

TRIQS

A Toolbox for Research in Interacting Quantum Systems

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European Research Council

Established by the European Commission

SIMONS FOUNDATION
Advancing Research in Basic Science and Mathematics

Team for the hands-on



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What is TRIQS ?

- A **Toolbox** in Python & C++ to build modern many body computations:
 - DMFT methods [Today's topic]
 - Ab-initio strongly correlated materials (DFT+ DMFT).
 - Cluster DMFT methods.
 - Third generation methods: self-consistency with two particles Green functions.
 - TRIQS is not limited to DMFT methods.
 - Other many-body computations.
e.g. Eliashberg type equations
(superconductivity, spin-fluctuations...)
 - Some tools are very general, beyond condensed matter physics.

Outline

- Start with an example :
I band DMFT with CT-INT QMC as an impurity solver.
- Overview of the TRIQS project
- A few highlights of the contents of the library in Python & C++
- Hands-on:
 - You will build a little IPT solver at half-filling with TRIQS and run it to recover a classical result for Mott transition in DMFT.
 - Then compare with TRIQS exact and general CT-HYB quantum impurity solver
 - Finish with a 2 orbital model, study the role of U vs J_{hund}

Motivating example

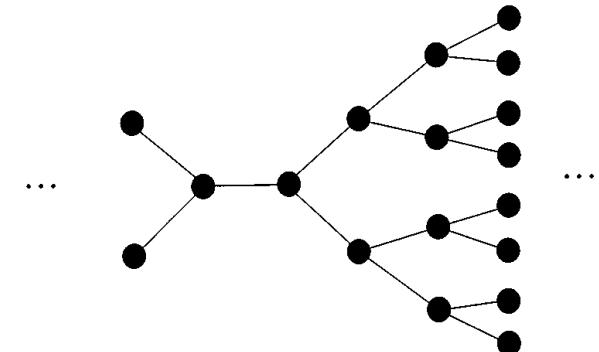
Let's do a DMFT computation
in Python using TRIQS

Reminder : DMFT on Bethe lattice

- DMFT equations (Cf D. Sénéchal's lecture yesterday).

$$H = - \sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U n_{i\uparrow} n_{i\downarrow}$$

$$G_c(\tau) = -\langle T c(\tau) c^\dagger(0) \rangle_{S_{\text{eff}}}$$



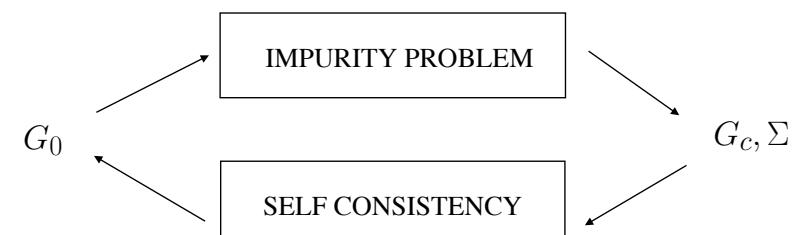
$$S_{\text{eff}} = - \int_0^\beta c_\sigma^\dagger(\tau) G_0^{-1}(\tau - \tau') c_\sigma(\tau') + \int_0^\beta d\tau U n_\uparrow(\tau) n_\downarrow(\tau)$$

- Bethe lattice with infinite connectivity. Semi-circular d.o.s.

A. Georges, G. Kotliar, W. Krauth and M. Rozenberg, Rev. Mod. Phys. 68, 13, (1996)

$$\Delta(i\omega_n) = t^2 G_c(i\omega_n)$$

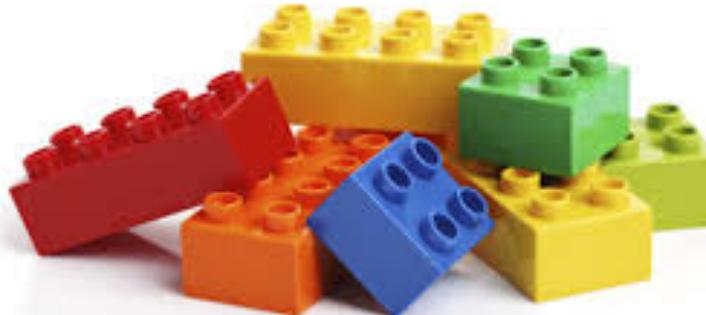
$$G_0^{-1}(i\omega_n) = i\omega_n + \mu - \Delta(i\omega_n)$$



- Goal: Solve DMFT equations, self-consistently with CT-INT.

How to do it ?

- Break the DMFT computation into small parts and assemble the computation.



- Which parts ?
 - Local Green functions
 - An impurity solver: e.g. the CT-INT solver.
 - Save the result.
 - Plot it.

Assemble a DMFT computation in 1 slide

- A complete code, using a CT-INT solver (one of the TRIQS apps).
- In Python, with parallelization included (mpi).
- Do not worry about the details of the syntax at this stage
(Cf the hands-on).
Get an idea of how to use TRIQS by example.

DMFT computation in 1 slide

```

from pytriqs.gf.local import *
from pytriqs.applications.impurity_solvers.ctint_tutorial import CtintSolver

U = 2.5          # Hubbard interaction
mu = U/2.0       # Chemical potential
half_bandwidth=1.0 # Half bandwidth (energy unit)
beta = 40.0       # Inverse temperature
n_iw = 128        # Number of Matsubara frequencies
n_cycles = 10000  # Number of MC cycles
delta = 0.1        # delta parameter
n_iterations = 21 # Number of DMFT iterations

S = CtintSolver(beta, n_iw) # Initialize the solver
S.G_iw << SemiCircular(half_bandwidth) # Initialize the Green's function

```

- Import some basic blocks (Green function, a solver) ...
- Define some parameters and declare a CT-INT solver S
- All TRIQS solvers contains G , G_0 , Σ as members with the correct β , dimensions, etc...
- Initialize $S.G_{\text{iw}}$ to a (the Hilbert transform of a) semi-circular dos.

DMFT computation in 1 slide

```

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S = CtintSolver(beta, n_iw) # Initialize the solver

S.G_iw << SemiCircular(half_bandwidth) # Initialize the Green's function

for sigma, G0 in S.G0_iw: # sigma = 'up', 'down'
    G0 << inverse(iOmega_n + mu - (half_bandwidth/2.0)**2 * S.G_iw[sigma] ) # Set G0

```

$$G_{0\sigma}^{-1}(i\omega_n) = i\omega_n + \mu - t^2 G_{c\sigma}(i\omega_n), \text{ for } \sigma = \uparrow, \downarrow$$

- Implement DMFT self-consistency condition

DMFT computation in 1 slide

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for sigma, G0 in S.G0_iw:
    G0 << inverse(iOmega_n + mu - (half_bandwidth/2.0)**2 * S.G_iw[sigma] ) # Set G0

S.solve(U, delta, n_cycles) # Solve the impurity problem

```

- Call the solver.
- From $G_0\sigma(i\omega_n)$ (and various parameters), it computes $G_\sigma(i\omega_n)$ for $\sigma=\uparrow,\downarrow$

DMFT computation in 1 slide

```

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delta = 0.1        # delta parameter
n_iterations = 21 # Number of DMFT iterations

S = CtintSolver(beta, n_iw) # Initialize the solver

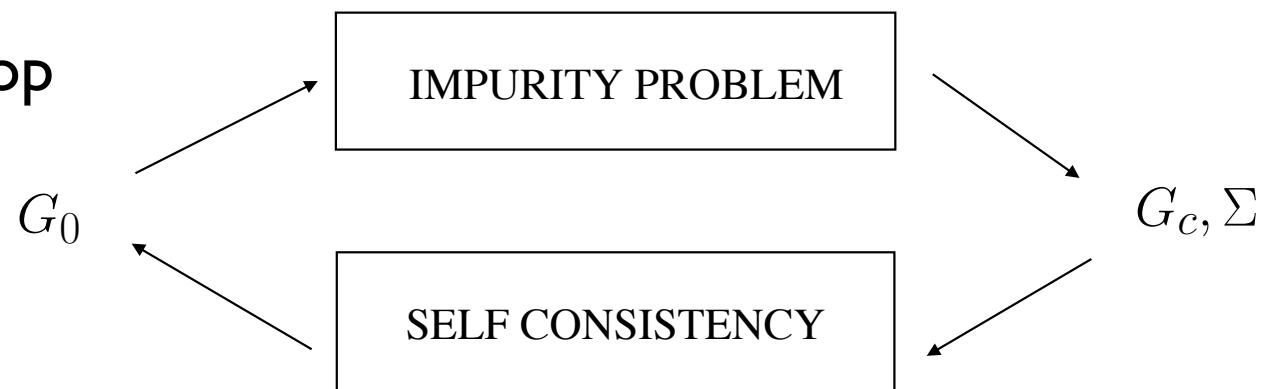
S.G_iw << SemiCircular(half_bandwidth) # Initialize the Green's function

for it in range(n_iterations): # DMFT loop
    for sigma, G0 in S.G0_iw:
        G0 << inverse(iOmega_n + mu - (half_bandwidth/2.0)**2 * S.G_iw[sigma] ) # Set G0

S.solve(U, delta, n_cycles) # Solve the impurity problem

```

- DMFT iteration loop



DMFT computation in 1 slide

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    for sigma, G0 in S.G0_iw:
        G0 << inverse(iOmega_n + mu - (half_bandwidth/2.0)**2 * S.G_iw[sigma] ) # Set G0

S.solve(U, delta, n_cycles) # Solve the impurity problem

G_sym = (S.G_iw['up'] + S.G_iw['down'])/2 # Impose paramagnetic solution
S.G_iw << G_sym

```

- Enforce the fact that the solution is paramagnetic, cf DMFT lecture.
(noise in the QMC would lead to a AF solution after iterations).

DMFT computation in 1 slide

```

from pytriqs.gf.local import *
from pytriqs.applications.impurity_solvers.ctint_tutorial import CtintSolver
from pytriqs.archive import HDFArchive

U = 2.5          # Hubbard interaction
mu = U/2.0       # Chemical potential
half_bandwidth=1.0 # Half bandwidth (energy unit)
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        G0 << inverse(iOmega_n + mu - (half_bandwidth/2.0)**2 * S.G_iw[sigma] ) # Set G0

S.solve(U, delta, n_cycles) # Solve the impurity problem

G_sym = (S.G_iw['up'] + S.G_iw['down'])/2 # Impose paramagnetic solution
S.G_iw << G_sym

with HDFArchive("dmft_bethe.h5",'a') as A:
    A['G%i'%it] = G_sym # Save G from every iteration to file as G1, G2, G3....

```

- Accumulate the various iterations in a (hdf5) file

DMFT computation in 1 slide

```

from pytriqs.gf.local import *
from pytriqs.applications.impurity_solvers.ctint_tutorial import CtintSolver
from pytriqs.archive import HDFArchive

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S = CtintSolver(beta, n_iw) # Initialize the solver

S.G_iw << SemiCircular(half_bandwidth) # Initialize the Green's function

for it in range(n_iterations): # DMFT loop
    for sigma, G0 in S.G0_iw:
        G0 << inverse(iOmega_n + mu - (half_bandwidth/2.0)**2 * S.G_iw[sigma] ) # Set G0

# Change random number generator on final iteration
random_name = 'mt19937' if it < n_iterations-1 else 'lagged_fibonacci19937'

S.solve(U, delta, n_cycles, random_name=random_name) # Solve the impurity problem

G_sym = (S.G_iw['up']+S.G_iw['down'])/2 # Impose paramagnetic solution
S.G_iw << G_sym

with HDFArchive("dmft_bethe.h5",'a') as A:
    A['G%i'%it] = G_sym # Save G from every iteration to file

```

- Change the random generator at the last iteration !

Look at the result (in IPython notebook)

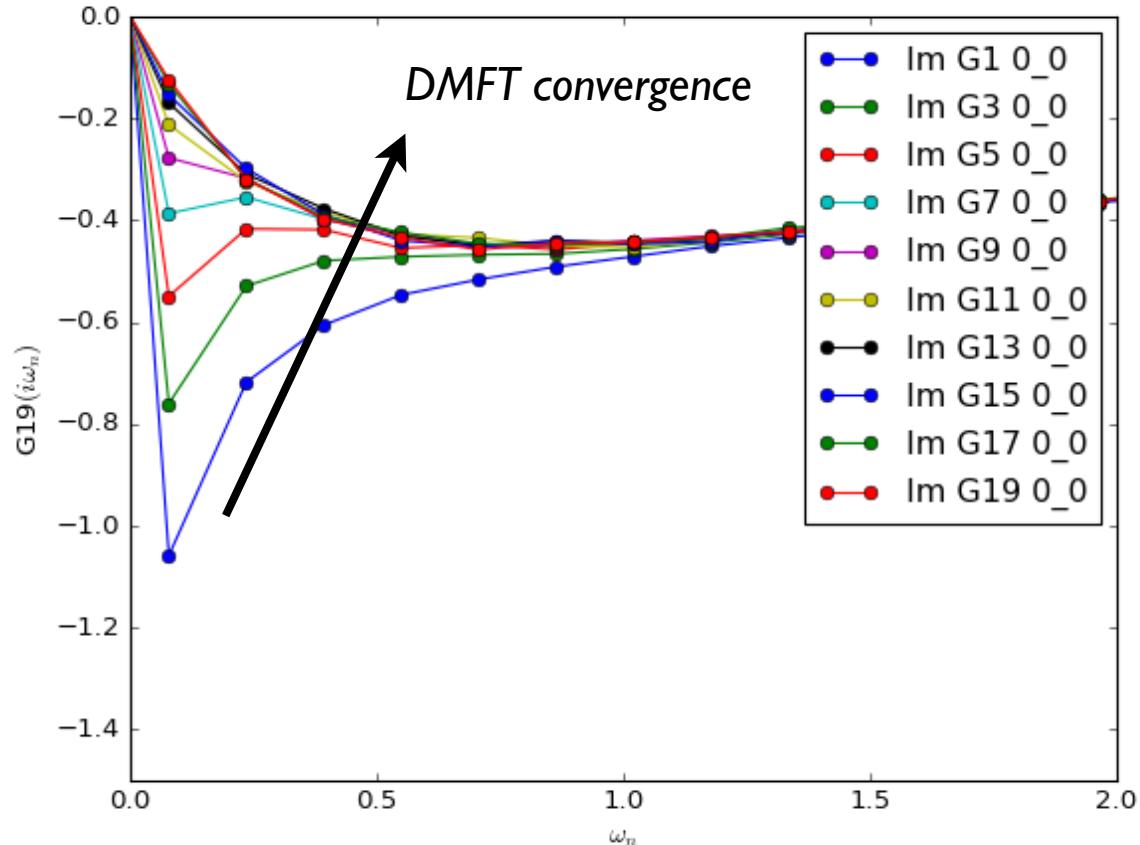
```
A = HDFArchive("dmft_bethe.h5", 'r') # Open file in read mode
for it in range(21):
    if it%2: # Plot every second result
        oplot(A['G%i'%it], '-o', mode='I', name='G%i'%it)
```

oplot can plot many TRIQS objects via matplotlib

Retrieve Gi from the file, and use it at once

Imaginary part only

NB
lines are guide to the eyes,
only Matsubara frequency point matters



Summarize what we have done so far

- A fully functional DMFT code.
- Computation & data analysis: all in Python.
- Green functions, solvers are used as Python classes.
- Since it is a script, it is **easy to change various details, e.g.**
 - Self-consistency condition: enforce paramagnetism
 - Change random generator
 - Change starting point (e.g. reload G from a file).
 - Improve convergence (e.g. mixing quantities over iterations).
 - Measure e.g. susceptibilities only at the end of the DMFT loop

FAQ : Where was the input file ?

- Traditional way: a monolithic program, input files, output files.
- Python/TRIQS way : write your own script with full control
- No input file as text : no need to parse it,
we can do operations on the fly, use Python to prepare data ...

```
from pytriqs.gf.local import *
from pytriqs.applications.impurity_solvers.ctint_tutorial import CtintSolver

U = 2.5          # Hubbard interaction
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```

• • •

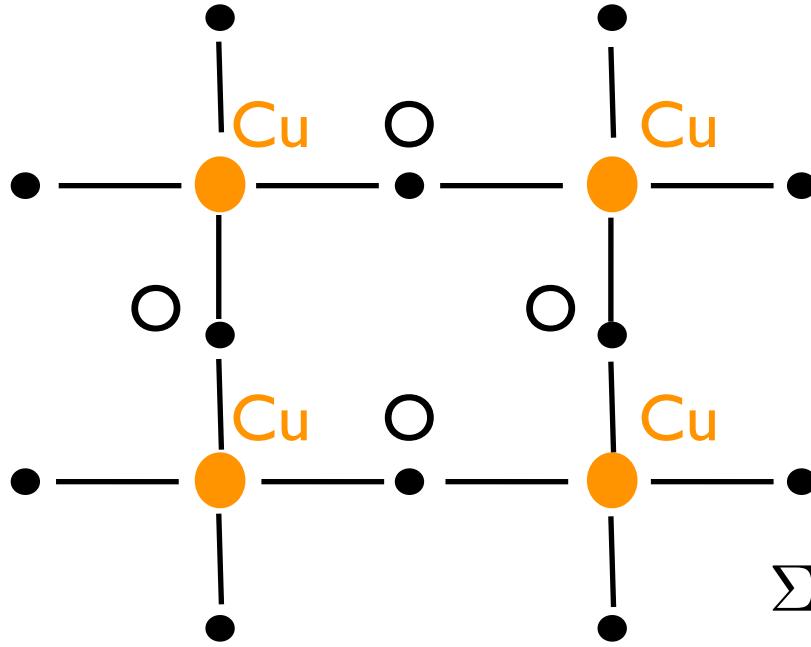
Why a library rather than a monolithic program ?

Library vs monolithic program

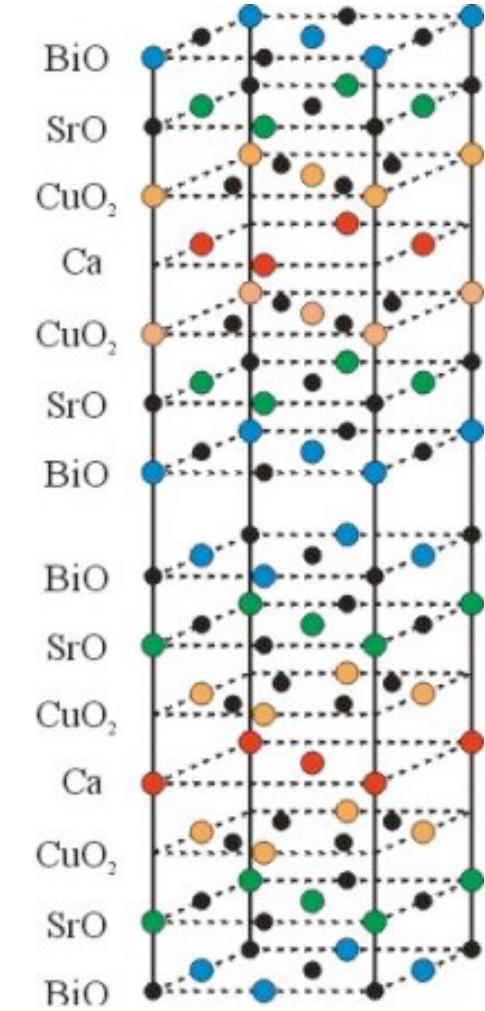
- Better to have a **simple language** to express your calculation.
- Since many-body approaches are quite versatile.
- Let me illustrate this point with examples of DMFT-like methods
 - Many impurity solvers (e.g. CT-QMC, ED, IPT, DMRG, NCA, ...)
 - Various impurity models (# of orbitals, symmetries, clusters)
 - Many self-consistency conditions ...

DMFT: correlated vs uncorrelated orbitals

- Select some correlated orbitals, even in models for cuprates, e.g.



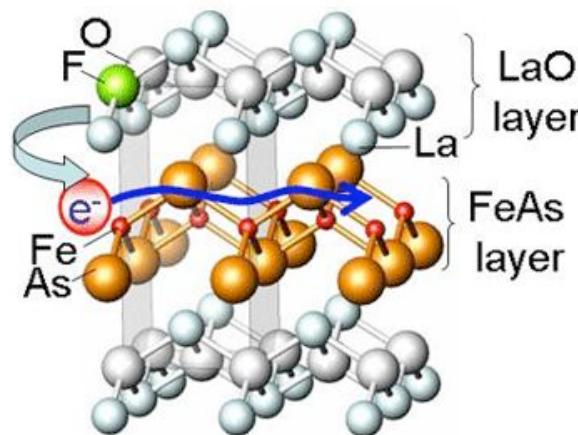
$$\Sigma(\omega) = \begin{pmatrix} \Sigma^{\text{imp}}(\omega) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



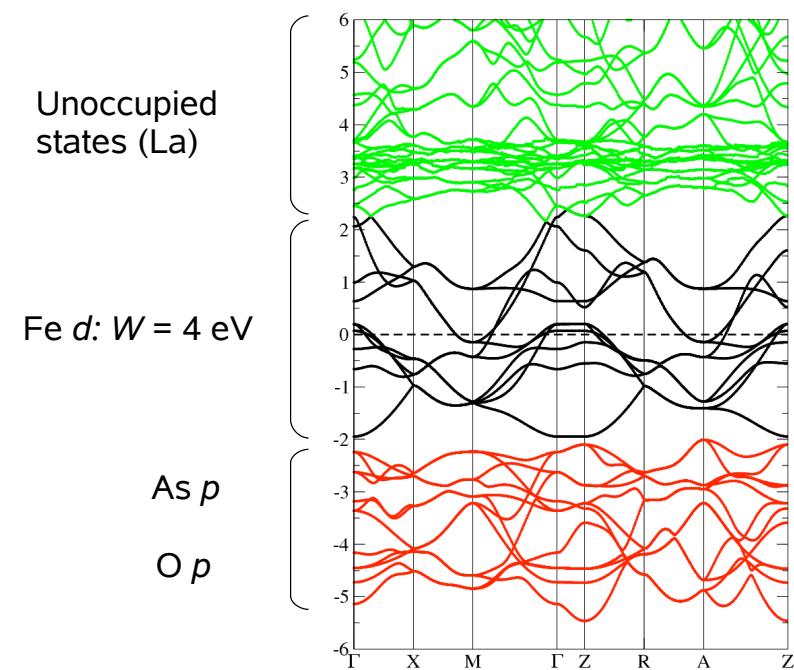
- Self-consistency in large unit cell (Cu + 2 O)
 $\Sigma_{ab}(\omega)$ a 3x3 matrix
- Interaction only on Cu.
 Impurity model is one band, with $\Sigma^{\text{imp}}(\omega)$ a 1x1 matrix

Mix with electronic structure codes

- Cf Lectures by Kotliar next week.
- (Much) more of the same kind of manipulations:
Extract the Green function of the correlated orbitals.
E.g. project on Wannier functions.
Embed the self-energy of the correlated orbital (downfolding).



Fe-Based (2008)



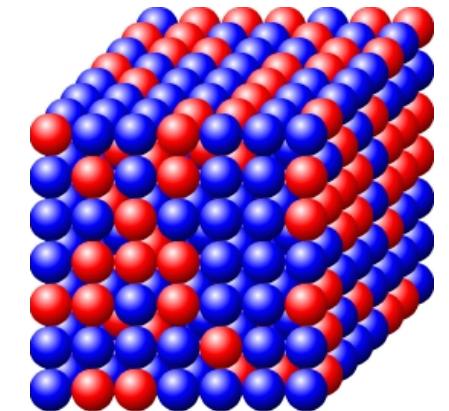
Role of geometry

- Real space DMFT : ultra-cold atoms

Cf lecture by D. Sénéchal.

- Disordered systems *Cf e.g. work of Dobrosavljević et al.*

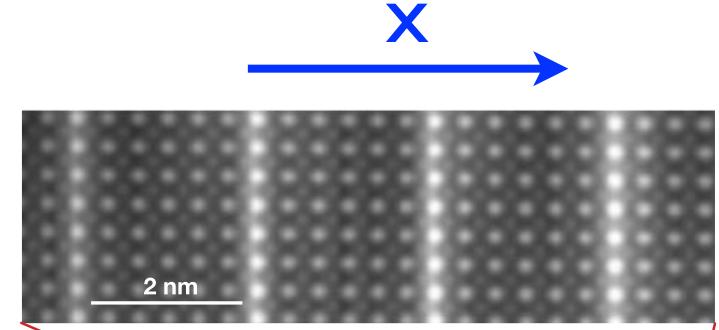
- One DMFT impurity for each part (red, blue).
- Linked by self-consistency condition, taking into account disorder



Alloy

- Correlated interfaces. *Cf e.g. Okamoto & Millis*

- One DMFT impurity for each layer \times
- Coupled by self-consistency condition



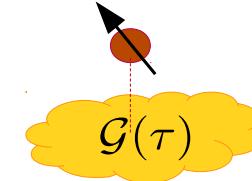
$SrTiO_3/LaTiO_3$
Ohtomo et al, Nature 2002

Cluster DMFT

Cf lecture by D. Sénéchal

- **DMFT:** 1 atom (Anderson impurity) + effective self-consistent bath

$$\Sigma(k, \omega) \approx \Sigma_{\text{impurity}}(\omega)$$



- Clusters = a systematic expansion around DMFT.
- Control parameter = size of cluster / momentum resolution.
Systematic benchmarks, cf J. LeBlanc et al., PRX 5 (2015)

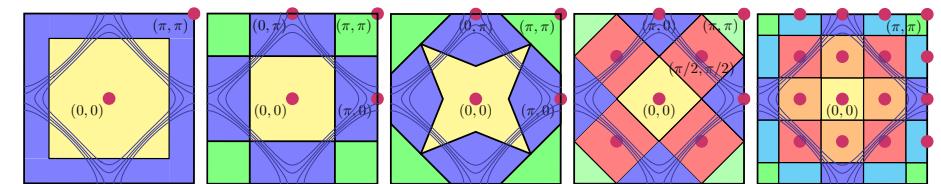
Real space clusters (C-DMFT)

*Lichtenstein, Katsnelson 2000
Kotliar et al. 2001*



*Reciprocal space (DCA)
clusters Brillouin zone patching*

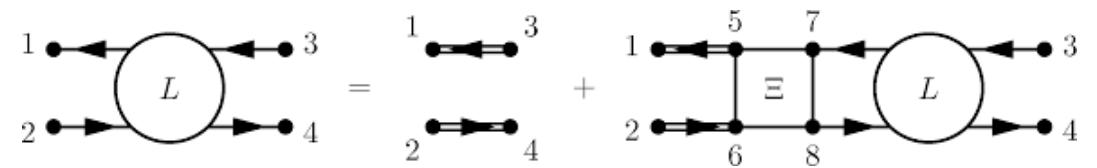
Hettler et al. '98, ...



Beyond DMFT

- Mix diagrammatic approximations, e.g. Eliashberg or spin-fluctuation theory, Bethe Salpeter equation, Parquet equations with local atomic (Mott) physics à la DMFT.
Dual fermions/bosons. *Rubtsov '08,'12, DΓA Toschi '07, Trilex, T.Ayral, O.Parcollet, 2015-2016*
- Study, compare these methods.
- Need to assemble more complex objects : $G(k,\omega)$, Vertex $\Gamma(\omega,v,v')$.

$$\Sigma(k, i\omega) \approx G + W^{\text{sp}}$$



Need for a library

- No “general” DMFT code.
- Same for other approaches (e.g. DMRG)
- I want a **simple language** able to write all of these, and much more.
- Even variants which you have not yet invented !
- Hence an open approach : a **library**
Extending Python/C++ to express the basic concepts of our field
(Green functions, impurity solvers, ...).

Reproducibility

- Methods & algorithms are more and more complex
- Do we have all the elements to reproduce easily a paper ?
Do we really do “**Reproducible Science**” ?
- Tools :
 - **Jupyter/IPython notebook**: a simple tool for reproducibility
 - Analysis script in the notebook, with text, latex formula.
 - A new kind of “paper” where the script from the raw data to the figure is provided, can be changed, reanalyzed.
 - **Version control (GitHub)**
- But the problem is deeper than that...

Status of numerics

- Numerical & analytical computations on the same footing.
- **Algorithmic Physics** : study algorithms e.g. for many-body problem.
- Scientific method requires openness as for analytical computations.
 - Codes should be published along with papers.
 - Therefore they should be written to be read !
 - Code review by peers. Improvements, reuse.
- In practice, not the case: writing/testing a code is long, costly
- Need efficient open source toolkits to quickly build computations, enable team work on codes, code review.

The TRIQS project

Goals and organization

TRIQS goals

- **Basic blocks** for many-body calculations, starting with DMFT and co.
- **Simplicity** : what is simple should be coded simply !
- **High performance** :
 - Human time : reduce the cost of writing codes.
 - Machine time : run fast.

TRIQS project : a modular structure

CTHYB
Impurity solver

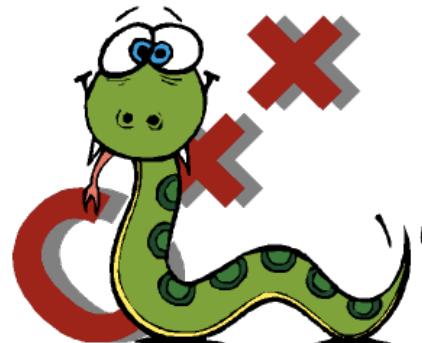
DFTTools
Interface to
electronic
structure
codes

Your app.

TRIQS library
The basic blocks



Python/C++



- **Python**

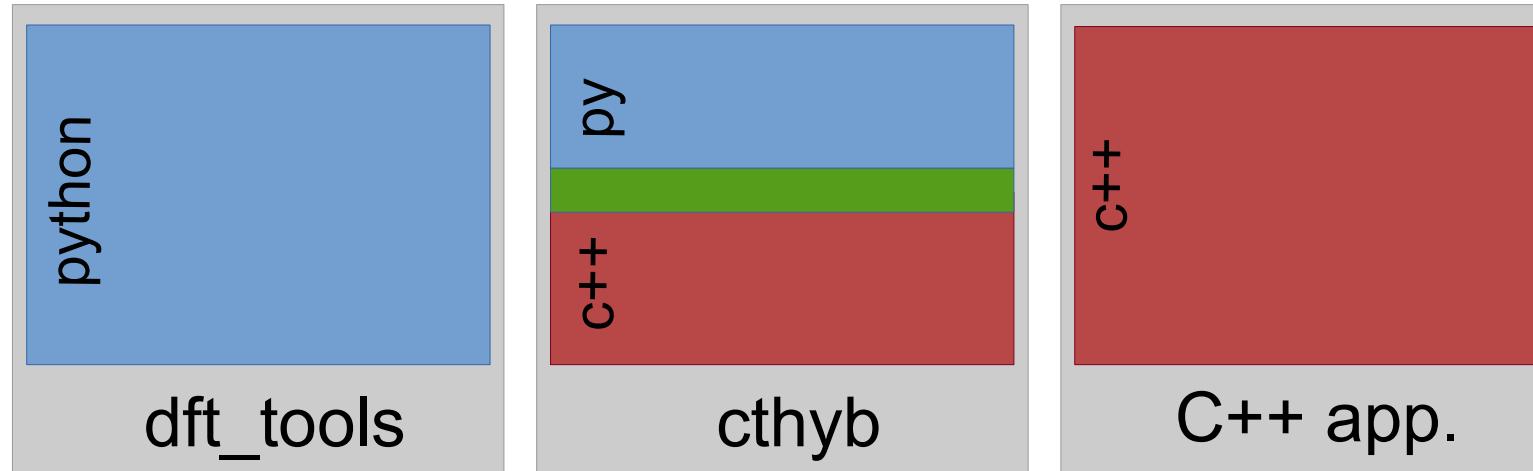
- Script language, easy to learn.
- High level part of the code (e.g. assemble a DMFT computation)
- Data analysis with the same objects/language
[ipython notebook](#)
- Glue with other codes (e.g. electronic structure codes)

- **C++**

- Computational intensive part, e.g. quantum impurity solvers (CTHYB, CTINT).

When Python is too slow.

TRIQS Python and C++ layers.



TRIQS: different levels of usage.

- Simplest usage
 - Take an existing template script, e.g. our DMFT computation, change parameters, run, plot, do physics ...
- With Python programming [THIS LECTURE & HANDS-ON]
 - Build your own DMFT self-consistency, using building blocks in Python, package solvers.
- With C++ programming
 - Use the C++ layer to develop a high-performance application.
 - Use the TRIQS tools to wrap it in Python and collaborate using Python as a common language.

TRIQS library.

Olivier Parcollet, Michel Ferrero, Thomas Ayral, Hartmut Hafermann,
Igor Krivenko, Laura Messio, Priyanka Seth
Comp. Phys. Comm. 196, 39 (2015), arXiv1504.01952

Main developers & contributors



O. Parcollet



Michel Ferrero



Thomas Ayral



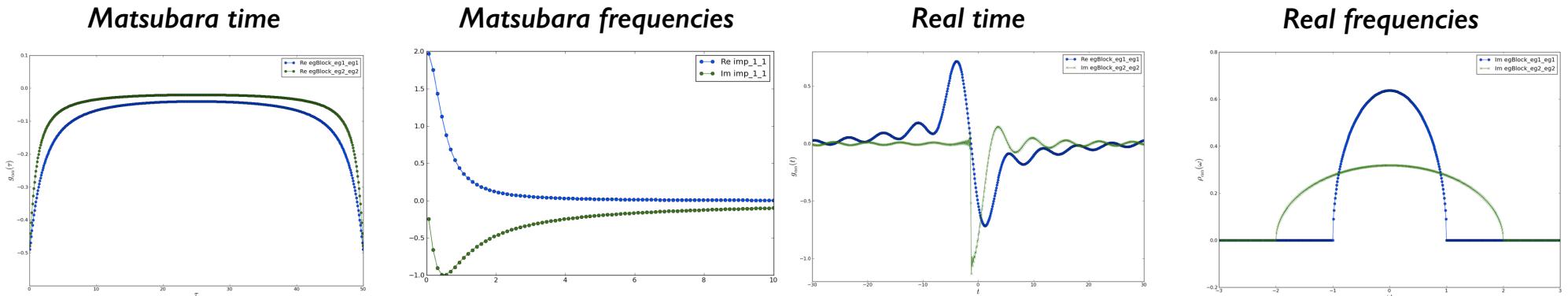
Priyanka Seth



Igor Krivenko

TRIQS library: Python

- Various kinds of Green functions and the associated functions.



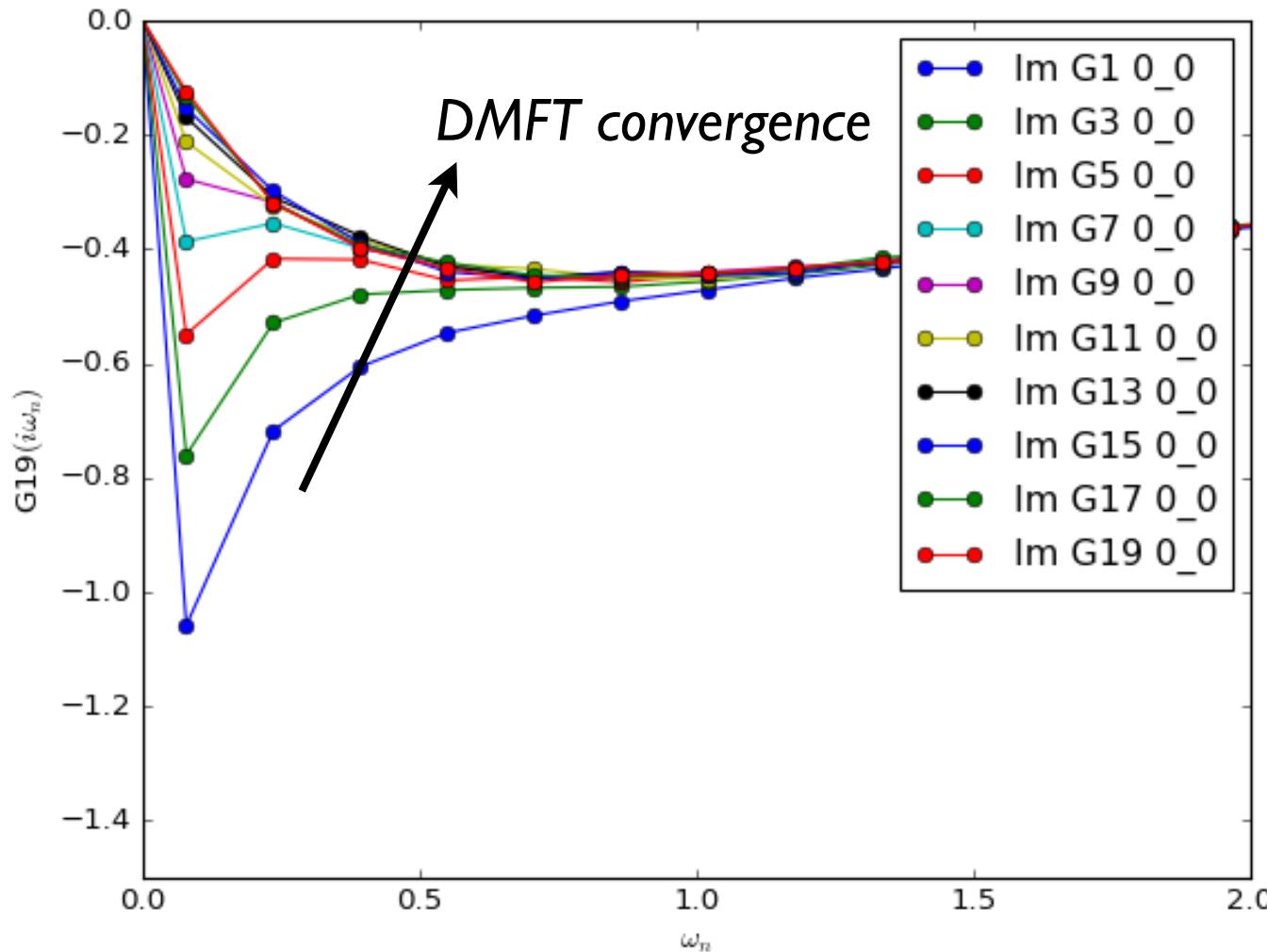
- Many-body operators to write Hamiltonians.
- Simple Bravais Lattices, Brillouin zone, density of states, Hilbert transform.
- Interfaces to save/load in HDF5 files, plot interactively in the ipython notebook.

TRIQS library: C++

- More libraries components:
 - “Green functions” containers (any function on a mesh)
 $G(\omega)$, $G(k,\omega)$, Vertex $\Gamma(\omega,v,v')$.
 - Generic Monte Carlo class & error analysis tools.
 - Determinant manipulations (for QMCs).
 - Lattice tools: Bravais Lattices, Brillouin zone,
 - Many-body operators.
- Basic blocks and tools , e.g.
 - Multidimensional array class
 - HDF5 light interface in C++

HDF5 file format ...

```
A = HDFArchive( "dmft_bethe.h5" , 'r' ) # Open file
for it in range(21):
    if it%2: oplot(A[ 'G%i'%it], 'o', mode="I", name='G%i'%it)
# Plot every second result
```



HDF5 file format

- Standard file format used by many projects, e.g. pytables, ALPS.
- Language agnostic (python, C/C++, F90).
- Binary format hence compact, but also portable.
- Dump & reload objects in one line.
Forget worrying about format, reading files, conventions.
- $G(\omega)(n_1, n_2)$ a 3d array of complex numbers, i.e. 4d array of reals.
No natural convention in a 2d text file.
- Also used by ALPS project
In progress : “standardize” details for even simpler data exchange between TRIQS & ALPS.

TRIQS/CTHYB.

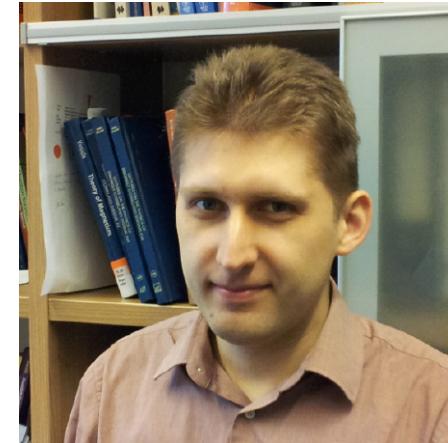
A Continuous-Time Quantum Monte Carlo Hybridization Expansion Solver for Quantum Impurity Problems

*Priyanka Seth, Igor Krivenko, Michel Ferrero, Olivier Parcollet
Comp. Phys. Comm. 200, 274 (2016), arXiv:1507.00175*

Developers



Priyanka Seth



Igor Krivenko



Michel Ferrero



O. Parcollet

TRIQS/CTHYB

<https://triqs.ipht.cnrs.fr/l.x/applications/cthyb/>



a generic quantum impurity solver based on the `triqs` library

Install

Documentation

Issues

About CTHYB

The hybridization-expansion solver

The [TRIQS-based](#) hybridization-expansion solver allows to solve the generic problem of a quantum impurity embedded in a conduction bath for an arbitrary local interaction vertex. The “impurity” can be any set of orbitals, on one or several atoms. To be more specific, the Hamiltonian of the problem has the form:

$$\hat{H} = \sum_{k,\alpha} \epsilon_{k,\alpha} c_{k,\alpha}^\dagger c_{k,\alpha} + \sum_{k,\alpha} V_{k,\alpha} (c_{k,\alpha}^\dagger d_\alpha + h.c.) - \mu \sum_\alpha d_\alpha^\dagger d_\alpha + \sum_{\alpha\beta} h_{\alpha\beta} d_\alpha^\dagger d_\beta + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} U_{\alpha\beta\gamma\delta} d_\alpha^\dagger d_\beta^\dagger d_\delta d_\gamma.$$

Here the operators c^\dagger construct a fermion in the bath, while the operators d^\dagger construct a fermion on the impurity. In this problem, the hybridization function Δ between the bath and the impurity is given by:

$$\Delta_{\alpha,\beta}(i\omega_n) = \sum_k \frac{V_{k,\alpha} V_{k,\beta}^*}{i\omega_n - \epsilon_{k,\alpha}},$$

so that the non-interacting Green's function of the impurity is:

$$\hat{G}_0^{-1}(i\omega_n) = i\omega_n + \mu - \hat{h} - \hat{\Delta}(i\omega_n).$$

With the knowledge of G_0 and the matrix $U_{\alpha\beta\gamma\delta}$, the quantum impurity solvers find the interacting Green's function G of the problem. Learn how to use it in the [Documentation](#).



Quick search

 Go

Enter search terms or a module, class or function name.

CT-HYB :Expansion in hybridization

P. Werner et al, PRL (2006)

$$S_{\text{eff}} = - \int_0^\beta c_a^\dagger(\tau) G_{0ab}^{-1}(\tau - \tau') c_b(\tau') + \int_0^\beta d\tau \mathbf{H}_{\text{local}}(\{c_a^\dagger, c_a\})(\tau)$$

$$G_{0ab}^{-1}(i\omega_n) = (i\omega_n + \mu)\delta_{ab} - \Delta_{ab}(i\omega_n)$$

$a, b = l, N$

- Expansion in hybridization :

$$Z = \sum_{n \geq 0} \int_< \prod_{i=1}^n d\tau_i d\tau'_i \sum_{a_i, b_i=1, N} \underbrace{\det_{1 \leq i, j \leq n} [\Delta_{a_i, b_j}(\tau_i - \tau'_j)]}_{w(n, \{a_i, b_i\}, \{\tau_i\})} \text{Tr} \left(\mathcal{T} e^{-\beta H_{\text{local}}} \prod_{i=1}^n c_{a_i}^\dagger(\tau_i) c_{b_i}(\tau'_i) \right)$$

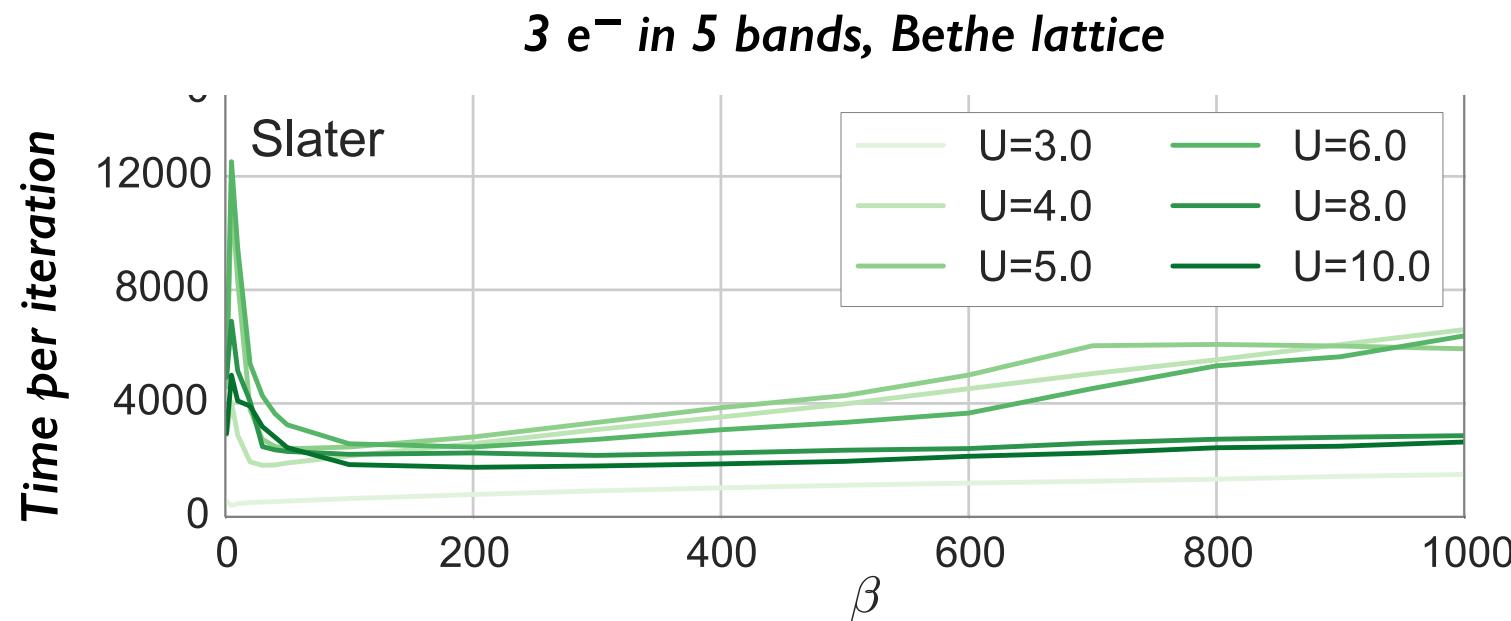
$$G_{ab}(\tau) = \frac{1}{Z} \frac{\delta Z}{\delta \Delta_{ba}(-\tau)}$$

Determinant manipulations

Atomic correlators

Key features of TRIQS/CTHYB

- Treat general Hamiltonian, not only density-density interaction.
≠ “segment picture code”
- New ‘auto-partitioning’ algorithm divides the impurity Hilbert space efficiently **without quantum numbers, i.e. no additional user input**.
- Use balanced tree (Gull 2008) and truncations on it (Yee, Sémond et al. 2014) for atomic correlators. **Much better scaling at low T**



Many-body operators

- Write Hamiltonian (or any operator) in a natural way, as polynomials of c, c^\dagger .
- Simple manipulation of fermionic second quantized operators with normal ordering and (basic) simplifications, Cf Tutorial.
- Part of TRIQS library.

```
from pytriqs.operators.operators import Operator, n, c_dag, c

# Spin operators
Sp = c_dag("up", 0)*c("dn", 0)          # S_+
Sz = 0.5*(n("up", 0) - n("dn", 0))      # S_z
S2 = Sz*Sz + (Sp*Sm + Sm*Sp)/2         # S^2

# The Hamiltonian of a half-filled Hubbard atom
U = 1.0
H = U*(n("up", 0) - 0.5)*(n("dn", 0) - 0.5) - U/4
```

Write more complex local Hamiltonians

- Atomic hamiltonian (impurities) can becomes quite complex, e.g.

$$H = \frac{1}{2} \sum_{ijkl, \sigma\sigma'} U_{ijkl} d_{i\sigma}^\dagger d_{j\sigma'}^\dagger d_{l\sigma'} d_{k\sigma}$$

- Corresponding code

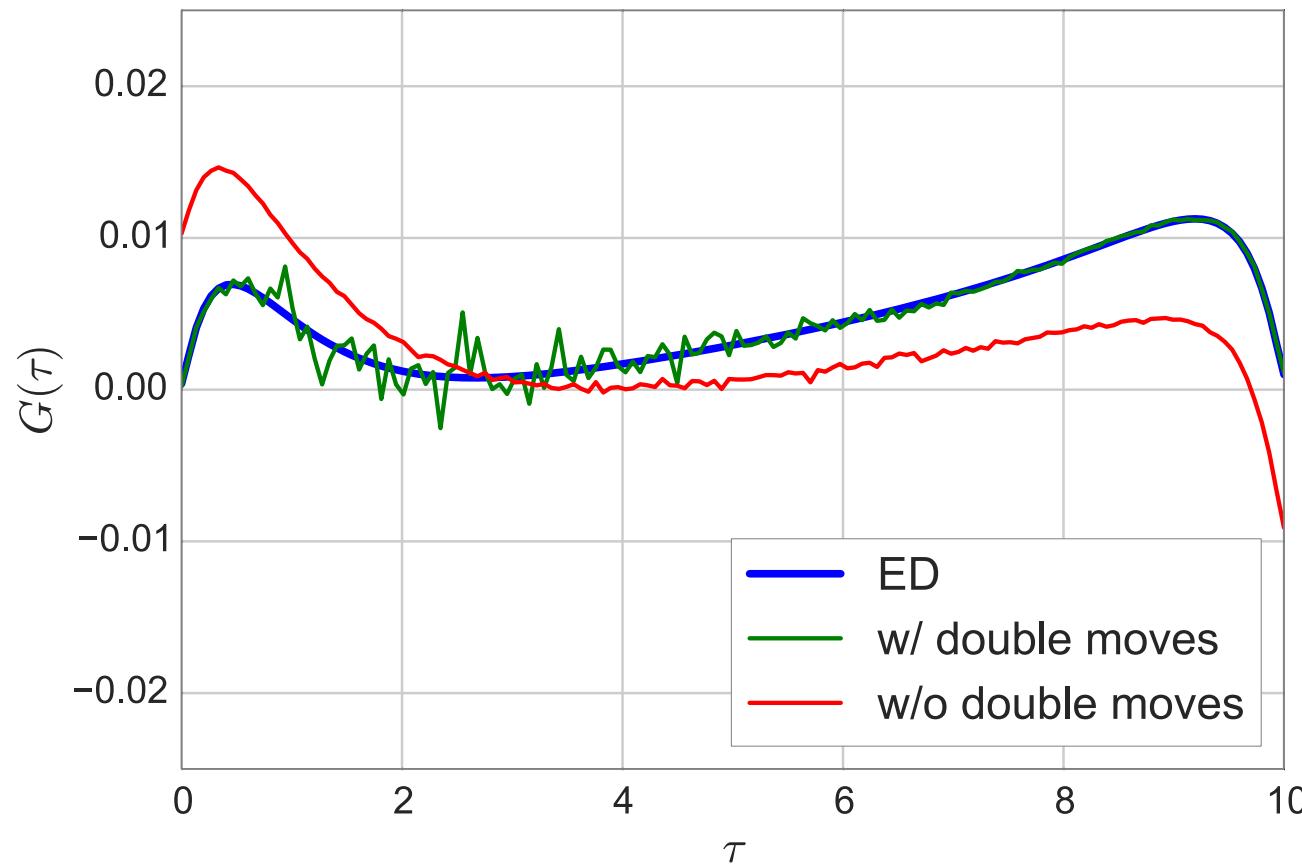
```
from pytriqs.operators import Operator, c, c_dag
spin, orb = ['up', 'down'], ['d0', 'd1', 'd2', 'd3', 'd5']

H = Operator()
for s1, s2 in product(spin, spin):
    for a1, a2, a3, a4 in product(orb, orb, orb, orb):
        U = U_matrix[orb.index(a1), orb.index(a2),
                      orb.index(a3), orb.index(a4)]
        H += 0.5 * U * c_dag(s1, a1) * c_dag(s2, a2) * c(s2, a4) * c(s1, a3)
```

- H is an input of the CTHYB solver

Features of TRIQS/CTHYB

- Four-operator insert/delete ensure ergodicity of common Hamiltonians (e.g. Kanamori), even in disordered phases (\neq Sémond et al. 2014)



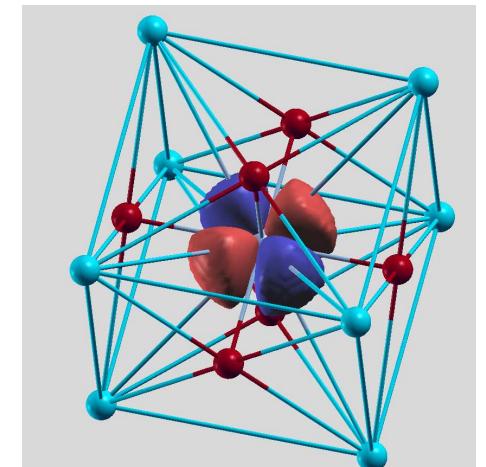
TRIQS/DFTTools.

Interface to electronic structure codes

*Markus Aichhorn, Leonid Pourovskii, Priyanka Seth, Veronica Vildosola, Manuel Zingl,
Oleg E. Peil, Xiaoyu Deng, Jernej Mravlje, Gernot J. Krabberger, Cyril Martins,
M. Ferrero, O. Parcollet
Comp. Phys. Comm. 204, 200 (2016), arXiv:1511.01302*

TRIQS / DFTTools

- Ab-initio DMFT + DFT.
Cf lectures by Kotliar & Haule next week.
Not covered in these hands-on.
- A TRIQS python interface with a growing number of electronic structure codes:
 - Wien2k
 - Wannier90
 - Vasp (in progress, to be published).
 - Fleur



SrVO_3

Developers & some contributors



Leonid Pourovskii



Markus Aichhorn



Veronica Vildosola



Michel Ferrero



Priyanka Seth



Jernej Mravlje



Gernot Kraberger



Manuel Zingl



Oleg Peil

Web sites

TRIQS web site

<https://triqs.ipht.cnrs.fr>



a Toolbox for Research on Interacting Quantum Systems

- Install
- Reference
- Tutorials
- Applications
- Issues
- About TRIQS

Welcome

TRIQS (Toolbox for Research on Interacting Quantum Systems) is a scientific project providing a set of C++ and Python libraries to develop new tools for the study of interacting quantum systems.

The goal of this toolkit is to provide high level, efficient and simple to use libraries in C++ and Python, and to promote the use of modern programming techniques.

TRIQS is free software distributed under the GPL license.

TRIQS applications

Based on the TRIQS toolkit, several [full-fledged applications](#) are also available. They allow for example to solve a generic quantum impurity model or to run a complete LDA+DMFT calculation.

Developed in a collaboration between IPHT Saclay and Ecole Polytechnique since 2005, the TRIQS library and applications have allowed us to address questions as diverse as:

- Momentum-selective aspects on cuprate superconductors (with various cluster DMFT methods)
- Degree of correlation in iron-based superconductors (within an LDA+DMFT approach)
- Fermionic Mott transition and exploration of Sarma phase in cold-atoms

Python & C++

The libraries exist at two complementary levels: on the one hand, C++ libraries allow to quickly develop high-performance low-level codes; on the other hand python libraries implement the most common many-body objects, like Green's functions, that can be manipulated easily in python scripts.

This duality is a real advantage in the development of new many-body tools. Critical parts where performance is essential can be written in C++ (e.g. a quantum impurity solver) while the data analysis, preparation of the inputs or interface with other programs can be done at the very user-friendly python level.

TRIQS 1.4-dev

This is the homepage of the TRIQS release 1.4 (in development). For the changes in 1.4, Cf [changelog page](#)

Quick search

Enter search terms or a module, class or function name.

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Documentation

- Python & C++. Mainly reference doc, with examples.
- Python tools: sphinx, rst (wiki + code + latex), automatic doc.

triqs
a Toolbox for Research on Interacting Quantum Systems

Install Reference **Tutorials** Applications Issues About TRIQS

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European Research Council
Established by the European Commission

Quick search

Enter search terms or a module, class or function name.

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The Github site

<https://github.com/TRIQS>

The screenshot shows the GitHub organization page for 'TRIQS'. At the top, there's a navigation bar with a search bar, a 'Pull requests' button, an 'Issues' button, a 'Gist' button, and user profile icons. A banner message says 'Working with your organization just got easier' and 'New customizable member privileges, fine-grained team permissions, and improved security'. Below the banner, the organization's logo (a blue pixelated alien head) and name 'TRIQS' are displayed. A navigation bar below the logo includes 'Repositories' (selected), 'People 11', 'Teams 7', and 'Settings'. On the left, two repository cards are shown: 'ctint_tutorial' (A tutorial code showing the full implementation of a CT-INT impurity solver, updated an hour ago) and 'cthyb' (A fast and generic hybridization-expansion solver, updated 22 hours ago). To the right, a 'People' section shows 11 members with their profile pictures. A 'New repository' button is located at the top right of the main content area.

This organization

Search

Pull requests Issues Gist

Working with your organization just got easier

New customizable member privileges, fine-grained team permissions, and improved security

Take the tour

TRIQS

Repositories People 11 Teams 7 Settings

Filters Find a repository... New repository

ctint_tutorial
A tutorial code showing the full implementation of a CT-INT impurity solver
Updated an hour ago

cthyb
A fast and generic hybridization-expansion solver
Updated 22 hours ago

C++ ★0 ⚡3

C++ ★8 ⚡4

People 11 >

Invite someone

How to contribute ?

- We are still a small team, help is welcome, in various ways
 - Cite TRIQS papers if you used some parts of it.
 - Issue reporting.
 - Short contribution : “pull request” on github
 - Larger contribution : coordinate with the developers.

Bug reporting

- When a problem is found in the library :
bugs, bad or unnatural behavior (API), speed issue, lack of doc.
Then :
 - Try to **narrow the problem**,
with a little test for us to **reproduce**.
 - Open an issue on github.
 - “It does not work” is **not** a bug report, it is **useless**.
- We will make a little exercise in hands-on #2
- We then fix the problem at its source (bug, design flaw) and add a non regression test.

TRIQS library for C++ programmers

The core of the TRIQS project

A few highlights

Zero cost abstraction

- What is simple should be coded simply
- Code should be as simple as math, high level **and yet fast.**
- High abstraction **and** speed.
- Contrary to a common wrong idea, it is possible.
- Let's illustrate this with 2 examples

C++ example with a Green function

- A Green function in Matsubara frequency, on a Brillouin zone.

$$G(k, i\omega_n) = \frac{1}{i\omega_n - 2t(\cos(k_x) + \cos(k_y)) - \frac{1}{i\omega_n + 2}}$$

- C++ code snippet

```
// A Brillouin zone of a Bravais lattice
auto bz = brillouin_zone{bravais_lattice{{{1, 0}, {0, 1}}}};

// A Green function G(k,omega)
double beta = 10;
auto g_k_iw = gf<cartesian_product<brillouin_zone, imfreq>>{
    {{bz, 100}, {beta, Fermion}},
    {1, 1}
};

clef::placeholder<0> iw_; // Dummy variables
clef::placeholder<1> k_;

g_k_iw(k_, iw_) << 1 / (iw_ - 2*t*(cos(k_(0)) + cos(k_(1)))) - 1 / (iw_ + 2);
```



- Loops are implicit, optimized, with proper memory traversal, ...

Lazy assignment techniques

- More complex example (from a *real* code in my group).

$$\chi_{\nu\nu'\omega}^{0\sigma\sigma'} = \beta(g_{\nu}^{0\sigma} g_{\nu'}^{0\sigma'} \delta_{\omega} - g_{\nu}^{0\sigma} g_{\nu+\omega}^{0\sigma} \delta_{\nu\nu'} \delta_{\sigma\sigma'})$$

- C++ code snippet

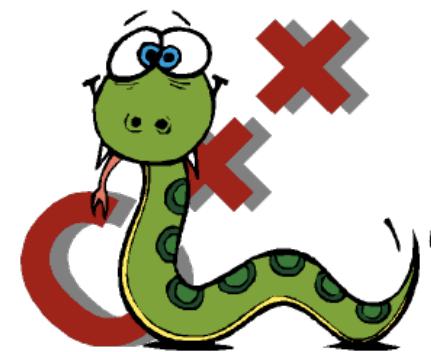
```
chi0(s_, sp_)(nu_, nup_, om_) <=
    beta * (g[s_](nu_) * g[sp_](nup_) * kronecker(om_))
  - beta * (g[s_](nu_) * g[s_](nu_ + om_) * kronecker(nu_, nup_) * kronecker(s_, sp_));
```

- Express intent, leave the details to the compiler & library writer.
- Save 5 loops, no need to think about bounds, memory traversal order.
- Easy parallelization, like parallel_for_each, gpu_for_each (TBB, AMP, ...)

CT-INT

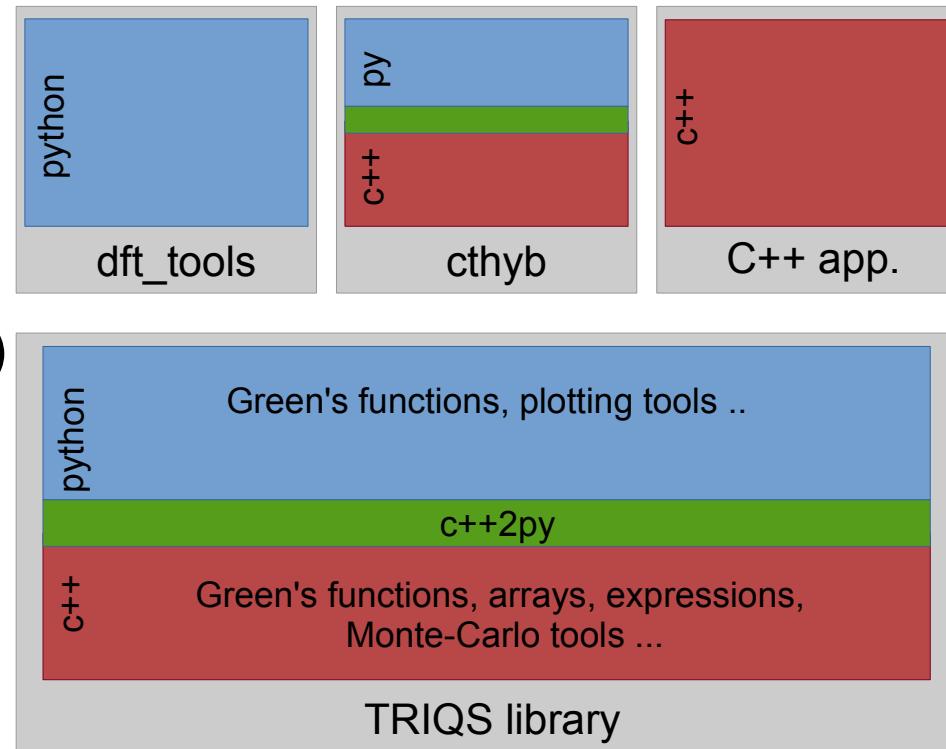
- A demo code given with the TRIQS paper.
- A full C++ implementation for CT-INT, for I band.
- Parallel (MPI), with its Python interface.
- Illustration of the use of TRIQS C++ lib, 200 lines of C++.
- git@github.com:TRIQS/ctint_tutorial.git

Gluing Python and C++



How to efficiently glue C++ & Python ?

- Issue: pass objects Python ↔ C++ (e.g. Green's functions, slices of them)
- It is **critical** for the TRIQS project.
- Not an easy problem in software engineering. Python & C++ are quite different.
- TRIQS solution : **c++2py**
- In most cases, it is entirely automatic.
 - Takes C++ code, extract the functions, classes, their documentation (thanks to libClang & Google compiler team !) ...
 - ... write glueing code, the documentation



Questions ?

Let us start the hands-on ...

Hands on: the menu



Files Running Clusters

Select items to perform actions on them.

<input type="checkbox"/>	<input type="checkbox"/>	
<input type="checkbox"/>	<input type="checkbox"/>	other
<input type="checkbox"/>	<input type="checkbox"/>	results_two_bands
<input type="checkbox"/>		00a-Introducing_the_ipython_notebook.ipynb
<input type="checkbox"/>		00b-Matplotlib_Examples.ipynb
<input type="checkbox"/>		01-Greens_functions.ipynb
<input type="checkbox"/>		02-Greens_functions_solution.ipynb
<input type="checkbox"/>		03-Archiving_your_data.ipynb
<input type="checkbox"/>		04-IPT_and_DMFT.ipynb
<input type="checkbox"/>		05-IPT_and_DMFT_solution.ipynb
<input type="checkbox"/>		06-Operators.ipynb
<input type="checkbox"/>		07-Introduction_to_the_CTHYB_solver.ipynb
<input type="checkbox"/>		08-Single-orbital_Hubbard_with_CTQMC.ipynb
<input type="checkbox"/>		09-Single-orbital_Hubbard_with_CTQMC_solution.ipynb
<input type="checkbox"/>		10-Two-orbital_Hubbard_with_CTQMC.ipynb
<input type="checkbox"/>		11-Two-orbital_Hubbard_with_CTQMC_solution.ipynb

Practical details

- TRIQS and apps are already installed on the server
- Everyone run his/her own ipython on one core on the server.
- Launch a ipython on a node (with 6h time limit).

```
qsub /soft/bin/run_jupyter
```

- Find the node attributed to you

```
more ~/jupyter_url.txt
```

```
> Starting jupyter sesion on https://10.0.XXX.XX:YYYY
```

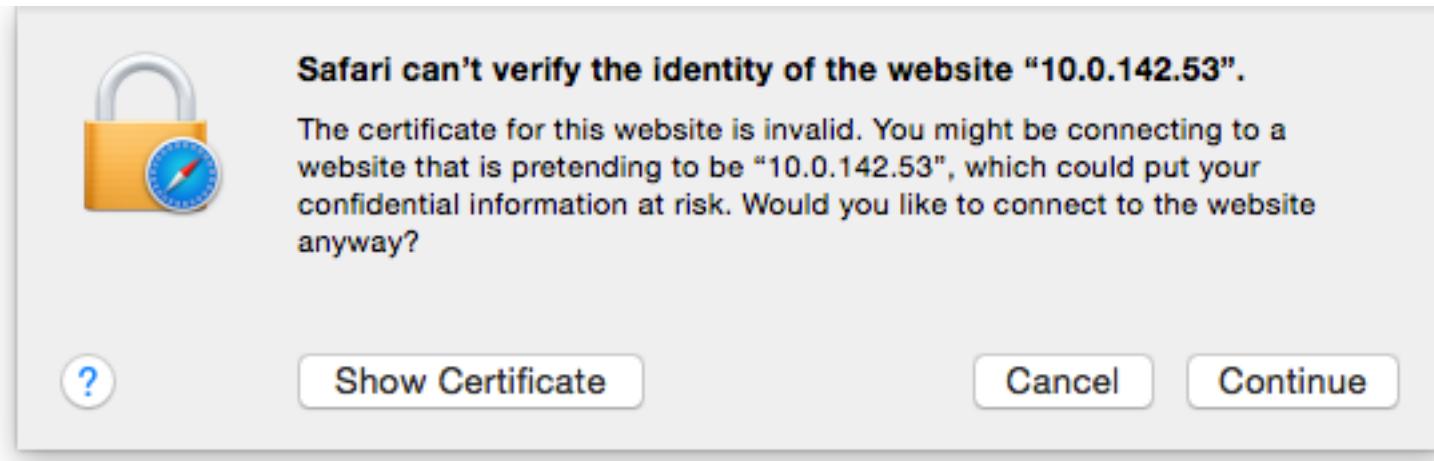
- Clone locally the tutorials (local copy of <https://github.com/TRIQS/tutorials.git>)

```
git clone /soft/public_soft/triqs/src/tutorials tutorials
```

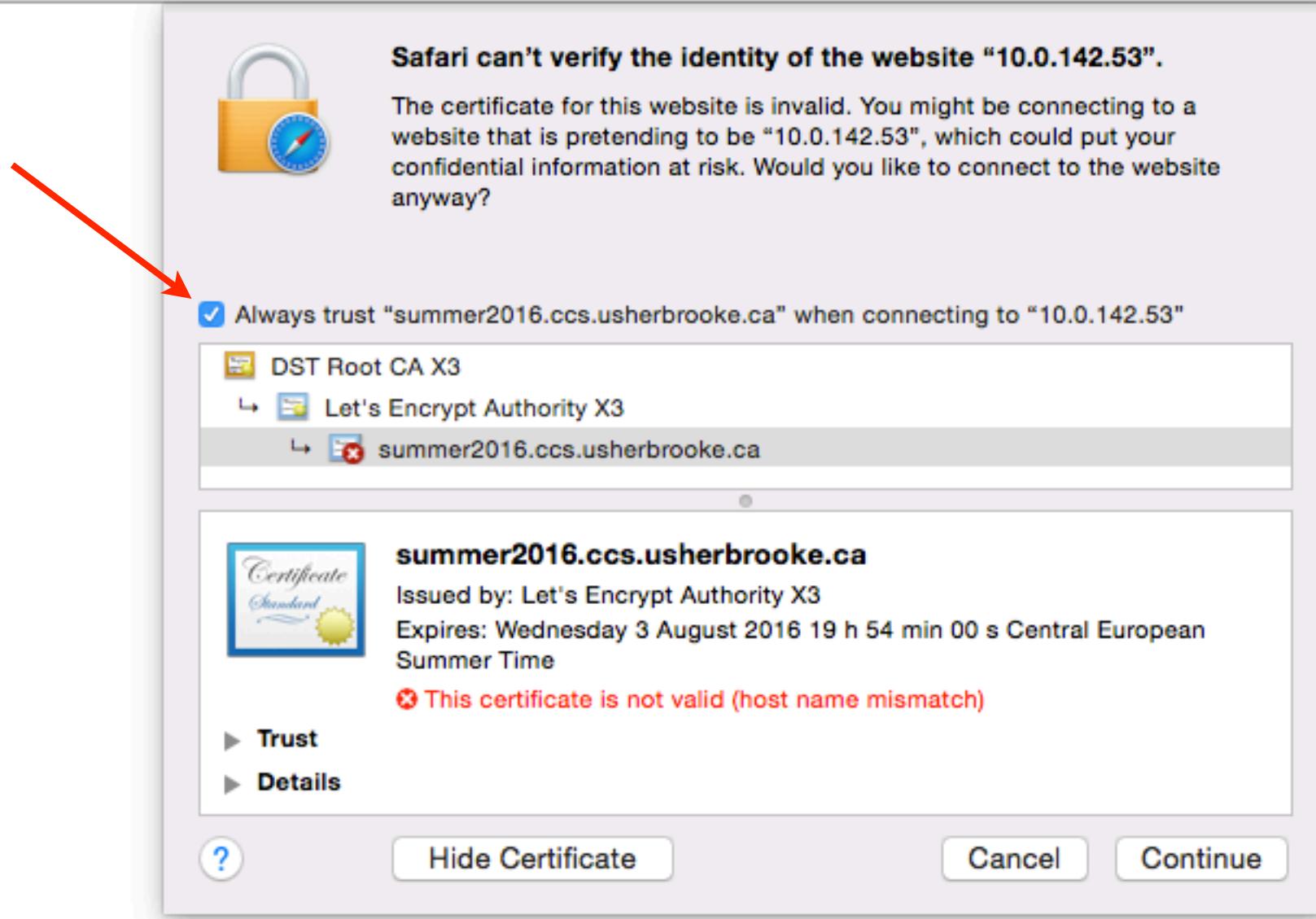
- Open browser at <https://10.0.XXX.XX:YYYY>
You need to reenter your password.

- At end, **qdel job_id** your ipython job (use **qstat** to the job id).

Safari on OS X



Safari on OS X



+ password to change Keychain Access

In the browser, you should see something like

The screenshot shows the Jupyter Notebook interface. At the top, there's a navigation bar with a logo, a "Logout" button, and tabs for "dashboard", "Files", "Running", and "Clusters". Below the dashboard tab, there's a message: "Select items to perform actions on them." On the right side of the interface, there are buttons for "Upload", "New", and a refresh icon. The main area is a file browser with a sidebar. The sidebar has a dropdown menu, a "Home" icon, and three entries: "Desktop", "tutorials", and "jupyter_url.txt".

- Click on tutorials, then python

Files

Running

Clusters

Select items to perform actions on them.

Upload

New ▾

 / tutorials / python

..

 00a-Introducing_the_ipython_notebook.ipynb 00b-Matplotlib_Examples.ipynb 01-Greens_functions.ipynb 02-Greens_functions_solution.ipynb 03-Archiving_your_data.ipynb 04-IPT_and_DMFT.ipynb 05-IPT_and_DMFT_solution.ipynb 06-Operators.ipynb 07-Introduction_to_the_CTHYB_solver.ipynb 08-Single-orbital_Hubbard_with_CTQMC.ipynb 09-Single-orbital_Hubbard_with_CTQMC_solution.ipynb 10-Two-orbital_Hubbard_with_CTQMC.ipynb 11-Two-orbital_Hubbard_with_CTQMC_solution.ipynb