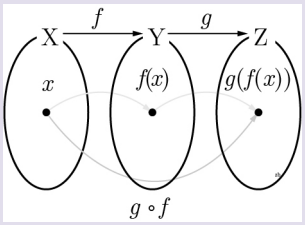
01 합성함수

두 함수 $f : X \to Y$, $g : Y \to Z$에 대하여

$(g \circ f)(x) = g(f(x))$



◪ **두 함수 $f(x)=3x-1$, $g(x)=x^2+2$에 대하여 다음을 구하시오.**

1. $(g \circ f)(x)$

2. $(f \circ f)(x)$

3. $(g \circ g)(x)$

4. $(f \circ g)(x)$

◪ **두 함수 $f(x)$, $g(x)$가 다음과 같을 때, 합성함수 $(g \circ f)(x)$를 구하여라.**

5. $f(x)=3x$, $g(x)=2x-3$

6. $f(x)=2x+1$, $g(x)=3x-2$

7. $f(x)=x-3$, $g(x)=2x^2+1$

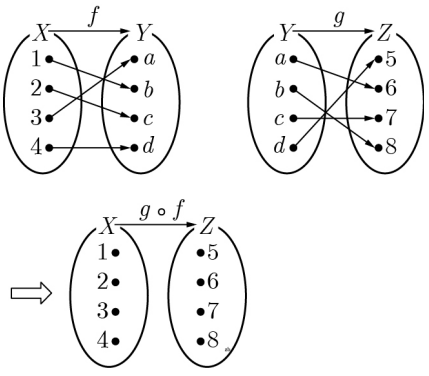
8. $f(x)=3x^2-1$, $g(x)=2x+5$

9. $f(x)=2x+1$, $g(x)=3x^2$

10. $f(x)=-x+3$, $g(x)=x^2-4$

◪ **두 함수 f, g가 다음 그림과 같을 때, 합성함수 $g \circ f$를 그림으로 나타내어라.**

11.



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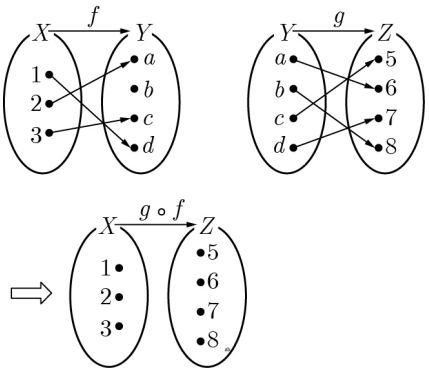

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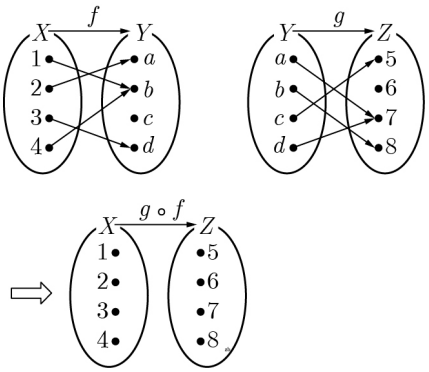
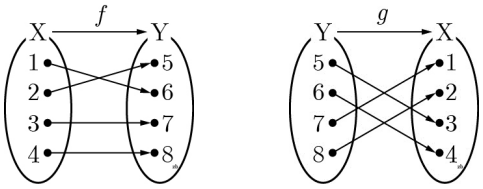


◪ **두 함수 $f(x)$, $g(x)$가 다음과 같을 때, 합성함수 $(g \circ f)(x)$를 구하여라.**

14. $f(x)=\frac{1}{2}x+4,\ g(x)=2x-3$

15. $f(x)=2x^2-1,\ g(x)=3x-2$

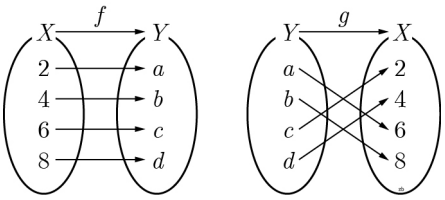
◪ **두 함수 f, g가 다음 그림과 같을 때, 다음을 구하시오.**



16. $(g \circ f)(2)$

17. $(f \circ g)(5)$

◪ **두 함수 f, g가 그림과 같을 때, 다음 값을 구하여라.**



18. $(g \circ f)(4)$

19. $(f \circ g)(c)$

20. $(g \circ f)(6)$

21. $(f \circ g)(b)$

◪ **두 함수** $f(x)=-3x+1,\ g(x)=\frac{1}{2}x+5$**에 대하여 다음 값을 구하여라.**

22. $(g\circ f)(-1)$

23. $(f\circ g)(2)$

24. $(g\circ g)(0)$

25. $(f\circ f)(3)$

◪ **두 함수** $f(x)=2x+1,\ g(x)=x^2$**에 대하여 다음을 구하여라.**

26. $(g\circ f)(-2)$

27. $(f\circ f)(5)$

28. $(f\circ g)\left(\frac{1}{2}\right)$

29. $(g\circ g)\left(-\frac{1}{2}\right)$

30. $(g\circ f)(x)$

31. $(f\circ f)(x)$

32. $(f\circ g)(x)$

33. $(g\circ g)(x)$

◪ **세 함수** $f(x)=2x-3,\ g(x)=3x+2,\ h(x)=x^2-1$**에 대하여 다음 값을 구하여라.**

34. $(f\circ f)(-1)$

35. $(g\circ f)\left(-\frac{1}{2}\right)$

36. $(f\circ g)(-2)$

37. $(f\circ h)(2)$

38. $(h\circ f)(2)$

39. $(g \circ h)(-1)$

40. $(h \circ g)\left(\frac{1}{3}\right)$

02 합성함수의 성질

세 함수 f, g, h에 대하여

(1) $g \circ f \neq f \circ g$

(2) $h \circ (g \circ f) = (h \circ g) \circ f$

(3) $f : X \to X$일 때, $f \circ I = I \circ f$ (단, I는 항등함수)

◪ **두 함수 $f(x)=3x+2$, $g(x)=x^2-1$에 대하여 다음 물음에 답하여라.**

41. $(f \circ g)(x)$**를 구하여라.**

42. $f \circ g = g \circ f$**인지 확인하여라.**

43. $(g \circ f)(x)$**를 구하여라.**

◪ **세 함수 $f(x)=x+3$, $g(x)=x^2+1$, $h(x)=x-2$에 대하여 다음 물음에 답하여라.**

44. $((h \circ g) \circ f)(x)$**를 구하여라.**

45. $(h \circ g) \circ f = h \circ (g \circ f)$**인지 확인하여라.**

46. $(h \circ (g \circ f))(x)$**를 구하여라.**

◪ **다음 두 함수 f, g에 대하여 $f \circ g = g \circ f$가 성립할 때, 상수 k의 값을 구하여라.**

47. $f(x)=2x$, $g(x)=3x+k$

48. $f(x)=kx-3$, $g(x)=\frac{2}{3}x+1$

49. $f(x)=3x+1$, $g(x)=-x+k$

50. $f(x)=x+k$, $g(x)=3x+10$

51. $f(x)=2x+k$, $g(x)=-x+3$

52. $f(x)=2x+3$, $g(x)=kx-1$

53. $f(x)=x+1$, $g(x)=kx+2$

54. $f(x)=2x+1$, $g(x)=-x-k$

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◪ **양의 실수 전체에서 정의된 세 함수**

$f(x)=3x-1,\ g(x)=x^2+2,\ h(x)=-\frac{1}{2}x+3$

에 대하여 다음 값을 구하여라.

55. $(g\circ f)(1)+(f\circ g)(1)$

56. $(f\circ g\circ h)(2)-(g\circ h\circ f)(1)$

57. **세 함수** $f(x)=2x^2-1,\ g(x)=x+3,$

$h(x)=\frac{1}{2}x+1$**에 대하여 다음 값을 구하여라.**

(1) $(f\circ g)(3)-(g\circ h)(2)$

(2) $(g\circ h\circ f)(1)-(g\circ f\circ h)(2)$

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정답 및 해설

1) $(g \circ f)(x) = 9x^2 - 6x + 3$
⇨ $(g \circ f)(x) = g(f(x)) = (3x-1)^2 + 2 = 9x^2 - 6x + 3$

2) $(f \circ f)(x) = 9x - 4$
⇨ $(f \circ f)(x) = f(f(x)) = 3(3x-1) - 1 = 9x - 4$

3) $(g \circ g)(x) = x^4 + 4x^2 + 6$
⇨ $(g \circ g)(x) = g(g(x)) = (x^2+2)^2 + 2 = x^4 + 4x^2 + 6$

4) $(f \circ g)(x) = 3x^2 + 5$
⇨ $(f \circ g)(x) = f(g(x)) = 3(x^2+2) - 1 = 3x^2 + 5$

5) $6x - 3$
⇨ $(g \circ f)(x) = g(f(x)) = g(3x)$
$= 2(3x) - 3 = 6x - 3$

6) $6x + 1$
⇨ $(g \circ f)(x) = g(f(x)) = g(2x+1)$
$= 3(2x+1) - 2 = 6x + 1$

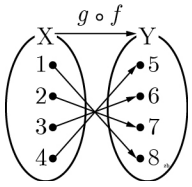
7) $2x^2 - 12x + 19$
⇨ $(g \circ f)(x) = g(f(x)) = g(x-3)$
$= 2(x-3)^2 + 1 = 2x^2 - 12x + 19$

8) $6x^2 + 3$
⇨ $(g \circ f)(x) = g(f(x)) = g(3x^2-1)$
$= 2(3x^2-1) + 5 = 6x^2 + 3$

9) $12x^2 + 12x + 3$
⇨ $(g \circ f)(x) = g(f(x)) = g(2x+1)$
$= 3(2x+1)^2 = 12x^2 + 12x + 3$

10) $x^2 - 6x + 5$
⇨ $(g \circ f)(x) = g(f(x)) = g(-x+3)$
$= (-x+3)^2 - 4 = x^2 - 6x + 5$

11)



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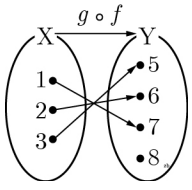

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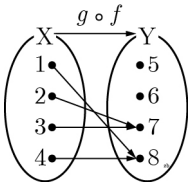

14) $(g \circ f)(x) = x + 5$
⇨ $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{2}x + 4\right)$
$= 2\left(\frac{1}{2}x + 4\right) - 3 = x + 5$

15) $(g \circ f)(x) = 6x^2 - 5$
⇨ $(g \circ f)(x) = g(f(x))$
$= g(2x^2-1)$
$= 3(2x^2-1) - 2 = 6x^2 - 5$

16) 3
⇨ $(g \circ f)(2) = g(f(2)) = g(5) = 3$

17) 7
⇨ $(f \circ g)(5) = f(g(5)) = f(3) = 7$

18) 8
⇨ $(g \circ f)(4) = g(f(4) = g(b) = 8$

19) a
⇨ $(f \circ g)(c) = f(g(c)) = f(2) = a$

20) 2
⇨ $(g \circ f)(6) = g(f(6) = g(c) = 2$

21) d
⇨ $(f \circ g)(b) = f(g(b)) = f(8) = d$

22) 7
⇨ $f(-1) = -3 \cdot (-1) + 1 = 4$이므로
$(g \circ f)(-1) = g(f(-1)) = g(4) = \frac{1}{2} \cdot 4 + 5 = 7$

23) -17
⇨ $g(2) = \frac{1}{2} \cdot 2 + 5 = 6$이므로
$(f \circ g)(2) = f(g(2)) = f(6) = -3 \cdot 6 + 1 = -17$

24) $\frac{15}{2}$
⇨ $g(0) = \frac{1}{2} \cdot 0 + 5 = 5$이므로
$(g \circ g)(0) = g(g(0)) = g(5) = \frac{1}{2} \cdot 5 + 5 = \frac{15}{2}$

25) 25
⇨ $f(3) = -3 \cdot 3 + 1 = -8$이므로
$(f \circ f)(3) = f(f(3)) = f(-8) = -3 \cdot (-8) + 1 = 25$

26) 9

⇨ $f(-2)=2\cdot(-2)+1=-3$이므로
$(g\circ f)(-2)=g(f(-2))=g(-3)=(-3)^2=9$

27) 23
⇨ $f(5)=2\cdot5+1=11$이므로
$(f\circ f)(5)=f(f(5))=f(11)=2\cdot11+1=23$

28) $\frac{3}{2}$
⇨ $g\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2=\frac{1}{4}$이므로
$(f\circ g)\left(\frac{1}{2}\right)=f\left(g\left(\frac{1}{2}\right)\right)=f\left(\frac{1}{4}\right)=2\cdot\frac{1}{4}+1=\frac{3}{2}$

29) $\frac{1}{16}$
⇨ $g\left(-\frac{1}{2}\right)=\left(-\frac{1}{2}\right)^2=\frac{1}{4}$
$(g\circ g)\left(-\frac{1}{2}\right)=g\left(g\left(-\frac{1}{2}\right)\right)=g\left(\frac{1}{4}\right)=\left(\frac{1}{4}\right)^2=\frac{1}{16}$

30) $4x^2+4x+1$
⇨ $(g\circ f)(x)=g(f(x))$
$=g(2x+1)=(2x+1)^2=4x^2+4x+1$

31) $4x+3$
⇨ $(f\circ f)(x)=f(f(x))$
$=f(2x+1)=2(2x+1)+1=4x+3$

32) $2x^2+1$
⇨ $(f\circ g)(x)=f(g(x))=f(x^2)=2x^2+1$

33) x^4
⇨ $(g\circ g)(x)=g(g(x))=g(x^2)=(x^2)^2=x^4$

34) -13
⇨ $f(-1)=2\cdot(-1)-3=-5$이므로
$(f\circ f)(-1)=f(f(-1))=f(-5)$
$=2\cdot(-5)-3=-13$

35) -10
⇨ $f\left(-\frac{1}{2}\right)=2\cdot\left(-\frac{1}{2}\right)-3=-4$이므로
$(g\circ f)\left(-\frac{1}{2}\right)=g\left(f\left(-\frac{1}{2}\right)\right)=g(-4)$
$(g\circ f)\left(-\frac{1}{2}\right)=3\cdot(-4)+2=-10$

36) -11
⇨ $g(-2)=3\cdot(-2)+2=-4$이므로
$(f\circ g)(-2)=f(g(-2))=f(-4)$
$=2\cdot(-4)-3=-11$

37) 3
⇨ $h(2)=2^2-1=3$이므로
$(f\circ h)(2)=f(h(2))=f(3)=2\cdot3-3=3$

38) 0
⇨ $f(2)=2\cdot2-3=1$이므로
$(h\circ f)(2)=h(f(2))=h(1)=1^2-1=0$

39) 2
⇨ $h(-1)=(-1)^2-1=0$이므로
$(g\circ h)(-1)=g(h(-1))=g(0)=3\cdot0+2=2$

40) 8
⇨ $g\left(\frac{1}{3}\right)=3\cdot\left(\frac{1}{3}\right)+2=3$이므로
$(h\circ g)\left(\frac{1}{3}\right)=h\left(g\left(\frac{1}{3}\right)\right)=h(3)=3^2-1=8$

41) $3x^2-1$
⇨ $(f\circ g)(x)=f(g(x))=f(x^2-1)$
$=3(x^2-1)+2=3x^2-1$

42) $f\circ g\neq g\circ f$
⇨ 41,42에서 $f\circ g\neq g\circ f$이다.

43) $9x^2+12x+3$
⇨ $(g\circ f)(x)=g(f(x))=g(3x+2)$
$=(3x+2)^2-1=9x^2+12x+3$

44) x^2+6x+8
⇨ $(h\circ g)(x)=h(g(x))=h(x^2+1)$
$=(x^2+1)-2=x^2-1$
∴ $((h\circ g)\circ f)(x)=(h\circ g)(f(x))$
$=(h\circ g)(x+3)$
$=(x+3)^2-1=x^2+6x+8$

45) $(h\circ g)\circ f=h\circ(g\circ f)$
⇨ 44에서 $(h\circ g)\circ f=h\circ(g\circ f)=x^2+6x+8$

46) x^2+6x+8
⇨ $(g\circ f)(x)=g(f(x))=g(x+3)$
$=(x+3)^2+1=x^2+6x+10$
∴ $(h\circ(g\circ f))(x)=h\circ((g\circ f)(x)$
$=h(x^2+6x+10)$
$=(x^2+6x+10)-2=x^2+6x+8$

47) 0
⇨ $(f\circ g)(x)=f(g(x))=f(3x+k)$
$=2(3x+k)=6x+2k$
$(g\circ f)(x)=g(f(x))=g(2x)$
$=3(2x)+k=6x+k$
$f\circ g=g\circ f$이므로 $2k=k$ ∴ $k=0$

48) 2
⇨ $(f\circ g)(x)=f(g(x))=f\left(\frac{2}{3}x+1\right)$

$=k\left(\frac{2}{3}x+1\right)-3=\frac{2}{3}kx+k-3$

$(g\circ f)(x)=g(f(x))=g(kx-3)$

$=\frac{2}{3}(kx-3)+1=\frac{2}{3}kx-1$

$f\circ g=g\circ f$이므로 $k-3=-1$ $\therefore\ k=2$

49) -1

⇨ $(f\circ g)(x)=f(g(x))=f(-x+k)$

$=3(-x+k)+1=-3x+3k+1$

$(g\circ f)(x)=g(f(x))=g(3x+1)$

$=-(3x+1)+k$

$=-3x+k-1$

$f\circ g=g\circ f$이므로 $3k+1=k-1$ $\therefore\ k=-1$

50) 0

⇨ $(f\circ g)(x)=(g(x))=f(3x+10)=3x+10+k$

$(g\circ f)(x)=g(f(x))=g(x+k)=3(x+k)+10$

$=3x+3k+10$

$f\circ g=g\circ f$이므로 $3x+10+k=3x+3k+10$

$10+k=3k+10$ $\therefore\ k=0$

51) $-\frac{3}{2}$

⇨ $(f\circ g)(x)=f(g(x))=f(-x+3)$

$=2(-x+3)+k=-2x+6+k$

$(g\circ f)(x)=g(f(x))=g(2x+k)$

$=-(2x+k)+3$

$=-2x-k+3$

$f\circ g=g\circ f$이므로 $6+k=-k+3$

$\therefore\ k=-\frac{3}{2}$

52) $\frac{2}{3}$

⇨ $(f\circ g)(x)=f(g(x))=f(kx-1)$

$=2(kx-1)+3=2kx+1$

$(g\circ f)(x)=g(f(x))=g(2x+3)=k(2x+3)-1$

$=2kx+3k-1$

$f\circ g=g\circ f$이므로 $2kx+1=2kx+3k-1$

$1=3k-1$ $\therefore\ k=\frac{2}{3}$

53) 1

⇨ $(f\circ g)(x)=f(g(x))=f(kx+2)$

$=(kx+2)+1=kx+3$

$(g\circ f)(x)=g(f(x))=g(x+1)$

$=k(x+1)+2=kx+k+2$

$f\circ g=g\circ f$이므로 $hx+3=kx+k+2$

$3=k+2$ $\therefore\ k=1$

54) 2

⇨ $(f\circ g)(x)=f(g(x))=f(-x-k)$

$=2(-x-h)+1=-2x-2k+1$

$(g\circ f)(x)=g(f(x))=g(2x+1)$

$=-(2x+1)-k=-2x-1-k$

$f\circ g=g\circ f$이므로 $-2x-2k+1=-2x-1-k$

$-2k+1=-1-k$ $\therefore\ k=2$

55) 14

⇨ $(g\circ f)(1)+(f\circ g)(1)=g(f(1))+f(g(1))$

$=g(2)+f(3)$

$=6+8=14$

56) 11

⇨ $(f\circ g\circ h)(2)-(g\circ h\circ f)(1)$

$=(f\circ g)(h(2))-(g\circ h)(f(1))$

$=(f\circ g)(2)-(g\circ h)(2)$

$=f(g(2))-g(h(2))$

$=f(6)-g(2)$

$=17-6=11$

57) (1) 66 (2) $-\frac{11}{2}$

⇨ (1) $(f\circ g)(3)-(g\circ h)(2)$

$=f(g(3))-g(h(2))$

$=f(3+3)-g\left(\frac{1}{2}\cdot 2+1\right)=f(6)-g(2)$

$=(2\cdot 6^2-1)-(2+3)=71-5=66$

(2) $(g\circ h\circ f)(1)-(g\circ f\circ h)(2)$

$=g(h(f(1)))-(g(f(h(2)))$

$=g(h(2\cdot 1^2-1))-g\left(f\left(\frac{1}{2}\cdot 2+1\right)\right)$

$=g(h(1))-g(f(2))$

$=g\left(\frac{1}{2}\cdot 1+1\right)-g(2\cdot 2^2-1)=g\left(\frac{3}{2}\right)-g(7)$

$=\left(\frac{3}{2}+3\right)-(7+3)=-\frac{11}{2}$

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