Berezinskii-Kosterlitz-Thouless transition and superconductor-to-insulator transition

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A U.S. Department of Energy laboratory managed by The University of Chicago

Symposium on *Spin Physics and Nanomagnetism*
Chudnovsky-Fest, March 13-14, 2009

Dear Eugene:

Warmest congratulations and love!

Observable signature of the Berezinskii Kosterlitz Thouless transition **in a planar lattice of Bose Einstein condensates**
Trombettoni, A.; Smerzi, A.; Sodano, P.
New Journal of Physics, Volume 7, Issue 1, pp. 57 (2005).

PHYSICAL REVIEW B **78**, 184520 (2008)

Vortex-dynamics approach to the Nernst effect in extreme type-II superconductors dominated by phase fluctuations

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[1]*Department of Physics, Stanford University, Stanford, California 94305, USA*
[2]*Department of Physics, University of California, Berkeley, California 94720, USA*
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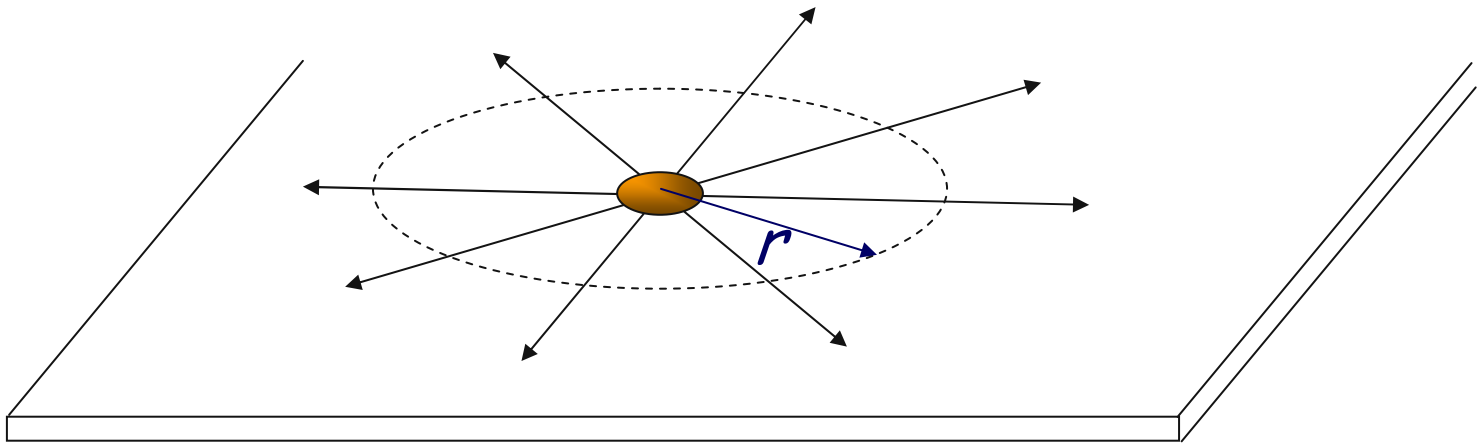
(Received 20 October 2008; published 26 November 2008)

We present a method to study the Nernst effect and diamagnetism of an extreme type-II superconductor dominated by phase fluctuations. We work directly with vortex variables and our method allows us to tune vortex parameters (e.g., core energy and number of vortex species). We find that diamagnetic response and transverse thermoelectric conductivity (α_{xy}) persist well above the Kosterlitz-Thouless transition temperature, and become more pronounced as the vortex core energy is increased. However, they *weaken* as the number of internal vortex states is increased. We find that α_{xy} closely tracks the magnetization ($-M/T$) over a wide range of parameters.

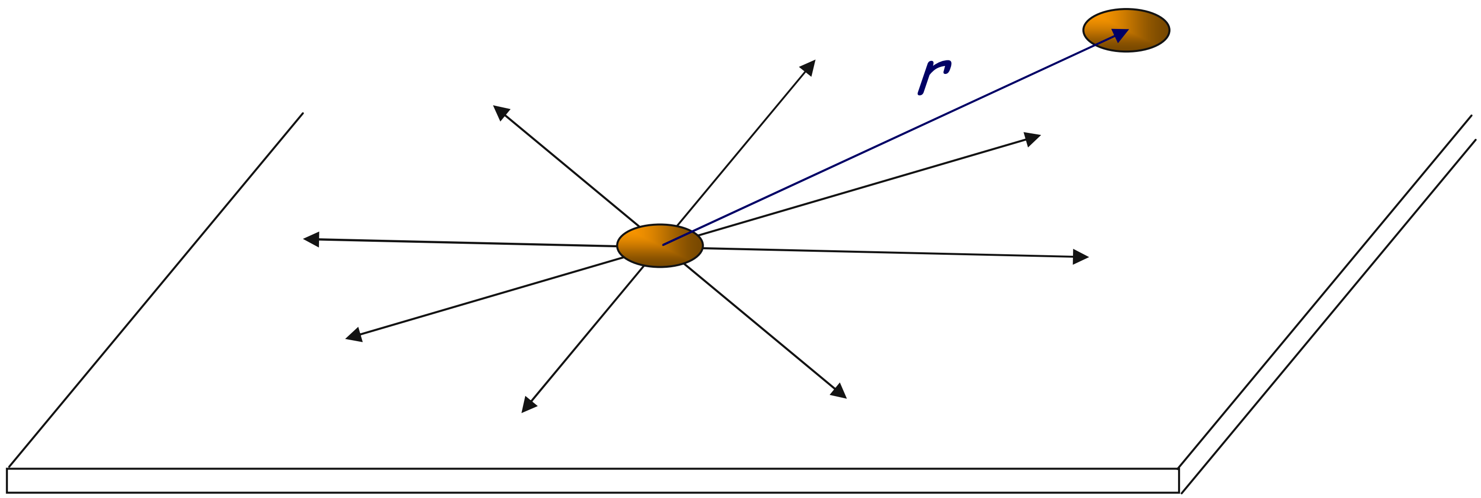
Two-dimensional Coulomb law I



Gauss law: $$2\pi r E = 4\pi e$$

$$E = \frac{2e}{r}$$

Two-dimensional Coulomb law II



Energy of the two charges interaction is thus:

$$U = e^2 \ln(r / r_0)$$

Two-dimensional electrolyt

J. Chem. Phys. **38**, 2587 (1

Equation of State for a Two-Dimensional Electrolyte*

ALLAN M. SALZBERG AND STEPHEN PRAGER
Department of Chemistry, University of Minnesota, Minneapolis 14, Minnesota

(Received 1 February 1963)

In view of recent interest in the equilibrium statistical mechanics of a one-dimensional system of point charges, it seems worth pointing out that, at least so far as the equation of state is concerned, the analogous two-dimensional system is extremely simple to treat.

$$U_{ij} = -q_i q_j \ln r_{ij}$$

$$Q = \left(\frac{1}{\prod N^{(l)}!}\right) \int_{\mathfrak{A}} \prod_{i<j} r_{ij}{}^{q_i q_j/kT} d\mathbf{x}$$

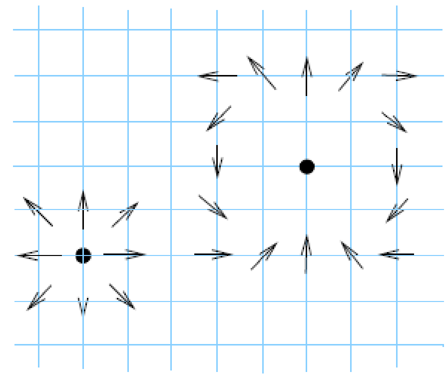
$$Q(\mathfrak{A}, T, N^{(l)}) = \mathfrak{A}^{[N+\frac{1}{2}\Sigma_{i<j} q_i q_j/kT]} Q^*(T, N^{(l)})$$

$$p = \left(\frac{NkT}{\mathfrak{A}}\right)\left[1 - (4kT)^{-1} \sum_l n^{(l)} q^{(l)2}\right]$$

Possibly what happens at this point may be interpreted as ion pair formation;

Berezinskii (1971): XY-magnetic model

V. Berezinskii, Zh. Eksp. Teor. Fiz. 59 (1970) 907 [Sov. Phys., JETP 32 (1971) 493].



$$E = -\sum_{\langle i,j \rangle} \vec{s}_i \vec{s}_j$$

Figure 1: Examples of charge-1 (left) and charge-2 (right) vortices in the XY model on a lattice. Spins rotate through 2π and 4π, respectively, as contours around the two bold points are traversed.

There, long-range correlations between spins at sites i and j (separated by a distance r, say) exist and are described by the correlation function whose leading behaviour in the thermodynamic infinite-volume limit is

$$G_\infty(r) \sim r^{-\eta(\beta)} \,. \qquad (T < T_c \text{ or } \beta > \beta_c)$$

Above this point $\quad G_\infty(r) \sim e^{-r/\xi_\infty(t)}$

Kosterlitz and Thouless

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

Ordering, metastability and phase transitions in two-dimensional systems

J M Kosterlitz and D J Thouless
Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT, UK

The energy of a single dislocation in a two dimensional system with lattice spacing a can be found from the theory of edge dislocations (Friedel 1964), and it is given by

$$E = \left(\frac{na^2(1+\tau)}{4\pi}\right)\ln\left(\frac{A}{A_0}\right) \tag{1}$$

Here n and τ are the two dimensional rigidity modulus and Poisson's ratio, A is the area of the system, and A_0 is an area of the order of a^2. The entropy of a dislocation is

$$S = k_B \ln(A/a^2) \tag{2}$$

At temperatures which satisfy the inequality

$$k_B T < k_B T_c = na^2(1+\tau)/4\pi \tag{3}$$

the logarithmically large energy dominates, and no isolated dislocation can be formed, so the system is rigid, but once this inequality is violated there are free dislocations in the equilibrium state, and viscous flow can occur.

BKT: superconducting films

B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. **36**, 599 (1979); S. Doniach and B. A. Huberman, Phys. Rev. Lett. **42**, 1169 (1979).

2D vortices ("pancakes") in a superconducting film: Logarithmic interaction:

KT transition in superconducting films: vortex binding-unbinding

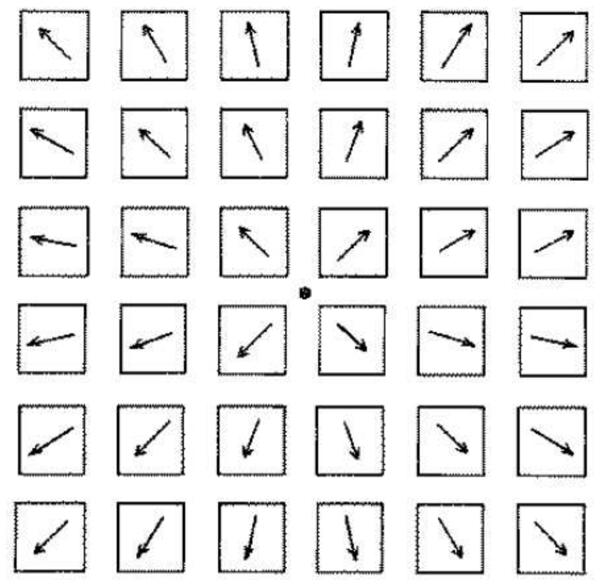
Vortices in JJA: BKT

$$E_v = \pi E_J \ln\left(\frac{L}{a}\right)$$

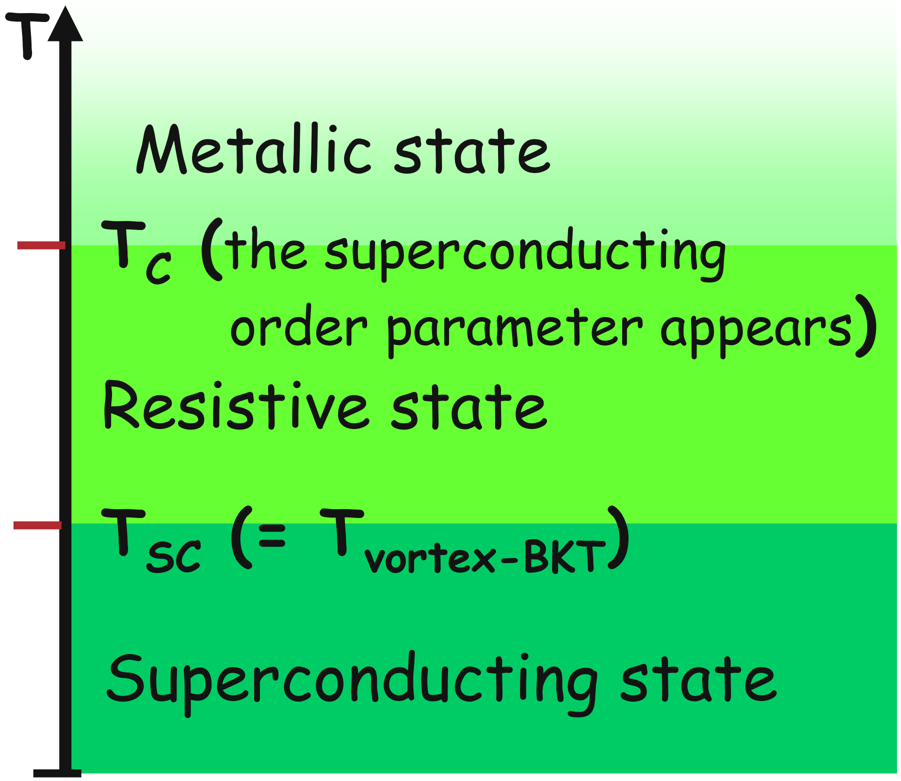
where L is the size of the system and a is the lattice spacing

$$T_{KT} = \frac{\pi E_J}{2k_B}$$

Phase configuration for a vortex.

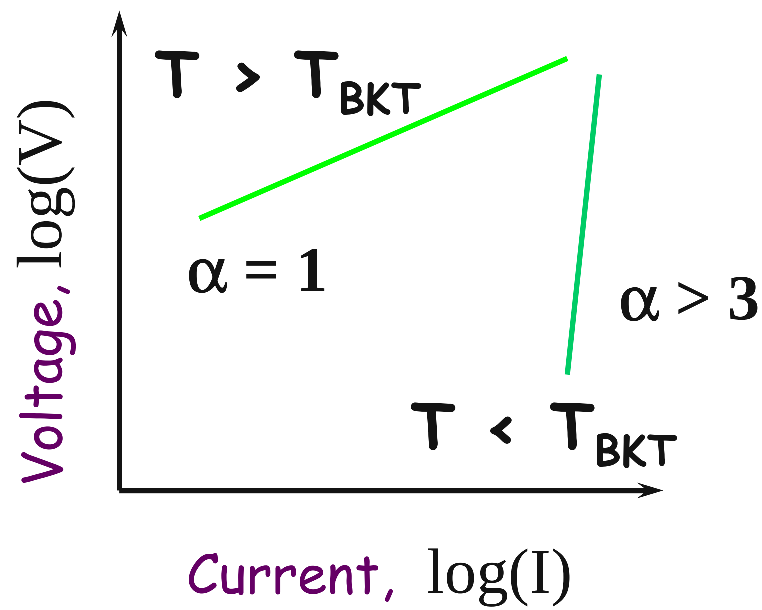


2D superconducting systems: experimental observation of BKT



in experiment:

$$V \propto I^{\alpha(T)}$$



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Jump in $\alpha(T)$ at T_{BKT}

Disordered superconducting films

PHYSICAL REVIEW B VOLUME 1, NUMBER 3 1 FEBRUARY 1970

Destruction of Superconductivity in Disordered Near-Monolayer Films*

MYRON STRONGIN, R. S. THOMPSON, O. F. KAMMERER AND J. E. CROW

T_c versus $R_\square$

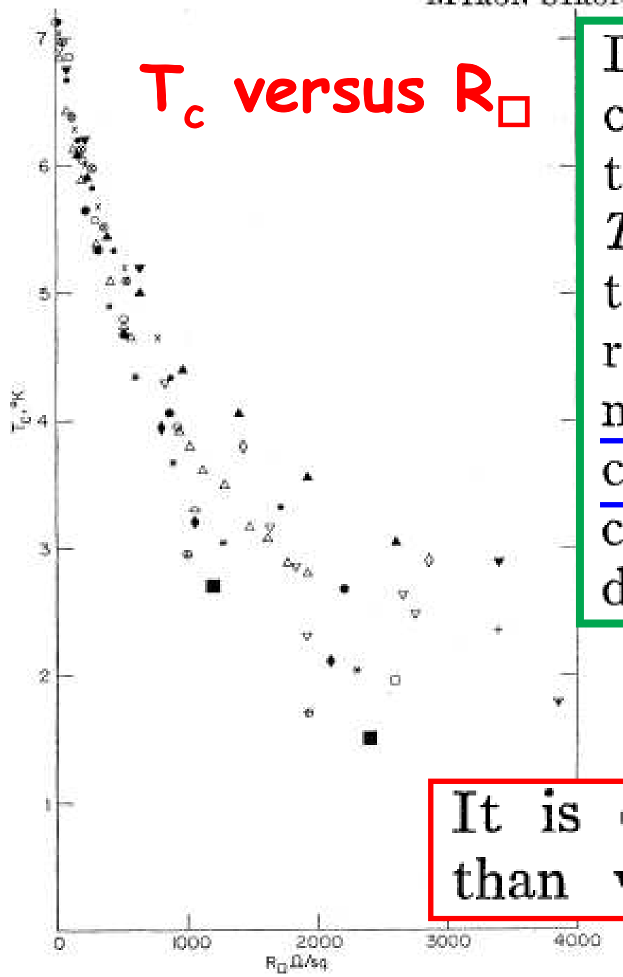


FIG. 5. T_c versus $R_\square$ where $R_\square$ is the resistance/sq area: ⊕, ■—Pb on Ge; +—Pb on Al_2O_3; ♦, ◇—Pb on Ge (deposited at room temperature); ∗—Bi on SiO; ∇, ⊗, ○, •, ●, △, ▲, ▼—Pb on SiO.

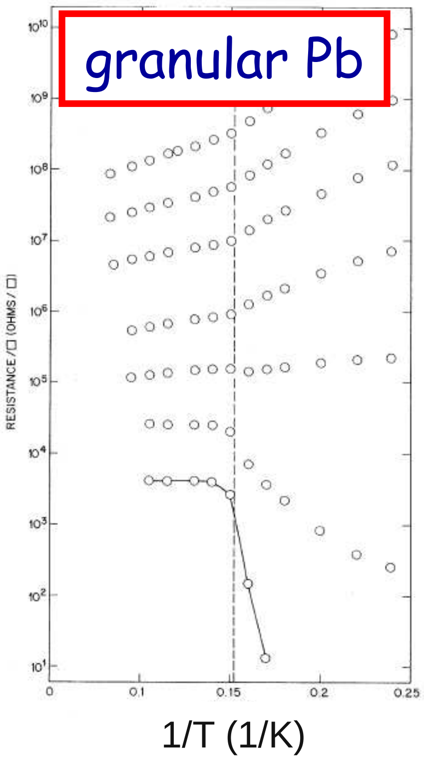
In the regime where the films first become electrically continuous, the resistance in the normal state is extremely high (greater than about 10 000 Ω/sq), and T_c is drastically reduced and is usually below the lowest temperature achievable with our cryostat. Also, in this regime where the films first become continuous, the normal-state resistance usually increases with decreasing T according to $R_0e^{+\langle E\rangle/kT}$. This, of course, is in contrast to the usual metallic behavior where R decreases with decreasing T.

activated behavior of resistance

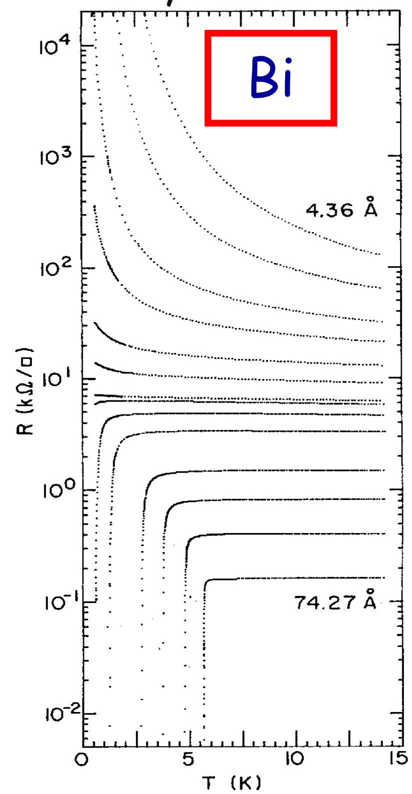
It is clear that T_c correlates much better with $R_\square$ than with thickness or ρ.

Disordered 2D systems: superconductor-insulator transition

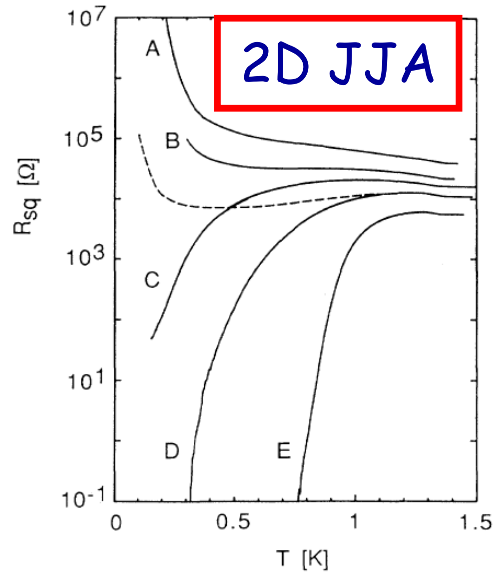
Altering various parameters of the system, such as tunnel resistance and transmittance in JJAs, conditions of deposition, chemical composition, and thickness of the films, one can drive the system directly from the superconducting to insulating state.



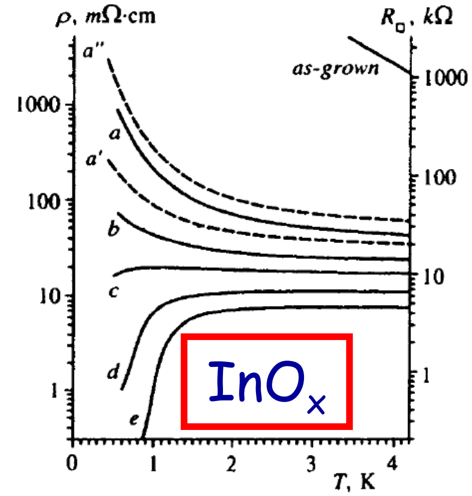
R.C. Dynes, J.P. Garno, and J.M. Rowell (1978)



D.B. Haviland, Y. Liu, and A.M. Goldman (1989)



L.J. Geerligs, M. Peters, L.E.M. de Groot, A. Verbruggen, and J.E. Mooij (1989).



V.F. Gantmakher, M.V. Golubkov, J.G.S. Lok, and A.K. Geim (1996)

Disordered 2D systems: superconductor-insulator transition

Altering various parameters of the system, such as tunnel resitance and transmittance in JJAs, conditions of deposition, chemical composition, and thickness of the films, one can drive the system directly from the superconducting to insulating state.

M. Strongin, R.S. Thompson, O. F. Kammerer, and J.E. Crow, Phys. Rev. B **1**, 1078 (1970).

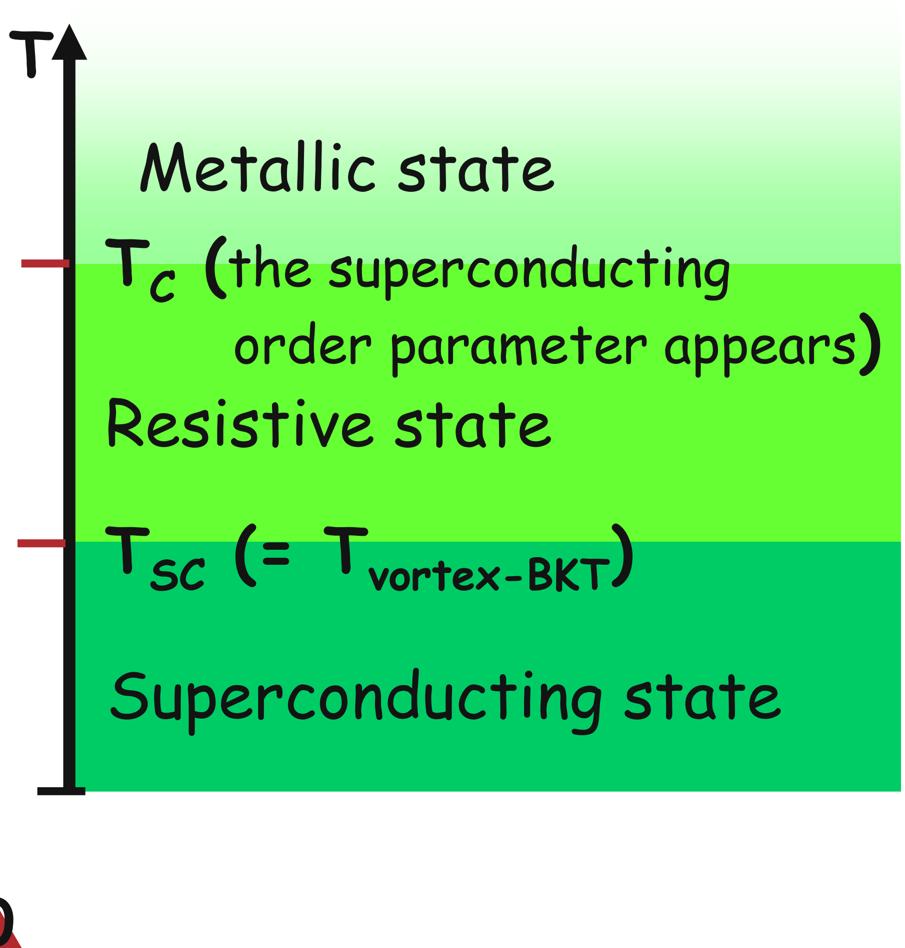
control parameter

Resistance

$R < R_c$	R_c	$R > R_c$
Superconductor	critical point	**Insulator**
Condensate of Cooper pairs		Localized Cooper pairs

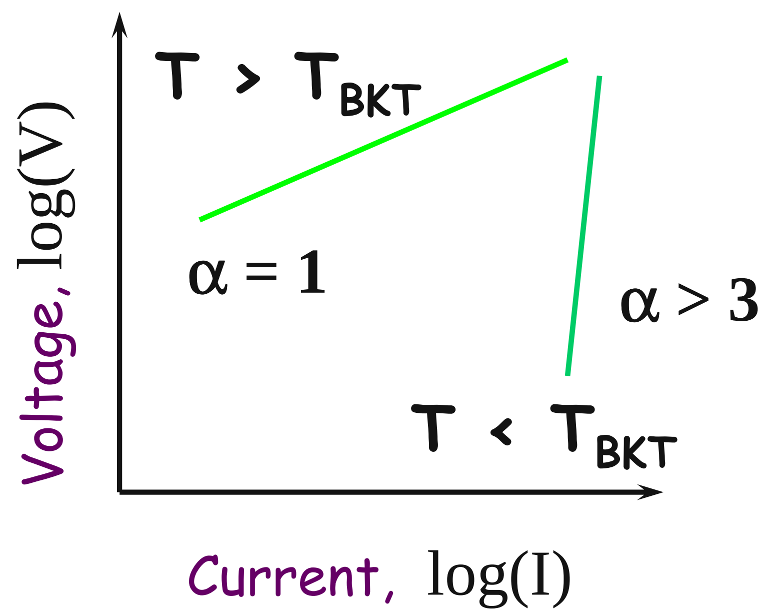
A. Gold, Z. Phys. B – Condensed Matter **52**, 1 (1983); Phys. Rev. A **33**, 652 (1986).
Matthew P.A. Fisher, G. Grinstein, S.M. Girvin , PRL **64**, 587 (1990).

2D superconducting systems

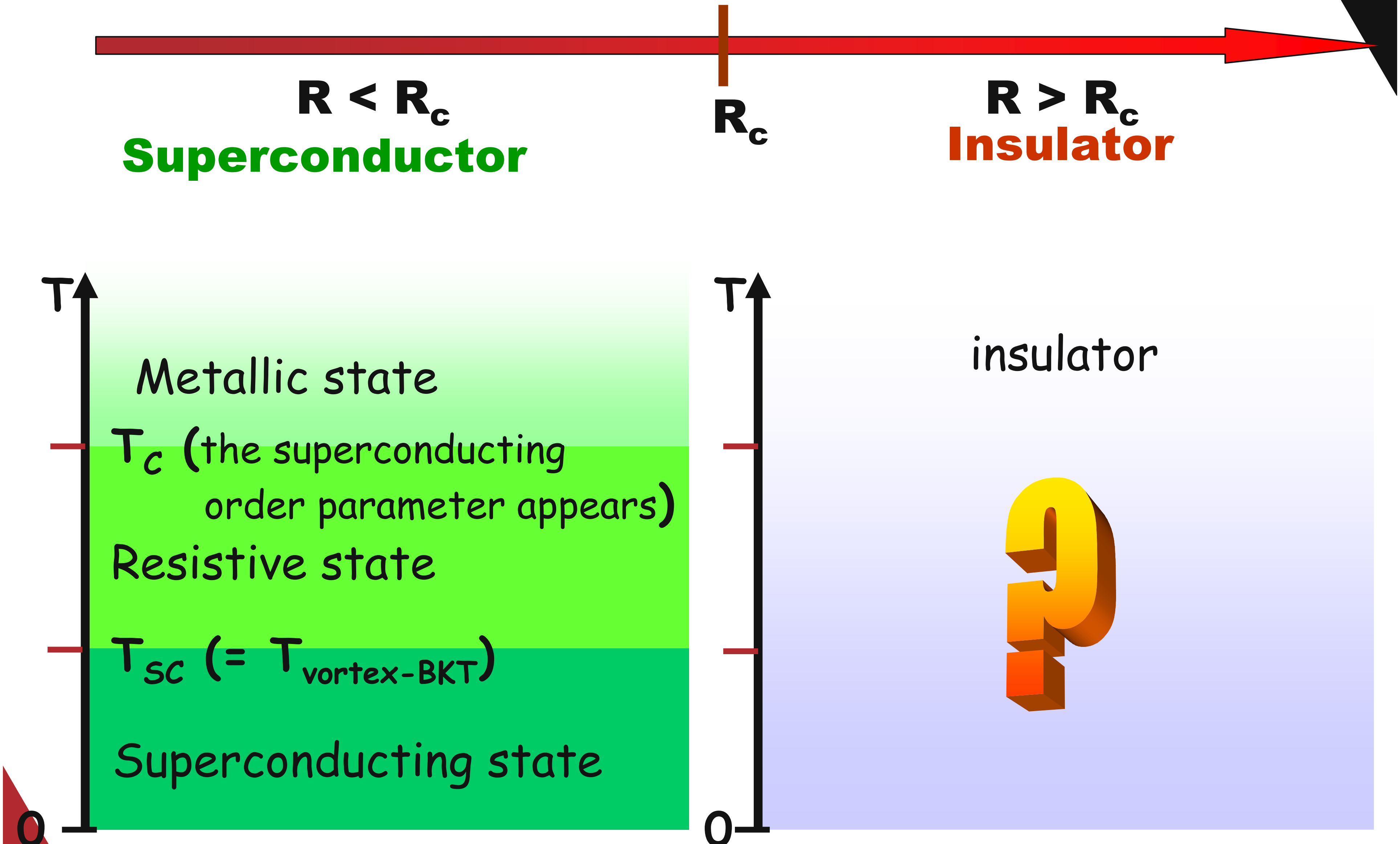


in experiment:

$$V \propto I^{\alpha(T)}$$



Disorder-driven superconductor-insulator transition



Superconductor-insulator and BKT transitions

PHYSICAL REVIEW B VOLUME 54, NUMBER 14 1 OCTOBER 1996-II

Quantum phase transitions in two dimensions: Experiments in Josephson-junction arrays

H. S. J. van der Zant, W. J. Elion, L. J. Geerligs, and J. E. Mooij
Department of Applied Physics and Delft Institute of Microelectronics and Submicron-technology (DIMES), Delft University of Technology Lorentzweg 1, 2628 CJ Delft, The Netherlands

(Received 21 May 1996)

We have studied two-dimensional superconducting networks coupled by Josephson junctions in the regime where the Josephson coupling energy is comparable in magnitude to the capacitive energy of charging an island with a Cooper pair. We have mapped out the dependence of quantum phase transitions on the ratio of these two energies (for different values of the applied magnetic field) and on the applied field for arrays both with square and triangular cells. Our experimental results are compared with existing theoretical predictions. [S0163-1829(96)07737-5]

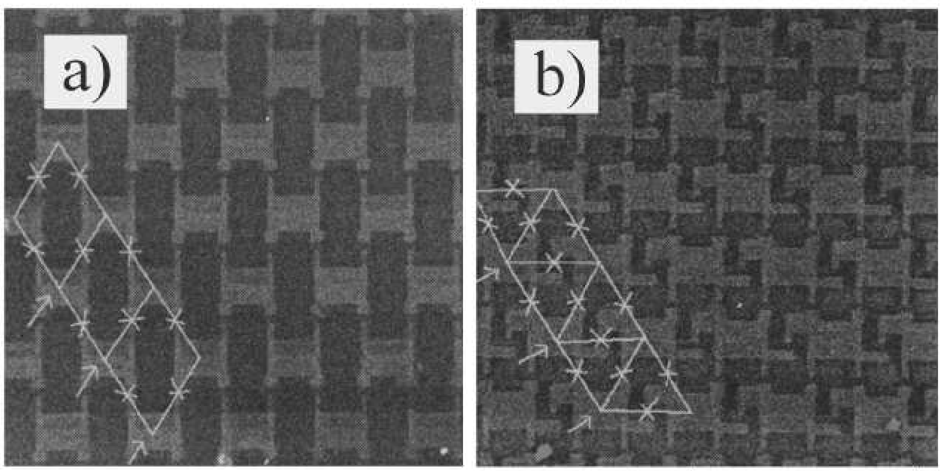


FIG. 2. Scanning-electron microscope photographs of a square (a) and a triangular (b) array. In the photographs, the schematic drawings of the arrays are also shown; crosses represent the junctions and arrows the way current is injected.

$$C_0 \approx 12 \times 10^{-18}\ \mathrm{F}$$

At low temperatures, the 2D flux penetration depth $\lambda_\perp(T) = \Phi_0/2\pi\mu_0 I_c(T)$ is much larger than the array sizes

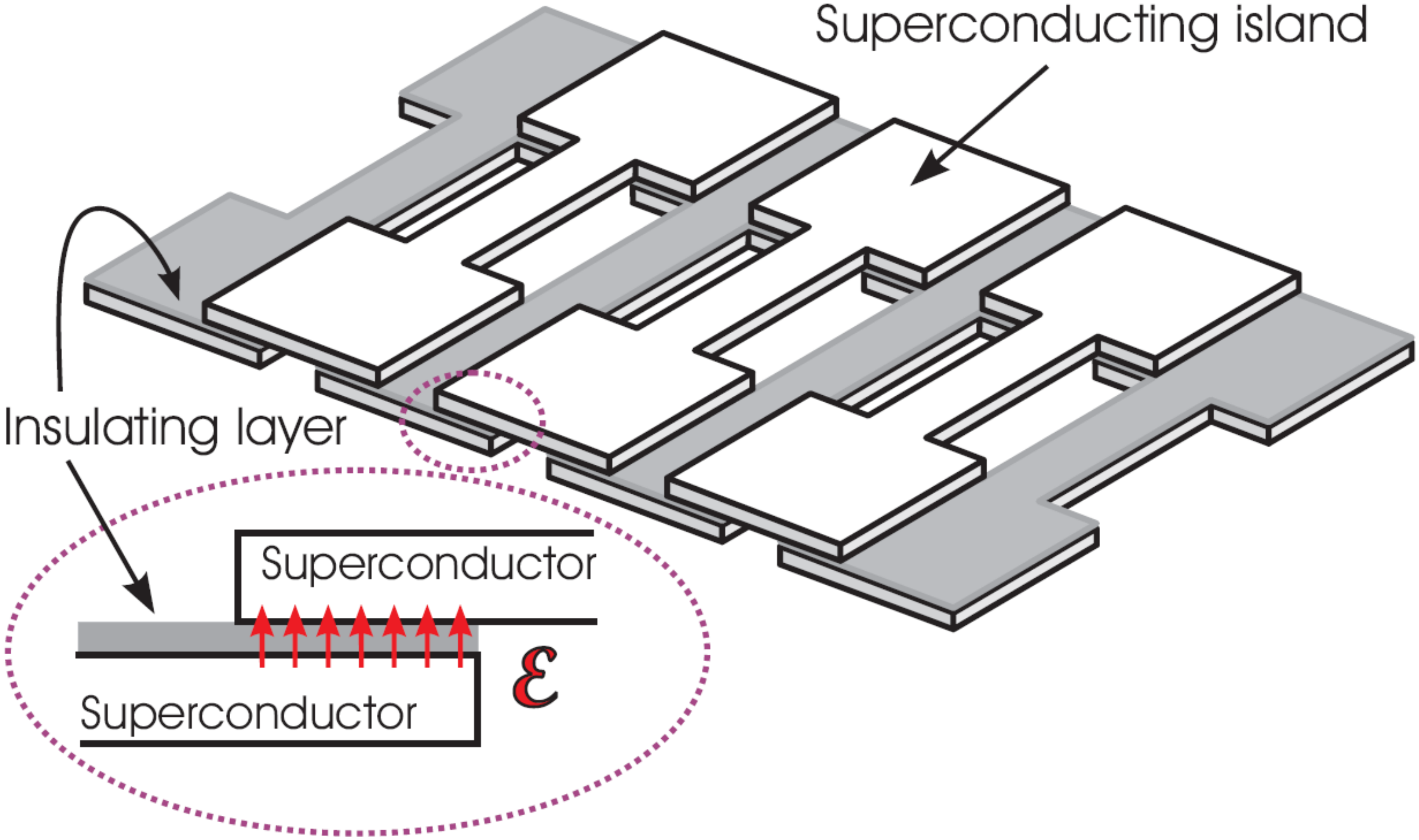
Superconductor-insulator and BKT transitions

IV. S–I TRANSITIONS AS A FUNCTION OF E_C/E_J

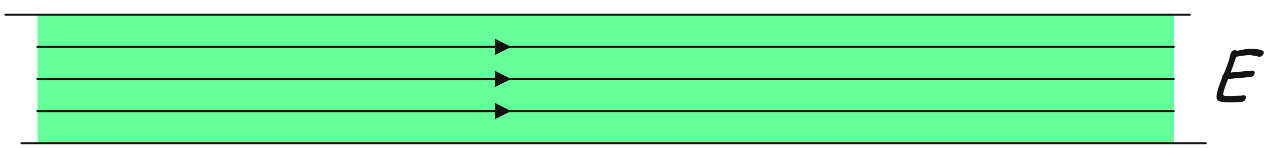
In zero magnetic field, classical arrays undergo a Kosterlitz-Thouless-Berenzinskii (KTB) phase transition[32] to the superconducting state. Below the KTB transition temperature $T_V \sim E_J/k_B$, arrays are superconducting because there are no free vortices. Only pairs of vortices and antivortices may be present. A necessary condition for a clear observation of a KTB phase transition is that vortices interact logarithmically over large distances. In arrays, vortices interact logarithmically over distances $\lambda_\perp$.

When $E_C \gg E_J$, a dual KTB transition for $2e$ charges is expected at a transition temperature $T_C \sim E_C/4k_B$.[14,33,34] When only C_0 is considered $2e$-charge pairs interact logarithmically over a normalized screening length of $\sqrt{C/C_0}$. In our arrays, $\sqrt{C/C_0} \approx 10$ so that the KTB transition will be smeared out. However, when the full capacitance matrix is considered logarithmic interactions persist over lengths of order C/C_0,[1,35] i.e., of the order of the array size. Therefore, one expects to observe the distinct features of a KTB transition for charges in our arrays.

Superconductor-insulator and BKT transitions



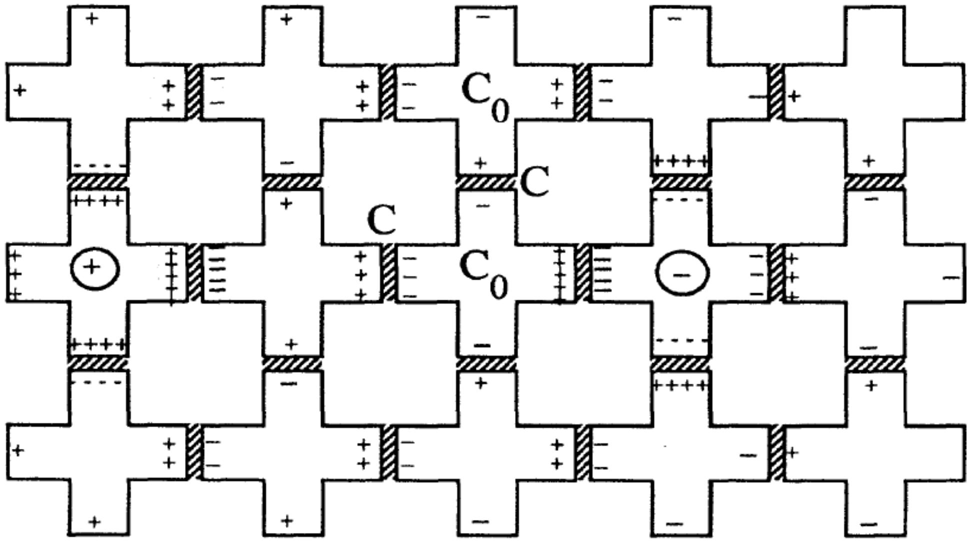
On a large spatial scale the 2D picture



PHYSICAL REVIEW B VOLUME 43, NUMBER 7 1 MARCH 1991

Charge and vortex dynamics in arrays of tunnel junctions

Rosario Fazio* and Gerd Schön



$E_J > E_C$
BKT transition:
Vortices – anti-vortices unbinding

$E_J < E_C$
Dual charge BKT transition:
charges – anti-charge unbinding

FIG. 1. A two-dimensional array of tunnel junction is shown. The capacitance is a nearest-neighbor capacitance located at the junction interfaces C and a capacitance to the ground C_0. Also shown is the polarization produced by discrete charges on two islands for $C_0 \ll C$.

This condition means that all the electric field is concentrated within the 2D array: logarithmic interaction of charges!

Disorder-driven Superconductor- Insulator transition in 2D-SC

$R < R_c$ **Superconductor**

R_c

$R > R_c$ **Insulator**

BKT transition

Vortice – antivortice binding-unbinding

Dual charge **BKT transition**

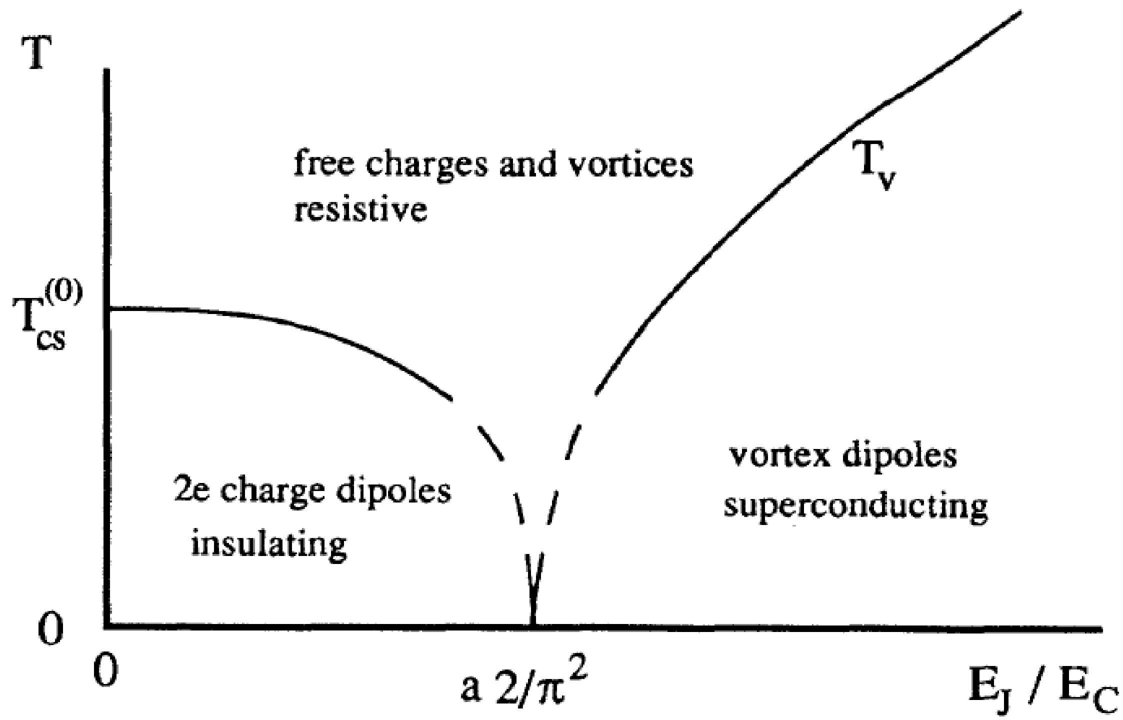
Charge – anticharge binding- unbinding

N. Yoshikawa, T. Akeyoshi, M. Kojima, and M. Sugahara, "Dual Conduction Characteristics Observed in Highly Resistive NbN Granular Thin Films", Proc. LT-18, Kyoto 1987, Jpn. J. Appl. Phys. **26** (Suppl. 26-3), 949 (1987).

A. Widom and S. Badjou, "Quantum displacement-charge transitions in two-dimensional granular superconductors", Phys. Rev. B **37**, 7915 (1988).

R. Fazio and G. Schon, "Charge and vortex dynamics in arrays of tunnel junctions", Phys. Rev. B **43**, 5307 (1991).

B.J. van Wees, "Duality between Cooper-pair and vortex dynamics in two-dimensional Josephson-junction arrays", Phys. Rev. B **44**, 7915 (1991).

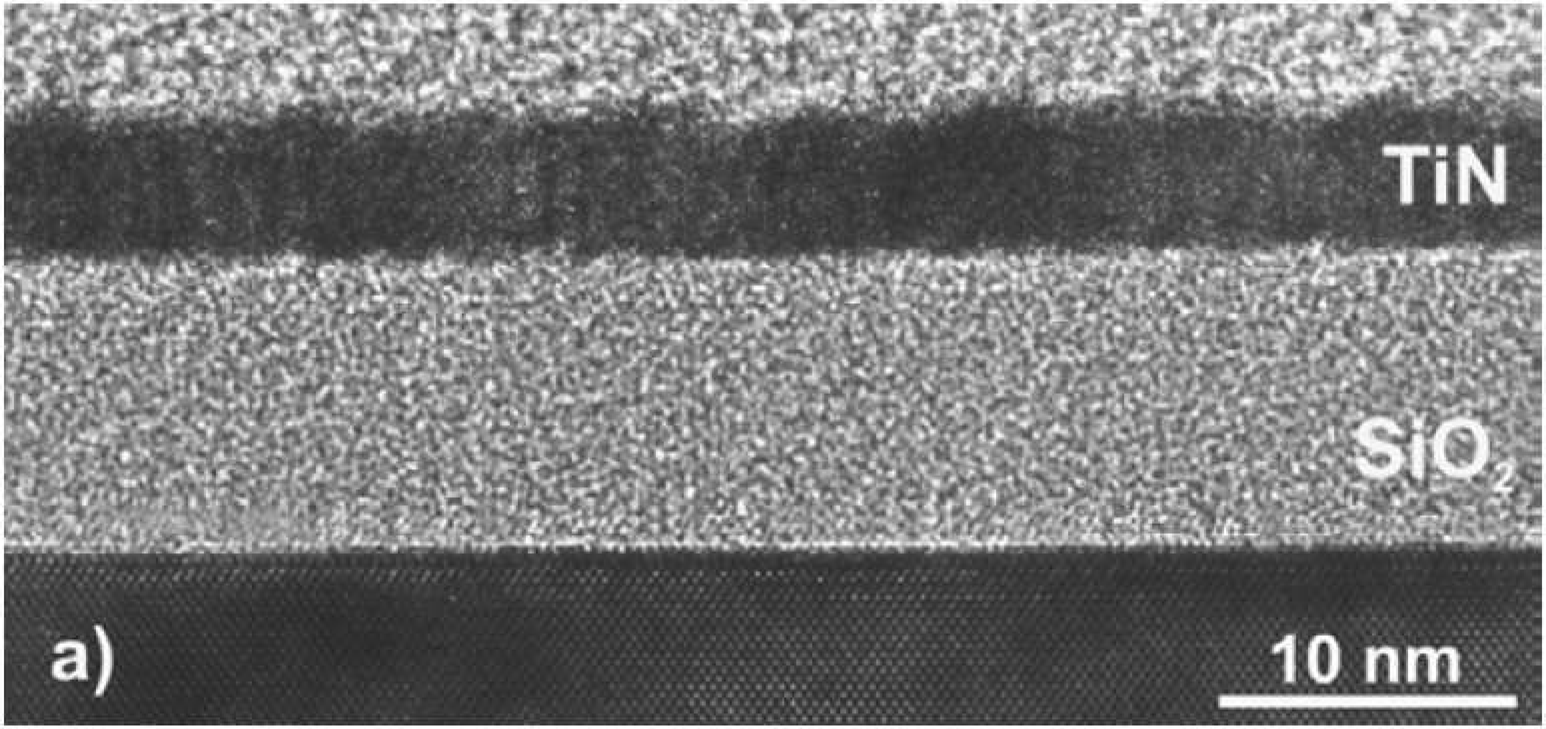


PHYSICAL REVIEW B VOLUME 43, NUMBER 7 1 MARCH 1991

Charge and vortex dynamics in arrays of tunnel junctions

Rosario Fazio* and Gerd Schön
Department of Applied Physics, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
(Received 7 August 1990)

Thin disordered SC films



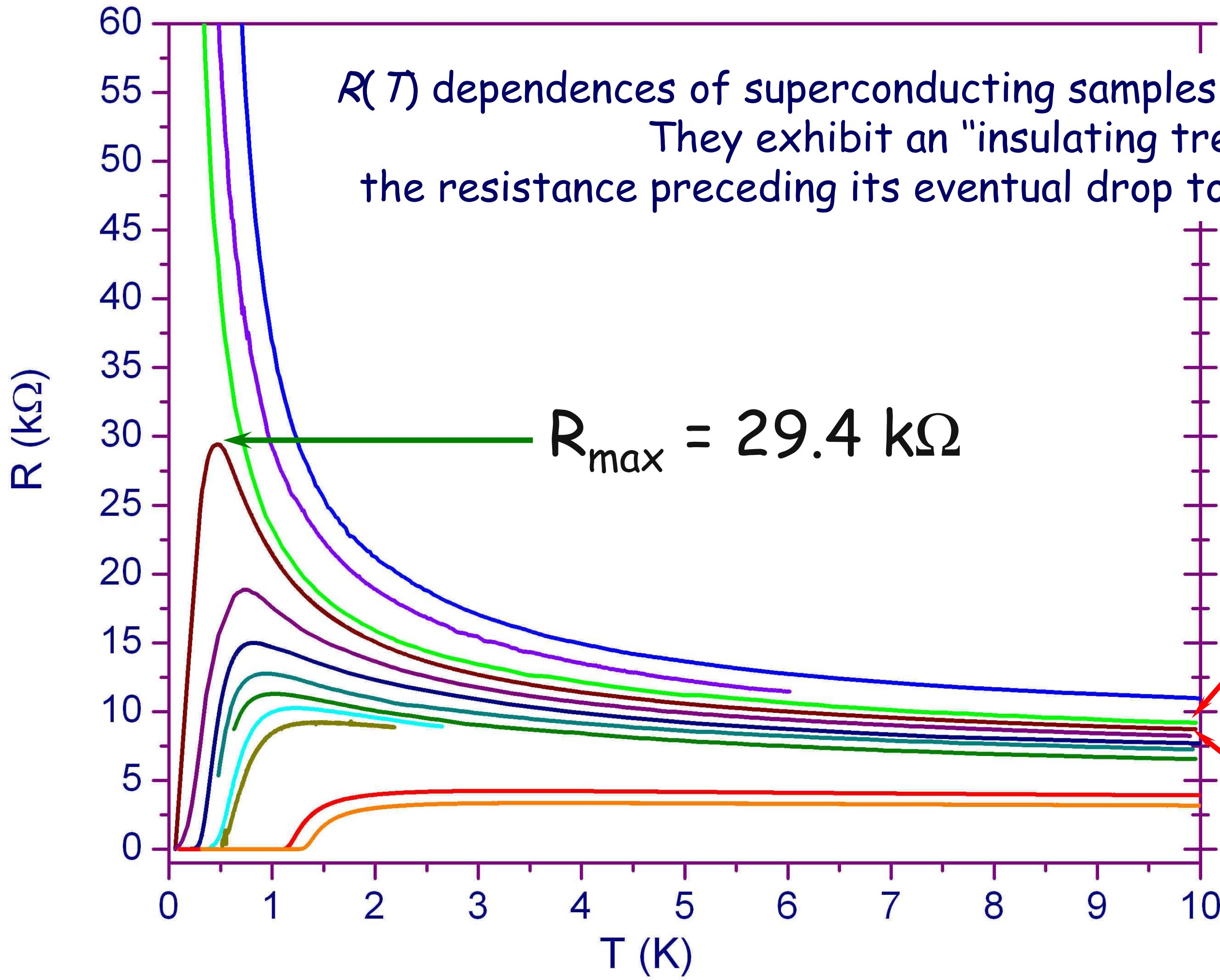
- TiN films were formed by atomic layer chemical vapor deposition onto a Si/SiO_2 substrate at 350 ^0C.
- Composition:

Ti	N	Cl
1	0.94	0.035

the thickness is 5 nm:

Films are 2D superconductors!

Disorder-driven SIT in TiN films (B=0)



R(*T*) dependences of superconducting samples are nonmonotonic. They exhibit an "insulating trend," i.e., an upward turn of the resistance preceding its eventual drop to zero at low temperatures.

T = 10 K

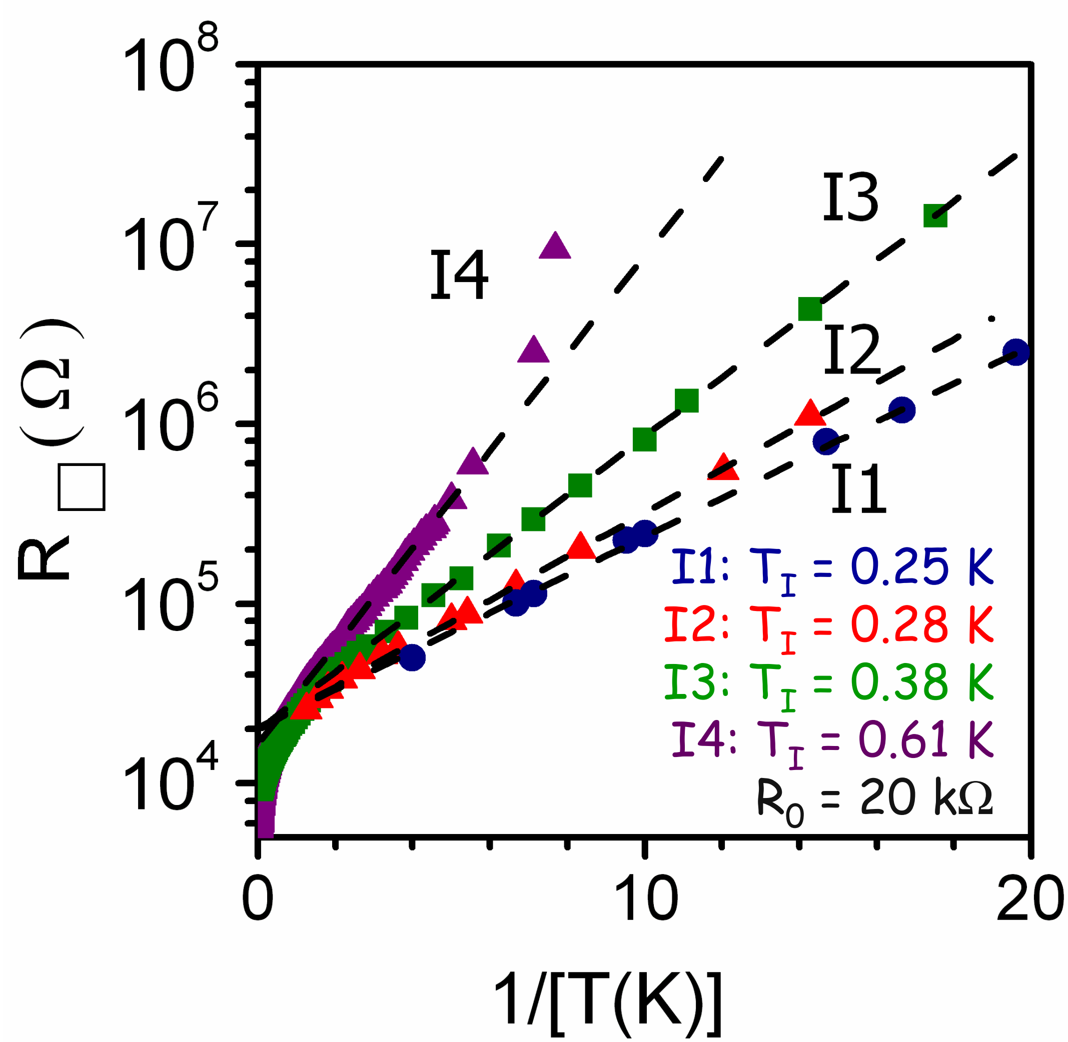
9.16 kΩ – I3

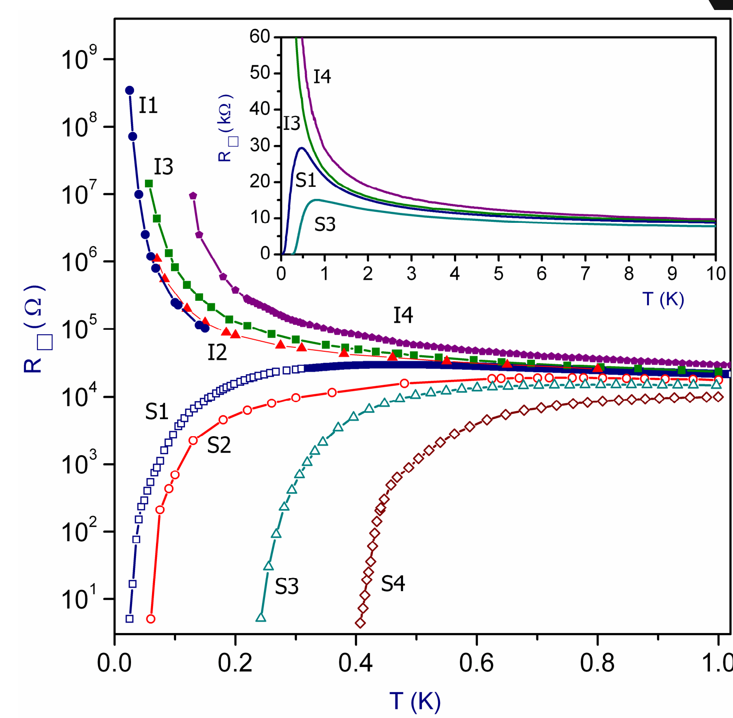
8.74 kΩ - S1

< 5% (!)

T. Baturina. T. B., C. Strunk, M.R. Baklanov, A. Satta, PRL 98, 127003 (2007)
T. Baturina, A.Yu. Mironov, V. Vinokur, M.R. Baklanov, C. Strunk, PRL 99, 257003 (2007)
T. Baturina, A. Bilušic, A.Yu. Mironov, V. Vinokur, M.R. Baklanov, C. Strunk, Physica C 468, 316 (2008)

Insulating side of the D-SIT in TiN films





$$R = R_0 \exp(T_I/T)$$

At low temperatures we observe an Arrhenius behavior of the resistance

T. Baturina, A.Yu. Mironov, V. Vinokur, M.R. Baklanov, C. Strunk, PRL 99, 257003 (2007)

T.B., A. Bilušic, A.Yu. Mironov, V. Vinokur, M.R. Baklanov, C. Strunk, Physica C 468, 316 (2008)

Activated resistance: equivalent capacitance

On the insulating side the activated resistance is determined by the relevant Coulomb energy

$$R \propto \exp(T_I / T) \quad k_B T_I = E_C = \frac{(2e)^2}{2C}$$

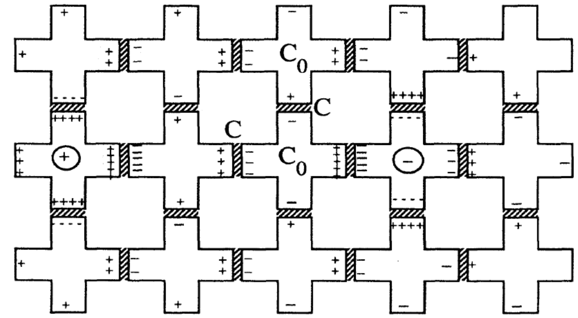
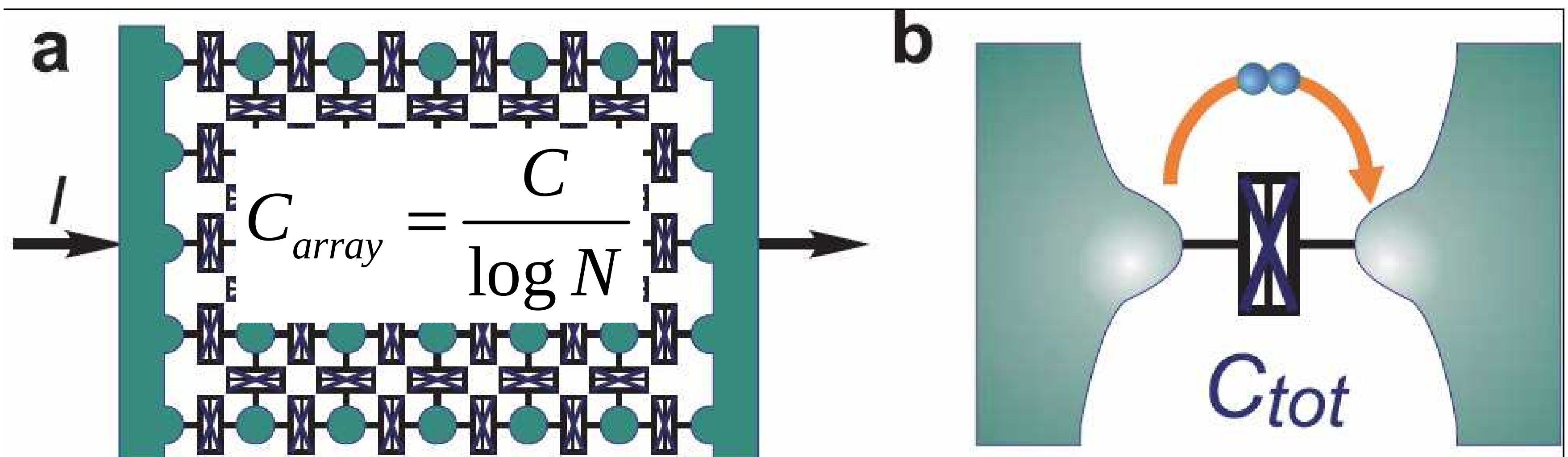
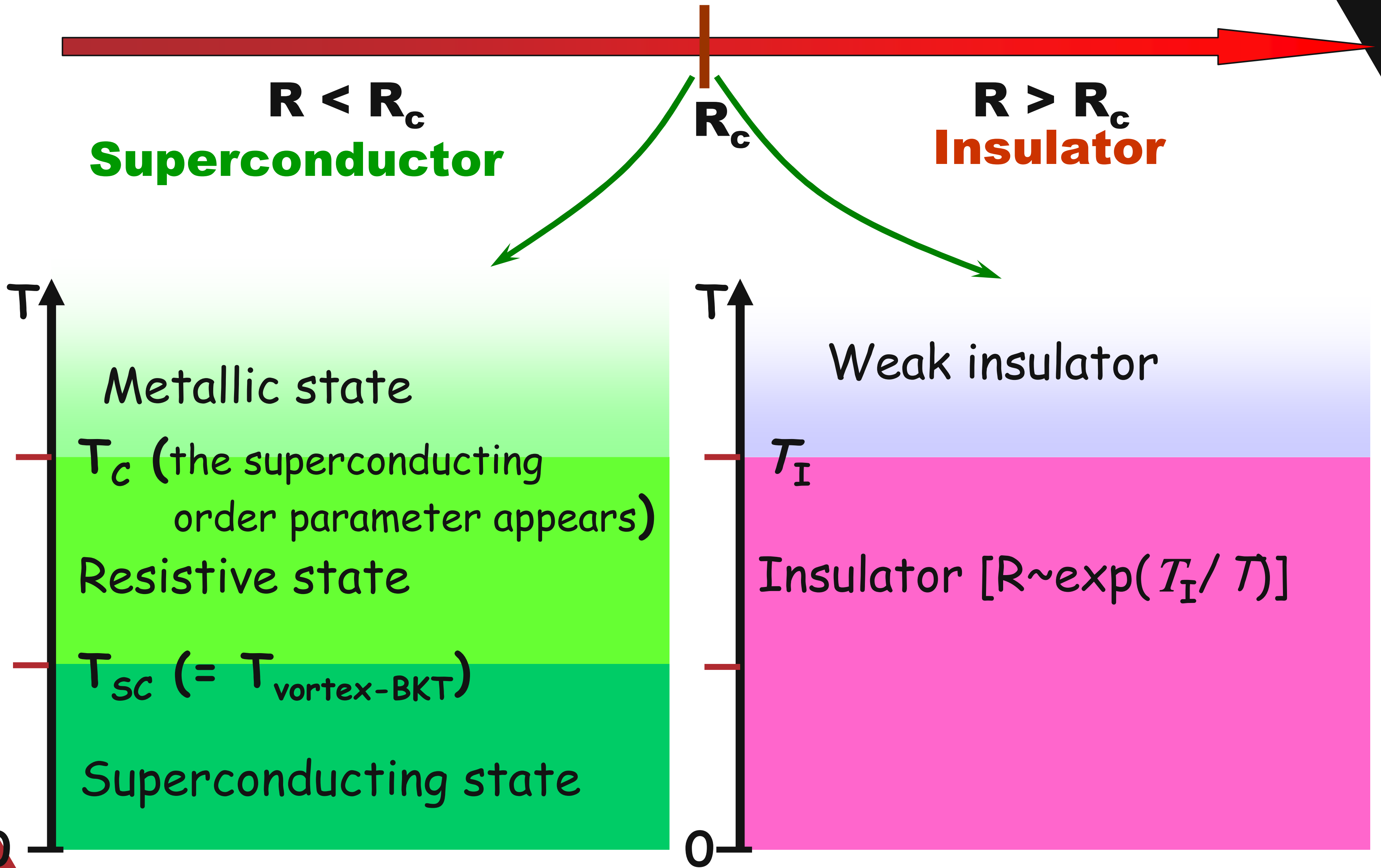


FIG. 1. A two-dimensional array of tunnel junction is shown. The capacitance is a nearest-neighbor capacitance located at the junction interfaces C and a capacitance to the ground C_0. Also shown is the polarization produced by discrete charges on two islands for $C_0 \ll C$.



$$k_B T_I \approx E_C \log N$$

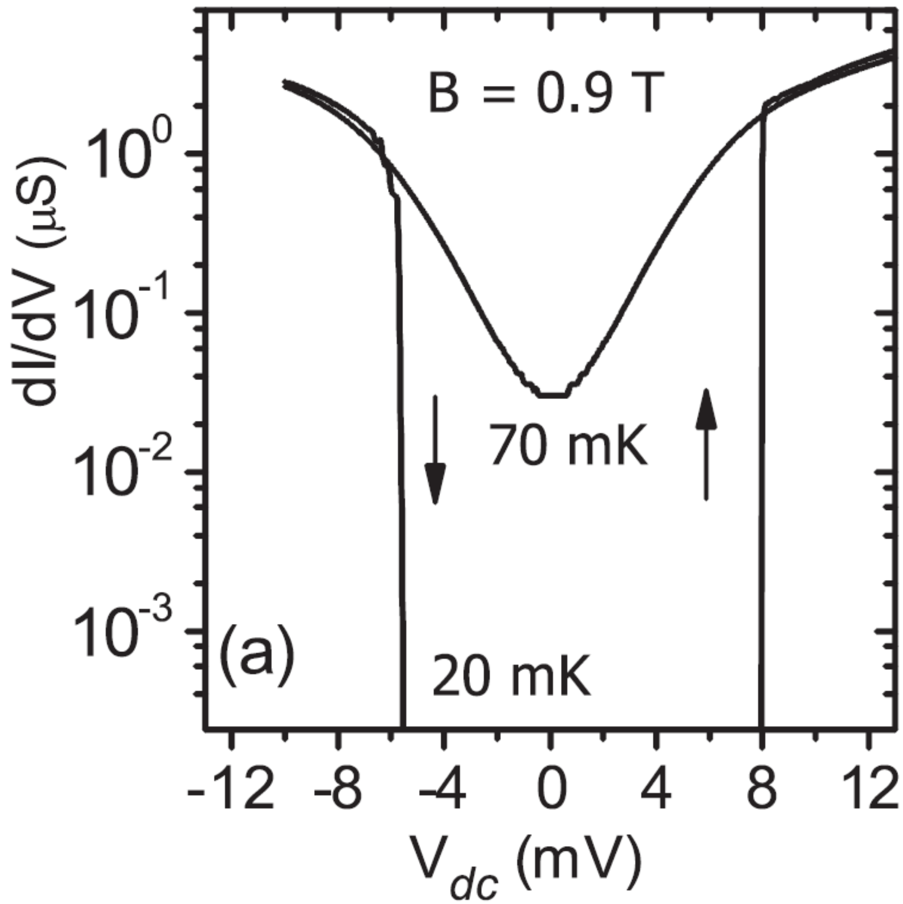
Disorder-driven SIT in and BKT in TiN films



PRL 99, 257003 (2007) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2007

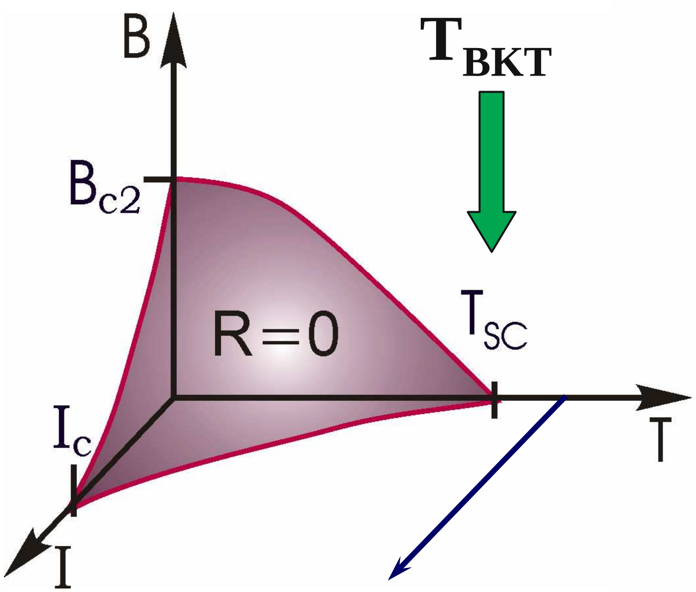
Localized Superconductivity in the Quantum-Critical Region of the Disorder-Driven Superconductor-Insulator Transition in TiN Thin Films

T. I. Baturina,[1,2] A. Yu. Mironov,[1,2] V. M. Vinokur,[3] M. R. Baklanov,[4] and C. Strunk[2]



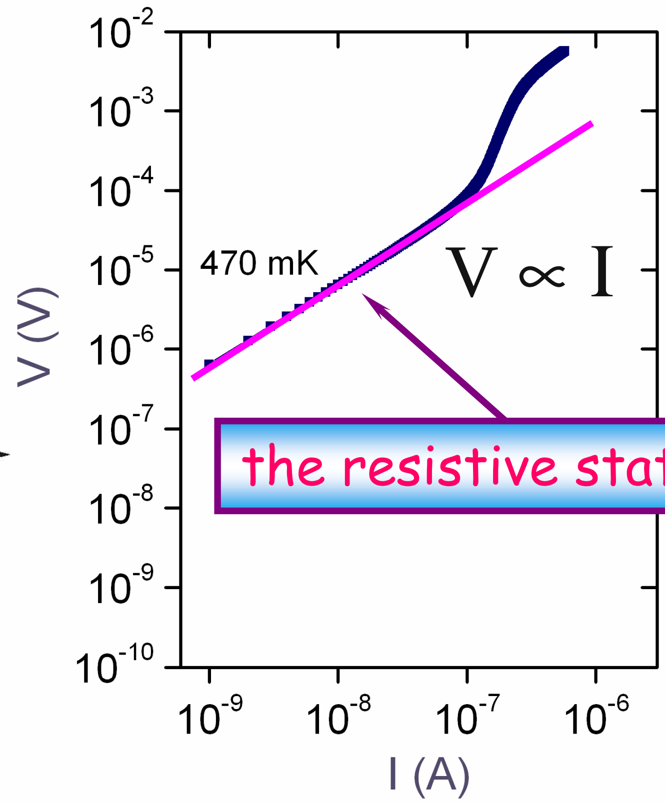
Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for Superconductor



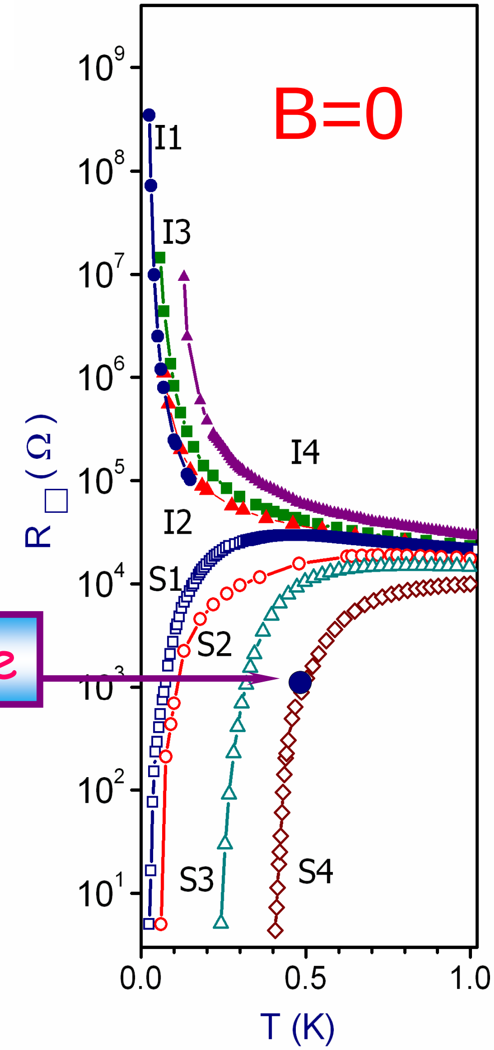
S4 at B = 0 T

V vs I



the resistive state

TiN films



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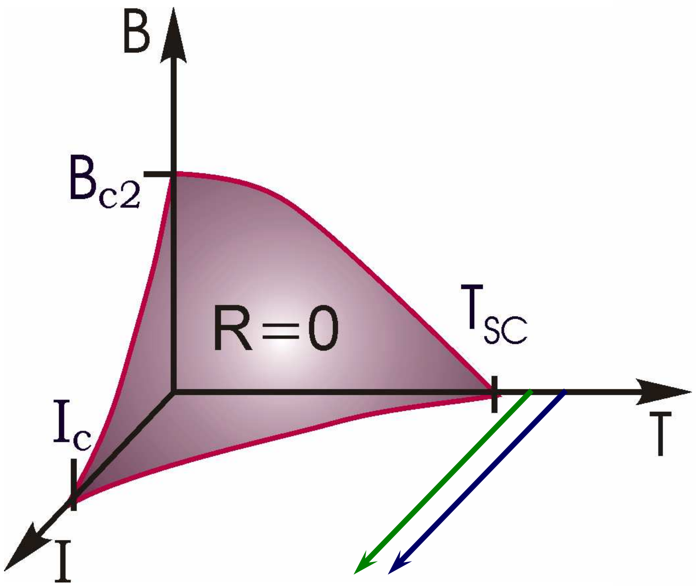
Phase diagram for Superconductor



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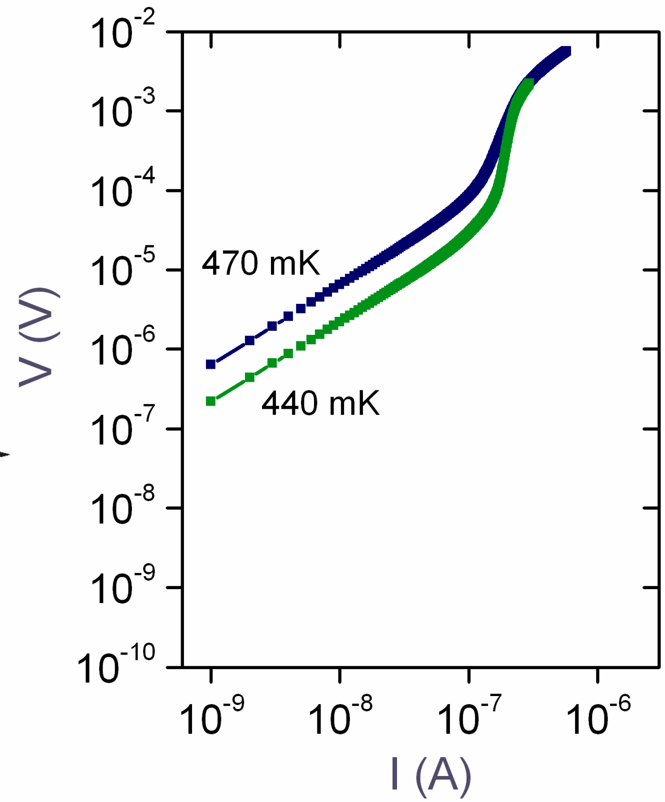
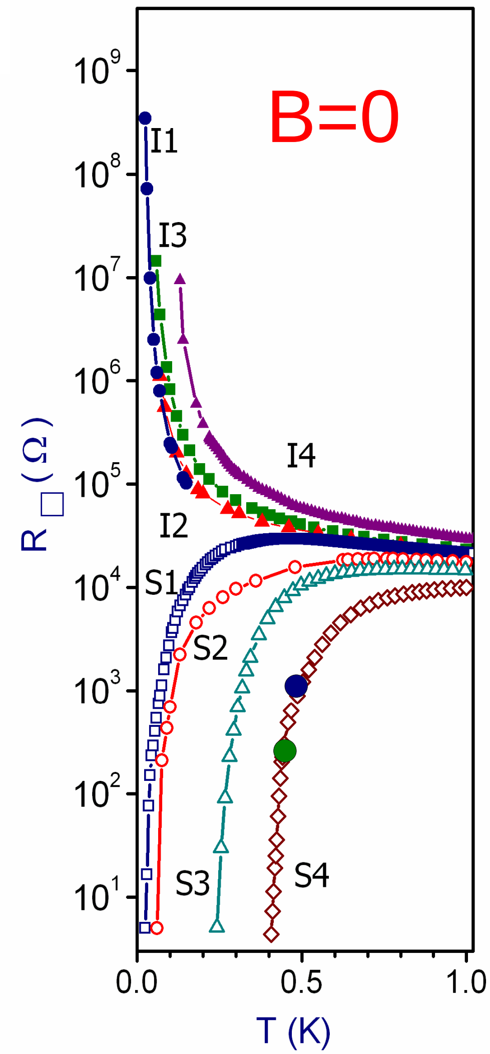


TiN films



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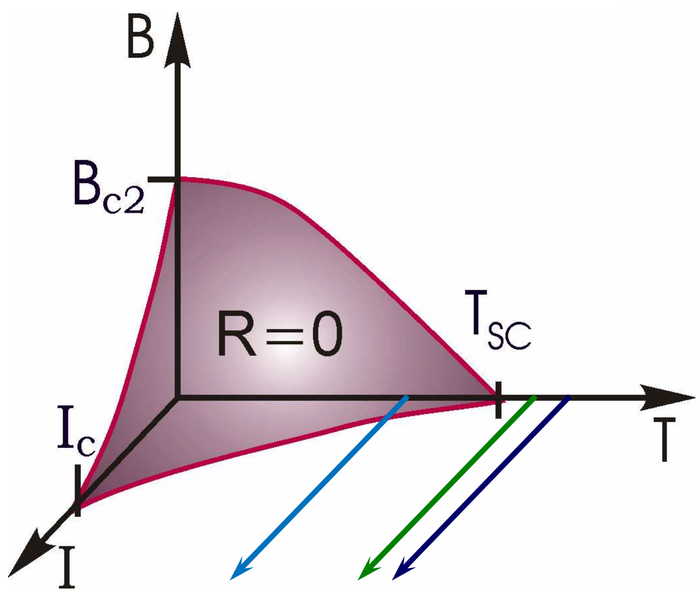
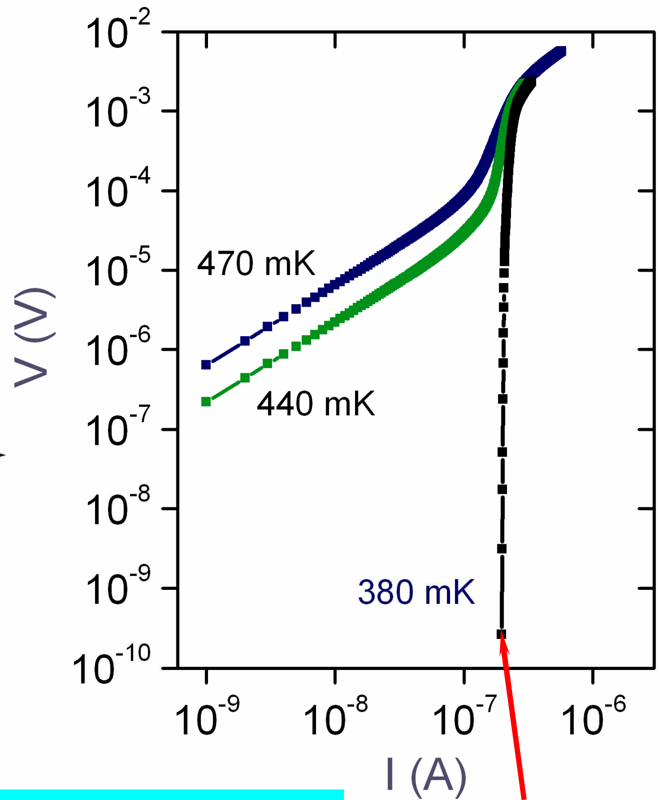
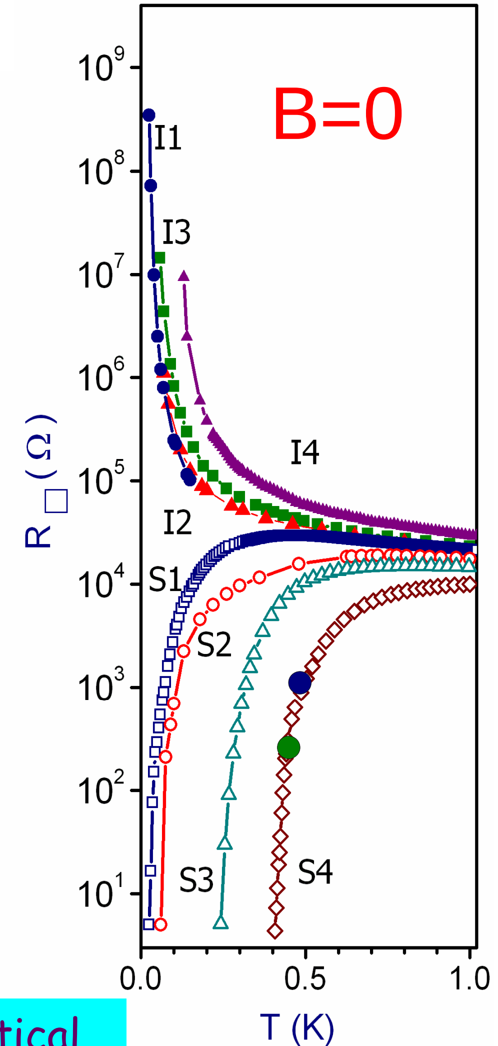


S4 at B = 0 T

V vs I



TiN films

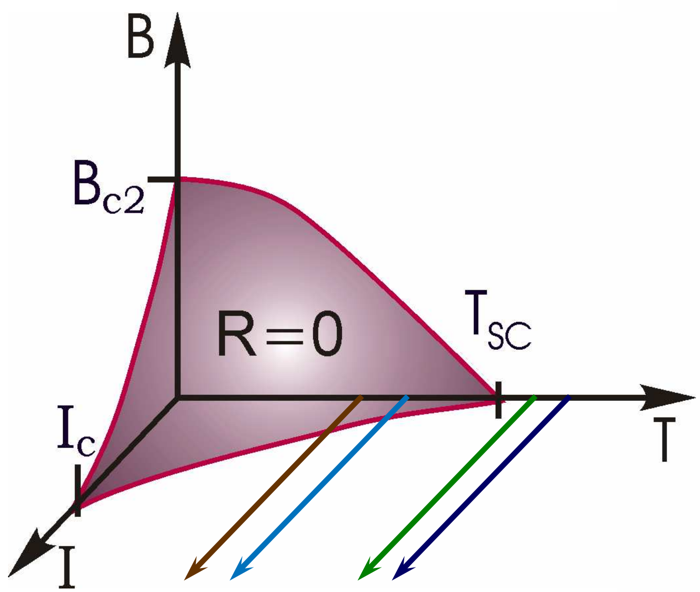


at $I<I_c(T)$ the resistance is immeasurably small

the critical current $I_c(T)$

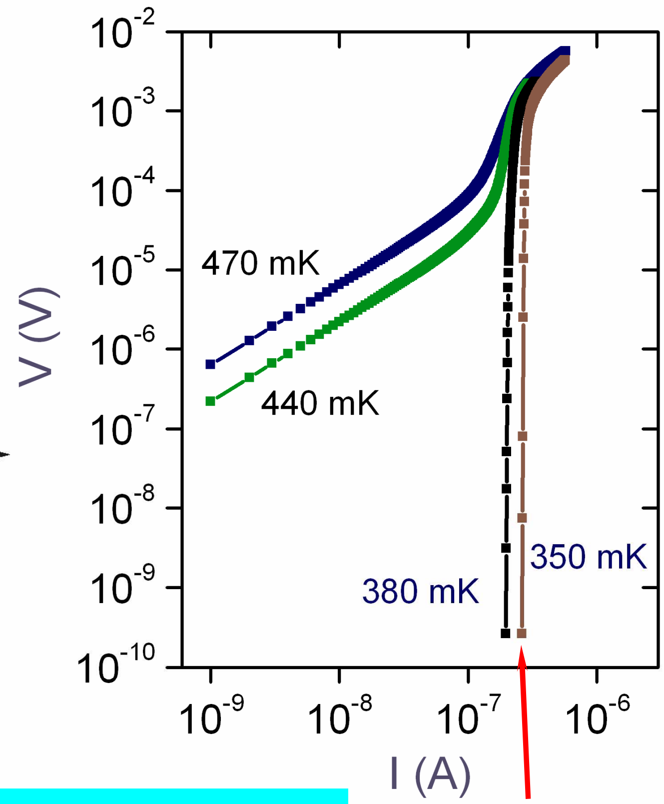
Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for Superconductor

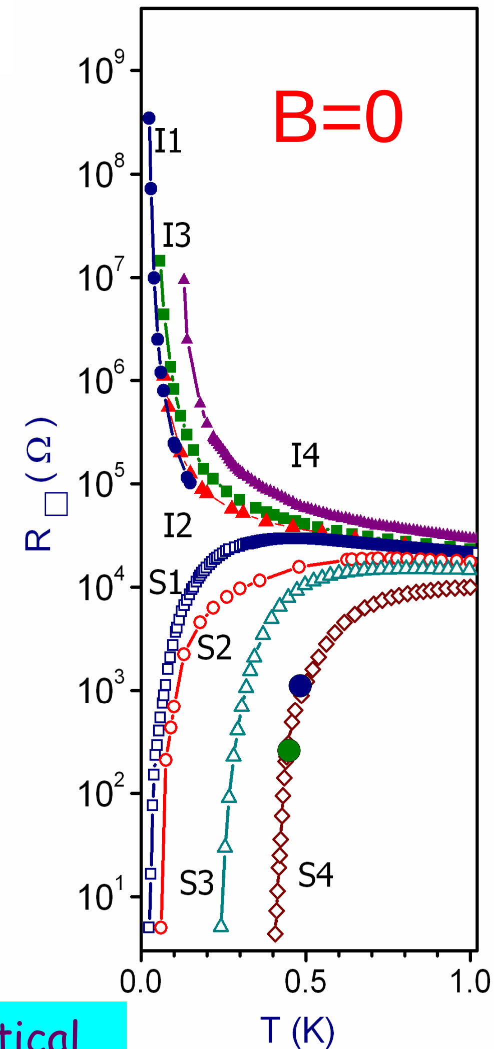


S4 at B = 0 T

V vs I



TiN films

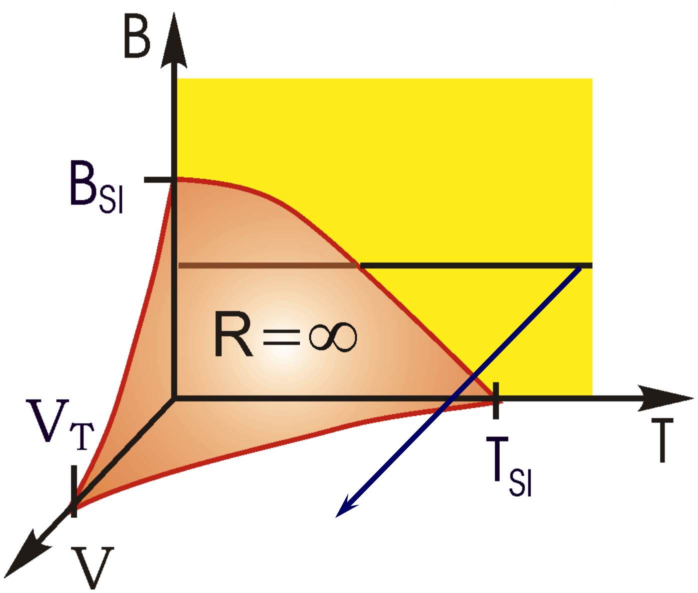


at $I<I_c(T)$ the resistance is immeasurably small

the critical current $I_c(T)$

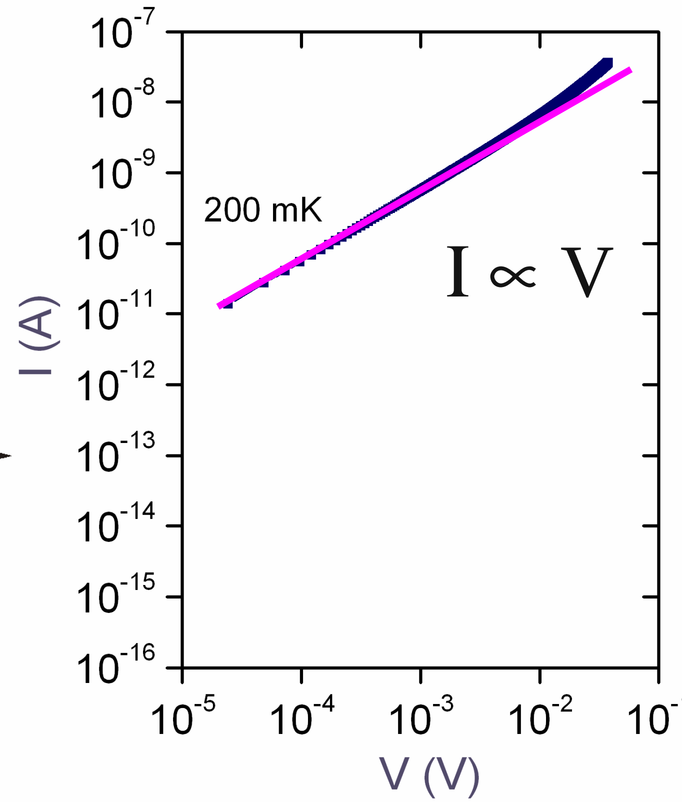
Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for insulating side

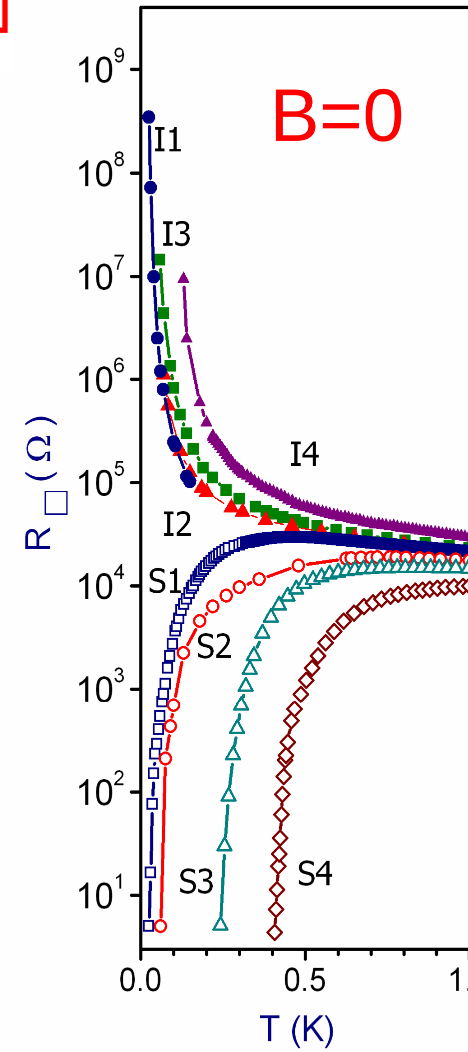


I2 at B = 0.87 T

I vs V

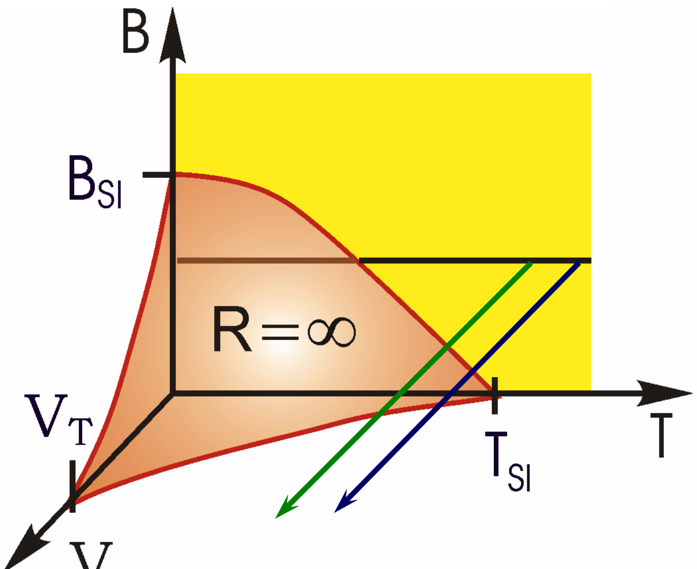


TiN films



Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for insulating side



I2 at B = 0.87 T

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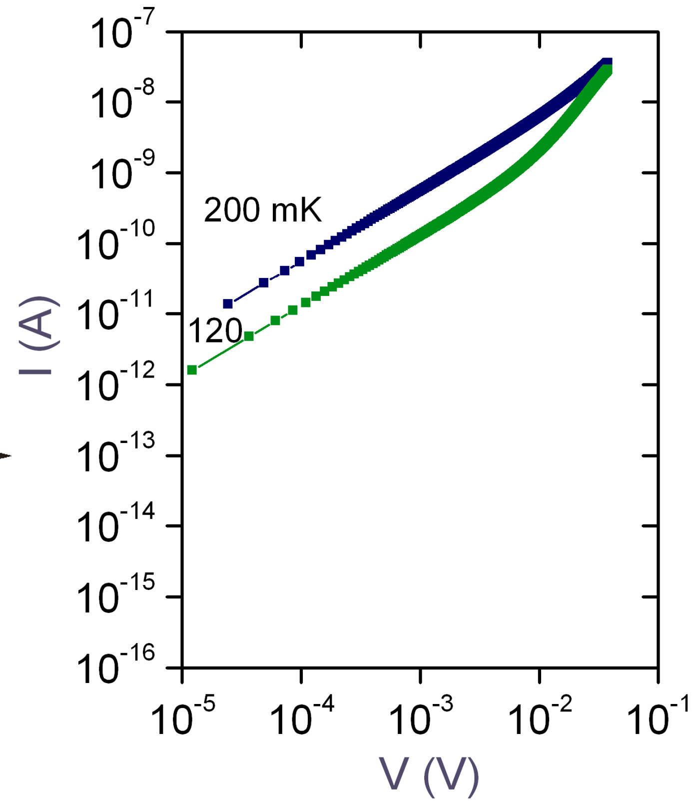
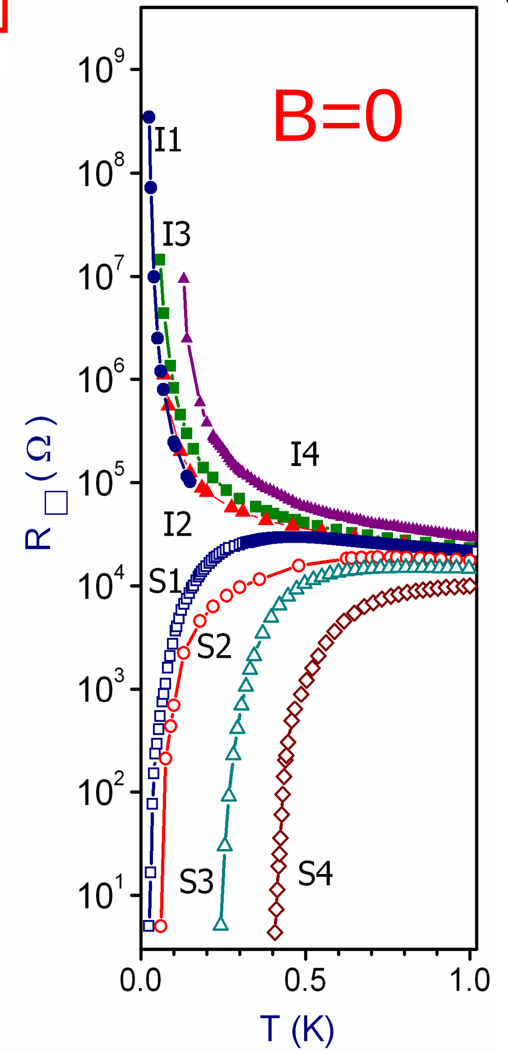
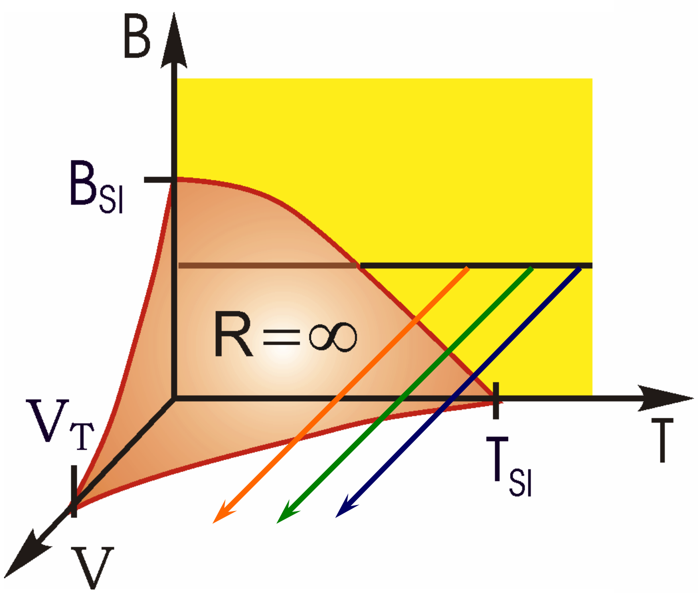


TiN films



Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for insulating side



I2 at B = 0.87 T

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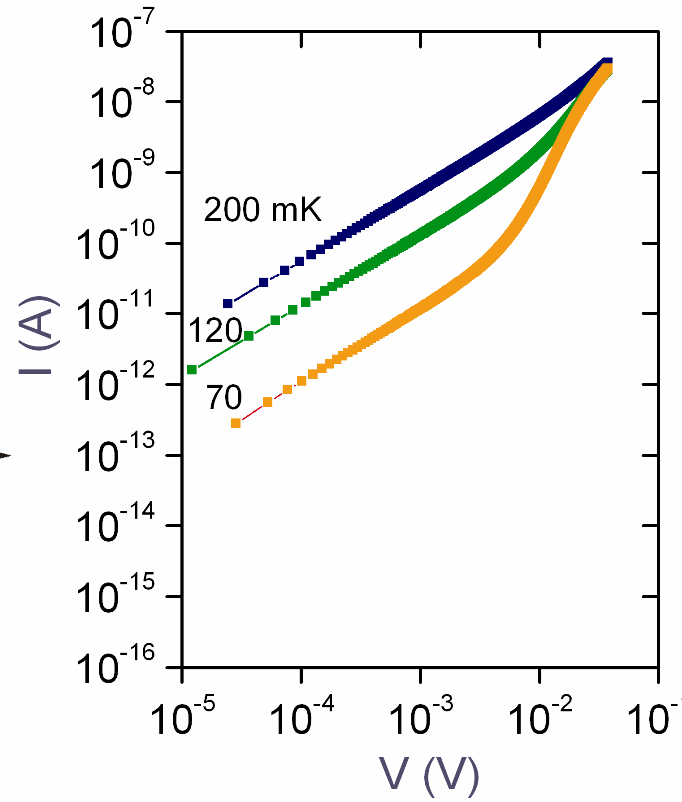
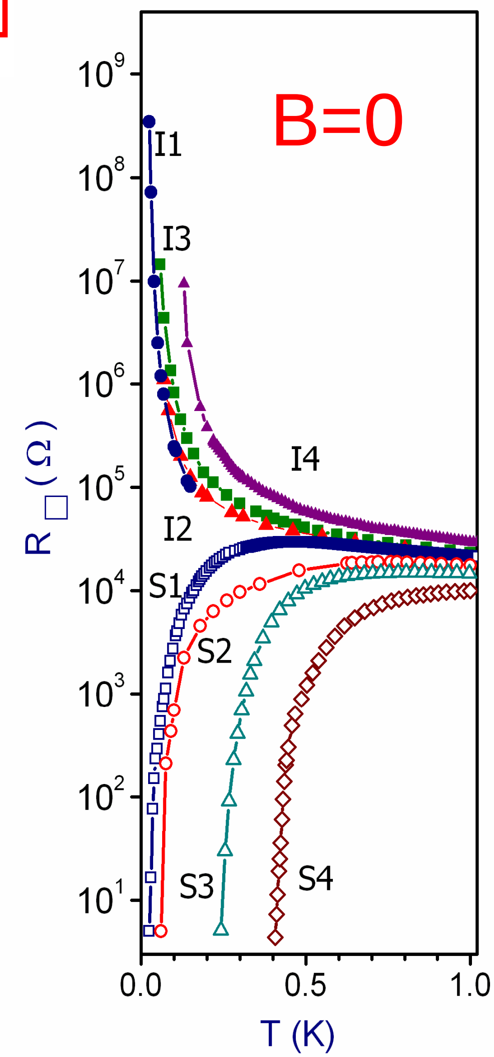
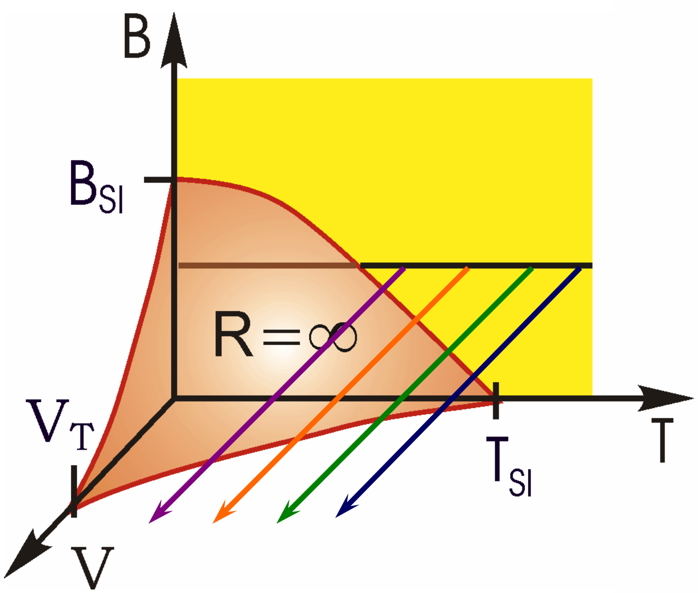


TiN films



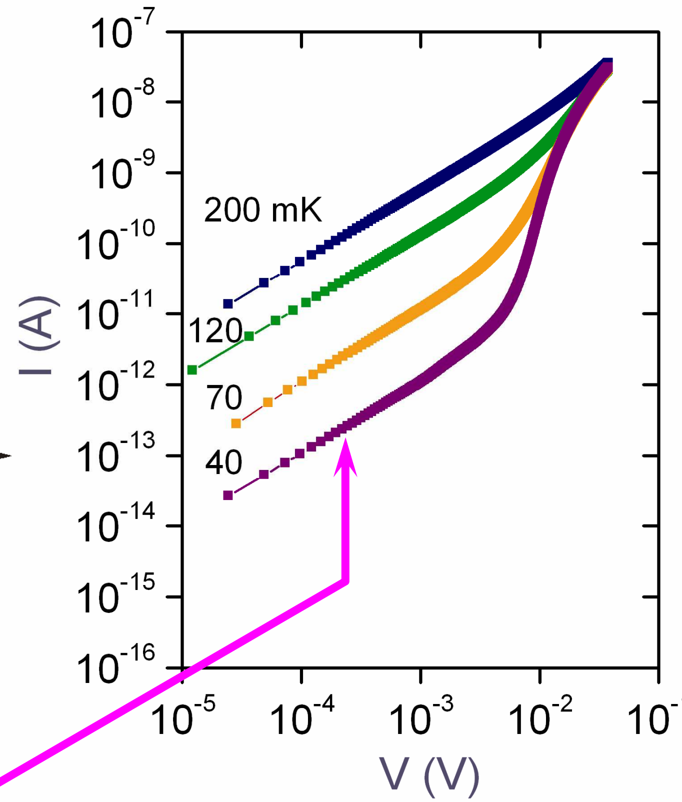
Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for insulating side

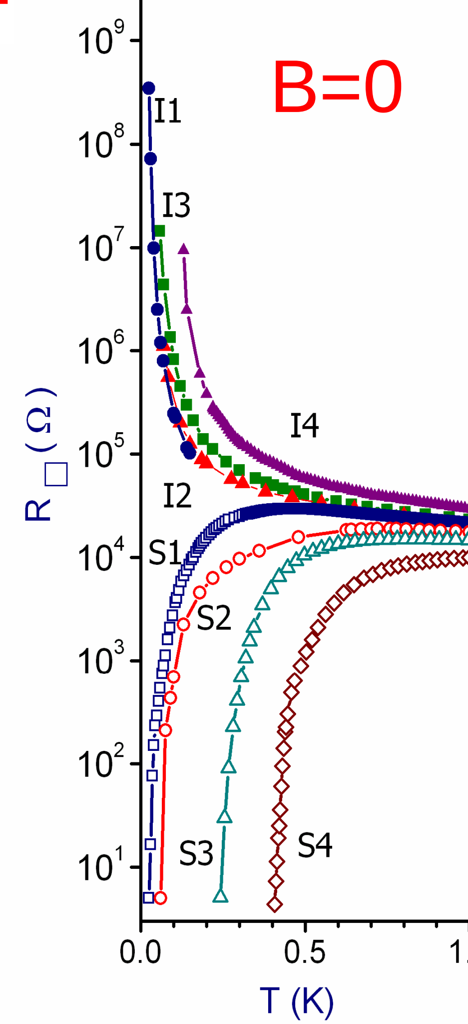


I2 at B = 0.87 T

I vs V



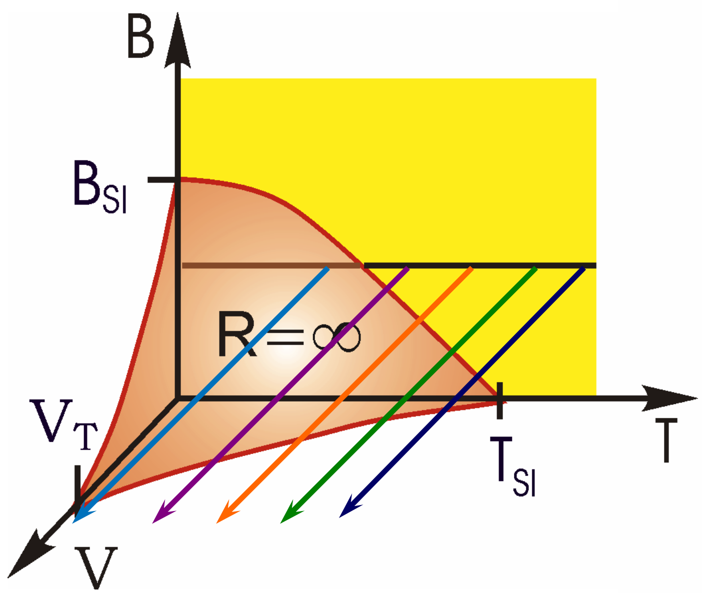
TiN films



Arrhenius behavior of the resistance

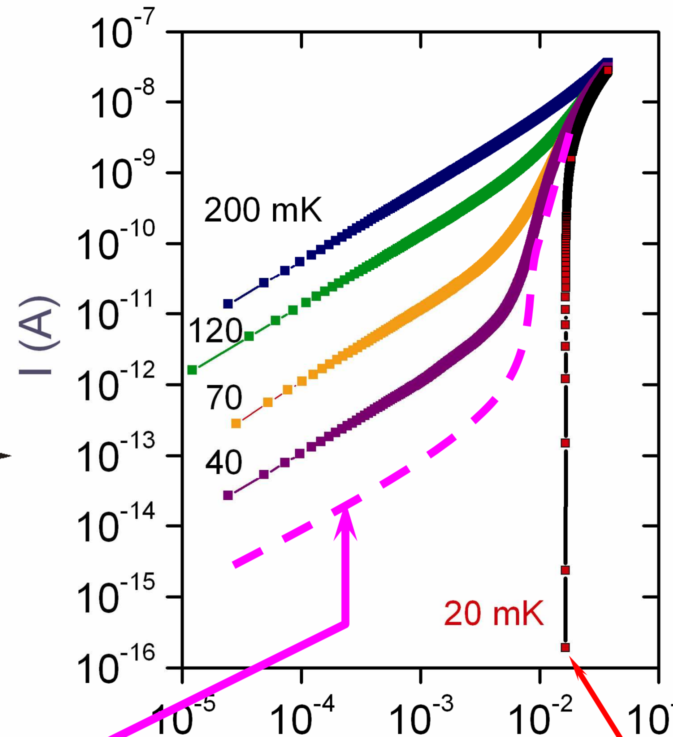
Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for insulating side

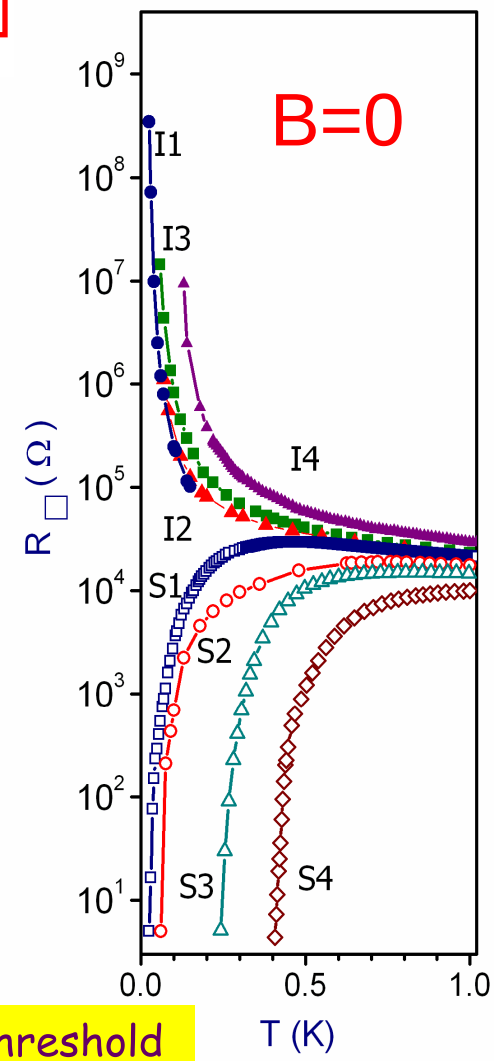


I2 at B = 0.87 T

I vs V



TiN films

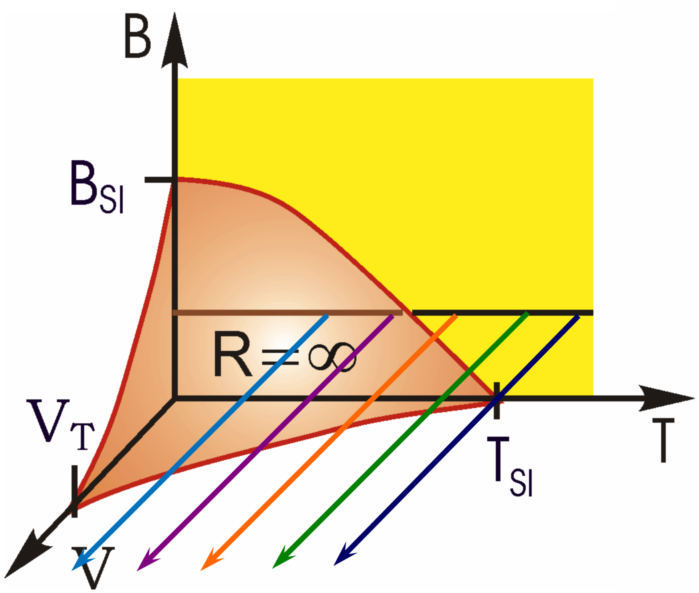


IF Arrhenius behavior of the resistance retained at 20 mK the I-V would have looked like that

the threshold voltage V_T

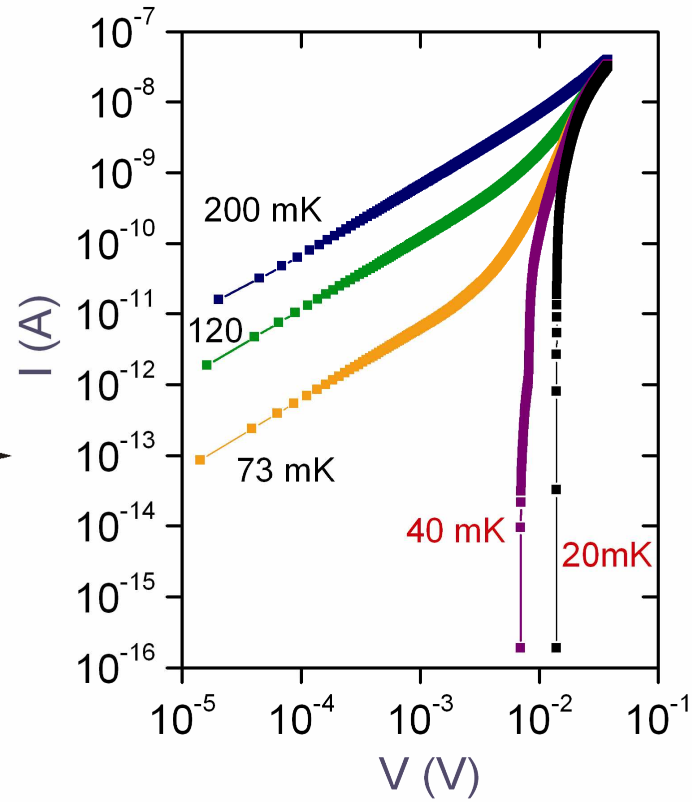
Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for insulating side

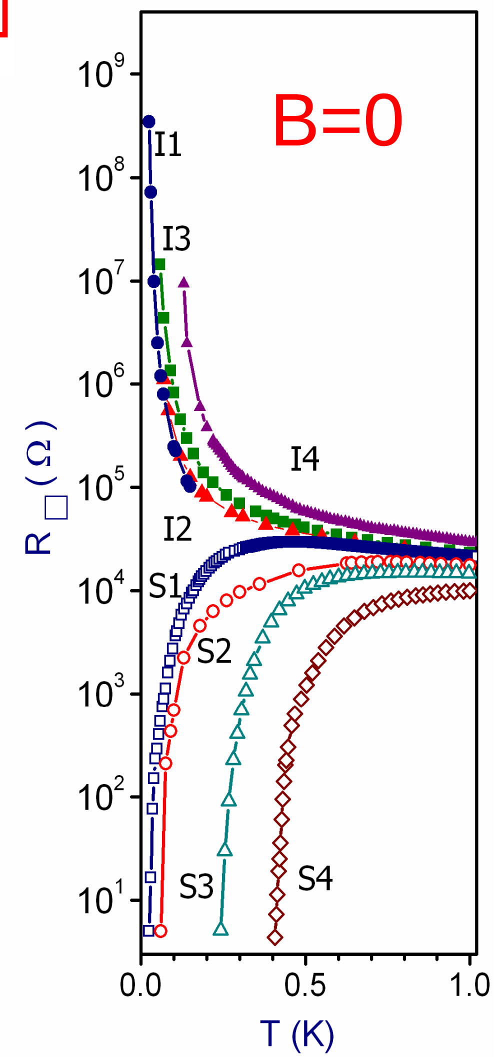


I2 at B = 0.37 T

I vs V



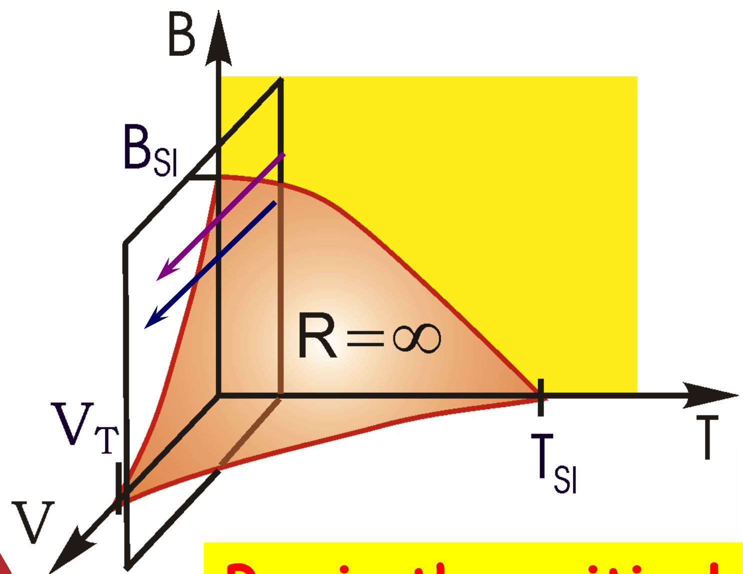
TiN films

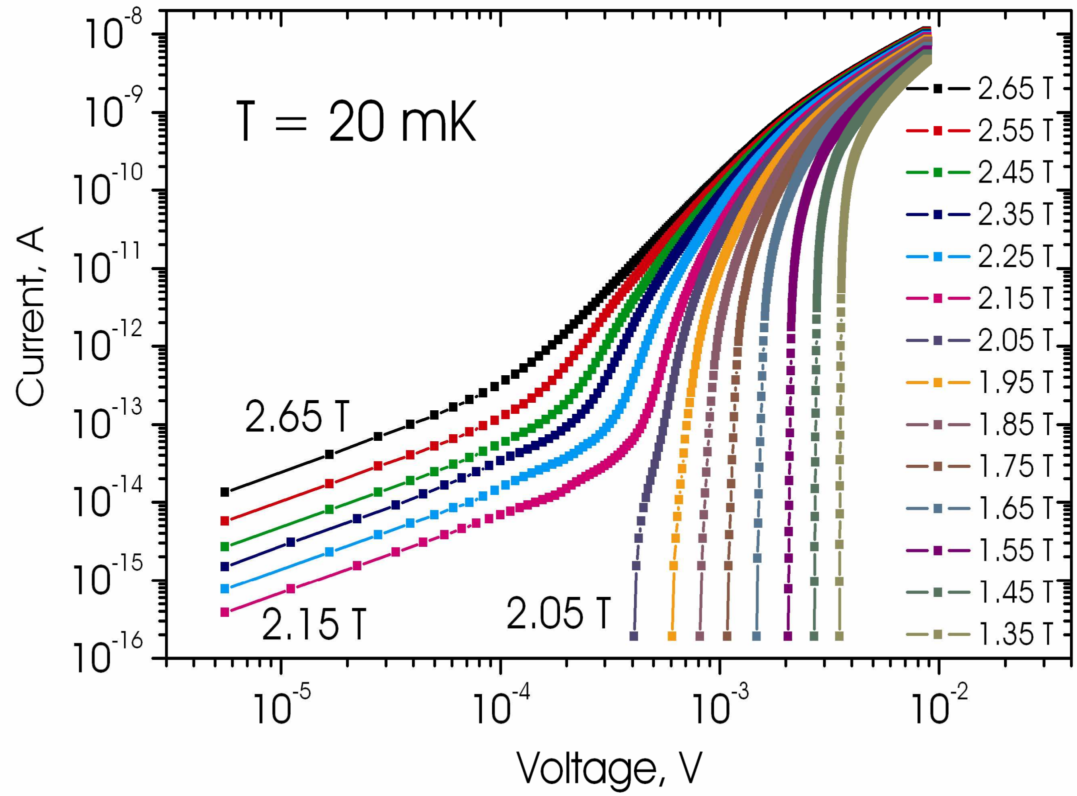


T_{SI} is magnetic field dependent

Disorder-driven SIT in and BKT in TiN films: I-V curves

Phase diagram for insulating side

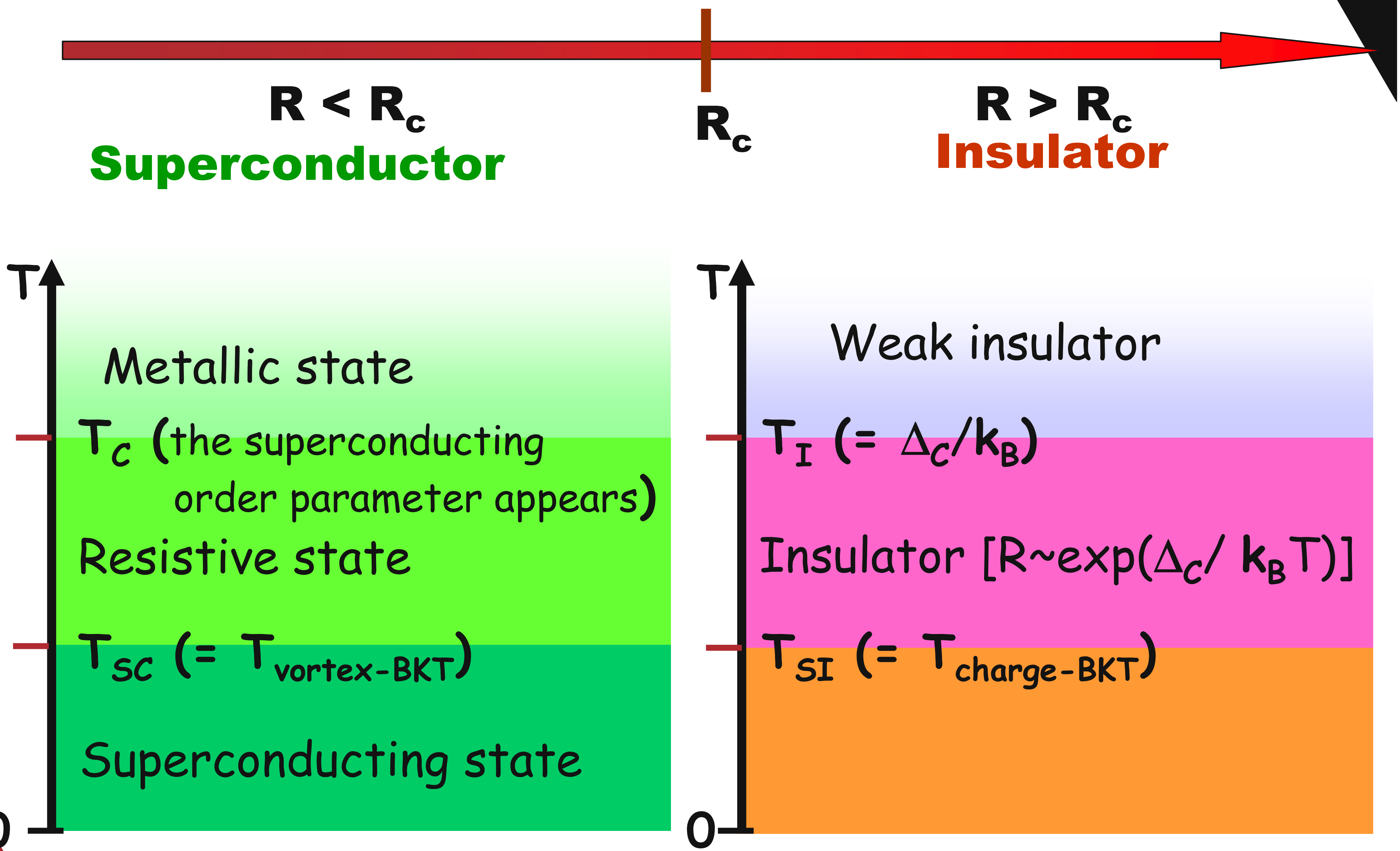




B_{SI} is the critical magnetic field

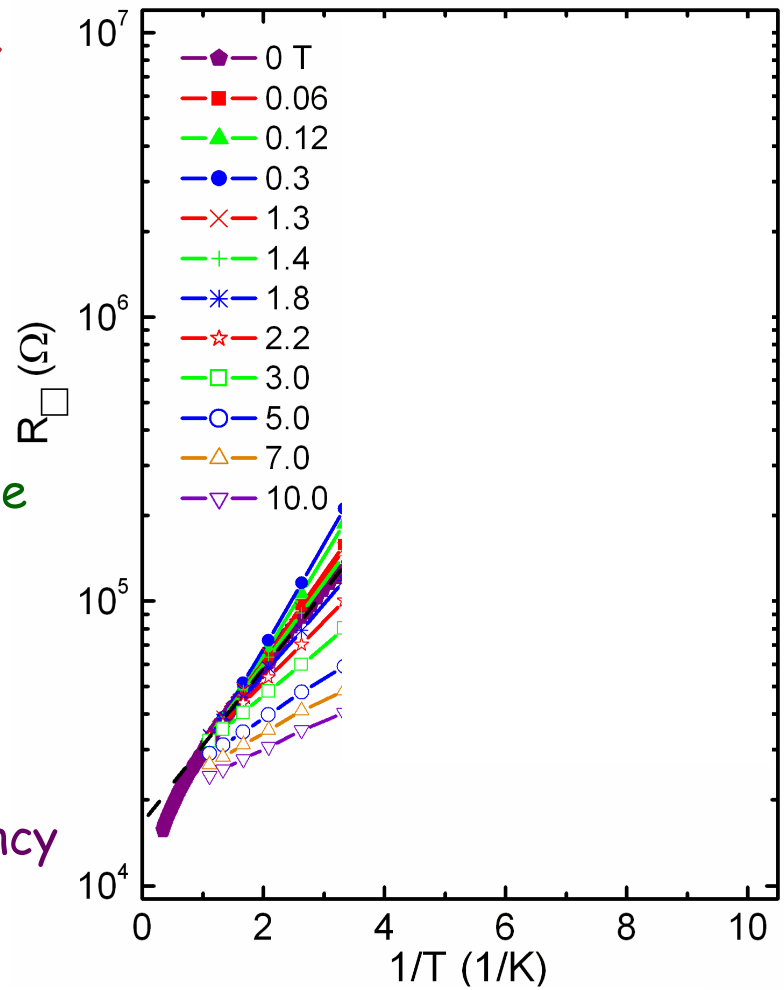
V. Vinokur, T. B., M.V. Fistul, A.Yu. Mironov, M.R. Baklanov, C. Strunk, Nature 452, 613 (2008)

Disorder-driven SIT in and BKT in TiN films



Hyperactivated behavior of the resistance in the low-T charge BKT phase

Arrhenius plots of the isomagnetic temperature dependences of the resistance.



✓ In relatively small magnetic fields the resistance curves shoot upwards from Arrhenius activation behavior below some magnetic field dependent temperature

✓ This hyperactivated behavior maintains till fields not exceeding 2 T.

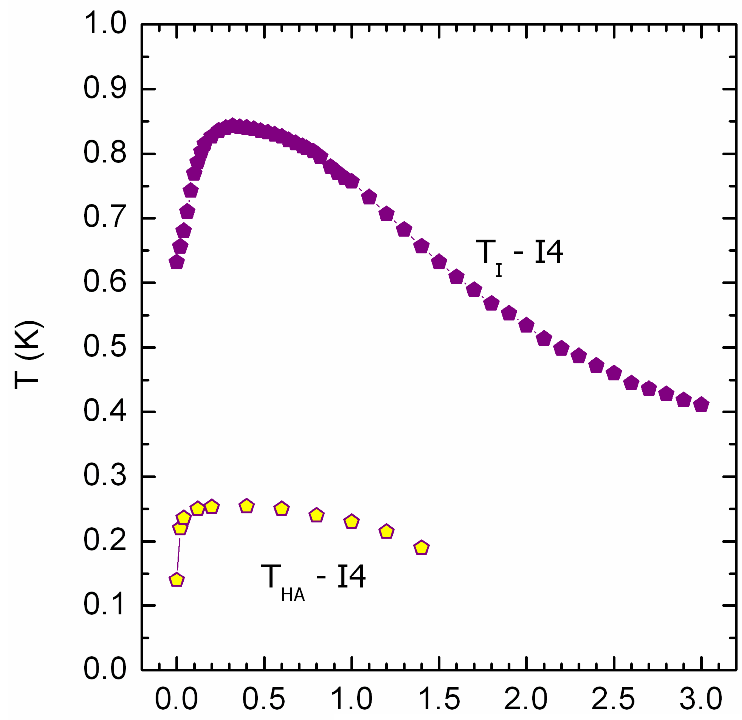
✓ At fields larger than 2 T the resistance curves bend downwards showing the tendency to saturation.

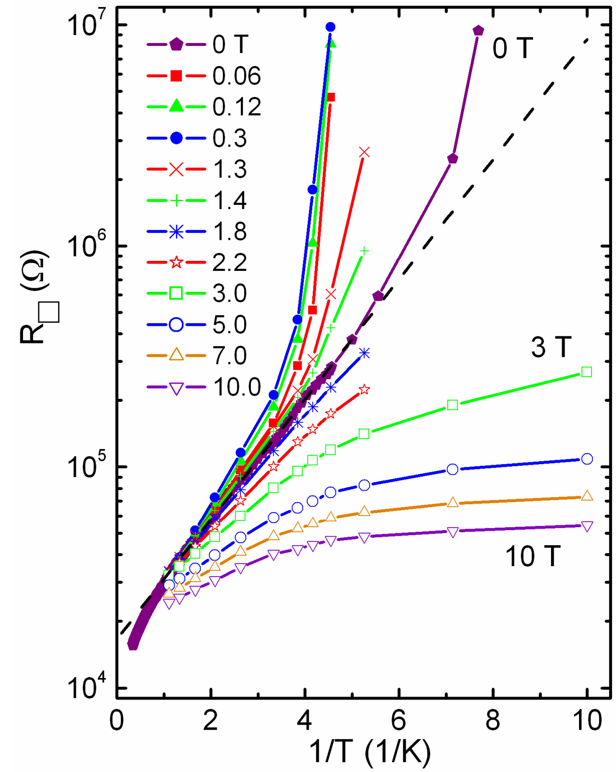
Pis'ma v ZhETF, vol. 88, iss. 11, pp. 867 – 872

Hyperactivated resistance in TiN films on the insulating side of the disorder-driven superconductor-insulator transition

T. I. Baturina[1)], A. Yu. Mironov, V. M. Vinokur[∇], M. R. Baklanov[+], C. Strunk[*]

Activated and hyperactivated behavior of the resistance





$R(T_{HA})=2R_0\exp(T_{HA}/T_I)$

We define the characteristic temperature of the transition to the hyperactivated regime by the relation $R(T_{HA})=2R_0\exp(T_{HA}/T_I)$, i.e., as the temperature at which the actually measured resistance exceeded two times the resistance obtained by extrapolation from the activated temperature interval.

Pis'ma v ZhETF, vol. 88, iss. 11, pp. 867 – 872

Hyperactivated resistance in TiN films on the insulating side of the disorder-driven superconductor-insulator transition

T. I. Baturina[1)], A. Yu. Mironov, V. M. Vinokur[∇], M. R. Baklanov[+], C. Strunk[*]

Journal of the Physical Society of Japan
Vol. 64, No. 1, January, 1995, pp. 19–21

LETTERS

Precursor of Charge KTB Transition in Normal and Superconducting Tunnel Junction Array

Akinobu KANDA and Shun-ichi KOBAYASHI

Department of Physics, School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113

(Received October 11, 1994)

Figure 1 shows the temperature dependence of resistance $R(T)$ at 50 μV in magnetic fields $H=0$ and 3 T. The array was ohmic up to 50 μV or higher. Both $R(T)$'s have upward curvatures, which are characteristic of the charge KTB transition.

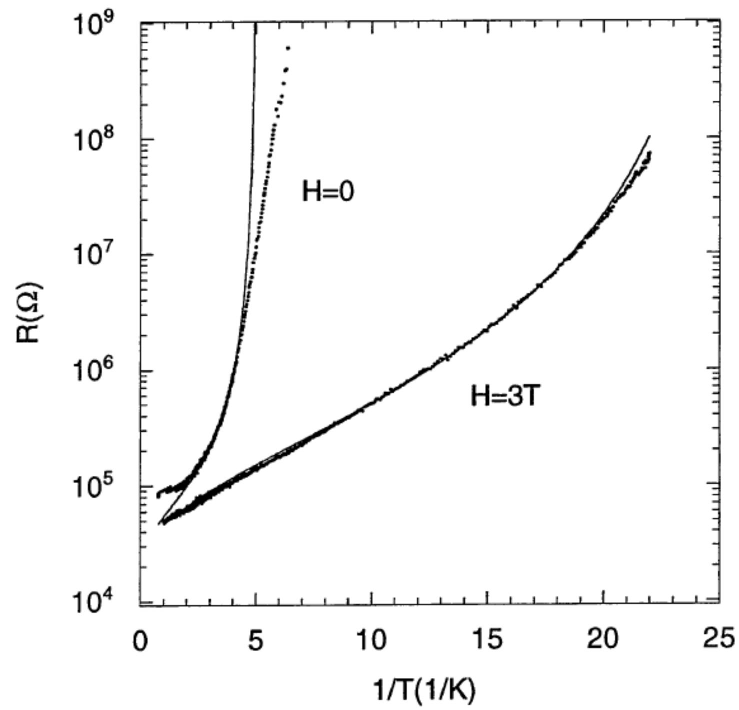
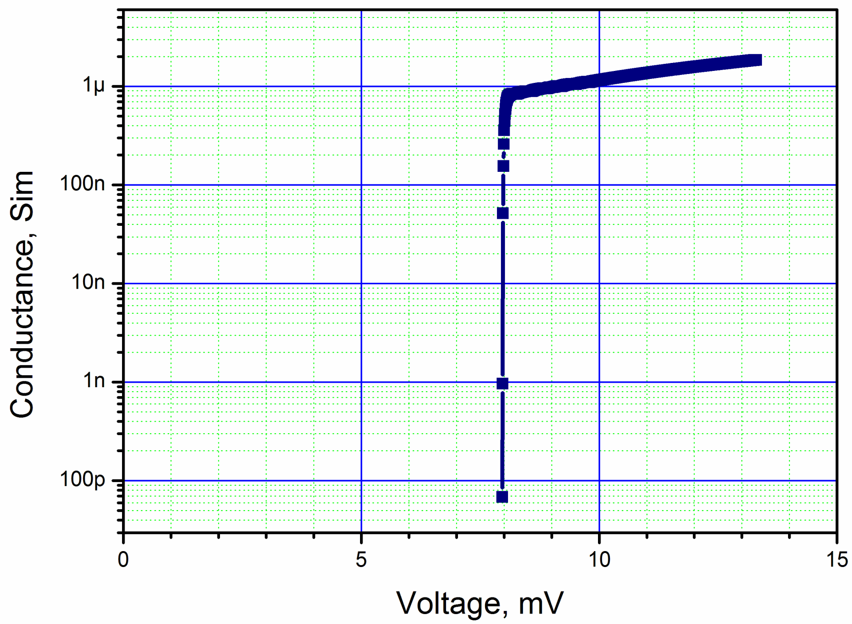
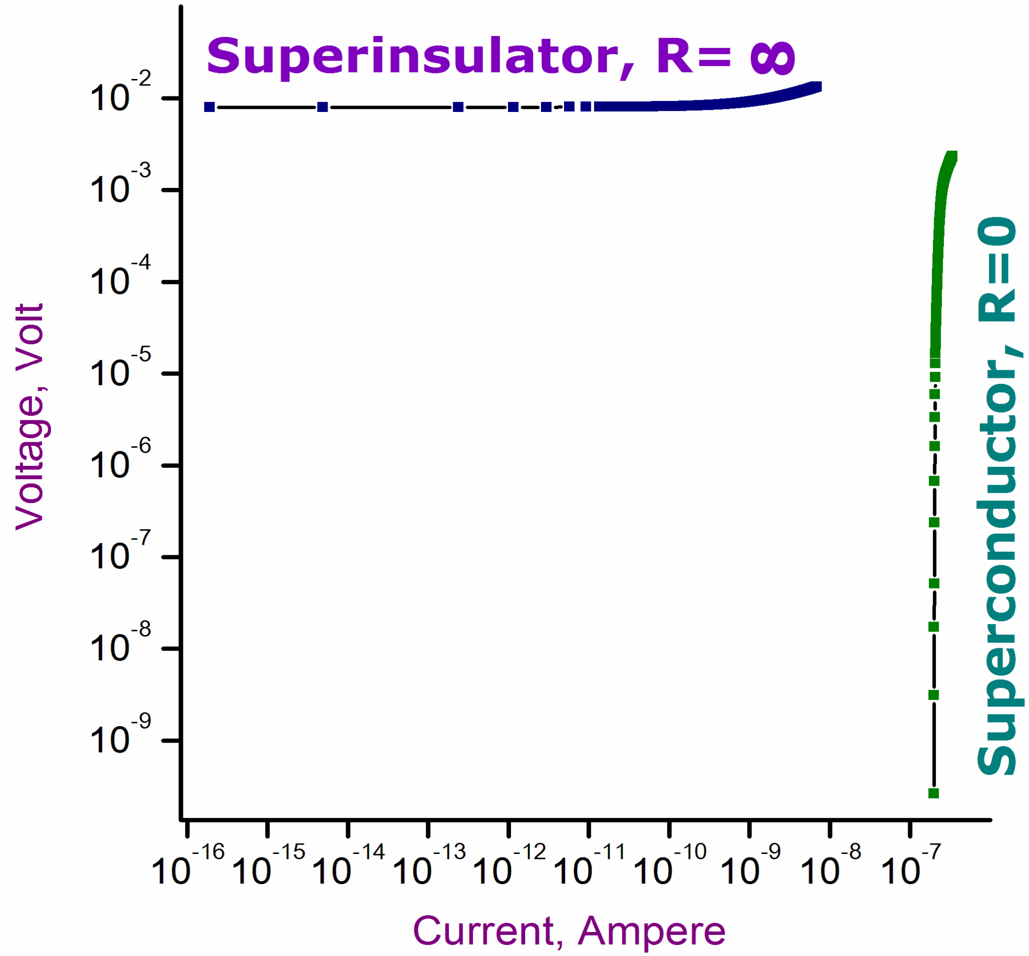


Fig. 1. Resistance at $V=50\,\mu$V as a function of $1/T$ in $H=0$ and 3 T. Solid lines are results of fitting with eq. (1). The values of fitting parameters are $K=1.6$ and $b=1.0$ in $H=0$, and $K=1.6$ and $b=2.2$ in $H=$ 3 T. For the values of T_{KT}, see the text.

Conclusions 1

Low-temperature BKT phase
on the insulating side of the
superconductor-insulator transition:
zero conductivity state!!





Joule loss: $P = I\ V = 0$

Conclusions 2

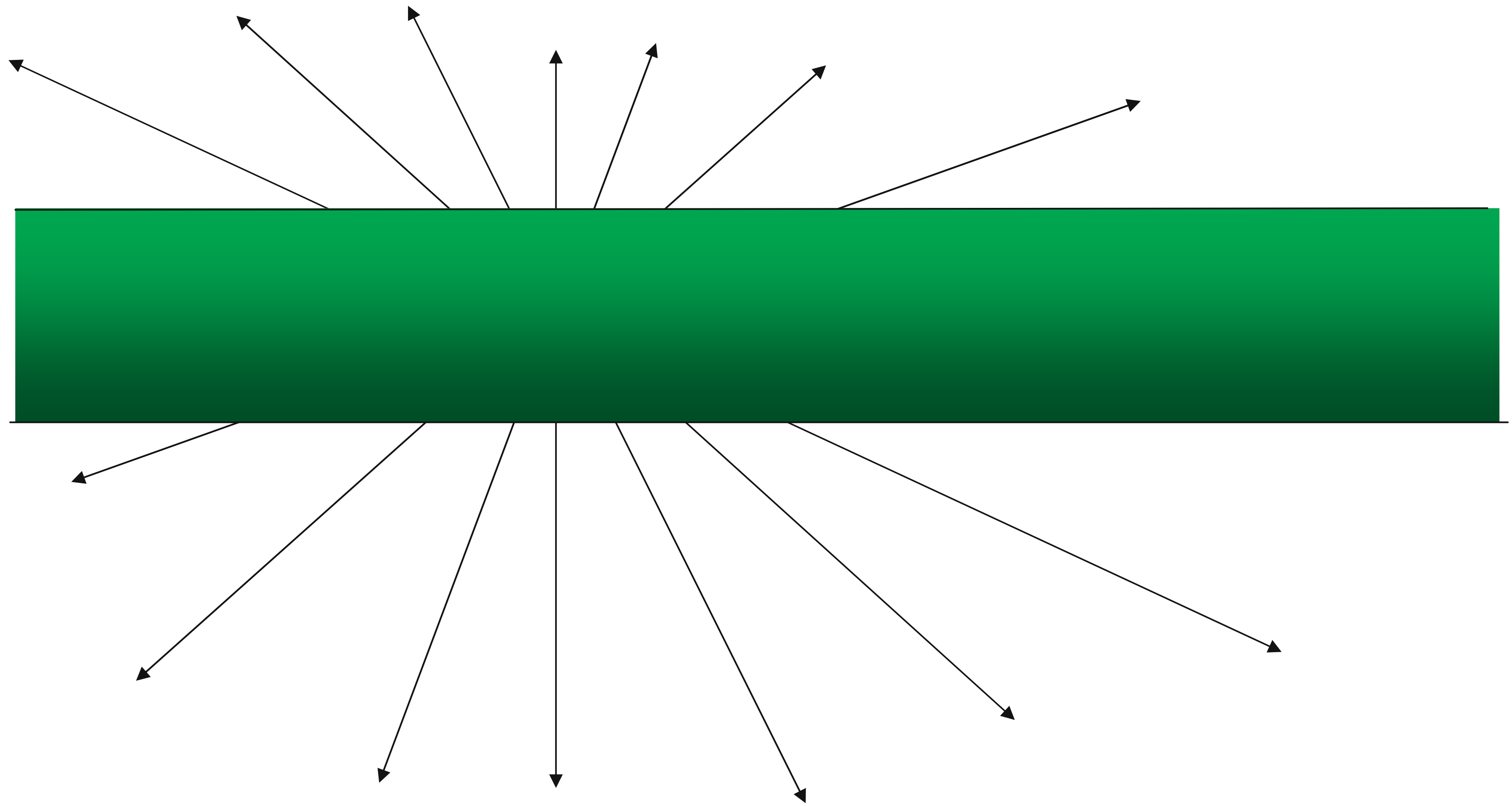
Questions:

How one can realize 2D Coulomb behavior in our 3D world?

What is the nature of transport in the low-temperature BKT phase?

How one can realize 2D Coulombbehavior in our 3D world?

- Trap all the field lines within the film.



How one can realize 2D Coulombbehavior in our 3D world?

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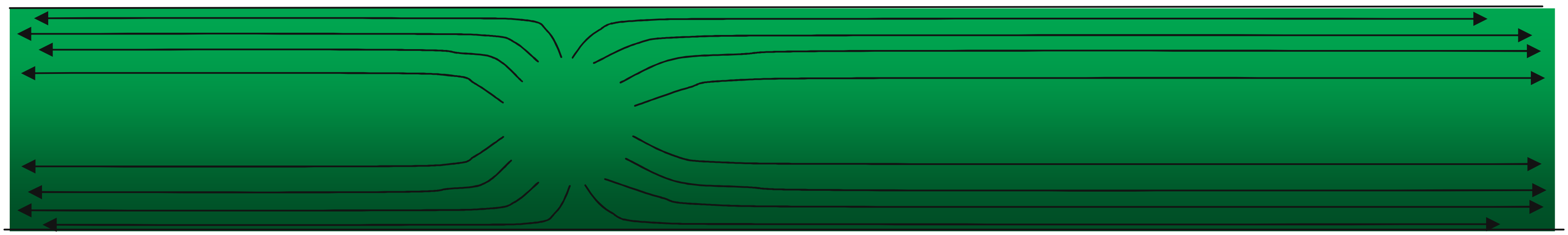


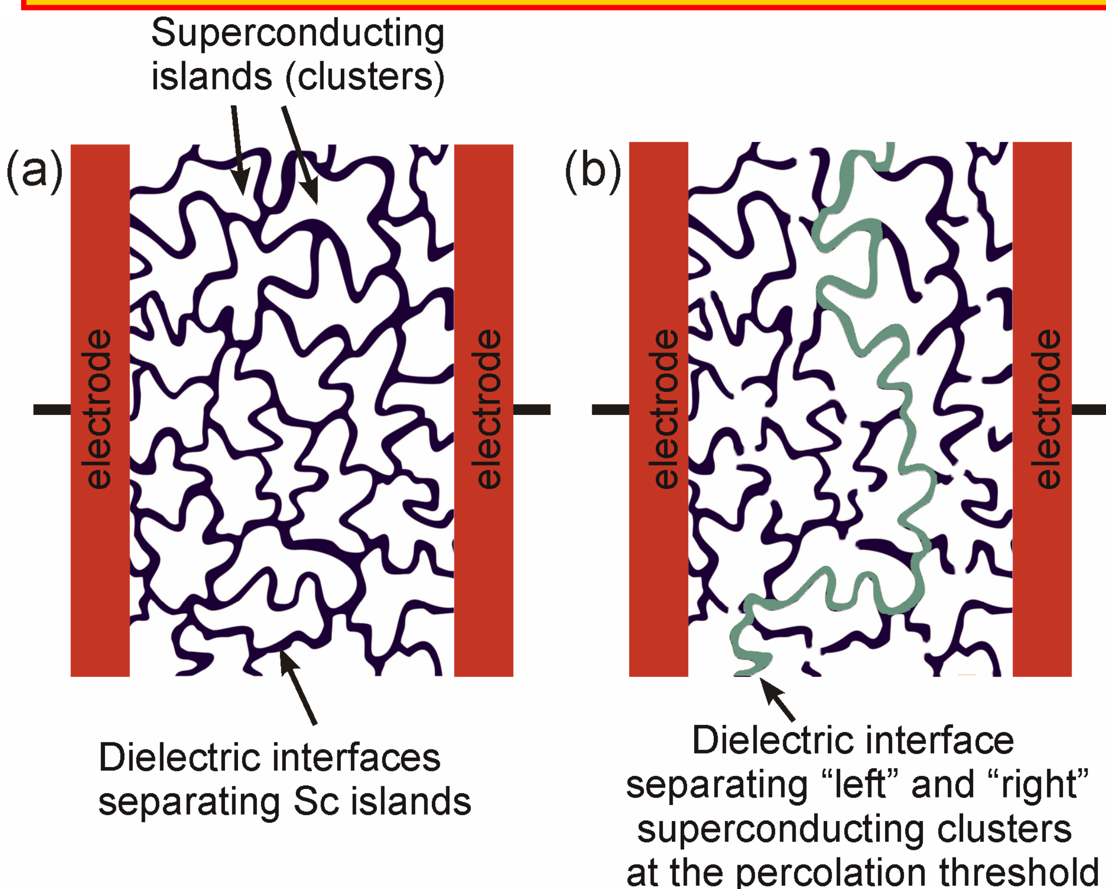
Field lines remain trapped over distances < εd

$\varepsilon \quad 1$

Why the films near the SIT realize 2D universe

Near the SIT the dielectric constant of the film $\varepsilon \to \infty$

Superconducting film near superconductor-insulator transition



$$C \propto \varepsilon l$$

Percolation transition

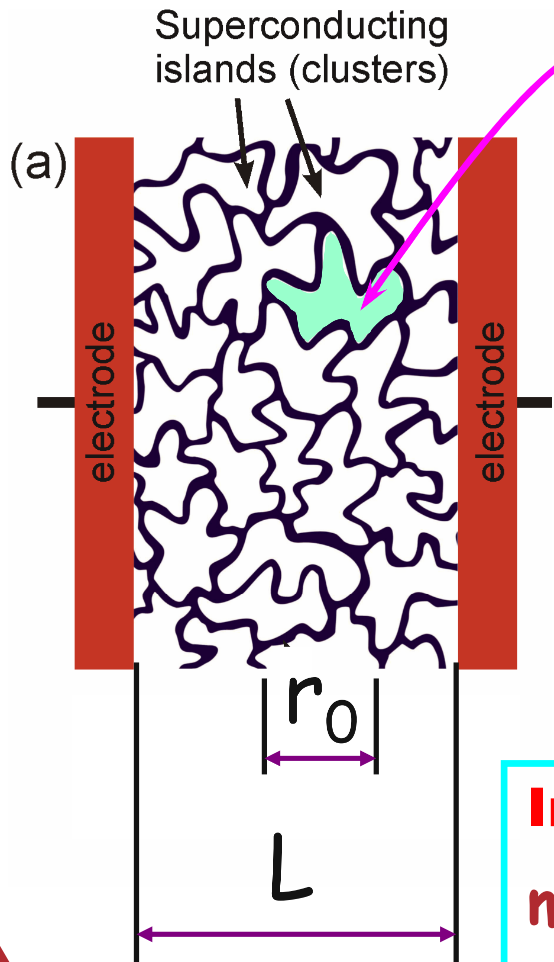
The capacitance between the two adjacent clusters is proportional to the length of the insulating layer separating them. Upon approaching the transition from the insulating side of the SIT, the length of this layer diverges infinitely. It results in the divergent growth of the effective capacitance of the system, implying the divergence of the dielectric constant $\varepsilon \to \infty$.

V.E. Dubrov, M.E. Levinshtein, and M.S. Shur, Sov. Phys. JETP 43, 1050 (1976).

Why the films near the SIT realize 2D universe

macroscopic Coulomb blockade energy Δ_C (2D)

E_C – charging energy of a single island (**local** charging energy)



$$\boxed{\Delta_C \approx E_C \ln(L/r_0)}$$

r_0 – characteristic size of a single island

L – the linear size of the system

Important:

macroscopic Coulomb blockade energy Δ_C well exceeds the **local** charging energy E_C !!

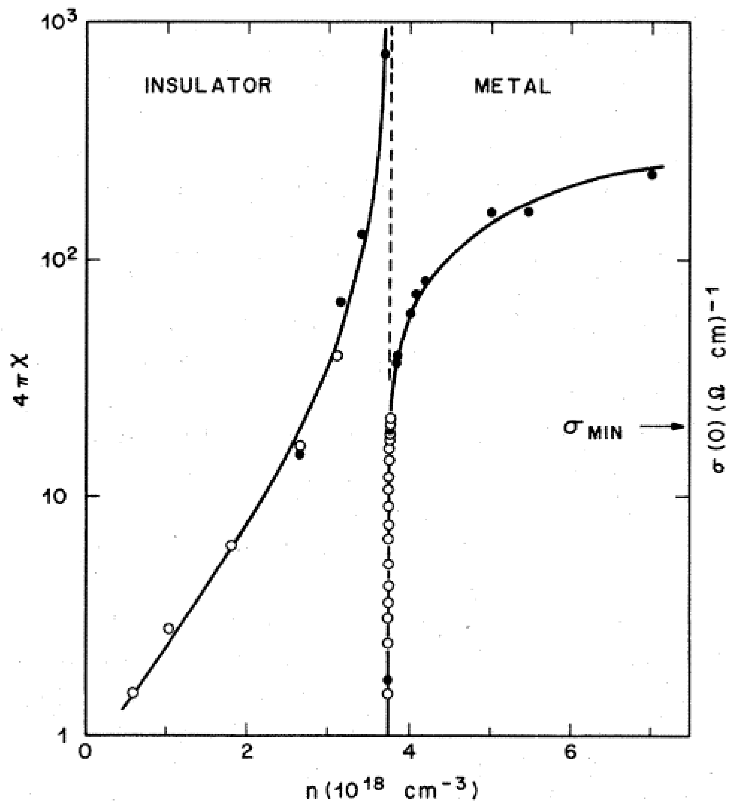
PHYSICAL REVIEW B VOLUME 25, NUMBER 8 15 APRIL 1982

Giant dielectric constants at the approach to the insulator-metal transition

Harald F. Hess and Keith DeConde
Joseph Henry Laboratory, Princeton, New Jersey 08544

T. F. Rosenbaum* and G. A. Thomas
Bell Laboratories, Murray Hill, New Jersey 07974

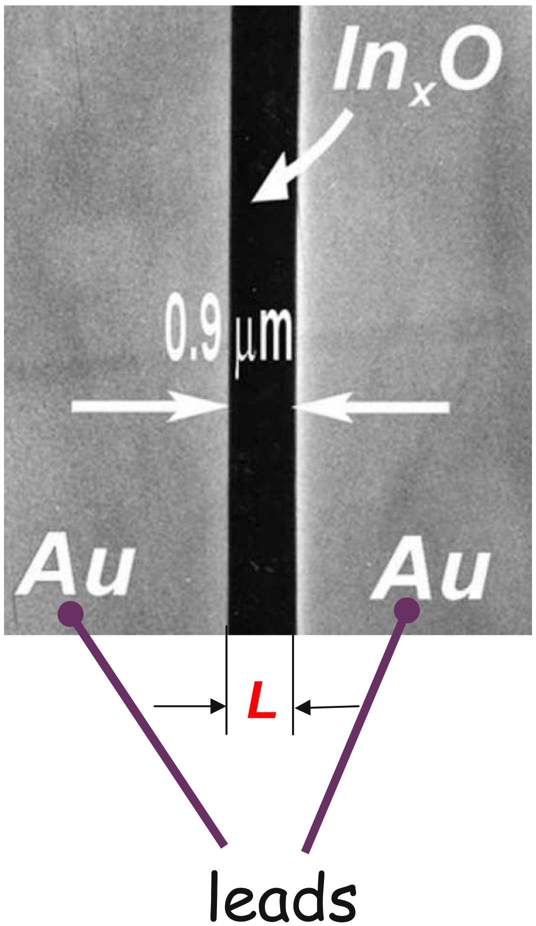
(Received 25 February 1982)



This is a particular case of critical divergence of physical quantities near the phase transition

More of experiment…

D. Kowal and Z. Ovadyahu, Physica C 468, 322 (2008).

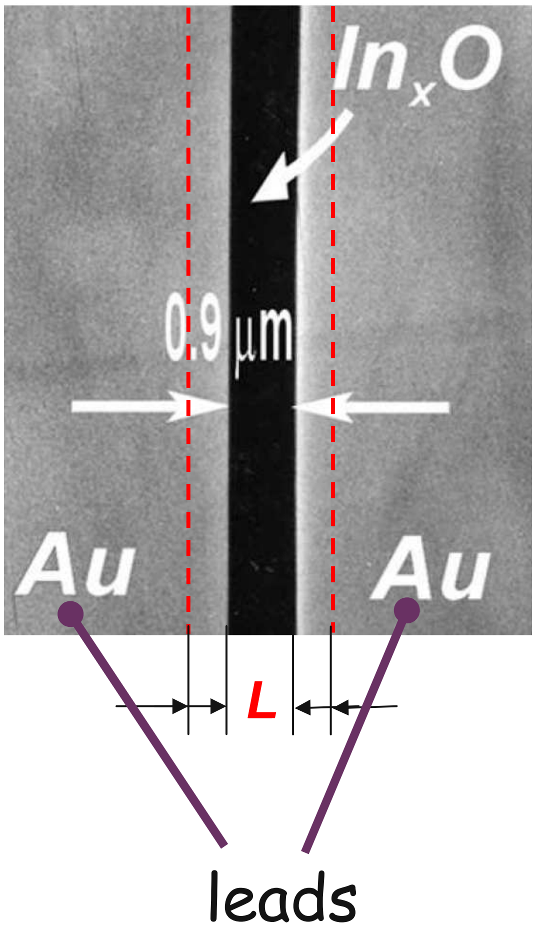


In_x O films

The distance between the contacts, L, varied, while preserving the width of the sample

More of experiment…

D. Kowal and Z. Ovadyahu, Physica C 468, 322 (2008).



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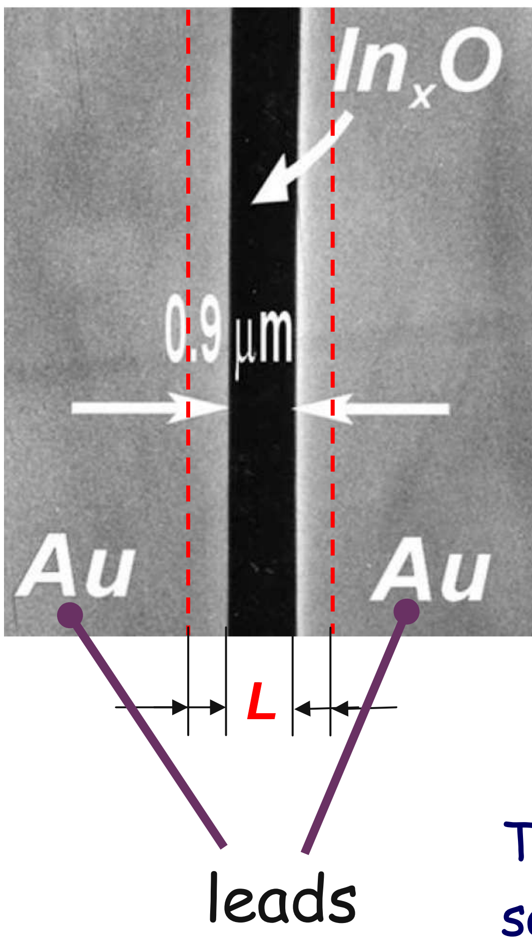
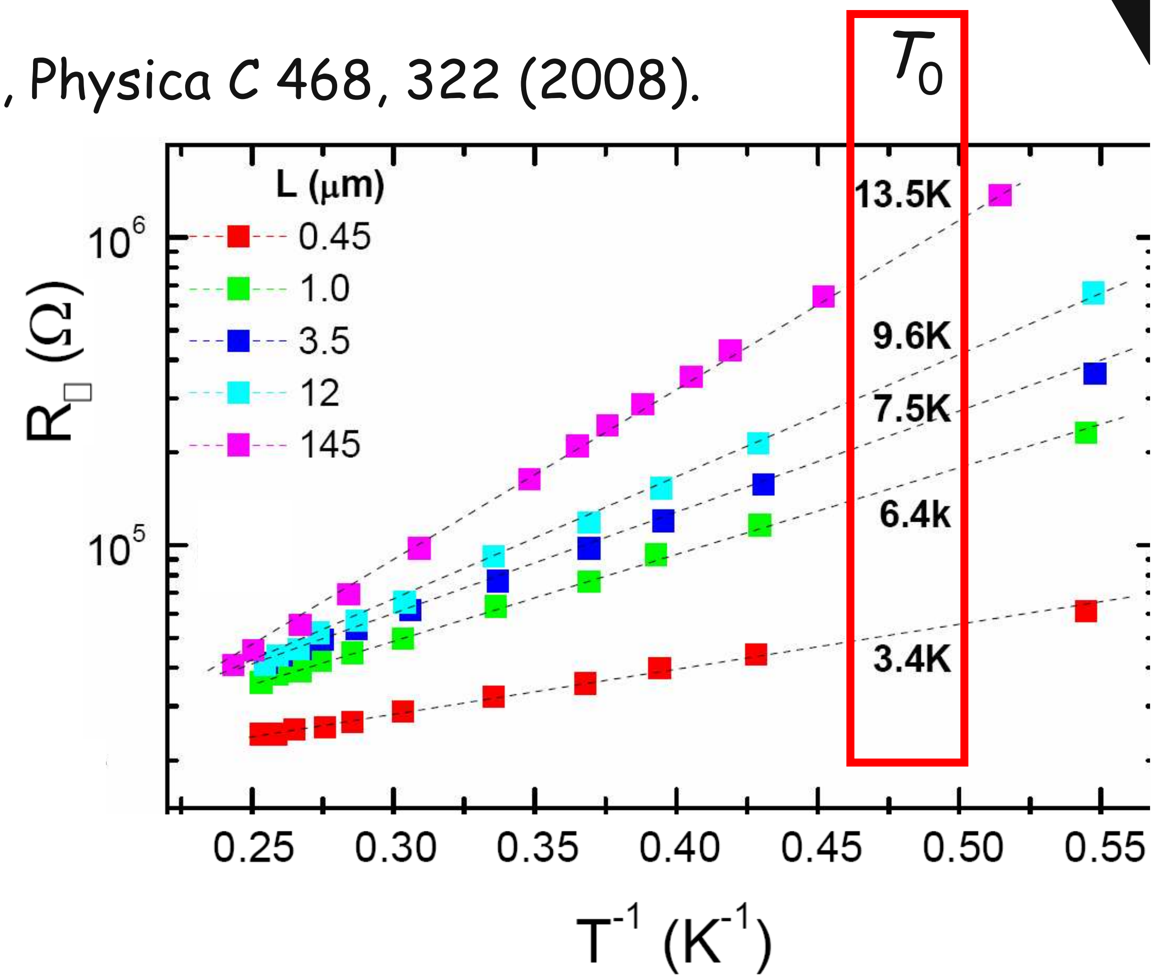
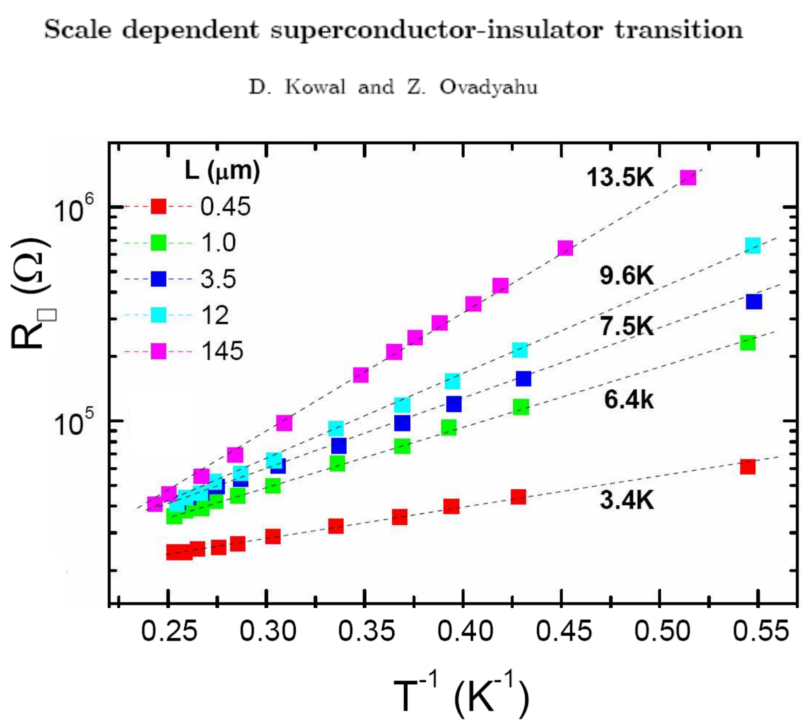
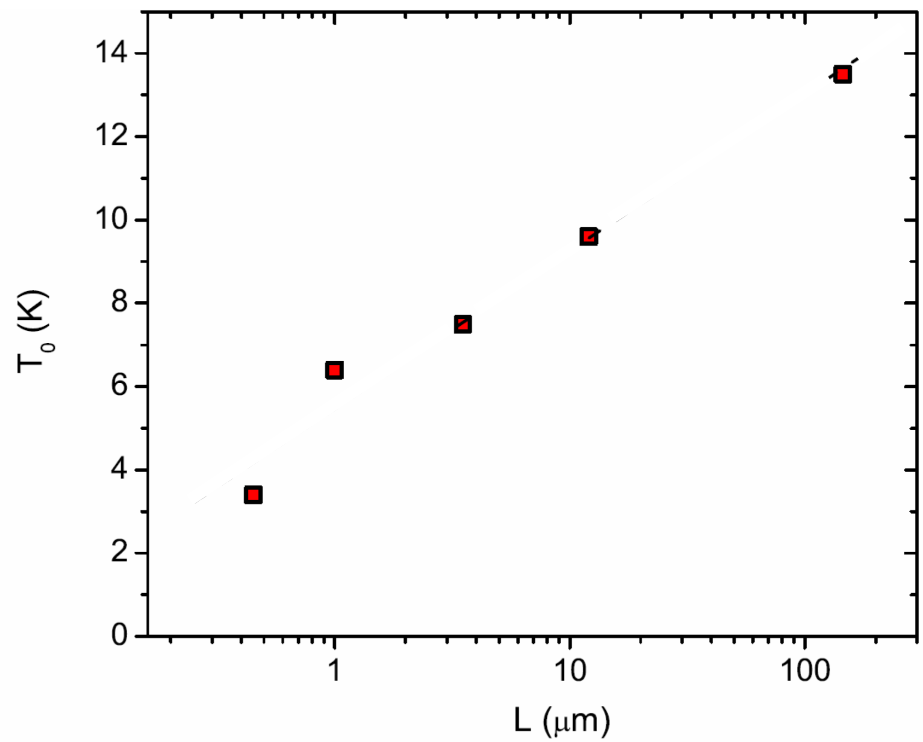




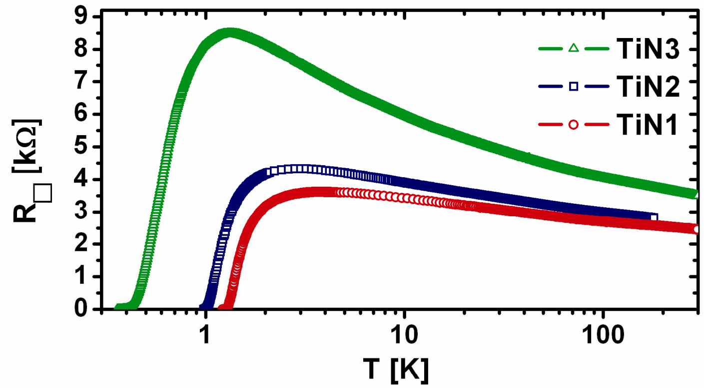
The activation energy T_0 appears to depend on the sample size: the larger the sample, the larger T_0, contrary to our experience in semiconductors.

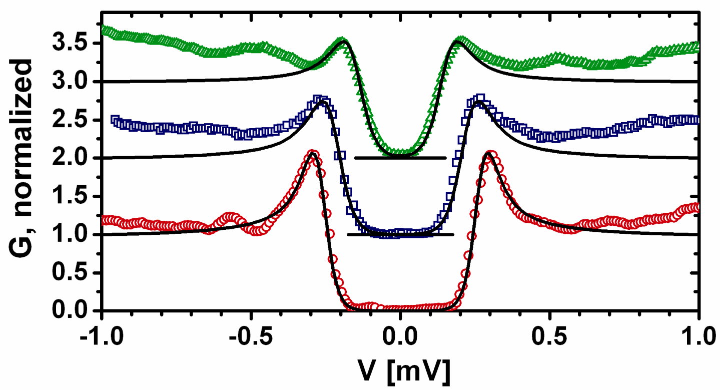
Scale dependent activation energy

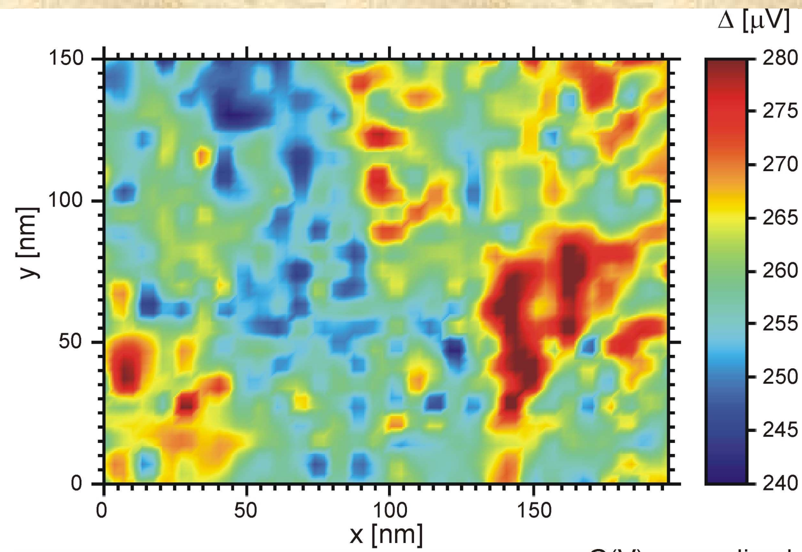


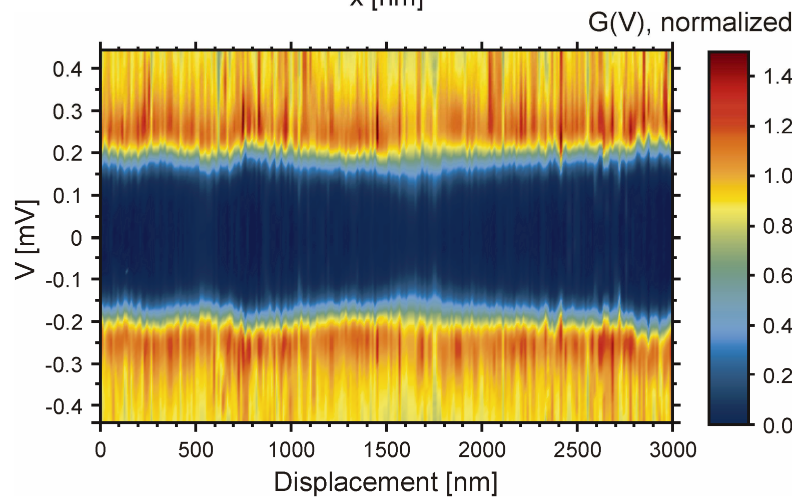


STM measurements of LDOS
inhomogeneous superconducting state (!)









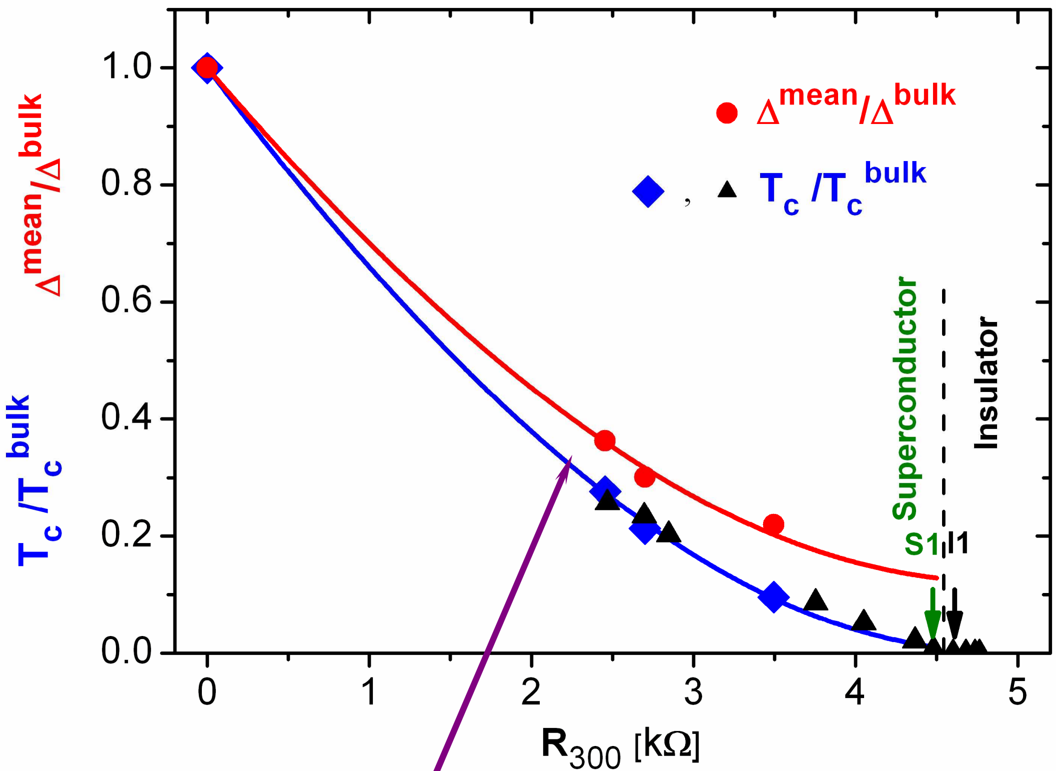
BCS fit

TiN1 – $\Delta = 260\ \mu eV$, $T_{eff} = 0.25$ K
TiN2 – $\Delta = 225\ \mu eV$, $T_{eff} = 0.32$ K
TiN3 – $\Delta = 154\ \mu eV$, $T_{eff} = 0.35$ K

in TiN3 the magnitudes of Δ scattered in the interval from 125 μeV to 215 μeV

B. Sacépé, C. Chapelier, T. B., V. Vinokur, M.R. Baklanov, M. Sanquer, PRL **101**, 157006 (2008)

T_c and Δ^{mean} : the resistance as control parameter



The observed trend of the faster decay in T_c than in Δ^{mean}, together with inhomogeneous state, offers a strong support to hypothesis that homogeneously disordered superconducting films near SIT can be viewed as granular-like superconducting structure.

T = 300 K 4.48 kΩ - S1

4.60 kΩ - I1

< 3% (!)

$$\ln\left(\frac{T_c}{T_c^{bulk}}\right) = \gamma + \frac{1}{\sqrt{2r}}\ln\left(\frac{1/\gamma + r/4 - \sqrt{r/2}}{1/\gamma + r/4 + \sqrt{r/2}}\right)$$

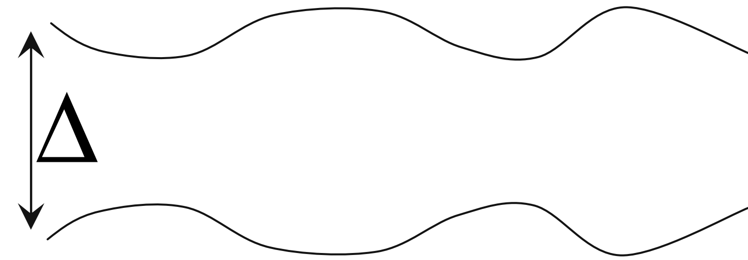
A.M. Finkel'stein, JETP Lett. 45, 46 (1987).

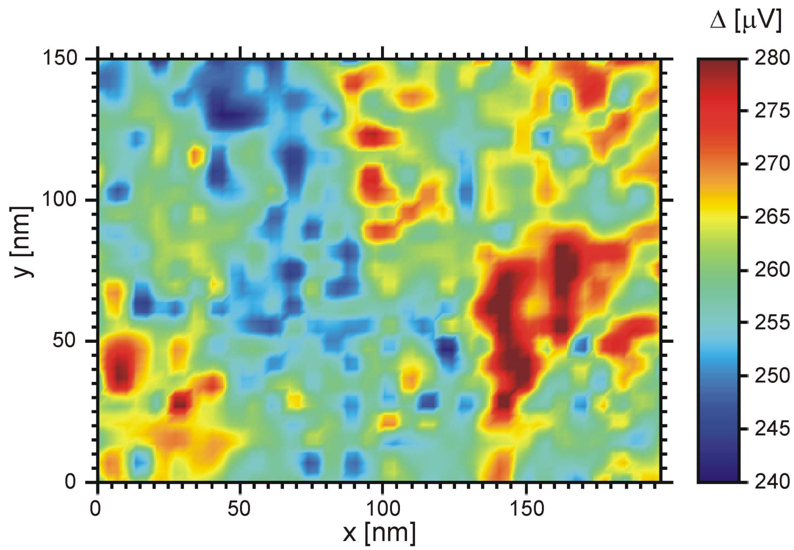
$r = R_\square e^2/(2\pi^2\hbar)$ the only fitting parameter: $\gamma \simeq 6.2$

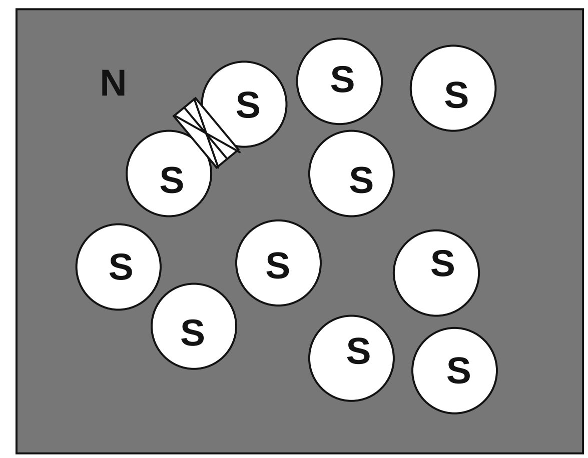
B. Sacépé, C. Chapelier, T. B., V. Vinokur, M.R. Baklanov, M. Sanquer, PRL **101**, 157006 (2008)

"Island" structure of disordered superconductor

Fluctuations in disorder lead to appearance of regions with enhanced T_c, i.e. to formation of superconducting "islands" above the 'bulk' T_c.







Localization in 2D Bose systems:

A. Larkin and V. Vinokur, PRL, 75, 4666 (1995)
A. Lopatin and V. Vinokur, PRL, 92, 67008 (2004)

The two-dimensional Coulomb gas: the model

charge distribution

$$f_{r_0}(r)$$

$$\int d^2r\, f_{r_0}(r) = 1$$

The limit $r_0 = 0$

$$f_{r_0=0}(r) = \delta(\mathbf{r})$$

With these definitions the interaction energy between two Coulomb gas charges i and j at positions $\mathbf{r}_i$ and $\mathbf{r}_j$ and with charges s_i and s_j, respectively, is given by $s_i s_j U(|\mathbf{r}_i - \mathbf{r}_j|)$, with the interaction U defined by

$$U(r) = \int d^2r' d^2r'' f_{r_0}(|\mathbf{r}-\mathbf{r}'|) V_{\lambda_c}(|\mathbf{r}'-\mathbf{r}''|) f_{r_0}(r'') \ .$$

The two-dimensional Coulomb gas

potential outside an infinitesimal test charge $\boxed{V_L(r)}$

The energy needed to separate two infinitesimal test charges with charges $+\delta t$ and $-\delta t$, respectively, by a distance r is related to $V_L(r)$ by

$$(\delta t)^2 \int d^2r' f_{r_0}(r')[V_L(r') - V_L(|\mathbf{r}-\mathbf{r}'|)]$$

$$\nabla^2 \delta t V_L = -\frac{2\pi\delta t}{\tilde{\varepsilon}} f_{r_0} - \frac{2\pi \delta t n_F}{\tilde{\varepsilon} T} V_L$$

$n_F = n_F^+ + n_F^-$ is the total density of free charges

$$\hat{V}_L(k) = \frac{1}{\tilde{\varepsilon}} \frac{2\pi}{k^2 + \lambda^{-2}} \hat{f}_{r_0}(k)$$

$$\lambda^{-2} = \frac{2\pi n_F}{\tilde{\varepsilon} T}$$

$$V_L \sim \begin{cases} \dfrac{1}{\sqrt{r}} e^{-r/\lambda}, & \lambda \neq \infty \,, \\ -(1/\tilde{\varepsilon}) \ln(r), & \lambda = \infty \end{cases}$$

The two-dimensional Coulomb gas: KT arguments

$$F_1 = \frac{1}{2\tilde{\varepsilon}} U(0) - TS$$

$$\simeq \frac{1}{2\tilde{\varepsilon}} \ln(R/r_0) - T\ln(R^2/r_0^2) \simeq \left[\frac{1}{4\tilde{\varepsilon}T} - 1\right] \ln(R^2/r_0^2)$$

$$\boxed{T_c = 1/4\tilde{\varepsilon}}$$

$$\langle \Delta n(r) \Delta n(0) \rangle \sim \begin{cases} \left[\dfrac{1}{r}\right]^{1/T\tilde{\varepsilon}}, & T < T_c \\ e^{-r/\lambda}, & T > T_c \end{cases}$$

screening length λ $\quad \lambda^{-2} - \lambda_c^{-2}/\tilde{\varepsilon} = 2\pi n_F/(T\tilde{\varepsilon})$

$$V_L(k=0) = \lambda_c^{-2} \text{ for } n_F = 0$$

The two-dimensional Coulomb gas

Fourier transforms $\hat{\varepsilon}(k)$ and $\hat{V}_L(k)$ are related by

$$\hat{V}_L(k) = \frac{2\pi}{k^2\hat{\varepsilon}(k)} \hat{f}_{r_0}(k)$$

in the $k \to 0$ limit

$$\frac{1}{\hat{\varepsilon}(0)} = \lim_{k\to 0} \frac{1}{\tilde{\varepsilon}} \frac{1}{1+(\lambda k)^{-2}}$$

$$= \begin{cases} \dfrac{1}{\tilde{\varepsilon}} & \text{for } T < T_c \, , \\ 0 & \text{for } T > T_c \end{cases}$$

The two-dimensional Coulomb gas: BKT summary

$$\hat{U}_{\text{eff}}(k) = \frac{1}{\tilde{\varepsilon}} \frac{2\pi(\hat{f}_{r_0})^2}{k^2+\lambda^{-2}}$$

Screening length

$$\lambda = r_0 \left[\frac{4\pi g z}{\tilde{\varepsilon} T} \right]^{1/[1/(2\tilde{\varepsilon}T)-2]}, \quad T > T_c$$

$$\lambda \sim r_0 e^{\text{const}/\sqrt{T-T_c}}, \quad T \to T_c^+$$

$$\lambda = \infty, \quad T < T_c^+$$

Dielectric constant due to bound pairs

$$\tilde{\varepsilon} = \frac{1}{4T_c}(1 + c_> \sqrt{|1-T/T_c|}), \quad T \to T_c^+$$

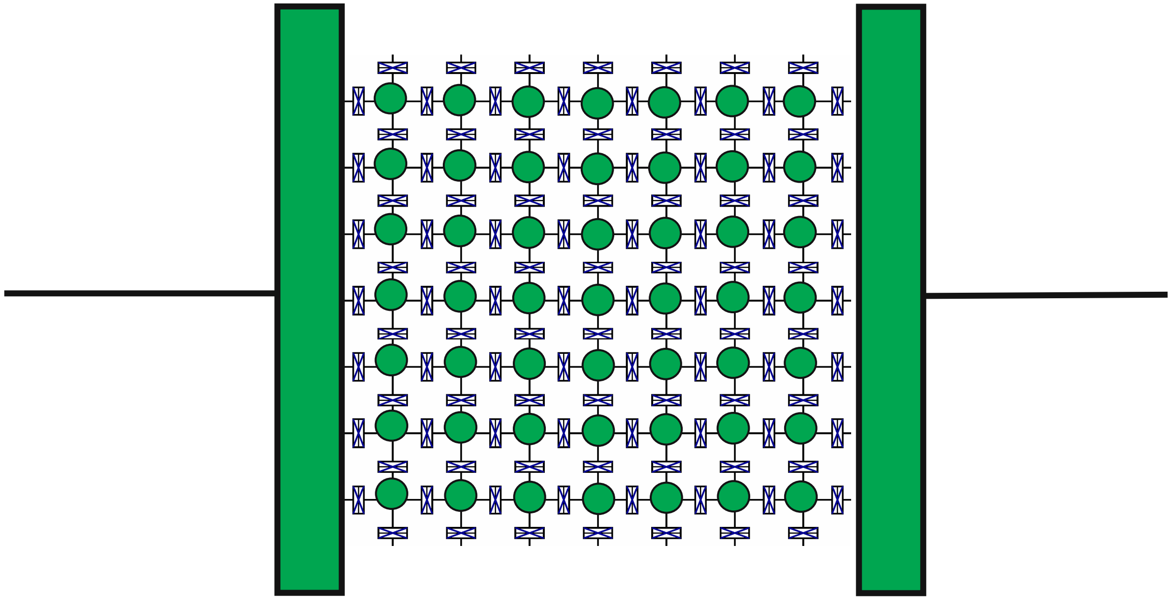
$$\tilde{\varepsilon} = \frac{1}{4T_c}(1 - c_< \sqrt{|1-T/T_c|}), \quad T \to T_c^-$$

Universal jump

$$1/[T\hat{\varepsilon}(0)] = 0, \quad T \to T_c^+$$

$$1/[T\hat{\varepsilon}(0)] = 4, \quad T \to T_c^-$$

An exemplary system for study BKT: Josephson junction array



Classical two-dimensional arrays ($E_C \ll E_J$) can be shown to be isomorphic to a two-dimensional XY spin system — they are physical representations of the XY model, which is a two-dimensional lattice of spins free to rotate in the XY plane. Since they are systems in which the various parameters are well known and easily controlled, they serve as models for investigating statistical mechanics — indeed, we can literally do "statistical mechanics on a chip."

$$H = -\sum_{\langle ij \rangle} E_J \cos\left(\phi_j - \phi_i - \frac{2\pi}{\Phi_o}\int_i^j A \cdot dr\right)$$

Vortices in JJA

$$2\pi\left(n - \frac{\Phi_{\text{total}}}{\Phi_o}\right) = \sum_{\text{junctions}} (\phi_j - \phi_i)$$

$$\boldsymbol{B} = 0.$$

$$\sum_{\text{junctions}} (\phi_j - \phi_i) = 0$$

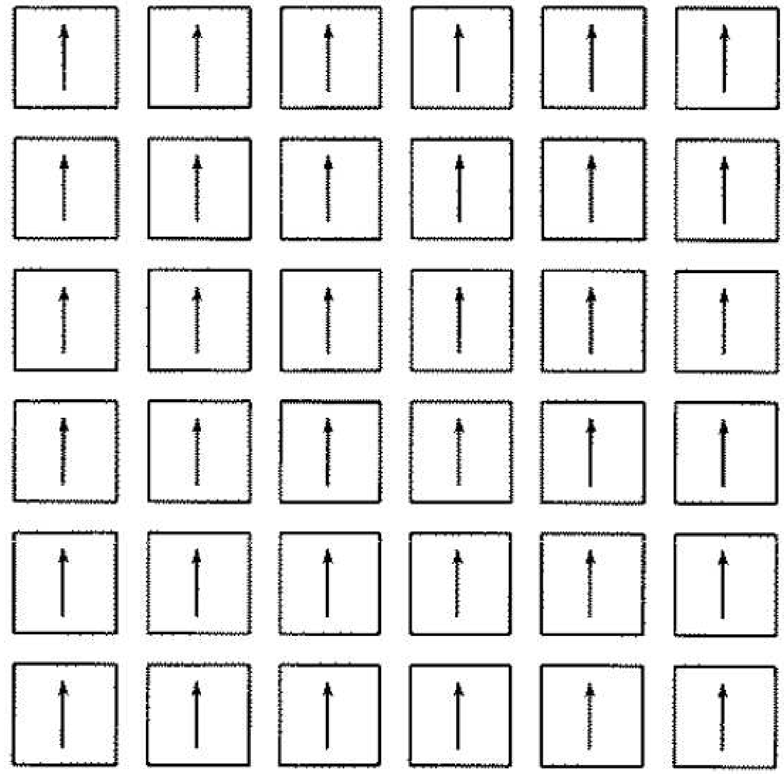
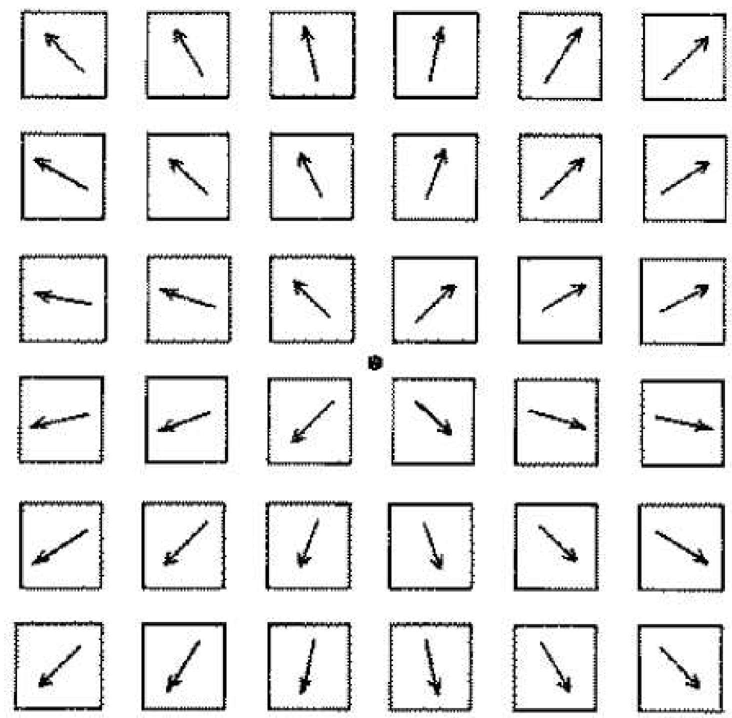


FIG. 13. Schematic of the uniform phase configuration state (the $f = \Phi/\Phi_o = 0$ ground state). The angle between the arrow on an island and the $+x$-axis represents the phase of the superconducting order parameter of that island. The junctions connecting the islands are omitted for simplicity.

Vortices in JJA

$$\sum_{\text{junctions}} (\phi_j - \phi_i) = 2\pi$$

Phase configuration for a vortex.



$$E_v = \pi E_J \ln\left(\frac{L}{a}\right)$$

where L is the size of the system and a is the lattice spacing

BKT physics

the energy of two vortices of opposite circulation, bound together a distance r apart

$$E_p = 2\pi E_J \ln\left(\frac{r}{a}\right)$$

$$\Delta F = E_v - T\Delta S_v$$

$$\Delta S_v = k_B \ln\left(\frac{L^2}{a^2}\right)$$

$$\Delta F = (\pi E_J - 2k_B T)\ln\left(\frac{L}{a}\right)$$

BKT physics

$$P_1 \propto e^{-\frac{1}{k_B T}(\pi E_J - 2k_B T)\ln\left(\frac{L}{a}\right)} = \left(\frac{L}{a}\right)^{2-\frac{\pi E_J}{k_B T}}$$

$$T_{KT} = \frac{\pi E_J}{2k_B}$$

the mean-square separation
of the two bound vortices

$$\langle r^2 \rangle = \frac{\int_a^\infty \exp[-2\pi E_J/k_B T] r^2 2\pi r dr}{\int_a^\infty \exp[-2\pi E_J/k_B T] 2\pi r dr} = a^2 \frac{2\pi E_J - 2k_B T}{2\pi E_J - 4k_B T}$$

BKT physics: I-V characteristics (vortices)

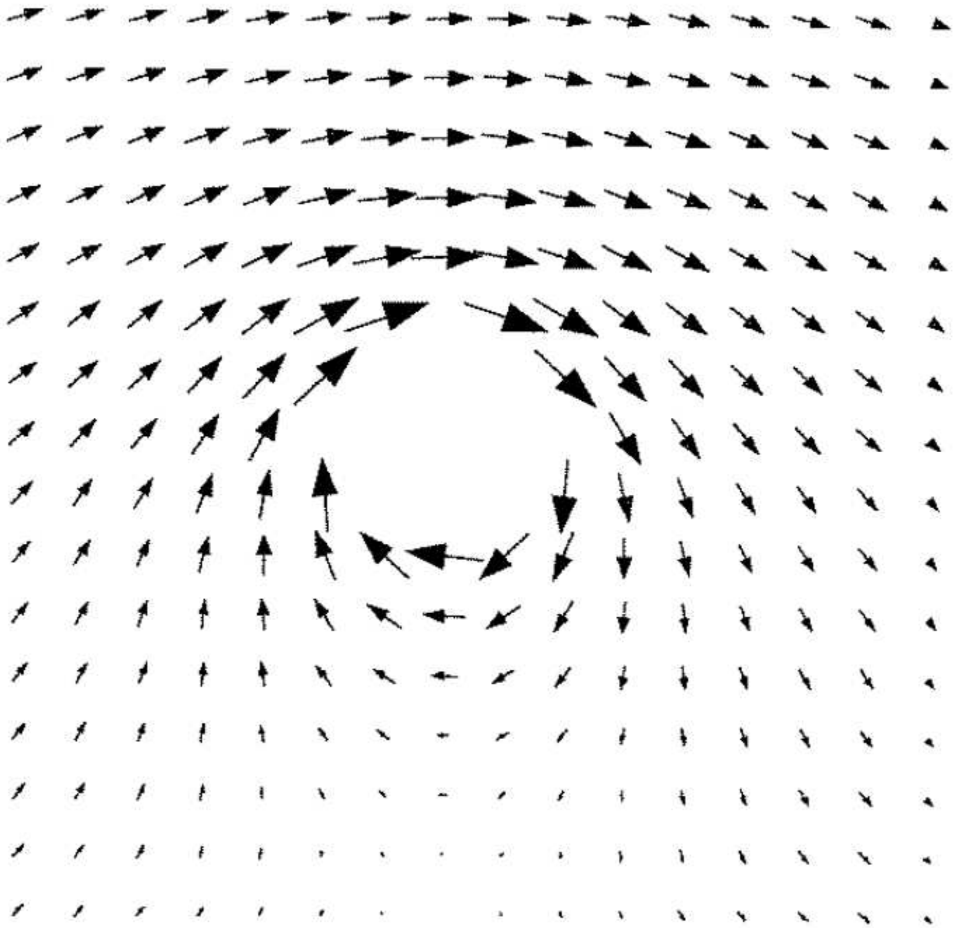


FIG. 16. The superposition of the circulating vortex current and an externally applied transport current. (In this figure arrows indicate the magnitude and direction of circulating current, not phases. The transport current is moving to the right.) The combination yields a current gradient perpendicular to the transport current, leading to a force analogous to a Bernoulli force in liquids or the Lorentz force on a current-carrying wire in a magnetic field.

Magnus force

$$F_L = \Phi_o j_{2d} = \left(\frac{h}{2e}\right)\left(\frac{i}{a}\right)$$

friction force

$$\boldsymbol{F}_D = -\eta \boldsymbol{v}_v$$

$$\eta = \left(\frac{\Phi_o}{a}\right)^2 \frac{1}{2R_o}$$

$$\langle v_v \rangle = \frac{1}{\eta}\frac{\Phi_o}{a} i.$$

BKT physics: I-V characteristics (vortices)

$$\langle V \rangle = 2R_o(n_f a^2)\frac{L}{W} I$$

$$n_f = \begin{cases} = 0 & T < T_{KT} \\ \geq 0 & T \geq T_{KT} \end{cases}$$

BKT physics: I-V characteristics (vortices)

THE RESISTIVITY ABOVE THE TRANSITION TEMPERATURE

$$\xi_+ = c_1 a e^{[c_2/(\tilde{T} - \tilde{T}_{KT})]^{1/2}}$$

where c_1 and c_2 are constants of order 1

$$\tilde{T} = \frac{2\pi k_B T}{\Phi_o i_c(T)} \qquad n_f(T) = b_1 a^{-2} e^{[b_2/(\tilde{T} - \tilde{T}_{KT})]^{1/2}}$$

$$b_2 = 4c_2$$

$$R(T) = \frac{V}{I} = \begin{cases} 2R_o \dfrac{L}{W} b_1 \exp[-b_2/(\tilde{T} - \tilde{T}_{KT})]^{1/2} & T > T_{KT} \\ 0 & T \leqslant T_{KT} \end{cases}$$

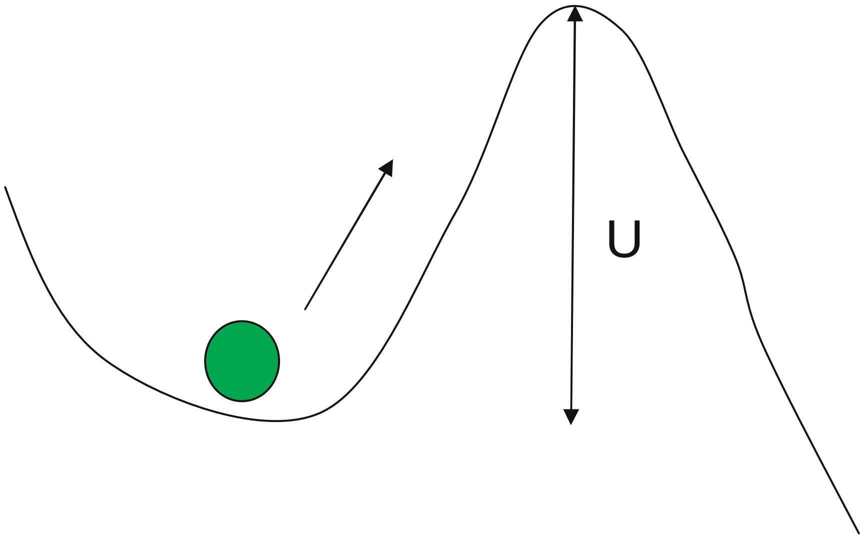
$$I_o^{-2}\left(\frac{E_J(T)}{10 k_B T}\right)$$

BKT physics: dynamics

Overdamped dynamics $$\gamma \frac{\partial \varphi}{\partial t} = -\frac{\delta H}{\delta \varphi} + \xi(\varphi, t)$$

Kolmogorov, 1937: crystal growth
Kramers, 1940: 1D diffusion
Zeldovich, 1943: decay of metastable state (nuclei growth) in 1-order phase transition
Langer, 1967: same
Ventzel and Fridland, 1969

Search for an instanton solution:

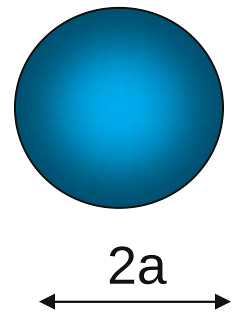


Mott VRH
Vortex creep
BKT dynamics

$$v \propto \exp(-U/T)$$

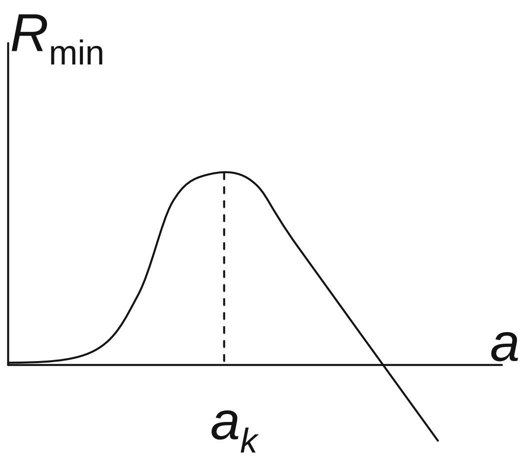
BKT physics: dynamics

$$f_0(a) \sim \exp\left\{-\frac{R_{\min}(a)}{T}\right\}$$



$$R_{\min} = -\frac{8\pi a^3 \alpha}{3a_\kappa} + 4\pi a^2 \alpha$$

$$R_{\min} = \frac{4\pi}{3}\alpha a_\kappa^2 - 4\pi\alpha(a - a_\kappa)^2$$



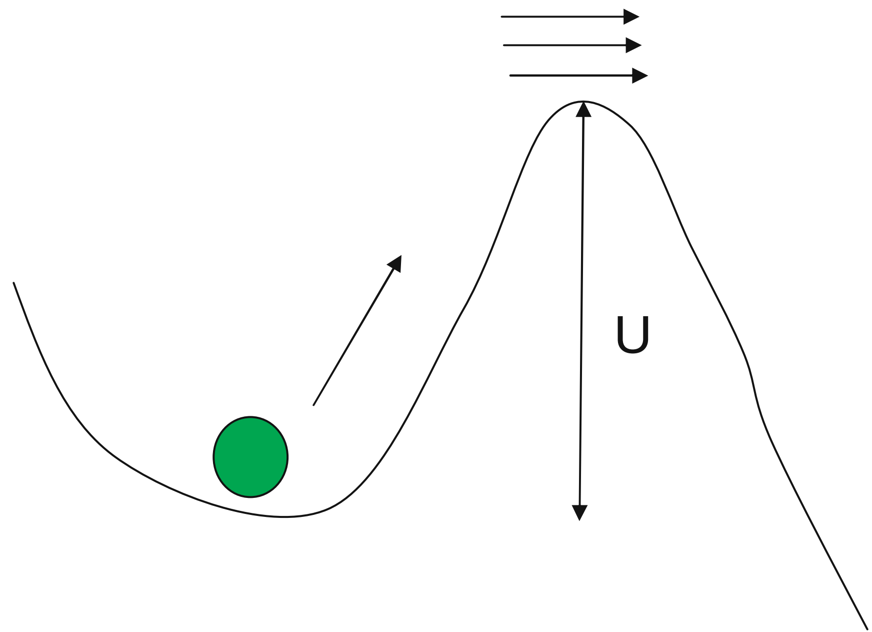
$$f_0(a) = f_0(a_\kappa)\exp\left\{\frac{4\pi\alpha}{T}(a - a_\kappa)^2\right\},$$

$$f_0(a_\kappa) = \mathrm{const}\cdot\exp\left\{-\frac{4\pi\alpha a_\kappa^2}{3T}\right\}$$

BKT physics: dynamics

$$\frac{\partial f}{\partial t} = -\frac{\partial s}{\partial a},$$

$$s = -B\frac{\partial f}{\partial a} + Af$$



$$A = -\frac{B}{T} R'_{\min}(a)$$

$$-Bf_0 \frac{\partial}{\partial a}\frac{f}{f_0} = s.$$

$$\frac{f}{f_0} = -s\int \frac{da}{Bf_0} + \text{const}.$$

BKT physics: dynamics

$$\frac{f}{f_0} = s \int_a^\infty \frac{da}{Bf_0}, \qquad \frac{1}{s} = \int_0^\infty \frac{da}{Bf_0}$$

$$s = 2\sqrt{\frac{\alpha}{T}}\, B(a_\kappa)\, f_0(a_\kappa)$$

$$f_0(a_\kappa) = \text{const} \cdot \exp\left\{-\frac{4\pi\alpha a_\kappa^2}{3T}\right\}$$

BKT physics: I-V characteristics (vortices)

$$j = I/Ma$$

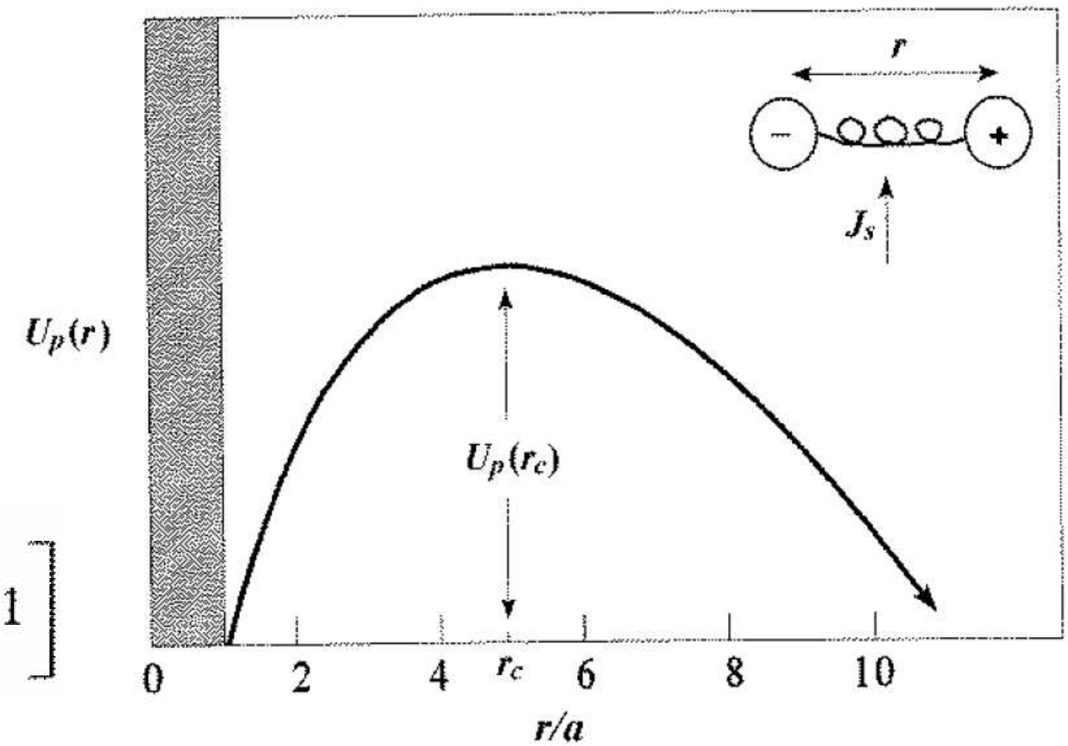
The Lorentz force on a vortex $\quad \boldsymbol{F}_L = \pm\Phi_o \boldsymbol{j} \times \hat{z}$

an additional potential energy $\quad U_L = -j\Phi_o r$

$$U_p = 2\pi E_J \ln\left(\frac{r}{a}\right) - j\Phi_o r$$

$$r_c = \left[\frac{i_c}{i}\right] a$$

$$U_c = U_p(r_c) = 2\pi E_J \left[\ln\frac{i_c}{i} - 1\right]$$



BKT physics: I-V characteristics (vortices)

$$R_e \cong \left(\frac{2ei_cR_o}{\hbar}\right)e^{-U_p/k_BT} = \left(\frac{2ei_cR_o}{\hbar}\right)\left(\frac{i_c}{i}\right)^{-2\pi E_J/k_BT}$$

recombination of pairs

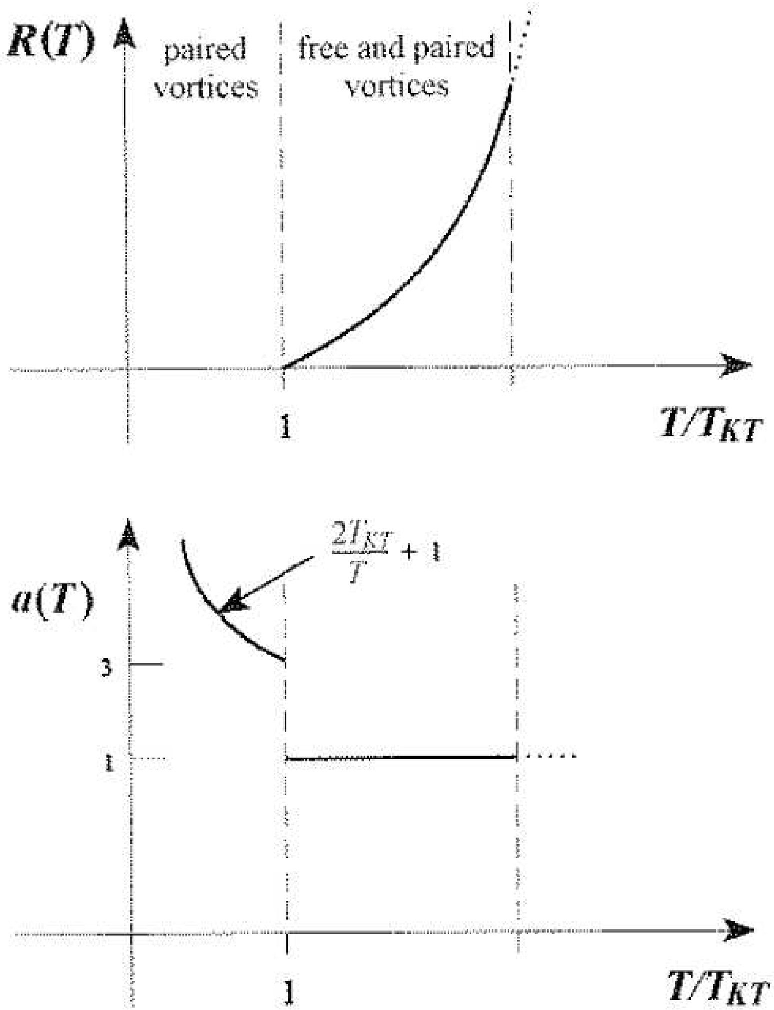
In equilibrium $\quad R_e - \alpha n_f^2(T < T_{KT}, I) = 0$

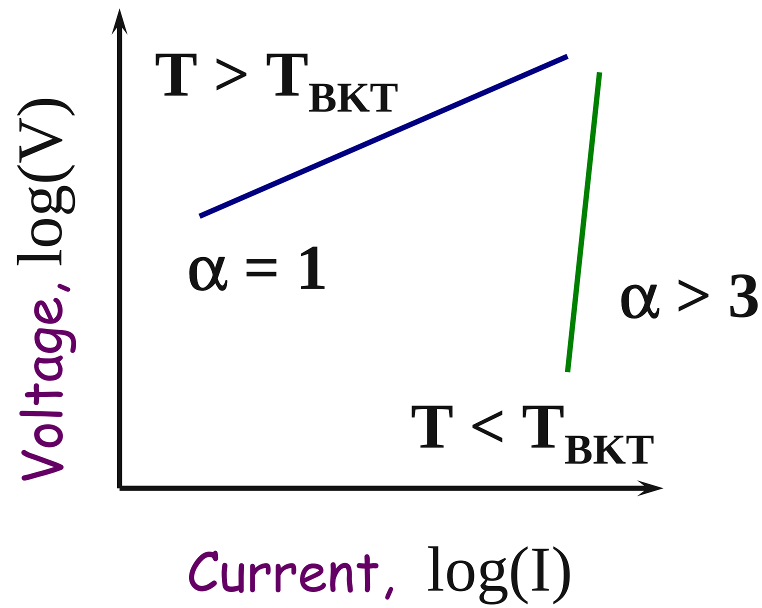
$$n_f(T < T_{KT}, I) = \left(\frac{R_e}{\alpha}\right)^{1/2} \approx \left(\frac{2\pi i_c(T)R_o}{\Phi_o\alpha}\right)^{1/2}\left(\frac{i}{i_c(T)}\right)^{\pi E_J/k_BT}$$

$$V = 2R_o^{3/2}La\left(\frac{2\pi}{\Phi_o\alpha}\right)^{1/2}(i_c(T))^{1/2-\pi E_J/k_BT}(i)^{\pi E_J/k_BT+1}$$

$$\boxed{V \propto I^{a(T)}} \qquad a(T) = \frac{\pi E_J(T)}{k_BT} + 1 = \frac{2T_{KT}}{T} + 1$$

BKT physics: *I-V* characteristics (vortices)



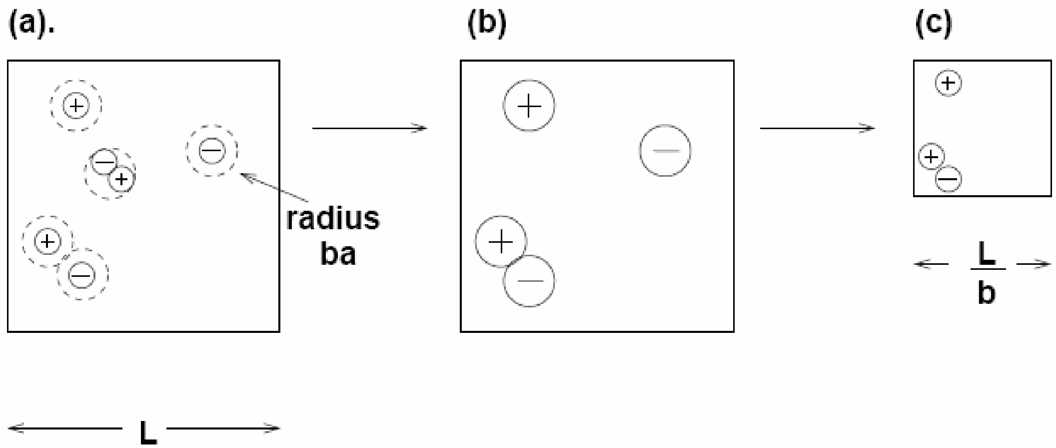


$$V \propto I^{\alpha(T)}$$

(Tatyana Baturina's presentation)

BKT physics: renormalization of dielectric constant

Fig. 1: steps in K.-T. renormalization



When we consider the statistics of a widely separated pair, as in the upper right corner, this will be controlled by the spin stiffness appropriate to that (nearly macroscopic) length scale. That stiffness will have been renormalized by the close pair fluctuations that occur on short length scales. The K.T. renormalization group asks us to imagine an infinite regression of such pairs-within-pairs. *

$$E_P(r) = \int_{r'=a}^{r'=r} \frac{2\pi E_J(T)}{\varepsilon(r')}\frac{1}{r'}\,dr' \qquad E_P(r, T) = 2\pi E_J^*(T)\ln(r/a)$$

BKT physics: superconducting films, finite size effects

interaction of vortices in superconducting films is only logarithmic over distances shorter than the Pearl screening length $\Lambda = \lambda^2/d$ where d is the film thickness and λ is the London penetration depth [5]. The size effects change the transport behavior at $T < T_{\mathrm{BKT}}$ where $E \propto I^{1+\alpha}$ only at currents $I > I_1 \sim c\epsilon/\phi_0$, for which the size of a dissociating vortex-antivortex pair, $\ell_c = 2cw\epsilon/\phi_0 I$, is smaller than the film width, w. Here, $\alpha = 2\epsilon/T$, $\epsilon = \phi_0^2/16\pi^2\Lambda$ is the vortex energy scale, ϕ_0 is the flux quantum, and c is the speed of light. For $I < I_1$, the E-I characteristic becomes Ohmic [2,3,6–8].

BKT physics: charges vs. vortices

charge: e

electric field: $\vec{E}$

force: $e\vec{E}$

driving force: electric field (voltage, V)

response: current I

vortices: "charge" is $\frac{\Phi_0}{c}$

current density: j

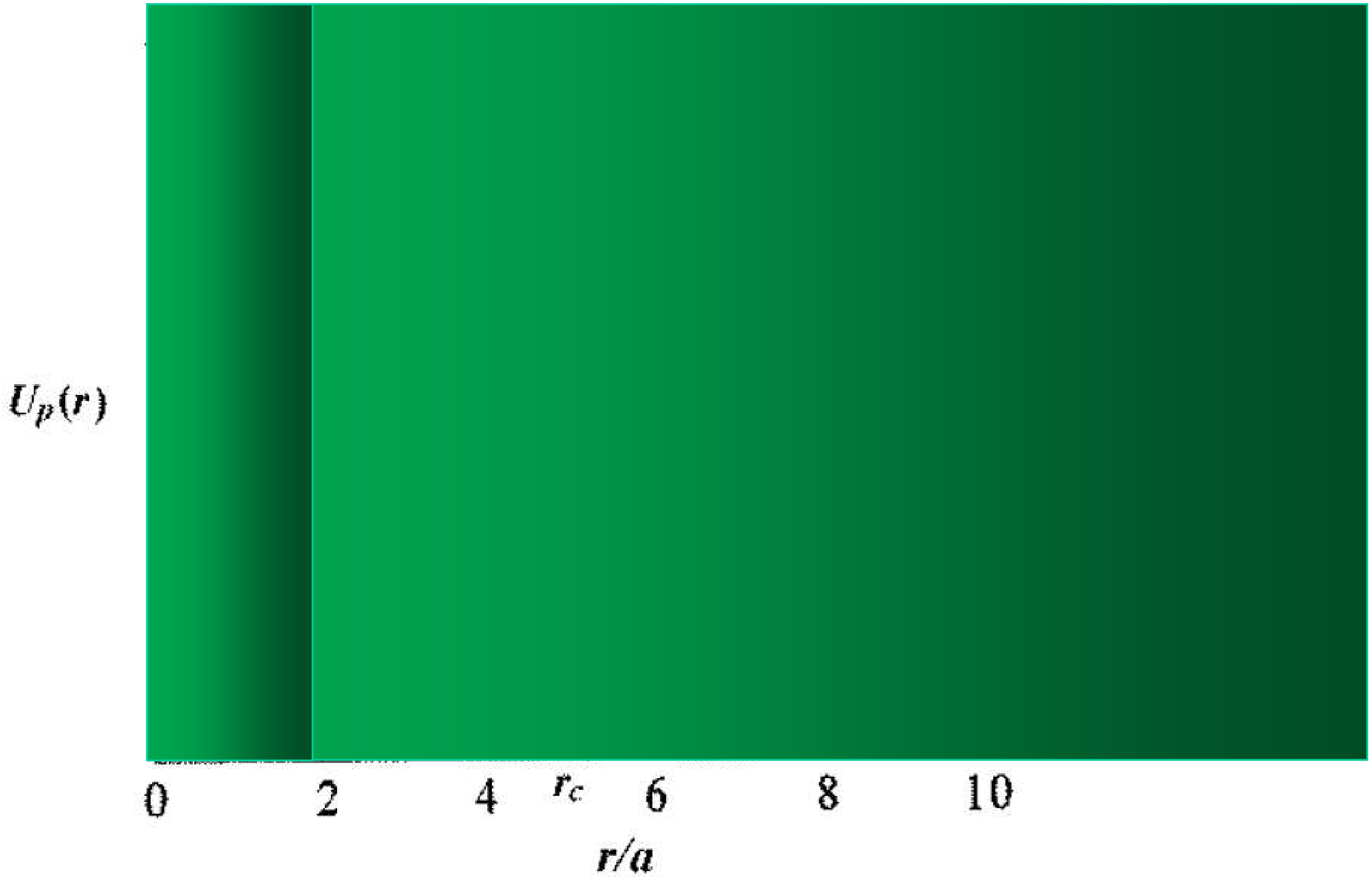
force (Magnus): $\frac{1}{c} j\Phi_0 L$

driving force: current

response: voltage, $V = \eta\left(\frac{1}{c} j\Phi_0 L\right) n_V$

(moving magnetic field induces electric field)

BKT physics: finite sample size effect



Q: How will the response change in a small sample?

A: The applied force is not sufficient to unbind the dipole, thus only the pairs that have already dissociated will contribute to transport

BKT physics: finite sample size – penetration from the edge

PRL **100**, 227007 (2008) PHYSICAL REVIEW LETTERS week ending 6 JUNE 2008

Size Effects in the Nonlinear Resistance and Flux Creep in a Virtual Berezinskii-Kosterlitz-Thouless State of Superconducting Films

A. Gurevich[1] and V. M. Vinokur.[2]

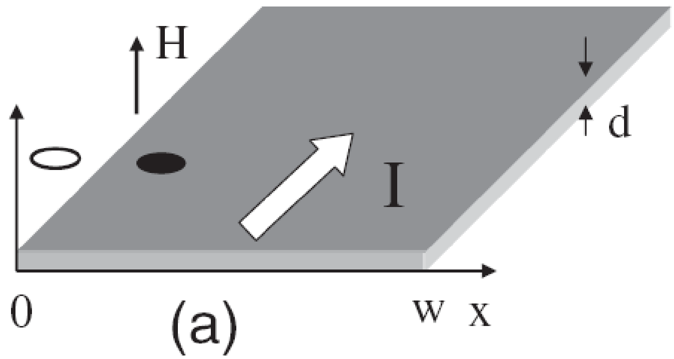


FIG. 1. A thin film in a perpendicular field H. The black dot shows a vortex moving across the film, and the empty circle shows the antivortex image (a). Geometries for probing the

$$\eta \dot{x} + U'(x) = \zeta$$

$\zeta(t)$ describes thermal noise

$$U(x) = U_0 - U_m$$

vortex self-energy $U_0(x)$

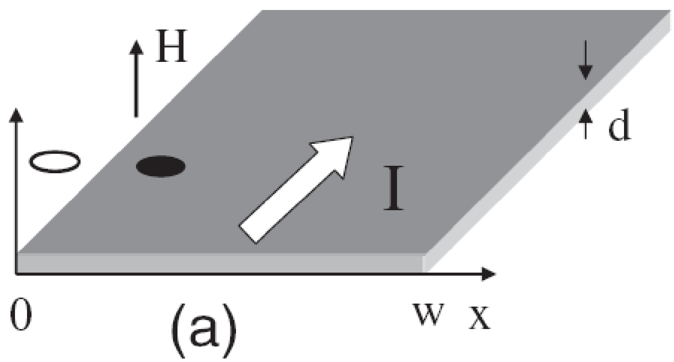
$$U_m = (\phi_0/c) \int_0^x J(u)du$$

$$J(x) \longrightarrow \int_0^w \frac{J(u)du}{u-x} + 4\pi\Lambda\partial_x J = -cH \qquad I = \int_0^w J(x)dx$$

BKT physics: finite sample size – penetration from the edge

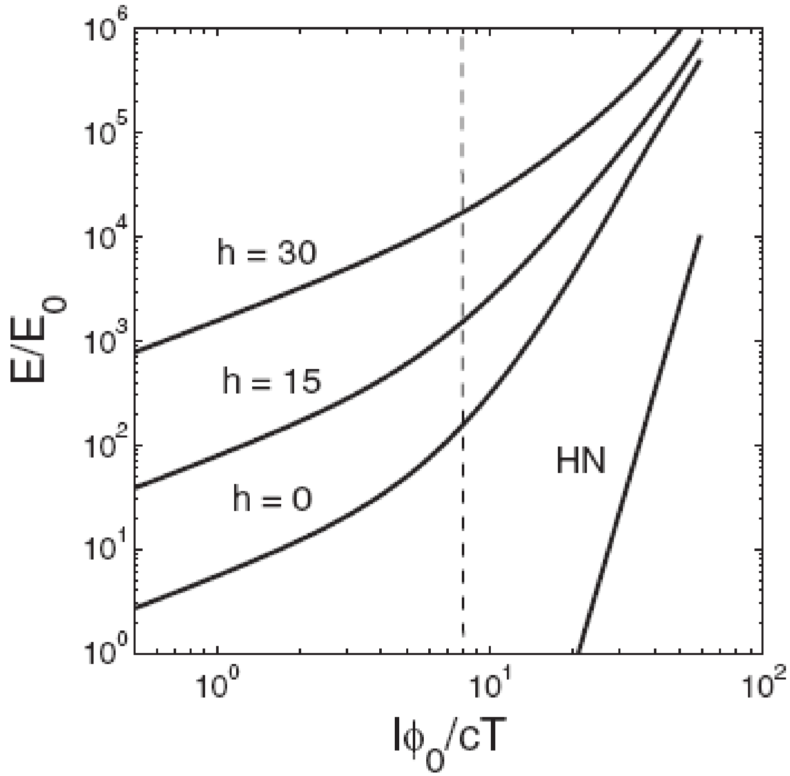
$$E \propto I^{1+\alpha}$$

$$\alpha = \begin{cases} 2\beta, & \text{classic BKT} \\ 2\beta + 1, & \text{wide films, } w > \Lambda \\ \beta, & \text{narrow films, } w < \Lambda \end{cases}$$



$$\beta = \epsilon / T$$

BKT physics: finite sample size – penetration from the edge

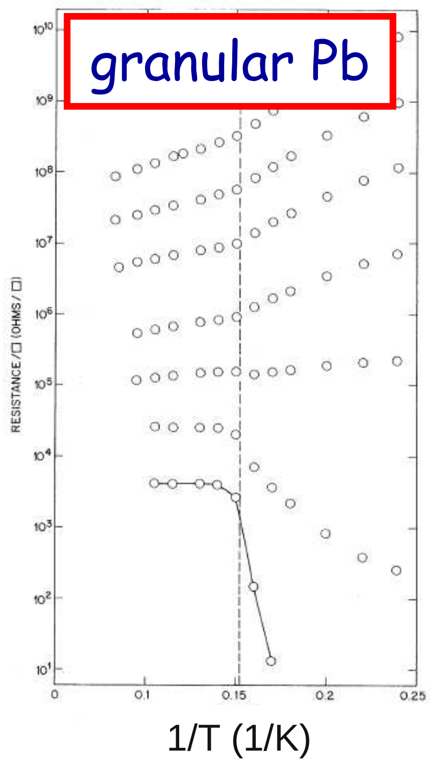


$$\beta = \epsilon / T$$

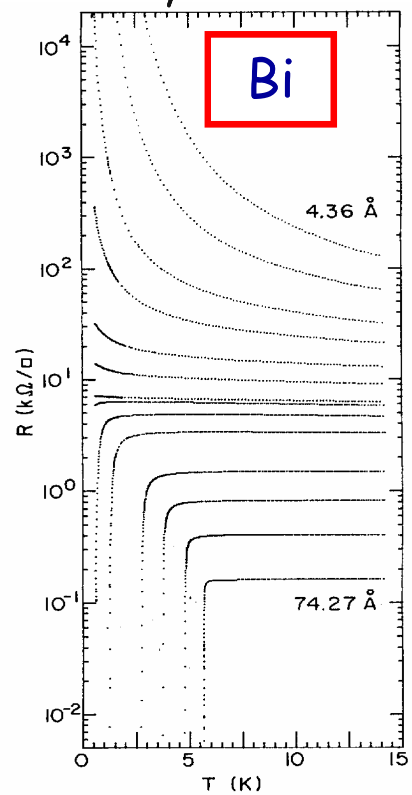
FIG. 3. E-J curves calculated from Eq. (8) for $\beta = 4$, $w = 20\xi \ll \Lambda$, and different fields $h = H\phi_0 w^2/8\pi\Lambda T$. Here, $j = I\phi_0/cT < j_s = 2\beta w/e\xi$ and $E_0 = \pi c \rho_n T(\beta - 1) \times (\pi\xi/w)^\beta / dw\phi_0$. The line labeled by HN shows the Halperin-Nelson result, $E_2(I)$ [2]. The critical pair length ℓ_c exceeds w in the region $j < 2\beta$ left of the dashed line.

Superconductor-insulator transition

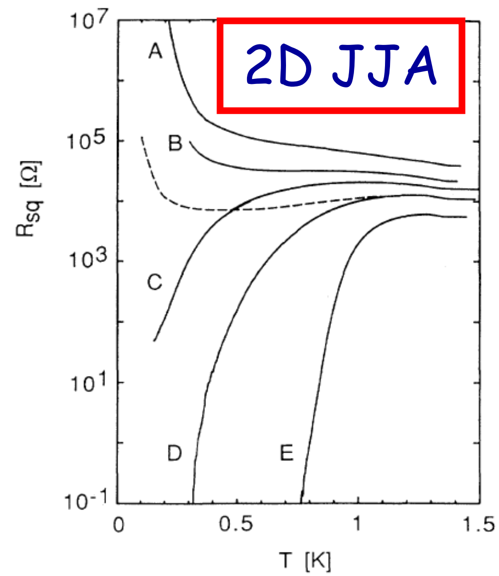
Altering various parameters of the system, such as tunnel resitance and transmittance in JJAs, conditions of deposition, chemical composition, and thickness of the films, one can drive the system directly from the superconducting to insulating state.



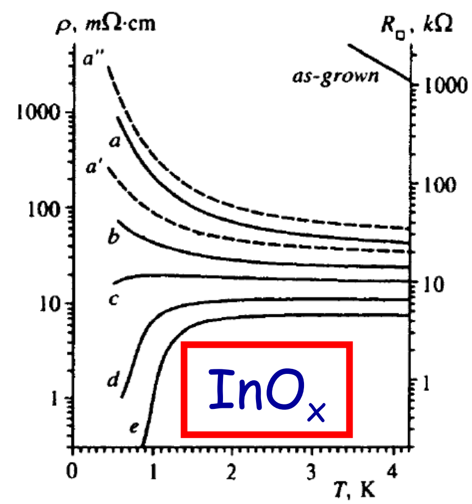
R.C. Dynes, J.P. Garno, and J.M. Rowell (1978)



D.B. Haviland, Y. Liu, and A.M. Goldman (1989)

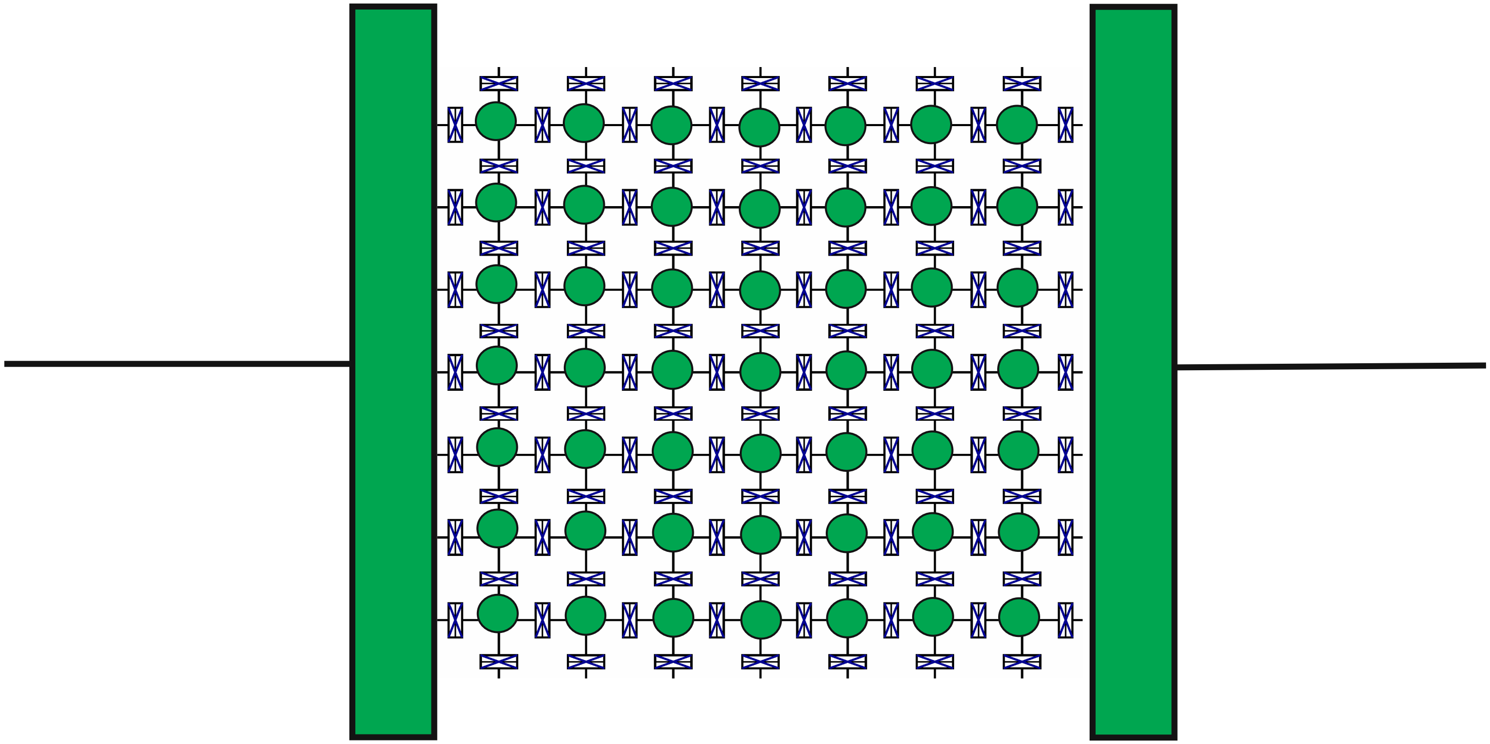


L.J. Geerligs, M. Peters, L.E.M. de Groot, A. Verbruggen, and J.E. Mooij (1989).



V.F. Gantmakher, M.V. Golubkov, J.G.S. Lok, and A.K. Geim (1996)

An exemplary system demonstrating SIT



Mechanism of the transition:

Coulomb blockade

$$E_c = \frac{(2e)^2}{2C} : \text{ charging energy}$$

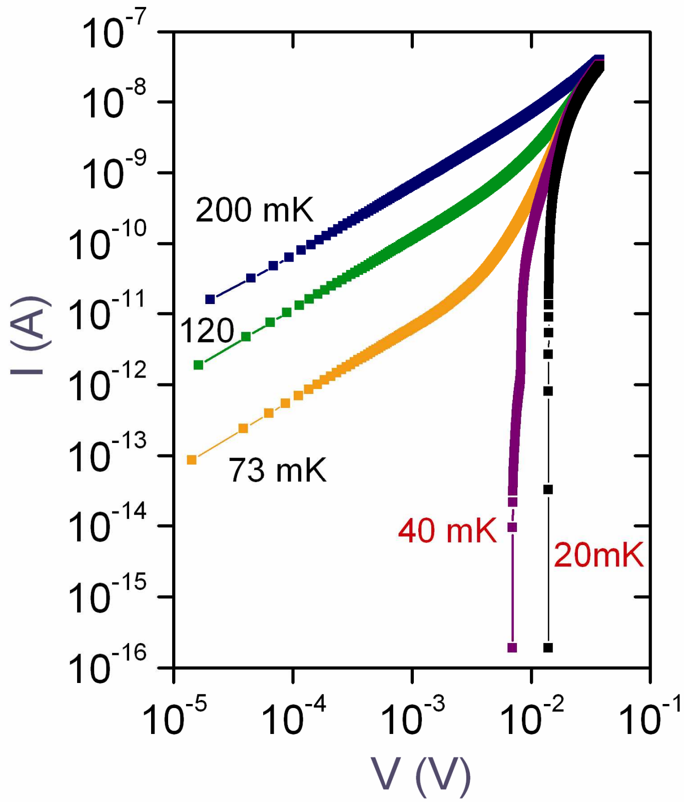
$$E_J : \text{ Josephson coupling energy}$$

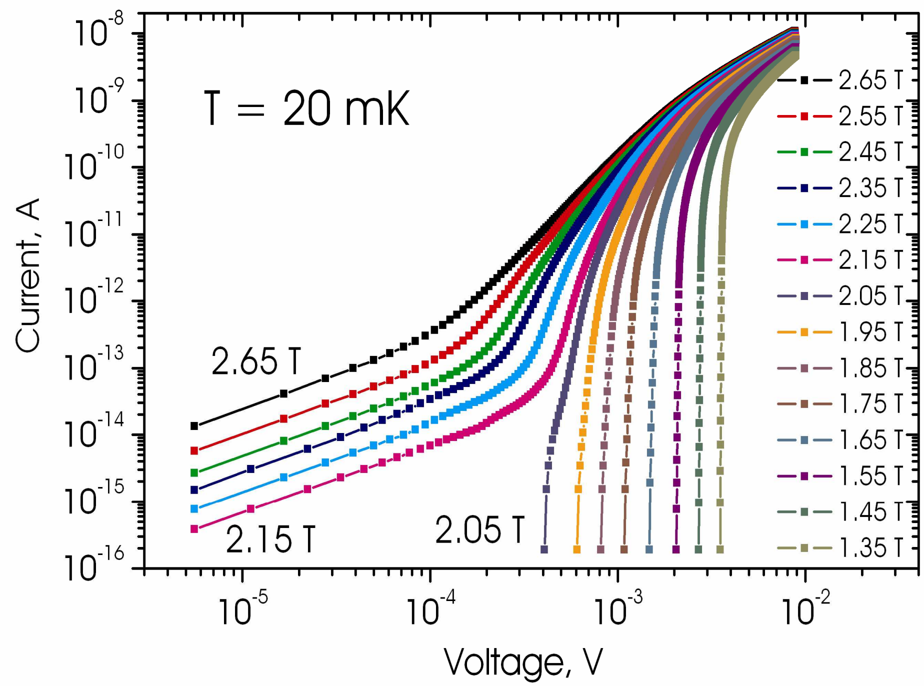
$$E_c > E_J : \text{ insulator}; \quad E_c < E_J : \text{ superconductor}$$

Disorder-driven Superconductor- Insulator transition in 2D-SC

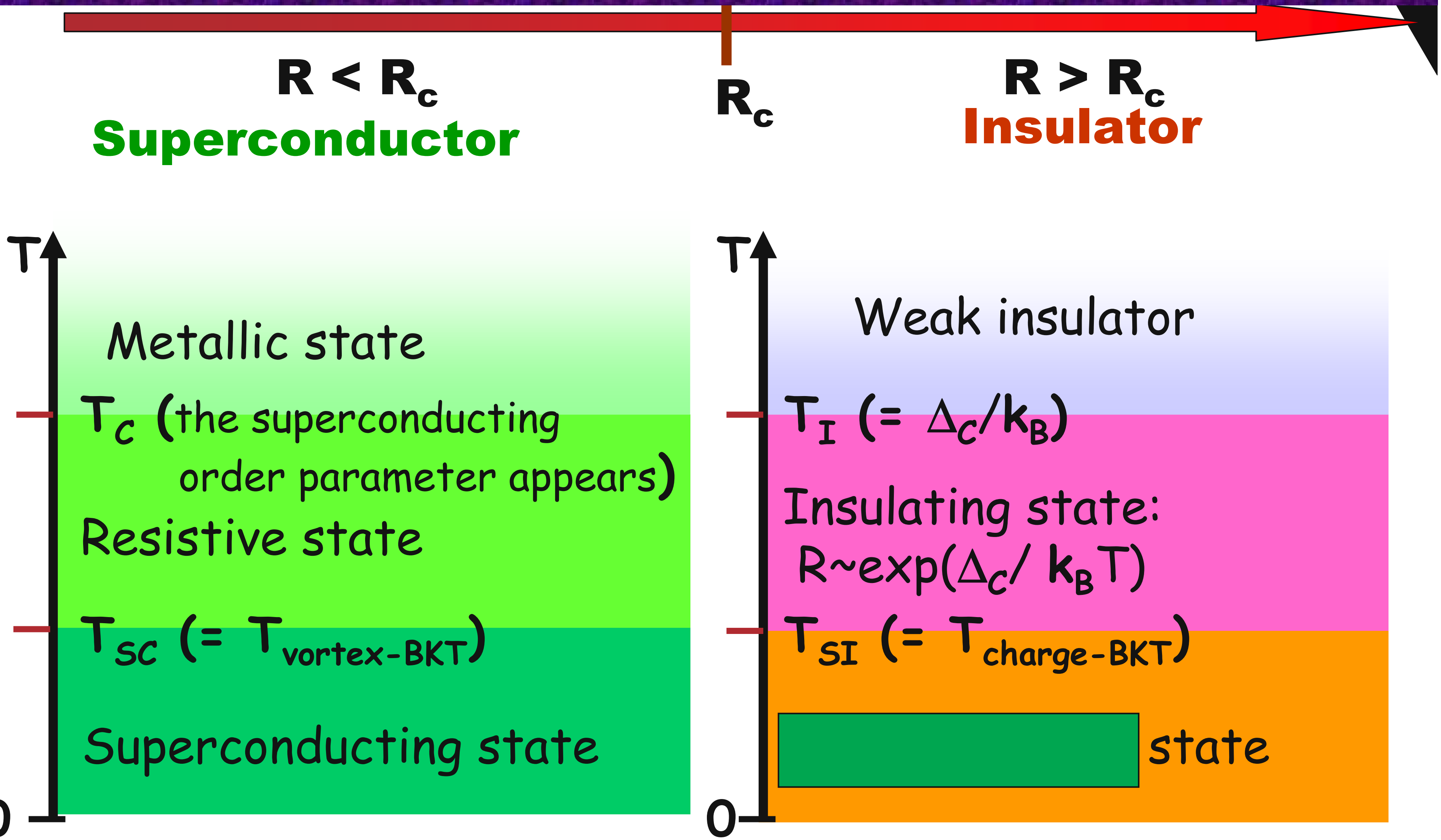
R < R_c — **R_c** — **R > R_c**

Superconductor | **Insulator**





Disorder-driven Superconductor- Insulator transition in 2D-SC:
The low-temperature BKT phase in the system of localized Cooper pairs



Transport in a superinsulating state

Q: How will the response change in a small sample?
A: The applied force is not sufficient to unbind the dipole, thus only the pairs that have already dissociated will contribute to transport

$$I = 2en_p v = 2e^2 n_p \eta V / L$$

Linear response approximation: we have to find n_p

Transport in a superinsulating state

Local Cooper pairs charge density $n_s(\mathbf{r})$:

$$\Delta_c = E_c \int d\mathbf{r}\, n_s^2(\mathbf{r})$$

probability for such a local density to appear

$$\exp[-n_s^2(\mathbf{r})/(2\langle \delta n^2 \rangle)]$$ $\langle \delta n^2 \rangle$ is the mean square fluctuation

$I_s \propto$ the total probability :

$$\aleph_s \propto \prod_{\mathbf{r}} \exp[-n_s^2(\mathbf{r})/(2\langle \delta n^2 \rangle)]$$

$$= \exp\{-[1/(2\langle \delta n^2 \rangle)] \int d\mathbf{r}\, n_s^2(\mathbf{r})\}$$

$$= \exp\{-[\Delta_c/(2E_c \langle \delta n^2 \rangle)]\}$$

Transport in a superinsulating state

$\langle \delta n^2 \rangle = f$: the filling factor of a SC island

$$f = \frac{1}{\exp(E_c/T) - 1}, \quad E_c \text{ is the energy of placing}$$

Cooper pair at the island

$$T \qquad E_c : \langle \delta n^2 \rangle = f \approx T/E_c \quad \Longrightarrow \quad R \propto \exp(\Delta_c / k_B T)$$

$$T \qquad E_c : \langle \delta n^2 \rangle = f \approx \exp(-E_c/T) \quad \Longrightarrow$$

$$\boxed{R \propto \exp\left\{\frac{\Delta_c}{E_c} \cdot \exp\left(\frac{E_c}{2k_B T}\right)\right\}}$$