◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시
1) 제작연월일 : 2016-10-25
2) 제작자 : 교육지대㈜
3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

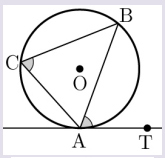
◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

계산시 참고사항

1. 접선과 현이 이루는 각

원의 접선과 그 접점을 지나는 현이 이루는 각의 크기는 그 각의 내부에 있는 호에 대한 원주각의 크기와 같다.

⇨ $\overleftrightarrow{AT}$가 원 O의 접선이면 $\angle BAT = \angle BCA$

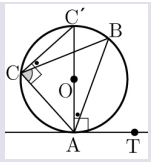


설명) 오른쪽 그림과 같이 원 O의 지름 AC′을 그으면

$\angle C'AT = \angle C'CA = 90°$

이 때, $\angle BAT = 90° - \angle C'AB$, $\angle BCA = 90° - \angle C'CB$이고

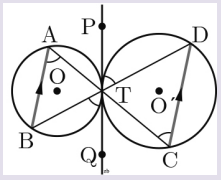
$\angle C'AB = \angle C'CB$($\overset{\frown}{C'B}$에 대한 원주각)이므로 $\angle BAT = \angle BCA$



참고
◉ 접선이 되기 위한 조건
원 O에서 $\angle BAT = \angle BCA$이면 직선 AT는 원의 접선이다.

2. 두 원에서 접선과 현이 이루는 각

두 원의 교점 T에서 접선 PQ가 다음 그림과 같을 때, 각 경우에 대하여 $\overline{AB} // \overline{CD}$이다.

1)


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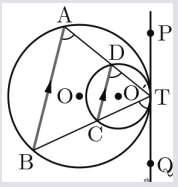
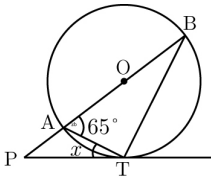


접선과 현이 이루는 각

◪ 다음 그림에서 직선 PT는 점 T를 접점으로 하는 원 O의 접선일 때, $\angle x$의 크기를 구하여라.

1.



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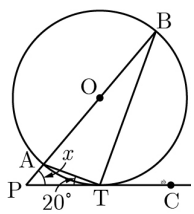

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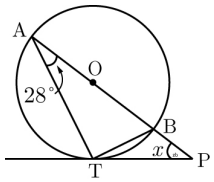

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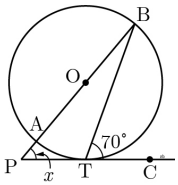

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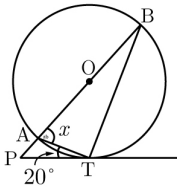

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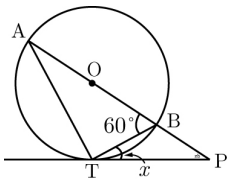

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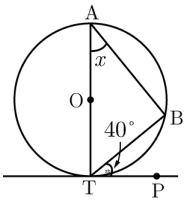

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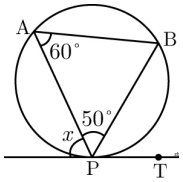

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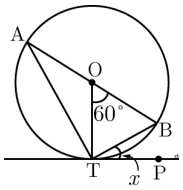

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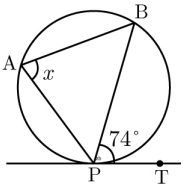

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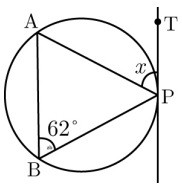

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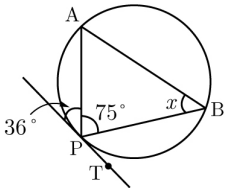

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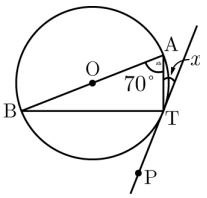

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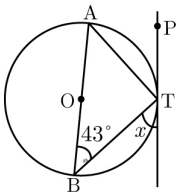

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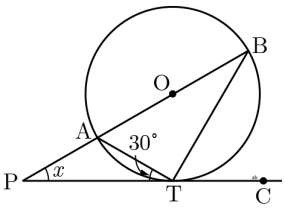

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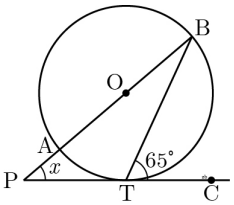

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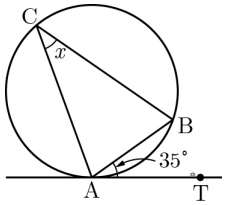
◪ 다음 그림에서 직선 AT가 원 O의 접선일 때, $\angle x$의 크기를 구하여라.

17.



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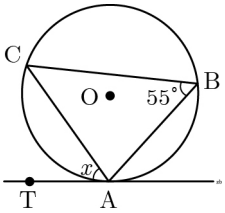

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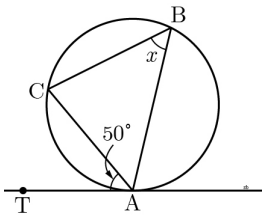

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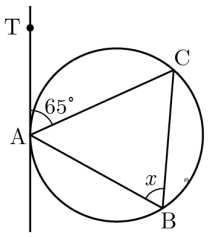

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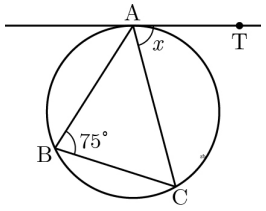

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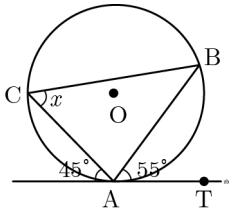

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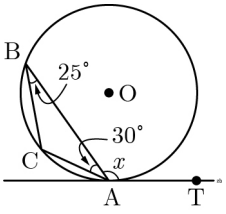

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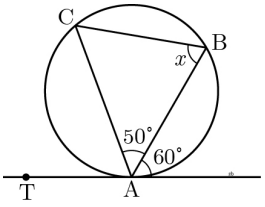

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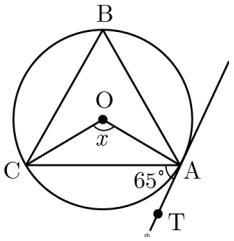

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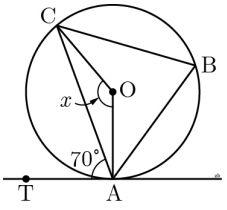

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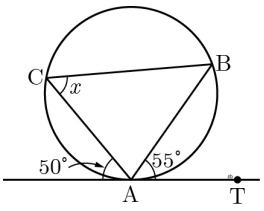

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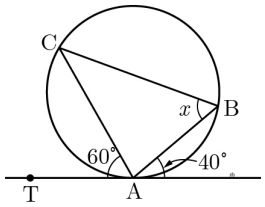

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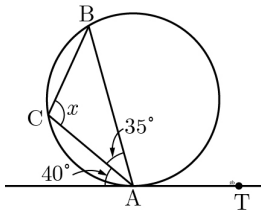

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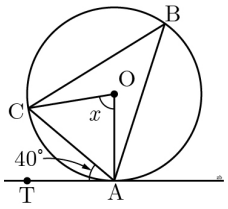

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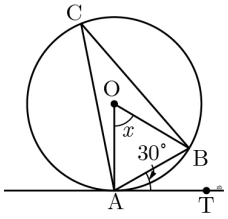

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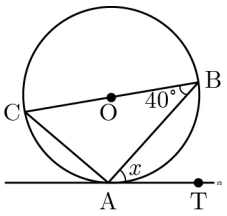

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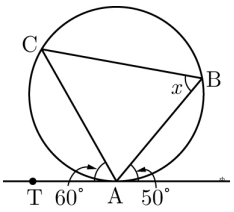

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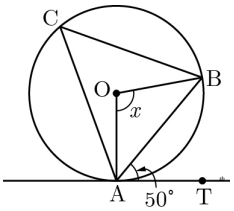

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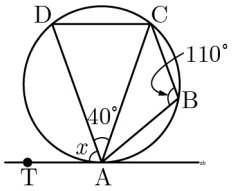

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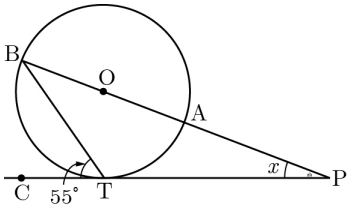

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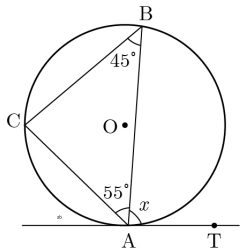
37.



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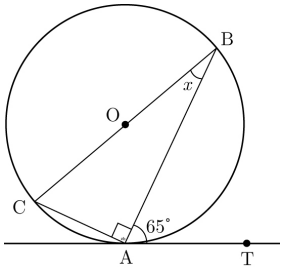

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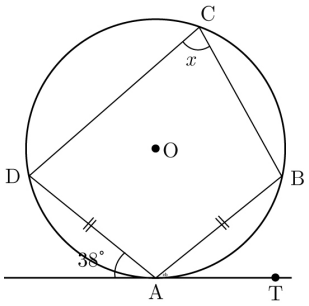

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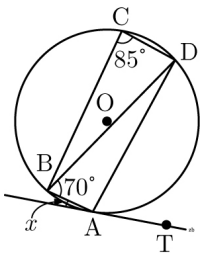
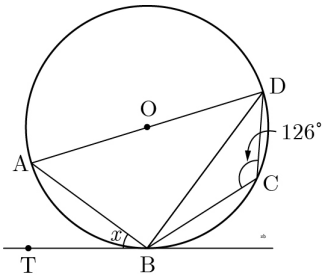


◪ **다음 그림에서 직선 BT가 원 O의 접선일 때, $\angle x$의 크기를 구하여라.**

41.



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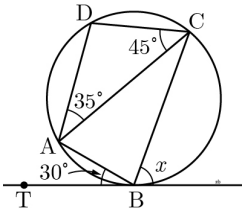

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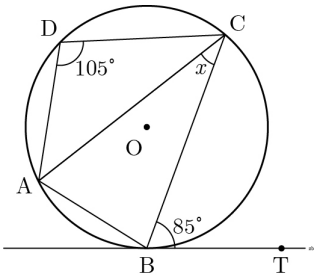

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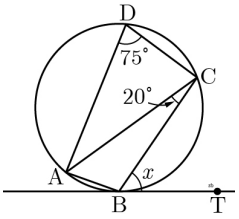

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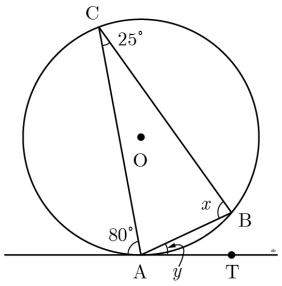
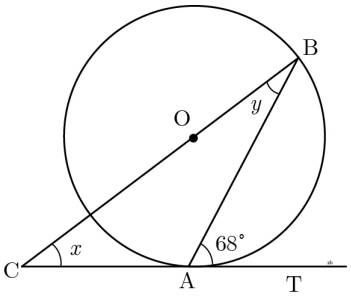
◪ 다음 그림에서 직선 AT가 원 O의 접선일 때, $\angle x$, $\angle y$의 크기를 각각 구하여라.

45.



46.



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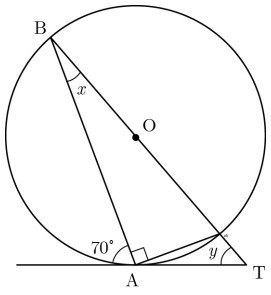

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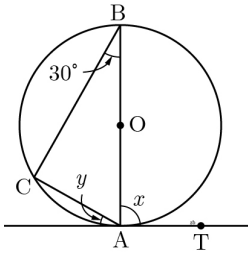

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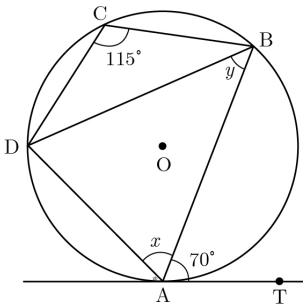

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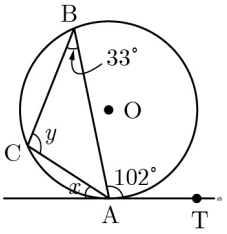

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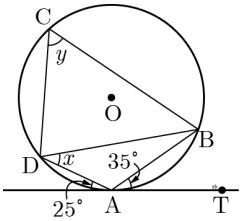

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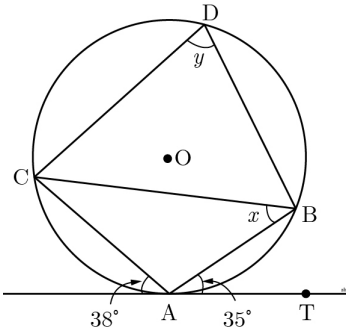

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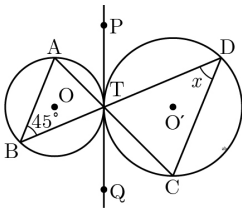
두 원에서 접선과 현이 이루는 각

◪ 다음 그림에서 직선 PQ가 두 원의 공통접선일 때, $\angle x$의 크기를 구하여라.

53.



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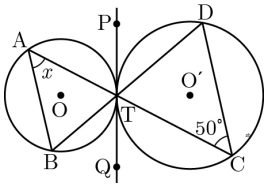

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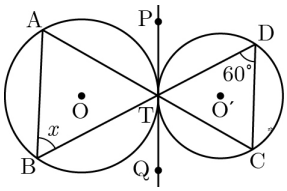

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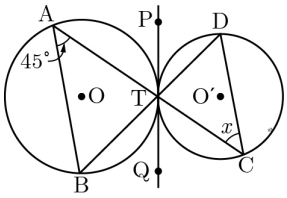

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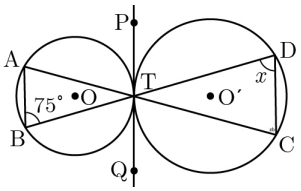

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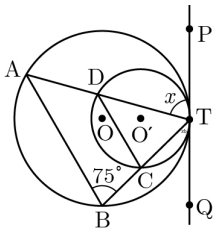

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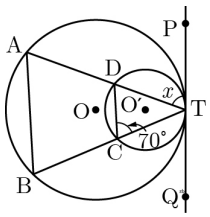

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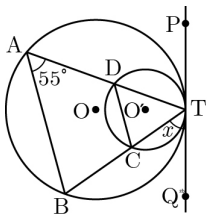

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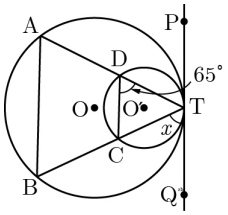

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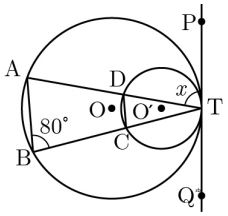

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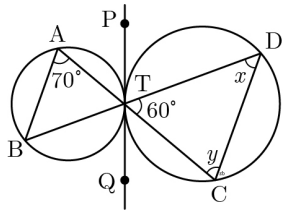
◪ 다음 그림에서 직선 PQ는 두 원의 공통인 접선일 때, $\angle x$, $\angle y$의 크기를 각각 구하여라.

63.



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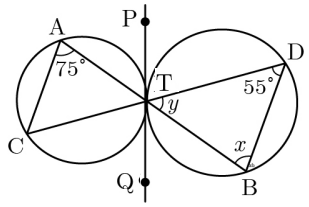

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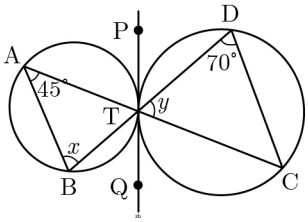

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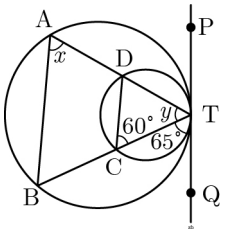
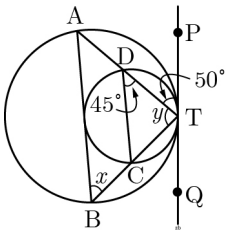
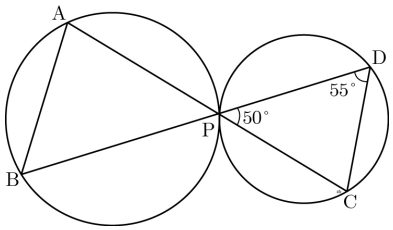


67.

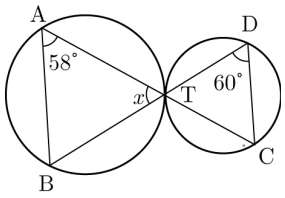


◪ 다음 물음에 답하여라.

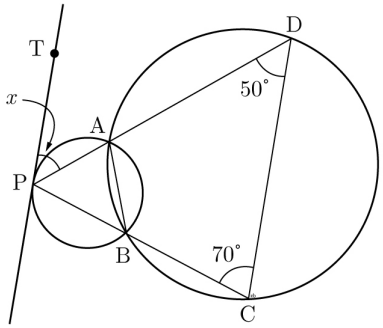
68. 두 원이 점 P에서 접하고, $\overline{\mathrm{AB}}$, $\overline{\mathrm{CD}}$는 각각 두 원의 현이다. $\angle \mathrm{D}=55^\circ$, $\angle \mathrm{DPC}=50^\circ$일 때, $\angle \mathrm{PAB}$의 크기를 구하여라.



69. 다음 그림과 같이 점 T에서 외접하는 두 원이 있다. 점 B를 지나는 두 직선이 원과 만나는 점을 A, B, C, D라 하고 $\angle \mathrm{BAT}=58^\circ$, $\angle \mathrm{CDT}=60^\circ$일 때, $\angle x$의 크기를 구하여라.



70. 다음 그림에서 $\overleftrightarrow{\mathrm{PT}}$가 작은 원의 접선일 때, $\angle x$의 크기를 구하여라.



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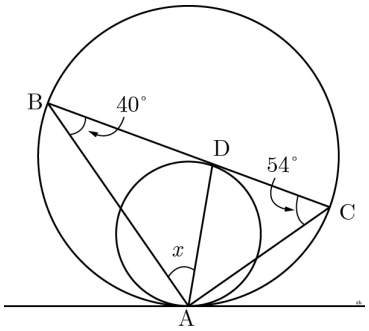
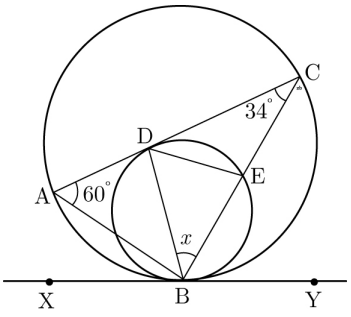
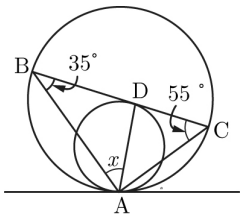
71. 두 원이 점 A에서 내접하고, $\overline{BC}$가 작은 원과 점 D에서 접하고 있다. $\angle CBA = 40^\circ$, $\angle ACB = 54^\circ$일 때, $\angle x$의 크기를 구하여라.



72. 다음 그림에서 직선 XY는 두 원의 공통접선이고, 큰 원의 현 AC는 작은 원과 점 D에서 접할 때, $\angle x$의 크기를 구하여라.



73. 두 원이 점A에서 내접하고, $\overline{BC}$가 작은 원과 점 D에서 접하고 있다. $\angle B = 35^\circ$, $\angle C = 55^\circ$일 때, $\angle x$의 크기를 구하여라.



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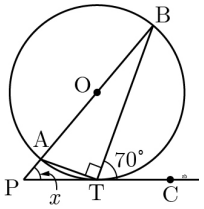
정답 및 해설

1) 25°
⇨ $\angle x = \angle ABT = 180^\circ - (65^\circ + 90^\circ) = 25^\circ$

2) 50°
⇨ $\overline{AB}$가 원 O의 지름이므로 $\angle ATB = 90^\circ$
원의 접선과 그 접점을 지나는 현이 이루는 각의 크기는 그 각의 내부에 있는 호에 대한 원주각의 크기와 같으므로 $\angle ABT = \angle ATP = 20^\circ$
$\triangle PBT$에서 $\angle x = 180^\circ - (20^\circ + 90^\circ + 20^\circ) = 50^\circ$

3) 34°
⇨ $\angle x = 180^\circ - (28^\circ + 90^\circ + 28^\circ) = 34^\circ$

4) 50°
⇨ 

보조선 AT를 그으면 $\overline{AB}$가 원 O의 지름이므로
$\angle ATB = 90^\circ$
$\angle ABT = \angle ATP = 180^\circ - (70^\circ + 90^\circ) = 20^\circ$
$\triangle PBT$에서 $\angle x = \angle BTC - \angle PBT = 70^\circ - 20^\circ = 50^\circ$

5) 70°
⇨ $\angle x = 180^\circ - (90^\circ + 20^\circ) = 70^\circ$

6) 30°
⇨ $\angle x = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$

7) 40°
⇨ $\angle x = \angle BTP = 40^\circ$

8) 70°
⇨ $\angle x = 180^\circ - (60^\circ + 50^\circ) = 70^\circ$

9) 30°
⇨ $\angle x = \angle BAT = \frac{1}{2}\angle BOT = \frac{1}{2} \times 60^\circ = 30^\circ$

10) 74°

11) 62°

12) 36°

13) 20°
⇨ $\angle x = \angle ABP = 180^\circ - (70^\circ + 90^\circ) = 20^\circ$

14) 47°
⇨ $\angle x = \angle BAT = 180^\circ - (43^\circ + 90^\circ) = 47^\circ$

15) 30°
⇨ $\overline{AB}$가 원 O의 지름이므로
$\angle ATB = 90^\circ$, $\angle ABT = \angle ATP = 30^\circ$
$\triangle PBT$에서 $\angle x = 180^\circ - (30^\circ + 90^\circ + 30^\circ) = 30^\circ$

16) 40°
⇨ 보조선 AT를 그으면 $\overline{AB}$가 원 O의 지름이므로
$\angle ATB = 90^\circ$, $\angle BAT = \angle BTC = 65^\circ$
$\triangle ATB$에서
$\angle ABT = \angle ATP = 180^\circ - (65^\circ + 95^\circ) = 25^\circ$
$\triangle PBT$에서 $\angle x = \angle BTC - \angle PBT = 65^\circ - 25^\circ = 40^\circ$

17) 35°

18) 55°
⇨ $\angle x = \angle ABC = 55^\circ$

19) 50°

20) 65°

21) 75°

22) 55°
⇨ $\angle x = \angle BAT = 55^\circ$

23) 125°
⇨ $\triangle ABC$에서 $\angle ACB = 180^\circ - (30^\circ + 25^\circ) = 125^\circ$
$\therefore \angle x = \angle ACB = 125^\circ$

24) 70°
⇨ $\angle CAT = 180^\circ - (50^\circ + 60^\circ) = 70^\circ$
$\therefore \angle x = \angle CAT = 70^\circ$

25) 130°
⇨ $\angle ABC = \angle CAT = 65^\circ$ 이므로
$\angle x = 2\angle ABC = 2 \times 65^\circ = 130^\circ$

26) 140°
⇨ 원의 접선과 그 접선을 지나는 현이 이루는 각의 크기는 그 각의 내부에 있는 호에 대한 원주각의 크기와 같으므로 $\angle CBA = \angle CAT = 70^\circ$
한 원에서 한 호에 대한 원주각의 크기는 그 호에 대한 중심각의 크기의 $\frac{1}{2}$이므로
$\angle x = 2\angle CBA = 2 \times 70^\circ = 140^\circ$

27) 55°
⇨ $\angle x = \angle BAT = 55^\circ$

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28) 60°
⇨ $\angle x = \angle CAT = 60^\circ$

29) 105°
⇨ $\angle BAT = 180^\circ - (40^\circ + 35^\circ) = 105^\circ$
$\therefore \angle x = \angle BAT = 105^\circ$

30) 80°
⇨ $\angle CBA = \angle CAT = 40^\circ$
$\therefore \angle x = 2\angle CBA = 2 \times 40^\circ = 80^\circ$

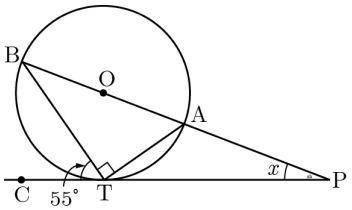
31) 60°
⇨ $\angle BCA = \angle BAT = 30^\circ$
$\therefore \angle x = 2\angle ACB = 2 \times 30^\circ = 60^\circ$

32) 50°
⇨ $\overline{BC}$는 원 O의 지름이므로 $\angle BAC = 90^\circ$
$\triangle ABC$에서 $\angle ACB = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$
$\therefore \angle x = \angle ACB = 50^\circ$

33) 60°
⇨ $\angle x = \angle CAT = 60^\circ$

34) 100°
⇨ $\angle BCA = \angle BAT = 50^\circ$
$\therefore \angle x = 2\angle BCA = 2 \times 50^\circ = 100^\circ$

35) $\angle x = 70^\circ$
⇨ $\angle CAT = \angle ABC = 110^\circ$ 이므로
$\angle x = 110^\circ - 40^\circ = 70^\circ$

36) 20°
⇨ 

보조건 AT를 그으면 $\overline{AB}$가 원 O의 지름이므로
$\angle ATB = 90^\circ$
$\angle ABT = \angle ATP = 180^\circ - (55^\circ + 90^\circ) = 35^\circ$
$\triangle PBT$에서 $\angle x = \angle BTC - \angle PBT = 55^\circ - 35^\circ = 20^\circ$

37) 80°

38) 25°

39) 76°
⇨ $\angle DCA = 38^\circ$ 이고 $\overline{AB} = \overline{AD}$ 이므로
$\angle DCA = \angle ACB = 38^\circ$ $\therefore \angle x = 76^\circ$

40) 15°
⇨ $\square ABCD$에서 $\angle BAD + \angle C = 180^\circ$ 이므로
$\angle BAD = 180^\circ - 85^\circ = 95^\circ$
$\triangle ABD$에서 $\angle ADB = 180^\circ - (70^\circ + 95^\circ) = 15^\circ$
$\therefore \angle x = \angle ADB = 15^\circ$

41) 36°

42) 70°
⇨ $\triangle DAC$에서 $\angle ADC = 180^\circ - (35^\circ + 45^\circ) = 100^\circ$
$\square ABCD$는 원에 내접하므로
$\angle ABC = 180^\circ - \angle ADC = 180^\circ - 100^\circ = 80^\circ$
직선 TB는 원의 접선이므로
$\angle ACB = \angle ABT = 30^\circ$
$\triangle ABC$에서 $\angle x = 180^\circ - (80^\circ + 30^\circ) = 70^\circ$

43) 20°

44) 55°
⇨ $\square ABCD$는 원에 내접하므로
$\angle ABC = 180^\circ - 75^\circ = 105^\circ$
$\triangle ABC$에서 $\angle BAC = 180^\circ - (20^\circ + 105^\circ) = 55^\circ$
$\therefore \angle x = \angle BAC = 55^\circ$

45) $\angle x = 80^\circ$, $\angle y = 25^\circ$

46) $\angle x = 46^\circ$, $\angle y = 22^\circ$

47) $\angle x = 20^\circ$, $\angle y = 50^\circ$

48) $\angle x = 90^\circ$, $\angle y = 30^\circ$

49) $\angle x = 65^\circ$, $\angle y = 45^\circ$

50) $\angle x = \angle ABC = 33^\circ$, $\angle y = \angle BAT = 102^\circ$

51) $\angle x = 35^\circ$, $\angle y = 60^\circ$
⇨ $\angle BAT = \angle BDA = 35^\circ$ 이므로 $\angle x = 35^\circ$
이때 $\angle DBA = 25^\circ$ 이고, 사각형 ABCD는 원 O에 내접하므로
$\angle y = 180^\circ - \angle DAB = 180^\circ - 120^\circ = 60^\circ$

52) $\angle x = 38^\circ$, $\angle y = 73^\circ$

53) 45°
⇨ $\angle x = \angle CTQ = \angle ATP = \angle ABT = 45^\circ$

54) 50°
⇨ $\angle x = \angle BTQ = \angle DTP = \angle DCT = 50^\circ$

55) 60°
⇨ $\angle x = \angle ATP = \angle CTQ = \angle CDT = 60^\circ$

56) 45°

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⇨ $\angle x = \angle DTP = \angle BTQ = \angle BAT = 45^\circ$

57) 75°
⇨ $\angle x = \angle CTQ = \angle ATP = \angle ABT = 75^\circ$

58) 75°
⇨ $\angle x = \angle ABT = 75^\circ$

59) 70°
⇨ $\angle x = \angle DCT = 70^\circ$

60) 55°
⇨ $\angle x = \angle BAT = 55^\circ$

61) 65°
⇨ $\angle x = \angle CDT = 65^\circ$

62) 80°
⇨ $\angle x = \angle ABT = 80^\circ$

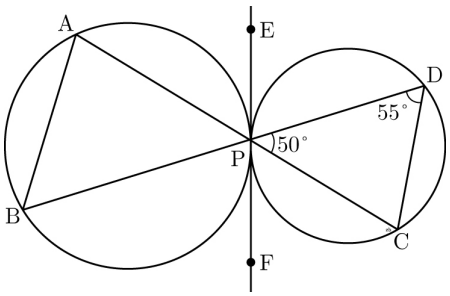
63) $\angle x = 50^\circ, \angle y = 70^\circ$
⇨ $\angle y = \angle DTP = \angle BTQ = \angle BAT = 70^\circ$
$\triangle CDT$에서 $\angle x = 180^\circ - (60^\circ + 70^\circ) = 50^\circ$

64) $\angle x = 75^\circ$, $\angle y = 50^\circ$
⇨ $\angle x = \angle DTP = \angle CTQ = \angle CAT = 75^\circ$
$\triangle BDT$에서 $\angle y = 180^\circ - (75^\circ + 55^\circ) = 50^\circ$

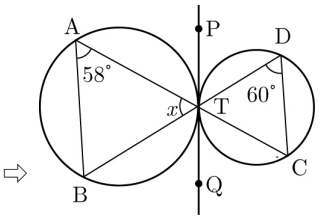
65) $\angle x = 70^\circ, \angle y = 65^\circ$
⇨ $\angle x = \angle ATP = \angle CTQ = \angle CDT = 70^\circ$
$\angle DCT = \angle DTP = \angle BTQ = \angle BAT = 45^\circ$ 이므로
$\triangle CDT$에서 $\angle y = 180^\circ - (70^\circ + 45^\circ) = 65^\circ$

66) $\angle x = 65^\circ, \angle y = 55^\circ$
⇨ $\angle x = \angle BTQ = 65^\circ$, $\angle DTP = \angle DCT = 60^\circ$ 이므로
$\angle y = 180^\circ - (65^\circ + 60^\circ) = 55^\circ$

67) $\angle x = 50^\circ, \angle y = 85^\circ$
⇨ $\angle x = \angle ATP = 50^\circ$
$\triangle CDT$에서 $\angle DCT = \angle DTP = 50^\circ$ 이므로
$\angle y = 180^\circ - (45^\circ + 50^\circ) = 85^\circ$

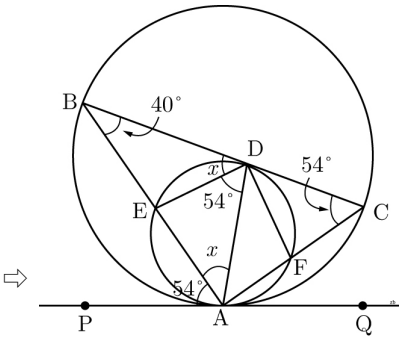
68) 75°
⇨ 

두 원의 접선을 $\overleftrightarrow{EF}$라 하면
$\angle PAB = \angle BPF = \angle DPE = \angle DCP$
$\triangle DCP$에서 $\angle DCP = 180^\circ - (50^\circ + 55^\circ) = 75^\circ$
$\therefore \angle PAB = 75^\circ$

69) 62°
⇨ 

두 원의 공통접선을 $\overleftrightarrow{PQ}$라 하면
$\angle ABT = \angle ATP = \angle CTQ = \angle CDT = 60^\circ$
따라서 $\triangle ABT$에서 $\angle x = 180^\circ - (58^\circ + 60^\circ) = 62^\circ$

70) 50°
⇨ $\angle TPA = x$이므로 $\angle ABP = x$이다. 사각형 ABCD가 원에 내접하므로 맞은편의 각끼리의 합은 180°이다.
따라서 $\angle ABC + \angle ADC = 180^\circ$이고
$\angle ABC = 180^\circ - x$, $\angle ADC = 50^\circ$이므로 $x = 50^\circ$이다.

71) 43°
⇨ 

큰 원과 접선 $\overline{PQ}$에서 $\angle BAP = \angle BCA = 54^\circ$
작은 원과 접선 $\overline{PQ}$에서 $\angle EDA = \angle EAP = 54^\circ$
또 작은 원과 접선 $\overline{BC}$에서 $\angle BDE = \angle EAD = \angle x$
따라서 $\triangle BDA$에서
$40^\circ + (\angle x + 54^\circ) + \angle x = 180^\circ$ 이므로
$\therefore \angle x = 43^\circ$

72) 43°
⇨ $\overline{BY}$가 접선이면 $\angle EBY = \angle BAC = 60^\circ$
$\overline{BX}$가 접선이면 $\angle ABX = \angle ACB = 34^\circ$
$\overline{CD}$가 접선이면 $\angle x = \angle CDE$
$\triangle CDE$에서 $\angle DEB = \angle x + 34$
$\triangle BDE$에서 $60 + x + 34 + x = 180$, $2x = 86$, $x = 43$
따라서 $\angle x = 43^\circ$ 가 된다.

73) 45°

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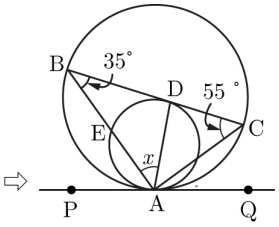

큰 원과 접선 PQ에서 $\angle BAP = \angle BCA = 55^\circ$
작은 원과 접선 PQ에서 $\angle EDA = \angle EAP = 55^\circ$
또 작은 원과 접선 BC에서 $\angle BDE = \angle EAD = \angle x$
따라서 $\triangle BDA$에서
$35^\circ + (\angle x + 55^\circ) + \angle x = 180^\circ$ 이므로
$\angle x = 45^\circ$

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