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Contents

1 Macroeconomics: Importing Data Files

Data file: A file in which datasets are stored. There are many types of formats, but the universal format for data files is the text format, because it can be read by all software.

Dataset: A collection of observed variables.

Missing values: They are missing observations in a dataset. In a data file, missing values are often represented by special characters.

Text file: It is a non formatted file that can be opened by any text editor. The most common extensions for text files are .txt and .csv.

Time series: It is a variable that we observe on a regular basis at different points in time. For example, the temperature in Toronto is a time series, because we observe it daily.

Variable: It is the general term used when we refer to measures. For example, inflation, unemployment rate, temperature, and CO_2 emission are four different variables.

1.1 What is Macroeconomics

Macroeconomics is the study of the economy as a whole. In macroeconomics, we are interested in the overall behaviour of an economy, which is the aggregated version of what we observe at the market level.

We need to address the measurement issue and distinguish different types of fluctuations that we observe in the data.

1.2 Data Manipulation: Loading the Data

1.2.1 Types of Files

Text Format (.txt): Stores the data by separating columns using spaces

Comma-Separated-Variable Format: Every column is separated by a comma.

1.2.2 Learning to Load Data Using Climate Series

1.2.3 Setting up the Data Software

You can use the `help` command to lookup commands.

1.2.4 Importing Data Files

To read a csv: `data <- read.csv(filename, skip=1, na.strings="***", header=TRUE)`. The skip argument is the number of lines to skip, na.strings is the missing values (you can specify more than one by `c(arg1, arg2)`), header tells us that the first row is the name of each column. `nrows` tells us the number of rows to load, sometimes there are comments at the end of the file to skip.

`head(table, n=k)` prints out the first k rows.

We can know the structure of each variable by either using the Environment tab in RStudio or type `str(table)`.

To read a txt: `data <- read.table(filename, skip=k, na.strings=-99.99, header=FALSE)`. We don't need the skip option because by default the comment.char option is set to # as comments.

Assign column names to the table by: `names(data) <- c(str1, str2, ...)`

Delete data: `rm(data)`

To save datasets use `save(data, file=filename)` and to load use `load(filename)`.

2 Mathematical Operations and Index Numbers

Aggregation: This is a technique used to combine multiple numbers to form a single one. The technique is used in time series analysis to reduce the frequency of a series: monthly to quarterly, quarterly to annual, etc.

Annualized growth rate: A monthly growth rate is annualized if it is calculated over one year by assuming that it remains the same every month. The same technique can be used to annualize quarterly growth rates.

Flow variable: A variable that is only defined over a period. A time period must therefore be associated with flow variables: income per month, consumption per quarter, etc.

Growth rate: This is defined as the percentage change of a value between two consecutive periods.

Index numbers: This is the ratio of the value of a variable at a given point in time over the value of the same variable at the base period, multiplied by 100. For example, if the base period is January 1960, the index is called index base 100 = January 1960.

Log-scale: The logarithm scale is a nonlinear transformation of a series using the natural logarithm function. When a series is expressed in logarithm, differences between consecutive observations are interpreted as growth rates.

Stock variable: A variable that measures a quantity at a given point in time. Since it is the result of an accumulation over a period, the date associated with stock variables is usually the last day of the period.

2.1 Preparing Your Data

Suppose we want to construct an hourly time series. We consider the flow variables to be an average flow over the period. If the label is 12:00 which means between 12:00 and 1:00, the we set the time of the average flow to the center of the period, which is 12:30. The stock is the total amount at the end of the period. The exact time for an observation would be 12:59.

If we have annual data, the exact date for flow variables is July 1 and for stock variables it is December 31. For quarterly data, flow is February 15 and stock is March 31.

Variables that Represent an Average: Stocks and flows are not the only types of variables.

Temperature is neither and usually we mean the average temperature over the month. Every time we have a variable that represents an average, we set the exact date to the middle of the month.

Creating a Monthly Time Series, use ts:
`timeSeries <- ts(data, frequency=k, start=c(startYear, startMonth))`
frequency is the number periods per year, 12 for monthly, 4 for quarterly, 1 for annual series. start is for the starting year and start month/quarter.

To define each column of a dataset as an annual time series, apply ts to the whole dataset.
`timeSeries <- ts(data, start=startYear)`
Default for the frequency and startMonth is 1.

To extract one time series, use `x <- timeSeries[,colName]`

Subsetting a Time Series with window: We can use window to get a specific subset.
`window(timeSeries[,"Jan"], start=startYear, end=endYear)`

The above gets us an annual time series. Suppose we want specific month to specific month, then
`window(timeSeries, start=c(startYear, startMonth), end=c(endYear, endMonth))`

Creating Montly Temperature Series: We want the transpose of the data, so use the t function.
`t(data[x:y, w:z])`

We want to put the solumns below each other, we use the function c (just like vector) but if we input a matrix, it returns one list by stack each column on top of each other.
`ts(c(t(data[,x:y])), frequency=12, start=c(startYear, startMonth))`

You can save by `save(data, otherVariables, file=filename)`

2.2 Mathematical Operations

$$^\circ C = (^\circ F - 32) \times \frac{5}{9}$$

$$1\,\text{in} = 2.54\,\text{cm}$$

$$1\,\text{CAD} = 0.75 - 0.8\,\text{USD}$$

$$1\,\text{CAD} = 0.69\,\text{EUR}$$

Growth Rate: Can be in decimal or percent and no units

$$g_t = \left(\frac{X_t - X_{t-1}}{X_{t-1}}\right)$$

$$g_t + 1 = \frac{X_t}{X_{t-1}}$$

Growth Rate of the product between two variables: Let X_t and Y_t be two variables. The growth rate of the product is

$$g_t = \frac{X_tY_t - X_{t-1}Y_{t-1}}{X_{t-1}Y_{t-1}} = (1+g_x)(1+g_y) - 1 \approx g_x + g_y$$

Growth Rate of X_t/Y_t:

$$g_t = \frac{1+g_x}{1+g_y} - 1 \approx g_x - g_y$$

Annualized Growth Rate: If we have monthly data, the annualized growth rate is

$$g_t = \left(1 + \frac{X_t - X_{t-1}}{X_{t-1}}\right)^{12} - 1 \approx 12\left(\frac{X_t - X_{t-1}}{X_{t-1}}\right)$$

for quarterly data

$$g_t = \left(1 + \frac{X_t - X_{t-1}}{X_{t-1}}\right)^{4} - 1 \approx 4\left(\frac{X_t - X_{t-1}}{X_{t-1}}\right)$$

Logarithm Scale: From the previous part,

$$\log(X_t) - \log(X_{t-1}) = \log\left(\frac{X_t}{X_{t-1}}\right) = \log(1+g_t)$$

For small g_t,

$$\log(1+g_t) = g_t$$

This is good enough for this course. So therefore,

$$g_t \approx \log(X_t) - \log(X_{t-1})$$

$$\log(X_tY_t) - \log(X_{t-1}Y_{t-1}) \approx g_x + g_y$$

Aggregation: The term aggregation means combining multiple numbers to form a single one. Therefore, when converting a monthly time series into a quarterly or annual series, we are aggregating the results. We can either sum up the variables or take the average of the sum depends on the variable.

2.3 Index Numbers

An index number is the ratio of a number of interest to a base value, often expressed in percentage. It is particularly useful when we want to compare the evolution of two time series with different units of measurement. Consumer Price Index (CPI) is a well known index. It is the ratio of the cost of a given basket of goods in a given year to the cost of the same basket in the base year.

If the current value is 120 and it was 80 in 2000, then we say that the index, base 100 = 2000, is 150. The term base 100 = 2000, means that the index is constructed using the value of the variable in year 2000 as base value and the index for 2000 is equal to 100 because $(80/80) \times 100 = 100$.

2.4 Operations in R

To compute growth rates we can do

```
diff(timeSeries)/lag(timeSeries,-1)
```

diff computes the difference $X_t - X_{t-1}$ and lag computes X_{t+s} for any s. So we choose -1.

To aggregate a time series,

```
aggregate(timeSeries, nfrequency=newFreq, FUN=funcForAggregate)
```

We can set FUN to sum to sum elements, and newFreq is the new frequency.
We need a complete serries to aggregate. If there are missing values, we have to use window to restrict the period.

To get the index number, we can just use regular operations. To get the base number we use window to restrict it to the same year.
`base <- window(timeSeries, year, year)`
However we need a vector of the base for R to be able to calculate, use c function:
`timeSeries/c(base)*100`

Userful Commands: We use `na.rm=TRUE` so R removes the missing values in the dataset.

1. mean
2. sum
3. rowSums
4. rowMeans
5. colSums
6. colMeans

3 Visualization Techniques

Bar plot: This is a chart used to compare values of a small number of observations. The X-axis represents the observations and the y-axis represents their values characterized by the height of the bars. For example, a bar plot can be used to illustrate the population by province. In this case, each bar is a province and its height represents the population of the province.

Comovement: This term characterizes the observed relationship between two series. A positive comovement means that the two series are moving in the same direction on average: they go up and down together. A negative comovement means that the two series move in opposite directions on average: when one series goes up, the other goes down. A comovement only characterizes observed relationships. The existence of a comovement does not imply that an actual relationship exists.

Histogram: This is a particular bar chart used to illustrate the distribution of a series. The range of the series (its minimum value to its maximum value) is divided into intervals and the height of the bars corresponds to the number of observations included in each interval. For example, the histogram of personal income is used to illustrate the income distribution. If we divide the the range of income by intervals of 10 thousand, the height of the first bar would be the number of individuals with income between 0 and 10 thousand, the height of the second would be the number of individuals with income between 10 and 20 thousand, and so on.

Line chart: This is a chart showing the evolution of a series through time. The X-axis represents time, the y-axis represents the value of the observations and the points are connected by lines.

Relationship: This term characterizes the link between two variables. A relationship exists if an underlined mechanism links the two variables. The relationship is direct if one variable is causing the other and it is indirect if one variable is causing the other through a third variable. The identification of a relationship requires advanced statistical techniques and/or strong theoretical background. In this course, we are not equipped to identify relationships.

Scatter plot: This is a chart used to illustrate comovements between two series. The X-axis represents the values of one series and the y-axis represents that values of another series. In general, the points are not connected by lines. However, if we want information about the evolution of the series over time, the points are connected and the date is added on top of each observation.

Short term fluctuations: This is the behaviour of a series over shorter periods of time. By shorter, we means shorter than the period spanned by our dataset. The types of short term fluctuations will be covered in details in the next module. For now, we define as the movement of a series over a few months, few quarters or few years.

Trending behaviour: This is the behaviour of a series on average over a long period of time. By long period, we mean the period spanned by our dataset. A positive trend means that the series in increasing on average over the period and a negative trend means that the series is decreasing on average over the period.

Volatility: This term characterizes the intensity of the short term fluctuations. If two series have the same measurement units and same scale, the one that fluctuates with higher ups and lower downs is more volatle than the other. We cannot compare the volatility of two series with different measurement units and different scale.

3.1 Visualizing Time Series

Line Charts: A line chart is a graph that presents successive observations connected by lines. The x-axis represents time in time series. To interpret two different time series in, it is easiest when the two things have the same units.

From the last module, we saw that the difference in logs is approximately equal to the growth rate. A linear trend implies a decreasing growth rate because $X_t - X_{t-1}$ is constant while X_{t-1} is increasing.

Scatter plots: A scatter plot is a way to illustrate comovements between two time series. A comovement or an observed relationship is not a direct or indirect relationship. Even though there is a comovement doesn't mean there is a relationship.

Bar plots: A bar plot is a visualization tool to compare a small number of observations. Using a bar plot instead of line charts is just a matter of preference.

Histogram: A histogram is visualization tool to illustrate how a variable is distributed. We can describe a histogram by 3 characteristics: the mode, the level of uniformity and the degree of symmetry.

To plot a line chart, we use: `plot(data[,colName])`
The plot knows the dates to put on the x-axis because the information is incorporated into the dataset. When you input a dataset for which no specific function has been written, the function `plot.default` is called. The function that is called when you input a time series is `plot.ts`.

`plot` arguments:

- main: Add a title, use string notation.

- xlab: Modify x-axis label. Default is Time.
- ylab: Modify y-axis label.
- type: Defines the type of charts. Default is "l" or line. If we do not want points connected, use "p" for points. If we want both, use "b" for both.
- col: Colour of line. Default is black.
- lty: Integer argument for type of line. 1: solid, 2: dashed, 3: dotted, 4: dotted-dashed, 5: long-dashed, 6: two-dashed. 1 is default.
- lwd: Thickness of lines. Higher the number the thicker the line. 1 is default.

Adding elements to existing graphs:

- lines: Adds a line to last graph created. Arguments type, col, lwd and lty are available.
- abline: Adds straight line to last graph created. h: horizontal, v: vertical, two integer arguments (a, b) define the equation $y = a + bx$
- text: Adds text at a specified coordinate. Another argument is pos where it defined position with respect to coordinates. 1, 2, 3, 4 for below, left, above, or to the right of. `text(x, y, "text")`
- legend: Adds legend to graph.
  `legend("topright", c(args...), col=c(args...), lty=c(args...), lwd=thick)`
- Other possibilities: arrow, points, grid, polygon

R will automatically handle different time periods, different frequencies (monthly, annually, etc.), and also plot a table of time series. To plot all time series in a table into one plot, we add the argument `plot.type="single"` into the plot function.

The function `cbind(ts1, ts2)` can be used to combine two or more series into a table of time series.

To create a scatter plot, we also use the command `plot`. R will know if we input two series. To produce a scatterplot we need to match the dates using `window`. The information about the start/end dates can be obtained with
`start(timeSeries)` and `end(timeSeries)`
`x <- window(timeSeries1, start(timeSeries2), end(timeSeries2))`

Then we plot by: `plot(ts1, ts2, ...)` where ts2 can be windowed like above. To control the type of points, use argument `pch` like `pch="*"`. pch can also be set from 21 to 24 where they are circles, squares, diamonds, and triangles respectively.

R will connect points if number of observations is small enough. We can aggregate time series to do that.
`temp <- window(aggregate(timeSeries, FUN=mean), start(ts1), end(ts1))`

`xy.labels = TRUE/FALSE` can show dates on the points and `xy.lines = TRUE/FALSE` makes the points connected.

We can use `points(ts, window(...), col=x, bg=y, pch=z)` to add points to an existing scatterplot.

`hist(ts, density=x, col=col, border=col, angle=ang, main=title, xlab="x-label")` to create a histogram. All histograms are filled with colour unless desnity is set then it will fill it with lines. angle tells what angle the lines of density should be.

`barplot` is used to make a barplot. Uses same arguments as histograms.

We can combine two or more barplots using `rbind(ts1, ts2, ...)` and when we plot we get them all. `beside=TRUE` tells R to put the bars side-by-side.

The command `expression` can be used to add math expressions to titles and labels. To add it to the title use `main=expression(...)`.

- Subscript: `expression(X[t])` makes X_t
- Superscript: `expression(X^2)` makes X^2
- Special characters: `expression(alpha+beta+gamma)` makes $\alpha+\beta+\gamma$, `0.1^degree*c` makes $0.1°C$
- Equation: As if typing into Console. == makes one equal sign.
  `expression(Y == log(T[t]*C[t]))` makes $Y = \log(T_t C_t)$
- Combining text/expressions: `text` is used to insert expressions in the graph.
  Text in an expression must be in quotes and ˜ (tilde) must be used to separate expressions and text.

4 Time Series Decomposition

Bias: An error is biased if it is not equal to 0 on average. It is positively biased if its average is positive, it is negatively biased is it is negative on average, and it is unbiased if it is equal to 0 on average.

Cycle: This is the medium low frequency of a time series. It represents the average movement of a series around its trend.

Detrended: A series is detrended when its trend component has be removed. A line chart of such series should no longer show very low frequency fluctuations.

High frequency: This type of fluctuations include the seasonal and irregular components only.

Irregular: This is the very high frequency component of a time series also known as the residual. It is the least important component for this course because it is the most likely to be affected by measurement errors.

Low frequency: This type of fluctuations include the trend and the cyclical components only.

Measurement Error: This is the error produced by the estimation of a variable. Every variable that we do not directly observe contains measurement errors.

Seasonally adjusted: This means that the seasonal fluctuations have be removed from the series. Such series is also called deseasonalized.

Seasonality: This is the medium high frequency of a time series. It represents the common fluctuations that we observe every year.

Trend: This is the very low frequency of a time series. It represents the average behaviour of a series over a long period of time.

4.1 Time Series Decomposition

We assume that a time series can be decomposed into four components:

1. A trend component T_t
2. A cyclical component C_t
3. A seasonal component S_t
4. An irregular component I_t, also known as the residual

Two Time Series Models: The models to link the components to the series are:
Additive model:

$$X_t = T_t + C_t + S_t + I_t$$

Multiplicative model:

$$X_t = T_t \times C_t \times S_t \times I_t$$

The difference is that in the additive model, the effect of any component on X_t is not related to the other components while the multiplicative model is.

Using logs and the multiplicative model, it implies the following additive model:

$$\log(X_t) = \log(T_t \times C_t \times S_t \times I_t) = \log(T_t) + \log(C_t) + \log(S_t) + \log(I_t)$$

The four components are ordered from lowest frequency T_t to very high frequency I_t.

This courses terminology:

- The very low frequency component: T_t
- The medium low frequency component: C_t
- The medium high frequency component: S_t
- The very high frequency component: I_t
- The low frequency component: $(T_t + C_t)$ for the additive model and $(T_t \times C_t)$ for the mult. model
- The high frequency component: $(S_t + I_t)$ for the additive model and $(S_t \times I_t)$ for the mult. model

A component becomes visible once the lower frequency components have been removed. We can do $X_t - T_t$. Then from that time series, we can get $X_t - T_t - C_t$, then $X_t - T_t - C_t - S_t$ which will finally get I_t.

The Trend component: Use the line of best fit to determine T_t.

The Cycle component: Fit C_t over the detrended series $X_t - T_t$. A positive value means the series is above the trend and negative value means it is below, and when it is one the trend, then $C_t = 0$. C_t shows how the series moves around its trend on average.

The Seasonal component: To understand the seasonal component, we first plot the high frequency component $(S_t + I_t) = (X_t - T_t - C_t)$, year by year. There will be a lot of lines for each year, but if we take the average of all the lines month by month, we obtain the season component S_t.
A positive S_t means that the series is higher than the low frequency $(T_t + C_t)$ on average, and inversely for negative S_t.
The component is called seasonal because it is the part of X_t that is affected by the season. However, it is not reserved to just the effects of the four seasons. For monthly time series, it is the effect of each month, and for quarterly time series, it is the effect of each quarter. There is no seasonal component in the annual time series.

Notice that not every series has a seasonal component. In particular, many macroeconomic time series are available in the deasonalized format.

Trend: A linear trend can be written as

$$T_t = a + bt$$

A quadratic trend can be written as

$$T_t = a + bt + ct^2$$

Log scale: In the previous module, a series with an increasing trend may be linear in the log-scale if its growth rate is <u>constant</u>. Therefore, we may want to use the multiplicative model and decompose $log(X_t)$ instead.

Cycle: One way to extract C_t is to apply a transformation that eliminates the components with higher frequency. This procedure is called smoothing, because it removes high fluctuations. We will use a simple smoothing technique called centered moving average (MA). Broadly speaking, every observation is replaced by the average of points around it. CMA of degree 5 is:

$$MA(5)_t = \frac{X_{t-2} + X_{t-1} + X_t + X_{t+1} + X_{t+2}}{5}$$

CMA of degree 11 is:

$$MA(13)_t = \frac{1}{13}(X_{t-5} + X_{t-4} + X_{t-3} + X_{t-2} + X_{t-1} + X_t + X_{t+1} + X_{t+2} + X_{t+3} + X_{t+4} + X_{t+5})$$

The MA(13) and MA(5) are good choices for monthly and quarterly time series respectively.

Seasonality: A seasonal component is a fluctuation that is influenced by either the month or the quarter. Many methods to obtain seasonal components require advanced statistical methods so we will assume constant seasonality across time in this course.

We assume that we are left with the high frequency component $(S_t + I_t)$. To get S_t, we find the averages of all the rows A_t where t are for example each month in a monthly series. To get S_t, we center A_t. To center A_t, we subtract each A_t by the average of all A_t.
When $S_t = 0$, which is the average of all S_t, there is no seasonal effect and the series is equal to the low frequency component $(C_t + T_t)$ on average. If it is positive, the series is above $(C_t + T_t)$ on average, and it is below when S_t is negative.

Additive Model Interpretation:

- $(X_t - T_t)$: A zero value means that the observation is equal to the trend, a positive value means that the observation is higher than the trend and inversely for negative values.
- C_t: A zero value means that the observation is equal to the trend **on average**, a positive value means that the observation is higher than the trend on average and inversely for negative values. In this context, on average means when the high frequency component $(S_t + I_t)$ is equal to 0.
- S_t: A zero value means that the observation is equal to the low frequency component $(T_t + C_t)$ on average, a postive value means that the observation is higher than $(T_t + C_t)$ on average and inversely for negative values. Here, on average means when the very high frequency component I_t is equal to 0.
- I_t: none for this course.

Multiplicative Model Interpretation:

- $[\log(X_t) - \log(T_t)]$: It is a difference in logs which implies that it measures the difference between X_t and T_t in percentage. e.g. 0.03 means X_t is approx. 3% higher than T_t.
- $\log(C_t)$: It has the same interpretation as $[\log(X_t) - \log(T_t)]$ but we need to add on average. e.g. 0.03 means X_t is approx. 3% higher than T_t on average. Here, on average means when the log of the high frequency component $(S_t \times I_t)$ or $[\log(S_t) + \log(I_t)]$ is equal to 0.
- $\log(S_t)$: If we ignore I_t, it is equal to $[\log(X_t) - \log(T_t \times C_t)]$. It is the percentage difference between X_t and the low frequency component on average. Here, on average means when the log of the very high frequency component I_t is equal to 0. e.g. 0.05 means X_t is 5% higher than the low frequency component $(T_t \times C_t)$ on average.
- $\log(I_t)$: none for this course.

Unbiased Errors: A measurement error is unbiased if it is equal to 0 on average. We expect when the measurement error is unbiased that the series has the same trending and cyclical behaviour as the true series. Errors are fluctuating around 0.

Biased Errors: A measurement error is biased if it is not zero on average.

4.2 Time Series Decomposition in R

Trend: First we need to convert all the dates into decimals. Use the `time` commands. If we want to follow convention for flow variables, use the argument `offset` and set to 0.5.

```
time(timeSeries, offset=0.5)
```

Fitting a linear trend: First we need to create the time series using `time`

```
t <- time(timeSeries, offset=0.5)
```

Use `lm` for linear model. The first argument is a formula by using something like `y ~ x+z+w`. We do not need to put coefficients on this equation so for a linear trend we can just do

```
fit <- lm(timeSeries ~ t)
```

To extract the coefficients for the component T_t we can use `coefT <- coef(fit)` to get the coefficients.

Therefore, to get the trend we do

```
trend <- coefT[1] + coefT[2]*t
```

We can then plot the time series, and use line to add the trend to the plot.

Fitting a quadratic trend: To fit the trend $T_t = a + bt + ct^2$ we do

```
fit <- lm(timeSeries ~ t + t^2)
coefT <- coef(fit)
trend <- coeft[1] + coefT[2]*t + coefT[3]*t^2
```

Fitting a trend with the log scale: First save the log of the time series

```
lco2M <- log(timeSeries)
lcoefT <- coef(lm(lco2M~t)) ## linear coefficients
lcoefT2 <- coef(lm(lco2M~t+t2)) ## quadratic coefficients
ltrend <- lcoefT[1] + lcoefT[2]*t ## linear trend
ltrend2 <- lcoefT2[1] + lcoefT2[2]*t + lcoefT2[3]*t2 ## quadratic trend
plot(lco2M, lwd=2, main="TITLE",
    ylab="log(Co2)")
lines(ltrend, col=2, lty=2, lwd=4)
lines(ltrend2, col=4, lty=3, lwd=4)
legend("topleft", c("log(Co2)", "Linear","Quadratic"), col=c(1,2,4), lty=1:3,
    lwd=c(2,4,4), bty='n')
```

Cycle: To detrend a series and plot, do

```
plot(timeSeries - trend, ...)
```

To get the cyclical component first we get the detrended series and calling it CSI for $C_t + S_t + I_t$:

```
CSI <- timeSeries - trend
```

To extract C_t, we use the function `filter`. The first argument is the series and the second is the filter, which is a list of weights.

Since we want all weights to be the same (from MA(5) and MA(13)), we can use the function `rep(val, numTimes)` to get repeated values. Therefore,

```
C <- filter(CSI, filter=rep(1/13, 13))
```

We can see the new C_t by

```
plot(CSI, lwd=2, main="TITLE", ylab="Co2 (ppm)")
lines(C, col=2, lwd=3, lty=2)
legend("topleft", c("Detrended Co2", "Cycle"), col=1:2, lty=1:2,
    lwd=2:3, bty='n')
```

We are often interesting in the low frequency fluctuations that include trend and cycle so:

```
CT <- trend + C
plot(CT, ...)
```

Seasonality: First we apply the `decompose` to the detrended series CSI and filter.

```
decomp <- decompose(CSI, filter=rep(1/13, 13))
```

Named Lists in R:

```
info <- list(names = c(...), education=c(...))
```

`info$names` or `info$education`
To check names use `names(info)`

The names produced by decompose are

```
names(Dec)
## [1] "x"        "seasonal" "trend"    "random"   "figure"   "type"
```

- trend: This is C_t. R defines trend differently. To save trend we can do
  `C <- decomp$trend`
- figure: This is S_t for one year. If it is monthly series, first 12 S_t are returned and first 4 for quarterly. It may not be in the right order if your time series starts on a different month other than January. If it starts on March for example, then we want 11, 12 and then 1 to 10.
  `S <- decomp$figure[c(11, 12, 1:10)]`
- seasonal: This is also seasonal component, but is repeated to match the period of the original series. This element is useful for removing the seasonal component of a series.
- random: This is I_t.

Deseasonalizing a Series: The process of removing the seasonal component of a series is called seasonal adjustment or deseasonalizing.

- Removing the trend:

  ```
  t <- time(co2M, offset=0.5)
  t2 <- t^2
  coefT <- coef(lm(co2M~t+t2))
  trend <- coefT[1] + coefT[2]*t + coefT[3]*t2
  CSI <- co2M - trend
  ```

- Extracting the seasonal component:
  `S <- decompose(CSI, filter=rep(1/13,13))$seasonal`
- Removing S_t from the original series:
  `Des_co2M <- co2M-S`

One purpose of decomposing series is to illustrate the comovement between two series at different frequencies.

5 National Accounts

Capital goods: This is a type of highly durable good used to produce goods and services. It may include residential houses (we can rent it to others or to ourselves), office buildings, factories, machinery, etc.

Consumption: This is the value of final goods and services purchased by households (C) or by the government (G).

Final goods and services: These are goods and services purchased at the end of the production chain. It means that they are not intermediate goods and services.

Gross domestic product (GDP): This is a measure of the aggregate economic activity taking place inside an economy.

Gross national product (GNP): This is a measure of the aggregate economic activity generated by the citizens of an economy.

Intermediate goods and services: These are goods and services used as input to produce other goods and/or services.

Investment (I): This is a flow variable that represents the value of capital goods purchased over a period. We need to distinguish gross investment, which is the purchase of new capital goods, from net investment, which represents the variation in the value of capital goods. Net investment is the gross investment minus the depreciation of existing capital goods.

Inflation rate: This is the growth rate of the general price level estimated by a price index.

Net export (NX): This is equal to the value of goods produced domestically and sold to other economies (exports or X) minus the values of goods produced in other economies purchased by domestic residents (imports or M).

Nominal measure: This a measure expressed in current dollars. Nominal wage is the amount of dollars received every week, nominal GDP is the dollar value of the production, nominal interest rate is the percentage return or payment on an amount expressed in current dollars.

Purchasing power: This term refers to the individual's ability to purchase goods with a given amount of dollars. Therefore, when prices increase, the purchasing power of each dollar decreases.

Price index: This is an index number defined as the ratio of two values of a given basket of goods from two different periods.

Quality adjustment bias: This bias explains partly why the consumer price index (CPI) inflation may overstate the change in the cost of living. The measures ignore the fact that a fraction of price increases may be explained by quality improvement. That part of price increases is not an increase in the cost of living.

Quantity index: This is an index number defined as the ratio of the value of two baskets of goods at a given set of prices.

Real measure: This is a measure for which the effect of price movements has been removed. Therefore, it measures changes in quantity. For example, an increase of real GDP is interpreted as an increase in economic activity (production), and an increase in real wage implies a higher purchasing power for the workers, which is not necessary true for an increase in nominal wage.

Substitution bias: This bias explains partly why the consumer price index (CPI) inflation may overstate the change in the cost of living. Since CPI uses a fixed basket of goods, substitutions from more expensive to less expensive goods as a response to changes in relative prices is not taken into account by the CPI.

5.1 The National Income Accounts

Measures of Economic Activity:

1. The product or value added approach: The amount of output produced, excluding used up in intermediate stages of production.
2. The income approach: The incomes received by the producers of output.
3. The expenditure approach: The amount of spending by the ultimate purchasers of output.

The three approaches reach the same conclusion.

The product or value added approach: The amount of output produced, ecluding output used up in intermediate stages of production. Equivalently, it is the value added by each producer.

The income approach: The total income includes wage, taxes (government income) and after-tax profit. If we add the three sources of income and the variation of the inventory, the economic activity.

The expenditure approach: The total expenditure on final goods and investment products (at market price). The total value of expenditure is $C + G + I + X - M$.

- Consumption (C): The total expenditure in final goods by private individuals (not used as intermediate goods to produce other goods)
- Government Expenditure (G): The total government expediture on final goods.
- Investment (I): The total values of capital goods purchased by the private and public sectors (equipment, buildings and inventory). There is no purchase of houses, buildings or equipment, but the stock of inventory has changed.
- Export (X): Total value of goods produced in the economy and purchased by individuals living in another economy.
- Import (M): Total value of goods produced outside the economy and purchased by individuals living in our economy.

The Canadian GDP: There are two tables using the income and expenditure approaches. They should be the same but the numbers in the tables are estimates. The GDP is then the average of the two tables X and the statistical discrepancy is $X - T_1$ and $X - T_2$.

The Measurement Error of GDP: Errors should be close to zero on average for high-quality data.

Sources of Error: There are two main sources of error. To summarize, the first source comes from the difficulty of measuring the value of nonmarket goods produced. Nonmarket goods include goods

produced in the underground economy (illegal activites, or legal activities that are not reported for the purpose of avoiding paying taxes) and other goods and services produced without being explicityly sold on a market.
The second source of error is not related to the ability of GDP to measure economic activity, but rather on its ability to measure the standard of living.

Nominal GDP is a poor measure of the economic activity over time becasue it is affected by economic activity and price movements. The average growth rate includes the growth rate of economic activity and the growth rate of prices. Suppose that Q is the economic activity and P is the price of what we produce. The value of the production, which is what we define as the nominal GDP, is $P \times Q$.
Suppose that P increases by 3% and Q increases by 4%. Then, the growth rate of the product is

$$[(1 + 0.03) \times (1 + 0.04) - 1] \times 100\% = 7.12\%$$

Therefore, the grwoth rate of nominal GDP is 7.12% and it is explained by a 3% increase in the economic activity and a 4% increase in the price level.

5.2 Index Numbers in Macroeconomics

Quantity Indexes

The objective of a quantity index is to measure the change in the value of a basket of goods at constant prices. Which set of constant prices we use is what distinguishes the different indexes.

Passche Quantity Index: A Paasche quantity index fixes the prices to the current period. Let the base year be 0, P_{it} be the price of good i at time t and Q_{it} be the quantity of good i at time t. If we have N goods, the Paasche quantity index (PQ) is

$$PQ_{t/0} = \frac{\sum_{i=1}^{N} P_{it} Q_{it}}{\sum_{i=1}^{N} P_{it} Q_{i0}} \times 100$$

It is the value of the basket of goods at time t using the current price, over the value of the basket of good at the base period using the same current prices.

We can use the formula from module 2,

$$X_t = X_{t-1}(1 + g)$$

where g is the growth rate of X between $t - 1$ and t from the $PQ_{t/0}$.

Important: When the value of a set of goods for different periods is computed using the prices of the same period, we callit the real value or the value at constant prices.

Laspeyres Quantity Index: The LQ index uses the prices of the base period. It is the value of the current basket of goods at the base year prices over the value of the basket of goods of the base year at the same base period prices. The formula is:

$$LQ_{t/0} = \frac{\sum_{i=1}^{N} P_{i0} Q_{it}}{\sum_{i=1}^{N} P_{i0} Q_{i0}} \times 100$$

Chained Quantity Index: The weakness of the Laspeyres and Paasche indexes is that they are both based on the value of a basket of goods using prices from a distant period. The solution is to never use prices/quantities that are separated by more than 1 period where 1 month period for monthly data and 1 quanter for quarterly data. The chained Paasche index base 100 = 0 for t is defined by the following formula:

$$CPQ_{t/0} = PQ_{1/0} \times PQ_{2/1} \times PQ_{3/2} \times \cdots \times PQ_{t/t-1} \times 100$$

where all indexes are not multiplied by 100. The chained Laspeyres quantity index (CLQ) is:

$$CLQ_{t/0} = LQ_{1/0} \times LQ_{2/1} \times LQ_{3/2} \times \cdots \times LQ_{t/t-1} \times 100$$

Fisher Quantity Index: The two chained indexes are closer but are still different. The FQ is defined as the geometric average of the Paasche and the Laspeyres indexes. The formula is:

$$FQ_{t/0} = 100 \times \sqrt{PQ_{t/0} \times LQ_{t/0}}$$

We can also construct the chained Fisher quantity index (CFQ) using the other chained indexes:

$$CFQ_{t/0} = 100 \times \sqrt{CPQ_{t/0} \times CLQ_{t/0}}$$

Paasche Price index

Price indexes are like quantity indexes with the exception that quantities are held fixed instead of prices. The objective is to measure the change in the value of a fixed basket of goods. A higher price index means that a given basket of goods cost more.

Paasche Price Index: For PP, it is the current basket of goods that we hold fixed. The formula is:

$$PP_{t/0} = \frac{\sum_{i=1}^{N} P_{it} Q_{it}}{\sum_{i=1}^{N} P_{i0} Q_{it}} \times 100$$

and the chained Paasche price index (CPP) iss

$$CPP_{t/0} = PP_{1/0} \times PP_{2/1} \times PP_{3/2} \times \cdots \times PQ_{t/t-1} \times 100$$

Laspeyres Price Index: For Laspeyres price indexes (LP), it is the basket of goods from the base period that we hold fixed. The formula is:

$$LP_{t/0} = \frac{\sum_{i=1}^{N} P_{it} Q_{i0}}{\sum_{i=1}^{N} P_{i0} Q_{i0}} \times 100$$

and the chained Laspeyres price index (CLP) is

$$CLP_{t/0} = LP_{1/0} \times LP_{2/1} \times LP_{3/2} \times \cdots \times LP_{t/t-1} \times 100$$

Fisher Price Index: The Fisher price index is the geometric mean of the Paasche and Laspeyres price indexes. The formula is:

$$FP_{t/0} = 100 \times \sqrt{PP_{t/0} \times LP_{t/0}}$$

The chained Fisher price index (CPQ) is:

$$CFP_{t/0} = 100 \times \sqrt{CPP_{t/0} \times CLP_{t/0}}$$

5.3 Real Versus Nominal and the Cost of Living

Nominal variables are measured based on current prices. Therefore, nominal GDP is the measure of economic activity at the current price. It increases when prices increase even if production remains constant. Real GDP is constructed in a way that if the production remains constant, it also remains constant even if the prices are changing. An increase in real GDP means an increase in production, not in price levels.

Using Quantity Indexes to Compute Real GDP: One way to obtain the real GDP is to construct a quantity index, and use that index to calculate the real GDP.

To get an index with a different base year, we divide all indexes by the index of the new base year and multiply by 100.

Real GDP is also called GDP at constant prices of a base year or simply at constant prices.

The Deflation Approach: The deflation approach consists of transforming nominal values into real values using price indexes. The formula is:

$$\text{Real value} = \frac{\text{Nominal value}}{\text{Price index}}$$

where the price index has not been multiplied by 100.
E.g. nominal value is 400 dollars in 2020 and 2020 price index base 100 = 2012 is 110.5, its real value at the prices of 2012 is

$$\text{Real value} = \frac{400}{1.105} = 361.99$$

The GDP Deflator: The GDP deflator is one index that measures the overall changes in the price of all goods included in the GDP. This is called the implicit GDP deflator because it is implied by the real GDP measure.

$$\text{GDP deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100$$

Consumer Price Index (CPI): The consumer price index is a Laspeyres type of price index in the sense that it compares the cost of a past fixed basket of goods between the base and current periods. The main difference between the CPI and a Laspeyres price index is that the price reference period is not the same as the basket reference period. (For this course, it is sufficient to assume it is a Laspeyres price index so use LPI).

Inflation Rate: Inflation rate is defined as the growth rate of the general price level. Inflation rate is generally based on the CPI and it is based on the GDP deflator when we want to measure the changes in the price level of goods produced in the economy. We can simply use the same formula for growth rate for inflation rate.

$$\pi_t = \frac{X_t - X_{t-1}}{X_{t-1}}$$

When given price index, we can calculate average annual interest rate by:

$$g = \left(\frac{X_2}{X_1}\right)^{\frac{1}{n}} - 1$$

where $X_2 = 100$ for the price index of that base year, X_1 is price index of considered year and n is the number of years between considered year and base year.

Nominal and Real Interest Rate: The interest rate publicized by your bank is the nominal interest rate. However, because of inflation, the purchasing power of each dollar is less. To convert the nominal interest rate into the real interest rate, we use:

$$r = \frac{1+i}{1+\pi} - 1 \approx i - \pi$$

where r is the real interest rate, i is the nominal interest rate and π is the inflation rate.

6 Unemployment

Acyclical: This is the property of a series when there is no comovement between its cyclical component and the cyclical component of the real GDP.

Countercyclical: This is the property of a series when there is a negative comovement between its cyclical component and the cyclical component of the real GDP.

Duration of unemployment (D): This term refers to the average duration of complete spells of unemployment for workers currently unemployed.

Employment (E): This is the number of workers who are currently employed (full or part time).

Employment rate (ER): This is the proportion of workers from the working age population that are employed (full or part time): E/WAP.

Harmonized unemployment rate (HUR): This is an adjusted version of the official unemployment rate. The purpose is have a measure that is comparable across countries.

Incidence of unemployment (I): This is the proportion of workers from the labour force who become unemployed at every period. It is the flow variable that affects the stock of unemployment.

Labour force (LF): This the number of workers who are either unemployed or employed: $LF = E + U$.

Participation rate (PR): This is the proportion of workers from the working age population that are in the labour force: LF/WAP.

Procyclical: This is the property of a series when there is a positive comovement between its cyclical component and the cyclical component of the real GDP.

Spell of unemployment: This is the period of time from the moment a worker becomes unemployed until the moment he finds a job or exits the labour force.

Unemployment (U): This the number of workers who can work, are looking for a job, but are not employed. This is an approximate definition. The exact definition is more complicated and varies across countries.

Unemployment rate (UR): This is the proportion of workers from the labour force that are unemployed: U/LF.

Working age population (WAP): This is the population 15 years and older.

6.1 Labour Force Characteristics

The Statistics Canada labour force characteristics table contains monthly series by sex, age group and provinces starting in 1976. It also includes seasonally adjusted and unadjusted series. The definition of the different series are

- Population (WAP): the number of persons of working age (15+)
- Employment (E): the number of employed workers (total, full-time, and part-time)
- Unemployment (U): the number of workers who are able and want to work but are not employed
- Labour force (LF): Employment + Unemployment
- Out of the labour force: It is not directly given, but it can be obtained using the other series. It is equal to Population minus Labour force. It includes discouraged workers, reitrees, students, etc.
- Unemployment rate (UR):
$$UR = \frac{U}{LF} \times 100\%$$
- Participation rate (PR):
$$PR = \frac{LF}{WAP} \times 100\%$$
- Employment rate (ER):
$$ER = \frac{E}{WAP} \times 100\%$$

The definition of unemployment varies from a country to another. The harmonized unemployment rate is an adjusted measure that makes the unemployment rate of different countries comparable.

The volatility around the unemployment rate series around its low frequency component is partly due to the fact that many economic sectors are seasonal (agriculture, fishery, tourism, etc.).

The Low Frequency Component of Unemployment Rate: Rates like UR (or inflation rate) are less likely to have a large trending behaviour, we can extract the lower frequency in one step by applying a centered moving average on the original series without detrending it first. Since the series is monthly, we can use a moving average of order 13. For better results, it is recommended to apply the moving average to the seasonally adjusted series provided by Statistics Canada.

Important: We do not analyze the comovement between the trend component of real GDP and the UR. Even if we observe a negative trend between 1980 and 2020, it cannot be explained by the trending behaviour of real GDP. If you look at Figure 16.1 of the module readings, you will see that UR has increased between 1945 and 1987 even if real GDP has increased. Understanding long run behaviour of UR requires more theoretical background.

When a variable has a negative comovement with the cyclical component of real GDP, we say that this variable is countercyclical, when it has a positive comovement we say that it is procyclical and it is acyclical when there is no comovement. Therefore, unemployment rate is countercyclical.

Viewing the harmonized unemployment rate, the cyclical components for Canada and the US are very similar.

There is a direct relationship between UR and the ratio ER/PR:

$$\begin{aligned}
UR &= \frac{U}{LF} \\
&= \frac{LF - E}{LF} \\
&= 1 - \frac{E}{LF} \\
&= 1 - \frac{E/Pop}{LF/Pop} \\
&= 1 - \frac{ER}{PR}
\end{aligned}$$

Labour Force Dynamics: The main message from the Labour Force Dynamics section of the readings is that the number of unemployed workers that find a job, the number of employed workers who quit their job (whether voluntary or not) and the number of workers that enter or leave the labour force every month is much bigger than what we would expect by only looking the UR fluctuations.

Incidence and Duration of Unemployment: The movement of workers in and out of any group is the flow that determines the stock of unemployed and employed workers. UR is decomposed into the incidence (I) and duration (D) of unemployment.

$$UR = D \times I$$

The incidence is the proportion of individuals that become unemployed and the duration is the average time each unemployed workers have been unemployed.

Important: In practice, it is much more difficult to calculate the incidence and duration of unemployment. The duration formula should only be used to understand the dynamics. It says that the stock (UR) is the flow (I) times the time (D), which is the usual relationship between stock and flow variables. What you need to retain is that the UR increases if the flow of unemployed workers increases and/or the time unemployed workers remain unemployed increases.

7 Income Inequality

Decile: When the population is divided into ten equal groups ordered with respect to a variable, the groups are called deciles. For example, the first income decile is one tenth (10%) of the population with the lowest income.

Economic growth: This is the growth rate of the real per capital GDP.

Economic mobility: This characterizes the ability for individuals to move from one part of the income distribution to another.

Gini coefficient: This is a global measure of inequality. It goes from 0 (perfect equality) to 1 (perfect inequality).

Income share: This is the proportion of total income that goes to an individual or a group. For example, if the income share of females is 55%, it means that 55% of the total income is earned by women.

Lorenz curve: This is a line chart that illustrates the cumulative income share of the population as a function of the proportion of the population. For example, the points (0.05, 0.10) and (0.60, 0.50) on the Lorenz curve mean that 10% of the poorest individuals share 5% of total income of the economy and 50% share 60% of total income.

Low income measure (LIM): This is equal to half the median of income. It is one of the measures of the poverty line. Individuals with income less than the LIM are considered below the poverty line.

Market basket measure (MBM): This is the value of a basket of goods used in one definition of the poverty line. Individuals with income less than the MBM are considered below the poverty line.

Median: This is a number that separates the distribution of a variable in two equal groups. For example, the median of income in Canada is \$40,000 if income is less then \$40,000 for 50% of the population and it is higher for the other 50%.

Per capita: This is a Latin word that means by head. For example, the per capita income is the average income by individual.

Quintile: When the population is divided into five equal groups ordered with respect to a variable, the groups are called quintiles. For example, the first income quintile is one fifth (20%) of the population with the lowest income.

7.1 Income Inequality

Economic growth is defined as the growth rate of the real per capita GDP defined as

$$\text{Real per capita GDP} = \frac{\text{Real GDP}}{\text{Population}}$$

The Lorenz Curve: The Lorenz Curve is a visualization tool to analyze income inequality. The plot the curve, we need the share of total income by quintile. We first need the cumulative share of household income. The Lorenz curve plots the values of the cumulative share of household income to the the quintiles.

The Gini Coefficient: The Gini coefficient is a measure based on the Lorenz curve. It is the area between the perfect equality line and the Lorenz curve divided by the area below the equality line. Since the equality line and the axes form a right triangle with a base of 1 (100%) and a height of 1 (100%), the area under the equality line is 0.5. Therefore, the Gini coefficient is twice the area between the equality line and the Lorenz curve. The coefficient is equal to 0 if the Lorenz curve is equal to the equality line and it increases as the allocation becomes more unequal. Let $h_i = (i/5 - s_i)$, for $i = 1, 2, 3, 4$, be the distance between the equality line and the Lorenz curve at 20%, 40%, 60% and 80% respectively, where s_i is the cumulative income share of quintiles 1 to i.

Then, the Gini coefficient is

$$\begin{aligned}\text{Gini} &= 0.40 \times (h_1 + h_2 + h_3 + h_4) \\ &= 0.8 - 0.4 \times (s_1 + s_2 + s_3 + s_4)\end{aligned}$$

Using this formula, the maximum Gini coefficient is reached when all shares are equal to zero which gives a coeffcient of 0.8. To have a coefficient of 1 in the most unequal allocation, we need to divide the population into more than 5 groups.

General Gini Coefficient: The Gini coefficient is not based on quintiles. It can be divided into n equal groups so the formula is

$$\text{Gini} = \frac{2}{n}\sum_{i=1}^{n-1}(\frac{i}{n} + s_i) = \frac{n-1}{n} - \frac{2}{n}\sum_{i=1}^{n-1} s_i$$

Measure of Poverty: A household is considered poor if its income falls below the poverty line. Two of the measures used by Stats Can are the market basket measure and the low income measure.

The Market Basket Measure: The Market Basket Measure (MBM) is based on the cost of a specific basket of goods and services representing a modest, basic standard of living. It includes the costs of food, clothing, shelter, transportation and other items for a reference family. These costs are compared to the disposable income of families to determine whether or not they fall below the poverty line.

The Low Income Measure: Another definition of the poverty line is the low income measure (LIM). It is equal to half the adjusted median household income (adjusted for household size). If the median household income was x, then households with income less than $x/2$ are below the poverty line.

Poverty Versus Per Capita GDP:

$$\text{Average income in decile } i = 10 \times (\text{Real per capita GDP}) \times (\text{income share of decile } i)$$

7.2 The Sources and Consequences of Inequality

Sources of Inequality:

1. Inequality in skills
2. Inequality in how skills are valued
3. Inequality in education
4. Technological advances
5. Value of education

The Effect of Inequality on Growth:

- Investment, affects growth through its impact on the accumulation of capital goods

- Education, affects the productivity of workers
- Taxation, effects incentives to work or produce
- Political stability, affects incentive to invest
- Level of criminality, implies wasting resources to prevent it instead of producing goods

Economic Mobility: Economic mobility is defined as the ability for individuals to move from one part of the income distribution to another. Two important factors that affect mobility are:

1. The accessibility of education: Children whose parents are poor have more chances to be in higher deciles if education is easily accessible.
2. The accessibility of health care: Health has an impact on school performance. Therefore, having an accessible health system helps poor children to acquire a higher level of education and move to a higher income decile than their parents.

8 Economic Growth

Bar density: This is a way of representing the distribution of variables. In a bar density, the area of each bar represents the proportion of observations that lie in the interval determined by its base. It is an alternative to histograms for which the height of the bars represents the number of observations.

Conditional convergence: This is a mechanism through which the standard of living of similar countries measured by the real per capita GDP converges to the same value. We observe conditional convergence when countries with low standard of living grow faster than similar countries with high standard of living. Being similar could mean having an equally educated or healthy population, having a similar political system, having an equivalent respect of human rights, and so on.

Convergence: This is a mechanism through which the standard of living of countries measured by the real per capita GDP converges to the same value over a long period of time. We observe convergence when countries with lower standard of living grow faster.

Correlation: This term is a synonym for comovement or observed relationship. A variable correlates with another if there is a positive or negative comovement between the two variables. To indicate the direction of the comovement, we say positive correlation when the variables move in the same direction and negative correlation if they move in opposite directions.

Economic growth: This is the growth rate of the standard of living measured by the real per capita GDP over a long period of time.

Human development index (HDI): This is an alternative measure of the aggregate standard of living. The index is based on education, life expectancy, the Gini coefficient, and the real per capita income.

Inequality adjusted HDI (IHDI): This is an alternative measure of the aggregate standard of living. The index is a modified version of the human development index HDI. It is constructed by taking into account inequality in education and health.

Smooth density: This is a line chart representing the distribution of a variable. It is approximately obtained by drawing a smooth line over a bar density. This tool is particularly useful to visualize multiple distributions on the same graph.

Standard of living: This is a subjective concept that characterizes the well being of individuals. The determinants of the standard of living may include health, education, quality of air, freedom, security, income, etc. The real per capita GDP is often used as an indicator of the standard of living because it is correlated with its determinants.

Weighted distribution: When we represent income distribution across countries using histograms, each country counts as one observation. To get a better representation of the distribution at the individual level, we need to take into account that the size of the population is not the same in each country. A weighted distribution will increase the importance of countries with larger population.

Welfare: This is a subjective concept that characterizes the well being of a country at the aggregate level. Welfare depends on the standard of living or each individual and on how it is distributed in the population. For example, if the standard of living of some individuals increases and it decreases for others, it is not clear what happens to the welfare of the economy.

8.1 Economic Growth

Economic growth and development: An area of economics that studies the long term behaviour of economic activity and its implications on the well being of the population. In particular, development economists are interested in understanding why some countries are poor compared with others and why some countries who were poor 50 years ago are now among the richest while others remain poor.

Real Per Capita GDP Versus Welfare: This module focuses on the long term evolution of the standard of living across countries mesasured by the annual real per capita GDP. This is a measure of how much goods and services individuals can purchase on average over a year. We already know that the real per capita GDP is not allocated equally across individuals and therefore hides the existence of inequality.

Human Development Index: The United Nation Development Program (UNDP) publishes the human development index (HDI) in an attempt to better measure welfare. It includes the real per capita GDP, a measure of the average health quality (life expectancy) and a measure of the average level of education.
Since 2010, they also publish an inequality adjusted index (IHDI) which includes inequality as a fourth component.

We will use real per capita GDP as an indicator of welfare. It is a relatively good indicator because it is positively related to most components that determine welfare (education, health, reduced inequality, clean water and air, gender equality, peace and justice, etc.).
High real per capita GDP does not necessarily imply welfare improvement.

Income Inequality: Total income is not allocated equally across countries.

Distribution Using Densities: A density is simply another way to measure the frequency of observations between each interval. Instead of counting the number of observations, we compute proportions.

Distribution Using Smooth Density: When we want to compare income distributions over time, it is better to use smooth density because line charts are easier to interpret than bar charts when multiple series are represented. A smooth density is simply a smooth line that covers the bars of the histogram.

Alternate Representation - Log Per Capita GDP Distribution: The distribution is not as skewed as the regular one. The shape of the distributions is different because large differences in dollars translate into small differences in logs. This is why the distribution looks more equally spread on both sides of the center when the variable is expressed in logs.
The difference in proportion is what we measure when we use the log scale.

Alternate Representation - Weighed Distribution

Importance of growth: The module is called economic growth (growth rate of the real per capita GDP) because growth is what allows poor countries to become richer.

Suppose that the initial real per capita GDP is X_0 and the annual growth rate is g for T years. It implies the following real per capita GDP after T years:

$$X_T = (1+g)^T X_0$$

Convergence: Models from economic growth theory predict that inequality across countries should eventually be reduced because poor countries should grow faster than rich countries. The result is called convergence because it implies that the standard living of poor and rich countries will eventually converge to the same.
Poorer countries grow faster than richer countries. The growth rate should decrease as the poor get richer.

We can verify that poor countries grow faster by looking at the scatterplot of the average annual growth rate over the whole period with respect to the initial real per capita GDP. The average growth rate g can be computed by average the log differences $(\log(X_t) - \log(X_{t-1}))$ for each country. A negative comovement between these two variables means that countries that were initally poor grow faster than countries that were initially rich, which is convergence.

Conditional Convergence: When we observe convergence among countries that share similar characteristics, we call it conditional convergence because the convergence is conditional on the countries having some similarities.

We cannot conclude that any of the variables analyzed help poor countries become richer. We did observe a relationship between education, health, economic freedom and the ability of poor countries to grow, but observing comovements does not imply that an actual relationship exists.

Determinants of Economic Growth: We want to see if a comovement exists between economic growth and some economic variables. The first set of variables is the government share (G divided by GDP), investment share (I divided by GDP), and the level of openness (X+M divided by GDP). We observe a positive comovement in all variables, but it is stronger for the investment share and level of openness than for the share of government spending. Economic theory would suggest that investment and trade are good for growth.

The next set of variables to compare with economic growth is education, literacy, and health. We also observe a positive comovement between those variables and economic growth. It is less strong for education, but used a narrowed measure.

We analyze the comovement between average economic growth and the economic freedom index. The scatterplot shows there is positive comovement between growth and economic freedom.

Summary:

- We use real per capita GDP as an indicator of welfare not because welfare only depends on it, but because countries with high real per capita GDP tend to have high values for variables that affect the quality of life. However, we have to keep in mind that the comovement is not perfect. Many countries have high real per capita GDP and low education and life expectancy.
- The real per capita GDP has increased in most countries between 1950 and 2017, but the inequality in the growth rates resulted in an increase of income inequality over the period. If we weight the country by their population to obtained the income distribution at the individual level, the increase of income inequality is reduced. This is mainly due to the fact that large countries such as China and India have experienced relatively high economic growth.
- We do not observe unconditional convergence: globally, the standard of living in poor countries is not converging to the standard of living of rich countries.
- We do observe conditional convergence: if we restrict our sample to countries with favorable characteristics (high education, life expectancy and economic freedom), we observe poor countries converging to rich countries on average. However, there are lots of variations which means that some poor countries with favorable characteristics do not grow enough to catch-up with the richer countries.
- Economic growth seems to positively correlate (positive comovement) with investment, openness, health, education and economic freedom. However, the observed variations suggest that it is not sufficient to have high investment, openness, health, education and economic freedom to grow.

9 International Finance

Canadian Effective Exchange Rate (CEER): This is the trade weighted exchange rate of Canada and it is computed by the Bank of Canada.

Capital inflows: This is the amount of domestic assets purchased by foreign investors over a period of time (flow variable). For example, a German investor buys Canadian government bonds.

Capital outflows: This is the amount of foreign assets purchased by domestic investors over a period of time (flow variable). For example, a Canadian buys stocks from a US company.

Net capital outflows: This is equal to the capital outflows minus the capital inflows.

Nominal exchange rate: Also called exchange rate, this is the price of trading one currency for another. It can be expressed as domestic currency per unit of the foreign currency (e.g. the amount of Canadian dollars per US dollar) or the foreign currency per unit of the domestic currency (e.g. the amount of the US dollars per Canadian dollar).

Purchasing power parity (PPP): This is a long term theory of real exchange rate. It argues that the cost of buying goods in two countries should eventually be the same. In other words, the real exchange rate should eventually be equal to 1.

Real exchange rate: This is a measure of the relative cost of buying goods between two countries. It takes into account the nominal exchange rate and the price levels in both countries.

Trade weighted exchange rate or effective exchange rate: This is an index that represents the average fluctuation of exchange rates between one country and its most important trading partners. This is also called multilateral exchange rate as opposed to regular exchange rates that are bilateral.

9.1 Nominal Exchange Rates

Nominal Exchange Rate: The price of a currency in terms of another currency.

Appreciation Vs. Depreciation: A currency appreciates with respect to a foreign currency if the price of purchasing the foreign currency decreases, and it depreciates if the price increases.
e.g. 1.3 CAD/USD $\rightarrow$ 1.4 CAD/USD means CAD is depreciating with respect to the USD.

Rule to follow: If the exchange rate increases, the currency in the numerator depreciates and the one in the denominator appreciatees and vice versa.

Effects of Appreciation:

- Imports become less expensive.
- Exports become more expensive: competition increases.

After detrending the series, there is a negative comovement between exchange rate and imports and a positive comovement with exchange rate and exports.

The Trade-Weighted Exchange Rate: A trade weighted exchange rate is a weighted average of several exchange rates, with each weight reflecting the importance of each country as a trading partner.

9.2 Real Exchange Rate

Nominal exchange rates are just prices of purchasing currencies. Appreciation is good for consumers because it lowers the price of imported goods. However, this is true only under the assumption that prices remain constant.

Real Exchange Rate, Version 1: The real exchange rate is an indicator of the relative price between two countries, taking into account the exchange rate and the price levels of the two countries. As for the nominal exchange rate, there are two formulas for the real exchange rate. If the nominal exchange rate e is expressed as the price of purchasing the foreign currency (e.g. CAD/USD), the formula for real exchange rate ε is

$$\varepsilon = e \times \frac{P^*}{P}$$

where P^* is the price level in the foreign country and P is the local price level.
If $\varepsilon = 1$ then foreign price equals local, $\varepsilon > 1$ means foreign is higher, and $\varepsilon < 1$ means foreign is lower.

Real Exchange Rate, Version 2: The second formula is based on the exchange rate expressed as the value of one CAD in terms of the foreign currency (e.g. USD/CAD). Formula is

$$\varepsilon = e \times \frac{P}{P^*}$$

If $\varepsilon = 1$ then local price equals foreign, $\varepsilon > 1$ means local is higher, and $\varepsilon < 1$ means local is lower.

Interpretation:

- For nominal exchange rate expressed in CAD/USD, $\varepsilon = eP^*/P$. If it is greater than one, the cost of US goods in CAD is higher than the cost of Canadian goods and vice versa if it is less than one.
- For nominal exchange rate expressed in USD/CAD, $\varepsilon = eP/P^*$. If it is greater than one, the cost of Canadian goods in USD is higher than the cost of US goods and vice versa if it is less than one.
- Notice that if the cost of Canadian or US goods is higher in CAD, it is also higher in USD. Therefore, the two versions carry the same information. We just need to be careful about how to interpret them.

Remember the units must cancel out in these formulas, so if rate is in Currency1/Currency2, then the numerator must be Currency2 and denominator is Currency1.

Real Appreciation and Depreciation: The appreciation is real when the number of goods a CAD can purchase in the US relative to the number of goods it can purchase in Canada increases. In version 1, an increase in the real exchange rate means a real depreciation of the CAD and a decrease means a real appreciation. Version 2 is reversed.

Evolution of Real Exchange in Canada: At the macroeconomic level, real exchange rate is computed using price indexes. The easiest way to obtain long time series of real exchange rates for several countries is to use the purchasing power parity (PPP) and nominal exchange rate series from the OECD. PPP series is simply P_i/P_{US} where P_i is the price level in country i and P_{US} is the price level in the US, and the exchange rate is expressed as the price of one USD. Therefore, the real exchange rate (v1) is the nominal exchange rate divided by the PPP series. e.g.

$$\frac{e}{P_{Can}/P_{US}} = e\frac{P_{US}}{P_{Can}} = e\frac{P^*}{P}$$

There is a positive comovement between the real and nominal exchange rates but they are usually very far from each other. The relative price level between US and Canada is stable (if P^*/P is constant, e and eP^*/P always move in the same direction).

Important: The PPP theory states that the real exchange should be equal to 1. For example, if the cost of US goods is higher than the cost of Canadian goods, ($\varepsilon > 1$) we should expect consumers to purchase more Canadian goods and fewer US goods, putting pressure on prices and nominal exchange rate until the costs are equal.

Real Canadian Effective Exchange Rate: As for the nominal exchange rate, the CAD/USD real exchange rate fails to take into account fluctuations of the exchange rates against other important Canadian trade partners.

Real Exchange Rate and Trade Balance: Since the real exchange rate is better than the nominal exchange rate to evaluate the relative cost of goods between two countries, we should see stronger comovement between the real exchange rate and either exports or imports. An appreciation of CAD (lower ε expressed in CAD/USD) means that the cost of US goods relative to the cost of Canadian goods decreases. We should observe a negative comovement between the real exchange rate and imports and a positive comovement with exports.
Instead of analyzing the two comovements separately, we can look at the comovement with the trade balance (exports-imports). We should observe a positive comovement between real exchange rates and trade balance: an appreciation (lower ε expressed in CAD/USD) should increase imports and decrease exports and then decrease exports minus imports.

Some Real Exchange Rate Theory: The PPP is a long run theory that explains why the real exchange rate should be equal to one on average. This theory is based on a supply and demand argument: when the cost of US goods is higher relative to Canadian goods (ε in CAD/USD greater than 1), the demand for US goods decreases. The lower demand puts a downward pressure on nominal exchange rate and/or US price levels until the costs are equal.

Short term fluctations can also be explained by supply and decmand arguments. The following summarizes what we need to know:

- For any good or asset (a currency is an asset), its price is determined by its demand and supply: the higher the demand for a given supply the higher the price (or the higher the value). For example, if the demand for apples increases but the production (supply) does not, we should expect the price of apples to increase.
- Capital outflows is the value of foreign assets purchased by domestic investors (e.g. a Canadian buys Toyota stocks). Capital inflows is the value of Canadian assets purchased by foreign investors (e.g. an American buys bonds issued by the Canadian government).
- The supply of CAD on the currency market is partly determined by imports (to buy a US good, a Canadian must first trade CAD for USD, which increases the quantity of CAD on the currency market) and capital outflows (a Canadian who buys a Toyota stock must first trade CAD which increases the supply of CAD on the currency market).
- The demand for CAD is partly determined by exports (an American who wants to buy Canadian goods must first buy CAD which increase the demand for CAD) and the capital inflows (an American who wants to buy Canadian assets must first buy CAD).
- To simplify, the demand for CAD is the demand from exports minus the supply from imports: Demand of CAD = Export - Import (trade balance or net exports or simply NX). The supply is the supply from capital outflows minus the demand from the capital inflows (net capital outflows or NCO).

We assum ethat real exchange rate only affects the relative cost of buying goods. The NCO (net purchase of foreign assets) is affected by factors other than the real exchange rate. Therefore, the supply is vertical (it is the same for all values of ε).

Using the version 2 ε (USD/CAD), if ε increases (appreciation), NX decreases which decreases the amount of CAD demanded.

The equilibrium real exchange rate is the exchange rate that equalizes the amount of CAD demanded and supplied (ε^* is the equilibrium real exchange rate and Q^* is the equilibrium amount of cAD which is also equal for the equilibrium NCO and NX).

If the supply of CAD increases (an increase of NCO), its value decreases (depreciation), which implies a lower ε expressed as USD/CAD (ε^{**}). It also increases the equilibrium NCO and NX (Q^{**}). Equivalently, if the supply of CAD decreases (decrease of NCO), its value increases (appreciation), which implies a higher ε expressed as USD/CAD (ε^{***}) and lower equilibrium NX and NCO (Q^{***}).

Therefore, shifts of the CAD supply create negative comovements between ε expressed as USD/CAD and NX. Therefore, it creates positive comovement between ε expressed as CAD/USD and NX, which is what we observed previously.

If the demand of CAD shifts right (an increase of NX due to factors other than the change in ε), its value increases (appreciation), which implies a higher ε expressed as USD/CAD (ε^{**}) . It does not change the equilibrium values of NCO and NX (Q^*) because they are determined in this model by the supply only. Equivalently, if the demand of CAD shifts left (decrease of NX due to factors other than the change in ε), its value decreases (depreciation), which implies a lower ε expressed as USD/CAD (ε^{***}). Again, this shift does not affect the equilibrium values of NX and NCO.

Shifts in the demand of CAD is different because it does not create comovements between real exchange rate and NX (NX in equilibrium is determined by the vertical supply). It only creates fluctuations unrelated to NX.

In the previous point, we mentioned that the right or left shift of the CAD demand corresponds to a change of NX not due to ε. But, what else can affect NX? The module readings offer different answers to this question. For example, if there is a recession in Canada, the consumption of Canadian and imported goods decreases which increases NX (X-M). On the other hand, if there is a recession in the US, exports go down which decreases NX.

Overall, the supply and demand theory of real exchange rate is in line with what we observed previously: we observed positive comovements between real exchange rate expressed as CAD/USD (supply shifts) and fluctuations not related to NX (demand shifts).

10 The Monetary System

Bank rate: This is the interest rate paid by commercial banks when they borrow money from the Bank of Canada. This is also known as the discount rate.

Central bank: This is an institution that manages the currency and money supply in an economy. For example, the central bank in Canada is the Bank of Canada and in the United Sates it is the Federal Reserve Bank.

Commodity money: This is a type of money with intrinsic value. An example is gold because it can be used as money and to make jewels.

Currency: This is the stock of bank notes and coins in circulation outside banks.

Fiat money: This is a type of money with no intrinsic value. Bank notes represent fiat money: by yourself on a desert island, the notes would only be useful to start a fire.

Monetary base: This is the value of all bank notes and coins inside and outside the banks plus the banks' deposits at the central bank. This is all known as the money base.

Monetary policy: This is a policy implemented by a central bank to control the creation of money.

Money: This is an item that can be used as medium of exchange, unit of account, and store of value.

Money supply: This is the stock of money in an economy.

Overnight rate: This is the interest rate on one-day loans between commercial banks.

Quantity equation: The equation is $M \times V = P \times Y$, where M is the money supply, V is the velocity of money, P is the GDP deflator, and Y is the real GDP.

Reserve: This is the value of deposits not loaned out by commercial banks.

Reserve ratio: This is the proportion of deposits kept as reserve by commercial banks.

Velocity of money: This is the average number of times each dollar is used for transaction over a given period.

10.1 The Stock of Money

Money: Money is defined by the following three functions:

- Medium of exchange: it is accepted as payment when purchasing goods and services. When an item is a medium of exchange, you are confident when you enter any store that it will be accepted. Therefore, even if one seller accepts to trade a good for another, it does not make that good a medium of exchange because it is only accepted by that particular seller. Cash (dollars for Canada) is a medium of exchange because it is accepted by all stores (even if the cash is tranferred using a debit card).
- Unit of account: It can be used to set the price of goods and services: the prices are set in dollars, not in goods.
- Store of value: It can be stored and used to purchase goods and services in the future.

The level of liquidity characterizes how easy it is to convert an item into a medium of exchange. e.g. If you have Mexican pesos, it may be difficult to find sellers that will accept them in Canada. Foreign currencies are assets, but they are not as liquid as Canadian dollars because not all Canadian sellers accept them. We need to use bank to convert. The level of liquidity of an assest is determined by the cost (in time or in dollars) to convert it into dollars (or any accepted medium of exchange).

We distinguish two types of money:

- Commodity money: This is an item with intrinsic value (it has a value by itself) that satisfies the three functions of money. Any not perishable good (to satisfy the store of value function) that is generally accepted in exchange for goods and services is a commodity money.

- Fiat money: It is an item without intrinsic value (it has no value by itself) that satifies the three functions of money. The value of a twenty dollar bill is determined entirely by how much goods it can purchase. As a peice of paper, it has no value.

Currency: The amount of paper bills and coins in circulation outside the commercial banks is called the stock of currency (or just currency). This is the most liquid definition of money becuase it is accepted almost everywhere as a medium of exchange.

M1 Stock of Money: Some stores only accept debit/credit cards. Therefore, stock of currency is not a good measure of the amount of money in circulation that can be used for transactions. M1 is a measure of stock of money that includes the stock of currency and deposits in chequable accounts.
There is more than one version of M1 like M1++ which includes non-chequable accounts for transactions.

M2 Stock of Money: M2 includes M1 and less liquid deposits such as saving bonds.

10.2 The Bank of Canada

Bank of Canada: The Canadian central bank. The following summarizes the roles of the Bank of Canada:

- Issue currency: The Bank of Canada is the only bank with the right to issue currencies.
- It is the banker of all banks: The Bank of Canada can lend money to commercial banks if they need it and commercial banks can deposit money at the Bank of Canada.
  The interest rate that commercial banks pay for loans is called the bank rate and the interest rate the Bank of Canada pays on deposits is the bank rate minus 0.50%. The bank rate determines the overnight reate, which is the rate at which commercial banks lend or borrow funds among themselves at the end of each day. It is somewhere in the interval [bank rate - 0.50%, bank rate].
- It is the banker of the Canadian Government: The Bank of Canada manages the government bank accounts and national debt.
- Monetary policy: The Bank of Canada can control the stock of money in the economy (money supply)

BOC Balance Sheet: The main components of the balance sheet of the BOC is:

- Government of Canada direct and guaranteed securities: This is the government securities (to borrow money, the gov't sells securities) that the BOC has purchased (from market or gov't).
- Securities purchased under resale agreements: This is one of the tools used by the BOC to control the amount of liquidity in the economy: Bank buys gov't securities help by financial institutions with a promise to resell them.
- Notes in circulation: Total amount of dollar bills in circulation. Dollar bills are liabilities for the BOC but coins are not.

- Canadian dollars deposits: Two important source of deposits are gov't of Canada and the members of payments Canada (commercial banks). Deposits from commercial banks are liquidity kept at the BOC to help them make payments to each other without having to borrow.

Controlling the Money Supply: In this simple economy, commercial banks create money by making loans. The amount of money they can create depends on how much currency individuals keep and the fraction of deposits the banks can or want to lend. There are a few cases.

Reserve Ratio: Calculated by taking reserve divided by deposits.

Case 1 - Reserve Ratio of 50% and Currency to Deposit Ratio of 0: In this scenario, individuals do not keep currency and 50% of all additional deposits are lent to individuals. The loan keeps going up by 50% from the previous period. The money injection is multiplied by 2 and money supply is 2 times the total reserve.

There is a formula to calculate the effect of a new deposit on the money supply.
Let X be the value of the new deposit and r be the reserve ratio (percentage of deposits kept in reserve), then after an infinite number of periods the money supply increases by:

$$\Delta M = \frac{X}{r}$$

Case 2 - Reserve Ratio of 50% and Currency to Deposit Ratio of 0.5: If we want the currency to deposit ratio to be 0.5, if we a certain loan, we divide the loan amount by $(1+0.5) = 1.5$ and the other to currency.

There is a formula to calculate the effect of a new deposit on the money supply if the currency-deposit ratio is c.
Let X be the value of new money injected in the economy and r be the reserve ratio. Here it makes no difference whether money is injected inside banks or among individuals. After an infinite number of periods, the effect of the injection on the money supply is:

$$\Delta M = X\frac{1+c}{r+c}$$

Monetary Policy: A monetary policy implemented by the BOC (or any central bank) to increase or decrease the money supply. The monetary policy is expansionary when the bank tries to increase the money supply (or its growth rate) and it is contractionary when it tries to reduce it.

Bank Rate: Bank rate or discount rate is the interest rate commercial banks pay if they borrow money from the BOC and the interest on deposits is the bank rate munus 0.5%. The interval [bank rate - 0.5%, bank rate] is called the **operating band**.
By controlling the band, the BOC controls the overnight rate, which is the rate on short term loans between commercial banks. Overnight rate is always inside the band.

Bank Rate vs Major Interest Rates: The bank rate impacts the reserves of commercial banks and therefore the money supply. A higher rate reduces the reserves and the money supply and vice versa for a lower bank rate. The bank rate also affects all major interest rates which affects the demand for loans and the money supply. The BOC controls all interest rates.

The following summarizes market operations that may be used by the BOC to control the money supply.

- Open market operation: BOC can purchase gov't bonds to inject liquidity in the market, which increases the money supply, or sell gov't bonds to decrease it.

- Quantitative easing: Like open market operation, but involves purchasing or selling other types of assets held by commercial banks. Effects on money supply is comparable to open market operations.

- Foreign exchange market operations: Foreign currencies are like any other asset. If BOC purchases USD and pay in CAD, money is injected in the economy and money supply increases.

- Sterilization: If BOC wants to control CAD exchange rate w/o affecting the money supply, it needs to combine a foreign exchange market operation with an open market operation. For e.g., bank can incrase value of CAD by purchasing CAD (demand in market increases which puts upward pressure on value of CAD) and buy assets with purchased dollars to avoid supply to decrease.

- Changing reserve requirements: If BOC requires commercial banks to hold a higher fraction of all deposits in reserve, it reduces their ability to make loans and then to create money.

Monetary Base: The main objective of different market operations and changes in the overnight rate is to control the monetary base. It is defined as the notes and coins inside and outside the banks plus the deposits at the BOC.

The monetary base is the first banking system measure to react to monetary policy. When a monetary policy is effective, the monetary base changes and then loans are created to change the money supply through its multiplicative effect.

The BOC has direct control of the monetary base, not of the money supply.

The Quantitiy Equation: The quantity equation is helpful to understand the relationship between money supply, price level, and real GDP. The equation is

$$M \times V = P \times Y$$

where V is the velocity of money, M is the money supply, P is the price level (GDP deflator) and Y is the real GDP. THe velocity is defined as the number of times each dollar is used over a period on average.
The nominal GDP is $P \times Y$.

$$g_M + g_V \approx \text{Inflation} + g_{\text{real GDP}}$$

or

$$g_M + g_V \approx g_{\text{Nominal GDP}}$$

Money Supply vs Nominal GDP: According to quantity equation, if the velocity is constant then the money supply should be proportional to nominal GDP.

Velocity of Money: The velocity seems to play an important role in identifying relationships between money supply and nominal GDP.

$$V = \frac{P \times Y}{M} = \frac{\text{Nominal GDP}}{\text{Money Supply}}$$

Money Supply vs Price Level: In monetary economics, the money is neutral if the money supply does not affect real variables. If we consider V and Y as real variables in the quantity equation (V is a frequency so it is not measured in dollars), the neutrality implies that changes in the money supply is matched by changes in the price level. This is something we can try to verify by comparing the cyclical components of the money supply and price level.

Growth Rate of Money Supply vs Inflation: The money is super-neutral if a change in its growth rate does not affect the growth rate of real variables. Therefore, under the super-neutrality assumption, a change in the growth rate of the money supply should be matched by a change in inflation. To find evidence of super-neutrality, we compared the cyclical components of the money supply and price level. For the super-neutrality assumption, we need to compare the growth rate of money supply and inflation.

Money Supply vs Real GDP: The neutrality assumption suggests that money only affects nominal variables (price in this case). Therefore, we should not observe any comovement between the cyclical components of the money supply and real GDP.

Growth of Money Supply vs Growth of Real GDP: We compared inflation with the growth rate of money supply in a previous section and found little evidence that the super-neutrality assumption is valid. The assumption can also be analyzed by comparing the growth rates of money supply and real GDP. If it is valid, the two series should be unrelated.

Summary of Results:

- The velocity of money is not constant: it fluctuates over short periods of time and has a negative trend between 1970 and 2020 (1990 and 2020 for M1++).
- We observe a strong comovement between the cyclical component of money supply and the cyclical components of price level and nominal GDP. However, the direction of the comovement is not always positive. Also, positive comovements are sometimes observed with lags (one variable increases and the other follows after a few periods).
- We observe a strong positive comovement between the growth rate of money supply and the growth rates of nominal GDP and price level (inflation). The positive comovement is observed over short period of times and on average over longer periods.
- We observe little or no comovement between the growth rate of the money supply and the growth rate of real GDP.
- We observe a comovement between the cyclical components of the money supply and real GDP, but it is negative for some period and positive for others.
- Overall, the money supply affects the price level more than the real GDP. Although we can not be certain, we did not find evidence that the money supply only affect nominal variables. In other words, we could not find evidence that the money is neutral or super-neutral.

The quantity equations is simple but the relationship between the four variables is complex.

Main Object of BOC: Besides supporting the banking system in periods of crisis, the sole objective of the BOC monetary policy is to control inflation. The BOC tries to keep it between 1% and 3%. If it gets close to 3%, then the bank implements a contractionary monetary policy (decrease money supply by increasing bank rate) and close to 1% is an expansionary monetary policy (increase money supply or lower bank rate).

Decision Making Process: The relationship between money supply and inflation is complex because all 4 variables of the quantity equation may be related to each other. It is therefore difficuly for the BOC to come up with the appropriate monetary policy.

11 Public Finance

Accumulated deficit: This it the sum of all past deficits (including the negative deficits) or minus the sum of all past surplus (including the negative surplus). It is also equal to the net debt minus the value of the government's non-financial assets.

Balanced budget: When total expenditure and total revenue are equal (or if the deficit and surplus are equal to 0), we say that the budget is balanced.

Collective consumption: This is a component of the government consumption expenditure that serves the interests of a society as a whole. It includes defence, justice, law enforcement etc.

Deficit: This is equal to total expenditure minus total revenue. When the deficit is positive, the government spends more than its revenue and must borrow money.

Fiscal policy: The purpose of this policy is to affect the economic activity by changing taxes and/or expenditures. For example, a government can support the economy in periods of recession by spending more or by reducing taxes.

Government transfer: This is part of the government expenditure but it does not involve the purchase of goods or services. It includes social assistance, child benefits, old age benefits, etc.

Gross debt: This is the value of all government bonds in circulation. It is also called the interest bearing debt because the interest payment is determined by it.

Individual consumption: This is a component of the government consumption expenditure that serves the interests of individuals. It includes education, health, etc.

Net debt: This is equal to the gross debt minus the government's financial assets. It is considered a better measure of the level of indebtedness because financial assets can be used to pay back part of the gross debt.

Surplus: This is the negative of the deficit: revenue minus the expenditure. A negative surplus implies a positive deficit.

11.1 Government Budget

Public finance (or public economics) is an area of economics that studies the impact of gov't interventions. The main objective is not to promote a specific gov't role but to predict if policies are in line with it. For exmaple, if one of the roles is to protect the population against poverty, public economists will compare different policies and propose the one that is the most likely to achieve this goal.

If we want to measure the size of a gov't in proportion to the size of the economy, we want to look at the expenditure in percentage of GDP. To compute this measure, we don't need real per capita GDP and expenditure. In fact, the nominal expenditure as a percentage of nominal GDP is identical to real per capita expenditure as a percentage of real per capita GDP:

$$\frac{\frac{\text{Expenditure}}{\text{Price}\times\text{Population}}}{\frac{\text{GDP}}{\text{Price}\times\text{Population}}} = \frac{\text{Expenditure}}{\text{GDP}}$$

Government Deficit: A deficit is defined as total expenditure minus total revenue over a given period of time. Therefore, a positive deficit means that the gov't spends more than its income. Similarly, a surplus is defined as the negative of the deficit: revenue minus expenditure. Therefore, a positive surplus implies a negative deficit and a negative surplus implies a positive deficit.

Nominal Deficit in Canada: When the deficit is positive, a gov't must borrow money by issuing bonds. The bonds can be purcahsed by anyone including the BOC. When the surplus is 0, we say that the gov't budget is balanced.

Real Per Capita Deficit in Canada: As for any variables measured in dollars, it is preferable to remove the effect of inflation. Also, it is better to look at the surplus per individual.

Deficit as a Percentage of GDP in Canada: We can also analyze the surplus or deficit as a percentage of GDP. It is a better measure because it takes into account the size of the economy. It is like analyzing deficits of individuals: if you spend 100 dollars more than your income in a given week, the impact is not the same on your finance if your weekly wage is 500 dollars or if it is 2000 dollars.

Government Debt: The deficit and surplus are flow variables because they represent the difference between two other flow variables: expenditure and revenue per period. Successive deficits result in an accumulation of gov't bonds in circulation that the gov't will eventually need to repurchase. This stock of bonds in circulation is the gov't debt.

Terms related to gov't debt:

- Gross debt: This is the sum of all gov't bonds in circulation. It is an important measure because interest payments on debt are based on it: the higher the gross debt, the higher the interest payments
- Net debt: This is the gross debt minus the value of the financial assets owned by the government: loans made by the government, taxes receivable, foreign exchange accounts, etc. It is considered a better measure of the financial difficulties that the government faces because it is what the gross debt would be if the government used all its financial assets to pay back part of the debt.
- Accumulated deficits: This is the net debt minus the value of the non-financial assets. It is often how federal debt is defined in the media. It can also be obtained by adding all past deficits and subtracting all past surpluses.

Types of debt: Nominal debt, real debt, real per capita debt, debt as a percentage of GDP.

Canada vs the rest of the world: The debt as a percentage of GDP is by far the best measure to evaluate the level of indebtedness of a gov't befcause it takes into account the change in price, population and national income. Also a ratio has no measurement units so we can easily use that measure to compare the level of indebtedness of countries.

Interest Payment: An indebted gov't is required to use a fraction of its revenue to pay interest on its debt. This could be costly for the population because it implies that the gov't has less money to provide them with services.

There are two factors that affect the amount of interst payments: for a given interest rate, the interest payments increase when the gross debt increases and for a given gross debt, the interest payments increase if the interest rate increases. THe latter is affected by the credit rating of the gov't. A bad rating means that a gov't is likely to not pay back its debt, making it riskier for investors to purchase its bonds. In that case, investors ask for higher returns to compensate for the risk.

Higher interest payments may be more problematic if the gov't debt is held by foreign investors because in that case a large fraction of the gov't revenue leaves the country.

11.2 The Government and the Economy

Fiscal policies are actions taken by the government to influence economic activity by either changing its expenditure or the amount of taxes collected.

Some theories will predict that for each dollar spent, the GDP increases by more than one dollar. This is called the multiplicative effect: when the gov't spends one dollar the GDP increases by one dollar, then the dollar received by the seller of the good purchased by the gov't is used to buy more goods which increases GDP even more. Summary: gov't can potentially affect GDP if needed.

Surpluses vs Business Cycle: The gov't surpluses decrease (or deficits increase) in periods of recessions. That could be caused by the gov't increasing its expenditure to support the economy. But even if gov't doesn't respond to the recession, surpluses always decrease in periods of recession: revenue from taxes decreases because fewer workers work and fewer goods and services are purchased.

By analyzing the components of gov't expenditure we realized that the gov't does not spend money for the sole purpose of stabilizing the business cycle. In fact, a large fraction of total expenditure is used to finance social programs. The list below are some public policies and their possible impacts on the economy:

- Promote economic growth: We saw that a small change in the economic growth could have large impacts on the per capita standard of living over a long period of time. The possible determinants of economic growth are education which has a positive impact on the productivity of workers and investment which increases the production capacity of the economy. How can a government promote growth is something covered in a growth and development course. For example, the government needs to maintain an effective system of justice that protects property rights in order to attract investors. Also, it needs to control the accessibility and quality of education.
- Social programs: The objective of social programs is to temporarily reduce poverty. Examples of such programs are social assistance, child tax benefits, social housing, old age securities, etc. These types of programs can be analyzed in different economic theory courses.

- Universal health care system: In a way, this is a policy that reduces poverty and inequality because it provides services to individuals that would otherwise be too poor to be able to purchase them. Economic theory can help determine the less costly health system that provides high quality services: should it be fully public, private with transfer payments to households or a mixture of both? Answers to these questions are covered in a health economics course.

- Minimum wage: The purpose of having a minimum wage is to reduce poverty and inequality. Is this an effective way to reduce poverty and inequality? Some believe that increasing the minimum wage increases unemployment among low income workers and therefore increases inequality. This is not a view that all economists share. Analyzing the impact of increasing the minimum wage on the labour market characteristics is one of the many topics covered in a labour economics course.

- Public education: This includes government subsidies to reduce University's tuition fees. This policy is likely to have an impact on poverty and inequality in the long run: by providing equal access to education, children from poor or rich families have an equal chance of becoming successful. Measuring the impact of education policies on poverty and inequality is addressed in education or labour economics courses.

- Employment insurance: This includes the resources provided by the government to help workers find jobs or receive training. This service helps to stabilize workers' income by providing temporary wages when they lose their jobs. It also reduces the cost for workers who are unsatisfied with their current job to search for a better one (they receive unemployment benefits while they search). Analyzing the effect of different employment insurance programs on the characteristics of the labour market (e.g. duration and incidence of unemployment) is a topic covered in a labour economics course.

- What else? The government is also responsible for producing public goods (bridges, roads, etc.), adopting policies to protect the environment (global warming, quality of air), protecting individuals against behaviour that may be harmful to them (mandatory seat belt or helmet, taxes on cigarettes, etc.) and so on. Not everyone agrees on how the government should be involved in these types of policies, but economic theory can always help to measure their impacts.

Notes:

Higher inflation means sell government bonds, low inflation means buy government bonds.

If BOC buys government bonds, this increases money supply. If they sell bonds, this decreases money supply.

According to the neutrality of money, if the money supply doubles, price index doubles but new real GDP stays the same.