Volatility Index for the Bitcoin Market

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Abstract

In this paper, we take a close look at one of the most common hedging tools known as Volatility Index (VIX). Bitcoin was first released in 2009 and is generally known as the first decentralized cryptocurrency. Bitcoin started gaining popularity in 2017 as it skyrocketed in value. At the same time, following the 2018 cryptocurrency crash, we investigate for tools to hedge this highly volatile asset. As there are no known Volatility Indexes for the cryptocurrency market, we attempt to construct one by following the method implemented by CBOE and discuss our findings. We also investigate another kind of volatility index known as the Simple Volatility Index (SVIX) which is priced using simple variance swaps, in contrast to variance swaps for VIX.

Author's Contributions

In this paper, we replicate the methods of VIX and variance swaps implemented by CBOE to construct a similar one for the Bitcoin market. At the same time, we also follow closely Ian Martin (2013)'s paper regarding simple variance swaps and the SVIX. We use the SVIX derived by Ian Martin to construct one for the Bitcoin currency. We also use a combination of linear interpolation and constant extrapolation schemes for generating for data points for the implied variance and strike prices. Ultimately, we construct a 30-day VIX for the Bitcoin currency and a 30-day SVIX for the Bitcoin currency. We also attempt to construct the indexes with inverse European options and discuss its viability.

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Chapter 1

Introduction

Market volatility of financial instruments measure the degree of variation of its trading price over time. Volatility is the measure of uncertainty that makes pricing financial instruments challenging and rewarding at the same time. Investors are always concerned about the risk they have to take when investing in an asset. In particular, investors are more concerned with downside risk and are always seeking means to minimise their risk while also maximising their returns. As of now, no volatility index exists to measure the volatility implied by Bitcoin options. Conventionally, the SPX500 uses the VIX as a measure of the 30-day implied volatility. Therefore, we want to adapt and test if the the VIX model is also a suitable one for the Bitcoin. At the same time, we also want to apply another alternative simple variance swap to the Bitcoin to see if it is a better fit due to the presence of large jumps and sharp falls in the Bitcoin market.

1.1 Historical Work

In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index (VIX), originally designed to measure the market's 30-day expected volatility implied by At-The-Money (ATM) S&P100 Index option prices. It very quickly became a notable benchmark for the US stock market volatility and was labeled as the "fear index".

In 2003, CBOE collaborated with Goldman Sachs to refine VIX. It is now based on the S&P 500 Index and is estimated by averaging the weighted prices of the Out-of-The-Money (OTM) option prices over a wide range of strike prices. This was better for investors because the underlying asset used is more widely traded than the previous S&P100 and the VIX calculation no longer relies on the Black-Scholes-Merton model. The new pricing mechanism is based directly on option prices which uses a the following pricing formula

$$\sigma^2 = \frac{2}{T}[rT - (\frac{S_0}{S_*}e^{rT} - 1) - ln(\frac{S_*}{S_0}) + e^{rT}(\int_0^{S_*} \frac{1}{K^2}P(K)dK + \int_{S_*}^{\infty} \frac{1}{K^2}C(K)dK)]. \tag{1.1}$$

S_0 is the current underlying price, S_* is an arbitrary stock price, which is typically chosen to be the forward price. $P(K)$ and $C(K)$ are the current fair values for the put and call options with strike price K respectively. Risk free rate is denoted as r and T is the time to maturity.

The new CBOE method uses all the OTM options, however, we use the discretized version of (1.1), which can be rewritten as the following

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2}e^{RT}Q(K_i) - \frac{1}{T}[\frac{F}{K_0} - 1]^2. \tag{1.2}$$

So that σ is the implied volatility across the OTM strike options, T is time to expiration, F is the forward index level desired from index option prices, K_0 is the first strike price below F. K_i is the strike price of the ith OTM option. In other words, a call if $K_i > K_0$ or a put if $K_i < K_0$ and both put and call if $K_i = K_0$. ΔK_i is the interval between strike prices, which is half the difference between the strike on K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}.$$

And if $i = 1$ or $i = n$, then the interval is simply the difference between its nearest strike neighbour.

1.2 Motivation

As there is currently no VIX for the crpytocurrency market, thus we would like to investigate the effectiveness of replicating a VIX for the Bitcoin market (BTC) as a financial instrument for hedging this highly volatile asset BTC. The cryptocurrency market has been notorious for having large jumps and large drops in the value. Therefore, due to its highly volatile nature, we want to investigate if constructing a simple variance swap index (SVIX) would be a better a measurement of volatility for the BTC market. In addition to that, since traditionally the vanilla European options have been used to calculate VIX, we would like to investigate and try if other less popular options such as the inverse European options are able to do the same as well.

1.3 Scope of This Paper

In chapter 2, we will present and derive the variance swaps and how it is closely linked to VIX. We also discuss how we modify the VIX pricing formula to accommodate the replication using inverse European options. In addition, inspired by

Ian Martin (2013)'s idea , we construct simple variance swaps and SVIX to deal with the presence of large jumps.

Chapter 3 will thoroughly discuss the algorithm implementation, data selection process, interpolation and extrapolation done to generate more data. At the same time, we will also discuss and interpret our data, alongside other papers that have touched upon this subject.

Finally for chapter 4, we will conclude our findings for the paper.

Chapter 2

Variance Swaps & VIX

2.1 Variance Swap

The newly defined VIX is very closely linked to the variance swap. A variance swap is an agreement to swap a fixed strike V for

$$\left(ln\frac{S_1}{S_0}\right)^2 + \left(ln\frac{S_2}{S_1}\right)^2 + \cdots + \left(ln\frac{S_T}{S_{T-1}}\right)^2 \tag{2.1}$$

at time T. More specifically, the annualized variance swap can be written as

$$\frac{252}{N}\sum_{i=1}^{N}(ln\frac{S_i}{S_{i-1}})^2. \tag{2.2}$$

We use daily prices S_i such that $\delta t = 1/252$ and N is the number of trading days from and including effective date to maturity date. We can then define the payoff of a variance swap as such:

$$NotionalAmount \times (\frac{252}{N}\sum_{i=1}^{N}(ln\frac{S_i}{S_{i-1}})^2 - X_{var}).$$

Without loss of generality, we will take the notional amount to be 1.

2.1.1 Historical volatility

Recall that the price process of an underlying asset following the Black-Scholes model is as such

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

which after using Ito's lemma, we can get the following as well

$$dlnS_t = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t.$$

Then consider the discretization of the above SDE, such that $t_i = i\delta t$, $i = 0, 1, ..., N$ and $S_i := S_{t_i}$. Then we have the following.

$$ln\frac{S_i}{S_{i-1}} = lnS_i - lnS_{i-1} \approx (\mu - \frac{\sigma^2}{2})\delta t + \sigma\phi\delta t^{0.5}, i = 1, 2, ...N$$

As $E(\phi) = 0, Var(\phi) = E(\phi^2) = 1$, we square both sides and obtain

$$(ln\frac{S_i}{S_{i-1}})^2 = \left[(\mu - \frac{\sigma^2}{2})\delta t + \sigma\phi\delta t^{0.5}\right]^2 \approx \sigma^2\phi^2\delta t.$$

Taking expectations on both sides, we get,

$$E\left[\left(ln\frac{S_i}{S_{i-1}})^2\right)\right] \approx \sigma^2\delta t.$$

Which means we can estimate the historical volatility as such

$$\sigma = \sqrt{\frac{1}{N}\frac{1}{\delta t}\sum_{i=1}^{N}\left(ln\frac{S_i}{S_{i-1}}\right)^2}.$$

However, in reality, volatility is not constant, then we can rewrite realised variance as,

$$\frac{252}{N}\sum_{i=1}^{N}\left(ln\frac{S_i}{S_{i-1}}\right)^2 \approx \frac{1}{N}\frac{1}{\delta t}\sum_{i=1}^{N}\sigma_{t_{i-1}}^2\delta t \to \frac{1}{T}\int_0^T \sigma_t^2 dt \tag{2.3}$$

as $N \to +\infty$.

Since at the beginning no cash has swapped hands, we must price the fair value of X_{var} such that the variance swap has 0 value at time 0. Using (2.3), we can set the payoff of the variance swap as 0, then we have

$$\hat{E}\left[\frac{1}{T}\int_0^T \sigma_t^2 dt - X_{var}\right] = 0$$

$$X_{var} = \hat{E}\left[\frac{1}{T}\int_0^T \sigma_t^2 dt\right]. \tag{2.4}$$

In the risk neutral world, we have

$$dlnS_t = \left(r - \frac{\sigma_t^2}{2}\right) dt + \sigma_t d\hat{W}_t.$$

Integrating both sides from 0 to T, we have

$$\begin{aligned}
\int_0^T dlnS_t &= \int_0^T \left[\left(r - \frac{\sigma_t^2}{2}\right) dt + \sigma_t d\hat{W}_t\right] \\
lnS_T - lnS_0 &= rT - \int_0^T \frac{\sigma_t^2}{2} dt + \int_0^T \sigma_t^2 d\hat{W}_t \\
\int_0^T \frac{\sigma_t^2}{2} dt &= lnS_0 - lnS_T + lne^{rT} + \int_0^T \sigma_t^2 d\hat{W}_t \\
&= lnF - lnS_T + \int_0^T \sigma_t^2 d\hat{W}_t \\
&= -ln\frac{S_T}{F} + \int_0^T \sigma_t^2 d\hat{W}_t.
\end{aligned}$$

Therefore, we can price a variance swap as

$$\frac{1}{T}\int_0^T \sigma_t^2 dt = \frac{2}{T}\left[-ln\frac{S_T}{F} + \int_0^T \sigma_t^2 d\hat{W}_t\right]. \tag{2.5}$$

Since $\int_0^T \sigma_t^2 d\hat{W}_t$ is a martingale in the risk-neutral world, its expectation is 0 and thus we can the payoff of the variance swap is

$$-ln\frac{S_T}{F}.$$

Using the following identity, for any twice differentiable function $f(x)$, we have

$$\begin{aligned} f(x) &= f(a) + f'(a)(x-a) + \int_a^x f''(K)(x-K)dK \\ &= f(a) + f'(a)(x-a) + \int_a^x f''(K)[(x-K)^+ - (K-x)^+]dK \\ &= f(a) + f'(a)(x-a) + \int_a^x f''(K)(x-K)^+dK + \int_x^a f''(K)(K-x)^+dK \\ &= f(a) + f'(a)(x-a) + \int_a^{+\infty} f''(K)(x-K)^+dK + \int_0^a f''(K)(K-x)^+dK. \end{aligned}$$

Hence, by choosing $f(x) = -ln\frac{x}{F}$, we have $a = F$ and $x = S_T$. Then we get the following

$$-ln\frac{S_T}{F} = -\frac{S_T - F}{F} + \int_F^{+\infty} \frac{1}{K^2}(S_T - K)^+dK + \int_0^F \frac{1}{K^2}(K - S_T)^+dK. \quad (2.6)$$

Which can be interpreted as holding a

1. a forward contract to short $\frac{1}{F}$ units of underlying asset S_T for F
2. a static position in $\frac{2}{K^2}dK$ OTM calls expiring at time T with strike K
3. a static position in $\frac{2}{K^2}dK$ OTM puts expiring at time T with strike K

Therefore, we can derive the VIX formula by using the above equation (2.4) and (2.5). We get

$$\begin{aligned} K_{var} &= \frac{2}{T}\hat{E}\left[-ln\frac{S_T}{F}\right] \\ &= \frac{2}{T}\hat{E}\left[-ln\frac{S_T}{K_0} + ln\frac{F}{K_0}\right] \\ &= \frac{2}{T}\hat{E}\left[-\frac{S_T - K_0}{K_0} + ln\frac{F}{K_0}\right] + \frac{2e^{rT}}{T}\left[\int_{K_0}^{+\infty} \frac{1}{K^2}C(S_0, 0; K, T)dK + \int_0^{K_0} \frac{1}{K^2}P(S_0, 0; K, T)dK\right] \end{aligned} \quad (2.7)$$

However, using taylor's expansion on the first term in (2.7), we get

$$\begin{aligned}\hat{E}\left[-\frac{S_T-K_0}{K_0}+ln\frac{F}{K_0}\right] &= -\frac{F-K_0}{K_0}+ln\frac{F}{K_0} \\ &= -\frac{F-K_0}{K_0}+\left(\frac{F}{K_0}-1\right)-0.5\left(\frac{F}{K_0}-1\right)^2+O\left(\frac{F}{K_0}-1\right)^3 \\ &= -0.5\left(\frac{F}{K_0}-1\right)^2+O\left(\frac{F}{K_0}-1\right)^3.\end{aligned} \tag{2.8}$$

Combining both (2.8) and (2.7), we can get the following VIX as below in (2.9).

2.2 VIX

The current VIX index which is a volatility index for S&P 500 is defined as

$$VIX = 100 \times \sqrt{\left[T_1\sigma_1^2\left(\frac{N_{T2}-N_{30}}{N_{T2}-N_{T1}}\right)+T_2\sigma_2^2\left(\frac{N_{30}-N_{T1}}{N_{T2}-N_{T1}}\right)\right]\times\frac{N_{365}}{N_{30}}}$$

where

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2}e^{RT}Q(K_i)-\frac{1}{T}\left[\frac{F}{K_0}-1\right]^2. \tag{2.9}$$

1. N_{T1} = Number of minutes to settlement of the near-term option. However to avoid microstructure effects, CBOE chooses the option such that its expiry is more than 9 days.

2. N_{T2} = Number of minutes to settlement of the next-term option. N_{T2} is the next closest maturity to N_{T1}.

3. N_{30} = Number of minutes in 30 days.

4. N_{365} = Number of minutes in 365 days.

5. $Q(K_i)$ = Midpoint of the bid-ask spread for each of the OTM option with strike price K_i.

6. F = The forward S&P 500 index level, calculated by using the put-call parity where the call and put options are closest to each other. Such that

$$F = e^{rT}[C(K) - P(K)] + K,$$

where $C(K) - P(K) = \min_i |C(K_i) - P(K_i)|$.

7. K_0 is the first strike price below F.

8. K_i is the strike price of the ith OTM option. In other words, K_i is the ith out of money call option if $K_i > K_0$ and is the ith out of money put option if $K_i < K_0$. If $K_i = K_0$, then $Q(K_i)$ refers to the average of the put and call at the K_i strike price.

9. $\Delta K_i = (K_{i+1} - K_{i-1})/2$ for $i = 2, ..., N-1$. However for the endpoints, we have $\Delta K_1 = K_2 - K_1$ and $\Delta K_N = K_N - K_{N-1}$.

Using equations (2.7), (2.8) and omitting the cubic terms onward from (2.8), we can see that the discretized version is the σ^2 in the VIX. At the same time, since we know that F is quite close to K_0, and T is relatively large, we can also write VIX as the following

$$VIX \equiv \frac{2}{T}\left[\int_{K_0}^{+\infty} \frac{1}{K^2} e^{rT} C(S_0, 0; K, T) dK + \int_0^{K_0} \frac{1}{K^2} e^{rT} P(S_0, 0; K, T) dK\right]. \tag{2.10}$$

Conventionally, VIX has always been calculated using vanilla European options as the underlying asset. We shall observe and see what happens when we change the underlying asset to inverse options. Firstly, we have the following formulas used for the pricing of an inverse option[1] with F being the spot price of the futures contract with a given strike value.

$$c = \frac{1}{K}N(-d_2) - \frac{1}{F}N(-d_1) \tag{2.11}$$

$$p = \frac{1}{F}N(d_1) - \frac{1}{K}N(d_2) \tag{2.12}$$

where,

$$d1 = \frac{ln(K/F) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d2 = d1 - \sigma\sqrt{T}.$$

Then we simply modify the expression (2.7) to accommodate the European inverse options payoffs. Let us define the payoffs for the inverse call and put options, denoting them as $call_{inverse}$ and $put_{inverse}$ respectively.

$$call_{inverse} = (\frac{1}{K} - \frac{1}{S_T})^+$$

Like the vanilla call, the inverse call option only has positive payoff when $S_T > K$.

$$put_{inverse} = (\frac{1}{S_T} - \frac{1}{K})^+$$

Like the vanilla put, the inverse put option only has a positive payoff when $S_T < K$.

[1] `https://quedex.net/edu/option_valuation`

Therefore, we can do a little bit of simple manipulation to modify the replication of VIX with vanilla options to the inverse options.

$$\begin{aligned} call_{vanilla} &= (S_T - K)^+ \\ &= \frac{S_T K}{S_T K}(S_T - K)^+ \\ &= S_T K(\frac{1}{K} - \frac{1}{S_T})^+ \\ &= S_T K call_{inverse}. \end{aligned} \tag{2.13}$$

Similarly for the European inverse put,

$$\begin{aligned} put_{vanilla} &= (K - S_T)^+ \\ &= \frac{S_T K}{S_T K}(K - S_T)^+ \\ &= S_T K(\frac{1}{S_T} - \frac{1}{K})^+ \\ &= S_T K put_{inverse}. \end{aligned} \tag{2.14}$$

Ultimately, we have the following result.

$$VIX \equiv \frac{2e^{rT}}{T}\left[\int_{K_0}^{+\infty} \frac{S_T}{K} C_{inverse} dK + \int_0^{K_0} \frac{S_T}{K} P_{inverse} dK\right] \tag{2.15}$$

The only difference with the vanilla VIX is the weights used for the OTM options. For the VIX with inverse options, the weights are $\frac{S_T}{K}$ while for the VIX with vanilla options, the weights are $\frac{1}{K^2}$.

2.2.1 Limitations

As we are working with the discretized version of VIX, the interval of integration is truncated from $[0, \infty]$ to $[K_L, K_H]$, where K_L and K_H is the lowest and highest OTM strike price available on the exchange used. This truncation will definitely lead to an underestimation of the volatility. At the same time, the taylor series used in equation (2.8) ultimately ignores terms of power 3 and higher. Therefore, we will use a method proposed by Jiang and Tian (2007) for a smooth interpolation-extrapolation of the implied volatility function to make up for the limited strike prices available. The implied volatility function is used on every interval between two consecutive strike prices. Linear extrapolation will be done below the minimum strike price and above the maximum strike price to generate a more accurate approximation. By setting the coordinates as $(d_2(K), \sigma^2(K))$, we will generate more points of implied variance and then, use the implied variance to generate the option prices for the respective strike prices. More details will be present in Chapter 4.

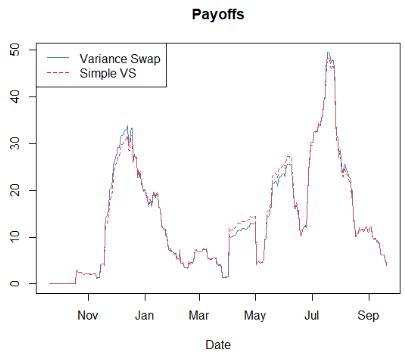
On the other hand, Bollerslev, Tauchen and Zhou (2009) and Drechsler and Yaron (2011) used the VIX^2 as the benchmark for the risk-neutral expectation of the quadratic variation of log returns. However, in the presence of large jumps, VIX does not correspond to the fair strike on a variance swap. In other words, the replicating portfolio (2.6) does not accurately replicate the variance swap payoff. Therefore, we can see that in the context of cryptocurrency where large jumps are ubiquitous, we need to look at another method of measuring the volatility. Therefore, in chapter 3, we will look at Simple Variance Swap Index (SVIX) as proposed by Ian Martin (2013).

2.3 Simple Variance Swap & SVIX

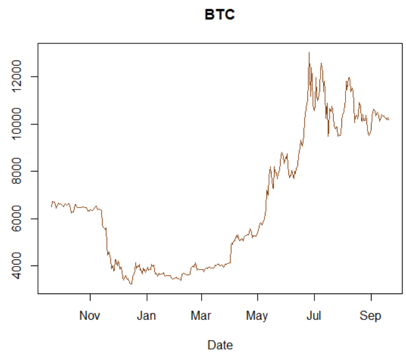
By definition of the variance swap, there exists a fundamental problem that causes the payoff of the variance swap to be ∞. More specifically, if the underlying asset used goes bankrupt, then the we have $S_t = 0$ some time before expiry T. Thus, resulting in an infinite payoff. Therefore, this motivates us to define the following definition of the simple variance swap. A simple variance swap is an agreement to exchange

$$V = \left(\frac{S_1 - S_0}{F_{0,0}}\right)^2 + \left(\frac{S_2 - S_1}{F_{0,1}}\right)^2 + \cdots + \left(\frac{S_T - S_{T-1}}{F_{0,T-1}}\right)^2 \tag{2.16}$$

for a fixed strike V at time T, such that F is the forward price of the underlying asset at time to time t, known at time 0 and $F_{0,t} = S_0 e^{rt}$.



(a) Payoffs of VS and SVS



(b) BTC Historical Price

Figure 2.1: VS and SVS Payoffs with BTC

We will show in detail the motivation for the simple variance swap. Figure 2.1a shows the respective payoffs for the variance swaps and simple variance swaps from the period of 10/18 to 10/19. As we can see in the period just before January, the variance swap has a larger payoff than the simple variance swap due to the large drop BTC prices. Similarly, the variance swap had a smaller payoff when the BTC rose sharply in the period of April to June. Overall, the payoffs for both instruments are very similar. We can refer to Figure 2.1b to see the BTC historical chart price for comparison. Figure 2.2 also shows the difference between the VS and SVS payoffs, more specifically payoffs (2.1) - (2.11).

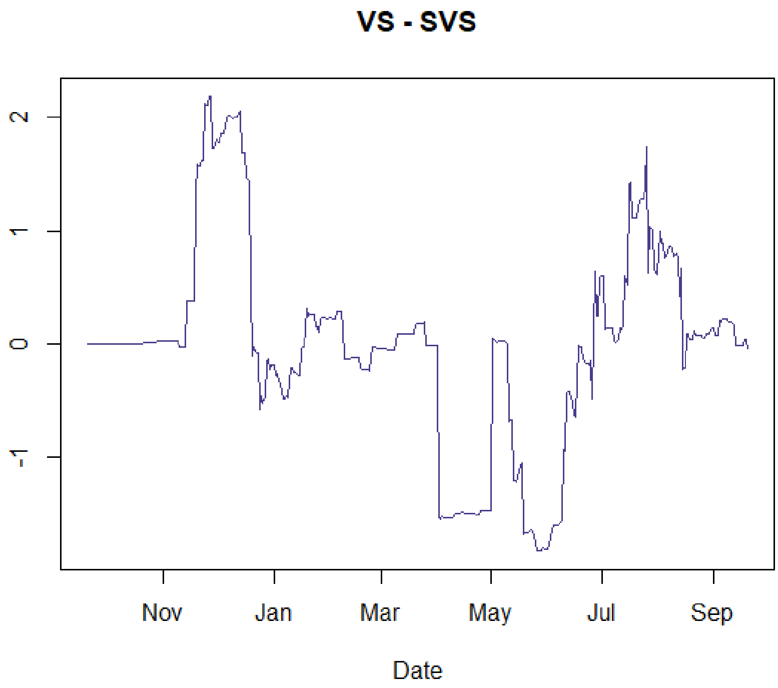


Figure 2.2

Annualized 30-day volatility was calculated for both payoffs of VS and SVS. Because Deribit Exchange does not keep readily available historical data for the Deribit Index, we took the BTC historical price from CoinMarketCap and used a range of 1 year data. Risk-free rates used were taken from the US treasury bill rates[2], which is consistently used throughout the whole paper. The payoffs calculated were ultimately multiplied by a factor of $\sqrt{12} * 100$, to be expressed as annual volatility in percentage.

[2] https://www.treasury.gov/resource-center/data-chart-center/interest-rates

The following results of simple variance swaps and SVIX are taken from Ian Martin (2013). We claim that the strike on a simple variance swap is

$$V = \frac{2e^{rT}}{F_{0,T}^2}\left[\int_0^{F_{0,T}} P(S_0, 0; K, T)dK + \int_{F_{0_T}}^{\infty} C(S_0, 0; K; T)dK\right]. \qquad (2.17)$$

Proof: The derivation of (2.12) Assuming the absence of arbitrage, we have a sequence of strictly positive stochastic discount factors M_Δ, such that a payoff $X_{k\Delta}$ at time $k\Delta$ has price $E_{i\Delta}\left[M_{(i+1)\Delta}M_{(i+2)\Delta}\dots M_{k\Delta}X_{k\Delta}\right]$ at time $i\Delta$, such that the subscript on the expectation operator implies that the expectation is conditional on the time $i\Delta$ information. At the same time, we denote $M_{k\Delta}^* \equiv M_\Delta M_{2\Delta}\dots M_{k\Delta}$.

Therefore, with respect to (2.11), we choose V such that the swap has 0 value at time 0, then we have

$$E\left[M_T^*\left(\left(\frac{S_1 - S_0}{F_{0,0}}\right)^2 + \left(\frac{S_2 - S_1}{F_{0,1}}\right)^2 + \dots + \left(\frac{S_T - S_{T-1}}{F_{0,T-1}}\right)^2 - V\right)\right] = 0. \quad (2.18)$$

We tackle the general terms in (2.13), using the law of iterated expectations and the fact that $E_{i\Delta}[M_{(i+1)\Delta}] = e^{-r\Delta}$. We also assume that if dividends are continuously reinvested into the underlying asset, then the investment of $e^{-\delta\Delta}S_{(i-1)\Delta}$ at time $(i-1)\Delta$ at time $i\Delta$ is worth $S_{i\Delta}$, thus implying that $E_{(i-1)\Delta}M_{i\Delta}S_{i\Delta} = e^{\delta\Delta}S_{(i-1)\Delta}$. Finally, we define $\Pi(i)$ to be the time-0 price of a claim to S_i^2 paid at time i. Then for each general term in (2.13), we have

$$\begin{aligned}
E[M_T^*(S_{i\Delta} - S_{(i-1)\Delta})^2] &= E\left[E_{i\Delta}\left(M_T^*(S_{i\Delta}^2 + S_{(i-1)\Delta}^2 - 2S_{i\Delta}S_{(i-1)\Delta})\right)\right] \\
&= E\left[e^{-r(T-i\Delta)}M_{i\Delta}^*(S_{i\Delta}^2 + S_{(i-1)\Delta}^2 - 2S_{i\Delta}S_{(i-1)\Delta})\right] \\
&= e^{-r(T-i\Delta)}E[M_{i\Delta}^*(S_{i\Delta}^2 + S_{(i-1)\Delta}^2 - 2S_{i\Delta}S_{(i-1)\Delta})] \\
&= e^{-r(T-i\Delta)}E[M_{i\Delta}^*S_{i\Delta}^2 + e^{-r\Delta}M_{(i-1)\Delta}^*S_{(i-1)\Delta}^2 - 2e^{-\delta\Delta}M_{(i-1)\Delta}^*S_{(i-1)\Delta}^2] \\
&= e^{-r(T-i\Delta)}[\Pi(i\Delta) - (2 - e^{-(r-\delta)\Delta})e^{-\delta\Delta}\Pi((i-1)\Delta)].
\end{aligned}$$

Replacing the respective i-th terms in (2.13) with the above, we have,

$$V = \sum_{i=1}^{T/\Delta} \frac{e^{ri\Delta}}{F_{0,(i-1)\Delta}^2}[\Pi(i\Delta) - (2 - e^{-(r-\delta)\Delta})e^{-\delta\Delta}\Pi((i-1)\Delta)]. \tag{2.19}$$

However, now it remains to calculate what $\Pi(t)$ is. With a clever trick suggested by Darrel Duffie, we can rewrite it as

$$\begin{aligned} S_t^2 &= 2\int_0^\infty E[(S_t - K)]^+ dK \\ \Pi(t) &= 2\int_0^\infty E[(S_t - K)]^+ dK \\ \Pi(t) &= 2\int_0^\infty C(S_0, 0; K, T) dK. \end{aligned}$$

In order to express $\Pi(t)$ in terms of OTM options, we have to use the put-call parity relationship, $C = P + S_0 - e^{-rt}K$.

$$\begin{aligned} \Pi(t) &= 2\int_0^\infty call_{0,t}(K)dK \\ \Pi(t) &= 2\int_0^{F_{0,t}} call_{0,t}(K)dK + 2\int_{F_{0,t}}^\infty call_{0,t}(K)dK \\ \Pi(t) &= 2\int_0^{F_{0,t}} put_{0,t}(K)dK + 2\int_{F_{0,t}}^\infty call_{0,t}(K)dK + 2\int_0^{F_{0,t}} e^{-rt}(F_{0,t} - K)dK \\ \Pi(t) &= 2\int_0^{F_{0,t}} put_{0,t}(K)dK + 2\int_{F_{0,t}}^\infty call_{0,t}(K)dK + 2e^{-rt}(F_{0,t}K - \frac{K^2}{2})\Big|_0^{F_{0,t}} \\ \Pi(t) &= 2\int_0^{F_{0,t}} put_{0,t}(K)dK + 2\int_{F_{0,t}}^\infty call_{0,t}(K)dK + e^{-rt}F_{0,t}^2. \end{aligned} \tag{2.20}$$

Then, let use denote $P(t)$ as such,

$$P(t) \equiv 2\left[\int_0^{F_{0,t}} put_{0,t}(K)dK + \int_{F_{0,t}}^\infty call_{0,t}(K)dK\right].$$

We can rewrite (2.14) as

$$V = \sum_{i=1}^{T/\Delta} \frac{e^{ri\Delta}}{F_{0,(i-1)\Delta}^2}[P(i\Delta) - (2 - e^{-(r-\delta)\Delta})e^{-\delta\Delta}P((i-1)\Delta)] + \frac{T}{\Delta}(e^{(r-\delta)\Delta} - 1)^2.$$

Once again, we tackle each general term in the above equation. Then for $0 < j < T/\Delta$, the coefficient on $P(j\Delta)$ can be written as,

$$\frac{e^{rj\Delta}}{F^2_{0,(j-1)\Delta}} - \frac{e^{r(j+1)\Delta}}{F^2_{0,j\Delta}}(2 - e^{-(r-\delta)\Delta})e^{-\delta\Delta}.$$

This uses the same argument when we were reducing the general terms in (2.13) as well. Thus it reduces to

$$\frac{e^{rj\Delta}}{F^2_{0,j\Delta}}(e^{(r-\delta)\Delta} - 1)^2.$$

Therefore, we can rewrite (2.14) as

$$V(\Delta) = \frac{e^{rT}}{F^2_{0,T-\Delta}}P(T) + \sum_{j=1}^{\frac{T}{\Delta}-1} \frac{e^{rj\Delta}}{F^2_{0,j\Delta}}(e^{(r-\delta)\Delta} - 1)^2 P(j\Delta) + \frac{T}{\Delta}(e^{(r-\delta)\Delta} - 1)^2. \quad (2.21)$$

The second term has T/Δ terms in the sum, each with magnitude of order Δ^2, therefore the total sum has order $O(\Delta)$. Equivalently, the third term has order $O(\Delta)$ as well, hence both second and third terms tend to 0 as Δ tends to 0. The first term tends to $\frac{e^{rT}}{F^2_{0,T-\Delta}}P(T)$.

There is a subtle difference between portfolios of variance swaps and simple variance swaps. The portfolio with simple variance swaps holds equal amounts of OTM options for all different strikes. On the other hand, the variance swaps holds increasingly larger positions in OTM puts with increasingly lower strike prices. Thus, implying that the commonly used variance swaps are more sensitive to sharp fall in prices in the underlying asset as compared to simple variance swaps. Then by analogy with VIX, we shall define a new index, SVIX

$$SVIX^2 \equiv \frac{2e^{rT}}{TF^2_{0,T}}\left[\int_0^{F_{0,T}} put_{0,T}(K)dK + \int_{F_{0,T}}^{\infty} call_{0,T}(K)dK\right]. \quad (2.22)$$

Chapter 3

Algorithm Implemented

This chapter describes in detail the algorithm used for computing VIX and how data was selected. Vanilla European options data are collected daily from Deribit exchange while inverse European options data are collected daily from Quedex exchange. Deribit exchange is generally quite liquid because it trades common vanilla European options and futures and is easy to use. However, Quedex only trades inverse European options and inverse futures, hence it is very illiquid and unpopular. Data has been collected from these two websites from August to September giving us about 2 months worth of data.

3.1 Data Selection

Step 1

We first identify the forward price level F and then clean the data again by only retaining a valid set of option prices.

- Firstly, we recalibrate the call and put prices by setting it to be the average of the bid/ask quotations.

- Next, take the absolute difference between all the Call and Put option prices available. Then we choose the strike price, call it K_0, to be the strike price such that it is the smallest absolute difference among the the pair of calls and puts.

- Apply the formula,

$$F = K_0 + e^{RT} \times (C(K) - P(K))$$

  to get the desired forward price level F.

- We now let K_0 to be the strike price just below F.

- Finally, we only retain relevant option prices. To be specific, for strike prices $K > K_0$, we select OTM call options and $K_0 \geq K$ we select OTM put options. At the same time, starting from K_0 and moving to lower strikes, we also exclude options whose bid prices are equal to zero and once two consecutive puts are found to have 0 bid prices, no puts with lower strikes are considered for inclusion. Similarly for calls, starting from K_0 upwards, we exclude calls with bid price of zero and any two consecutive calls with zero bid will not be considered from then on.

3.2 Bisection Method for Implied Volatility

Step 2

Since the data extracted from the exchanges as mentioned above is not perfect, we need to generate more strikes and respective strike prices so that we can have a better estimation of the variance. We calculate implied variance using the option data given in step 1, so ultimately we can generate a good interpolation and extrapolation of data.

- For each option data, since we only know $S_0, K, r, T, C(K), P(K)$, we can derive the σ which represents the implied volatility for each option at strike price K. Unfortunately, it is impossible to invert the Black-Scholes formula so that σ is expressed as a function of $S_0, K, r, T, C(K), P(K)$. Therefore, we will use the bisection method to calculate the implied volatility.

- For the bisection method, we set the error bound to be 10^{-5}, initial lower and upper bound to be 0 and 2 respectively.

- The algorithm runs until the absolute difference between calculated and market option price is bounded above by 10^{-5}. We do this for all the option prices retained in step 1.

3.3 Interpolation & Extrapolation

Step 3

After getting the implied volatilities for each option, we create data points (d_2, σ^2). We define d_2 as the following

$$d_2(K) = \frac{-ln\frac{K}{F} - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}.$$

We can see that $d_2(K)$ is a decreasing function on K. Thus, we have to ensure its monotonicity.

- Starting from the OTM put with the highest strike, call it K_i, we go to the next lower strike price K_{i-1}, and compare their respective $d_2(K)$ and if the monotonic decreasing property fails, we discard all data points that have strike prices smaller than K_i. In other words, if $d_2(K_i) > d_2(K_{i-1})$, then we discard all strike prices $\leq K_{i-1}$.

- Similarly, starting from the OTM call with the lowest strike, call it K_i, we go to the next higher strike price K_{i+1}, and compare their respective $d_2(K)$. This time we ensure that the monotonic increasing property holds, otherwise we do the same as above. In other words, if $d_2(K_i) < d_2(K_{i+1})$, then we discard all strike prices $\geq K_{i+1}$.

- Finally with the final data points, we construct a cubic polynomial between each two consecutive points. As for the intervals $(-\infty, d_2(K_N))$ and $(d_2(K_1), +\infty)$, we will use a combination of linear and constant extrapolation.

- We sort the data points in ascending order of d_2, such that for N data points, we have (x_i, y_i) and $x_1 \leq x_2 \leq \cdots \leq x_n$.

- Define the slope at each interim point to be

$$y'(x_i) = -\left(\frac{x_{i+1}-x_i}{l_{i+1}} - \frac{x_i - x_{i-1}}{l_i}\right) \div \left(\frac{y_{i+1}-y_i}{l_{i+1}} - \frac{y_i - y_{i-1}}{l_i}\right),$$

$$l_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}.$$

  By defining the slopes this way, the resulting approximation function will be smooth across each data point. At the end points, we define $y'(x_1) = \frac{y_1}{x_1}$ and $y'(x_n) = \frac{y_n}{x_n}$.

- For every $1 \leq i \leq N-1$, the cubic polynomial on $[x_i, x_{i+1}]$ is defined as

$$y(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, x_i \leq x \leq x_{i+1}$$

  where

$$a_i = y_i$$

$$b_i = y'(x_i)$$

$$c_i = (3\Delta y_i - \Delta x_i y'(x_{i+1}) - 2\Delta x_i y'(x_i)) / (\Delta x_i)^2$$

$$d_i = (-2\Delta y_i + \Delta x_i y'(x_{i+1}) + \Delta x_i y'(x_i)) / (\Delta x_i)^3$$

$$\Delta x_i = x_{i+1} - x_i$$

$$\Delta y_i = y_{i+1} - y_i.$$

- For extrapolation, we start with the lower end. We identify a x_0 such that $\sigma_0^2 = \min(2, 2 * y(x_1)) = y'(x_1) * x_0$.

- On the interval $(x_0, x_1]$ we use linear extrapolation and from $(-\infty, x_0]$, we use constant extrapolation such that

$$y(x) = \begin{cases} y'(x_1) * x_0 & x \in (\infty, x_0) \\ y'(x_1) * x & x \in [x_0, x_1) \end{cases}$$

- For extrapolation on the upper end, we identify a x_{N+1} such that $\sigma^2_{N+1} = \min(2, 2 * y(x_n)) = y'(x_N) * x_{N+1}$.

- Similar to the above, we have

$$y(x) = \begin{cases} y'(x_N) * x & x \in (x_N, x_{N+1}] \\ y'(x_N) * x_0 & x \in (x_{N+1}, +\infty) \end{cases}$$

- We implement our algorithm as stated above and fit an approximation for the implied variance. Figure 1 shows a sample plot of the approximated variance

Ultimately, we want to have a well defined mapping of $(d_2(K), \sigma^2(K))$. Then, we will use the implied variance calculated from the respective strike price K for the Black-Scholes pricing formula for vanilla options (3.1), (3.2) and inverse options (3.3), (3.4). By doing these for all the interpolation and extrapolations, we can generate a larger set of more "complete" data with larger range of strike prices and estimated option prices. Thus, we can have a better approximation of our VIX.

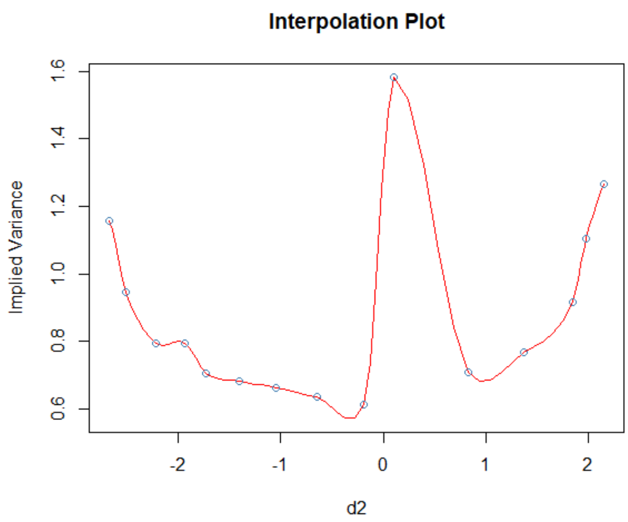


Figure 3.1: Interpolation Plot blue dots represent the actual points obtained from data while the red lines are the cubic polynomials generated.

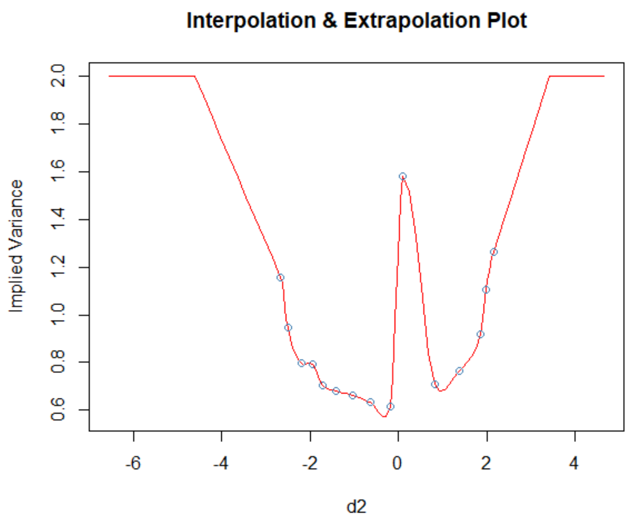


Figure 3.2: Interpolation Plot blue dots represent the actual points obtained from data while the red lines are the cubic polynomials generated.

3.4 Simulation Results & Interpretation

In this chapter, we will discuss and interpret of our simulated results.

3.4.1 VIX vs VIXBTC

The VIX index is a measurement of the expected future volatility implied by the set of call and put OTM options. Traders often quote implied volatility of an option rather than its price because the implied volatility tends to be less variable than the price itself. At the same time, historical volatility is backward looking as compared to implied volatility which is forward looking, thus giving valuable information.[1].

As we can see in the Figure 3.3 below, the 30 day annualized volatility for SPX500[2] is very close to the SPX500 VIX. The data used in Figure 3.3 was from weeks 2-3 of September. To be more specific, the first column was calculated using equation (2.2), second column was calculated using (2.9) and the third column was simply the VIX/vol ratio. Overall, we can see that the historical volatility and the VIX are relatively close in range.

[1] Hull J.C. - Options Futures and Other Derivatives 9th Edition

[2] We use SPX500, S&P500 interchangeably

30-day vol	VIX	VIX / vol
19.56	18.98	0.97
19.58	19.66	1.00
19.80	17.33	0.88
20.05	16.27	0.81
20.00	15.00	0.75
19.95	15.27	0.77
19.84	15.20	0.77
19.94	14.61	0.73
19.94	14.22	0.71
19.70	13.74	0.70
19.55	14.67	0.75
19.45	14.44	0.74
17.44	13.95	0.80
17.04	14.05	0.82

Figure 3.3: First column: 30-day historical volatility, second column: VIX, third column: VIX / vol ratio.

Similarly with respect to Figure 3.4, VIXBTC was also relatively in close range to the 30-day annualized volatility for BTC returns. However, we can clearly see that the VIXBTC is much higher than the 30-day historical volatility, whereas in the case of the SPX500, the VIX was smaller than the 30-day historical volatility. We believe that this discrepancy is due to the highly volatile nature of the Bitcoin currency. At the same time, the price of SPX500 was rather stable during the period of the data, therefore resulting in a lower implied volatility. However, the price of BTC was spiking hard and was relatively unstable, thus resulting in its high VIXBTC.

30-day vol	VIXBTC	VIXBTC / vol
66.97	83.86	1.25
67.43	83.06	1.23
65.64	79.75	1.21
58.02	79.37	1.37
57.45	79.04	1.38
56.46	76.87	1.36
56.11	77.89	1.39
56.21	78.54	1.40
56.21	75.13	1.34
55.00	74.81	1.36
55.00	76.17	1.38
55.12	77.07	1.40
39.26	76.46	1.95
40.14	81.57	2.03

Figure 3.4: First column: 30-day historical volatility, second column: VIXBTC, third column: VIXBTC / vol ratio.

The average VIXBTC is about 80% from our data. So how do we interpret this number? To put it simply, a VIXBTC of 80% means that the expected 1 month change in BTC is less than 80% up or down with a probability of 68% (1 s.d). This is no surprise for the Bitcoin market due to its highly volatile nature. Being a relatively new currency in the market, cryptocurrency has been going through a lot of ups and downs in the presence of news and regulations. At the same time, there isn't any real tangible value backed by Bitcoin. It is merely a digital currency.

One thing we do know is that, a high VIX implies that investors are predicting sharp moves in the market whether it be upward or downward. If investors truly believe that the BTC is currently fairly priced and not volatile, only then will we see the implied volatility of the call and put options to be lower, therefore lowering the VIXBTC.

Let us revisit the Black-Scholes Pricing formula for vanilla European options.

$$c = S_0N(d_1) - Ke^{-rT}N(d_2) \tag{3.1}$$

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1) \tag{3.2}$$

where,

$$d1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d2 = d1 - \sigma\sqrt{T}.$$

Let us take a closer look at the pricing formulas above. For equation (3.1), the term $N(d_2)$ is the probability that the call option will be exercised in a risk neutral world. The term $N(d_1)$ is not so easy to interpret. However, the term $S_0N(d_1)$ can be interpreted as the expected stock price at time T in a risk-neutral world when stock prices less than the strike price are noted as zero. Similarly for the put option (3.2), $N(-d_2)$ is the probability that the put option will be exercises in a risk neutral world. Using this knowledge, we can now interpret the "outliers" in Figure 3.1.

As seen in Figure 3.1, the blue dots represent the actual price and implied variance that was taken from Deribit Exchange. Throughout the 80 runs we've had, there always exists one "outlier" that is the blue dot hovering just above $d_2 = 0$. Consequently, the strike price that the outlier d_2 has is extremely close to the forward price level F, and also happens to be the closest data point to the forward price level F.

Given the definition of d_2,

$$d_2(K) = \frac{-ln\frac{K}{F} - \frac{1}{2}\sigma^2T}{\sigma\sqrt{T}}$$

and the fact that we are only working with OTM options,we can say that the more negative d_2 values represent the OTM call options while the more positive d_2 values represent the OTM put options. In spite of being an OTM option, investors are willing to pay a relatively larger premium for it because they believe that there is an equally high probability that the option might be in the money when it expires, this is especially true for the strike prices very very near the forward price F. Therefore, resulting in a relatively high implied variance. Despite the other points looking regular, their implied variance hovers between 0.7 to 1.2, which is still extremely high for options.

3.4.2 SVIX vs VIX

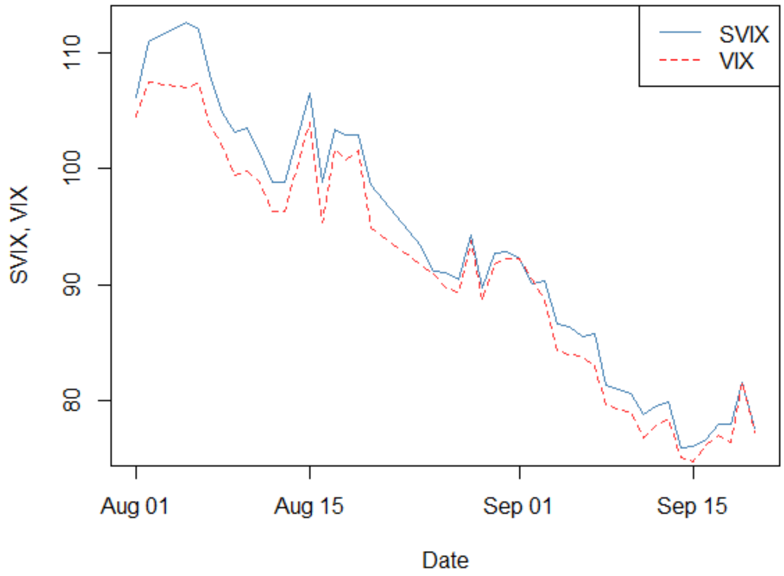


Figure 3.5: SVIX & VIX from 1/8/19 - 20/9/15

Figure 3.5 plots the SVIX and VIX from start of August to mid-September 2019. It shows the time series of VIX and SVIX for each day. We can see that the VIX and SVIX move in tandem, specifically when there were sharp changes in prices. This pattern is very similar to that of Ian Martin's. At the scale of the figure, it is hard to see any difference between the two. However, Figure 3.6 shows the difference between SVIX and VIX, more specifically $SVIX - VIX$. Comparing Figure 3.5 to Appendix A, we see that the large spike in BTC is accounted for the in large VIX and SVIX at the beginning of August. However, in the long run, we can see the VIX and SVIX slowly decreasing, this is a result of the BTC prices starting to stabilize relatively for the period of September.

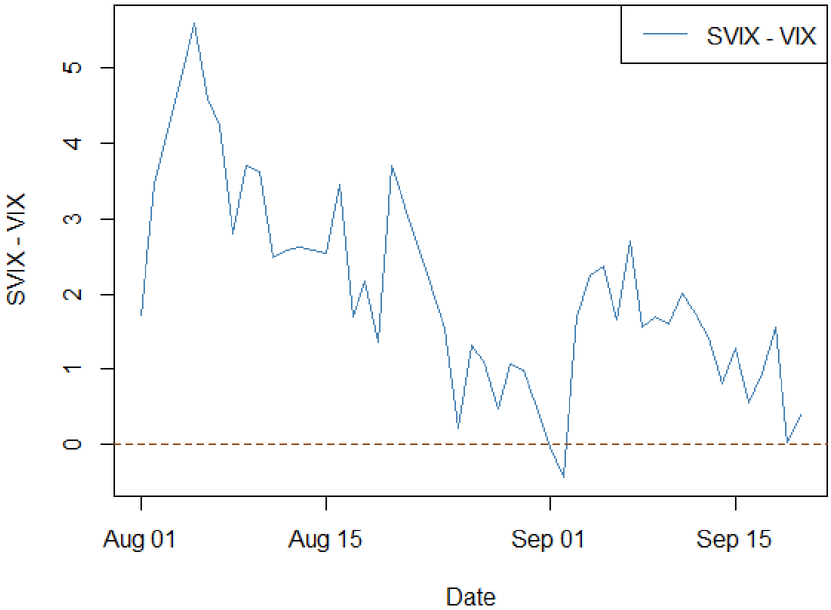


Figure 3.6: SVIX minus VIX from 1/8/19 - 20/9/15

At first glance, this is unexpected because with reference to Ian Martin[3], we expect $SVIX \leq VIX$. Briefly speaking, Ian Martin calculated the VIX and SVIX for the SPX500 from 1996 to 2012. His results showed that VIX was always greater than SVIX and he loosely concluded that this was indeed direct evidence that we do not live in a lognormal world[4].

However, for the VIXBTC, we had $SVIX \geq VIX$, which contradicts Ian Martin's observations. How do we explain this? Well, for a start, we compare the equations (2.10) and (2.11). We can clearly see that the only difference between VIX and SVIX is the weights of each OTM options.

[3]Simple Variance Swaps by Ian Martin 2013

[4]More details in Simple Variance Swaps Ian Martin 2013

- For VIX, the weight of the OTM options are inversely proportional to K^2.
- For SVIX, the weight of the OTM options are inversely proportional to F^2.

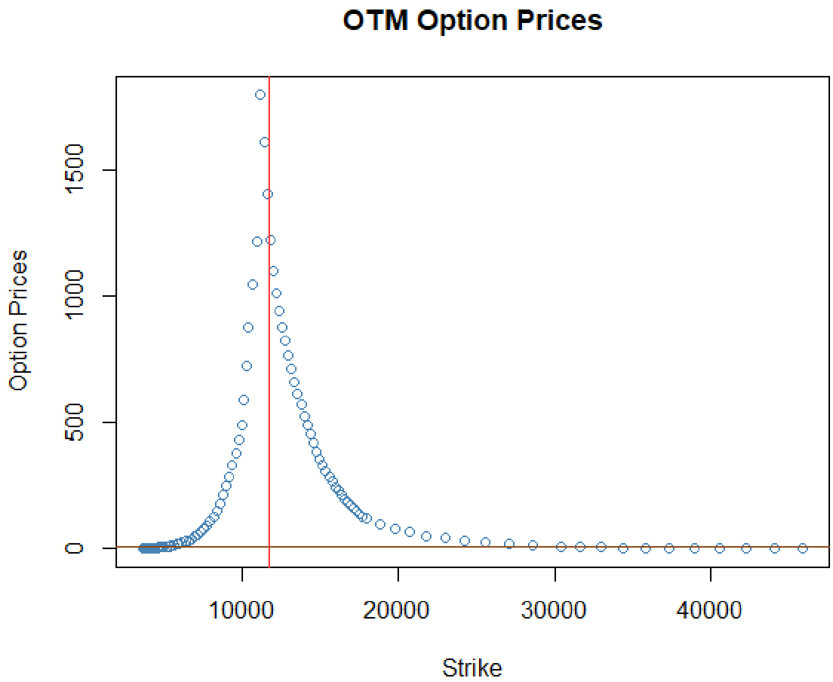


Figure 3.7: OTM Option Prices for 2/8/2019

Figure 3.7 shows the interpolated OTM option prices of BTC from Deribit Exchange. The vertical red line denotes where the forward price level F is. Therefore, everything to the right of the vertical red line represents the OTM call options, while equivalently, everything to the left of the vertical red line represents the OTM put options. Blue dots in the figure are the data points representing (Strike, Option Price). The brown horizontal line is set at 10.

Clearly, we can see that majority of the OTM put options are priced relatively low, very close to 10. However, for the OTM call options, majority of the options are priced relatively high. As previously mentioned, investors are very willing to pay a large premium for the OTM options due to the highly volatile nature of BTC. Hence, we expect more contribution from the OTM call options. However, because the OTM call option strike prices K and forward prices F have the relation $F \leq K$, the large contribution from OTM call options are evidently dampened by the factor $1/K^2$ weight for VIX. On the other hand, the large contribution from the OTM call options are less inhibited by the factor $1/F^2$, which explains the result $SVIX > VIX$ for VIXBTC.

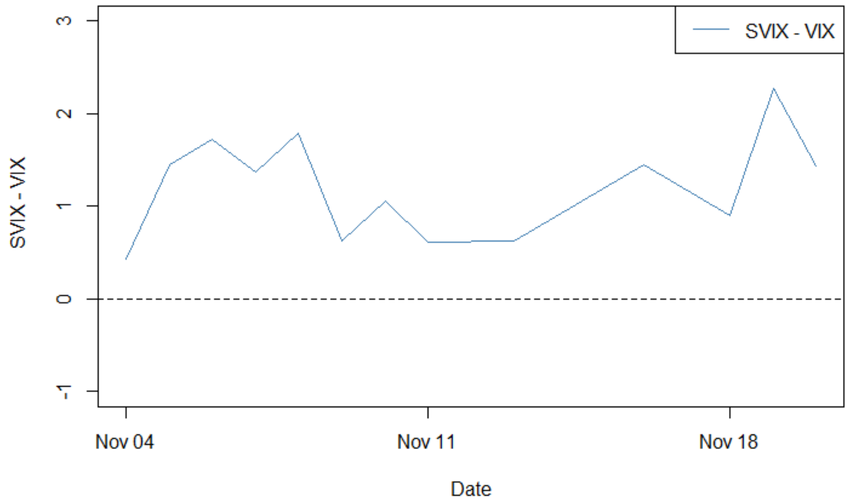


Figure 3.8: OTM Option Prices for 2/8/2019

We made another interesting observation. We noticed that the difference between SVIX and VIX grew larger when the Bitcoin market was having a more bullish run. On the other hand, when the Bitcoin market was experiencing a bearish session, we can see VIX got very close to SVIX but is always smaller than it. The data taken from Figure 3.6 was taken from August to mid September, we which had ups and downs but if we observe carefully, we can see that the spike in difference between SVIX and VIX correlates with the spike in BTC prices. With reference to Figure 3.8, for the period of November, where the BTC prices were constantly declining, we can see that the difference between SVIX and VIX was relatively lower. Hence we feel that the difference between SVIX and VIX might be a good indicator of the market's sentiments on BTC market.

Based on the respective formulas of VIX and SVIX, we can say that the VIX has more weight from the OTM puts while the SVIX equally weights all the OTM call and put options. Hence, VIX is more sensitive to the left tail of the return distribution because it places more weight on the OTM put options. On the other hand, SVIX is equally sensitive to both tails of the return distribution, because of its equally placed weights. In conclusion, we feel that the SVIX is a better measure of return volatility.

3.4.3 Vanilla vs Inverse

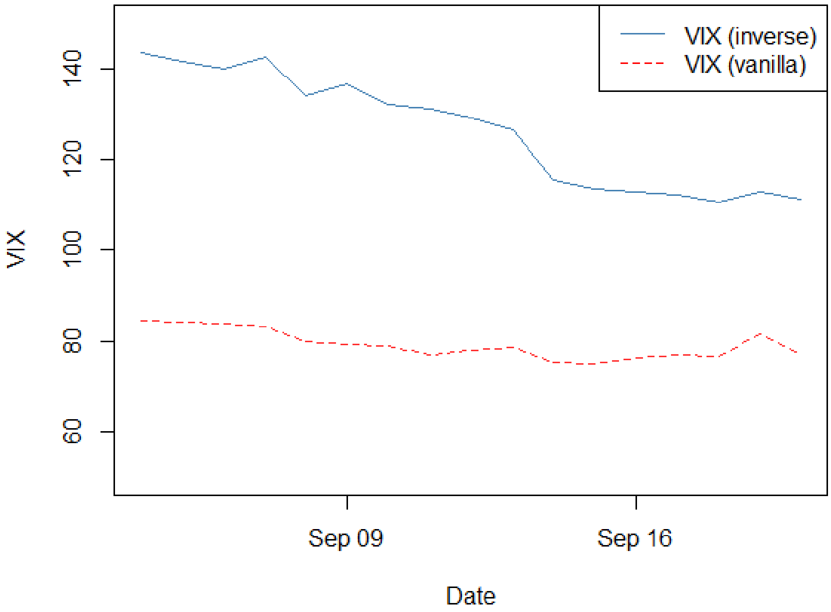


Figure 3.9: VIX with different underlying assets

Figure 3.9 shows the VIX for BTC calculated using two different payoff functions. There is a huge disparity between the two. Let us denote VIX1 to be the VIX replicated with vanilla OTM options and VIX2 to be the VIX replicated with inverse OTM options. However, the trends between the two are still relatively similar. It does look like VIX2 is more sensitive to the jumps than VIX1 is but overall they are both decreasing. We will discuss and explain the difference below.

3.4.4 Limitations

VIX2 was calculated using data from Quedex. One of the limitations of using Quedex was that the BTC inverse options were quite rarely traded. After spending a few days on the website, we found that the options were highly illiquid. For simulation purposes, the average of the closest bid and ask for each respective strike was taken to be the option price simply because there were rarely any last done prices. This in turn affects the interpolation and extrapolation results that are then used to calculate the overall contribution to volatility by the OTM options.

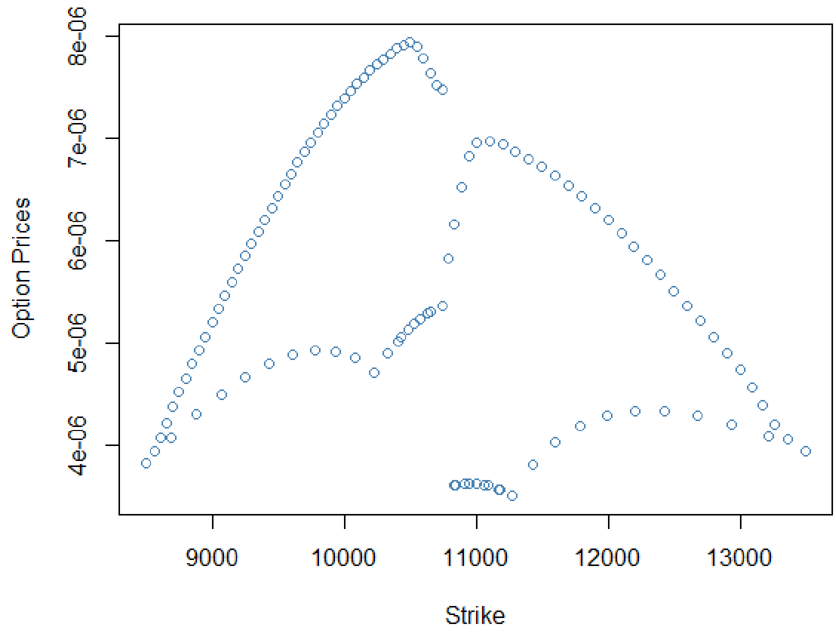


Figure 3.10: Inverse OTM Option Prices

However, another interesting observation we saw was that the interpolated and actual inverse option prices were very similar across the large range of strike prices. This is evident in Figure 3.10. We can see that the option prices were very close to the magnitude of 10^{-6}. Obviously, the option prices are not very accurate as you can see in Figure 3.10. On the interval $[0, F]$, we should be expecting a trend of increasing option price as the strike price approaches the forward price level (keep in mind that the options are OTM). Then on the interval from (F, ∞), we should expect the option prices to be decreasing due to the nature of its payoff, where higher strike prices and higher S_T imply a small option price. In short, something like a $-x^2$ graph.

For small values of strike price K, the OTM put payoffs would be $(1/S_T - 1/K)$, thus if BTC falls in price such that the put option is ITM (in-the-money), then the payoff would be relatively larger than if there were an increase in BTC for an inverse call option. If there is an equal increase in BTC, such that the inverse call option is ITM, then the payoff $(1/K - 1/S_T)$ would be smaller than that of the inverse OTM put, because of the decreasing magnitude of the payoff with increasing K and S_T.

Hence, we conclude that the high VIX2 is due to the result of inaccurate pricing data that deviated too far from the fair value of the options, thus distorting option prices and ultimately VIX2.

Chapter 4

Conclusion

The main goal of this paper is to construct a suitable volatility index for the Bitcoin market. We first begin by introducing the variance swap and its analogous Volatility Index (VIX). Explaining in detail the derivation for the formulas and the assumptions needed stated for VIX. Next, we introduced a similar financial instrument, the simple variance swap and its analogous Simple Volatility Index (SVIX). We also dived deep into the proof of derivation for the equations needed for SVIX. We then discussed the process of collecting and cleaning data needed for the calculation of VIX and SVIX. More importantly, we discussed how to generate more data points for a larger range of strike prices in order to improve the accuracy after discretizing the original form. We used a combination of interpolation and extrapolation to produce an approximation for the implied variance function, ultimately reducing discretization error in CBOE's method for calculating volatility index, VIX. We also attempt to generate the VIX for Bitcoin currency by using inverse European options.

In conclusion, we feel that the SVIX is a much better tool for measuring Bitcoin's volatility, because simple variance swaps can be hedged in the presence of jumps. On the other hand, the presence of jumps severely disrupts the pricing of variance swaps. SVIX is equally sensitive to both tails of the return distribution as it places equal weights for all the OTM options, whereas VIX is more sensitive to the left tail of the return distribution. We also noted that the difference between SVIX and VIX was a good indicator of the market's outlook on BTC. In other words, the more bullish the market was, the larger the magnitude of $SVIX - VIX$. Conversely, the more bearish the market was, the smaller the magnitude of $SVIX - VIX$. Lastly, we feel that the data for inverse European options for Bitcoin market is too inconsistent due to its illiquidity.

Appendix A

Appendix A is referenced in subsection 3.4.2 and is used to explain the jumps in SVIX and VIX. The historical chart is on the next page.

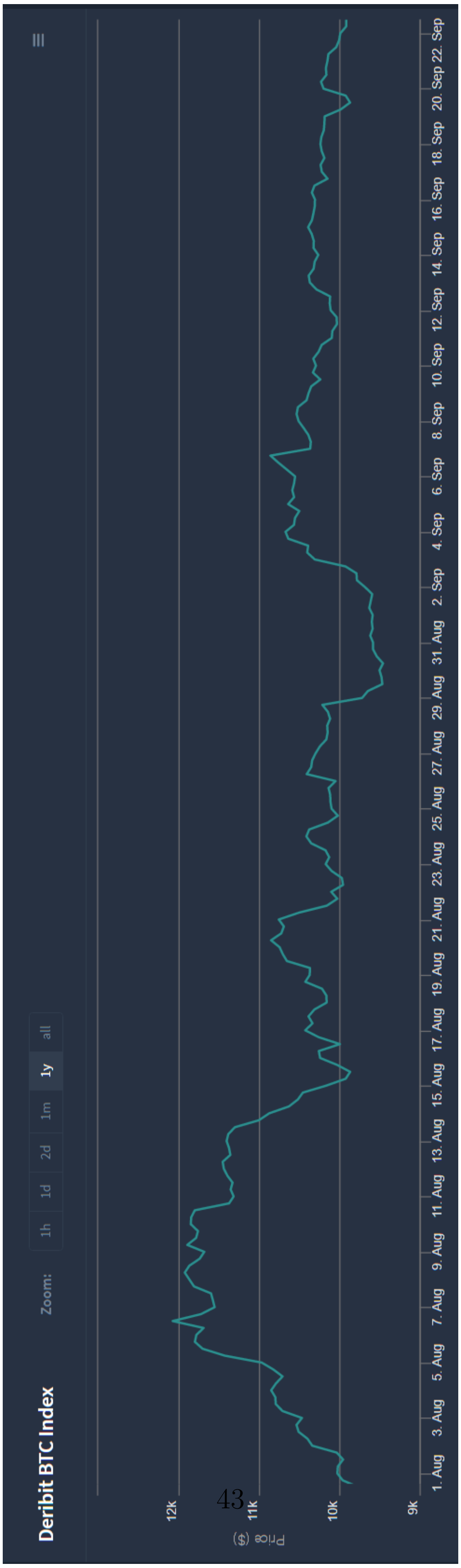


Figure A.1: BTC Historical Price

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