01 구간에 따라 다르게 정의된 정적분의 계산

함수 $f(x)=\begin{cases} g(x) & (x \le c) \\ h(x) & (x \ge c) \end{cases}$가 닫힌구간 $[a,b]$에서 연속이고 $a<c<b$일 때,

$$\int_a^b f(x)dx=\int_a^c g(x)dx+\int_c^b h(x)dx$$

◪ 다음 정적분의 값을 구하여라.

1. $f(x)=\begin{cases} x^2-1 & (x \le 1) \\ x-1 & (x>1) \end{cases}$일 때, $\int_0^2 f(x)dx$

2. $f(x)=\begin{cases} -x^2+1 & (x \le 1) \\ x-1 & (x>1) \end{cases}$일 때, $\int_0^2 f(x)dx$

3. $f(x)=\begin{cases} x^2 & (x \le 0) \\ 2x & (x>0) \end{cases}$일 때, $\int_{-1}^1 f(x)dx$

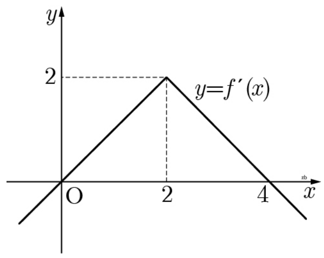
4. $f(x)=\begin{cases} x^2 & (x \le 1) \\ 2-x & (x>1) \end{cases}$일 때, $\int_{-2}^2 f(x)dx$

5. $f(x)=\begin{cases} 2 & (x \le 0) \\ -3x+2 & (x>0) \end{cases}$일 때, $\int_{-1}^1 xf(x)dx$

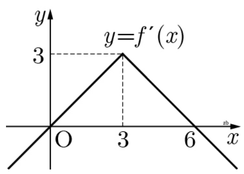
6. $f(x)=\begin{cases} x & (x<1) \\ -x+2 & (x \ge 1) \end{cases}$일 때, $\int_1^3 xf(x-1)dx$

◪ 다음 정적분의 값을 구하여라.

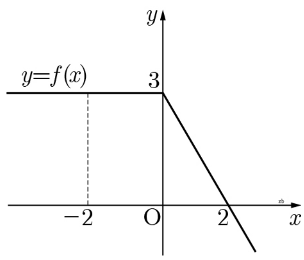
7. 연속함수 $f(x)$의 도함수 $y=f'(x)$의 그래프가 다음 그림과 같을 때, $f(4)-f(0)$의 값을 구하여라.



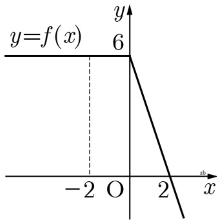
8. 연속함수 $f(x)$의 도함수 $y=f'(x)$의 그래프가 다음 그림과 같을 때, $f(6)-f(0)$의 값을 구하여라.



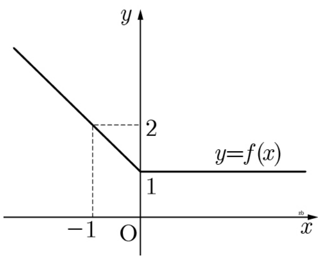
9. 함수 $y=f(x)$의 그래프가 다음 그림과 같을 때, 정적분 $\int_{-1}^1 xf(x)\,dx$의 값을 구하여라.



10. 함수 $y=f(x)$의 그래프가 다음 그림과 같을 때, 정적분 $\int_{-2}^{2} xf(x)dx$의 값을 구하여라.



11. 함수 $y=f(x)$의 그래프가 다음 그림과 같을 때, 정적분 $\int_{-1}^{1} xf(x)\,dx$의 값을 구하여라.



02 절댓값 기호를 포함한 함수의 정적분의 계산

절댓값 기호를 포함한 함수 $y=|f(x)|$의 정적분은 절댓값 기호 안의 식의 값이 0이 되는 x의 값을 경계로 적분 구간을 나눈다.

◪ 다음 정적분의 값을 구하여라.

12. $\int_{-2}^{1} |x+1|dx$

13. $\int_{0}^{2} |x-1|\,dx$

14. $\int_{-1}^{2} |x-1|\,dx$

15. $\int_{0}^{3} |x-1|dx$

16. $\int_{0}^{4} |x-1|dx$

17. $\int_{0}^{3} |x-2|dx$

18. $\int_{0}^{6} |x-4|dx$

19. $\int_{0}^{2} |x^2-1|dx$

20. $\int_{-2}^{2} |x^2-1|dx$

21. $\int_{0}^{3} |x(x-2)|dx$

22. $\int_{0}^{4}|x^2-2x|\,dx$

23. $\int_{1}^{3}|x(x-2)|\,dx$

24. $\int_{1}^{3}|3x^2-6x|\,dx$

25. $\int_{0}^{2}|x^2+x-2|\,dx$

26. $\int_{0}^{2}|x^2-4x+3|\,dx$

27. $\int_{-1}^{2}x|x-1|\,dx$

28. $\int_{0}^{3}6x|x-1|\,dx$

03 우함수와 기함수의 정적분

함수 $f(x)$가 닫힌구간 $[a,b]$에서 연속일 때,
(1) $f(-x)=f(x)$이면, 즉 $f(x)$가 **우함수**이면

$$\int_{-a}^{a}f(x)dx=2\int_{0}^{a}f(x)dx$$

(2) $f(-x)=-f(x)$이면, 즉 $f(x)$가 **기함수**이면

$$\int_{-a}^{a}f(x)dx=0$$

◪ **다음 함수 중 $\int_{a}^{b}f(x)dx=\int_{-b}^{-a}f(x)dx$를 만족하는 것에는 '우', $\int_{a}^{b}f(x)dx=-\int_{-b}^{-a}f(x)dx$를 만족하는 것에는 '기'라고 괄호 안에 써 넣어라.**

29. $f(x)=-x^3-2x$ ( )

30. $f(x)=x^4+x^2$ ( )

31. $f(x)=x^5+x$ ( )

32. $f(x)=x^8+1$ ( )

◪ **다음 정적분의 값을 구하여라.**

33. $\int_{-1}^{1}(3x^2+4)dx$

34. $\int_{-1}^{1}(x+1)(3x+2)\,dx$

35. $\int_{0}^{2}(2x^2+2x)dx-2\int_{0}^{-2}(x+x^2)dx$

36. $\int_{-1}^{1}(x^2+x+1)dx-\int_{-1}^{1}(x^2-x+1)dx$

37. $\int_{-2}^{1}(6x^2-4x+1)dx+\int_{2}^{1}(-6x^2+4x-1)dx$

38. $\int_{-1}^{0}(3x^2+4x^3+5x^4)dx-\int_{1}^{0}(3x^2+4x^3+5x^4)dx$

39. $\int_{-2}^{2}(3x^2+|x|)dx$

40. $\int_{-1}^{1}(x^3+3x)dx$

41. $\int_{-1}^{1}(x+1)(x^2-x+1)dx$

42. $\int_{-2}^{2}(x^3-3x^2-2x+4)dx$

43. $\int_{-1}^{1}(5x^3-6x^2-7x+8)dx$

44. $\int_{-2}^{2}(100x^3+3x^2-90x+1)dx$

45. $\int_{-3}^{3}(-x^3+3x^2+4x-2)\,dx$

46. $\int_{-1}^{0}(x^3-3x^2+2)dx+\int_{0}^{1}(x^3-3x^2+2)dx$

47. $\int_{-1}^{0}(4x^3-2x+1)dx+\int_{0}^{1}(4x^3-2x+1)dx$

48. $\int_{-2}^{1}(x^3+6x^2+2x-1)dx$

$+\int_{1}^{2}(x^3+6x^2+2x-1)dx$

49. $\int_{-1}^{0}(1+2x+3x^2+4x^3)dx$

$+\int_{0}^{1}(1+2x+3x^2+4x^3)dx$

50. $\int_{-1}^{1} x(1-x)^2 dx$

51. $\int_{-2}^{2} (5x^4+3x^3-7x-1)\,dx$

52. $\int_{-1}^{1} (5x^4-6x^3+3x^2-7x+3)dx$

53. $\int_{-1}^{1} (5x^4+4x^3+3x^2+2x+1)dx$

54. $\int_{-1}^{4} (5x^4+4x^3+3x^2)dx-\int_{1}^{4}(5x^4+4x^3+3x^2)dx$

55. $\int_{-1}^{1} (x^5+2x)dx$

56. $\int_{-2}^{2} (x^5-2x^3+3x^2-1)dx$

57. $\int_{-5}^{5} (6x^5-8x^3+3x^2-2)dx$

58. $\int_{-2}^{2} (6x^5-4x^3+3x^2+2x+1)dx$

59. $\int_{-2}^{1} (x^5+3x^2-2x)dx-\int_{2}^{1}(x^5+3x^2-2x)dx$

60. $\int_{-1}^{1} x^3(1+x)^2 dx$

61. $\int_{-1}^{1} (8x^9-x^5+10x^4-3x^2+4)dx$

62. $\int_{-1}^{1} (11x^{10}+12x^{11}+13x^{12}+14x^{13})dx$

63. $\int_{-1}^{1} (x^{11}+10x^9-25x^7+3x^2-9x+2)dx$

정답 및 해설

1) $-\frac{1}{6}$

$\Rightarrow \int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$= \int_0^1 (x^2-1)dx + \int_1^2 (x-1)dx$

$= \left[\frac{1}{3}x^3 - x\right]_0^1 + \left[\frac{1}{2}x^2 - x\right]_1^2$

$= -\frac{2}{3} + \frac{1}{2} = -\frac{1}{6}$

2) $\frac{7}{6}$

$\Rightarrow \int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$= \int_0^1 (-x^2+1)dx + \int_1^2 (x-1)dx$

$= \left[-\frac{1}{3}x^3 + x\right]_0^1 + \left[\frac{1}{2}x^2 - x\right]_1^2$

$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$

3) $\frac{4}{3}$

$\Rightarrow \int_{-1}^1 f(x)dx = \int_{-1}^0 f(x)dx + \int_0^1 f(x)dx$

$= \int_{-1}^0 x^2 dx + \int_0^1 2x dx$

$= \left[\frac{1}{3}x^3\right]_{-1}^0 + \left[x^2\right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$

4) $\frac{7}{2}$

$\Rightarrow \int_{-2}^2 f(x)dx = \int_{-2}^1 f(x)dx + \int_1^2 f(x)dx$

$= \int_{-2}^1 x^2 dx + \int_1^2 (2-x)dx$

$= \left[\frac{1}{3}x^3\right]_{-2}^1 + \left[2x - \frac{1}{2}x^2\right]_1^2 = 3 + \frac{1}{2} = \frac{7}{2}$

5) -1

$\Rightarrow f(x) = \begin{cases} 2 & (x \le 0) \\ -3x+2 & (x > 0) \end{cases}$ 이므로

$xf(x) = \begin{cases} 2x & (x \le 0) \\ -3x^2+2x & (x > 0) \end{cases}$

$\int_{-1}^1 xf(x)\,dx = \int_{-1}^0 2x\,dx + \int_0^1 (-3x^2+2x)dx$

$= \left[x^2\right]_{-1}^0 + \left[-x^3+x^2\right]_0^1 = -1 + 0 = -1$

6) 2

$\Rightarrow f(x-1) = \begin{cases} x-1 & (x < 2) \\ -x+3 & (x \ge 2) \end{cases}$ 이므로

$xf(x-1) = \begin{cases} x^2-x & (x < 2) \\ -x^2+3x & (x \ge 2) \end{cases}$

$\int_1^3 xf(x-1)dx$

$= \int_1^2 (x^2-x)dx + \int_2^3 (-x^2+3x)dx$

$= \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_1^2 + \left[-\frac{1}{3}x^3 + \frac{3}{2}x^2\right]_2^3$

$= \frac{5}{6} + \frac{7}{6} = 2$

7) 4

$\Rightarrow f'(x) = \begin{cases} x & (x \le 2) \\ 4-x & (x > 2) \end{cases}$ 이므로

$\int_0^4 f'(x)dx = \int_0^2 x dx + \int_2^4 (4-x)dx$

$= \left[\frac{x^2}{2}\right]_0^2 + \left[4x - \frac{x^2}{2}\right]_2^4 = 2 + 2 = 4$

$\int_0^4 f'(x)dx = [f(x)]_0^4 = f(4) - f(0)$ 이므로

$f(4) - f(0) = 4$

8) 9

$\Rightarrow \int_0^6 f'(x)dx = [f(x)]_0^6 = f(6) - f(0)$ 이고

$f'(x) = \begin{cases} x & (x \le 3) \\ 6-x & (x > 3) \end{cases}$ 이므로

$\int_0^6 f'(x)dx = \int_0^3 x dx + \int_3^6 (6-x)dx$

$= \left[\frac{1}{2}x^2\right]_0^3 + \left[6x - \frac{1}{2}x^2\right]_3^6$

$= \frac{9}{2} + \frac{9}{2} = 9$

$\therefore\ f(6) - f(0) = 9$

9) $-\frac{1}{2}$

$\Rightarrow f(x) = \begin{cases} 3 & (x \le 0) \\ -\frac{3}{2}x+3 & (x > 0) \end{cases}$ 이므로

$\int_{-1}^1 xf(x)dx = \int_{-1}^0 3x\,dx + \int_0^1 \left(-\frac{3}{2}x^2 + 3x\right)dx$

$= \left[\frac{3}{2}x^2\right]_{-1}^0 + \left[-\frac{1}{2}x^3 + \frac{3}{2}x^2\right]_0^1$

$= -\frac{3}{2} + 1 = -\frac{1}{2}$

10) -8

$\Rightarrow f(x) = \begin{cases} 6 & (x \le 0) \\ -3x+6 & (x > 0) \end{cases}$ 이므로

$$\int_{-2}^{2} xf(x)dx = \int_{-2}^{0} 6xdx + \int_{0}^{2}(-3x^2+6x)dx$$
$$= [3x^2]_{-2}^{0} + [-x^3+3x^2]_{0}^{2}$$
$$= -12+4 = -8$$

11) $-\frac{1}{3}$

⇨ $f(x) = \begin{cases} -x+1 & (x \le 0) \\ 1 & (x > 0) \end{cases}$ 이므로

$$\int_{-1}^{1} xf(x)dx = \int_{-1}^{0}(-x^2+x)dx + \int_{0}^{1} xdx$$
$$= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{-1}^{0} + \left[\frac{1}{2}x^2\right]_{0}^{1} = -\frac{5}{6} + \frac{1}{2} = -\frac{1}{3}$$

12) $\frac{5}{2}$

⇨ $|x+1| = \begin{cases} -x-1 & (x \le -1) \\ x+1 & (x > -1) \end{cases}$ 이므로

$$\int_{-2}^{1} |x+1|dx = \int_{-2}^{-1}(-x-1)dx + \int_{-1}^{1}(x+1)dx$$
$$= \left[-\frac{1}{2}x^2 - x\right]_{-2}^{-1} + \left[\frac{1}{2}x^2 + x\right]_{-1}^{1}$$
$$= \frac{1}{2} + 2 = \frac{5}{2}$$

13) 1

⇨ $|x-1| = \begin{cases} x-1 & (x \ge 1) \\ -x+1 & (x \le 1) \end{cases}$ 이므로

$$\int_{0}^{2} |x-1|\,dx = \int_{0}^{1}(-x+1)\,dx + \int_{1}^{2}(x-1)\,dx$$
$$= \left[-\frac{1}{2}x^2 + x\right]_{0}^{1} + \left[\frac{1}{2}x^2 - x\right]_{1}^{2}$$
$$= \frac{1}{2} + \frac{1}{2} = 1$$

14) $\frac{5}{2}$

⇨ $|x-1| = \begin{cases} x-1 & (x \ge 1) \\ -x+1 & (x \le 1) \end{cases}$ 이므로

$$\int_{-1}^{2} |x-1|\,dx$$
$$= \int_{-1}^{1}(-x+1)\,dx + \int_{1}^{2}(x-1)\,dx$$
$$= \left[-\frac{1}{2}x^2 + x\right]_{-1}^{1} + \left[\frac{1}{2}x^2 - x\right]_{1}^{2}$$
$$= 2 + \frac{1}{2} = \frac{5}{2}$$

15) $\frac{5}{2}$

⇨ $|x-1| = \begin{cases} 1-x & (x \le 1) \\ x-1 & (x > 1) \end{cases}$ 이므로

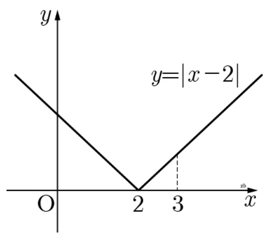
$$\int_{0}^{3} |x-1|dx = \int_{0}^{1}(1-x)dx + \int_{1}^{3}(x-1)dx$$
$$= \left[x - \frac{1}{2}x^2\right]_{0}^{1} + \left[\frac{1}{2}x^2 - x\right]_{1}^{3}$$
$$= \frac{1}{2} + 2 = \frac{5}{2}$$

16) 5

⇨ $\int_{0}^{4} |x-1|dx$

$$= \int_{0}^{1}(-x+1)dx + \int_{1}^{4}(x-1)dx$$
$$= \left[-\frac{1}{2}x^2 + x\right]_{0}^{1} + \left[\frac{1}{2}x^2 - x\right]_{1}^{4} = \frac{1}{2} + \frac{9}{2} = 5$$

17) $\frac{5}{2}$

⇨ 

$|x-2| = \begin{cases} -(x-2) & (0 \le x \le 2) \\ x-2 & (2 < x \le 3) \end{cases}$ 이므로

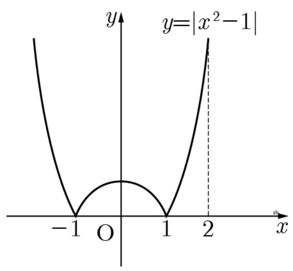
$$\int_{0}^{3} |x-2|dx$$
$$= \int_{0}^{2}(-x+2)dx + \int_{2}^{3}(x-2)dx$$
$$= \left[-\frac{1}{2}x^2 + 2x\right]_{0}^{2} + \left[\frac{1}{2}x^2 - 2x\right]_{2}^{3} = 2 + \frac{1}{2} = \frac{5}{2}$$

18) 10

⇨ $\int_{0}^{6} |x-4|dx$

$$= \int_{0}^{4}(-x+4)dx + \int_{4}^{6}(x-4)dx$$
$$= \left[-\frac{1}{2}x^2 + 4x\right]_{0}^{4} + \left[\frac{1}{2}x^2 - 4x\right]_{4}^{6} = 8 + 2 = 10$$

19) 2

⇨ 

$$\int_{0}^{2} |x^2-1|dx$$
$$= \int_{0}^{1}(-x^2+1)dx + \int_{1}^{2}(x^2-1)dx$$
$$= \left[-\frac{x^3}{3} + x\right]_{0}^{1} + \left[\frac{x^3}{3} - x\right]_{1}^{2} = \frac{2}{3} + \frac{4}{3} = 2$$

20) 4

⇨ $\int_{-2}^{2}|x^2-1|dx$

$=\int_{-2}^{-1}(x^2-1)dx+\int_{-1}^{1}(-x^2+1)dx+\int_{1}^{2}(x^2-1)dx$

$=\left[\frac{1}{3}x^3-x\right]_{-2}^{-1}+\left[-\frac{1}{3}x^3+x\right]_{-1}^{1}+\left[\frac{1}{3}x^3-x\right]_{1}^{2}$

$=\frac{4}{3}+\frac{4}{3}+\frac{4}{3}=4$

21) $\frac{8}{3}$

⇨ $|x(x-2)|=\begin{cases}2x-x^2 & (0\le x\le 2)\\ x^2-2x & (x<0 \text{ 또는 } x>2)\end{cases}$ 이므로

$\int_{0}^{3}|x(x-2)|dx$

$=\int_{0}^{2}(2x-x^2)dx+\int_{2}^{3}(x^2-2x)dx$

$=\left[x^2-\frac{1}{3}x^3\right]_{0}^{2}+\left[\frac{1}{3}x^3-x^2\right]_{2}^{3}$

$=\frac{4}{3}+\frac{4}{3}=\frac{8}{3}$

22) 8

⇨ $|x(x-2)|=\begin{cases}x^2-2x & (x\le 0 \text{ 또는 } x\ge 2)\\ -x^2+2x & (0\le x\le 2)\end{cases}$ 이므로

$\int_{0}^{4}|x^2-2x|\,dx$

$=\int_{0}^{2}(-x^2+2x)\,dx+\int_{2}^{4}(x^2-2x)\,dx$

$=\left[-\frac{1}{3}x^3+x^2\right]_{0}^{2}+\left[\frac{1}{3}x^3-x^2\right]_{2}^{4}$

$=\frac{4}{3}+\frac{20}{3}=8$

23) 2

⇨ $|x(x-2)|=\begin{cases}x^2-2x & (x\le 0 \text{ 또는 } x\ge 2)\\ -x^2+2x & (0\le x\le 2)\end{cases}$ 이므로

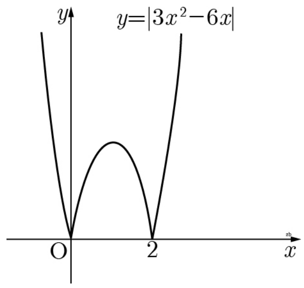
$\int_{1}^{3}|x(x-2)|\,dx$

$=\int_{1}^{2}(-x^2+2x)\,dx+\int_{2}^{3}(x^2-2x)\,dx$

$=\left[-\frac{1}{3}x^3+x^2\right]_{1}^{2}+\left[\frac{1}{3}x^3-x^2\right]_{2}^{3}$

$=\frac{2}{3}+\frac{4}{3}=2$

24) 6

⇨ 

$\int_{1}^{3}|3x^2-6x|dx$

$=\int_{1}^{2}(-3x^2+6x)dx+\int_{2}^{3}(3x^2-6x)dx$

$=[-x^3+3x^2]_{1}^{2}+[x^3-3x^2]_{2}^{3}$

$=2+4=6$

25) 3

⇨ $|x^2+x-2|=\begin{cases}-x^2-x+2 & (-2\le x\le 1)\\ x^2+x-2 & (x<-2 \text{ 또는 } x>1)\end{cases}$

이므로

$\int_{0}^{2}|x^2+x-2|dx$

$=\int_{0}^{1}(-x^2-x+2)dx+\int_{1}^{2}(x^2+x-2)dx$

$=\left[-\frac{1}{3}x^3-\frac{1}{2}x^2+2x\right]_{0}^{1}+\left[\frac{1}{3}x^3+\frac{1}{2}x^2-2x\right]_{1}^{2}$

$=\frac{7}{6}+\frac{11}{6}=3$

26) 2

⇨ $|x^2-4x+3|=\begin{cases}x^2-4x+3 & (x<1 \text{ 또는 } x>3)\\ -x^2+4x-3 & (1\le x\le 3)\end{cases}$

이므로

$\int_{0}^{2}|x^2-4x+3|dx$

$=\int_{0}^{1}(x^2-4x+3)dx+\int_{1}^{2}(-x^2+4x-3)dx$

$=\left[\frac{1}{3}x^3-2x^2+3x\right]_{0}^{1}+\left[-\frac{1}{3}x^3+2x^2-3x\right]_{1}^{2}$

$=\frac{4}{3}+\frac{2}{3}=2$

27) $\frac{1}{6}$

⇨ $x|x-1|=\begin{cases}x(1-x) & (x\le 1)\\ x(x-1) & (x>1)\end{cases}$ 이므로

$\int_{-1}^{2}x|x-1|dx$

$=\int_{-1}^{1}x(1-x)dx+\int_{1}^{2}x(x-1)dx$

$=\int_{-1}^{1}(x-x^2)dx+\int_{1}^{2}(x^2-x)dx$

$=\left[\frac{1}{2}x^2-\frac{1}{3}x^3\right]_{-1}^{1}+\left[\frac{1}{3}x^3-\frac{1}{2}x^2\right]_{1}^{2}$

$=-\frac{2}{3}+\frac{5}{6}=\frac{1}{6}$

28) 29

⇨ $6x|x-1|=\begin{cases}6x(1-x) & (x\le 1)\\ 6x(x-1) & (x>1)\end{cases}$이므로

$\int_0^3 6x|x-1|dx$

$=\int_0^1 6x(1-x)dx+\int_1^3 6x(x-1)dx$

$=\int_0^1 (6x-6x^2)dx+\int_1^3 (6x^2-6x)dx$

$=[3x^2-2x^3]_0^1+[2x^3-3x^2]_1^3=1+28=29$

29) 기

⇨ $\int_a^b f(x)dx=-\int_{-b}^{-a} f(x)dx$를 만족하면 함수 $y=f(x)$의 그래프는 원점에 대하여 대칭이다.
$f(x)=-x^3-2x$는 원점에 대하여 대칭이므로 '기'이다.

30) 우

⇨ $\int_a^b f(x)dx=\int_{-b}^{-a} f(x)dx$를 만족하면 함수 $y=f(x)$의 그래프는 y축에 대하여 대칭이다.
$f(x)=x^4+x^2$은 y축에 대하여 대칭이므로 '우'이다.

31) 기

⇨ $\int_a^b f(x)dx=-\int_{-b}^{-a} f(x)dx$를 만족하면 함수 $y=f(x)$의 그래프는 원점에 대하여 대칭이다.
$f(x)=x^5+x$는 원점에 대하여 대칭이므로 '기'이다.

32) 우

⇨ $\int_a^b f(x)dx=\int_{-b}^{-a} f(x)dx$를 만족하면 함수 $y=f(x)$의 그래프는 y축에 대하여 대칭이다.
$f(x)=x^8+1$는 y축에 대하여 대칭이므로 '우'이다.

33) 10

⇨ $\int_{-1}^1 (3x^2+4)\,dx=2\int_0^1 (3x^2+4)dx$

$=2[x^3+4x]_0^1=10$

34) 6

⇨ $\int_{-1}^1 (x+1)(3x+2)\,dx=\int_{-1}^1 (3x^2+5x+2)\,dx$

$=\int_{-1}^1 (3x^2+2)\,dx+\int_{-1}^1 5x\,dx$

$=2\int_0^1 (3x^2+2)\,dx$

$=2[x^3+2x]_0^1$

$=2\cdot 3=6$

35) $\frac{32}{3}$

⇨ $\int_0^2 (2x^2+2x)dx-2\int_0^{-2} (x+x^2)dx$

$=\int_0^2 (2x^2+2x)dx+\int_{-2}^0 (2x+2x^2)dx$

$=\int_{-2}^2 (2x^2+2x)dx$

$=2\int_0^2 2x^2dx=2\left[\frac{2}{3}x^3\right]_0^2=\frac{32}{3}$

36) 0

⇨ $\int_{-1}^1 (x^2+x+1)dx-\int_{-1}^1 (x^2-x+1)dx$

$=\int_{-1}^1 2xdx=0$

37) 36

⇨ $\int_{-2}^1 (6x^2-4x+1)dx+\int_2^1 (-6x^2+4x-1)dx$

$=\int_{-2}^1 (6x^2-4x+1)dx+\int_1^2 (6x^2-4x+1)dx$

$=\int_{-2}^2 (6x^2-4x+1)dx$

$=2\int_0^2 (6x^2+1)dx$

$=2[2x^3+x]_0^2$

$=2(16+2)=36$

38) 4

⇨ $\int_{-1}^0 (3x^2+4x^3+5x^4)dx-\int_1^0 (3x^2+4x^3+5x^4)dx$

$=\int_{-1}^0 (3x^2+4x^3+5x^4)dx+\int_0^1 (3x^2+4x^3+5x^4)dx$

$=\int_{-1}^1 (3x^2+4x^3+5x^4)dx=2\int_0^1 (3x^2+5x^4)dx$

$=2[x^3+x^5]_0^1=4$

39) 20

⇨ $y=|x|$는 y축에 대하여 대칭인 함수이므로

$\int_{-2}^2 (3x^2+|x|)dx=2\int_0^2 (3x^2+x)dx$

$=2\left[x^3+\frac{1}{2}x^2\right]_0^2$

$=2\times 10=20$

40) 0

⇨ $\int_{-1}^1 (x^3+3x)dx=0$

41) 2

⇨ $\int_{-1}^{1}(x+1)(x^2-x+1)dx$

$=\int_{-1}^{1}(x^3+1)dx=2\int_{0}^{1}1dx$

$=2[x]_0^1=2$

42) 0

⇨ $\int_{-2}^{2}(x^3-3x^2-2x+4)dx=2\int_{0}^{2}(-3x^2+4)dx$

$=2[-x^3+4x]_0^2=0$

43) 12

⇨ $\int_{-1}^{1}(5x^3-6x^2-7x+8)dx$

$=2\int_{0}^{1}(-6x^2+8)dx$

$=2[-2x^3+8x]_0^1$

$=2(-2+8)=12$

44) 20

⇨ $\int_{-2}^{2}(100x^3+3x^2-90x+1)dx$

$=2\int_{0}^{2}(3x^2+1)dx$

$=2[x^3+x]_0^2$

$=2(8+2)=20$

45) 42

⇨ $\int_{-3}^{3}(-x^3+3x^2+4x-2)\,dx$

$=\int_{-3}^{3}(3x^2-2)\,dx+\int_{-3}^{3}(-x^3+4x)\,dx$

$=2\int_{0}^{3}(3x^2-2)\,dx=2[x^3-2x]_0^3$

$=2\cdot 21=42$

46) 2

⇨ $\int_{-1}^{0}(x^3-3x^2+2)dx+\int_{0}^{1}(x^3-3x^2+2)dx$

$=\int_{-1}^{1}(x^3-3x^2+2)dx=2\int_{0}^{1}(-3x^2+2)dx$

$=2[-x^3+2x]_0^1=2\times 1=2$

47) 2

⇨ $\int_{-1}^{0}(4x^3-2x+1)dx+\int_{0}^{1}(4x^3-2x+1)dx$

$=\int_{-1}^{1}(4x^3-2x+1)dx=2\int_{0}^{1}1dx$

$=2[x]_0^1=2$

48) 28

⇨ $\int_{-2}^{1}(x^3+6x^2+2x-1)dx$

$+\int_{1}^{2}(x^3+6x^2+2x-1)dx$

$=\int_{-2}^{2}(x^3+6x^2+2x-1)dx$

$=2\int_{0}^{2}(6x^2-1)dx$

$=2[2x^3-x]_0^2$

$=2(16-2)=28$

49) 4

⇨ $\int_{-1}^{0}(1+2x+3x^2+4x^3)dx$

$+\int_{0}^{1}(1+2x+3x^2+4x^3)dx$

$=\int_{-1}^{1}(1+2x+3x^2+4x^3)dx$

$=2\int_{0}^{1}(1+3x^2)dx$

$=2[x+x^3]_0^1=2\times 2=4$

50) $-\frac{4}{3}$

⇨ $\int_{-1}^{1}x(1-x)^2dx=\int_{-1}^{1}(x^3-2x^2+x)dx$

$=2\int_{0}^{1}(-2x^2)dx$

$=2\left[-\frac{2}{3}x^3\right]_0^1=-\frac{4}{3}$

51) 60

⇨ $\int_{-2}^{2}(5x^4+3x^3-7x-1)\,dx$

$=\int_{-2}^{2}(5x^4-1)\,dx+\int_{-2}^{2}(3x^3-7x)\,dx$

$=2\int_{0}^{2}(5x^4-1)\,dx=2[x^5-x]_0^2$

$=2\cdot 30=60$

52) 10

⇨ $\int_{-1}^{1}(5x^4-6x^3+3x^2-7x+3)dx$

$=2\int_{0}^{1}(5x^4+3x^2+3)dx=2[x^5+x^3+3x]_0^1$

$=2\times 5=10$

53) 6

⇨ $\int_{-1}^{1}(5x^4+4x^3+3x^2+2x+1)dx$

$=2\int_{0}^{1}(5x^4+3x^2+1)dx=2[x^5+x^3+x]_0^1=6$

54) 4

⇨ $\int_{-1}^{4}(5x^4+4x^3+3x^2)dx-\int_{1}^{4}(5x^4+4x^3+3x^2)dx$

$=\int_{-1}^{4}(5x^4+4x^3+3x^2)dx+\int_{4}^{1}(5x^4+4x^3+3x^2)dx$

$=\int_{-1}^{1}(5x^4+4x^3+3x^2)dx$

$=2\int_{0}^{1}(5x^4+3x^2)dx$

$=2\left[x^5+x^3\right]_0^1$

$=2(1+1)=4$

55) 0

⇨ $\int_{-1}^{1}(x^5+2x)dx=0$

56) 12

⇨ $\int_{-2}^{2}(x^5-2x^3+3x^2-1)dx$

$=2\int_{0}^{2}(3x^2-1)dx$

$=2\left[x^3-x\right]_0^2$

$=2(8-2)=12$

57) 230

⇨ $\int_{-5}^{5}(6x^5-8x^3+3x^2-2)dx$

$=2\int_{0}^{5}(3x^2-2)dx$

$=2\left[x^3-2x\right]_0^5$

$=2(125-10)=230$

58) 20

⇨ $\int_{-2}^{2}(6x^5-4x^3+3x^2+2x+1)dx$

$=2\int_{0}^{2}(3x^2+1)dx$

$=2\left[x^3+x\right]_0^2$

$=2(8+2)=20$

59) 16

⇨ $\int_{-2}^{1}(x^5+3x^2-2x)dx-\int_{2}^{1}(x^5+3x^2-2x)dx$

$=\int_{-2}^{1}(x^5+3x^2-2x)dx+\int_{1}^{2}(x^5+3x^2-2x)dx$

$=\int_{-2}^{2}(x^5+3x^2-2x)dx$

$=2\int_{0}^{2}3x^2dx$

$=2\left[x^3\right]_0^2=2\times 8=16$

60) $\frac{4}{5}$

⇨ $\int_{-1}^{1}x^3(1+x)^2dx$

$=\int_{-1}^{1}x^3(1+2x+x^2)dx$

$=2\int_{0}^{1}2x^4dx=2\left[\frac{2}{5}x^5\right]_0^1=\frac{4}{5}$

61) 10

⇨ $\int_{-1}^{1}(8x^9-x^5+10x^4-3x^2+4)dx$

$=2\int_{0}^{1}(10x^4-3x^2+4)dx$

$=2\left[2x^5-x^3+4x\right]_0^1$

$=2(2-1+4)=10$

62) 4

⇨ $\int_{-1}^{1}(11x^{10}+12x^{11}+13x^{12}+14x^{13})dx$

$=2\int_{0}^{1}(11x^{10}+13x^{12})dx=2\left[x^{11}+x^{13}\right]_0^1=4$

63) 6

⇨ $\int_{-1}^{1}(x^{11}+10x^9-25x^7+3x^2-9x+2)dx$

$=2\int_{0}^{1}(3x^2+2)dx$

$=2\left[x^3+2x\right]_0^1$

$=2(1+2)=6$