



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시

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3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

**01** / 로그의 밑의 변환 공식 $a > 0, a \neq 1, b > 0$ 일 때

(1)  $\log_a b = \frac{\log_c b}{\log_c a}$  (단,  $c > 0, c \neq 1$ )

(2)  $\log_a b = \frac{1}{\log_b a}$  (단,  $b \neq 1$ )

■ 다음 식을 간단히 하여라.

1.  $\log_2 9 \times \log_3 8$

2.  $\log_2 3 \times \log_3 2$

3.  $\log_2 \sqrt{5} \times \log_{25} 8$

4.  $\log_{25} 9 \times \log_{27} 5$

5.  $\log_5 3 \times \log_3 \sqrt{5}$

6.  $\log_{16} 25 \times \log_9 8 \times \log_5 81$

7.  $\log_3 5 \times \log_5 7 \times \log_7 9$

8.  $\log_5 3 \times \log_2 5 \times \log_3 2$

9.  $\log_{25} 8 \times \log_8 81 \times \log_9 125$

10.  $\log_3 49 \times \log_5 27 \times \log_7 \sqrt{5}$

11.  $\log_4 27 \times \log_9 25 \times \log_5 16$

12.  $\log_9 25 + \log_{27} 625$

13.  $\log_{12} 16 + \frac{1}{\log_9 12}$

14.  $\log_3 5 \times \log_5 7 \times \log_7 9 \times \log_9 3$

15.  $\log_8 81 \times \log_9 125 \times \log_5 64$

$$16. \log_{25} 3 \times \log_{16} 5 \times \log_9 32$$

$$17. (\log_6 3)^2 + \frac{\log_6 18}{\log_2 6}$$

$$18. \log_2 20 - \frac{1}{\log_5 2}$$

$$19. \log_3 63 - \frac{1}{\log_7 3}$$

$$20. \log_6 24 + \frac{1}{\log_9 6}$$

$$21. \log_3 \sqrt{10} - \frac{1}{2} \log_3 \frac{1}{6} - \frac{3}{2} \log_3 \sqrt[3]{20}$$

$$22. \log_6 27 \times \log_9 6 + \log_2 4 + \log_{\frac{1}{3}} 27$$

$$23. \log_8 27 \times \log_3 25 \times \log_5 32$$

$$24. \log_2 \frac{1}{3} + 2 \log_2 \sqrt{3} - \log_2 3 \cdot \log_3 4$$

$$25. \log_2 (\log_2 3) + \log_2 (\log_3 4)$$

$$26. (\log_2 5 + \log_4 125)(\log_5 4 - \log_{25} 8)$$

$$27. (\log_2 3 + \log_8 27)(\log_3 4 + \log_{\sqrt{3}} 2)$$

$$28. (\log_3 5 - \log_9 125)(\log_{25} 27 + \log_5 9)$$

$$29. (\log_2 15 - \log_2 3)(\log_5 24 - \log_5 3)$$

$$30. (\log_2 3 + \log_4 9)(\log_3 4 - \log_9 8)$$

$$31. \log_2 3 \times \log_3 4 \times \log_4 5 \times \cdots \times \log_{31} 32$$

$$32. \log_6 (\log_3 2) + \log_6 (\log_4 3) + \log_6 (\log_5 4) + \cdots + \log_6 (\log_{64} 63)$$

**02** / 로그의 정의를 이용하여 식을 문자로 나타내기

$x = \log_{10} a \Leftrightarrow 10^x = a$ 임을 이용하여 주어진 식을 문자로 나타낼 수 있다.

■  $10^x = a, 10^y = b$ 일 때, 다음을  $x, y$ 로 나타내어라.

33.  $\log_b \sqrt{a}$

34.  $\log_a b^2$

35.  $\log_{10} \frac{b^2}{a}$

36.  $\log_{10} ab$

■  $10^a = 2, 10^b = 3$ 일 때, 다음을  $a, b$ 로 나타내어라.

37.  $\log_{12} \sqrt{6}$

38.  $\log_4 81$

39.  $\log 54$

■  $10^x = a, 10^y = b, 10^z = c$ 일 때, 다음을  $x, y, z$ 로 나타내어라.

40.  $\log_{a^2bc} \sqrt[3]{abc^2}$

41.  $\log_{b^2c} \sqrt{a^3b^5}$

42.  $\log_{ab} \sqrt[3]{bc^2}$

■  $\log_{10} 2 = a, \log_{10} 3 = b$ 일 때, 다음을  $a, b$ 로 나타내어라.

43.  $\log_2 3$

44.  $\log_5 24$

45.  $\log_2 \sqrt{27}$

46.  $\log_6 18$

47.  $\log_6 12$

48.  $\log_3 12$

49.  $\log_6 5$

50.  $\log_3 \sqrt{18}$

51.  $\log_4 \sqrt{54}$

52.  $\log_{12} 72$

■ 다음 물음에 답하여라.

53.  $\log_2 3 = a, \log_3 5 = b$ 일 때,  $\log_{10} 5$ 를  $a, b$ 로 나타내어라.

54.  $7^a = 2, 7^b = 3$ 일 때,  $\log_6 24$ 을  $a, b$ 로 나타내어라.

55.  $8^x = 3, 9^y = 2$ 일 때,  $xy$ 의 값을 구하여라.

56.  $\log_a x = 3, \log_a y = 5$ 일 때,  $\log_{xy} a$ 의 값을 구하여라.

57.  $7^a = 2, 7^b = 3$ 일 때,  $\log_7 \frac{16}{3}$ 을  $a, b$ 로 나타내어라.

58.  $\log_{10} 2 = a, \log_{10} 3 = b$ 일 때,  $\log_{12} \sqrt{24}$ 를  $a, b$ 로 나타내어라.

59.  $3^a = 2, 3^b = 5$ 일 때,  $\log_{18} 15$ 를  $a, b$ 로 나타내어라.

60.  $\log_2 3 = a, \log_2 5 = b$ 일 때,  $\log_{45} 100$ 을  $a, b$ 로 나타내면?

61.  $\log_2 3 = a, \log_5 2 = b$ 일 때,  $\log_{12} 270$ 을  $a, b$ 로 나타내어라.

62.  $\log_{10} 15 = a, \log_{10} 45 = b$ 일 때,  $\log_{10} \frac{5}{3}$ 를  $a, b$ 로 나타내어라.

63.  $\log_{10} 36 = a, \log_{10} 24 = b$ 일 때,  $\log_{10} 45$ 를  $a, b$ 로 나타내어라.

64.  $2^a = 10, 10^b = 3$ 일 때,  $\log_6 45$ 를  $a, b$ 로 나타내어라.

65.  $2^a = 10$ ,  $3^b = 10$ 일 때,  $\log_3 5$ 을  $a$ ,  $b$ 로 나타내어라.

66.  $3^a = 2$ ,  $3^b = 5$ 일 때,  $\log_{12} 50$ 을  $a$ ,  $b$ 로 나타내어라.

67.  $2^a = 3$ ,  $3^b = 5$ 일 때,  $\log_{18} 30$ 을  $a$ ,  $b$ 로 나타내어라.

68.  $10^x = a$ ,  $10^y = b$ ,  $10^z = c$ 일 때,  $\log_{10} \frac{b^5 c}{a^3}$ 를  $x$ ,  $y$ ,  $z$ 로 나타내어라.



## 정답 및 해설

1) 6

$$\begin{aligned}\Rightarrow \log_2 9 \cdot \log_3 8 &= \frac{\log_{10} 9}{\log_{10} 2} \cdot \frac{\log_{10} 8}{\log_{10} 3} \\ &= \frac{\log_{10} 3^2}{\log_{10} 2} \cdot \frac{\log_{10} 2^3}{\log_{10} 3} \\ &= \frac{2 \log_{10} 3}{\log_{10} 2} \cdot \frac{3 \log_{10} 2}{\log_{10} 3} = 6\end{aligned}$$

2) 1

$$\Rightarrow \log_2 3 \times \log_3 2 = \log_2 3 \times \frac{1}{\log_2 3} = 1$$

3)  $\frac{3}{4}$ 

$$\begin{aligned}\Rightarrow \log_2 \sqrt{5} \cdot \log_{25} 8 &= \frac{\log_{10} \sqrt{5}}{\log_{10} 2} \cdot \frac{\log_{10} 8}{\log_{10} 25} \\ &= \frac{\log_{10} 5^{\frac{1}{2}}}{\log_{10} 2} \cdot \frac{\log_{10} 2^3}{\log_{10} 5^2} \\ &= \frac{\frac{1}{2} \log_{10} 5}{\log_{10} 2} \cdot \frac{3 \log_{10} 2}{2 \log_{10} 5} = \frac{3}{4}\end{aligned}$$

4)  $\frac{1}{3}$ 

$$\begin{aligned}\Rightarrow \log_{25} 9 \times \log_{27} 5 &= \frac{\log_{10} 9}{\log_{10} 25} \times \frac{\log_{10} 5}{\log_{10} 27} \\ &= \frac{\log_{10} 3^2}{\log_{10} 5^2} \times \frac{\log_{10} 5}{\log_{10} 3^3} \\ &= \frac{2 \log_{10} 3}{2 \log_{10} 5} \times \frac{\log_{10} 5}{3 \log_{10} 3} \\ &= \frac{1}{3}\end{aligned}$$

5)  $\frac{1}{2}$ 

$$\begin{aligned}\Rightarrow \log_5 3 \times \log_3 \sqrt{5} &= \log_5 3 \times \frac{1}{2} \log_3 5 \\ &= \frac{1}{2} \log_5 3 \times \frac{1}{\log_5 3} = \frac{1}{2}\end{aligned}$$

6) 3

$$\begin{aligned}\Rightarrow \log_{16} 25 \cdot \log_9 8 \cdot \log_5 81 &= \frac{\log_{10} 25}{\log_{10} 16} \cdot \frac{\log_{10} 8}{\log_{10} 9} \cdot \frac{\log_{10} 81}{\log_{10} 5} \\ &= \frac{\log_{10} 5^2}{\log_{10} 2^4} \cdot \frac{\log_{10} 2^3}{\log_{10} 3^2} \cdot \frac{\log_{10} 3^4}{\log_{10} 5} \\ &= \frac{2 \log_{10} 5}{4 \log_{10} 2} \cdot \frac{3 \log_{10} 2}{2 \log_{10} 3} \cdot \frac{4 \log_{10} 3}{\log_{10} 5} = 3\end{aligned}$$

7) 2

$$\begin{aligned}\Rightarrow \log_3 5 \times \log_5 7 \times \log_7 9 &= \frac{\log_{10} 5}{\log_{10} 3} \times \frac{\log_{10} 7}{\log_{10} 5} \times \frac{\log_{10} 9}{\log_{10} 7} \\ &= \frac{\log_{10} 9}{\log_{10} 3} = \frac{2 \log_{10} 3}{\log_{10} 3} = 2\end{aligned}$$

8) 1

$$\begin{aligned}\Rightarrow \log_5 3 \cdot \log_2 5 \cdot \log_3 2 &= \frac{\log_{10} 3}{\log_{10} 5} \cdot \frac{\log_{10} 5}{\log_{10} 2} \cdot \frac{\log_{10} 2}{\log_{10} 3} = 1\end{aligned}$$

9) 3

$$\begin{aligned}\Rightarrow \log_{25} 8 \times \log_8 81 \times \log_9 125 &= \log_{25} 81 \cdot \log_9 125 \\ &= 2 \log_{25} 9 \cdot \log_9 125 \\ &= 2 \log_{25} 125 \\ &= 2 \log_{5^2} 5^3 \\ &= 2 \times \frac{3}{2} \\ &= 3\end{aligned}$$

10) 3

$$\begin{aligned}\Rightarrow \log_3 49 \cdot \log_5 27 \cdot \log_7 \sqrt{5} &= \frac{\log_{10} 49}{\log_{10} 3} \cdot \frac{\log_{10} 27}{\log_{10} 5} \cdot \frac{\log_{10} \sqrt{5}}{\log_{10} 7} \\ &= \frac{\log_{10} 7^2}{\log_{10} 3} \cdot \frac{\log_{10} 3^3}{\log_{10} 5} \cdot \frac{\log_{10} 5^{\frac{1}{2}}}{\log_{10} 7} \\ &= \frac{2 \log_{10} 7}{\log_{10} 3} \cdot \frac{3 \log_{10} 3}{\log_{10} 5} \cdot \frac{\frac{1}{2} \log_{10} 5}{\log_{10} 7} = 3\end{aligned}$$

11) 6

12)  $\frac{7}{3} \log_3 5$ 

$$\begin{aligned}\Rightarrow \log_9 25 + \log_{27} 625 &= \log_{3^2} 5^2 + \log_{3^3} 5^4 \\ &= \frac{2}{2} \log_3 5 + \frac{4}{3} \log_3 5 \\ &= \frac{7}{3} \log_3 5\end{aligned}$$

13) 2

$$\begin{aligned}\Rightarrow \log_{12} 16 + \frac{1}{\log_9 12} &= \log_{12} 16 + \log_{12} 9 = \log_{12} (16 \times 9) \\ &= \log_{12} 144 = \log_{12} 12^2 = 2\end{aligned}$$

14) 1

15) 12

$$\Rightarrow \log_8 81 \times \log_9 125 \times \log_5 64$$

$$= \frac{4}{3} \log_2 3 \times \frac{3}{2} \log_3 5 \times 6 \log_5 2 = 12$$

$$16) \frac{5}{16}$$

$$\begin{aligned} \Rightarrow \log_{25} 3 \cdot \log_{16} 5 \cdot \log_9 32 \\ &= \frac{\log_{10} 3}{\log_{10} 25} \cdot \frac{\log_{10} 5}{\log_{10} 16} \cdot \frac{\log_{10} 32}{\log_{10} 9} \\ &= \frac{\log_{10} 3}{\log_{10} 5^2} \cdot \frac{\log_{10} 5}{\log_{10} 2^4} \cdot \frac{\log_{10} 2^5}{\log_{10} 3^2} \\ &= \frac{\log_{10} 3}{2 \log_{10} 5} \cdot \frac{\log_{10} 5}{4 \log_{10} 2} \cdot \frac{5 \log_{10} 2}{2 \log_{10} 3} = \frac{5}{16} \end{aligned}$$

$$17) 1$$

$$18) 2$$

$$\begin{aligned} \Rightarrow \log_2 20 - \frac{1}{\log_5 2} &= \log_2 20 - \log_2 5 \\ &= \log_2 \frac{20}{5} = \log_2 4 = 2 \end{aligned}$$

$$19) 2$$

$$\begin{aligned} \Rightarrow \log_3 63 - \frac{1}{\log_7 3} &= \log_3 63 - \log_3 7 \\ &= \log_3 \frac{63}{7} = \log_3 3^2 = 2 \end{aligned}$$

$$20) 3$$

$$\begin{aligned} \Rightarrow \log_6 24 + \frac{1}{\log_9 6} &= \log_6 24 + \log_6 9 \\ &= \log_6 (24 \times 9) = \log_6 6^3 = 3 \end{aligned}$$

$$21) \frac{1}{2}$$

$$\begin{aligned} \Rightarrow \log_3 \sqrt{10} - \frac{1}{2} \log_3 \frac{1}{6} - \frac{3}{2} \log_3 \sqrt[3]{20} \\ &= \log_3 \sqrt{10} - \log_3 \frac{1}{\sqrt{6}} - \log_3 \sqrt{20} \\ &= \log_3 \frac{\sqrt{10} \times \sqrt{6}}{\sqrt{20}} = \log_3 \sqrt{3} = \frac{1}{2} \end{aligned}$$

$$22) \frac{1}{2}$$

$$\begin{aligned} \Rightarrow \log_6 27 \times \log_9 6 + \log_2 4 + \log_{\frac{1}{3}} 27 \\ &= 3 \log_6 3 \times \frac{1}{2} \log_3 6 + 2 - 3 = \frac{3}{2} - 1 = \frac{1}{2} \end{aligned}$$

$$23) 10$$

$$\begin{aligned} \Rightarrow \log_8 27 \times \log_3 25 \times \log_5 32 &= \log_2 3 \times 2 \log_3 5 \times 5 \log_5 2 \\ &= 10 \end{aligned}$$

$$24) -2$$

$$\Rightarrow \log_2 \frac{1}{3} + 2 \log_2 \sqrt{3} - \log_2 3 \cdot \log_3 4$$

$$= -\log_2 3 + \log_2 3 - \log_2 3 \cdot \frac{\log_2 4}{\log_2 3} = -2$$

$$25) 1$$

$$\begin{aligned} \Rightarrow \log_2 (\log_2 3) + \log_2 (\log_3 4) \\ &= \log_2 (\log_2 3 \times \log_3 4) \\ &= \log_2 (\log_2 3 \times 2 \log_3 2) \\ &= \log_2 \left( \log_2 3 \times \frac{2}{\log_2 3} \right) \\ &= \log_2 2 = 1 \end{aligned}$$

$$26) \frac{5}{4}$$

$$\begin{aligned} \Rightarrow (\log_2 5 + \log_4 125)(\log_5 4 - \log_{25} 8) \\ &= (\log_2 5 + \log_{2^2} 5^3)(\log_5 2^2 - \log_{5^2} 2^3) \\ &= \left( \log_2 5 + \frac{3}{2} \log_2 5 \right) \left( 2 \log_5 2 - \frac{3}{2} \log_5 2 \right) \\ &= \left( \frac{5}{2} \log_2 5 \right) \left( \frac{1}{2} \log_5 2 \right) \\ &= \frac{5}{2} \log_2 5 \cdot \frac{1}{2 \log_2 5} = \frac{5}{4} \end{aligned}$$

$$27) 8$$

$$\begin{aligned} \Rightarrow (\log_2 3 + \log_8 27)(\log_3 4 + \log_{\sqrt{3}} 2) \\ &= (\log_2 3 + \frac{3}{2} \log_2 3)(2 \log_3 2 + 2 \log_3 2) \\ &= 2 \log_2 3 \cdot 4 \log_3 2 = 8 \end{aligned}$$

$$28) -\frac{7}{4}$$

$$\begin{aligned} \Rightarrow (\log_3 5 - \log_9 125)(\log_{25} 27 + \log_5 9) \\ &= \left( \frac{\log_{10} 5}{\log_{10} 3} - \frac{\log_{10} 125}{\log_{10} 9} \right) \left( \frac{\log_{10} 27}{\log_{10} 25} + \frac{\log_{10} 9}{\log_{10} 5} \right) \\ &= \left( \frac{\log_{10} 5}{\log_{10} 3} - \frac{3 \log_{10} 5}{2 \log_{10} 3} \right) \left( \frac{3 \log_{10} 3}{2 \log_{10} 5} + \frac{2 \log_{10} 3}{\log_{10} 5} \right) \\ &= \frac{2 \log_{10} 5 - 3 \log_{10} 5}{2 \log_{10} 3} \cdot \frac{3 \log_{10} 3 + 4 \log_{10} 3}{2 \log_{10} 5} \\ &= \frac{-\log_{10} 5}{2 \log_{10} 3} \cdot \frac{7 \log_{10} 3}{2 \log_{10} 5} = -\frac{7}{4} \end{aligned}$$

$$29) 3$$

$$\begin{aligned} \Rightarrow (\log_2 15 - \log_2 3)(\log_5 24 - \log_5 3) \\ &= \log_2 \frac{15}{3} \cdot \log_5 \frac{24}{3} = \log_2 5 \cdot \log_5 8 \\ &= \frac{\log_{10} 5}{\log_{10} 2} \cdot \frac{\log_{10} 8}{\log_{10} 5} = \frac{\log_{10} 5}{\log_{10} 2} \cdot \frac{3 \log_{10} 2}{\log_{10} 5} = 3 \end{aligned}$$

$$30) 1$$

$$\begin{aligned} \Rightarrow (\log_2 3 + \log_4 9)(\log_3 4 - \log_9 8) \\ &= \left( \frac{\log_{10} 3}{\log_{10} 2} + \frac{\log_{10} 9}{\log_{10} 4} \right) \left( \frac{\log_{10} 4}{\log_{10} 3} - \frac{\log_{10} 8}{\log_{10} 9} \right) \end{aligned}$$

$$\begin{aligned}
 &= \left( \frac{\log_{10} 3}{\log_{10} 2} + \frac{2 \log_{10} 3}{2 \log_{10} 2} \right) \left( \frac{2 \log_{10} 2}{\log_{10} 3} - \frac{3 \log_{10} 2}{2 \log_{10} 3} \right) \\
 &= \frac{2 \log_{10} 3 + 2 \log_{10} 3}{2 \log_{10} 2} \cdot \frac{4 \log_{10} 2 - 3 \log_{10} 2}{2 \log_{10} 3} \\
 &= \frac{2 \log_{10} 3}{\log_{10} 2} \cdot \frac{\log_{10} 2}{2 \log_{10} 3} = 1
 \end{aligned}$$

31) 5

$$\begin{aligned}
 \Rightarrow \log_2 3 \times \log_3 4 \times \log_4 5 \times \cdots \times \log_{31} 32 \\
 = \log_2 3 \times \frac{\log_2 4}{\log_2 3} \times \frac{\log_2 5}{\log_2 4} \times \cdots \times \frac{\log_2 32}{\log_2 31} \\
 = \log_2 32 = \log_2 2^5 = 5
 \end{aligned}$$

32) -1

$$\begin{aligned}
 \Rightarrow (\text{준식}) &= \log_6 (\log_2 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \cdots \cdot \log_{64} 63) \\
 &= \log_6 \frac{\log_6 2}{\log_6 64} = \log_6 \frac{1}{6} = -1
 \end{aligned}$$

33)  $\frac{x}{2y}$ 

$$\begin{aligned}
 \Rightarrow 10^x = a, 10^y = b \text{에서 } x = \log_{10} a, y = \log_{10} b \\
 \log_b \sqrt{a} = \log_b a^{\frac{1}{2}} = \frac{1}{2} \log_b a = \frac{1}{2} \cdot \frac{\log_{10} a}{\log_{10} b} = \frac{x}{2y}
 \end{aligned}$$

34)  $\frac{2y}{x}$ 

$$\begin{aligned}
 \Rightarrow 10^x = a, 10^y = b \text{에서 } x = \log_{10} a, y = \log_{10} b \\
 \log_a b^2 = \frac{\log_{10} b^2}{\log_{10} a} = \frac{2 \log_{10} b}{\log_{10} a} = \frac{2y}{x}
 \end{aligned}$$

35)  $2y - x$ 

$$\begin{aligned}
 \Rightarrow 10^x = a, 10^y = b \text{에서 } x = \log_{10} a, y = \log_{10} b \\
 \log_{10} \frac{b^2}{a} = \log_{10} b^2 - \log_{10} a = 2 \log_{10} b - \log_{10} a = 2y - x
 \end{aligned}$$

36)  $x + y$ 

$$\begin{aligned}
 \Rightarrow 10^x = a, 10^y = b \text{에서 } x = \log_{10} a, y = \log_{10} b \\
 \log_{10} ab = \log_{10} a + \log_{10} b = x + y
 \end{aligned}$$

37)  $\frac{a+b}{4a+2b}$ 

$$\begin{aligned}
 \Rightarrow \log_{12} \sqrt{6} &= \frac{\log \sqrt{6}}{\log 12} = \frac{\log (2 \times 3)^{\frac{1}{2}}}{\log (2^2 \times 3)} \\
 &= \frac{\frac{1}{2} (\log 2 + \log 3)}{2 \log 2 + \log 3} = \frac{a+b}{4a+2b}
 \end{aligned}$$

38)  $\frac{2b}{a}$ 

$$\Rightarrow \log_4 81 = \frac{\log 81}{\log 4} = \frac{\log 3^4}{\log 2^2} = \frac{4 \log 3}{2 \log 2} = \frac{2b}{a}$$

39)  $a + 3b$ 

$$\begin{aligned}
 \Rightarrow \log 54 &= \log (2 \times 3^3) = \log 2 + \log 3^3 \\
 &= \log 2 + 3 \log 3 = a + 3b
 \end{aligned}$$

40)  $\frac{x+y+2z}{6x+3y+3z}$ 

$$\begin{aligned}
 \Rightarrow 10^x = a, 10^y = b, 10^z = c \text{에서} \\
 x = \log_{10} a, y = \log_{10} b, z = \log_{10} c \text{이므로}
 \end{aligned}$$

$$\begin{aligned}
 \log_{a^2bc} \sqrt[3]{abc^2} &= \frac{\log_{10} \sqrt[3]{abc^2}}{\log_{10} a^2bc} \\
 &= \frac{\frac{1}{3} (\log_{10} a + \log_{10} b + 2 \log_{10} c)}{2 \log_{10} a + \log_{10} b + \log_{10} c} \\
 &= \frac{x+y+2z}{3(2x+y+z)} = \frac{x+y+2z}{6x+3y+3z}
 \end{aligned}$$

41)  $\frac{3x+5y}{4y+2z}$ 

$$\begin{aligned}
 \Rightarrow 10^x = a, 10^y = b, 10^z = c \text{에서} \\
 x = \log_{10} a, y = \log_{10} b, z = \log_{10} c \text{이므로}
 \end{aligned}$$

$$\begin{aligned}
 \log_{b^2c} \sqrt{a^3b^5} &= \frac{\log_{10} \sqrt{a^3b^5}}{\log_{10} b^2c} \\
 &= \frac{\frac{1}{2} (3 \log_{10} a + 5 \log_{10} b)}{2 \log_{10} b + \log_{10} c} = \frac{3x+5y}{2(2y+z)} = \frac{3x+5y}{4y+2z}
 \end{aligned}$$

42)  $\frac{y+2z}{3x+3y}$ 

$$\begin{aligned}
 \Rightarrow 10^x = a, 10^y = b, 10^z = c \text{에서} \\
 x = \log_{10} a, y = \log_{10} b, z = \log_{10} c \text{이므로}
 \end{aligned}$$

$$\begin{aligned}
 \log_{ab} \sqrt[3]{bc^2} &= \frac{\log_{10} \sqrt[3]{bc^2}}{\log_{10} ab} \\
 &= \frac{\frac{1}{3} (\log_{10} b + 2 \log_{10} c)}{\log_{10} a + \log_{10} b} = \frac{y+2z}{3(x+y)} = \frac{y+2z}{3x+3y}
 \end{aligned}$$

43)  $\frac{b}{a}$ 

$$\Rightarrow \log_2 3 = \frac{\log_{10} 3}{\log_{10} 2} = \frac{b}{a}$$

44)  $\frac{3a+b}{1-a}$ 

$$\begin{aligned}
 \Rightarrow \log_5 24 &= \frac{\log_{10} 24}{\log_{10} 5} = \frac{\log_{10} (2^3 \times 3)}{\log_{10} \frac{10}{2}} \\
 &= \frac{3 \log_{10} 2 + \log_{10} 3}{\log_{10} 10 - \log_{10} 2} = \frac{3a+b}{1-a}
 \end{aligned}$$

45)  $\frac{3b}{2a}$ 

$$\Rightarrow \log_2 \sqrt{27} = \log_2 3^{\frac{3}{2}} = \frac{3}{2} \log_2 3 = \frac{3}{2} \times \frac{\log_{10} 3}{\log_{10} 2} = \frac{3b}{2a}$$



$$46) \frac{a+2b}{a+b}$$

$$\begin{aligned} \Rightarrow \log_6 18 &= \frac{\log_{10} 18}{\log_{10} 6} = \frac{\log_{10} (2 \times 3^2)}{\log_{10} (2 \times 3)} \\ &= \frac{\log_{10} 2 + 2 \log_{10} 3}{\log_{10} 2 + \log_{10} 3} = \frac{a+2b}{a+b} \end{aligned}$$

$$47) \frac{2a+b}{a+b}$$

$$\begin{aligned} \Rightarrow \log_6 12 &= \frac{\log_{10} 12}{\log_{10} 6} = \frac{\log_{10} (2^2 \times 3)}{\log_{10} (2 \times 3)} \\ &= \frac{\log_{10} 2^2 + \log_{10} 3}{\log_{10} 2 + \log_{10} 3} = \frac{2 \log_{10} 2 + \log_{10} 3}{\log_{10} 2 + \log_{10} 3} = \frac{2a+b}{a+b} \end{aligned}$$

$$48) \frac{2a+b}{b}$$

$$\begin{aligned} \Rightarrow \log_3 12 &= \frac{\log_{10} 12}{\log_{10} 3} = \frac{\log_{10} (2^2 \times 3)}{\log_{10} 3} \\ &= \frac{2 \log_{10} 2 + \log_{10} 3}{\log_{10} 3} = \frac{2a+b}{b} \end{aligned}$$

$$49) \frac{1-a}{a+b}$$

$$\begin{aligned} \Rightarrow \log_6 5 &= \frac{\log_{10} 5}{\log_{10} 6} = \frac{\log_{10} \frac{10}{2}}{\log_{10} (2 \times 3)} \\ &= \frac{\log_{10} 10 - \log_{10} 2}{\log_{10} 2 + \log_{10} 3} = \frac{1-a}{a+b} \end{aligned}$$

$$50) \frac{a+2b}{2b}$$

$$\begin{aligned} \Rightarrow \log_3 \sqrt{18} &= \frac{1}{2} \log_3 (2 \times 3^2) = \frac{1}{2} (\log_3 2 + 2 \log_3 3) \\ &= \frac{1}{2} \left( \frac{\log_{10} 2}{\log_{10} 3} + 2 \right) = \frac{1}{2} \left( \frac{a}{b} + 2 \right) = \frac{a+2b}{2b} \end{aligned}$$

$$51) \frac{a+3b}{4a}$$

$$\begin{aligned} \Rightarrow \log_4 \sqrt{54} &= \log_4 54^{\frac{1}{2}} = \frac{1}{2} \log_4 54 \\ &= \frac{1}{2} \cdot \frac{\log_{10} 54}{\log_{10} 4} = \frac{\log_{10} (2 \times 3^3)}{2 \log_{10} 2^2} \\ &= \frac{\log_{10} 2 + 3 \log_{10} 3}{4 \log_{10} 2} = \frac{a+3b}{4a} \end{aligned}$$

$$52) \frac{3a+2b}{2a+b}$$

$$\begin{aligned} \Rightarrow \log_{12} 72 &= \frac{\log_{10} 72}{\log_{10} 12} = \frac{\log_{10} (2^3 \times 3^2)}{\log_{10} (2^2 \times 3)} \\ &= \frac{3 \log_{10} 2 + 2 \log_{10} 3}{2 \log_{10} 2 + \log_{10} 3} = \frac{3a+2b}{2a+b} \end{aligned}$$

$$53) \frac{ab}{1+ab}$$

$$\begin{aligned} \Rightarrow \log_2 3 &= a \quad \text{이므로} \quad \log_3 2 = \frac{1}{a} \\ \log_3 5 &= b \\ \therefore \log_{10} 5 &= \frac{\log_3 5}{\log_3 10} = \frac{\log_3 5}{\log_3 2 + \log_3 5} \\ &= \frac{\frac{b}{\frac{1}{a} + b}}{\frac{1}{a} + b} = \frac{b}{\frac{1}{a} + b} = \frac{ab}{1+ab} \end{aligned}$$

$$54) \frac{3a+b}{a+b}$$

$$\begin{aligned} \Rightarrow 7^a &= 2, \quad 7^b = 3 \text{에서} \quad a = \log_7 2, \quad b = \log_7 3 \\ \log_6 24 &= \frac{\log_7 24}{\log_7 6} = \frac{\log_7 (2^3 \times 3)}{\log_7 (2 \times 3)} \\ &= \frac{3 \log_7 2 + \log_7 3}{\log_7 2 + \log_7 3} = \frac{3a+b}{a+b} \end{aligned}$$

$$55) \frac{1}{6}$$

$$\begin{aligned} \Rightarrow 8^x &= 3 \text{에서} \quad x = \log_8 3 \\ 9^y &= 2 \text{에서} \quad y = \log_9 2 \\ \therefore xy &= \log_8 3 \times \log_9 2 = \frac{\log_2 3}{\log_2 8} \times \frac{\log_2 2}{\log_2 9} \\ &= \frac{\log_2 3}{3} \times \frac{1}{2 \log_2 3} = \frac{1}{6} \end{aligned}$$

$$56) \frac{1}{8}$$

$$\Rightarrow \log_{xy} a = \frac{1}{\log_a xy} = \frac{1}{\log_a x + \log_a y} = \frac{1}{3+5} = \frac{1}{8}$$

$$57) 4a-b$$

$$\begin{aligned} \Rightarrow 7^a &= 2, \quad 7^b = 3 \text{에서} \quad a = \log_7 2, \quad b = \log_7 3 \\ \log_7 \frac{16}{3} &= \log_7 16 - \log_7 3 = \log_7 2^4 - \log_7 3 \\ &= 4 \log_7 2 - \log_7 3 = 4a - b \end{aligned}$$

$$58) \frac{3a+b}{4a+2b}$$

$$\begin{aligned} \Rightarrow \log_{12} \sqrt{24} &= \log_{12} 24^{\frac{1}{2}} = \frac{1}{2} \log_{12} 24 \\ &= \frac{1}{2} \cdot \frac{\log_{10} 24}{\log_{10} 12} = \frac{\log_{10} (2^3 \times 3)}{2 \log_{10} (2^2 \times 3)} \\ &= \frac{3 \log_{10} 2 + \log_{10} 3}{2(2 \log_{10} 2 + \log_{10} 3)} = \frac{3a+b}{4a+2b} \end{aligned}$$

$$59) \frac{b+1}{a+2}$$

$$\begin{aligned} \Rightarrow 3^a &= 2, \quad 3^b = 5 \text{에서} \quad a = \log_3 2, \quad b = \log_3 5 \\ \therefore \log_{18} 15 &= \frac{\log_3 15}{\log_3 18} = \frac{\log_3 (3 \times 5)}{\log_3 (2 \times 3^2)} \end{aligned}$$

$$= \frac{\log_3 3 + \log_3 5}{\log_3 2 + 2 \log_3 3} = \frac{b+1}{a+2}$$

$$60) \frac{2+2b}{2a+b}$$

$$61) \frac{3ab+b+1}{ab+2b}$$

$$62) 3a-2b$$

$$\Rightarrow \log_{10} 15 = \log_{10} (3 \times 5) = \log_{10} 3 + \log_{10} 5$$

$$\therefore a = \log_{10} 3 + \log_{10} 5 \quad \cdots \cdots \textcircled{7}$$

$$\log_{10} 45 = \log_{10} (3^2 \times 5) = 2 \log_{10} 3 + \log_{10} 5$$

$$\therefore b = 2 \log_{10} 3 + \log_{10} 5 \quad \cdots \cdots \textcircled{8}$$

⑦, ⑧을 연립하여 풀면

$$\log_{10} 3 = -a + b, \log_{10} 5 = 2a - b$$

$$\begin{aligned} \therefore \log_{10} \frac{5}{3} &= \log_{10} 5 - \log_{10} 3 = (2a - b) - (-a + b) \\ &= 3a - 2b \end{aligned}$$

$$63) \frac{7}{4}a - \frac{3}{2}b + 1$$

$$\Rightarrow \log_{10} 36 = \log_{10} (2^2 \times 3^2) = 2 \log_{10} 2 + 2 \log_{10} 3$$

$$\therefore a = 2 \log_{10} 2 + 2 \log_{10} 3 \quad \cdots \cdots \textcircled{9}$$

$$\log_{10} 24 = \log_{10} (2^3 \times 3) = 3 \log_{10} 2 + \log_{10} 3$$

$$\therefore b = 3 \log_{10} 2 + \log_{10} 3 \quad \cdots \cdots \textcircled{10}$$

⑨, ⑩을 연립하여 풀면

$$\log_{10} 2 = -\frac{1}{4}a + \frac{1}{2}b, \log_{10} 3 = \frac{3}{4}a - \frac{1}{2}b$$

$$\begin{aligned} \therefore \log_{10} 45 &= \log_{10} (5 \times 3^2) \\ &= \log_{10} 5 + 2 \log_{10} 3 \\ &= 1 - \log_{10} 2 + 2 \log_{10} 3 \\ &= 1 - \left( -\frac{1}{4}a + \frac{1}{2}b \right) + 2 \left( \frac{3}{4}a - \frac{1}{2}b \right) \\ &= \frac{7}{4}a - \frac{3}{2}b + 1 \end{aligned}$$

$$64) \frac{2ab+a-1}{ab+1}$$

$$\Rightarrow \log_2 10 = a \text{ 이므로 } \log_{10} 2 = \frac{1}{a}$$

$$\log_{10} 3 = b$$

$$\begin{aligned} \therefore \log_6 45 &= \frac{\log_{10} 45}{\log_{10} 6} = \frac{2 \log_{10} 3 + \log_{10} 5}{\log_{10} 2 + \log_{10} 3} \\ &= \frac{2 \log_{10} 3 + \log_{10} 10 - \log_{10} 2}{\log_{10} 2 + \log_{10} 3} \\ &= \frac{2b + 1 - \frac{1}{a}}{\frac{1}{a} + b} = \frac{\frac{2ab + a - 1}{a}}{\frac{1 + ab}{a}} = \frac{2ab + a - 1}{1 + ab} \end{aligned}$$

$$65) \frac{ab-b}{a}$$

$$\Rightarrow 2^a = 10 \Rightarrow a = \log_2 10 \text{ 이므로 } \frac{1}{a} = \log 2$$

$$3^b = 10 \Rightarrow b = \log_3 10 \text{ 이므로 } \frac{1}{b} = \log 3$$

$$\therefore \log_3 5 = \frac{\log 5}{\log 3} = \frac{\log \frac{10}{2}}{\log 3} = \frac{\log 10 - \log 2}{\log 3} = \frac{1 - \frac{1}{a}}{\frac{1}{b}}$$

$$\begin{aligned} &\frac{a-1}{\frac{a}{b}} = \frac{ab-b}{a} \end{aligned}$$

$$66) \frac{a+2b}{2a+1}$$

$$\Rightarrow 3^a = 2 \text{ 이므로 } \log_3 2 = a \text{ 이다.}$$

$$3^b = 5 \text{ 이므로 } \log_3 5 = b \text{ 이다.}$$

$$\begin{aligned} \therefore \log_{12} 50 &= \frac{\log_3 50}{\log_3 12} = \frac{\log_3 (5^2 \cdot 2)}{\log_3 (2^2 \cdot 3)} \\ &= \frac{2 \log_3 5 + \log_3 2}{2 \log_3 2 + \log_3 3} = \frac{2b + a}{2a + 1} \end{aligned}$$

$$67) \frac{1+a+ab}{1+2a}$$

$$68) -3x+5y+z$$

$$\Rightarrow 10^x = a, 10^y = b, 10^z = c \text{ 에서}$$

$$x = \log_{10} a, y = \log_{10} b, z = \log_{10} c$$

$$\begin{aligned} \log_{10} \frac{b^5 c}{a^3} &= 5 \log_{10} b + \log_{10} c - 3 \log_{10} a \\ &= -3x + 5y + z \end{aligned}$$