UAV-Enable Federated Learning and Radar

***Abstract*—**

***Index Terms*—.**

I. INTRODUCTION

II. SYSTEM MODEL

We consider a device-based FL network as shown in Fig. 1. The network includes a set $\mathcal{N}$ of N autonomous devices which are single-antenna ground devices, while one UAV keeps cruising for realizing wide-range model aggregation from the devices. Here, besides model aggregation, the UAV is expected to be a source device that sends an adaptive IEEE 802.11ad single-carrier physical layer (SC PHY) frame to a target device and uses the reflections from the target device to derive its range and velocity. In addition, the local model which contained a large of number bits need to be quantized before being transmitted.

A. FL Model

This subsection briefly investigates the basics of FL in UAV-enabled networks. Let $\mathbf{w}$ denote the vector including global model parameters. Each device i has a local dataset $\mathcal{D}_i$ with D_i data samples. Each data sample k consists of an input $\mathbf{x}_{ik}$ and its corresponding output $\mathbf{y}_{ik}$. We introduce the loss function $g_i(\mathbf{w}, \mathbf{x}_{ik}, \mathbf{y}_{ik})$ that captures the FL performance. The total loss function of device i is given by

$$G_i(\mathbf{w}) = \frac{1}{D_i} \sum_{k=1}^{D_i} g_i(\mathbf{w}, \mathbf{x}_{ik}, \mathbf{y}_{ik}). \tag{1}$$

The goal of the process is to find the global parameter $\mathbf{w}$. Therefore, the UAV need to solve the following distributed learning problem

$$\min_{\mathbf{w}} G(\mathbf{w}) = \sum_{i=1}^{N} p_i G_i(\mathbf{w}) \tag{2}$$

where $p_i = \frac{D_i}{\sum_{i=1}^{N} D_i}$.

Assume that the global model has a size of S bits. In the FL algorithm, we denote T as the number of global rounds, $t \in \mathcal{T} = \{0, \ldots, T-1\}$. During the t-th global round, each device receives the global parameter $\mathbf{w}(t)$ from the UAV, and perform ν steps of the stochastic gradient descent (SGD) method.

In particular, in digital communication systems, the model parameters need to be quantized before transmitted, which brings quantization errors (QEs) to the learned model.

B. Quantization Model

In order to prevent a wasteful overuse of resources, we assume that devices send to the UAV a quantized version of $\mathbf{w}_i(t)$, which is denoted as $Q(\mathbf{w}_i(t))$. where Q denotes the quantization function.

Following that, in each global round t, for each element parameter j of $\mathbf{w}_i(t)$, it holds $|w_{i,j}(t)| \in [\underline{w_{i,j}(t)}, \overline{w_{i,j}(t)}]$, and is quantized by the stochastic quantization method [20]. In each global round, with $B_i(t)$ quantization bits, device i can devide the interval $[\underline{w_{i,j}(t)}, \overline{w_{i,j}(t)}]$ into the following ζ intervals: $I_1 = [s_0, s_1], I_2 = [s_1, s_2], \ldots, I_\zeta = [s_{\zeta-1}, s_\zeta]$, where $\zeta = 2^{B_i(t)} - 1$ and

$$s_u = \underline{w_{i,j}(t)} + u \times \frac{\underline{w_{i,j}(t)} - \overline{w_{i,j}(t)}}{2^{B_i(t)} - 1}, \tag{3}$$

where $u = 0, 1, \ldots, 2^{B_i(t)} - 1$. Then, if the parameter $w_{i,j}(t)$ falls into the interval I_u, it will be quantized as

$$\mathcal{Q}(w_{i,j}(t)) = \begin{cases} \text{sign}(w_{i,j}(t)) \cdot s_{u-1}, & \text{with prob } \frac{s_u - |w_{i,j}(t)|}{s_u - s_{u-1}} \\ \text{sign}(w_{i,j}(t)) \cdot s_u, & \text{with prob } \frac{|w_{i,j}(t)| - s_{u-1}}{s_u - s_{u-1}} \end{cases} \tag{4}$$

Let μ be the number of bits used to specify the values of sign$(w_{i,j}(t))$, $\underline{w_{i,j}(t)}$, and $\overline{w_{i,j}(t)}$. Then, the quantized model $\mathcal{Q}(w_{i,j}(t))$ of the overall m-dimensional local model parameters is expressed by a total number of

$$S_i(t) = m(B_i(t) + 1) + \mu(\text{bits}) \tag{5}$$

since each element of the quantized model vector is represented by $B_i(t)$ bits plus one bit for the sign specification.

Now, we analyze the convergence of FL with Stochastic Quantization and Stochastic Gradient Function. First of all, we refer to a conception named "unbiased estimator". In machine learning, an unbiased estimator refers to a statistical estimator that, on average, produces parameter estimates that are close to the true population parameters. In other words, an unbiased estimator has an expected value equal to the true parameter value. $\mathcal{Q}(\mathbf{w}_i(t))$ is quantization parameters of $(\mathbf{w}_i(t))$ and E means unbiased estimator. To maintain the accuracy of the parameterized model, the quantized parameters need to have values that are close to the original parameters. It means:

$$\mathbb{E}\left[\mathcal{Q}(\mathbf{w}_i(t))\right] \triangleq \mathbf{w}_i(t) \tag{6}$$

Therefore, the quantization error is given by

$$\mathbb{E}\left[\left|\mathcal{Q}(\mathbf{w}_i(t)) - \mathbf{w}_i(t)\right|^2\right] \leq \frac{\delta_i^2(t)}{(2^{B_i(t)} - 1)^2} \triangleq J_i^2(t) \tag{7}$$

where

$$\delta_i(t) \triangleq \sqrt{\frac{1}{4} \sum_{j=1}^{m} \left(\underline{w_{i,j}(t)} - \overline{w_{i,j}(t)}\right)^2} \tag{8}$$

The proof can be found in [18], [21].

C. FL Latency

We denote T as the time for the whole FL process. We denote τ as the duration of each global iteration. Then, during τ, the FL process consists of four steps: global model broadcasting, local training at the users, local model update transmissions of the users and result aggregation at the BS. We assume that the global model broadcasting, local training at the users, and local model update transmission of the users need to be finished in τ_g, τ_l, and τ_u, respectively.

1) Global model broadcasting: To estimate the parameters of the target, the UAV takes (copies) a beginning part of the bit sequence representing the global model and then places this part at the beginning of the original bit sequence. As such, the copied part and the part the UAV extracts are the two same sequences. Therefore, they can be used for the target parameter estimation. We denote αS as the part that the UAV extracts from the original bit sequence, where $0 < \alpha \leq S$ is the the factor. We denote $h_{U,i}$ as the channel gain between the UAV and the device i. The data rate achieved by device i is given by

$$R_i = B \log_2 \big(1 + \frac{P|h_{U,i}|^2}{B\sigma^2}\big), \tag{9}$$

where P is the transmit power of the UAV, B is the bandwidth, σ^2 is the white Gaussian noise spectral density at device i. Then, the time required for transmitting the global model from the UAV to device i is given by

$$\tau_i^g = \frac{(1+\alpha)S}{R_i}. \tag{10}$$

2) Local training latency: The dedicated time for local computations by each device i, for ν SGD steps, in order to generate the local model, is given by

$$\tau_i^c = \nu \frac{c_i D_i}{f_i}, \forall i \in \mathcal{N}, \tag{11}$$

where f_i is the CPU cycle frequency of device i, D_i is the size of the mini-batch (in bits), while c_n denotes the number of CPU cycles for device i to perform one sample of data during the local model training.

3) Model upload latency: The data rate achieved by the UAV as device i transmits its local model is

$$r_i = b_i \log_2 \Big(1 + \frac{p_i|h_{i,U}|^2}{b_i\sigma_{i,U}^2)}\Big), \tag{12}$$

where $b_i = B/N$ is the bandwidth assigned to device i, and $\sigma_{i,U}^2 = b_i\sigma_0^2$ where σ_0^2 is the one-sided power spectral density level of the noise at the UAV. Then, the time required for transmitting the local model from device i to the BS is given by

$$\tau_i^l = \frac{S_i}{r_i}, \tag{13}$$

where S_i is the size of quantized local model of device i that is presented in (5).

4) Aggregation latency:

D. Target Radar Tracking

The UAV is equipped with a dual radar and communication function as shown in Fig. **??**. The communication function acts as a communication transmitter and a communication receiver. For this, a full-duplex technique is used for the communication function so that a single antenna is able to transmit and receive signal simultaneously. The transmitter of the communication function transmits the bit sequence representing the global model to the devices. The receiver of the communication function receives the local updates from the devices for the federated learning aggregation. The radar function is equipped with a radar receiver with a different antenna. The radar receiver receives the signal returned from the target due to the global sequence transmission. Fig. **??** shows the modules for the target parameter estimation.

After the analog RF combining and matched filtering can be represented as

$$y_r(t) = \sqrt{P} h_0 x(t-\tau_0) e^{j4\pi vt/\lambda} + n_r(t), \tag{14}$$

where P is the transmit power of the UAV, λ is the wavelength of the transmitted signal, v is the relative velocity of the target with respect to the UAV, $n_r(t)$ is the Gaussian noise at the UAV, and h_0 is the round-trip channel between the UAV and the target that is given by [1]

$$h_0 = \frac{G_U^{\mathrm{R}} G_T^{\mathrm{R}} \lambda^2 \sigma^{\mathrm{RCS}}}{(4\pi)^3 d^4}, \tag{15}$$

where G_U^{R} and G_T^{R} are the antenna gains of radar of the UAV and the target device, respectively, σ^{RCS} is the Radar Cross Section (RCS) from target to radar of UAV. In addition, $\tau_0 = 2d/c$ is the round-trip delay of the target with c is the speed of light, d being the distance between the UAV and the target that is determined as follows

$$d = \sqrt{(x_U - x_T)^2 + (y_U - y_T)^2 + (z_U - z_T)^2}, \tag{16}$$

where (x_U, y_U, z_U) and (x_T, y_T, z_T) are the coordinates of the UAV and the target, respectively. Note that the target is on the ground, and thus $z_T = 0$. We assume that the coordinate of the target at the time t is $(x_T(t), y_T(t), z_T(t))$. The location of the target at the time t can be expressed as follows

$$\begin{cases} x_T(t) = x_T(0) + v_x t, \\ y_T(t) = y_T(0) + v_y t, \\ z_T(t) = 0, \end{cases} \tag{17}$$

where v_x and v_y are the velocity of the target along Ox and Oy axis, respectively, $x_T(0)$ and $y_T(0)$ are the initial location of the target.

1) Target velocity estimation: As the communication function transmits the bit sequence of the global model, the radar receiver is triggered to determine the round-trip delay of the bit sequence to estimate the range between the UAV and the target. There are different techniques to determine the round-trip delay. The first one is that the UAV measures the energy of the returned signal and determine the round-trip delay and the corresponding distance between the UAV and the target. The range estimation technique based on energy measurement is simple, but it is not highly accuracy as the

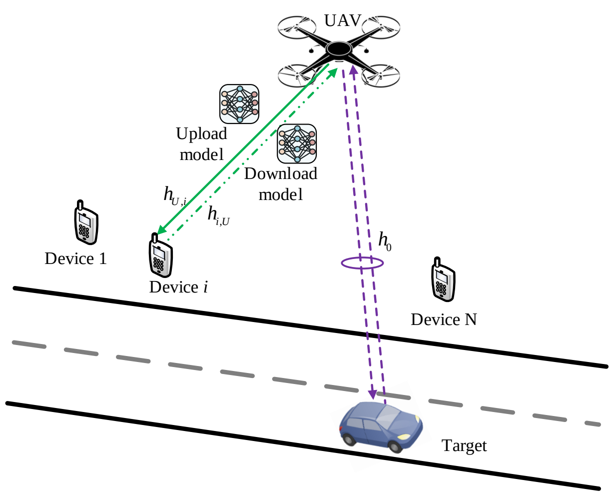

Fig. 1. A UAV-enabled Federated Learning.

noise and interference are high. Moreover, it is difficult to set the detection threshold properly. An effective technique is to use auto-correlation functions. The key idea is that the transmitter transmits at least two sequences that are the same in terms of size and bit value. For example, the first sequence is ”1011”, and the second sequence is also ”1011”. In other words, the second bit sequence is the copy of the first bit sequence. The radar receiver then performs an auto-correlation function between any two received sequences. The arrival time of the first sequence is determined by searching a value at which the auto-correlation function achieves the maximum. From the arrival time, the round-trip and the range between the UAV and target are determined. Although the auto-correlation-based range estimation technique requires at least two same sequences, but it addresses the issues of the energy detection based technique. More important, the auto-correction function can be used for the velocity estimation.

We denote $\mathbf{Y} = [y_r(0)^{(1)}, y_r(1)^{(1)}, \ldots, y_r(\alpha S - 1)^{(1)}]$ as the signal samples of the first sequence returned from the target.

In this work, we assume that the delay estimation is correctly implemented. In other words, the UAV correctly detects the arrival of the returned Upon detecting the arrival of the returned signal, the velocity estimation is determined,

2) Cramer-Rao lower bound (CRB): We leverage the Cramer-Rao lower bound (CRB) as a lower bound on the velocity estimation error. and the auto-correlation function, the CRB is given by

$$\mathtt{CRB}_v = \frac{6\lambda^2}{16\pi^2\alpha^3 S^3 T_s^2 \mathtt{SNR}_r}, \tag{18}$$

where $T_s \approx 1/B$ is the symbol duration, $\mathtt{SNR}_r$ is the SNR at the radar receiver of the UAV determined by

$$\mathtt{SNR}_r = \frac{P|h_0|^2}{B\sigma^2} \tag{19}$$

.

3) Radar processing time: The radar receiver implements the auto-correlation function on αS bits. Thus, the time for implementing this function is $\alpha S T_s$. Since the radar receiver processes the auto-correlation function after the round-trip delay, the total radar processing time is

$$t_r = \tau_0 + \alpha S T_s \tag{20}$$

E. Problem Formulation 1

We aim to determine the UAV power, the transmit power of the devices for uploading their local models, the fraction α, the CPU cycle frequency of each device, the number of quantization bit at each device to minimize the average quantization error and CRB over T global iterations. Here, T should be fixed, e.g., 250. The optimization problem is mathematically formulated as follows

$$\min_{P,p_i,\alpha,f_i,B_i} \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{i\in\mathcal{N}} J_i^2(t) + \mathtt{CRB}_v(t)\right), \tag{21a}$$
$$\text{s.t. } P \le P_U^{\max}, \tag{21b}$$
$$p_i \le p_i^{\max}, \forall i \in \mathcal{N}, \tag{21c}$$
$$f_i \le f_i^{\max}, \forall i \in \mathcal{N}, \tag{21d}$$
$$\tau_i^g + \tau_i^c + \tau_i^l \le \tau^g, \forall i \in \mathcal{N}, \tag{21e}$$
$$\tau_0 + \alpha S T_s \le \min_{i\in\mathcal{N}}\left\{\tau_i^g + \tau_i^c\right\}, \tag{21f}$$

where $P_U^{\max}$ is the maximum transmit power of the UAV, $p_i^{\max}$ is the maximum transmit power of device i, $f_i^{\max}$ is the maximum CPU cycle frequency of device i, and τ^g is the required global iteration time. The constraint in (23f) implies that the total radar processing time should be less than the training time of the device that finishes the local training first.

F. Problem Formulation 2

We aim to determine the UAV power, the transmit power of the devices for uploading their local models, the fraction α, the CPU cycle frequency of each device, the number of quantization bit at each device to minimize the overall training time over I global iterations subject to the constraints of the quantization error and CRB. Here, I should be fixed, e.g., 250. We denote T_i as total training latency of device i over I global iterations. Then, we have

$$T_i = \sum_{t=1}^{I}\left(\tau_i^g(t) + \tau_i^c(t) + \tau_i^l(t)\right). \tag{22}$$

The optimization problem is mathematically formulated as follows

$$\min_{P,p_i,\alpha,f_i,B_i} \max_{i\in\mathcal{N}} T_i, \tag{23a}$$
$$\text{s.t. } P \le P_U^{\max}, \tag{23b}$$
$$p_i \le p_i^{\max}, \forall i \in \mathcal{N}, \tag{23c}$$
$$f_i \le f_i^{\max}, \forall i \in \mathcal{N}, \tag{23d}$$
$$J_i^2 \le J_0^2, \forall i \in \mathcal{N}, \tag{23e}$$
$$\tau_0 + \alpha S T_s \le \min_{i\in\mathcal{N}}\left\{\tau_i^g + \tau_i^c\right\}, \tag{23f}$$
$$\mathtt{CRB}_v \le \mathtt{CRB}_0, \tag{23g}$$
$$\tag{23h}$$

where $P_U^{\max}$ is the maximum transmit power of the UAV, $p_i^{\max}$ is the maximum transmit power of device i, $f_i^{\max}$ is the maximum CPU cycle frequency of device i, and τ^g is the required global iteration time. The constraint in (23f) implies that the total radar processing time should be less than the training time of the device that finishes the local training first. The constraint in (23g) is the CRB requirement for the velocity estimation of the BS.

III. Alternating Descent Algorithm

IV. Simulation Results

We denote $h_{u,i}$ and $h_{i,u}$ as the channel gains between the device i and the UAV u, respectively, which are determined as follows.

$h_{i,u}$ is composed of the line-of-sight (LoS) component, denoted by $h_{i,u}^{\text{LoS}}(t)$, and the non-LoS component, denoted by $h_{i,u}^{\text{NLoS}}(t)$. Note that the NLoS component exists due to the shadowing effect and reflection of signals from the obstacles. In particular, the components $h_{i,u}^{\text{LoS}}(t)$ and $h_{i,u}^{\text{NLoS}}(t)$ can be determined as follows:

$$h_{i,u}^{\text{LoS}}(t) =\Big(\frac{4\pi f}{v}\Big)^{-2}\mu_{\text{LoS}} d_{g,u}^{-\beta_{\text{LoS}}}(t)|h_{g,u}^{\text{Rice}}(t)|^2 \tag{24}$$

$$h_{i,u}^{\text{NLoS}}(t) =\Big(\frac{4\pi f}{v}\Big)^{-2}\mu_{\text{NLoS}} d_{g,u}^{-\beta_{\text{NLoS}}}(t)|h_{i,u}^{\text{Rayleigh}}(t)|^2, \tag{25}$$

where f is the carrier frequency, v is the speed of light, β_{LoS} and β_{NLoS} are the path loss exponents of the LoS and NLoS components, respectively, μ_{LoS} and μ_{NLoS} are the attenuation factors of the LoS and NLoS components, respectively, $h_{g,u}^{\text{Rice}}(t) \sim \text{Rice}(\nu,\delta)$ and $h_{g,u}^{\text{Rayleigh}}(t) \sim \text{Rayleigh}(0,\delta)$ are the small-scale fading coefficients of the LoS and NLoS components, respectively, and $d_{i,u}$ is the distance between the device i and the UAV. Here, $d_{i,u}(t) = \sqrt{(x_u(t)-x_i(t))^2+(y_u(t)-y_i(t))^2+l_u(t)^2}$, where $(x_u(t),y_u(t),l_u(t))$ and $(x_i(t),y_i(t),0)$ are the coordinates of the UAV and the device i, respectively. Then, the channel between the device i and the UAV is determined by

$$h_{i,u}(t) = \omega_{i,u}^{\text{LoS}}(t)h_{i,u}^{\text{LoS}}(t)+\omega_{i,u}^{\text{NLoS}}(t)h_{i,u}^{\text{NLoS}}(t), \tag{26}$$

where $\omega_{i,u}^{\text{LoS}}(t)$ and $\omega_{i,u}^{\text{NLoS}}(t)$ are the weights of the LoS and NLoS components, respectively. In particular, $\omega_{i,u}^{\text{NLoS}}(t) = 1-\omega_{i,u}^{\text{LoS}}(t)$, where $\omega_{i,u}^{\text{LoS}}(t)$ is determined based on the elevation angle $\rho_{i,u}(t)$ (see Fig. **??**) as follows:

$$\omega^{\text{LoS}} = \frac{a-b}{1+\big(\frac{\rho_{i,u}(t)-c}{d}\big)^e}, \tag{27}$$

where $a,b,c,d,$ and e are the constants that are the empirical parameters related to the environments that the UAVs are working. For example, in dense urban environments, the typycal values of a,b,c,d,e are $187.3, 0, 0, 82.10, 1.478$, respectively. $\rho_{g,u}(t) = \arcsin\big(\frac{l_u(t)}{d_{g,u}(t)}\big)$ is the elevation angle in time slot t as shown in Fig. 1. In order for the weights to have their values in the interval of [0, 1], the value of ω^{LoS} is normalized as follows:

$$\omega_{i,u}^{\text{LoS}}(t) = \frac{\omega^{\text{LoS}}-\omega_{\min}^{\text{LoS}}}{\omega_{\max}^{\text{LoS}}-\omega_{\min}^{\text{LoS}}}, \tag{28}$$

where $\omega_{\max}^{\text{LoS}} = a-b$ and $\omega_{\min}^{\text{LoS}} = \frac{a-b}{1+\frac{\pi}{2}}$ are the maximum and minimum values of ω^{LoS}, respectively.

V. Conclusions

Appendix A: Fundamental inequalities

The function $f(u,v) = \log_2(1+\frac{1}{uv})$ is convex on $\mathbb{R}_+^2$, we have [**?**]

$$\begin{aligned}\log_2(1+\frac{1}{uv}) &\geq f(\bar{u},\bar{v}) + \langle\nabla f(\bar{u},\bar{v}),(u,v)-(\bar{u},\bar{v})\rangle \\ &\geq \log_2(1+1/\bar{u}\bar{v}) + \frac{1/\bar{u}\bar{v}}{(1+1/\bar{u}\bar{v})\ln 2}(2-\frac{u}{\bar{u}}-\frac{v}{\bar{v}}).\end{aligned} \tag{A.1}$$

By replacing $u \to 1/u$ and $\bar{u} \to 1/\bar{u}$, we have

$$\log_2(1+\frac{u}{v}) \geq \log_2(1+\bar{u}/\bar{v}) + \frac{\bar{u}/\bar{v}}{(1+\bar{u}/\bar{v})\ln 2}(2-\frac{\bar{u}}{u}-\frac{v}{\bar{v}}). \tag{A.2}$$

Moreover, the function $f(u,v,z) = \frac{1}{z}\log_2(1+\frac{1}{uv})$ is convex on $\mathbb{R}_+^3$ [**?**]. Therefore

$$\begin{aligned}\frac{1}{z}\log_2(1+\frac{1}{uv}) \geq& f(\bar{u},\bar{v},\bar{z}) + \langle\nabla f(\bar{u},\bar{v},\bar{z}),(u,v,z)-(\bar{u},\bar{v},\bar{z})\rangle \\ \geq& \frac{2}{\bar{z}}\log_2(1+1/\bar{u}\bar{v}) + \frac{1/\bar{u}\bar{v}}{\bar{z}(1+1/\bar{u}\bar{v})\ln 2}(2 \\ &-\frac{u}{\bar{u}}-\frac{v}{\bar{v}}) - \frac{\log_2(1+1/\bar{u}\bar{v})}{\bar{z}^2}z.\end{aligned} \tag{A.3}$$

By replacing $u \to 1/u$ and $\bar{u} \to 1/\bar{u}$, we have

$$\begin{aligned}\frac{1}{z}\log_2(1+\frac{u}{v}) \geq& \frac{2}{\bar{z}}\log_2(1+\bar{u}/\bar{v}) + \frac{\bar{u}/\bar{v}}{\bar{z}(1+\bar{u}/\bar{v})\ln 2}(2 \\ &-\frac{\bar{u}}{u}-\frac{v}{\bar{v}}) - \frac{\log_2(1+\bar{u}/\bar{v})}{\bar{z}^2}z.\end{aligned} \tag{A.4}$$

References

[1] P. Kumari, S. A. Vorobyov, and R. W. Heath, "Adaptive virtual waveform design for millimeter-wave joint communication–radar," *IEEE Transactions on Signal Processing*, vol. 68, pp. 715–730, 2019.