



◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시
 1) 제작연월일 : 2018-02-15
 2) 제작자 : 교육지대(주)
 3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초
 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호
 되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무
 단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법
 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 / 무리식의 뜻

(1) 무리식 : 근호 안에 문자가 포함된 식 중에서
 유리식으로 나타낼 수 없는 식을 무리식이라 한다.

예 ㉠ \sqrt{x} , $\sqrt{2x+1}$, $\frac{x}{\sqrt{x-1}}$ 는 무리식이다.

(2) 무리식을 계산할 때에는
 (근호 안의 식의 값) ≥ 0 , (분모) $\neq 0$
 이 되는 문자의 값의 범위에서만 생각한다.

■ 다음 중 유리식인 것에는 '유리식', 무리식인 것에는
 '무리식'을 써넣어라.

1. $\sqrt{x}+1$ ()

2. $\sqrt{2x-1}$ ()

3. $\frac{x+1}{x}$ ()

4. $\sqrt{x^2+2x}$ ()

5. $\sqrt{2}x+1$ ()

6. $\frac{\sqrt{3}x+1}{2}$ ()

7. $2x-\frac{1}{3}$ ()

8. $1-\frac{\sqrt{3}}{x}$ ()

9. $\frac{1}{\sqrt{x}-\sqrt{x+1}}$ ()

10. $\frac{\sqrt{1-x}}{\sqrt{1+x}}$ ()

11. $\sqrt{2+x}+\sqrt{2-x}$ ()

12. $\frac{2}{3\sqrt{x+1}}$ ()

■ 다음 무리식의 값이 실수가 되도록 하는 실수 x 의
 값의 범위를 구하시오.

13. $\sqrt{3-x}+x$

14. $\sqrt{2x-3}$

15. $\sqrt{3x+5}$

16. $1 + \sqrt{3-2x}$

17. $\frac{1}{\sqrt{x-2}}$

18. $\sqrt{2x+8} - \sqrt{1-x}$

19. $\frac{1}{\sqrt{1+x}}$

20. $\sqrt{2-x} - \sqrt{x+3}$

21. $\sqrt{x-2} + \frac{1}{\sqrt{6-x}}$

22. $\sqrt{2x+1} - \sqrt{3-2x}$

23. $\sqrt{3-x} + \sqrt{3x+6}$

24. $\sqrt{x+2} + \sqrt{1-x}$

02 / 제곱근의 성질

(1) 제곱근 : a 가 실수일 때

① $(\sqrt{a})^2 = a$, $(-\sqrt{a})^2 = a$

② $\sqrt{a^2} = |a| = \begin{cases} a & (a \geq 0) \\ -a & (a < 0) \end{cases}$

(2) 제곱근의 성질 : $a > 0$, $b > 0$ 일 때

① $\sqrt{a} \sqrt{b} = \sqrt{ab}$ ② $\sqrt{a^2 b} = a \sqrt{b}$

③ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ ④ $\sqrt{\frac{a}{b^2}} = \frac{\sqrt{a}}{b}$

■ 다음 식을 간단히 하여라.

25. $\sqrt{(-a)^2}$ (단, $a > 0$)

26. $\sqrt{a^2} - \sqrt{b^2}$ (단, $a < b < 0$)

27. $\sqrt{(-b)^2}$ (단, $b < 0$)

28. $\sqrt{(-a)^2} + \sqrt{b^2}$ (단, $0 < a < b$)

29. $\sqrt{(a-b)^2}$ (단, $0 < a < b$)

30. $\sqrt{a^2} + |-b|$ (단, $a > 0$, $b > 0$)

31. $\sqrt{(a-b)^2} - |b|$ (단, $a < b < 0$)

$$32. \sqrt{(a+b)^2} - |ab| \quad (\text{단, } a > 0, b > 0)$$

$$33. \sqrt{(b-a)^2} - |ab| \quad (\text{단, } 0 < a < b)$$

$$34. \sqrt{a^2+2a+1} + \sqrt{a^2-4a+4} \quad (\text{단, } -1 < a < 2)$$

03 / 무리식의 계산

(1) 분모의 유리화

$a > 0, b > 0$ 일 때

$$\textcircled{1} \frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{\sqrt{b}\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

$$\textcircled{2} \frac{c}{\sqrt{a} + \sqrt{b}} = \frac{c(\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} = \frac{c(\sqrt{a} - \sqrt{b})}{a - b}$$

(단, $a \neq b$)

$$\textcircled{3} \frac{c}{\sqrt{a} - \sqrt{b}} = \frac{c(\sqrt{a} + \sqrt{b})}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} = \frac{c(\sqrt{a} + \sqrt{b})}{a - b}$$

(단, $a \neq b$)

■ 다음 식을 간단히 하여라.

$$35. \frac{3}{\sqrt{x+3} - \sqrt{x}}$$

$$36. \frac{x}{\sqrt{x+1} + 1}$$

$$37. \frac{1}{\sqrt{x} + \sqrt{x+2}}$$

$$38. \frac{1}{\sqrt{x-1} - \sqrt{x}}$$

$$39. \frac{x}{1 + \sqrt{x+1}}$$

$$40. \frac{x-2}{1 - \sqrt{x-1}}$$

$$41. \frac{x}{\sqrt{x^2-x} + x}$$

$$42. \frac{2}{\sqrt{x+1} + \sqrt{x+3}}$$

$$43. \frac{\sqrt{x+4}-2}{\sqrt{x+4}+2}$$

$$44. x + \sqrt{x^2+1} - \frac{1}{x + \sqrt{x^2+1}}$$

$$45. \frac{x^2}{\sqrt{x^2+4}+2} - \frac{x^2}{\sqrt{x^2+4}-2}$$

$$46. \frac{1-\sqrt{x}}{1+\sqrt{x}} + \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

$$47. \frac{x}{\sqrt{x+4}+2} - \frac{x}{\sqrt{x+4}-2}$$

$$48. \frac{x}{\sqrt{x+9}+3} - \frac{x}{\sqrt{x+9}-3}$$

$$49. \frac{1}{\sqrt{x+2}-\sqrt{x-2}} + \frac{1}{\sqrt{x+2}+\sqrt{x-2}}$$

$$50. \frac{x}{1+\sqrt{x+1}} + \frac{x}{1-\sqrt{x+1}}$$

$$51. \frac{1}{x+\sqrt{x^2-1}} + \sqrt{x^2-1}$$

$$52. \frac{\sqrt{x}}{\sqrt{x}+\sqrt{x+1}} + \frac{\sqrt{x}}{\sqrt{x}-\sqrt{x+1}}$$

$$53. (\sqrt{x+1}+\sqrt{x-1})(\sqrt{x+1}-\sqrt{x-1})$$

$$54. (\sqrt{x+3}+1)(\sqrt{x+3}-1)(x-1)$$

■ 다음 등식이 성립하는 실수 x 의 값의 범위를 구하여라.

$$55. \sqrt{x-5} \sqrt{1-x} = -\sqrt{-x^2+6x-5}$$

$$56. \frac{\sqrt{x+1}}{\sqrt{x-1}} = -\sqrt{\frac{x+1}{x-1}}$$

$$57. \frac{\sqrt{x+1}}{\sqrt{x}} = -\sqrt{\frac{x+1}{x}}$$

$$58. \frac{\sqrt{x+2}}{\sqrt{x-1}} = -\sqrt{\frac{x+2}{x-1}}$$

$$59. \sqrt{x-1} \sqrt{x} = -\sqrt{x^2-x}$$

$$60. \frac{\sqrt{x+3}}{\sqrt{x-3}} = -\sqrt{\frac{x+3}{x-3}}$$

$$61. \sqrt{x-2} \sqrt{x+1} = -\sqrt{x^2-x-2}$$

$$62. \sqrt{x-3} \sqrt{2-x} = -\sqrt{-x^2+5x-6}$$

■ 다음 식의 값을 구하여라.

$$63. x=2+\sqrt{2}, y=2-\sqrt{2} \text{ 일 때, } x+y \text{의 값}$$

$$64. x=\sqrt{2} \text{ 일 때, } \frac{1-\sqrt{x}}{1+\sqrt{x}} + \frac{1+\sqrt{x}}{1-\sqrt{x}} \text{의 값}$$

$$65. x=2+\sqrt{2}, y=2-\sqrt{2} \text{ 일 때, } x-y \text{의 값}$$

$$66. x=\sqrt{3} \text{ 일 때, } \frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} \text{의 값}$$

67. $x = \sqrt{2} + 1$ 일 때, $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1}$ 의 값

68. $x = \sqrt{2}$ 일 때, $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1}$ 의 값

69. $x = \frac{\sqrt{2}}{2}$ 일 때, $\frac{1}{1-\sqrt{x}} + \frac{1}{1+\sqrt{x}}$ 의 값

70. $x = \sqrt{2}$ 일 때, $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}}$ 의 값

71. $x = \sqrt{3}$ 일 때, $\frac{1}{x + \sqrt{x^2+1}} + \frac{1}{x - \sqrt{x^2+1}}$ 의 값

72. $x = 2 + \sqrt{2}$, $y = 2 - \sqrt{2}$ 일 때, xy 의 값

73. $x = \sqrt{2} + 1$ 일 때, $\frac{1}{1-\sqrt{x+1}} + \frac{1}{1+\sqrt{x+1}}$ 의
값

74. $x = \frac{2}{\sqrt{3}+1}$ 일 때, $\frac{\sqrt{x}+1}{\sqrt{x}-1} + \frac{\sqrt{x}-1}{\sqrt{x}+1}$ 의 값

75. $x = 2 + \sqrt{2}$, $y = 2 - \sqrt{2}$ 일 때, $\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$ 의 값

76. $x = \frac{1}{\sqrt{2}+1}$ 일 때, $\frac{\sqrt{x}-1}{\sqrt{x}+1} + \frac{\sqrt{x}+1}{\sqrt{x}-1}$ 의 값

77. $x = \frac{1}{\sqrt{2}+1}$ 일 때, $\frac{1+\sqrt{x}}{1-\sqrt{x}} + \frac{1-\sqrt{x}}{1+\sqrt{x}}$ 의 값



정답 및 해설

1) 무리식

2) 무리식

3) 유리식

4) 무리식

5) 유리식

6) 유리식

7) 유리식

8) 유리식

9) 무리식

10) 무리식

11) 무리식

12) 무리식

13) $x \leq 3$ $\Rightarrow 3-x \geq 0$ 이어야 하므로 $x \leq 3$ 14) $x \geq \frac{3}{2}$ $\Rightarrow 2x-3 \geq 0$ 이어야 하므로 $x \geq \frac{3}{2}$ 15) $x \geq -\frac{5}{3}$ $\Rightarrow \sqrt{3x+5}$ 의 값이 실수하려면
 $3x+5 \geq 0$ 이어야 하므로

$$x \geq \boxed{-\frac{5}{3}}$$

16) $x \leq \frac{3}{2}$ $\Rightarrow 3-2x \geq 0$ 이어야 하므로 $3 \geq 2x \quad \therefore x \leq \frac{3}{2}$ 17) $x > 2$ $\Rightarrow x-2 > 0 \quad \therefore x > 2$ 18) $-4 \leq x \leq 1$ $\Rightarrow 2x+8 \geq 0$ 에서 $x \geq -4$
 $1-x \geq 0$ 에서 $x \leq 1$
 $\therefore -4 \leq x \leq 1$ 19) $x > -1$ $\Rightarrow 1+x > 0$ 이어야 하므로 $x > -1$ 20) $-3 \leq x \leq 2$ $\Rightarrow 2-x \geq 0$ 이고 $x+3 \geq 0$ 이어야 하므로

$$-3 \leq x \leq 2$$

21) $2 \leq x < 6$ $\Rightarrow x-2 \geq 0, 6-x > 0$ 이어야 하므로 $x \geq 2, x < 6$
 $\therefore 2 \leq x < 6$ 22) $-\frac{1}{2} \leq x \leq \frac{3}{2}$ $\Rightarrow 2x+1 \geq 0$ 이고 $3-2x \geq 0$ 이어야 하므로
 $-\frac{1}{2} \leq x \leq \frac{3}{2}$ 23) $-2 \leq x \leq 3$ $\Rightarrow \sqrt{3-x} + \sqrt{3x+6}$ 의 값이 실수하려면
 $3-x \geq 0$ 이고, $3x+6 \geq 0$ 이어야 하므로
 $\boxed{-2} \leq x \leq \boxed{3}$ 24) $-2 \leq x \leq 1$ $\Rightarrow x+2 \geq 0$ 이고, $1-x \geq 0$ 이어야 하므로
 $x \geq -2$ 이고, $x \leq 1$
 $\therefore -2 \leq x \leq 1$ 25) a $\Rightarrow a > 0$ 에서 $-a < 0$ 이므로
 $\sqrt{(-a)^2} = |-a| = -(-a) = a$ 26) $-a+b$ $\Rightarrow a < 0, b < 0$ 이므로
 $\sqrt{a^2} - \sqrt{b^2} = -a - (-b) = -a + b$ 27) $-b$ $\Rightarrow b < 0$ 에서 $-b > 0$ 이므로
 $\sqrt{(-b)^2} = -b$ 28) $a+b$ $\Rightarrow a > 0, b > 0$ 에서 $-a < 0$ 이므로
 $\sqrt{(-a)^2} + \sqrt{b^2} = -(-a) + b = a + b$ 29) $-a+b$ $\Rightarrow a < b$ 에서 $a-b < 0$ 이므로
 $\sqrt{(a-b)^2} = -(a-b) = -a + b$ 30) $a+b$ $\Rightarrow a > 0$ 이고, $b > 0$ 에서 $-b < 0$ 이므로
 $\sqrt{a^2} + |-b| = a - (-b) = a + b$ 31) $-a+2b$ $\Rightarrow a < b < 0$ 에서 $a-b < 0$ 이므로
 $\sqrt{(a-b)^2} - |b| = -(a-b) - (-b)$
 $= -a + 2b$ 32) $a+b-ab$ $\Rightarrow a > 0, b > 0$ 에서 $a+b > 0, ab > 0$ 이므로
 $\sqrt{(a+b)^2} - |ab| = |a+b| - |ab|$
 $= a + b - ab$

33) $b - a - ab$

$\Rightarrow 0 < a < b$ 에서 $b - a > 0$, $ab > 0$ 이므로

$$\sqrt{(b-a)^2} - |ab| = b - a - ab$$

34) 3

$\Rightarrow -1 < a < 2$ 에서 $a+1 > 0$, $a-2 < 0$ 이므로

$$\begin{aligned}\sqrt{a^2+2a+1} + \sqrt{a^2-4a+4} &= \sqrt{(a+1)^2} + \sqrt{(a-2)^2} \\ &= a+1 - (a-2) \\ &= 3\end{aligned}$$

35) $\sqrt{x+3} + \sqrt{x}$

$$\begin{aligned}\Rightarrow \frac{3}{\sqrt{x+3} - \sqrt{x}} &= \frac{3(\sqrt{x+3} + \sqrt{x})}{(\sqrt{x+3} - \sqrt{x})(\sqrt{x+3} + \sqrt{x})} \\ &= \frac{3(\sqrt{x+3} + \sqrt{x})}{(x+3) - x} \\ &= \sqrt{x+3} + \sqrt{x}\end{aligned}$$

36) $\sqrt{x+1} - 1$

$$\begin{aligned}\Rightarrow \frac{x}{\sqrt{x+1} + 1} &= \frac{x(\sqrt{x+1} - 1)}{(\sqrt{x+1} + 1)(\sqrt{x+1} - 1)} \\ &= \frac{x(\sqrt{x+1} - 1)}{x} \\ &= \boxed{\sqrt{x+1} - 1}\end{aligned}$$

37) $\frac{\sqrt{x+2} - \sqrt{x}}{2}$

$$\begin{aligned}\Rightarrow \frac{1}{\sqrt{x} + \sqrt{x+2}} &= \frac{\sqrt{x} - \sqrt{x+2}}{(\sqrt{x} + \sqrt{x+2})(\sqrt{x} - \sqrt{x+2})} \\ &= \frac{\sqrt{x} - \sqrt{x+2}}{x - (x+2)} \\ &= \frac{\sqrt{x+2} - \sqrt{x}}{2}\end{aligned}$$

38) $-\sqrt{x-1} - \sqrt{x}$

$$\begin{aligned}\Rightarrow \frac{1}{\sqrt{x-1} - \sqrt{x}} &= \frac{\sqrt{x-1} + \sqrt{x}}{(\sqrt{x-1} - \sqrt{x})(\sqrt{x-1} + \sqrt{x})} \\ &= \frac{\sqrt{x-1} + \sqrt{x}}{(x-1) - x} \\ &= -\sqrt{x-1} - \sqrt{x}\end{aligned}$$

39) $\sqrt{x+1} - 1$

$$\begin{aligned}\Rightarrow \frac{x}{1 + \sqrt{x+1}} &= \frac{x(1 - \sqrt{x+1})}{(1 + \sqrt{x+1})(1 - \sqrt{x+1})} \\ &= \frac{x(1 - \sqrt{x+1})}{1 - (x+1)} \\ &= \frac{x(1 - \sqrt{x+1})}{-x} \\ &= \sqrt{x+1} - 1\end{aligned}$$

40) $-1 - \sqrt{x-1}$

$$\Rightarrow \frac{x-2}{1 - \sqrt{x-1}} = \frac{(x-2)(1 + \sqrt{x-1})}{(1 - \sqrt{x-1})(1 + \sqrt{x-1})}$$

$$\begin{aligned}&= \frac{(x-2)(1 + \sqrt{x-1})}{1 - (x-1)} \\ &= \frac{(x-2)(1 + \sqrt{x-1})}{2-x} \\ &= -1 - \sqrt{x-1}\end{aligned}$$

41) $x - \sqrt{x^2 - x}$

$$\begin{aligned}\Rightarrow \frac{x}{\sqrt{x^2 - x} + x} &= \frac{x(\sqrt{x^2 - x} - x)}{(\sqrt{x^2 - x} + x)(\sqrt{x^2 - x} - x)} \\ &= \frac{x(\sqrt{x^2 - x} - x)}{(x^2 - x) - x^2} \\ &= \frac{x(\sqrt{x^2 - x} - x)}{-x} \\ &= x - \sqrt{x^2 - x}\end{aligned}$$

42) $\sqrt{x+3} - \sqrt{x+1}$

$$\begin{aligned}\Rightarrow \frac{2}{\sqrt{x+1} + \sqrt{x+3}} &= \frac{2(\sqrt{x+1} - \sqrt{x+3})}{(\sqrt{x+1} + \sqrt{x+3})(\sqrt{x+1} - \sqrt{x+3})} \\ &= \frac{2(\sqrt{x+1} - \sqrt{x+3})}{(x+1) - (x+3)} \\ &= \sqrt{x+3} - \sqrt{x+1}\end{aligned}$$

43) $\frac{x+8-4\sqrt{x+4}}{x}$

$$\begin{aligned}\Rightarrow \frac{\sqrt{x+4} - 2}{\sqrt{x+4} + 2} &= \frac{(\sqrt{x+4} - 2)^2}{(\sqrt{x+4} + 2)(\sqrt{x+4} - 2)} \\ &= \frac{x+4-4\sqrt{x+4}+4}{x} \\ &= \frac{x+8-4\sqrt{x+4}}{x}\end{aligned}$$

44) $2x$

$$\begin{aligned}\Rightarrow x + \sqrt{x^2 + 1} - \frac{1}{x + \sqrt{x^2 + 1}} \\ &= x + \sqrt{x^2 + 1} - \frac{x - \sqrt{x^2 + 1}}{(x + \sqrt{x^2 + 1})(x - \sqrt{x^2 + 1})} \\ &= x + \sqrt{x^2 + 1} - \frac{x - \sqrt{x^2 + 1}}{x^2 - (x^2 + 1)} \\ &= x + \sqrt{x^2 + 1} + (x - \sqrt{x^2 + 1}) \\ &= 2x\end{aligned}$$

45) -4

$$\begin{aligned}\Rightarrow \frac{x^2}{\sqrt{x^2 + 4} + 2} - \frac{x^2}{\sqrt{x^2 + 4} - 2} \\ &= \frac{x^2(\sqrt{x^2 + 4} - 2) - x^2(\sqrt{x^2 + 4} + 2)}{(\sqrt{x^2 + 4} + 2)(\sqrt{x^2 + 4} - 2)} \\ &= \frac{-4x^2}{x^2 + 4 - 4} \\ &= -4\end{aligned}$$

$$\begin{aligned}
 46) \quad & \frac{2x+2}{-x+1} \\
 \Rightarrow & \frac{1-\sqrt{x}}{1+\sqrt{x}} + \frac{1+\sqrt{x}}{1-\sqrt{x}} = \frac{(1-\sqrt{x})^2 + (1+\sqrt{x})^2}{(1+\sqrt{x})(1-\sqrt{x})} \\
 & = \frac{(1-2\sqrt{x}+2) + (1+2\sqrt{x}+x)}{1-x} \\
 & = \frac{2x+2}{-x+1}
 \end{aligned}$$

$$\begin{aligned}
 47) \quad & -4 \\
 \Rightarrow & \frac{x}{\sqrt{x+4}+2} - \frac{x}{\sqrt{x+4}-2} \\
 & = \frac{x(\sqrt{x+4}-2) - x(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)(\sqrt{x+4}-2)} \\
 & = \frac{x(\sqrt{x+4}-2) - x(\sqrt{x+4}+2)}{(x+4)-4} \\
 & = \frac{-4x}{x} = -4
 \end{aligned}$$

$$\begin{aligned}
 48) \quad & -6 \\
 \Rightarrow & \frac{x}{\sqrt{x+9}+3} - \frac{x}{\sqrt{x+9}-3} \\
 & = \frac{x(\sqrt{x+9}-3) - x(\sqrt{x+9}+3)}{(\sqrt{x+9}+3)(\sqrt{x+9}-3)} \\
 & = \frac{x\sqrt{x+9}-3x-x\sqrt{x+9}-3x}{(x+9)-9} \\
 & = \frac{-6x}{x} = -6
 \end{aligned}$$

$$\begin{aligned}
 49) \quad & \frac{\sqrt{x+2}}{2} \\
 \Rightarrow & \frac{1}{\sqrt{x+1}-\sqrt{x-2}} + \frac{1}{\sqrt{x+2}+\sqrt{x-2}} \\
 & = \frac{\sqrt{x+2}+\sqrt{x-2}+\sqrt{x+2}-\sqrt{x-2}}{(\sqrt{x+2}-\sqrt{x-2})(\sqrt{x+2}+\sqrt{x-2})} \\
 & = \frac{2\sqrt{x+2}}{(x+2)-(x-2)} \\
 & = \frac{\sqrt{x+2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 50) \quad & -2 \\
 \Rightarrow & \frac{x}{1+\sqrt{x+1}} + \frac{x}{1-\sqrt{x+1}} \\
 & = \frac{x(1-\sqrt{x+1}) + x(1+\sqrt{x+1})}{(1+\sqrt{x+1})(1-\sqrt{x+1})} \\
 & = \frac{2x}{1-(x+1)} \\
 & = \frac{2x}{-x} = -2
 \end{aligned}$$

$$\begin{aligned}
 51) \quad & x \\
 \Rightarrow & \frac{1}{x+\sqrt{x^2-1}} + \sqrt{x^2-1}
 \end{aligned}$$

$$\begin{aligned}
 & = \frac{x-\sqrt{x^2-1}}{(x+\sqrt{x^2-1})(x-\sqrt{x^2-1})} + \sqrt{x^2-1} \\
 & = \frac{x-\sqrt{x^2-1}}{x^2-(x^2-1)} + \sqrt{x^2-1} \\
 & = (x-\sqrt{x^2-1}) + \sqrt{x^2-1} \\
 & = x
 \end{aligned}$$

$$\begin{aligned}
 52) \quad & -2x \\
 \Rightarrow & \frac{\sqrt{x}}{\sqrt{x}+\sqrt{x+1}} + \frac{\sqrt{x}}{\sqrt{x}-\sqrt{x+1}} \\
 & = \frac{\sqrt{x}(\sqrt{x}-\sqrt{x+1}) + \sqrt{x}(\sqrt{x}+\sqrt{x+1})}{(\sqrt{x}+\sqrt{x+1})(\sqrt{x}-\sqrt{x+1})} \\
 & = \frac{2x}{x-(x+1)} \\
 & = -2x
 \end{aligned}$$

$$\begin{aligned}
 53) \quad & 2 \\
 \Rightarrow & (\sqrt{x+1}+\sqrt{x-1})(\sqrt{x+1}-\sqrt{x-1}) \\
 & = (\sqrt{x+1})^2 - (\sqrt{x-1})^2 \\
 & = (x+1) - (x-1) \\
 & = 2
 \end{aligned}$$

$$\begin{aligned}
 54) \quad & x^2+x-2 \\
 \Rightarrow & (\sqrt{x+3}+1)(\sqrt{x+3}-1)(x-1) \\
 & = \{(\sqrt{x+3})^2-1^2\}(x-1) \\
 & = (x+2)(x-1) \\
 & = x^2+x-2
 \end{aligned}$$

$$\begin{aligned}
 55) \quad & 1 \leq x \leq 5 \\
 \Rightarrow & \sqrt{x-5}\sqrt{1-x} = -\sqrt{-x^2+6x-5} \\
 & = -\sqrt{-(x^2-6x+5)} \\
 & = -\sqrt{-(x-1)(x-5)} \\
 & = -\sqrt{(x-5)(1-x)} \\
 \text{예시} \quad & x-5 \leq 0, 1-x \leq 0 \\
 \therefore & 1 \leq x \leq 5
 \end{aligned}$$

$$\begin{aligned}
 56) \quad & -1 \leq x < 1 \\
 \Rightarrow & \frac{\sqrt{x+1}}{\sqrt{x-1}} = -\sqrt{\frac{x+1}{x-1}} \quad \text{예시} \\
 & x+1 \geq 0, x-1 < 0 \\
 \therefore & -1 \leq x < 1
 \end{aligned}$$

$$\begin{aligned}
 57) \quad & -1 \leq x < 0 \\
 \Rightarrow & \frac{\sqrt{x+1}}{\sqrt{x}} = -\sqrt{\frac{x+1}{x}} \quad \text{예시} \\
 & x+1 \geq 0, x < 0 \quad \therefore -1 \leq x < 0
 \end{aligned}$$

$$\begin{aligned}
 58) \quad & -2 \leq x < 1 \\
 \Rightarrow & \frac{\sqrt{x+2}}{\sqrt{x-1}} = -\sqrt{\frac{x+2}{x-1}} \quad \text{예시} \\
 & x+2 \geq 0, x-1 < 0 \quad \therefore -2 \leq x < 1
 \end{aligned}$$

$$59) \quad x \leq 0$$

$$\Rightarrow \sqrt{x-1} \sqrt{x} = -\sqrt{x^2-2} = -\sqrt{x(x-1)} \text{에서}$$

$$x \leq 0, x-1 \leq 0 \quad \therefore x \leq 0$$

$$60) -3 \leq x < 3$$

$$\Rightarrow \frac{\sqrt{x+3}}{\sqrt{x-3}} = -\sqrt{\frac{x+3}{x-3}} \text{에서}$$

$$x+3 \geq 0, x-3 < 0$$

$$\therefore -3 \leq x < 3$$

$$61) x \leq -1$$

$$\Rightarrow \sqrt{x-2} \sqrt{x+1} = -\sqrt{x^2-x-2}$$

$$= -\sqrt{(x-2)(x+1)}$$

$$\text{에서 } x-2 \leq 0, x+1 \leq 0 \quad \therefore x \leq -1$$

$$62) 2 \leq x \leq 3$$

$$\Rightarrow \sqrt{x-3} \sqrt{2-x} = -\sqrt{-x^2+5x-6}$$

$$= -\sqrt{-(x^2-5x+6)}$$

$$= -\sqrt{-(x-2)(x-3)}$$

$$= -\sqrt{(x-3)(2-x)}$$

$$\text{에서 } x-3 \leq 0, 2-x \leq 0 \quad \therefore 2 \leq x \leq 3$$

$$63) 4$$

$$\Rightarrow x+y = (2+\sqrt{2}) + (2-\sqrt{2}) = 4$$

$$64) -6-4\sqrt{2}$$

$$\Rightarrow \frac{1-\sqrt{x}}{1+\sqrt{x}} + \frac{1+\sqrt{x}}{1-\sqrt{x}} = \frac{(1-\sqrt{x})^2 + (1+\sqrt{x})^2}{(1+\sqrt{x})(1-\sqrt{x})}$$

$$= \frac{(1-2\sqrt{x}+x) + (1+2\sqrt{x}+x)}{1-x}$$

$$= \frac{2+2x}{1-x}$$

$$\text{이때, } x = \sqrt{2} \text{ 이므로}$$

$$(\text{주어진 식}) = \frac{2+2\sqrt{2}}{1-\sqrt{2}} = \frac{2(1+\sqrt{2})^2}{(1-\sqrt{2})(1+\sqrt{2})}$$

$$= \frac{2(1+2\sqrt{2}+2)}{1-2}$$

$$= -6-4\sqrt{2}$$

$$65) 2\sqrt{2}$$

$$\Rightarrow x-y = (2+\sqrt{2}) - (2-\sqrt{2}) = 2\sqrt{2}$$

$$66) \sqrt{3}+1$$

$$\Rightarrow \frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = \frac{\sqrt{x}+1 - (\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{2}{x-1}$$

$$x = \sqrt{3} \text{ 을 대입하면}$$

$$\frac{2}{x-1} = \frac{2}{\sqrt{3}-1} = \frac{2(\sqrt{3}+1)}{2} = \sqrt{3}+1$$

$$67) \sqrt{2}$$

$$\Rightarrow \frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = \frac{2}{x-1}$$

$$x = \sqrt{2}+1 \text{ 을 대입하면}$$

$$\frac{d}{x-1} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2} \cdot \sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$68) 2\sqrt{2}+2$$

$$\Rightarrow \frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = \frac{(\sqrt{x}+1) - (\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{2}{x-1}$$

$$\text{이때, } x = \sqrt{2} \text{ 이므로}$$

$$(\text{주어진 식}) = \frac{2}{\sqrt{2}-1} = \frac{2(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{2\sqrt{2}+2}{2-1} = 2\sqrt{2}+2$$

$$69) 4+2\sqrt{2}$$

$$\Rightarrow \frac{1}{1-\sqrt{x}} + \frac{1}{1+\sqrt{x}} = \frac{(1+\sqrt{x}) + (1-\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})}$$

$$= \frac{2}{1-x}$$

$$\text{이때, } x = \frac{\sqrt{2}}{2} \text{ 에서}$$

$$1-x = 1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$$

$$\frac{1}{1-x} = \frac{2}{2-\sqrt{2}} = \frac{2(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = 2+\sqrt{2} \text{ 이므로}$$

$$(\text{주어진 식}) = \frac{2}{1-x} = 2(2+\sqrt{2}) = 4+2\sqrt{2}$$

$$70) \sqrt{2}+1$$

$$\Rightarrow \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}}$$

$$= \frac{(\sqrt{x+1} + \sqrt{x-1})^2}{(\sqrt{x+1} - \sqrt{x-1})(\sqrt{x+1} + \sqrt{x-1})}$$

$$= \frac{x+1+2\sqrt{x+1}\sqrt{x-1}+x-1}{x+1-(x-1)}$$

$$= x + \sqrt{x^2-1}$$

$$\text{이때, } x = \sqrt{2} \text{ 이므로}$$

$$(\text{주어진 식}) = x + \sqrt{x^2-1} = \sqrt{2}+1$$

$$71) -2\sqrt{3}$$

$$\Rightarrow \frac{1}{x+\sqrt{x^2+1}} + \frac{1}{x-\sqrt{x^2+1}}$$

$$= \frac{(x-\sqrt{x^2+1}) + (x+\sqrt{x^2+1})}{(x+\sqrt{x^2+1})(x-\sqrt{x^2+1})}$$

$$= \frac{2x}{x^2-(x^2+1)} = -2x$$

$$\text{이때, } x = \sqrt{3} \text{ 이므로}$$

$$(\text{주어진 식}) = -2\sqrt{3}$$

$$72) 2$$

$$\Rightarrow xy = (2+\sqrt{2})(2-\sqrt{2}) = 4-2 = 2$$

$$73) 2-2\sqrt{2}$$

$$\begin{aligned} \Rightarrow & \frac{1}{1-\sqrt{x+1}} \frac{+1}{1+\sqrt{x+1}} \\ &= \frac{(1+\sqrt{x+1})+(1-\sqrt{x+1})}{(1-\sqrt{x+1})(1+\sqrt{x+1})} \\ &= \frac{2}{1-(x+1)} = -\frac{2}{x} \end{aligned}$$

이때, $x = \sqrt{2}+1$ 이므로

$$\begin{aligned} (\text{주어진 식}) &= -\frac{2}{\sqrt{2}+1} = -\frac{2(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} \\ &= -\frac{2\sqrt{2}-2}{2-1} = 2-2\sqrt{2} \end{aligned}$$

$$74) -6-4\sqrt{3}$$

$$\begin{aligned} \Rightarrow & \frac{\sqrt{x}+1}{\sqrt{x}-1} + \frac{\sqrt{x}-1}{\sqrt{x}+1} = \frac{(\sqrt{x}+1)^2 + (\sqrt{x}-1)^2}{(\sqrt{x}-1)(\sqrt{x}+1)} \\ &= \frac{(x+2\sqrt{x}+1) + (x-2\sqrt{x}+1)}{x-1} \\ &= \frac{2x+2}{x-1} \end{aligned}$$

이때, $x = \frac{2}{\sqrt{3}+1} = \frac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \sqrt{3}-1$ 이므로

$$\begin{aligned} (\text{주어진 식}) &= \frac{(2\sqrt{3}-1)+2}{(\sqrt{3}-1)-1} = \frac{2\sqrt{3}}{\sqrt{3}-2} \\ &= \frac{2\sqrt{3}(\sqrt{3}+2)}{(\sqrt{3}-1)(\sqrt{3}+2)} = \frac{6+4\sqrt{3}}{3-4} \\ &= -6-4\sqrt{3} \end{aligned}$$

$$75) \sqrt{2}+1$$

$$\begin{aligned} \Rightarrow & \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} \\ &= \frac{(\sqrt{x}+\sqrt{y})^2}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} = \frac{x+y+2\sqrt{xy}}{x-y} \\ &= \frac{4+2\sqrt{2}}{2\sqrt{2}} = \sqrt{2}+1 \end{aligned}$$

$$76) -2-2\sqrt{2}$$

$$\begin{aligned} \Rightarrow x &= \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sqrt{2}-1 \text{이므로} \\ \frac{\sqrt{x}-1}{\sqrt{x}+1} + \frac{\sqrt{x}+1}{\sqrt{x}-1} &= \frac{2(x+1)}{x-1} = \frac{2\sqrt{2}}{\sqrt{2}-2} \\ &= \frac{2\sqrt{2}(\sqrt{2}+2)}{(\sqrt{2}-2)(\sqrt{2}+2)} = \frac{2(2+2\sqrt{2})}{-2} \\ &= -2-2\sqrt{2} \end{aligned}$$

$$77) 2\sqrt{2}+2$$

$$\begin{aligned} \Rightarrow & \frac{1+\sqrt{x}}{1-\sqrt{x}} + \frac{1-\sqrt{x}}{1+\sqrt{x}} = \frac{(1+\sqrt{x})^2 + (1-\sqrt{x})^2}{(1-\sqrt{x})(1+\sqrt{x})} \\ &= \frac{(1+2\sqrt{x}+x) + (1-2\sqrt{x}+x)}{1-x} \\ &= \frac{2+2x}{1-x} \end{aligned}$$

이때, $x = \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sqrt{2}-1$ 이므로

$$\begin{aligned} (\text{주어진 식}) &= \frac{2+2x}{1-x} = \frac{2+2(\sqrt{2}-1)}{1-(\sqrt{2}-1)} \\ &= \frac{2\sqrt{2}}{2-\sqrt{2}} \\ &= \frac{2\sqrt{2}(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} \\ &= 2\sqrt{2}+2 \end{aligned}$$