



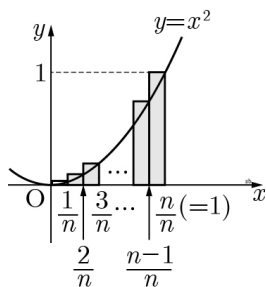
◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시  
1) 제작연월일 : 2019-08-13  
2) 제작자 : 교육지대(주)  
3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

## 01 / 구분구적법

- (1) 주어진 도형을  $n$ 개의 기본 도형으로 세분한다.
- (2)  $n$ 개의 기본 도형의 넓이의 합  $S_n$  또는 부피의 합  $V_n$ 을 구한다.
- (3)  $\lim_{n \rightarrow \infty} S_n$  또는  $\lim_{n \rightarrow \infty} V_n$ 을 구한다.

1. 다음은 곡선  $y = x^2$ 과  $x$ 축 및 직선  $x=1$ 로 둘러싸인 도형의 넓이  $S$ 를 구분구적법을 이용하여 구하는 과정이다.



닫힌구간  $[0, 1]$ 을  $n$ 등분 한 각 구간의 오른쪽 끝 점의  $x$ 좌표는 차례로

$\frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, \frac{n}{n}(=1)$ 이고, 이에 대응하는  $y$ 의 값은 각각

$$\left(\frac{1}{n}\right)^2, \left(\frac{2}{n}\right)^2, \dots, \left(\frac{n-1}{n}\right)^2, \left(\frac{n}{n}\right)^2$$

이므로 색칠한 직사각형의 넓이의 합을  $S_n$ 이라 하면

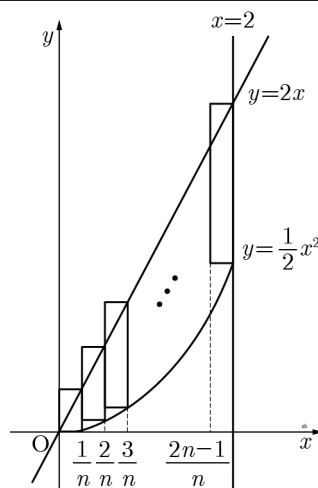
$$S_n = \boxed{(가)} \times \left(\frac{1}{n}\right)^2 + \boxed{(나)} \times \left(\frac{2}{n}\right)^2 + \dots + \boxed{(다)} \times \left(\frac{n}{n}\right)^2$$

$$= \sum_{k=1}^n \boxed{(라)}$$

$$\therefore S = \lim_{n \rightarrow \infty} S_n = \boxed{(마)}$$

(가), (나), (다), (라), (마)에 알맞은 식

2. 다음은 곡선  $y = \frac{1}{2}x^2$  ( $0 \leq x \leq 2$ )과 두 직선  $y=2x$ ,  $x=2$ 로 둘러싸인 도형의 넓이를 구하는 과정이다.



두 함수  $f(x)$ 와  $g(x)$ 를  $f(x) = \frac{1}{2}x^2$ ,  $g(x) = 2x$ 라 하자.

그림과 같이 닫힌 구간  $[0, 2]$ 를  $2n$ 등분하여 구간  $\left[0, \frac{1}{n}\right], \left[\frac{1}{n}, \frac{2}{n}\right], \left[\frac{2}{n}, \frac{3}{n}\right], \dots, \left[\frac{2n-1}{n}, 2\right]$ 를 얻는다.

각 구간에서 가로의 길이가  $\frac{1}{n}$ 이고 구간의 오른쪽 끝점에서의 두 함수값의 차를 세로의 길이로 하는 직사각형을 만든다. 왼쪽에서  $k$ 번째 직사각형의 넓이를  $S_k$ 라 하면

$$S_k = \boxed{(가)}$$

직사각형  $n$ 개의 넓이의 합은

$$\sum_{k=1}^{2n} S_k = \sum_{k=1}^{2n} \boxed{(가)}$$

구하는 도형의 넓이는

$$\lim_{n \rightarrow \infty} \sum_{k=1}^{2n} S_k = \boxed{(나)}$$

$$= \int_0^2 \boxed{(다)} dx = \int_0^1 \boxed{(라)} dx$$

(가), (나), (다), (라)에 알맞은 식을 써라.

3. 다음은 정적분의 정의를 이용하여  $\int_0^2 x^2 dx$ 의 값을 구하는 과정이다.

$f(x)=x^2$ 이라고 하면 함수  $f(x)$ 는 닫힌구간  $[0, 2]$ 에서 연속이다. 정적분의 정의에서  $a=0$ ,  $b=2$ 라고 하면

$$\Delta x = \frac{b-a}{n} = \frac{2}{n}, \quad x_k = a + k\Delta x = \frac{2k}{n}$$

$$f(x_k) = \frac{(가)}{n^2} \text{이므로}$$

$$\int_0^2 x^2 dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$$

$$= \lim_{n \rightarrow \infty} \frac{(나)}{n^3} \sum_{k=1}^n k^2$$

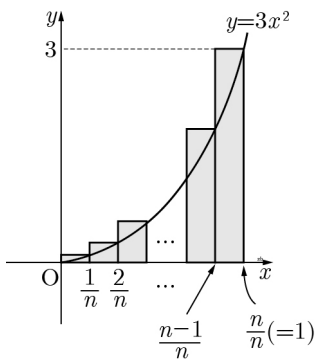
$$= \lim_{n \rightarrow \infty} \left\{ (다) \cdot \frac{n(n+1)(2n+1)}{6} \right\}$$

$$= \frac{4}{3} \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right) \left( 2 + \frac{1}{n} \right) = (라)$$

(가), (나), (다), (라)에 알맞은 식을 써라.

4. 다음은 곡선  $y=3x^2$ 과  $x$ 축 및 직선  $x=1$ 로 둘러싸인 도형의 넓이를 구분구적법을 이용하여 구하는 과정이다. 안에 알맞은 식을 구하여라.

<보기>



구간  $[0,1]$ 을  $n$ 등분하면 양 끝점과 각 분점의  $x$ 좌표는 앞에서부터 차례대로

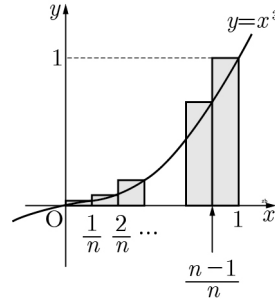
$$0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, 1$$

이므로 위의 그림의 직사각형의 넓이의 합을  $S_n$ 이라고 하면 구하는 넓이  $S$ 는

$$S = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \boxed{\phantom{000}} = 1$$

■ 다음 물음에 답하여라.

5. 곡선  $y=x^3$ 과  $x$ 축 및 직선  $x=1$ 로 둘러싸인 부분의 넓이를 구분구적법으로 구하여라.



## 02 정적분과 급수

(1) 함수  $f(x)$ 가 닫힌구간  $[a, b]$ 에서 연속일 때,

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x = \int_a^b f(x) dx$$

$$(\text{단, } \Delta x = \frac{b-a}{n}, x_k = a + k \Delta x)$$

(2) 정적분과 급수의 합의 관계

$$\textcircled{1} \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(a + \frac{(b-a)k}{n}\right) \cdot \frac{b-a}{n} = \int_a^b f(x) dx$$

$$\textcircled{2} \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(a + \frac{p}{n}k\right) \cdot \frac{p}{n} = \int_a^{a+p} f(x) dx = \int_0^p f(a+x) dx$$

$$\textcircled{3} \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{p}{n}k\right) \cdot \frac{p}{n} = \int_0^p f(x) dx$$

$$\textcircled{4} \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{k}{n}\right) \cdot \frac{1}{n} = \int_0^1 f(x) dx$$

■ 정적분을 이용하여 다음 극한값을 구하여라.

$$6. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{2k}{n} \right)^2 \times \frac{2}{n}$$

$$7. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( 4 + \frac{2k}{n} \right) \times \frac{2}{n}$$

$$8. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n^2 + 4k^2}$$

$$9. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^3 \frac{4}{n}$$

$$10. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^2(3n-k)}{n^4}$$

$$11. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left\{ \left( \frac{2k}{n} \right)^2 + 1 \right\} \frac{2}{n}$$

$$12. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( 2 + \frac{k}{n} \right)^3 \frac{4}{n}$$

$$13. \lim_{n \rightarrow \infty} \sum_{k=1}^n (-n+3k)^5 \cdot \frac{4}{n^6}$$

$$14. \lim_{n \rightarrow \infty} \sum_{k=1}^{4n} \left( 1 + \frac{k}{2n} \right)^4 \frac{2}{n}$$

$$15. \lim_{n \rightarrow \infty} \sum_{k=1}^n (n+2k)^3 \cdot \frac{3}{n^4}$$

$$16. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4k}{n^2} e^{\frac{k}{n}}$$

$$17. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k(k-2n)}{n^3} \left( \frac{n-k}{n} \right)$$

$$18. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( 1 + \frac{2k}{n} \right)^5 \cdot \frac{3}{n}$$

$$19. \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \sin^2 \frac{k\pi}{n}$$

$$20. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2}{n+k}$$

$$21. \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} \frac{k^2(5k^2+7)}{n^3(n^2+3)}$$

■ 다음 물음에 답하여라.

22.  $\int_1^5 x^4 dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{a}{n} \left(1 + \frac{ak}{n}\right)^4$  일 때, 상수  $a$ 의 값을 구하여라.

23.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{3k}{n}\right)^2 \frac{6}{n} = a \int_1^b x^2 dx$  일 때, 두 자연수  $a, b$ 의 합을 구하여라.

24.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^2 \frac{4}{n} = \int_1^b ax^2 dx$  일 때, 자연수  $a, b$ 에 대한 곱  $ab$ 의 값을 구하여라.

25.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{3k}{n}\right)^3 \times \frac{1}{n} = \frac{q}{p}$  일 때,  $p+q$ 의 값을 구하여라. (단,  $p, q$ 는 서로소인 자연수)

26.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(3 + \frac{8k}{n}\right)^2 \cdot \frac{4}{n} = a \int_0^b (3+2x)^2 dx$  일 때, 상수  $a, b$ 에 대하여  $a+b$ 의 값을 구하여라.

27. 등식  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left\{ \frac{n+2k}{n^2} \ln \left(1 + \frac{2k}{n}\right) \right\} = a \ln 3 + b$ 를 만족하는 두 유리수  $a, b$ 에 대하여  $a+b$ 의 값을 구하여라.

■ 다음 함수  $f(x)$ 에 대하여 알맞은 값을 구하여라.

28. 함수  $f(x) = x^4$ 에 대하여  $\lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right) \frac{10}{n}$ 의 값을 구하여라.

29. 함수  $f(x) = x^2 + 2x$ 에 대하여  $\lim_{n \rightarrow \infty} \frac{3}{n} \sum_{k=1}^n f\left(\frac{2k}{n} - 1\right)$ 의 값을 구하여라.

30. 함수  $f(x) = e^x + 2x$ 에 대하여  $\lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{k}{n}\right) \frac{1}{n} + \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} f\left(1 + \frac{k}{n}\right) \frac{1}{n}$ 의 값을 구하여라.

31. 함수  $f(x) = \frac{1}{x^2 + x}$ 에 대하여  $\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right)$ 의 값을 구하여라.

32. 함수  $f(x) = x^3 + x + 1$ 에 대하여  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right)$ 의 값을 구하여라.

■ 정적분을 이용하여 다음 극한값을 구하여라.

$$33. \lim_{n \rightarrow \infty} \frac{\pi}{n} \left( \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{n\pi}{n} \right)$$

$$34. \lim_{n \rightarrow \infty} \frac{3}{n} \left\{ \left( \frac{n+1}{n} \right)^2 + \left( \frac{n+2}{n} \right)^2 + \dots + \left( \frac{n+n}{n} \right)^2 \right\}$$

$$35. \lim_{n \rightarrow \infty} \frac{1}{n} \left( \sqrt{\frac{n}{1}} + \sqrt{\frac{n}{2}} + \sqrt{\frac{n}{3}} + \dots + \sqrt{\frac{n}{n}} \right)$$

$$36. \lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{n \sqrt{n}}$$

$$37. \lim_{n \rightarrow \infty} \left( \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right)$$

$$38. \lim_{n \rightarrow \infty} \left( \frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \frac{3^2}{n^3+3^3} + \dots + \frac{n^2}{n^3+n^3} \right)$$

$$39. \lim_{n \rightarrow \infty} \frac{2}{n} \left( e^{1+\frac{4}{n}} + e^{1+\frac{8}{n}} + \dots + e^{1+\frac{4n}{n}} \right)$$

$$40. \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left( \frac{n+1}{n} \times \frac{n+2}{n} \times \frac{n+3}{n} \times \dots \times \frac{2n}{n} \right)$$

$$41. \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n^2} (\sqrt{n+1} + \sqrt{n+2} + \sqrt{n+3} + \dots + \sqrt{n+n})$$

$$42. \lim_{n \rightarrow \infty} \frac{\sqrt{3} + \sqrt{6} + \dots + \sqrt{3n}}{n \sqrt{n}}$$

$$43. 2 \lim_{n \rightarrow \infty} \left( \frac{1}{n^2+1^2} + \frac{2}{n^2+2^2} + \dots + \frac{n}{2n^2} \right)$$

$$44. \lim_{n \rightarrow \infty} \frac{(n+1)^5 + (n+2)^5 + (n+3)^5 + \dots + (n+n)^5}{1^5 + 2^5 + 3^5 + \dots + n^5}$$

$$45. \lim_{n \rightarrow \infty} \frac{n(1^3 + 2^3 + 3^3 + \dots + n^3)}{(1+2+\dots+n)(1^2+2^2+\dots+n^2)}$$

$$46. \lim_{n \rightarrow \infty} \frac{1}{n} \left( e^{\frac{1}{n}-1} + e^{\frac{2}{n}-1} + \dots + e^{\frac{n-1}{n}-1} \right)$$

$$47. \lim_{n \rightarrow \infty} \frac{e^{\frac{1}{n}} + 2e^{\frac{2}{n}} + \dots + (n-1)e^{\frac{n-1}{n}} + ne}{n^2}$$

$$48. \lim_{n \rightarrow \infty} \left\{ \frac{2}{n} \cdot e^{\frac{2}{n}} + \frac{4}{n} \cdot e^{\frac{4}{n}} + \dots + \frac{2n}{n} \cdot e^{\frac{2n}{n}} \right\} \frac{2}{n}$$

$$49. \lim_{n \rightarrow \infty} \frac{1}{n^2} \left\{ \frac{1}{\sqrt[n]{e}} + \frac{2}{\sqrt[n]{e^2}} + \frac{3}{\sqrt[n]{e^3}} + \dots + \frac{n}{\sqrt[n]{e^n}} \right\}$$

$$50. \lim_{n \rightarrow \infty} \left\{ \frac{6 \cdot 2^2}{n^3 + 2^3} + \frac{6 \cdot 4^2}{n^3 + 4^3} + \frac{6 \cdot 6^2}{n^3 + 6^3} + \dots + \frac{6(2n)^2}{n^3 + (2n)^3} \right\}$$

$$51. \lim_{n \rightarrow \infty} \frac{1}{n} \left( e^{1+\frac{2}{n}} + e^{1+\frac{4}{n}} + e^{1+\frac{6}{n}} + \dots + e^{1+\frac{2n}{n}} \right)$$

$$52. \lim_{n \rightarrow \infty} \frac{27\pi^2}{n^2} \left( \sin \frac{3\pi}{n} + 2\sin \frac{6\pi}{n} + 3\sin \frac{9\pi}{n} + \dots + n \sin 3\pi \right)$$

$$53. \lim_{n \rightarrow \infty} \frac{1}{n} \left( \sin \frac{\pi}{2n} + \sin \frac{2\pi}{2n} + \sin \frac{3\pi}{2n} + \dots + \sin \frac{\pi}{2} \right)$$

$$54. \lim_{n \rightarrow \infty} \frac{2}{n} \left( \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{n\pi}{n} \right)$$

$$55. \lim_{n \rightarrow \infty} \frac{\pi^2}{n^2} \left( \sin \frac{\pi}{n} + 2\sin \frac{2\pi}{n} + 3\sin \frac{3\pi}{n} + \dots + n \sin \frac{n\pi}{n} \right)$$

$$56. \lim_{n \rightarrow \infty} \frac{1}{n^2} \left( \cos \frac{\pi}{n} + 2\cos \frac{2\pi}{n} + 3\cos \frac{3\pi}{n} + \dots + n \cos \frac{n\pi}{n} \right)$$

57.

$$\lim_{n \rightarrow \infty} \frac{\pi^3}{8n^3} \left( \cos \frac{\pi}{2n} + 4 \cos \frac{2\pi}{2n} + 9 \cos \frac{3\pi}{2n} + \cdots + n^2 \cos \frac{n\pi}{2n} \right)$$

58.

$$\lim_{n \rightarrow \infty} \left\{ \frac{\pi}{n} \tan \left( \frac{\pi}{4} + \frac{\pi}{12n} \right) + \frac{\pi}{n} \tan \left( \frac{\pi}{4} + \frac{2\pi}{12n} \right) + \cdots + \frac{\pi}{n} \tan \left( \frac{\pi}{4} + \frac{n\pi}{12n} \right) \right\}$$

$$59. \lim_{n \rightarrow \infty} \frac{6}{n} \ln \left( \frac{n+3}{n} \times \frac{n+6}{n} \times \frac{n+9}{n} \times \cdots \times \frac{4n}{n} \right)$$

$$60. \lim_{n \rightarrow \infty} \left[ \left( 1 + \frac{1}{n} \right) \left( 1 + \frac{2}{n} \right) \cdots \left( 1 + \frac{n}{n} \right) \right]^{\frac{1}{n}}$$

61.

$$\lim_{n \rightarrow \infty} n \left\{ \frac{\ln(n+1) - \ln n}{(2n+1)^2} + \frac{\ln(n+2) - \ln n}{(2n+2)^2} + \cdots + \frac{\ln 2n - \ln n}{(2n+n)^2} \right\}$$

■ 다음 물음에 답하여라.

$$62. \lim_{n \rightarrow \infty} \frac{\pi}{n} \left( \cos \frac{\pi}{n} + \cos \frac{2\pi}{n} + \cos \frac{3\pi}{n} + \cdots + \cos \frac{n\pi}{n} \right) \text{를}$$

정적분으로 나타내면  $\int_0^a \cos x dx$  일 때, 상수  $a$ 의 값을 구하여라.

63. 정적분을 이용하여

$$a = \lim_{n \rightarrow \infty} \frac{(n+1)^4 + (n+2)^4 + (n+3)^4 + \cdots + (n+n)^4}{n^5} \text{일 때, } 5a \text{의 값을 구하여라.}$$

■ 다음 함수  $f(x)$ 에 대하여 알맞은 값을 구하여라.

64. 함수  $f(x) = x^4$ 에 대하여

$$\lim_{n \rightarrow \infty} \frac{1}{n^5} \{ f(n+1) + f(n+2) + \cdots + f(2n) \} \text{의 값을 구하여라.}$$

65. 함수  $f(x) = e^{2x}$ 에 대하여

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + f\left(\frac{3}{n}\right) + \cdots + f\left(\frac{n}{n}\right) \right\} \text{의 값을 구하여라.}$$

66. 함수  $f(x) = \tan x$ 에 대하여

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left\{ f\left(\frac{\pi}{n}\right) \right\}^2 + \left\{ f\left(\frac{2\pi}{n}\right) \right\}^2 + \left\{ f\left(\frac{3\pi}{n}\right) \right\}^2 + \cdots + \left\{ f\left(\frac{n\pi}{n}\right) \right\}^2 \right]$$

의 값을 구하여라.

67. 함수  $f(x) = 2xe^{x^2}$ 에 대하여

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ f\left(\frac{2}{n}\right) + f\left(\frac{4}{n}\right) + f\left(\frac{6}{n}\right) + \cdots + f\left(\frac{2n}{n}\right) \right\}$$

의 값을 구하여라.





## 정답 및 해설

$$1) \frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \frac{k^2}{n^3}, \frac{1}{3}$$

$\Rightarrow S_n$ 은 밑변의 길이가  $\frac{1}{n}$ 이고 높이가 각각  $\left(\frac{1}{n}\right)^2, \left(\frac{2}{n}\right)^2, \dots, \left(\frac{n+1}{n}\right)^2, \left(\frac{n}{n}\right)^2$ 인 직사각형의 넓이의 합이므로

$$\begin{aligned} S_n &= \left[\frac{1}{n}\right] \times \left(\frac{1}{n}\right)^2 + \left[\frac{1}{n}\right] \times \left(\frac{2}{n}\right)^2 + \dots + \left[\frac{1}{n}\right] \times \left(\frac{n}{n}\right)^2 \\ &= \sum_{k=1}^n \frac{1}{n} \times \left(\frac{k}{n}\right)^2 \\ &= \sum_{k=1}^n \frac{k^2}{n^3} \end{aligned}$$

$$\begin{aligned} \therefore S &= \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^2}{n^3} \\ &= \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6n^3} \\ &= \left[\frac{1}{3}\right] \end{aligned}$$

$$2) \text{ (가) } \frac{1}{n} \left( \frac{2k}{n} - \frac{1}{2} \left( \frac{k}{n} \right)^2 \right)$$

$$\text{ (나) } \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} \frac{1}{n} \left( \frac{2k}{n} - \frac{1}{2} \left( \frac{k}{n} \right)^2 \right)$$

$$\text{ (다) } 2x - \frac{1}{2}x^2$$

$$\text{ (라) } 4x - 2x^2$$

$$\Rightarrow S_k = \frac{1}{n} \left( \frac{2k}{n} - \frac{1}{2} \left( \frac{k}{n} \right)^2 \right)$$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^{2n} S_k = \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} \frac{1}{n} \left( \frac{2k}{n} - \frac{1}{2} \left( \frac{k}{n} \right)^2 \right)$$

$$= \int_0^2 2x dx - \frac{1}{2} \int_0^2 x^2 dx$$

$$= \int_0^2 \left( 2x - \frac{1}{2}x^2 \right) dx = 2 \int_0^1 (4x - 2x^2) dx$$

$$3) \text{ (가) } 4k^2 \quad \text{ (나) } 8 \quad \text{ (다) } \frac{8}{n^3} \quad \text{ (라) } \frac{8}{3}$$

$$\Rightarrow x_k = \frac{2k}{n} \text{ 이므로 } f(x_k) = \frac{4k^2}{n^2} \quad \therefore \text{ (가) } = 4k^2$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4k^2}{n^2} \cdot \frac{2}{n} = \lim_{n \rightarrow \infty} \frac{8}{n^3} \sum_{k=1}^n k^2 \\ &= \lim_{n \rightarrow \infty} \left\{ \frac{8}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right\} \\ &= \frac{4}{3} \lim_{n \rightarrow \infty} \left\{ \frac{n(n+1)(2n+1)}{n^3} \right\} = \frac{4}{3} \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{n^2} \\ &= \frac{4}{3} \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right) \left( 2 + \frac{1}{n} \right) = \frac{8}{3} \end{aligned}$$

$$\text{이므로 (나)} = 8, \text{ (다)} = \frac{8}{n^3}, \text{ (라)} = \frac{8}{3}$$

$$4) \frac{3}{n} \sum_{k=1}^n \left( \frac{k}{n} \right)^2$$

$$\Rightarrow k\text{번째 직사각형의 넓이는 } 3 \left( \frac{k}{n} \right)^2 \frac{1}{n}$$

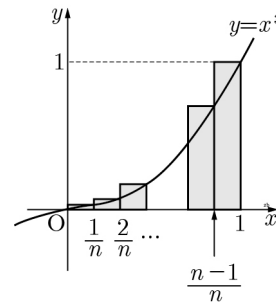
$n$ 개의 직사각형의 넓이의 합은

$$S_n = \sum_{k=1}^n 3 \left( \frac{k}{n} \right)^2 \frac{1}{n}$$

$$S = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n 3 \left( \frac{k}{n} \right)^2 \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{k=1}^n \left( \frac{k}{n} \right)^2$$

$$5) \frac{1}{4}$$

$\Rightarrow$



그림과 같이 곡선 위에 만든 직사각형의 넓이의 합을  $T_n$ 이라고 하면

$$T_n = \frac{1}{n} \left( \frac{1}{n} \right)^3 + \frac{1}{n} \left( \frac{2}{n} \right)^3 + \dots + \frac{1}{n} \left( \frac{n}{n} \right)^3$$

$$= \frac{1}{n^4} (1^3 + 2^3 + \dots + n^3)$$

$$= \frac{1}{n^4} \left\{ \frac{n(n+1)}{2} \right\}^2 = \frac{1}{4} \left( 1 + \frac{1}{n} \right)^2$$

따라서 구하는 도형의 넓이는

$$\lim_{n \rightarrow \infty} T_n = \lim_{n \rightarrow \infty} \frac{1}{4} \left( 1 + \frac{1}{n} \right)^2 = \frac{1}{4}$$

$$6) \frac{8}{3}$$

$$\Rightarrow f(x) = x^2, \quad a=0, \quad b=2 \text{로 놓으면}$$

$$\Delta x = \frac{2}{n}, \quad x_k = \frac{2k}{n}$$

$$\begin{aligned} \therefore \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{2k}{n} \right)^2 \times \frac{2}{n} &= \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x \\ &= \int_0^2 x^2 dx \\ &= \left[ \frac{1}{3} x^3 \right]_0^2 = \frac{8}{3} \end{aligned}$$

$$7) 10$$

$$\Rightarrow f(x) = 4+x, \quad a=0, \quad b=2 \text{로 놓으면}$$

$$\Delta x = \frac{2}{n}, \quad x_k = \frac{2k}{n}$$

$$\begin{aligned}\therefore \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(4 + \frac{2k}{n}\right) \times \frac{2}{n} &= \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x \\ &= \int_0^2 (4+x) dx \\ &= \left[4x + \frac{1}{2}x^2\right]_0^2 = 10\end{aligned}$$

$$8) \frac{1}{8} \ln 5$$

$$\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n^2 + 4k^2} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\frac{k}{n}}{1 + 4\left(\frac{k}{n}\right)^2} \cdot \frac{1}{n}$$

$$= \int_0^1 \frac{x}{1+4x^2} dx$$

$$\text{이때, } 1+4x^2=t \text{로 놓으면 } 8x = \frac{dt}{dx}$$

$$x=0 \text{일 때 } t=1, \quad x=1 \text{일 때 } t=5$$

$$\begin{aligned}\int_0^1 \frac{x}{1+4x^2} dx &= \frac{1}{8} \int_1^5 \frac{1}{t} dt \\ &= \frac{1}{8} \left[ \ln |t| \right]_1^5 \\ &= \frac{1}{8} \ln 5\end{aligned}$$

$$9) 40$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^3 \frac{4}{n} &= 2 \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^3 \frac{2}{n} \\ &= 2 \int_1^3 x^3 dx = 2 \left[ \frac{1}{4} x^4 \right]_1^3 = 2 \left( \frac{81}{4} - \frac{1}{4} \right) = 40\end{aligned}$$

$$10) \frac{3}{4}$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n}\right)^2 \left(3 - \frac{k}{n}\right) \frac{1}{n} &= \int_0^1 \{x^2(3-x)\} dx \\ &= \int_0^1 (3x^2 - x^3) dx = \left[ x^3 - \frac{1}{4} x^4 \right]_0^1 = \frac{3}{4}\end{aligned}$$

$$11) \frac{14}{3}$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left\{ \left(\frac{2k}{n}\right)^2 + 1 \right\} \frac{2}{n} &= \int_0^2 (x^2 + 1) dx \\ &= \left[ \frac{1}{3} x^3 + x \right]_0^2 = \frac{8}{3} + 2 = \frac{14}{3}\end{aligned}$$

$$12) 65$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{k}{n}\right)^3 \frac{4}{n} &= 4 \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{k}{n}\right)^3 \frac{1}{n} \\ &= 4 \int_2^3 x^3 dx = 4 \left[ \frac{1}{4} x^4 \right]_2^3 = 4 \times \left( \frac{81}{4} - \frac{16}{4} \right) = 65\end{aligned}$$

$$13) 14$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n (-n+3k)^5 \cdot \frac{4}{n^6} &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{-n+3k}{n}\right)^5 \cdot \frac{4}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(-1 + \frac{3k}{n}\right)^5 \cdot \frac{3}{n} \cdot \frac{4}{3} \\ &= \frac{4}{3} \int_{-1}^2 x^5 dx = \frac{4}{3} \left[ \frac{1}{6} x^6 \right]_{-1}^2 = \frac{4}{3} \left( \frac{64}{6} - \frac{1}{6} \right) = 14\end{aligned}$$

$$14) \frac{968}{5}$$

$\Rightarrow$

$$\begin{aligned}\lim_{n \rightarrow \infty} \sum_{k=1}^{4n} \left(1 + \frac{k}{2n}\right)^4 \frac{2}{n} &= 4 \lim_{n \rightarrow \infty} \sum_{k=1}^{4n} \left(1 + \frac{k}{2n}\right)^4 \frac{1}{2n} = 4 \int_1^3 x^4 dx \\ &= 4 \left[ \frac{1}{5} x^5 \right]_1^3 = 4 \left( \frac{243}{5} - \frac{1}{5} \right) = \frac{968}{5}\end{aligned}$$

$$15) 30$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n (n+2k)^3 \cdot \frac{3}{n^4} &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^3 \cdot \frac{3}{n} \\ &= \frac{3}{2} \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^3 \cdot \frac{2}{n} \\ &= \frac{3}{2} \int_1^3 x^3 dx\end{aligned}$$

$$= \frac{3}{2} \left[ \frac{1}{4} x^4 \right]_1^3 = \frac{3}{2} \times \frac{1}{4} \times (81 - 1) = 30$$

$$16) 4$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4k}{n^2} e^{\frac{k}{n}} &= 4 \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n} e^{\frac{k}{n}} \cdot \frac{1}{n} \\ &= 4 \int_0^1 x e^x dx \\ &= 4 \left[ x e^x - e^x \right]_0^1 = 4\end{aligned}$$

$$17) -\frac{1}{4}$$

$$\begin{aligned}\Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k(k-2n)}{n^3} \left(\frac{n-k}{n}\right) &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \cdot \frac{k}{n} \cdot \left(\frac{k}{n} - 2\right) \cdot \left(1 - \frac{k}{n}\right) \\ &\quad \left(\frac{k}{n} = x \text{로 치환하면}\right) \\ &= \int_0^1 x(x-2)(1-x) dx \\ &= \int_0^1 (-x^3 + 3x^2 - 2x) dx \\ &= \left[ -\frac{1}{4} x^4 + x^3 - x^2 \right]_0^1 \\ &= -\frac{1}{4}\end{aligned}$$

18) 182

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^5 \cdot \frac{3}{n} \\ = \frac{3}{2} \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^5 \cdot \frac{2}{n} \\ = \frac{3}{2} \int_1^3 x^5 dx = \frac{3}{2} \left[ \frac{1}{6} x^6 \right]_1^3 = \frac{1}{4} \times (729 - 1) = 182 \end{aligned}$$

19)  $\frac{1}{2}$ 

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \sin^2 \frac{k\pi}{n} \\ = \int_0^1 \sin^2 \pi x dx \\ = \int_0^1 \left( \frac{1 - \cos 2\pi x}{2} \right) dx \\ = \frac{1}{2} \left[ x - \frac{1}{2\pi} \sin 2\pi x \right]_0^1 \\ = \frac{1}{2} \end{aligned}$$

20)  $\ln 4$ 

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2}{n+k} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2}{1 + \frac{k}{n}} \cdot \frac{1}{n} = \int_0^1 \frac{2}{1+x} dx \\ = [2\ln|1+x|]_0^1 = 2\ln 2 = \ln 4 \end{aligned}$$

21) 32

 $\Rightarrow$ 

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} \frac{k^2(5k^2+7)}{n^3(n^2+3)} = \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} \frac{\left(\frac{k}{n}\right)^2 \left(5\left(\frac{k}{n}\right)^2 + \frac{7}{n^2}\right)}{n\left(1 + \frac{3}{n^2}\right)} \\ = \int_0^2 x^2(5x^2) dx = \int_0^2 5x^4 dx = [x^5]_0^2 = 32 \end{aligned}$$

22) 4

$$\Rightarrow \int_1^5 x^4 dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{4k}{n}\right)^4 \cdot \frac{4}{n} \quad \therefore a=4$$

23) 6

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{3k}{n}\right)^2 \frac{6}{n} = 2 \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{3k}{n}\right)^2 \frac{3}{n} \\ = 2 \int_1^4 x^2 dx \end{aligned}$$

따라서  $a=2$ ,  $b=4$ 이므로  $a+b=2+4=6$ 이다.

24) 6

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^2 \frac{4}{n} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^2 \cdot \frac{2}{n} \cdot 2 \\ = \int_1^3 2x^2 dx \end{aligned}$$

따라서  $a=2$ ,  $b=3$ 이므로  $ab=6$ 이다.

25) 207

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{3k}{n}\right)^3 \frac{1}{n} \\ = \int_0^1 (2+3x)^3 dx = \left[ \frac{1}{3} \times \frac{1}{4} (2+3x)^4 \right]_0^1 \\ = \frac{5^4 - 2^4}{12} = \frac{625 - 16}{12} = \frac{203}{4} = \frac{q}{p} \end{aligned}$$

따라서  $p=4$ ,  $q=203$ 이므로  $p+q=207$ 

26) 5

 $\Rightarrow x = \frac{4k}{n}$ 로 놓으면

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(3 + \frac{8k}{n}\right)^2 \cdot \frac{4}{n} = \int_0^4 (3+2x)^2 dx \\ \text{따라서 } a=1, b=4 \text{이므로 } a+b=5 \text{이다.} \end{aligned}$$

27)  $\frac{5}{4}$ 

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \left\{ \frac{1}{n} \left(1 + 2\frac{k}{n}\right) \ln \left(1 + \frac{2k}{n}\right) \right\} = \frac{1}{2} \int_1^3 x \ln x dx \\ = \frac{1}{2} \left[ \frac{x^2}{2} \ln x - \frac{x^2}{4} \right]_1^3 = \frac{1}{2} \left( \frac{9}{2} \ln 3 - \frac{9}{4} + \frac{1}{4} \right) = \frac{9}{4} \ln 3 - 1 \\ a = \frac{9}{4}, b = -1 \quad \therefore a+b = \frac{5}{4} \end{aligned}$$

28) 242

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right) \frac{10}{n} = \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right) \frac{2}{n} \cdot 5 \\ = 5 \int_1^3 f(x) dx = 5 \int_1^3 x^4 dx \\ = 5 \left[ \frac{1}{5} x^5 \right]_1^3 = 5 \left( \frac{243}{5} - \frac{1}{5} \right) = 5 \times \frac{242}{5} = 242 \end{aligned}$$

29) 1

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{k=1}^n f\left(\frac{2k}{n} - 1\right) = \frac{3}{2} \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{k=1}^n f\left(\frac{2k}{n} - 1\right) \\ = \frac{3}{2} \int_{-1}^1 f(x) dx = \frac{3}{2} \times 2 \int_0^1 x^2 dx \\ = 3 \left[ \frac{1}{3} x^3 \right]_0^1 = 3 \times \frac{1}{3} \times 1 = 1 \end{aligned}$$

30)  $e^3 + 8$ 

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{k}{n}\right) \frac{1}{n} = \int_0^1 f(x) dx, \\ \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} f\left(1 + \frac{k}{n}\right) \frac{1}{n} = \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} f\left(1 + \frac{2k}{2n}\right) \frac{2}{2n} \\ = \int_1^3 f(x) dx \end{aligned}$$

이므로

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{k}{n}\right) \frac{1}{n} + \lim_{n \rightarrow \infty} \sum_{k=1}^{2n} f\left(1 + \frac{k}{n}\right) \frac{1}{n}$$

$$\begin{aligned}
&= \int_0^1 f(x)dx + \int_1^3 f(x)dx \\
&= \int_0^3 f(x)dx \\
&= \int_0^3 (e^x + 2x)dx \\
&= \left[ e^x + x^2 \right]_0^3 = e^3 + 8
\end{aligned}$$

31)  $\ln 3 - \ln 2$ 

$$\begin{aligned}
\Rightarrow f(x) &= \frac{1}{x^2 + x} = \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1} \text{ 이므로} \\
&= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right) \\
&= \int_1^3 f(x)dx \\
&= \int_1^3 \left( \frac{1}{x} - \frac{1}{x+1} \right) dx \\
&= \left[ \ln |x| - \ln |x+1| \right]_1^3 \\
&= \ln 3 - \ln 2
\end{aligned}$$

32) 13

$$\begin{aligned}
\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right) &= \frac{1}{2} \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(1 + \frac{2k}{n}\right) \cdot \frac{2}{n} \\
&= \frac{1}{2} \int_1^3 f(x)dx \\
&= \frac{1}{2} \int_1^3 (x^3 + x + 1)dx = \frac{1}{2} \left[ \frac{1}{4}x^4 + \frac{1}{2}x^2 + x \right]_1^3 \\
&= \frac{1}{2} \left\{ \left( \frac{81}{4} + \frac{9}{2} + 3 \right) - \left( \frac{1}{4} + \frac{1}{2} + 1 \right) \right\} \\
&= \frac{1}{2} (20 + 4 + 2) = \frac{1}{2} \times 26 = 13
\end{aligned}$$

33) 2

$$\begin{aligned}
\Rightarrow \lim_{n \rightarrow \infty} \frac{\pi}{n} \left( \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{n\pi}{n} \right) \\
= \lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{k=1}^n \sin \frac{k\pi}{n} \\
f(x) = \sin x, \quad a = 0, \quad b = \pi \text{로 놓으면} \\
\Delta x = \frac{\pi}{n}, \quad x_k = \frac{k\pi}{n} \\
\therefore (\text{주어진 식}) = \int_0^\pi \sin x dx = [-\cos x]_0^\pi = 2
\end{aligned}$$

34) 7

$$\begin{aligned}
\Rightarrow \lim_{n \rightarrow \infty} \frac{3}{n} \left\{ \left( \frac{n+1}{n} \right)^2 + \left( \frac{n+2}{n} \right)^2 + \dots + \left( \frac{n+n}{n} \right)^2 \right\} \\
= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{k=1}^n \left( 1 + \frac{k}{n} \right)^2 \\
= 3 \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( 1 + \frac{k}{n} \right)^2 \times \frac{1}{n} \\
f(x) = x^2, \quad a = 1, \quad b = 2 \text{로 놓으면}
\end{aligned}$$

$$\Delta x = \frac{1}{n}, \quad x_k = 1 + \frac{k}{n}$$

$$\therefore (\text{주어진 식}) = 3 \int_1^2 x^2 dx = 3 \left[ \frac{1}{3} x^3 \right]_1^2 = 7$$

35) 2

$$\begin{aligned}
\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \left( \sqrt{\frac{n}{1}} + \sqrt{\frac{n}{2}} + \sqrt{\frac{n}{3}} + \dots + \sqrt{\frac{n}{n}} \right) \\
= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \sqrt{\frac{n}{k}} \\
= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{\frac{k}{n}}} \cdot \frac{1}{n} \\
= \int_0^1 \frac{1}{\sqrt{x}} dx \\
= \left[ 2\sqrt{x} \right]_0^1 = 2
\end{aligned}$$

36)  $\frac{2}{3}$ 

$$\begin{aligned}
\Rightarrow \lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{n\sqrt{n}} \\
= \lim_{n \rightarrow \infty} \frac{1}{n} \left( \sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \dots + \sqrt{\frac{n}{n}} \right) \\
= \lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{\frac{k}{n}} \cdot \frac{1}{n} \\
= \int_0^1 \sqrt{x} dx \\
= \left[ \frac{2}{3} x^{\frac{3}{2}} \right]_0^1 = \frac{2}{3}
\end{aligned}$$

37)  $\ln 2$ 

$$\begin{aligned}
\Rightarrow \lim_{n \rightarrow \infty} \left( \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right) \\
= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n+k} \\
= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{1 + \frac{k}{n}} \cdot \frac{1}{n} \\
= \int_0^1 \frac{1}{1+x} dx = \left[ \ln |1+x| \right]_0^1 \\
= \ln 2
\end{aligned}$$

38)  $\frac{1}{3} \ln 2$ 

$$\begin{aligned}
\Rightarrow \lim_{n \rightarrow \infty} \left( \frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \frac{3^2}{n^3+3^3} + \dots + \frac{n^2}{n^3+n^3} \right) \\
= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^2}{n^3+k^3} \\
= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\left( \frac{k}{n} \right)^2}{1 + \left( \frac{k}{n} \right)^3} \cdot \frac{1}{n}
\end{aligned}$$

$$= \int_0^1 \frac{x^2}{1+x^3} dx$$

이때,  $1+x^3=t$ 로 놓으면  $3x^2 = \frac{dt}{dx}$

$x=0$ 일 때  $t=1$ ,  $x=1$ 일 때  $t=2$

$$\begin{aligned} \int_0^1 \frac{x^2}{1+x^3} dx &= \frac{1}{3} \int_1^2 \frac{1}{t} dt \\ &= \frac{1}{3} \left[ \ln |t| \right]_1^2 \\ &= \frac{1}{3} \ln 2 \end{aligned}$$

39)  $\frac{1}{2}(e^5 - e)$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{2}{n} \left( e^{1+\frac{4}{n}} + e^{1+\frac{8}{n}} + \dots + e^{1+\frac{4n}{n}} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{k=1}^n e^{1+\frac{4k}{n}} \\ &= \frac{1}{2} \lim_{n \rightarrow \infty} \sum_{k=1}^n e^{1+\frac{4k}{n}} \cdot \frac{4}{n} \\ &= \frac{1}{2} \int_1^5 e^x dx \\ &= \frac{1}{2} \left[ e^x \right]_1^5 \\ &= \frac{1}{2}(e^5 - e) \end{aligned}$$

40)  $2\ln 2 - 1$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left( \frac{n+1}{n} \times \frac{n+2}{n} \times \frac{n+3}{n} \times \dots \times \frac{2n}{n} \right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left( \ln \frac{n+1}{n} + \ln \frac{n+2}{n} + \ln \frac{n+3}{n} + \dots + \ln \frac{2n}{n} \right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \ln \frac{n+k}{n} \\ &= \int_1^2 \ln x dx \\ &= \left[ x \ln x - x \right]_1^2 = 2\ln 2 - 1 \end{aligned}$$

41)  $\frac{2}{3}(2\sqrt{2} - 1)$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n^2} (\sqrt{n+1} + \sqrt{n+2} + \dots + \sqrt{n+n}) \\ &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \sqrt{1+\frac{k}{n}} \right) \frac{1}{n} = \int_1^2 \sqrt{x} dx = \left[ \frac{2}{3} x \sqrt{x} \right]_1^2 = \frac{2}{3}(2\sqrt{2} - 1) \end{aligned}$$

42)  $\frac{2\sqrt{3}}{3}$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{\sqrt{3} + \sqrt{6} + \dots + \sqrt{3n}}{n\sqrt{n}} \\ &= \lim_{n \rightarrow \infty} \left( \sqrt{\frac{3}{n}} + \sqrt{\frac{6}{n}} + \dots + \sqrt{\frac{3n}{n}} \right) \frac{1}{n} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{\frac{3k}{n}} \times \frac{1}{n} = \int_0^1 \sqrt{3x} dx \\ &= \sqrt{3} \left[ \frac{2}{3} x \sqrt{x} \right]_0^1 = \frac{2\sqrt{3}}{3} \end{aligned}$$

43)  $\ln 2$

$$\begin{aligned} \Rightarrow 2 \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{k}{n^2+k^2} \right) &= 2 \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left( \frac{\frac{k}{n}}{1+\left(\frac{k}{n}\right)^2} \right) \\ &= 2 \int_0^1 \frac{x}{1+x^2} dx \quad (1+x^2=t \text{로 치환하자. } 2x dx = dt) \\ &= \int_1^2 \frac{1}{t} dt = [\ln |t|]_1^2 = \ln 2 \end{aligned}$$

44) 63

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{(n+1)^5 + (n+2)^5 + (n+3)^5 + \dots + (n+n)^5}{1^5 + 2^5 + 3^5 + \dots + n^5} \\ &= \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n (n+k)^5}{\sum_{k=1}^n k^5} = \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n \left(1+\frac{k}{n}\right)^5 \frac{1}{n}}{\sum_{k=1}^n \left(\frac{k}{n}\right)^5 \frac{1}{n}} \\ &= \frac{\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1+\frac{k}{n}\right)^5 \frac{1}{n}}{\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n}\right)^5 \frac{1}{n}} = \frac{\int_1^2 x^5 dx}{\int_0^1 x^5 dx} \\ &= \frac{\left[ \frac{1}{6} x^6 \right]_1^2}{\left[ \frac{1}{6} x^6 \right]_0^1} = \frac{\frac{64}{6} - \frac{1}{6}}{\frac{1}{6}} = 63 \end{aligned}$$

45)  $\frac{3}{2}$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{n(1^3+2^3+3^3+\dots+n^3)}{(1+2+\dots+n)(1^2+2^2+\dots+n^2)} \\ &= \lim_{n \rightarrow \infty} \frac{n \sum_{k=1}^n k^3}{\sum_{k=1}^n k \cdot \sum_{k=1}^n k^2} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^5} \cdot n \sum_{k=1}^n k^3}{\left( \frac{1}{n^2} \sum_{k=1}^n k \right) \cdot \left( \frac{1}{n^3} \sum_{k=1}^n k^2 \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n \left( \frac{k}{n} \right)^3 \cdot \frac{1}{n}}{\left( \sum_{k=1}^n \frac{k}{n} \cdot \frac{1}{n} \right) \cdot \left( \sum_{k=1}^n \left( \frac{k}{n} \right)^2 \cdot \frac{1}{n} \right)} \\ &= \frac{\int_0^1 x^3 dx}{\int_0^1 x dx \cdot \int_0^1 x^2 dx} = \frac{\left[ \frac{1}{4} x^4 \right]_0^1}{\left[ \frac{1}{2} x^2 \right]_0^1 \cdot \left[ \frac{1}{3} x^3 \right]_0^1} \\ &= \frac{\frac{1}{4}}{\frac{1}{2} \cdot \frac{1}{3}} = \frac{6}{4} = \frac{3}{2} \end{aligned}$$

$$46) 1 - \frac{1}{e}$$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \left( e^{\frac{1}{n}-1} + e^{\frac{2}{n}-1} + \dots + e^{\frac{n}{n}-1} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n e^{\frac{k}{n}-1} = \int_0^1 e^{x-1} dx \\ = [e^{x-1}]_0^1 = 1 - \frac{1}{e} \end{aligned}$$

$$47) 1$$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n} \cdot \frac{1}{n} = \int_0^1 x e^x dx = [x e^x]_0^1 - \int_0^1 e^x dx \\ = e - (e - 1) = 1 \end{aligned}$$

$$48) e^2 + 1$$

$\Rightarrow$  주어진 식을 정리하면

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{2k}{n} e^{\frac{2k}{n}} \right) \frac{2}{n} = \int_0^2 x e^x dx \\ = [x e^x]_0^2 - \int_0^2 e^x dx = 2e^2 - (e^2 - 1) = e^2 + 1 \end{aligned}$$

$$49) \frac{e-2}{e}$$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\frac{k}{n}}{e^{\frac{k}{n}}} \cdot \frac{1}{n} = \int_0^1 \frac{x}{e^x} dx = \int_0^1 x e^{-x} dx \\ = [-x e^{-x}]_0^1 + \int_0^1 e^{-x} dx \\ = -\frac{1}{e} + [-e^{-x}]_0^1 = -\frac{1}{e} - \frac{1}{e} + 1 = 1 - \frac{2}{e} = \frac{e-2}{e} \end{aligned}$$

$$50) 2 \ln 3$$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{6(2k)^2}{n^3 + (2k)^3} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{24 \left( \frac{k}{n} \right)^2}{1 + 8 \left( \frac{k}{n} \right)^3} \times \frac{1}{n} \\ = \int_0^1 \frac{24x^2}{1+8x^3} dx = [\ln|1+8x^3|]_0^1 = \ln 9 = 2 \ln 3 \end{aligned}$$

$$51) \frac{1}{2}(e^3 - e)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n e^{1+\frac{2k}{n}} = \frac{1}{2} \int_1^3 e^x dx = \frac{1}{2}(e^3 - e)$$

$$52) 9\pi$$

$\Rightarrow$  주어진 극한식을 급수로 표현하면

$$\begin{aligned} 27\pi^2 \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n} \sin \frac{3k\pi}{n} \frac{1}{n} \\ = 27\pi^2 \int_0^1 x \sin 3\pi x dx = 27\pi^2 \times \frac{1}{3\pi} = 9\pi \end{aligned}$$

가 된다.

$$53) \frac{2}{\pi}$$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \sin \frac{k\pi}{2n} = \int_0^1 \sin \frac{\pi}{2} x dx \\ = \frac{2}{\pi} \left[ -\cos \frac{\pi}{2} x \right]_0^1 = \frac{2}{\pi} \end{aligned}$$

$$54) \frac{4}{\pi}$$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} \frac{2}{n} \left( \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{n\pi}{n} \right) \\ = \frac{2}{\pi} \lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{k=1}^n \sin \frac{k\pi}{n} \\ = \frac{2}{\pi} \int_0^\pi \sin x dx \\ = \frac{2}{\pi} [-\cos x]_0^\pi = \frac{4}{\pi} \end{aligned}$$

$$55) \pi$$

$$\begin{aligned} \Rightarrow \sum_{k=1}^n \left( \frac{k\pi}{n} \sin \frac{k\pi}{n} \right) \frac{\pi}{n} = \int_0^\pi x \sin x dx \\ = [-x \cos x]_0^\pi + \int_0^\pi \cos x dx = \pi \end{aligned}$$

$$56) -\frac{2}{\pi^2}$$

$\Rightarrow$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n^2} \left( \cos \frac{\pi}{n} + 2 \cos \frac{2\pi}{n} + \dots + n \cos \frac{n\pi}{n} \right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{k}{n} \cos \frac{k\pi}{n} \right) \frac{1}{n} \\ = \int_0^1 x \cos \pi x dx = \left[ \frac{1}{\pi} x \sin \pi x \right]_0^1 - \frac{1}{\pi} \int_0^1 \sin \pi x dx \\ = \frac{1}{\pi^2} [\cos \pi x]_0^1 = -\frac{2}{\pi^2} \end{aligned}$$

$$57) \frac{\pi^2}{4} - 2$$

$\Rightarrow$  주어진 식을 정리하면

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{\pi^3}{8n^3} \sum_{k=1}^n k^2 \cos \frac{k\pi}{2n} \\ = \lim_{n \rightarrow \infty} \frac{\pi^3}{8} \sum_{k=1}^n \frac{k^2}{n^2} \cos \frac{\pi}{2} \frac{k}{n} \frac{1}{n} \\ = \frac{\pi^3}{8} \int_0^1 x^2 \cos \frac{\pi}{2} x dx \\ = \frac{\pi^3}{8} \left( \frac{2}{\pi} - \frac{2}{\pi} \int_0^1 2x \sin \frac{\pi}{2} x dx \right) \\ = \frac{\pi^3}{8} \left\{ \frac{2}{\pi} + \left( \frac{2}{\pi} \right)^2 \int_0^1 2 \left( -\cos \frac{\pi}{2} x \right) dx \right\} \\ = \frac{\pi^2}{4} - 2 \end{aligned}$$

이다.

$$58) 6 \ln 2$$

$$\Rightarrow \frac{\pi}{4} + \frac{\pi k}{12n} \rightarrow x, \quad \frac{\pi}{12n} \rightarrow dx \text{로 놓으면,}$$

$$\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$$

$$\therefore (\text{주어진 극한}) = 12 \lim_{n \rightarrow \infty} \sum_{k=1}^n \tan\left(\frac{\pi}{4} + \frac{\pi k}{12n}\right) \cdot \frac{\pi}{12n}$$

$$= 12 \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x dx = 12 [-\ln |\cos x|]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= 12 \left( -\ln \frac{1}{2} + \ln \frac{\sqrt{2}}{2} \right) = 6 \ln 2$$

59)  $16 \ln 2 - 6$  $\Rightarrow$  주어진 극한식을 정리하면

$$\lim_{n \rightarrow \infty} \frac{6}{n} \sum_{k=1}^n \ln\left(1 + \frac{3k}{n}\right) = \lim_{n \rightarrow \infty} 6 \sum_{k=1}^n \ln\left(1 + \frac{3k}{n}\right) \frac{1}{n}$$

$$= 6 \int_0^1 \ln(1+3x) dx, \quad 1+3x=t$$

$$= 16 \ln 2 - 6$$

60)  $\frac{4}{e}$  $\Rightarrow$  주어진 극한을 P라 놓으면

$$\ln P = \lim_{n \rightarrow \infty} \sum_{k=1}^n \ln\left(1 + \frac{k}{n}\right) \cdot \frac{1}{n} = \int_1^2 \ln x dx$$

$$= [x \ln x - x]_1^2 = 2 \ln 2 - 1 = \ln \frac{4}{e}$$

$$\therefore P = \frac{4}{e}$$

61)  $\frac{5}{3} \ln 2 - \ln 3$  $\Rightarrow$  주어진 식은

$$n \sum_{k=1}^n \left( \frac{\ln\left(\frac{n+k}{n}\right)}{(2n+k)^2} \right) = \sum_{k=1}^n \ln\left(1 + \frac{k}{n}\right) \frac{1}{\left(2 + \frac{k}{n}\right)^2 n^2} \times n$$

$$= \int_0^1 \frac{\ln(1+x)}{(2+x)^2} dx = \left[ -\frac{\ln(1+x)}{2+x} \right]_0^1 + \int_0^1 \frac{1}{(1+x)(2+x)} dx$$

$$= -\frac{\ln 2}{3} + \int_0^1 \left( \frac{1}{1+x} - \frac{1}{2+x} \right) dx$$

$$= -\frac{\ln 2}{3} + [\ln(x+1) - \ln(x+2)]_0^1 = -\frac{\ln 2}{3} + \ln 2 - \ln 3 + \ln 2$$

$$= \frac{5}{3} \ln 2 - \ln 3$$

62)  $\pi$ 

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\pi}{n} \left( \cos \frac{\pi}{n} + \cos \frac{2\pi}{n} + \cos \frac{3\pi}{n} + \dots + \cos \frac{n\pi}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{k=1}^n \cos \frac{k\pi}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n \cos \frac{k\pi}{n} \cdot \frac{\pi}{n}$$

$$= \int_0^\pi \cos x dx$$

$$\therefore a = \pi$$

63) 31

$$\Rightarrow a = \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \left( \frac{n+1}{n} \right)^4 + \left( \frac{n+2}{n} \right)^4 + \dots + \left( \frac{n+n}{n} \right)^4 \right\}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( 1 + \frac{k}{n} \right)^4 \frac{1}{n} = \int_1^2 x^4 dx = \left[ \frac{1}{5} x^5 \right]_1^2 = \frac{32}{5} - \frac{1}{5}$$

$$= \frac{31}{5}$$

$$\therefore 5a = 5 \times \frac{31}{5} = 31$$

64)  $\frac{31}{5}$ 

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n^5} \{f(n+1) + f(n+2) + \dots + f(2n)\}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^5} \sum_{k=1}^n f(n+k) = \lim_{n \rightarrow \infty} \frac{1}{n^5} \sum_{k=1}^n (n+k)^4$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left( 1 + \frac{k}{n} \right)^4$$

$$= \int_1^2 x^4 dx = \left[ \frac{1}{5} x^5 \right]_1^2 = \frac{32-1}{5} = \frac{31}{5}$$

65)  $\frac{1}{2}e^2 - \frac{1}{2}$ 

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + f\left(\frac{3}{n}\right) + \dots + f\left(\frac{n}{n}\right) \right\}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{k}{n}\right) \cdot \frac{1}{n}$$

$$= \int_0^1 f(x) dx = \int_0^1 e^{2x} dx$$

$$= \left[ \frac{1}{2} e^{2x} \right]_0^1 = \frac{1}{2} e^2 - \frac{1}{2}$$

66) -1

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left\{ f\left(\frac{\pi}{n}\right) \right\}^2 + \left\{ f\left(\frac{2\pi}{n}\right) \right\}^2 + \left\{ f\left(\frac{3\pi}{n}\right) \right\}^2 + \dots + \left\{ f\left(\frac{n\pi}{n}\right) \right\}^2 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left\{ f\left(\frac{k\pi}{n}\right) \right\}^2$$

$$= \frac{1}{\pi} \lim_{n \rightarrow \infty} \sum_{k=1}^n \left\{ f\left(\frac{k\pi}{n}\right) \right\}^2 \cdot \frac{\pi}{n}$$

$$= \frac{1}{\pi} \int_0^\pi \{f(x)\}^2 dx = \frac{1}{\pi} \int_0^\pi \tan^2 x dx$$

이때,  $1 + \tan^2 x = \sec^2 x$ 에서  $\tan^2 x = \sec^2 x - 1$  이므로

$$\frac{1}{\pi} \int_0^\pi \tan^2 x dx = \frac{1}{\pi} \int_0^\pi (\sec^2 x - 1) dx$$

$$= \frac{1}{\pi} \left[ \tan x - x \right]_0^\pi$$

$$= \frac{1}{\pi} \cdot (-\pi)$$

$$=-1$$

$$67) \frac{1}{2}(e^4-1)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ f\left(\frac{2}{n}\right) + f\left(\frac{4}{n}\right) + f\left(\frac{6}{n}\right) + \cdots + f\left(\frac{2n}{n}\right) \right\}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{2k}{n}\right) \frac{1}{n}$$

$$= \frac{1}{2} \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{2k}{n}\right) \frac{2}{n}$$

$$= \frac{1}{2} \int_0^2 f(x) dx$$

$$= \frac{1}{2} \int_0^2 2xe^{x^2} dx$$

$$= \frac{1}{2} \left[ e^{x^2} \right]_0^2$$

$$= \frac{1}{2}(e^4-1)$$