A Novel Optimization Algorithm: Space Gravitational Optimization

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***Abstract*—A new concept for the optimization of nonlinear functions is proposed. For most of the proposed evolutionary optimization algorithms, such as particle swarm optimization and ant colony optimization, they search the solution space by sharing known knowledge. The proposed algorithm is based on the Einstein's general theory of relativity, which we utilize the concept of gravitational field to search for the global optimal solution for a given problem. In this paper, detail procedure of the proposed algorithm is introduced. The proposed algorithm has been tested on an application that is known difficult with promising and exciting results.**

***Index Terms*—Optimization theory, Einstein's general theory of relativity, gravitational field, and PID controller.**

I. Introduction

THIS paper introduces a new concept for the optimization of nonlinear functions. In recently decades, there are many optimization algorithms have been proposed, such as ant colony optimization (ACO) [1] and particle swarm optimization (PSO) [2-3]. The main feature of these algorithms is that their searching behaviors are emulating from the social behaviors of artificial life, such as ants creeping and birds flocking, and these artificial life search for best solution by sharing known knowledge to each other, so that they shall gather together at the best solution point. However, the defect of this behavior is that once a searching agent falls into a local optimal solution and the solution is good enough, it will attract others searching agents fall into the local optimal solution as well, instead of keep searching other undiscovered solution space.

For another heuristic optimization technique, the simulated annealing (SA) [4], that is derived by a natural analogy with the statistical physics of random systems. In SA, the pseudo-temperature of the system is slowly descending till it reaches to the freezing point. According to the descending pseudo-temperature, the moving speed (energy) of an atom is slowly decelerating as well; finally, it shall convergences to a global optimal solution. However, the defect of this approach is that if the atom falls into a solution of local optima, it is difficult for the atom to escape from it. This problem is highly significant when the solution space of the given problem is very big.

In this work, we utilize principles of astrophysics to design a new optimization algorithm. The proposed algorithm is inspired by a simulation of several asteroids shifting within a universe to search for the body with heaviest mass. By referring to the Einstein's general theory of relativity, the space is curved by the gravitational field. Therefore, the asteroid will be able to accelerate toward the heavy mass around it by the variations in geometry of spacetime. Then the asteroid shall be sling-shot by the heavy mass that attract it, and then keep searching for the other heavy masses within the universe.

According to the concepts described in the above paragraph, a new optimization algorithm, called space gravitational optimization, has been developed. The space gravitation optimization was developed by very simple concepts of the general theory of relativity that makes the algorithm computational inexpensive in term of both memory requirement and computational power. The most important feature of the proposed algorithm is that the possibility of falling into a local optimal solution is very small, because the searching agents (asteroids) have a great possibility to be sling-shot out of the local optimal solution, and typically will never stop searching for other solution in the entire solution space.

In order to demonstrate the validity and efficiency of the proposed algorithm, the proposed algorithm has been applied to an application of designing of the PID controllers. Several simulation results obtained by the other optimization techniques are also provided for comparison.

This paper is organized as follows: the concept of the Einstein's general theory of relativity is briefly introduced in Section II. The detail procedures and searching strategy of the proposed algorithm are described in Section III. The affections of some adjustable coefficients in the proposed algorithm are been experimented and discussed in Section IV. The benchmark of the proposed algorithm is provided in Section V. Finally, the conclusions are drawn in the last section.

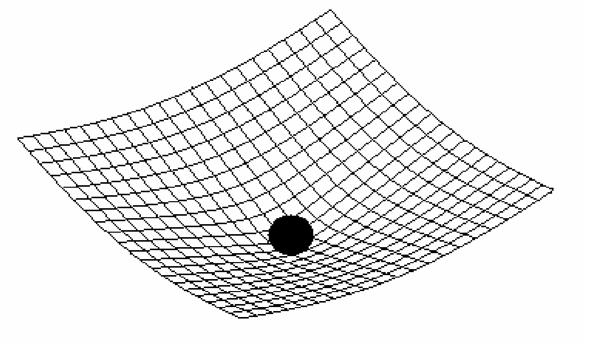

Fig. 1. Illustration of the curvature of the geometry of spacetime.

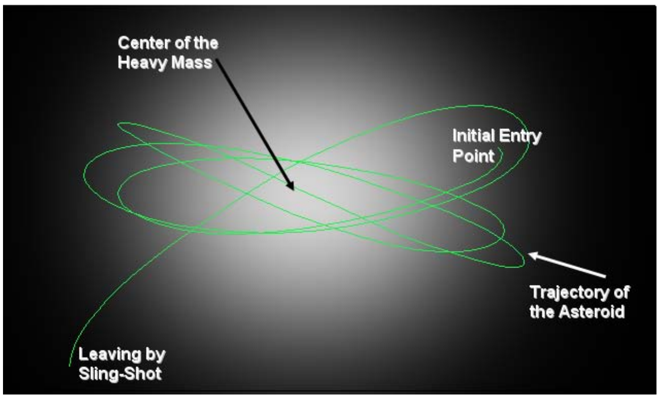

Fig. 2. Illustration of the trajectory of an asteroid sling-shot by a heavy mass.

II. EINSTEIN'S GENERAL THEORY OF RELATIVITY

Einstein's general theory of relativity was developed in the early years of the twentieth century, and reached it's final form in 1916. The base of the Einstein's general theory of relativity was that the inertial and free-falling systems are entirely equivalent. The Einstein's principle of equivalence declares that the acceleration of a free-falling laboratory cancels completely the gravitational effect. Then, we can realize that the geometry and the gravity have many properties in common if we illustrate the gravitational field that is as shown in fig. 1. The geometry of spacetime is mapped into the curved grid as in Fig. 1. The black sphere, in the middle of fig. 1, indicates the heavy mass in the space. According to the Einstein's general theory of relativity, the geometrical description of the spacetime distortion is as follow:

variation in geometry of spacetime = *stress*, *mass-energy* and *momentum* of source. (1)

This is the famous Einstein's gravitational field equation. It is similar to the Maxwell's equation, which relates the four independent vectors as well. The detail descriptions of the Einstein's general theory of relativity and astrophysics can be obtained in [5-7].

As we all know that our universe contains innumerous stars and planets, and the mass of these bodies are usually huge enough to curve the spacetime. Therefore, according to the Newton's gravity of law, the strength of gravity existing between two standard mass with standard separation, and the strength of gravity is the same throughout the universe at all time. The Newton's law of gravity may be written as follow:

$$Force = \frac{GMm}{r^2} = mg \tag{2}$$

where M and m are the two gravitating masses, r is the distance between them, G is a constant with value of $6.67\times10^{-11}\mathrm{Nm^2kg^{-2}}$, and g is the gravitational acceleration rate charges on m. In this work, we assume the absolute position of M is constant; therefore, the equation (2) can be rewritten as follow:

$$g = \frac{GM}{r^2} \tag{3}$$

By detecting the variation of the equation (3), we shall be able to determine the direction of the gravitational acceleration for the asteroid. The variation of the gravity, that is assumed to be the same as equation (1), is proportional to the inverse square of the variation of distance between M and m.

The possibility for an asteroid to enter the orbiting trajectory of a heavy mass in the universe is extremely small. Therefore, there are two possible results for an asteroid that is captured by the gravity of a heavy mass: One, the asteroid might circum the heavy mass for several circles, and then it might be sling-shot out of the system by the gravity of the heavy mass; the other result is that the asteroid couldn't escape from the heavy mass, and finally crashes on the heavy mass. However, in this work, we assume that the real body of the heavy mass does not physically exist. Therefore, when the asteroid is closed up toward the center of the heavy mass, the kinetic energy of the asteroid shall be significantly increased by the gravity of the heavy mass, and also provides enough speed for the asteroid to directly sling-shot out of the gravitational field of the heavy mass, as shown in Fig. 2.

III. PROCEDURES OF THE SPACE GRAVITATIONAL OPTIMIZATION ALGORITHM

A. General Description

The procedures of the space gravitational optimization algorithm are explained in this section. This algorithm is inspired by a simulation of several asteroids shifting within a universe to search for the body of the heaviest mass (optimal solution). According to the conceptual description of last section, the algorithm was being thought as a simplified model of astrophysics. In this algorithm, the searching agents are treated as asteroids in the universe. These agents travel through the space by the Newton's law of gravity, and the solution space is formulated as a curved spacetime according to the concept of the Einstein's general theory of relativity.

The asteroids intent to search for the bodies of heavy masses, therefore, in this paper, the optimal solutions of a given problem was formulated as the heavy masses in the universe. Differ from the other methods, the proposed algorithm intents to search the solution space without any inertia parameters (such as evaporation rate in ACO, inertia weighting in PSO, and pseudo-temperature in SA). Therefore, the proposed algorithm is able to avoid to be trapped in a solution of local optima.

B. Initialization

A group of *n* asteroids was randomly initialized with a position *x[]* and *y[]* for each with velocities on x axis and y axis, which are *vx[]* and *vy[]*. The initial acceleration rate for each asteroid on x axis and y axis were all settled to zero, such that *ax[]* = 0 and *ay[]* = 0. The global optimal solution G_{best} is settled to minus infinity. Each asteroid was represented as a point with a unique color within the two-dimensional solution space.

C. Searching Variation of Spacetime

According to the conceptual descriptions of the Einstein's general theory of relativity given in section II, we can understand that the spacetime of the universe is curved by the gravitational field, and the variation in geometry of spacetime shall causes acceleration or deceleration on asteroids (searching agents). Therefore, because we don't know the exactly position of the heavy mass (low costs), in order to determine the trajectories for the asteroids, we have to detect the variation in geometry of spacetime.

A range of detection is denoted as r_d, and the strength of gravity is represented by *G*. The summation of the variations in geometry of spacetime on directions of x axis and y axis is the acceleration rate for the asteroids:

$$ax[n] = G \cdot \left\{ \left[f\left(x[n], y[n]\right) - f\left(x[n]+r_d, y[n]\right) \right] + \left[f\left(x[n]-r_d, y[n]\right) - f\left(x[n], y[n]\right) \right] \right\} \tag{4}$$

$$ay[n] = G \cdot \left\{ \left[f\left(x[n], y[n]\right) - f\left(x[n], y[n]+r_d\right) \right] + \left[f\left(x[n], y[n]-r_d\right) - f\left(x[n], y[n]\right) \right] \right\} \tag{5}$$

where *ax[n]* and *ay[n]* are the acceleration rate on x axis and y axis of asteroid *n*, respectively. *f*(*x[n]*,*y[n]*) is the cost function that used to evaluate the goodness of the solution contains by the asteroid *n*. According to equation (4) and (5), the asteroid can be attracted to heavy masses (solutions with low costs).

D. Speeding Up/Slowing Down

After we obtained the acceleration rate of each axis, the speed of the asteroid *n* can be simply updated by following equations:

$$vx[n] = vx[n] + ax[n] \tag{6}$$

$$vy[n] = vy[n] + ay[n] \tag{7}$$

The position of the asteroid *n* in the solution space is then be updated by following equations:

$$x[n] = x[n] + vx[n] \tag{8}$$

$$y[n] = y[n] + vy[n] \tag{9}$$

E. Update Optimal Solution and Check for Stop Criterion

If the solution obtained by asteroid *n* is better than the global optimal solution G_{best}, then update G_{best} as following rule:

$$\text{if } f(x[n],y[n])<G_{best} \text{ then } G_{best}=f(x[n],y[n]) \quad G_{best}\_x=x[n] \text{ and } G_{best}\_y=y[n] \tag{10}$$

where G_{best}_x and G_{best}_y is the optimal solution on the x axis and y axis at that time.

IV. Modification of the Algorithm

Several concepts of cosmology were used to modify the equations of the proposed algorithm to explore the potential capability of the original concepts that have already used to develop the algorithm.

A. Gravitational Effect between Asteroids

According to the Newton's gravity of law, a strength of gravity existing between two standard mass with standard separation, and the strength of gravity is the same throughout the universe at all time. Therefore, it is reasonable for the gravitational effect between asteroids to join the equations (4) and (5), which can be rewritten as follows:

$$ax[n] = G \cdot \left\{ \left[f\left(x[n], y[n]\right) - f\left(x[n]+r_d, y[n]\right) \right] + \left[f\left(x[n]-r_d, y[n]\right) - f\left(x[n], y[n]\right) \right] \right\} + \left(\alpha \cdot x_c\right)/r_n^2 \tag{11}$$

$$ay[n] = G \cdot \left\{ \left[f\left(x[n], y[n]\right) - f\left(x[n], y[n]+r_d\right) \right] + \left[f\left(x[n], y[n]-r_d\right) - f\left(x[n], y[n]\right) \right] \right\} + \alpha \cdot y_c / r_n^2 \tag{12}$$

where x_c and y_c are coordinates for the center of mass of all asteroids, which can be simply obtained by the following equations:

$$x_c = \left(\sum_n x[n] \right) \Big/ n \tag{13}$$

$$y_c = \left(\sum_n y[n] \right) \Big/ n \tag{14}$$

and α is a constant that determine the influence of the gravitational effect between all asteroids, r_n is the distance between asteroid n and the center of mass of all asteroids. Typically, it is not necessary to set the value of α to a big value, because the asteroids might gather together. However, it is useful to set the value of α to a reasonable small value to increase the searching efficiency of the proposed algorithm.

B. Opening, Flat, and Closing Universe

As we all known that our universe might be one of three type of the universe: Opening universe, flat universe, and closing universe. For opening universe, the universe will keep expending until all energy of the universe are vaporized. Due to the expending effect, the distances between stars are exponentially increasing. For flat universe, the size of the universe is constant, as well as the distances between stars. Moreover, the concept of the closing universe is that the size of the universe will keep compressing until all matters are combined in a singular point, therefore, the distances between stars are also exponentially decreasing.

According to the concept of the different type of the universe, the equation (6) and (7) can be rewritten as follows:

$$vx[n] = \beta \cdot vx[n] + ax[n] \tag{15}$$

$$vy[n] = \beta \cdot vy[n] + ay[n] \tag{16}$$

where β can determine the type of the universe. If β is equal to 1, then it determines the solution space to be as the characteristic of the flat universe. If β is greater than 1, then it turns the solution space into a closing universe, because of everything seems to be approaching to the asteroid. Last, if β is less than 1, then the characteristic of the solution space is turned into an open universe, because everything seems to leaving from the asteroid.

We found that when β is a little greater than 1, it can help the algorithm escape from the solution of local optima, significantly. But there is no point to set β to a big value, because of that it makes the algorithm ineffective if the speed of an asteroid is too high.

V. Experimental Results

A. Optimal Design of PID Controller

The proposed algorithm has been implemented and tested on application of designing of PID controller under a given plant. All the simulation is implemented with MATLAB/Simulink on a P4 3.06 GHz computers with 1GB RAM. The values of the parameters in proposed algorithm are $n = 100$, $G = 0.5$, $\alpha = 0.05$, $\beta = 1.01$, $r_d = 0.05$, and the maximum generation N_g is 10 epochs, respectively. Moreover, the solution space consists of three variables, which are K_p, K_i, and K_d. The primary objective of this test is to find the optimal solutions for PID parameters K_p, K_i, and K_d. We utilized three plants, as listed as follow, to demonstrate the performance and efficient of the proposed algorithm:

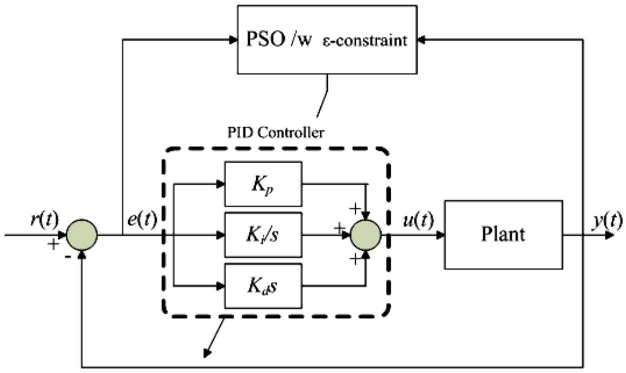

Fig 3. Conceptual Block Diagram of a PID control system under a given plant.

TABLE 1
Summary of simulation result

Plant		Zeigler-Nichols	Kitamori	Fuzzy	ACO	SGO
$G_{P1}(s)$	K_p	2.808	2.212		2.4911	2.6609
	K_i	1.712	1.085		0.8158	0.74822
	K_d	1.151	1.148		1.3540	1.5366
	y_{mo}	42.95%	14.51%	6.0%	4.41%	0.8%
	t_r	0.7500	1.0240	3.09	1.0070	0.9870
	t_s	3.8530	3.6400		1.9240	1.9080
	IAE	1.8304	1.5466	1.18	1.3776	1.3403
$G_{P2}(s)$	K_p	2.190			4.5721	1.4922
	K_i	2.126			1.2351	0.84654
	K_d	0.565			2.2814	0.0198
	y_{mo}	16.47%		6.1%	6.15%	4.67%
	t_r	0.7300		5.01	0.8000	1.0980
	t_s	5.3720			4.2770	3.3610
	IAE	0.9595		1.01	1.0585	1.3220
$G_{P3}(s)$	K_p	3.072	2.2357		3.1619	3.1661
	K_i	2.272	1.429		1.1032	1.1063
	K_d	1.038	0.976		1.6639	1.8244
	y_{mo}	32.55%	10.77%	1.9%	3.72%	3.72%
	t_r	0.6650	0.8270	2.632	0.9040	0.9410
	t_s	3.7210	2.2980		1.5210	1.5780
	IAE	1.1245	0.8815	0.811	1.0883	1.1221

$$G(s) = \frac{e^{-0.5S}}{(s+1)^2} \tag{17}$$

$$G_{P2}(s) = \frac{4.228}{(s+0.5)(s^2+1.64s+8.456)} \tag{18}$$

$$G_{P3}(s) = \frac{27}{(s+1)(s+3)^3} \tag{19}$$

Fig. 4, 5, and 6 shows the step response on the $G_{P1}(s)$, $G_{P1}(s)$, and $G_{P1}(s)$, respectively. The results obtained by using Ziegler-Nichols [8], Kitamori [9], Fuzzy [10] and ACO [11] method are also presented for comparison. Table 1 summaries the simulation results obtained on each plant. In Table 1, y_{mo} denotes the percent maximum overshoot, t_s represents the 5 percent settling time, t_r denotes the rise time and IAE are the integral of the absolute error.

For $G_{P1}(s)$, as we can see that the step response obtained by the proposed algorithm (SGO) produces significantly better

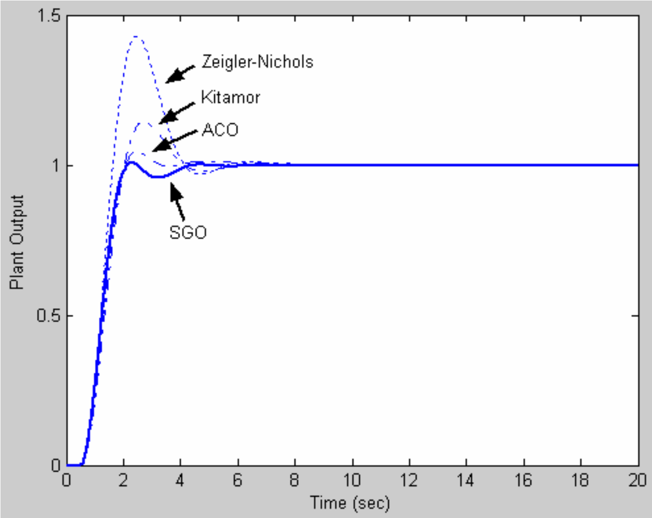

Fig. 4. Comparison of step response on plant $G_{P1}(s)$.

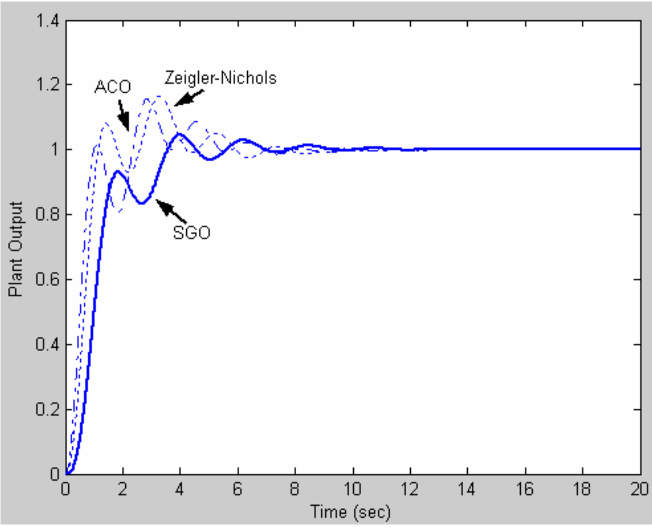

Fig. 5. Comparison of step response on plant $G_{P2}(s)$.

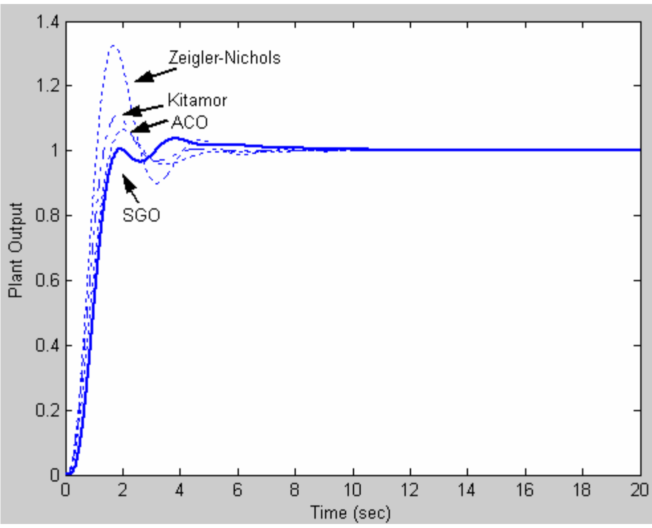

Fig. 6. Comparison of step response on plant $G_{P3}(s)$.

response than that obtained using the Ziegler-Nichols, Kitamori and Fuzzy, and ACO method on all performance indexes. For $G_{P2}(s)$, although the Ziegler-Nichols and ACO method produce smaller rising time response, the proposed method yields a smaller overshoot and settling time on $G_{P2}(s)$, and the rest performance indexes are also good enough to compare with other known methods. For $G_{P3}(s)$, the proposed algorithm apparently outperformed the Ziegler-Nichols, Kitamori, and Fuzzy methods, and the response obtained by the proposed algorithm is also good enough to compare with the ACO method.

It is necessary to notice that the proposed algorithm only requires 20 epochs to produce the high quality parameters for a PID controller under a given plant.

B. Typical Optimization Problem

In order to demonstrate the searching ability of the proposed algorithm, the proposed algorithm has also applied on a typical optimization problem given as follow:

$$\min \ f(x_1,x_2)=\sum_{i=1}^{2}\left(x_i^4-16x_i^2+0.5x_i\right) \qquad (20)$$

$$\text{subject to } -50.0 \le x_1, x_2 \le 50.0$$

The values of the parameters in proposed algorithm are n=1000, G=1.0, $\alpha = 0.05$, β=1.01, r_d=0.1, the maximum generation N_g is 10000 epochs, respectively. The illustration of the objective function $f(x_1,x_2)$ is as shown in Fig. 7. The success rate of each generation of simulation is as shown in Fig. 8. The most remarkable feature of this simulation result is that the proposed algorithm only needs approximately 40 seconds to complete the simulation (10,000,000 times of total asteroid movements) with good success rate for the optimization problem (20), that indicates the computational complexity of the proposed algorithm is very small.

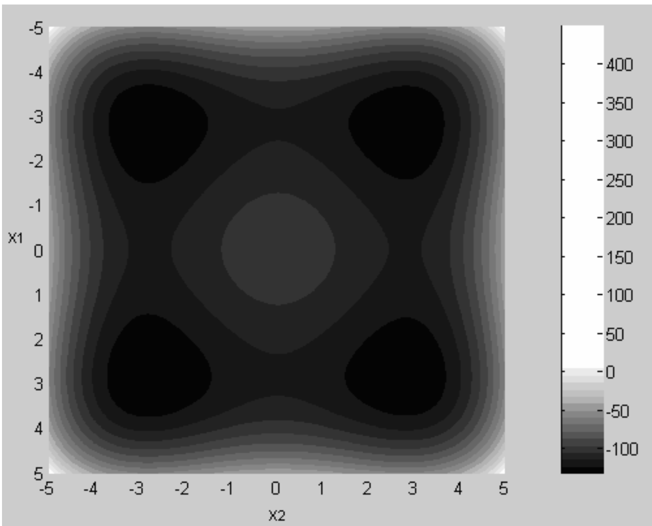

Fig. 7. Illustration of objective function for optimization problem (20).

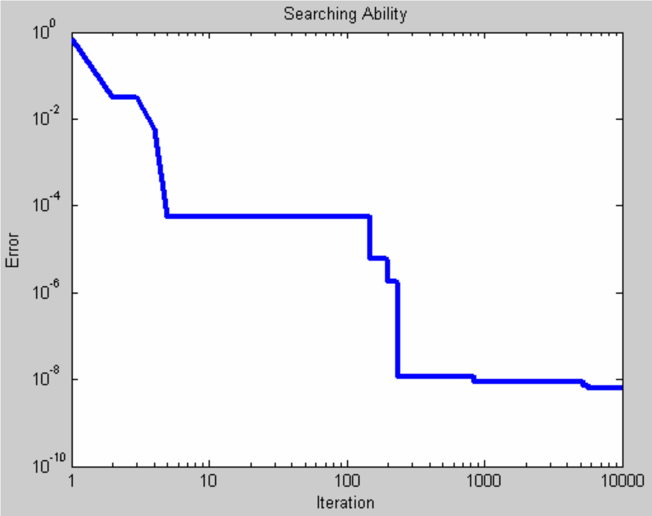

Fig. 8. Searching ability of the proposed algorithm for problem (20).

VI. Conclusions

In this paper, a novel optimization algorithm, called space gravitational optimization, has been proposed. We utilize principles of astrophysics to design the proposed new optimization algorithm. The proposed algorithm is inspired by a simulation of several asteroids shifting within a universe to search for the body with heaviest mass. The concepts of Einstein's general theory of relativity and Newton's gravity of law are also briefly introduced to explain the searching behaviors of the proposed algorithm. Conceptually, the proposed algorithm was developed as a simplified model for asteroids shifting in a curved spacetime, and the asteroids are shifting almost independently, therefore, the computational complexity of the proposed algorithm is extremely small, and the possibility of an searching agent to be trapped in the solution of local minima is very small.

The proposed algorithm has applied on an application of designing of PID controller under a given plant to verify the efficient and validation of the proposed algorithm. The result obtained by the proposed algorithm is outperformed other known methods with small amount of searching agents and lesser epoch cycles.

The goal of developing the proposed algorithm is to avoid the searching agents from trapped in the solutions of local minima, and keeping the algorithm extremely simple and robust. And the goal seems to be successfully achieved. The algorithm can be implemented in a few lines of codes. Therefore, the most basic strength of the universe has provided us an elegantly technique to process information.

VII. Acknowledgments

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