

## Roles of Nonequilibrium Conduction Electrons on the Magnetization Dynamics of Ferromagnets

S. Zhang and Z. Li

*Department of Physics and Astronomy, University of Missouri-Columbia, Columbia, Missouri 65211, USA*  
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The mutual dependence of spin-dependent conduction and magnetization dynamics of ferromagnets provides the key mechanisms in various spin-dependent phenomena. We compute the response of the conduction electron spins in a spatial and time varying magnetization  $\mathbf{M}(\mathbf{r}, t)$  in the time-dependent semiclassical transport theory. We show that the induced nonequilibrium conduction spin density in turn generates four spin torques acting on the magnetization—with each torque playing a different role in magnetization dynamics. By comparing with recent theoretical models, we find that one of these torques which has not been previously identified is crucial to consistently interpreting experimental data on domain wall motion.

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Recently, there has been emerging interests in the interplay between spin-dependent transport properties and the magnetization dynamics of ferromagnets. The giant magnetoresistive effect in magnetic multilayers [1] is one of the examples where the spin transport is controlled by magnetization dynamics (or configurations). Spin angular momentum transfer [2], or spin torque, manifests the magnetization dynamics controlled by spin-polarized conduction electrons. There are quite a few closely related phenomena reported recently, e.g., enhancement of damping parameters due to spin pumping [3,4] and reaction spin torques [5], dynamic RKKY interaction [6], spin echo [7], adiabatic spin torques in a domain wall [8], and current-induced global and local pressures on a domain wall [9]. These proposed or observed phenomena motivated us to look for a theoretical framework which is capable to address the above phenomena on an equal footing. The essence of the above phenomena is to recognize two types of electrons: spin-dependent transport is provided by electrons at or near the Fermi level and the magnetization dynamics can involve electrons below the Fermi sea. While it is impossible to unambiguously separate electrons of transport from electrons of magnetization in a real ferromagnet, it has conventionally been modeled in an “s-d” Hamiltonian,

$$H_{sd} = -J_{ex} \mathbf{s} \cdot \mathbf{S} \quad (1)$$

where  $\mathbf{s}$  and  $\mathbf{S}$  are the (dimensionless) spins of itinerant and localized electrons, and  $J_{ex}$  is the exchange coupling strength between them. In this Letter, we show that the above simple s-d model in fact captures most of the physics on the interplay between spin-polarized transport of itinerant electrons and the magnetization dynamics of local moments. We will first derive a linear response function for the conduction electron spin in the presence of a time and spatially varying local moment, and then by using the same s-d model to calculate the spin torque on

the magnetization dynamics. Among other things, we have found four distinct spin torques on the magnetization. Three of them are closely related to previously derived torques by using different methods. One of the derived torque is new; it describes the mistracking between the conduction electron spin and the spatially varying local moment. We further show that our formulation can be conveniently applied to study the magnetization dynamics. An example of domain wall motion is presented at the end of the Letter.

The dynamics of the conduction electron will be considered separately from that of local magnetization. We treat the itinerant spin  $\mathbf{s}$  as a full quantum mechanical operator whose equation of motion is governed by a transport equation, but we approximate  $\mathbf{S}$  as a classical magnetization vector whose dynamics is much slower than that of itinerant spins, i.e., we replace  $\mathbf{S}$  by a classical magnetization  $\mathbf{S}/S = -\mathbf{M}(\mathbf{r}, t)/M_s$  so that

$$H_{sd} = \frac{SJ_{ex}}{M_s} \mathbf{s} \cdot \mathbf{M}(\mathbf{r}, t) \quad (2)$$

where  $|\mathbf{M}(\mathbf{r}, t)| = M_s$  is the saturation magnetization. We first determine the induced spin density for a given  $\mathbf{M}(\mathbf{r}, t)$  and then derive the reaction of the induced spin density to the magnetization.

In the present Letter, the nonequilibrium conduction electrons are generated by applying either a DC electric field or a time-dependent magnetic field. While the electric field directly generates the charge and spin currents in conducting ferromagnets, the time-dependent magnetic field is to drive the magnetization motion that induces a nonequilibrium spin density via the “s-d” interaction. The conduction electron spin operator satisfies the generalized spin continuity equation,

$$\frac{\partial \mathbf{s}}{\partial t} + \nabla \cdot \hat{\mathbf{j}} = \frac{1}{i\hbar} [\mathbf{s}, H_{sd}] - \Gamma_{re}(\mathbf{s}) \quad (3)$$

where  $\hat{J}$  is the spin current operator, and  $\Gamma_{re}(\mathbf{s})$  represents the spin relaxation due to scattering with impurities, electrons, etc., By defining electron spin density  $\mathbf{m}(\mathbf{r}, t) = \langle \mathbf{s} \rangle$  and spin current density  $\mathbf{J}(\mathbf{r}, t) = \langle \hat{J} \rangle$  where  $\langle \rangle$  represents the average over all occupied electronic states, e.g.,  $\langle \mathbf{s} \rangle = \text{Tr}(\rho \mathbf{s})$  where the trace is over all electronic as well as spin states, and  $\rho$  is the density operator, one obtains a semiclassical Bloch equation for the conduction electron spin density  $\mathbf{m}$ ,

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot \mathbf{J} = -\frac{1}{\tau_{ex} M_s} \mathbf{m} \times \mathbf{M}(\mathbf{r}, t) - \langle \Gamma(\mathbf{s}) \rangle \quad (4)$$

where the commutator in Eq. (3) has been explicitly calculated by utilizing Eq. (2), and we have defined  $\tau_{ex} = \hbar/SJ_{ex}$ .

Next, we separate the induced spin density  $\mathbf{m}$  into two terms,

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(\mathbf{r}, t) + \delta \mathbf{m}(\mathbf{r}, t) = n_0 \frac{\mathbf{M}(\mathbf{r}, t)}{M_s} + \delta \mathbf{m}(\mathbf{r}, t) \quad (5)$$

where  $n_0$  is the local equilibrium spin density whose direction is parallel to the magnetization. The first term in Eq. (5) represents the adiabatic spin density when the conduction electron spin relaxes to its equilibrium value at an instantaneous time  $t$ . Since the dynamics of the magnetization is slow compared to that of conduction electrons, it is reasonable to assume the spin of the conduction electrons approximately follows the direction of the local moment, which is known as the adiabatic process. The second term represents the deviation from this adiabatic process. Similarly, we write the spin current density as

$$\begin{aligned} \mathbf{J}(\mathbf{r}, t) &= \mathbf{J}_0(\mathbf{r}, t) + \delta \mathbf{J}(\mathbf{r}, t) \\ &= -(\mu_B P/e) \mathbf{j}_e \otimes \frac{\mathbf{M}(\mathbf{r}, t)}{M_s} + \delta \mathbf{J}(\mathbf{r}, t) \end{aligned} \quad (6)$$

where  $e$  is the electron charge,  $j_e$  is the current density,  $\mu_B$  is the Bohr magneton, and  $P$  is the spin current polarization of the ferromagnet. Note that the spin current is a tensor that consists of two vectors: the charge current and the spin polarization of the current. The first term in Eq. (6) is the spin current whose spin polarization is parallel to the local magnetization  $\mathbf{M}(\mathbf{r}, t)$ . To solve for the nonequilibrium spin density in a closed form, we assume the following simplifications. First, we use a simple relaxation time approximation to model the relaxation term in Eq. (4), i.e., we write  $\langle \Gamma(\mathbf{s}) \rangle = \delta \mathbf{m}(\mathbf{r}, t)/\tau_{sf}$  where  $\tau_{sf}$  is the spin-flip relaxation time. This approximation is necessary in order to obtain a simple analytic expression. Second, we only consider the linear response of  $\delta \mathbf{m}$  to the electric current  $j_e$  and to the time derivative of magnetization  $\partial \mathbf{M}/\partial t$ . Since  $\delta \mathbf{m}$  is already the first order,  $\partial \delta \mathbf{m}/\partial t$  will be the order of  $j_e \partial \mathbf{M}/\partial t$  or  $\partial^2 \mathbf{M}/\partial t^2$  and thus it can be discarded. Within

the semiclassical picture of the transport, the nonadiabatic current density  $\delta \mathbf{J}$  is related to the nonequilibrium spin density  $\delta \mathbf{m}$  via  $\delta \mathbf{J} = -D_0 \nabla \delta \mathbf{m}$  where  $D_0$  is the diffusion constant. By inserting Eqs. (5) and (6) into (4) and utilizing the above simplification, we obtain the closed form for the nonequilibrium spin density

$$\begin{aligned} D_0 \nabla^2 \delta \mathbf{m} - \frac{1}{\tau_{ex} M_s} \delta \mathbf{m} \times \mathbf{M} - \frac{\delta \mathbf{m}}{\tau_{sf}} \\ = \frac{n_0}{M_s} \cdot \frac{\partial \mathbf{M}}{\partial t} - \frac{\mu_B P}{e M_s} (\mathbf{j}_e \cdot \nabla) \mathbf{M}. \end{aligned} \quad (7)$$

One immediately realizes that the nonequilibrium spin density is created by two source terms on the right side of Eq. (7): one is the time variation and the other is the spatial variation of the magnetization. The solution of the above differential equation depends on the detail structure of the magnetization vector. Here we assume that the magnetization varies slowly in space, i.e., the domain wall width  $W$  of the magnetization is much larger than the transport length scale defined in the footnote [10]. In this case, the spatial derivative, the first term in Eq. (7), can be discarded [10]. Then Eq. (7) becomes a simple vector algebraic equation and by using elementary vector manipulations we readily obtain an explicit expression for the nonequilibrium spin density

$$\begin{aligned} \delta \mathbf{m} = \frac{\tau_{ex}}{(1 + \xi^2)} \left[ -\frac{\xi n_0}{M_s} \frac{\partial \mathbf{M}}{\partial t} - \frac{n_0}{M_s^2} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \frac{\mu_B P \xi}{e M_s} (\mathbf{j}_e \cdot \nabla) \mathbf{M} \right. \\ \left. + \frac{\mu_B P}{e M_s^2} \mathbf{M} \times (\mathbf{j}_e \cdot \nabla) \mathbf{M} \right] \end{aligned} \quad (8)$$

where  $\xi = \tau_{ex}/\tau_{sf}$ . The above induced spin density in turn exerts a spin torque on the magnetization. From Eq. (2), the torque is  $\mathbf{T} = -(SJ_{ex}/\hbar M_s) \mathbf{M} \times \mathbf{m} = -(1/\tau_{ex} M_s) \mathbf{M} \times \delta \mathbf{m}$ . By using Eq. (8), we have

$$\begin{aligned} \mathbf{T} = \frac{1}{1 + \xi^2} \left[ -\frac{n_0}{M_s} \frac{\partial \mathbf{M}}{\partial t} + \frac{\xi n_0}{M_s^2} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} - \frac{\mu_B P}{e M_s^3} \mathbf{M} \right. \\ \left. \times [\mathbf{M} \times (\mathbf{j}_e \cdot \nabla) \mathbf{M}] - \frac{\mu_B P \xi}{e M_s^2} \mathbf{M} \times (\mathbf{j}_e \cdot \nabla) \mathbf{M} \right] \end{aligned} \quad (9)$$

There are four terms; the first two are from magnetization variation in time and the last two describe the magnetization variation in space. Interestingly, the first two terms are independent of the current. The last two terms represent the current-driven effect since they are proportional to the current. Before we calculate the role of these spin torques, we point out a significant difference of the spin torques in spin valves or multilayers. The nonadiabatic torque we propose here relies on the approximation that the spin transport length scale, e.g.,  $\lambda$  defined in [10], is much smaller than the domain wall width. In multilayers, the magnetization abruptly changes at the interface and the “equivalent” domain wall width is zero. Therefore, the approximation of dropping the first term of left-handside of Eq. (7) is not valid. In fact, the length scale

of the spin torque in multilayers is solely determined by the spin transport length scale.

The standard Landau-Lifshitz-Gilbert (LLG) equation consists of a precessional term due to an effective field and a phenomenological damping term. In addition to these two torques, the above torque  $\mathbf{T}$  is now added to the LLG equation,

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \mathbf{T}, \quad (10)$$

where  $\gamma$  is the gyromagnetic ratio,  $\mathbf{H}_{eff}$  is the effective magnetic field,  $\alpha$  is the Gilbert damping parameter. We immediately realize that the first term in Eq. (9) is simply to renormalize the gyromagnetic ratio while the second term is to renormalize the damping parameter. Thus if we introduce an effective gyromagnetic ratio  $\gamma'$  and the damping parameter  $\alpha'$ ,

$$\gamma' = \gamma(1 + \eta)^{-1}; \quad \gamma' \alpha' = \gamma(\alpha + \xi \eta)$$

where we have defined  $\eta = (n_0/M_s)/(1 + \xi^2)$ , the LLG equation remains in the same form. We point out that the modification of the gyromagnetic ratio and the damping parameter through the present mechanism is rather small in transition metal ferromagnets. For a typical ferromagnet (Ni, Co, Fe and their alloys),  $J_{ex} \approx 1$  eV,  $S = 2$ ,  $\tau_{sf} \approx 10^{-12}$  s,  $n_0/M_s \approx 10^{-2}$ ,  $\xi \approx 10^{-2}$  and thus  $\eta$  is about  $10^{-2}$  and  $\xi \eta$  is of the order of  $10^{-4}$ —much smaller than the typical damping parameter of the order of  $10^{-2}$ . Therefore, we conclude that the temporal spin torque driven by the exchange interaction only slightly modifies the damping parameter and can not be identified as a leading mechanism for magnetization damping.

At this point, we should compare other theories on the spin torque to this Letter. Tserkovnyak *et al.* [3,7] proposed an adiabatic spin pumping mechanism to explain the enhancement of Gilbert damping parameters. Ho *et al.* suggested a radiation field induced by precessional motion of magnets [5]. Most recently, a similar s-d model in the presence of the time-dependent magnetization has been considered [11]. The present approach reduces to these theories in the simple limit considered for these two terms. In fact, the idea of this temporal spin torque had been suggested earlier: when the magnetization varies in time, the spin of the conduction electrons tends to follow the direction of the magnetization with a time delay given by spin relaxation time. This latter phenomenon was named as “breathing Fermi surface” [12]. We are now able to consider this physics of the enhanced damping on the equal footing as the current-induced spin torques. We also point out that Waintal and Viret [9] have recently proposed a nonadiabatic torque based on a possible Larmor precession of the conduction electron spin around the axis of the local magnetization vector.

Our main focus here is the spin torque due to the spatially nonuniform magnetization vector, the last two

terms in Eq. (9). Since the temporal spin torques can be completely absorbed by the redefinition of the gyromagnetic ratio and damping constant, we shall now ignore them and concentrate on the role of spin torque generated by the nonuniform magnetization. We thus write the full equation for the magnetization dynamics below

$$\begin{aligned} \frac{\partial \mathbf{M}}{\partial t} = & -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} - \frac{b_J}{M_s^2} \mathbf{M} \\ & \times \left( \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x} \right) - \frac{c_J}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x} \end{aligned} \quad (11)$$

where we assume the direction of current  $x$ -direction ( $\mathbf{j}_e = j_e \mathbf{e}_x$ ),  $b_J = P j_e \mu_B / e M_s (1 + \xi^2)$ , and  $c_J = P j_e \mu_B \xi / e M_s (1 + \xi^2)$ . Note that  $b_J$  and  $c_J$  have the units of velocity. The “ $b_J$ ” term has been already proposed by Bazaliy *et al.* [13] when they consider a ballistic motion of conduction electrons in the half-metal materials. Recently Tataru and Kohn also derived a similar expression [8]. We have seen that this term describes the adiabatic process of the nonequilibrium conduction electrons. The “ $c_J$ ” term is completely new; it is related to the spatial mistracking of spins between conduction electrons and local magnetization. While this term is known in the physics of domain wall resistance [14–16], it also gives rise a nonadiabatic spin torque, the last term in Eq. (11). At first sight, one might think that this “ $c_J$ ” term may be discarded since it is much smaller than the “ $b_J$ ” term ( $c_J/b_J = \xi \approx 10^{-2}$ ). We will show below that the terminal velocity of a domain wall is *independent* of the strength of “ $b_J$ ”, rather it is controlled by this small “ $c_J$ ” term. Thus, experimental analysis on the domain wall motion must include this new “ $c_J$ ” term.

To make a concrete prediction on the domain wall dynamics from Eq. (11), we consider a Néel-wall in a magnetic nanowire whose magnetization vector only depends on the position along the wire, i.e.,  $\mathbf{M} = \mathbf{M}(x, t)$ . The effective field entering Eq. (11) is modeled by

$$\mathbf{H}_{eff} = \frac{H_K M_x}{M_s} \mathbf{e}_x + \frac{2A}{M_s^2} \nabla^2 \mathbf{M} - 4\pi M_z \mathbf{e}_z + H_{ext} \mathbf{e}_x \quad (12)$$

where  $H_K$  is the anisotropy field,  $A$  is the exchange constant, and  $4\pi M_z$  is the demagnetization field. In the presence of the spin torque, we follow the Walker’s prescription of the domain wall motion by introducing a trial function  $\mathbf{M}(\theta, \varphi)$  where  $(\theta, \phi)$  are polar angles in the following form [17],

$$\varphi = \varphi(t); \quad \tan \frac{\pi - \theta}{2} = \frac{1}{W(t)} \left[ x - \int_0^t v(\tau) d\tau \right] \quad (13)$$

The first equation assumes that the projection of the magnetization vector in the domain wall on the  $yz$  plane is independent of the position. The second equation in Eq. (13) postulates that the domain wall shape remains a standard Néel-wall form except that the wall width  $W(t)$  varies with time and the wall moves at velocity  $v(t)$ . By

placing Eqs. (12) and (13) into Eq. (11), and by assuming the domain wall width changes slowly as in the Walker's theory, we can find two coupled differential equations for determining the domain wall distortion parameters  $\varphi(t)$  and  $W(t)$ . Interestingly, the expression for the velocity of the domain wall at the initial application of the current is [18]

$$v(0) = -\frac{1}{1+\alpha^2}[\alpha\gamma H_{\text{ext}}W(0) + b_J + \alpha c_J] \quad (14)$$

while the terminal velocity of the domain wall is

$$v_T \equiv v(\infty) = -\frac{1}{\alpha}[\gamma H_{\text{ext}}W(\infty) + c_J] \quad (15)$$

where  $W(\infty)$  is the terminal wall width that is slightly smaller than the initial Néel-wall width  $W(0)$ . Eqs. (14) and (15) reveal the different roles played by the adiabatic ( $b_J$  term) and nonadiabatic ( $c_J$  term) spin torques: the adiabatic torque is most important at the initial motion of the wall while the nonadiabatic  $c_J$  controls the terminal velocity of the domain wall. The adiabatic torque causes the domain wall distortion. The distorted domain wall is able to completely absorb the adiabatic spin angular momentum so that the net effect of the adiabatic torque on the domain wall velocity becomes null, i.e., the domain wall stops. In contrast, the nonadiabatic spin torque behaves as a nonuniform magnetic field  $c_J\partial\mathbf{M}/\partial x$  that can sustain a steady state wall motion. Although the magnitude of the nonadiabatic torque  $c_J$  is about 2 orders of magnitude smaller than adiabatic torque  $b_J$ , the terminal velocity is inversely proportional to the damping parameter which makes the velocity comparable to  $b_J$ .

Finally, we emphasize that the present study has resolved an outstanding mystery between the recent experimental observation [20] and the theoretical prediction based on the adiabatic spin torque. It has been recognized that a critical current density of the order of  $10^9 - 10^{10}$  A/cm<sup>2</sup> is required to move a perfect domain wall [8,19] if we only use the adiabatic spin torque  $b_J$ . Experimentally, a velocity about 3 m/s was observed in a NiFe nanowire when a current density  $1.2 \times 10^8$  A/cm<sup>2</sup> was applied. This velocity had been assumed to relate with  $b_J$  [20] in spite of the apparent *qualitative and quantitative disagreement* between theory and experiment. Here, we have pointed out that  $b_J$  is simply an initial velocity of the domain wall and the measured velocity was the terminal velocity. For the experimental current density of  $1.2 \times 10^8$  A/cm<sup>2</sup>, the adiabatic spin torque alone is unable to sustain a constant velocity. By including a small nonadiabatic torque  $c_J$ , we find the domain wall velocity is now  $c_J/\alpha$  in the absence of the magnetic field, see Eq. (15). Although the numerical

values of both the exchange constant  $J_{\text{ex}}$  and the damping parameter  $\alpha$  are not precisely known in ferromagnets, we estimate that the wall velocity should be  $6 \sim 60$  (m/s) for the above current density if we use the parameters indicated before (taking  $\alpha = 0.01 \sim 0.1$  for permalloy). While the experimental velocity is smaller than our estimated value, it is reasonable since we did not include any defects which would reduce the velocity significantly.

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