



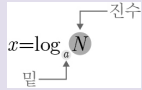
◇「콘텐츠산업 진흥법 시행령」제33조에 의한 표시
 1) 제작연월일 : 2019-02-13
 2) 제작자 : 교육지대(주)
 3) 이 콘텐츠는 「콘텐츠산업 진흥법」에 따라 최초 제작일부터 5년간 보호됩니다.

◇「콘텐츠산업 진흥법」외에도「저작권법」에 의하여 보호되는 콘텐츠의 경우, 그 콘텐츠의 전부 또는 일부를 무단으로 복제하거나 전송하는 것은 콘텐츠산업 진흥법 외에도 저작권법에 의한 법적 책임을 질 수 있습니다.

01 / 로그의 정의

(1) 로그의 정의

: $a > 0, a \neq 1$ 일 때, 양수 N 에 대하여 $a^x = N$ 을 만족시키는 실수 x 를 a 를 밑으로 하는 N 의 로그라 하고 기호로는 $\log_a N$ 과 같이 나타낸다. 이때 N 을 $\log_a N$ 의 진수라 한다.



(2) 로그의 밑과 진수의 조건

- ① 밑의 조건 : $a > 0, a \neq 1$
 ② 진수의 조건 : $N > 0$

■ 다음 등식을 로그를 사용하여 나타내어라.

- $125^{\frac{1}{3}} = 5$
- $5^{\frac{1}{2}} = \sqrt{5}$
- $3^4 = 81$
- $10^{-3} = 0.001$
- $2^{-3} = \frac{1}{8}$
- $10^4 = 10000$
- $\left(\frac{1}{5}\right)^{-3} = 125$

■ 다음 등식을 만족하는 x 의 값을 구하여라.

- $\log_x 2 = 4$
- $\log_x 16 = 4$
- $\log_{\frac{1}{3}} x = -2$
- $\log_3 x = 4$
- $\log_x 100000 = 5$
- $\log_3 x = -4$
- $\log_4 \frac{1}{64} = x$
- $\log_2 1024 = x$
- $\log_x \frac{1}{100} = -2$

■ 다음 값이 존재하기 위한 실수 x 의 값의 범위를 구하여라.

17. $\log_{x+2} 3$

18. $\log_2 (x+1)$

19. $\log_{x-1} 7$

20. $\log_3 (x+1)$

21. $\log_{x-2} (5-x)$

22. $\log_{x+3} (x-1)$

23. $\log_x (x^2+2x-3)$

24. $\log_{x-1} (-x^2+2x+24)$

25. $\log_{x+1} (x^2-2x-8)$

26. $\log_5 (x^2-x-12)$

27. $\log_3 (x-1)(x-4)$

28. $\log_{x-2} (x^2-8x+15)$

29. $\log_{x-1} (-x^2+3x+10)$

02 로그의 성질

$a > 0, a \neq 1, M > 0, N > 0$ 일 때

(1) $\log_a 1 = 0, \log_a a = 1$

(2) $\log_a MN = \log_a M + \log_a N$

(3) $\log_a \frac{M}{N} = \log_a M - \log_a N$

(4) $\log_a M^k = k \log_a M$ (단, k 는 실수)

■ 다음 식의 값을 구하여라.

30. $\log_5 1$

31. $\log_2 2^{\frac{3}{7}}$

32. $\log_{10} \sqrt{0.01}$

33. $\log_7 7^{-4}$

34. $\log_2 0.25$

35. $\log_{49} \sqrt{343}$

36. $\log_2 \sqrt[3]{16}$

37. $\log_{12} \frac{1}{144}$

38. $2 \log_5 \sqrt{5}$

39. $\log_{0.25} 4$

40. $\log_{2\sqrt{2}} \sqrt[4]{32}$

41. $\log_{125} \sqrt[3]{25}$

42. $\log_3 63 - \log_3 7$

43. $9^{\frac{3}{2}} + \log_3 81$

44. $\log_6 3 + \log_6 12$

45. $\log_2 18 - 2 \log_2 6$

46. $\log_4 2 + \log_4 8$

47. $\log_2 16 + \log_2 \frac{1}{8}$

48. $\log_3 24 + 3 \log_3 \frac{3}{2}$

49. $\log_{12} 4 + \log_{12} 3$

50. $\log_3 36 + \log_3 \frac{1}{4}$

51. $\log_6 \frac{8}{3} + \log_6 \frac{27}{2}$

52. $\log_8 \frac{1}{2} - \log_8 \frac{1}{128}$

53. $\log_2 \sqrt[3]{128} + \log_2 \sqrt[3]{4}$

54. $\log_{\frac{1}{2}} \frac{1}{3} - \log_{\frac{1}{2}} \frac{1}{24}$

$$55. \log_6 72 - \log_6 \frac{1}{3}$$

$$56. \log_3 6 + \log_3 2 - \log_3 4$$

$$57. \log_3 \sqrt{27} - \log_3 \frac{1}{\sqrt{3}}$$

$$58. \log_2 \frac{4}{3} + 2 \log_2 \sqrt{12}$$

$$59. \frac{1}{\sqrt[3]{8}} \times \log_3 81$$

$$60. \log_2 \sqrt{3} + \frac{1}{2} \log_2 \frac{1}{2} - \frac{3}{2} \log_2 \sqrt[3]{6}$$

$$61. \log_2 12 + \log_2 \frac{1}{3} + \log_2 6 - 2 \log_2 \sqrt{3}$$

$$62. \log_{\sqrt{3}} 2 \sqrt{3} - \log_9 \frac{9}{16} + 2 \log_{\frac{1}{3}} \frac{4}{3}$$

$$63. \log_5 \sqrt{6} - \log_{25} 3 - \frac{3}{2} \log_5 \sqrt[3]{2}$$

$$64. 3 \log_3 2 + \frac{1}{2} \log_3 5 - \log_3 8 \sqrt{5}$$

$$65. \log_6 45 + \log_6 20 - 2 \log_6 5$$

$$66. \frac{1}{2} \log_2 12 + \log_2 \sqrt{6} - \log_2 \frac{3}{2}$$

$$67. \log_2 3 - \log_2 \frac{9}{2} + \log_2 12$$

$$68. \log_{\frac{1}{3}} \frac{\sqrt{3}}{2} + \log_{\frac{1}{3}} 2 \sqrt{3}$$

$$69. \frac{2}{3} \log_3 \sqrt{10} - \frac{2}{3} \log_3 \frac{1}{2} + 2 \log_3 \sqrt[3]{5}$$

$$70. \log_2 (\log_3 25) + \log_2 (\log_5 9)$$

▣ $\log_{10} 2 = a$, $\log_{10} 3 = b$ 일 때, 다음을 a , b 로 나타내어라.

71. $\log_{10} 5$

72. $\log_{10} 45$

73. $\log_{10} 60$

74. $\log_{10} \sqrt[4]{15}$

75. $\log_{10} \left(\frac{4}{5}\right)^3$

76. $\log_{10} 0.072$

77. $\log_{10} \sqrt{72}$

78. $\log_{10} 24$

79. $\log_{10} \frac{8}{27}$

80. $\log_{10} \frac{9}{32}$

81. $\log_{10} \frac{1}{125}$



정답 및 해설

1) $\log_{125} 5 = \frac{1}{3}$

2) $\log_5 \sqrt{5} = \frac{1}{2}$

3) $4 = \log_3 81$

4) $\log_{10} 0.001 = -3$

5) $\log_2 \frac{1}{8} = -3$

6) $\log_{10} 10000 = 4$

7) $\log_{\frac{1}{5}} 125 = -3$

8) $\sqrt[4]{2}$

$$\Rightarrow \log_x 2 = 4 \text{에서 } x^4 = 2$$

$$\therefore x = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

9) 2

$$\Rightarrow \log_x 16 = 4 \text{에서 } x^4 = 16$$

$$\therefore x = 16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2$$

10) 9

$$\Rightarrow \log_{\frac{1}{3}} x = -2 \text{에서 } x = \left(\frac{1}{3}\right)^{-2} = 9$$

11) 81

$$\Rightarrow \log_3 x = 4 \text{에서 } x = 3^4 = 81$$

12) 10

$$\Rightarrow \log_x 100000 = 5 \text{에서 } x^5 = 100000$$

$$\text{이때, } 100000 = 10^5 \text{이므로 } x^5 = 10^5 \therefore x = 10$$

13) $\frac{1}{81}$

$$\Rightarrow \log_3 x = -4 \text{에서 } x = 3^{-4} = \frac{1}{81}$$

14) -3

$$\Rightarrow \log_4 \frac{1}{64} = x \text{에서 } 4^x = \frac{1}{64}$$

$$\text{이때, } \frac{1}{64} = 4^{-3} \text{이므로 } 4^x = 4^{-3} \therefore x = -3$$

15) 10

$$\Rightarrow \log_2 1024 = x \text{에서 } 2^x = 1024$$

$$\text{이때, } 1024 = 2^{10} \text{이므로 } 2^x = 2^{10} \therefore x = 10$$

16) 10

$$\Rightarrow \log_x \frac{1}{100} = -2 \text{에서 } x^{-2} = \frac{1}{100}$$

$$\frac{1}{x^2} = \left(\frac{1}{10}\right)^2$$

$$x^2 = 10^2 \therefore x = \pm 10$$

그런데 $x > 0$ 이므로 $x = 10$

17) $-2 < x < -1$ 또는 $x > -1$

$$\Rightarrow \text{밑 조건에서 } x+2 > 0, x+2 \neq 1$$

$$\therefore x > -2, x \neq -1$$

$$\therefore -2 < x < -1 \text{ 또는 } x > -1$$

18) $x > -1$

$$\Rightarrow \text{진수의 조건에서 } x+1 > 0 \therefore x > -1$$

19) $1 < x < 2$ 또는 $x > 2$

$$\Rightarrow \text{밑의 조건에서 } x-1 > 0, x-1 \neq 1$$

$$\text{즉, } x > 1, x \neq 2 \text{이므로 } 1 < x < 2 \text{ 또는 } x > 2$$

20) $x > -1$

$$\Rightarrow \text{진수의 조건에서 } x+1 > 0 \therefore x > -1$$

21) $2 < x < 3$ 또는 $3 < x < 5$

$$\Rightarrow \text{밑의 조건에서 } x-2 > 0, x-2 \neq 1$$

$$\text{즉, } x > 2, x \neq 3 \text{이므로}$$

$$2 < x < 3 \text{ 또는 } x > 3 \quad \dots\dots \textcircled{7}$$

$$\text{또, 진수의 조건에서 } 5-x > 0$$

$$\therefore x < 5 \quad \dots\dots \textcircled{8}$$

$$\textcircled{7}, \textcircled{8} \text{의 공통 범위이므로}$$

$$2 < x < 3 \text{ 또는 } 3 < x < 5$$

22) $x > 1$

$$\Rightarrow \text{밑의 조건에서 } x+3 > 0, x+3 \neq 1$$

$$\text{즉, } x > -3, x \neq -2 \text{이므로}$$

$$-3 < x < -2 \text{ 또는 } x > -2 \quad \dots\dots \textcircled{7}$$

$$\text{또, 진수의 조건에서 } x-1 > 0 \therefore x > 1 \quad \dots\dots \textcircled{8}$$

$$\textcircled{7}, \textcircled{8} \text{의 공통 범위이므로 } x > 1$$

23) $x > 1$

$$\Rightarrow \text{(i) 밑 조건에서 } x > 0, x \neq 1$$

$$\text{(ii) 진수 조건에서 } x^2 + 2x - 3 > 0$$

$$(x+3)(x-1) > 0$$

$$\therefore x < -3 \text{ 또는 } x > 1$$

$$\text{(i), (ii)에서 } x > 1$$

24) $1 < x < 2$ 또는 $2 < x < 6$

$$\Rightarrow \text{밑의 조건에서 } x-1 > 0, x-1 \neq 1$$

$$\text{즉, } x > 1, x \neq 2 \text{이므로}$$

$$1 < x < 2 \text{ 또는 } x > 2 \quad \dots\dots \textcircled{7}$$

$$\text{또, 진수의 조건에서 } -x^2 + 2x + 24 > 0,$$

$$x^2 - 2x - 24 < 0 \quad \dots\dots \textcircled{8}$$

$$\textcircled{7}, \textcircled{8} \text{의 공통 범위이므로}$$

$$1 < x < 2 \text{ 또는 } 2 < x < 6$$

25) $x > 4$

⇒ 밑의 조건에서 $x+1 > 0$, $x+1 \neq 1$
 즉, $x > -1$, $x \neq 0$ 이므로
 $-1 < x < 0$ 또는 $x > 0$ ㉞
 또, 진수의 조건에서
 $x^2 - 2x - 8 > 0$, $(x+2)(x-4) > 0$
 $\therefore x < -2$ 또는 $x > 4$ ㉟
 ㉞, ㉟의 공통 범위이므로 $x > 4$

26) $x < -3$ 또는 $x > 4$

⇒ 진수의 조건에서 $x^2 - x - 12 > 0$
 $(x+3)(x-4) > 0 \quad \therefore x < -3$ 또는 $x > 4$

27) $x < 1$ 또는 $x > 4$

⇒ 진수의 조건에서 $(x-1)(x-4) > 0$
 $\therefore x < 1$ 또는 $x > 4$

28) $2 < x < 3$ 또는 $x > 5$

⇒ 밑의 조건에서 $x-2 > 0$, $x-2 \neq 1$
 즉, $x > 2$, $x \neq 3$ 이므로
 $2 < x < 3$ 또는 $x > 3$ ㉞
 또, 진수의 조건에서 $x^2 - 8x + 15 > 0$
 $(x-3)(x-5) > 0$
 $\therefore x < 3$ 또는 $x > 5$ ㉟
 ㉞, ㉟의 공통범위이므로 $2 < x < 3$ 또는 $x > 5$

29) $1 < x < 2$ 또는 $2 < x < 5$

⇒ (i) 밑 조건에서 $x-1 > 0$, $x-1 \neq 1$
 $\therefore x > 1$, $x \neq 2$
 (ii) 진수 조건에서 $-x^2 + 3x + 10 > 0$
 $x^2 - 3x - 10 < 0$
 $(x+2)(x-5) < 0 \quad \therefore -2 < x < 5$
 (i), (ii)에서 $1 < x < 2$ 또는 $2 < x < 5$

30) 0

31) $\frac{3}{7}$

32) -1

⇒ $\log_{10} \sqrt{0.01} = \log_{10} (10^{-2})^{\frac{1}{2}} = \log_{10} 10^{-1} = -1$

33) -4

34) -2

⇒ $\log_2 0.25 = \log_2 \frac{1}{4} = \log_2 2^{-2} = -2 \log_2 2 = -2$

35) $\frac{3}{4}$

⇒ $\log_{49} \sqrt{343} = x$ 로 놓으면 로그의 정의에 의하여
 $49^x = \sqrt{343}$ 이므로 $(7^2)^x = \sqrt{7^3}$
 $7^{2x} = 7^{\frac{3}{2}}$ 에서 $2x = \frac{3}{2} \quad \therefore x = \frac{3}{4}$

$$\therefore \log_{49} \sqrt{343} = \frac{3}{4}$$

36) $\frac{4}{3}$

$$\Rightarrow \log_2 \sqrt[3]{16} = \log_2 16^{\frac{1}{3}} = \log_2 2^{\frac{4}{3}} = \frac{4}{3}$$

37) -2

$$\Rightarrow \log_{12} \frac{1}{144} = \log_{12} \left(\frac{1}{12} \right)^2 = \log_{12} 12^{-2} = -2$$

38) 1

$$\Rightarrow 2 \log_5 \sqrt{5} = \log_5 (\sqrt{5})^2 = \log_5 5 = 1$$

39) -1

⇒ $\log_{0.25} 4 = x$ 로 놓으면 로그의 정의에 의하여

$$0.25^x = 4 \text{ 이므로 } \left(\frac{1}{4} \right)^x = 4$$

$$4^{-x} = 4 \quad \therefore x = -1$$

$$\therefore \log_{0.25} 4 = -1$$

40) $\frac{5}{6}$

⇒ $\log_{2\sqrt{2}} \sqrt[4]{32} = x$ 로 놓으면 로그의 정의에 의하여

$$(2\sqrt{2})^x = \sqrt[4]{32} \text{ 이므로 } (2^{\frac{3}{2}})^x = 2^{\frac{5}{4}}$$

$$2^{\frac{3}{2}x} = 2^{\frac{5}{4}} \text{ 에서}$$

$$\frac{3}{2}x = \frac{5}{4} \quad \therefore x = \frac{5}{6}$$

$$\therefore \log_{2\sqrt{2}} \sqrt[4]{32} = \frac{5}{6}$$

41) $\frac{2}{9}$

⇒ $\log_{125} \sqrt[3]{25} = x$ 로 놓으면 로그의 정의에 의하여

$$125^x = \sqrt[3]{25} \text{ 이므로 } (5^3)^x = 5^{\frac{2}{3}}$$

$$5^{3x} = 5^{\frac{2}{3}} \text{ 에서}$$

$$3x = \frac{2}{3} \quad \therefore x = \frac{2}{9}$$

$$\therefore \log_{125} \sqrt[3]{25} = \frac{2}{9}$$

42) 2

$$\Rightarrow \log_3 63 - \log_3 7 = \log_3 \frac{63}{7} = \log_3 9 = \log_3 3^2 = 2$$

43) 31

$$\Rightarrow 9^{\frac{3}{2}} + \log_3 81 = (3^2)^{\frac{3}{2}} + \log_3 3^4 = 3^{2 \times \frac{3}{2}} + 4 \log_3 3$$

$$= 27 + 4 = 31$$

44) 2

$$\begin{aligned}\Rightarrow \log_6 3 + \log_6 12 &= \log_6 (3 \times 12) = \log_6 36 \\ &= \log_6 6^2 = 2 \log_6 6 = 2\end{aligned}$$

45) -1

$$\begin{aligned}\Rightarrow \log_2 18 - 2 \log_2 6 \\ &= \log_2 (2 \cdot 3^2) - 2 \log_2 (2 \cdot 3) \\ &= \log_2 2 + 2 \log_2 3 - 2(\log_2 2 + \log_2 3) \\ &= 1 + 2 \log_2 3 - 2 - 2 \log_2 3 \\ &= -1\end{aligned}$$

46) 2

47) 1

$$\Rightarrow \log_2 16 + \log_2 \frac{1}{8} = \log_2 \left(16 \times \frac{1}{8} \right) = \log_2 2 = 1$$

48) 4

$$\begin{aligned}\Rightarrow \log_3 24 + 3 \log_3 \frac{3}{2} \\ &= \log_3 (2^3 \cdot 3) + 3(\log_3 3 - \log_3 2) \\ &= \log_3 2^3 + \log_3 3 + 3 \log_3 3 - 3 \log_3 2 \\ &= 3 \log_3 2 + 1 + 3 - 3 \log_3 2 \\ &= 4\end{aligned}$$

49) 1

$$\Rightarrow \log_{12} 4 + \log_{12} 3 = \log_{12} (4 \times 3) = \log_{12} 12 = 1$$

50) 2

$$\Rightarrow \log_3 36 + \log_3 \frac{1}{4} = \log_3 \left(36 \times \frac{1}{4} \right) = \log_3 9 = \log_3 3^2 = 2$$

51) 2

$$\begin{aligned}\Rightarrow \log_6 \frac{8}{3} + \log_6 \frac{27}{2} &= \log_6 \left(\frac{8}{3} \times \frac{27}{2} \right) \\ &= \log_6 36 = \log_6 6^2 = 2\end{aligned}$$

52) 2

$$\begin{aligned}\Rightarrow \log_8 \frac{1}{2} - \log_8 \frac{1}{128} \\ &= \log_8 \left(\frac{1}{2} \div \frac{1}{128} \right) = \log_8 \left(\frac{1}{2} \times 128 \right) \\ &= \log_8 64 = \log_8 8^2 = 2\end{aligned}$$

53) 3

$$\begin{aligned}\Rightarrow \log_2 \sqrt[3]{128} + \log_2 \sqrt[3]{4} \\ &= \log_2 (\sqrt[3]{128} \times \sqrt[3]{4}) = \log_2 \sqrt[3]{512} \\ &= \log_2 2^{\frac{9}{3}} = \log_2 2^3 = 3\end{aligned}$$

54) -3

$$\Rightarrow \log_{\frac{1}{2}} \frac{1}{3} - \log_{\frac{1}{2}} \frac{1}{24}$$

$$\begin{aligned}&= \log_{\frac{1}{2}} \left(\frac{1}{3} \div \frac{1}{24} \right) = \log_{\frac{1}{2}} \left(\frac{1}{3} \times 24 \right) \\ &= \log_{\frac{1}{2}} 8 = \log_{\frac{1}{2}} \left(\frac{1}{2} \right)^{-3} = -3\end{aligned}$$

55) 3

$$\begin{aligned}\Rightarrow \log_6 72 - \log_6 \frac{1}{3} &= \log_6 \left(72 \div \frac{1}{3} \right) = \log_6 (72 \times 3) \\ &= \log_6 216 = \log_6 6^3 = 3\end{aligned}$$

56) 1

$$\Rightarrow \log_3 6 + \log_3 2 - \log_3 4 = \log_3 \frac{6 \times 2}{4} = \log_3 3 = 1$$

57) 2

$$\begin{aligned}\Rightarrow \log_3 \sqrt{27} - \log_3 \frac{1}{\sqrt{3}} \\ &= \log_3 \frac{\sqrt{27}}{\frac{1}{\sqrt{3}}} = \log_3 \sqrt{81} = \log_3 9 \\ &= \log_3 3^2 = 2\end{aligned}$$

58) 4

$$\begin{aligned}\Rightarrow \log_2 \frac{4}{3} + 2 \log_2 \sqrt{12} \\ &= \log_2 \frac{4}{3} + \log_2 12 = \log_2 \left(\frac{4}{3} \times 12 \right) \\ &= \log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4\end{aligned}$$

59) 2

$$\Rightarrow \frac{1}{\sqrt[3]{8}} \times \log_3 81 = \frac{1}{\sqrt[3]{2^3}} \times \log_3 3^4 = \frac{1}{2} \times 4 = 2$$

60) -1

61) 3

$$\begin{aligned}\Rightarrow \log_2 12 + \log_2 \frac{1}{3} + \log_2 6 - 2 \log_2 \sqrt{3} \\ &= \log_2 \left(\frac{12 \cdot \frac{1}{3} \cdot 6}{3} \right) = \log_2 8 = 3\end{aligned}$$

62) 2

$$\begin{aligned}\Rightarrow \log_{\sqrt{3}} 2\sqrt{3} - \log_9 \frac{9}{16} + 2 \log_{\frac{1}{3}} \frac{4}{3} \\ &= \log_{\frac{1}{3}^{\frac{1}{2}}} 2\sqrt{3} - \log_{3^2} \frac{9}{16} + 2 \log_{3^{-1}} \frac{4}{3} \\ &= \log_3 12 - \log_3 \frac{3}{4} - \log_3 \frac{16}{9} \\ &= \log_3 \left(\frac{12}{\frac{3}{4} \cdot \frac{16}{9}} \right) \\ &= \log_3 9 = 2\end{aligned}$$

63) 0

$$\begin{aligned}
 &\Rightarrow \log_5 \sqrt{6} - \log_{25} 3 - \frac{3}{2} \log_5 \sqrt{2} \\
 &= \frac{1}{2} \log_5 6 - \frac{1}{2} \log_5 3 - \frac{1}{2} \log_5 2 \\
 &= \frac{1}{2} \log_5 \left(\frac{6}{3 \cdot 2} \right) \\
 &= \frac{1}{2} \log_5 1 = 0
 \end{aligned}$$

64) 0

65) 2

$$\begin{aligned}
 &\Rightarrow \log_6 45 + \log_6 20 - 2 \log_6 5 \\
 &= \log_6 45 + \log_6 20 - \log_6 5^2 \\
 &= \log_6 \frac{45 \times 20}{25} = \log_6 36 \\
 &= \log_6 6^2 = 2
 \end{aligned}$$

66) $\frac{5}{2}$

$$\begin{aligned}
 &\Rightarrow \frac{1}{2} \log_2 12 + \log_2 \sqrt{6} - \log_2 \frac{3}{2} \\
 &= \log_2 12^{\frac{1}{2}} + \log_2 \sqrt{6} - \log_2 \frac{3}{2} \\
 &= \log_2 \sqrt{12} + \log_2 \sqrt{6} - \log_2 \frac{3}{2} \\
 &= \log_2 \left(\sqrt{12} \times \sqrt{6} \div \frac{3}{2} \right) \\
 &= \log_2 4 \sqrt{2} = \log_2 2^{\frac{5}{2}} = \frac{5}{2}
 \end{aligned}$$

67) 3

$$\begin{aligned}
 &\Rightarrow \log_2 3 - \log_2 \frac{9}{2} + \log_2 12 = \log_2 \frac{3 \times 12}{\frac{9}{2}} = \log_2 8 \\
 &= \log_2 2^3 = 3
 \end{aligned}$$

68) -1

$$\begin{aligned}
 &\Rightarrow \log_{\frac{1}{3}} \frac{\sqrt{3}}{2} + \log_{\frac{1}{3}} 2\sqrt{3} = \log_{\frac{1}{3}} \left(\frac{\sqrt{3}}{2} \times 2\sqrt{3} \right) \\
 &= \log_{\frac{1}{3}} 3 = \log_{\frac{1}{3}} \left(\frac{1}{3} \right)^{-1} = -1
 \end{aligned}$$

69) $\log_3 10$

$$\begin{aligned}
 &\Rightarrow \frac{2}{3} \log_3 \sqrt{10} - \frac{2}{3} \log_3 \frac{1}{2} + 2 \log_3 \sqrt[3]{5} \\
 &= \log_3 \left(10^{\frac{1}{2}} \right)^{\frac{2}{3}} - \log_3 (2^{-1})^{\frac{2}{3}} + \log_3 \left(5^{\frac{1}{3}} \right)^2 \\
 &= \log_3 \left\{ \frac{(10)^{\frac{1}{3}} \cdot (5)^{\frac{2}{3}}}{(2)^{-\frac{2}{3}}} \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \log_3 \left\{ \frac{(2)^{\frac{1}{3}} \cdot (5)^{\frac{1}{3}} \cdot (5)^{\frac{2}{3}}}{(2)^{-\frac{2}{3}}} \right\} \\
 &= \log_3 (2 \cdot 5) \\
 &= \log_3 10
 \end{aligned}$$

70) 2

$$\begin{aligned}
 &\Rightarrow \log_2 (\log_3 25) + \log_2 (\log_5 9) \\
 &= \log_2 \{ (\log_3 25) \times (\log_5 9) \} \\
 &= \log_2 \{ (2 \log_3 5) \times (\log_5 9) \} \\
 &= \log_2 (2 \log_3 9) \\
 &= \log_2 (4 \log_3 3) \\
 &= \log_2 4 \\
 &= 2
 \end{aligned}$$

71) $1-a$

$$\Rightarrow \log_{10} 5 = \log_{10} \frac{10}{2} = \log_{10} 10 - \log_{10} 2 = 1 - a$$

72) $-a+2b+1$

$$\begin{aligned}
 &\Rightarrow \log_{10} 45 = \log_{10} (5 \times 3^2) = \log_{10} 5 + \log_{10} 3^2 \\
 &= \log_{10} \frac{10}{2} + 2 \log_{10} 3 \\
 &= \log_{10} 10 - \log_{10} 2 + 2 \log_{10} 3 \\
 &= -a + 2b + 1
 \end{aligned}$$

73) $a+b+1$

$$\begin{aligned}
 &\Rightarrow \log_{10} 60 = \log_{10} (2 \times 3 \times 10) \\
 &= \log_{10} 2 + \log_{10} 3 + \log_{10} 10 = a + b + 1
 \end{aligned}$$

74) $\frac{1}{4}(-a+b+1)$

$$\begin{aligned}
 &\Rightarrow \log_{10} \sqrt[4]{15} = \frac{1}{4} \log_{10} 15 = \frac{1}{4} \log_{10} (3 \times 5) \\
 &= \frac{1}{4} (\log_{10} 3 + \log_{10} 5) \\
 &= \frac{1}{4} (\log_{10} 3 + \log_{10} 10 - \log_{10} 2) \\
 &= \frac{1}{4} (-a + b + 1)
 \end{aligned}$$

75) $9a-3$

$$\begin{aligned}
 &\Rightarrow \log_{10} \left(\frac{4}{5} \right)^3 = 3 (\log_{10} 4 - \log_{10} 5) \\
 &= 3 (\log_{10} 2^2 - \log_{10} 5) \\
 &= 3 \left(2 \log_{10} 2 - \log_{10} \frac{10}{2} \right) \\
 &= 3 (2 \log_{10} 2 - (\log_{10} 10 - \log_{10} 2)) \\
 &= 3 \{ 2a - (1 - a) \} = 3(3a - 1) \\
 &= 9a - 3
 \end{aligned}$$

76) $3a+2b-3$

$$\begin{aligned}
 \Rightarrow \log_{10} 0.072 &= \log_{10} \frac{72}{1000} = \log_{10} \frac{2^3 \times 3^2}{10^3} \\
 &= \log_{10} 2^3 + \log_{10} 3^2 - \log_{10} 10^3 \\
 &= 3 \log_{10} 2 + 2 \log_{10} 3 - 3 \\
 &= 3a + 2b - 3
 \end{aligned}$$

$$77) \frac{3}{2}a + b$$

$$\begin{aligned}
 \Rightarrow \log_{10} \sqrt{72} &= \log_{10} 72^{\frac{1}{2}} = \frac{1}{2} \log_{10} 72 \\
 &= \frac{1}{2} \log_{10} (2^3 \times 3^2) \\
 &= \frac{1}{2} (3 \log_{10} 2 + 2 \log_{10} 3) \\
 &= \frac{1}{2} (3a + 2b) = \frac{3}{2}a + b
 \end{aligned}$$

$$78) 3a + b$$

$$\begin{aligned}
 \Rightarrow \log_{10} 24 &= \log_{10} (2^3 \times 3) = \log_{10} 2^3 + \log_{10} 3 \\
 &= 3 \log_{10} 2 + \log_{10} 3 \\
 &= 3a + b
 \end{aligned}$$

$$79) 3a - 3b$$

$$\begin{aligned}
 \Rightarrow \log_{10} \frac{8}{27} &= \log_{10} \frac{2^3}{3^3} = \log_{10} 2^3 - \log_{10} 3^3 \\
 &= 3 \log_{10} 2 - 3 \log_{10} 3 = 3a - 3b
 \end{aligned}$$

$$80) 2b - 5a$$

$$\begin{aligned}
 \Rightarrow \log_{10} \frac{9}{32} &= \log_{10} \frac{3^2}{2^5} = \log_{10} 3^2 - \log_{10} 2^5 \\
 &= 2 \log_{10} 3 - 5 \log_{10} 2 = 2b - 5a
 \end{aligned}$$

$$81) 3a - 3$$

$$\begin{aligned}
 \Rightarrow \log_{10} \frac{1}{125} &= \log_{10} 5^{-3} = -3 \log_{10} 5 \\
 &= -3 \log_{10} \frac{10}{2} = -3(1 - a) = 3a - 3
 \end{aligned}$$