frac{2}{9}e^3 + e^2 + \frac{19}{9}$$

$\Rightarrow f(x) = \ln x, g'(x) = x^2 + 4x + 1$ 로 놓으면

$$f'(x) = \frac{1}{x}, g(x) = \frac{1}{3}x^3 + 2x^2 + x$$

$$\begin{aligned} & \int (x^2 + 4x + 1) \ln x dx \\ &= \left(\frac{1}{3}x^3 + 2x^2 + x \right) \cdot \ln x - \int \frac{1}{x} \cdot \left(\frac{1}{3}x^3 + 2x^2 + x \right) dx \\ &= \left(\frac{1}{3}x^3 + 2x^2 + x \right) \cdot \ln x - \int \left(\frac{1}{3}x^2 + 2x + 1 \right) dx \\ &= \left(\frac{1}{3}x^3 + 2x^2 + x \right) \cdot \ln x - \frac{1}{9}x^3 - x^2 - x + C \end{aligned}$$

이때, $f(1) = 0$ 이므로

$$\left(\frac{1}{3} \cdot 1^3 + 2 \cdot 1^2 + 1 \right) \cdot \ln 1 - \frac{1}{9} \cdot 1^3 - 1^2 - 1 + C = 0$$

$$\therefore C = \frac{19}{9}$$

$$\therefore f(x) = \left(\frac{1}{3}x^3 + 2x^2 + x \right) \cdot \ln x - \frac{1}{9}x^3 - x^2 - x + \frac{19}{9}$$

따라서 구하는 값은

$$\begin{aligned} f(e) &= \left(\frac{1}{3}e^3 + 2e^2 + e \right) \cdot \ln e - \frac{1}{9}e^3 - e^2 - e + \frac{19}{9} \\ &= \frac{2}{9}e^3 + e^2 + \frac{19}{9} \end{aligned}$$

$$84) 2e^3$$

\Rightarrow 부분적분법에 의해

$$f(x) = xe^x - \int e^x dx = xe^x - e^x + C$$

$$f(1) = 0 \text{이므로 } C = 0$$

따라서 $f(x) = xe^x - e^x$ 이므로 $f(3) = 2e^3$ 이다.

$$85) \frac{\pi}{3} + 1$$

\Rightarrow

$$u : x, u' : 1, v' : \sin 3x, v : -\frac{1}{3} \cos 3x \text{라 하면}$$

$$f(x) = -\frac{1}{3}x \cos 3x + \frac{1}{3} \int \cos 3x dx$$

$$= -\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + C$$

$$f(0) = C = 1 \text{이므로 } f(\pi) = \frac{\pi}{3} + 1$$

$$86) \frac{1}{2}$$

$\Rightarrow u = \cos x, v' = e^x$ 로 놓으면

$$u' = -\sin x, v = e^x$$

$$\begin{aligned} f(x) &= \int e^x \cos x dx \\ &= e^x \cos x + \int e^x \sin x dx \cdots \cdots \textcircled{7} \end{aligned}$$

$$\int e^x \sin x dx \text{에서}$$

$f = \sin x, g' = e^x$ 로 놓으면

$$f' = \cos x, g = e^x$$

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx \cdots \cdots \textcircled{8}$$

$\textcircled{7}, \textcircled{8}$ 에서

$$\begin{aligned} \int e^x \cos x dx &= e^x \cos x + \left(e^x \sin x - \int e^x \cos x dx \right) \\ &= e^x \cos x + e^x \sin x - \int e^x \cos x dx \end{aligned}$$

$$\int e^x \cos x dx = \frac{e^x}{2} (\sin x + \cos x) + C$$

$$f(x) = \frac{e^x}{2} (\sin x + \cos x) + C \text{이고}$$

$$f\left(\frac{\pi}{2}\right) = \frac{e^{\frac{\pi}{2}}}{2} \text{이므로}$$

$$f\left(\frac{\pi}{2}\right) = \frac{e^{\frac{\pi}{2}}}{2} + C = \frac{e^{\frac{\pi}{2}}}{2}, C = 0$$

$$\text{따라서 } f(x) = \frac{e^x}{2} (\sin x + \cos x) \text{이므로 } f(0) = \frac{1}{2}$$

$$87) \frac{\sqrt{2}}{2}$$

$\Rightarrow \int e^x \cos x dx$ 에서

$f(x) = \cos x, g'(x) = e^x$ 으로 놓으면

$f'(x) = -\sin x, g(x) = e^x$ 이므로

$$\int e^x \cos x dx = e^x \cos x + \int e^x \sin x dx \cdots \textcircled{9}$$

$$\int e^x \sin x dx \text{에서}$$

$u(x) = \sin x, v'(x) = e^x$ 으로 놓으면

$u'(x) = \cos x, v(x) = e^x$ 이므로

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx \cdots \textcircled{10}$$

$\textcircled{9}$ 을 $\textcircled{10}$ 에 대입하면

$$\int e^x \cos x dx = e^x \cos x + \left(e^x \sin x - \int e^x \cos x dx \right)$$

$$\therefore h(x) = \int e^x \cos x dx = \frac{1}{2} e^x (\cos x + \sin x) + C$$

$$h(0) = \frac{1}{2} \text{이므로 } \frac{1}{2} + C = \frac{1}{2} \therefore C = 0$$

$$\text{따라서 } h(x) = \frac{1}{2} e^x (\cos x + \sin x) \text{이므로}$$

$$h(\pi) = \frac{1}{2} e^{\pi} (\cos \pi + \sin \pi) = -\frac{1}{2} e^{\pi}$$

$$88) -1$$

$\Rightarrow f(x) = \int \ln x dx$ 에서

$$u = \ln x, v' = 1 \text{로 놓으면 } u' = \frac{1}{x}, v = x$$

$$\begin{aligned} f(x) &= \int \ln x dx = \ln x \cdot x - \int \frac{1}{x} \cdot x dx \\ &= x \ln x - x + C \end{aligned}$$

$$f(e) = 0 \text{이므로}$$

$$f(e) = e \ln e - e + C = 0, C = 0$$

따라서 $f(x) = x \ln x - x$ 이므로

$$f(1) = 1 \cdot \ln 1 - 1 = -1$$

89) 2

$$\Rightarrow F'(x) = f(x)$$

주어진 식의 양변을 미분하자.

$$f(x) = f(x) + xf'(x) - 2x \sin x - x^2 \cos x$$

$$f'(x) = 2 \sin x + x \cos x$$

$$f(x) = \int (2 \sin x + x \cos x) dx$$

$$\int (x \cos x) dx = x \sin x - \int \sin x dx$$

$$\int x \cos x dx = x \sin x + \cos x + C$$

$$f(x) = -\cos x + x \sin x + C$$

$$f(0) = 0 \quad \therefore C = 1$$

$$f(x) = -\cos x + x \sin x + 1$$

$$\therefore f(\pi) = 1 + 0 + 1 = 2$$

90) 4

$\Rightarrow F(x) = xf(x) - x \ln x$ 에서 양변을 x 에 대하여 미분하면

$$F'(x) = f(x) + xf'(x) - \ln x - 1$$

$$F'(x) = f(x) \text{ 이므로}$$

$$xf'(x) = \ln x + 1,$$

$$f'(x) = \frac{\ln x}{x} + \frac{1}{x} \quad (\because x \neq 0)$$

$$\begin{aligned} f(x) &= \int f'(x) dx = \int \left(\frac{\ln x}{x} + \frac{1}{x} \right) dx \\ &= \int \frac{\ln x}{x} dx + \int \frac{1}{x} dx \end{aligned}$$

이때, $\int \frac{\ln x}{x} dx$ 에서 $u = \ln x$, $v' = \frac{1}{x}$ 로 놓으면

$$u' = \frac{1}{x}, \quad v = \ln x$$

$$\int \frac{\ln x}{x} dx = (\ln x)^2 - \int \frac{\ln x}{x} dx$$

$$2 \int \frac{\ln x}{x} dx = (\ln x)^2$$

$$\therefore \int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2}$$

$$\begin{aligned} \text{즉, } f(x) &= \int \frac{\ln x}{x} dx + \int \frac{1}{x} dx \\ &= \frac{(\ln x)^2}{2} + \ln x + C \end{aligned}$$

$$f(1) = 0 \text{ 이므로 } C = 0$$

$$\therefore f(x) = \frac{(\ln x)^2}{2} + \ln x$$

따라서 구하는 값은

$$f(e^2) = \frac{(\ln e^2)^2}{2} + \ln e^2 = 4$$