Solution of then-channel Kondo problem (scaling and integrability)

Article *in* Zeitschrift für Physik B Condensed Matter · September 1984

DOI: 10.1007/BF01319184

2 authors:

A. M. Tsvelik
Brookhaven National Laboratory

Paul Wiegmann
University of Chicago

Z. Phys. B – Condensed Matter 54, 201-206 (1984)

Condensed Matter
Zeitschrift für Physik B

Solution of the n-Channel Kondo Problem (Scaling and Integrability)

A.M. Tsvelick and P.B. Wiegmann
L.D. Landau Institute for Theoretical Physics, USSR Academy of Sciences, Moscow, USSR

Received November 14, 1983

The exact solution of the Kondo model for n-flavours of electrons with the spin 1/2 scattered by the S-spin impurity is presented. For $n=2S=5$ the model describes manganese impurities dissolved in a metal. It is shown that at $n>2S$ the effective exchange coupling approaches a finite fixed point as the energy scale decreases. It means that at $n>2S$ the Gell-Mann-Low function turns to zero in this point and the scaling behaviour of physical quantities is observed. The scaling behaviour, first obtained in the $1D$ quantum field theory, can be analyzed on the basis of the exact solution. In the case $n \leqq 2S$ the effective coupling becomes infinitely strong at low energies.

1. Introduction

In all known asymptotically free $1D$ quantum field theories the effective interaction rapidly increases with a decrease of the energy scale until the unitary limit is achieved. Such a dramatical role of the interaction in the formation of the ground state of the system is primarily due to one-dimensionality and the common lore is that in such systems the fixed point can be either zero or infinite.

Recently Nozieres and Blandin [1] have pointed out the first exception to this rule. They have given simple plausible arguments that the fixed point of the exchange Hamiltonian (4), describing the scattering of n-flavours of 1/2-spin electrons by the S-spin impurity (the so-called n-channel Kondo problem), at $n>2S$ corresponds to a finite J^*. As a consequence, all physical quantities should have the scaling power law at small energy scales. Their prediction has been confirmed by the numerical renormalization group calculations [2].

In this paper we give the Exact Solution of the Nozieres-Blandin Hamiltonian. We show that the scaling can actually exist in the $1D$ quantum field theory and, moreover, the scaling behaviour can be investigated in the framework of a nontrivial completely integrable system.

Below we calculate only the magnetic field dependence of the impurity magnetization $M_{\mathrm{imp}}(H)$. According to the general properties of renormalizability, what kind of low field behaviour should be expected? The magnetization satisfies the equation

$$\frac{dM_{\mathrm{imp}}}{d\ln H}=f(z) \tag{1}$$

where the effective coupling z is defined by the Gell-Mann-Low function

$$\Phi(z)=\ln H/T_K; \tag{2}$$

or

$$\frac{dz}{d\ln H}=\beta(z); \qquad \beta(z)=\left(\frac{d\Phi(z)}{dz}\right)^{-1}$$

If in the case we are interested in, the β-function has a zero at some $z=z^*$

$$\beta(z)=\beta_0\cdot(z-z^*) \quad \text{at } |z-z^*|\ll 1$$

then as a consequence, [3, 4]

$$z-z^* \propto (H/T_K)^{\beta_0} \quad \text{at } H\to 0$$

and

$$M_{\mathrm{imp}}(H)\propto\left(\frac{H}{T_K}\right)^{f(z^*)} \quad \text{at } H\to 0 \tag{3}$$

Below we calculate the impurity magnetization for an arbitrary magnetic field and parameters n and S.

2. The "n-Channel" Kondo Hamiltonian

$$\mathscr{H} = \sum_{k,m,\sigma} v_F(k-k_F)\, c^+_{km\sigma}\, \sigma_{km\sigma} + \mathscr{J} \sum_{\substack{k,p\\ m,\sigma,\sigma'}} c^+_{km\sigma}\, \bar{\sigma}_{\sigma\sigma'}\, c_{pm\sigma'} \cdot \bar{S} \tag{4}$$

Here $c^+_{km\sigma}$ and $c_{km\sigma}$ are the creation and annihilation operators of an electron in the state characterized by the projection $m=-l\dots+l$, of the orbital moment $l=(n-1)/2$ and the spin $\sigma=\pm\frac{1}{2}$ (only the l-partial wave is scattered by the impurity which is an orbital singlet). For $n=2S$ the Hamiltonian (4) is realistic and describes the Mn ($n=5$) alloys (and Co ($n=3$), V ($n=2$) alloys in a strong cubic crystal field) [2, 5]. The ground state of these ions is an orbital singlet $({}^{1+n}S_{n/2})$. For the Hamiltonian (4) can serve only as a model*.

We derive the following formula for the impurity magnetization at $T=0$:

$$M_{\mathrm{imp}}(H) = -\frac{i n}{4\pi^{\frac{3}{2}}} \int_{-\infty}^{+\infty} \frac{dy}{y-i0}\, e^{2iy\ln\frac{H}{T_H}} \left(\frac{iyn+0}{e}\right)^{iyn} \cdot \frac{\Gamma(1+iy)\,\Gamma(\frac{1}{2}-iy)}{\Gamma(1+iyn)} \left(e^{-\pi|y|\,|n-2S|} - e^{-\pi|y|(n+2S)}\right) \cdot \left(1-e^{-2\pi n|y|}\right)^{-1} + (S-n|2)\,\theta(S-n|2). \tag{5}$$

Here $T_H = \dfrac{2\pi(n/2e)^{n/2}}{\Gamma(n/2)} T_K$, T_K being the Kondo temperature.

Equation (5) generalizes the result, obtained in [6] (for review see [7, 8]).

At $n<2S$ the ground state of the impurity is $(2S+1-n)$-fold degenerate: $M_{\mathrm{imp}}(0)=0$.

At $n \geqq 2S$ the ground state is *a singlet:* $M_{\mathrm{imp}}(0)=0$. However polarization of this singlet is different for $n=2S$ and $n>2S$: the well-known *Fermi liquid law:*

$M_{\mathrm{imp}}(H) \backsim H/T_K$ at $n=2S$

is replaced at $n>2S$ by *the scaling law:*

$M_{\mathrm{imp}}(H) \backsim (H/T_K)^{2/n}$.

3.

Attempts to diagonalize the Hamiltonian (4) by the Bethe-Ansatz technique encounter a difficulty which is absent in other integrable models of magnetic impurities in a metal. The linear spectrum approximation and point-like interaction, which are necessary for amodel to be integrable, lead to the independence of the "bare" S-matrix upon the energy of particles. In our case

$$S^{m'\sigma';s'}_{m\sigma;s} = (\exp(i\mathscr{J}\bar{\sigma}\cdot\bar{S}))^{\sigma's'}_{\sigma s} \cdot \delta_{mm'} \tag{6}$$

$(m=-(n-1)/2, \dots, (n-1)/2;\ \sigma=\pm\frac{1}{2},\ s=-S\dots+S.)$

and is the tensor product of matrices acting in the spin and orbital spaces due to the symmetry of the Hamiltonian $SU(2)$ and $U(n)$. As a result of the two-particle factorization, many-particle scattering processes in spin and flavour channels seem to be independent as well. This conclusion for the physical particles is drawn by Furuya and Lowenstein [9]. To make sure that this result is invalid, suffice it to calculate the two-loop diagram in the conventional perturbation theory where the spin and flavour processes are coupled. The thing is that in the relativistic version of the Hamiltonian it is impossible to correctly take into account the high energy processes and the axial flavour anomaly related to them.

4.

To solve the problem, we consider the "nonrelativistic" integrable version of the exchange Hamiltonian (4) – the orbital degenerate Anderson model of the special type:

$$\mathscr{H} = \sum_{k,m,\sigma} v_F(k-k_F)\, c^+_{km\sigma} c_{km\sigma} + V \sum_{k,m,\sigma} (c^+_{km\sigma} d_{m\sigma} + \text{h.c.}) + \mathscr{H}_{\text{atom}} \tag{7a}$$

where

$$\mathscr{H}_{\text{atom}} = \varepsilon_d \sum_{m,\sigma} d^+_{m\sigma} d_{m\sigma} - \frac{U}{2} \sum_{\substack{m,m'\\ \sigma,\sigma'}} d^+_{m\sigma} d^+_{m'\sigma'} d_{m'\sigma} d_{m\sigma'} \tag{7b}$$

In fact, in the absence of hybridization, the ground state of the atomic shell at

$0 < U(n-1)/2 < \varepsilon_d < U(n+2)/2$

is the orbital singlet ($n_d=n$; $S=n/2$; $L=0$). For the sake of simplicity we shall deal with only the symmetric case and consider hybridization of only the states $n_d=n$ and $n_d=n-1$, i.e., the parameters satisfy the condition

$$U \gg \varepsilon_d - U(n-1)/2 \gg n\Gamma; \quad (\Gamma = \pi\rho(\varepsilon_F) V^2) \tag{8}$$

Under these conditions the models (4) and (7) are equivalent at $n=2S$.

* This model may be applied to the alloys with some Mn-isotopes where the hyperfine interaction is comparable with the Kondo temperature

5.

It is not difficult to find the Bethe-Ansatz solution of the model (7). The two-particle S-matrix of this model is also a tensor product

$$S(k,p)=S^{(\sigma)}(k,p)\otimes S^{(f)}(k,p)$$

Here $S^{(\sigma)}$ is the S-matrix of the nondegenerate Anderson model with the electron repulsion on the atomic shell, it acts in the spin space; $S^{(f)}$ is the S-matrix of the degenerate Anderson model with the attractive interaction [10, 11], it acts in the flavour space:

$$S^{(a)}(k,p)=\frac{k-p\pm i\frac{2\Gamma}{n}P^{(a)}}{k-p\pm i\frac{2\Gamma}{n}};\qquad (a=f,\sigma)$$

where $P^{(a)}$ is the permutation operator.
Using the Bethe-Ansatz technique, one can obtain the spectral equations glueing together the spin and flavour Bethe-Ansatzes:

$$\exp(i k_j L)\cdot\frac{k_j-\varepsilon_d-i\Gamma}{k_j-\varepsilon_d+i\Gamma}=t^{(\sigma)}(k_j)\cdot t^{(f)}(k_j);\tag{9}$$

where $t^{(a)}$ are the eigenvalues of the operators

$$T^{(a)}(k_j)=\prod_{p=j+1}^{\mathcal{N}} S^{(a)}(k_j,k_p)\prod_{p=1}^{j-1} S^{(a)}(k_j,k_p):\quad (a=f,\sigma).$$

The eigenvalues of the Hamiltonian (7) are

$$E=\sum_{j=1}^{\mathcal{N}} k_j;$$

and they do not split into independent spin and flavour parts. They are coupled by (9) due to the k-dependence of the bare S-matrix. The eigenvalue $t^{(a)}$ are determined by the Bethe-Ansatz hierarchies

$$t^{(\sigma)}(k_j)=\prod_{\alpha=1}^{M} e_1(k_j-\lambda_\alpha);\tag{10a}$$

$$\prod_{j=1}^{\mathcal{N}} e_1(-\lambda_\alpha+k_j)=\prod_{\beta=1}^{M} e_2(\lambda_\alpha-\lambda_\beta);\tag{10b}$$

$$t^{(f)}(k_j)=\prod_{\alpha=1}^{m^{(1)}} e_1^{-1}(k_j-\mu_\alpha^{(1)});\tag{11a}$$

$$\prod_{\tau=\pm1}\prod_{\beta=1}^{m^{(j+\tau)}} e_1(\mu_\alpha^{(j)}-\mu_\beta^{(j+\tau)})=\prod_{\beta=1}^{m^{(j)}} e_2(\mu_\alpha^{(j)}-\mu_\beta^{(j)});\tag{11b}$$

here $e_n(x)=\dfrac{x+i n/2}{x-i n/2}$; $\mu^{(0)}=k$; $\mathcal{N}$ is the total number of particles, $M=\dfrac{\mathcal{N}}{2}-S^z$ is the number of particles with up-spins

$$m^{(j)}=\sum_{k=j+1}^{n} n_k$$

where n_k is the number of the k-flavour particles. Below we consider only the orbital singlet state when

$$n_k=\mathcal{N}/n.$$

We derive the solution of (10), (11) in the limit (8) and, as a result, the correct solution of the n-channel Kondo problem at $n=2S$. Unfortunately we do not know the "nonrelativistic" version of the Hamiltonian (4) for $n\neq 2S$. Therefore to solve the problem, we use the following observation.

6.

First of all consider the integrable exchange model, describing the S_e-spin electrons scattered by S_i spin impurity:

$$\mathcal{H}=\sum_{k,s} v_F(k-k_F)\,c_{ks}^{+}c_{ks}+\sum_{\substack{k,k'\\ s,s'}} c_{ks}^{+}P((\bar S_e)_{ss'}\cdot(\bar S_i))\,c_{k's'};\tag{12}$$

where $P(x)$ is a polynomial. The model is integrable if

$$P(x)=\frac{1}{i}\ln R(x,\mathcal{J}^{-1});$$

where $R(x,\lambda)$ is the matrix for the S-presentation for the $SU(2)$ symmetry group, satisfying the Yang-Baxter (triangle) equations, λ is the spectral parameter, $\mathcal{J}$ is the coupling constant. The explicit form of this matrix was obtained [12, 13]

$$R(x,\lambda)=-\sum_{l=|S_i-S_e|}^{S_i+S_e}\prod_{K=0}^{l}\frac{\lambda-ik}{\lambda+ik}P_l(x)$$

where P_l is the projection operator on the state with the total spin

$$P_l|l'\rangle=\delta_{ll'}|l\rangle$$
$$\bar S_e\cdot\bar S_i|l\rangle=(\tfrac{1}{2}l(l+1)-S(S+1))|l\rangle.$$

The Bethe-Ansatz technique applied directly to the exchange Hamiltonian (12) does not encounter the difficulties mentioned above. The solution has a form

$$\exp(i k_j L)=\prod_{\alpha=1}^{M} e_{2S_e}(-\lambda_\alpha) \quad (13a)$$

$$[e_{2S_e}(-\lambda_\alpha)]^{\mathcal{N}} e_{2S_i}(-\lambda_\alpha-1/\mathcal{J})=\prod_{\beta=1}^{M} e_2(\lambda_\beta-\lambda_\alpha). \quad (13b)$$

The reasons to consider such an exotic model are as follows. Solving (12), on the one hand, and (10), (11), on the other hand, we show that the both solutions coincide and, as a consequence, the both models (4) and (12) are equivalent at $S_e=S_i=n/2$. Next, we assume that the models (4) and (12) are equivalent for arbitrary $n=2S_e$; $S=S_i$★.
To prove the equivalence, we consider here the ground state properties of the both models in the presence of a magnetic field.

7.

Let us start with the model (12). It is clear that the ground state formed by the $n=2S_e$ - string solution

$$\operatorname{Im}\lambda=ik; \quad k=-(n-1)/2\ldots(n-1)/2.$$

The distribution of the real parts of the strings satisfies the equation:

$$\int_{-\infty}^{B} A_{nn}(\lambda-\lambda')\,\rho(\lambda')\,d\lambda' = \int_{-\infty}^{+\infty}\left(A_{nn}(\lambda')+\frac{1}{\mathcal{N}}A_{n,2S}(\lambda')\right)\frac{d\lambda'}{2\cosh\pi(\lambda-\lambda')}; \quad (14)$$

with the condition

$$\frac{L}{2\pi\mathcal{N}}\cdot H+M_{\text{imp}}(H)=S_e+\frac{1}{\mathcal{N}}S_i-\int_{-\infty}^{B}\rho(\lambda)\,d\lambda. \quad (15)$$

where

$$\int A_{nm}(\lambda)\,e^{i\omega\lambda}\,d\lambda = \coth|\omega|\,(e^{-|\omega||n-m|}-e^{-(n+m)|\omega|})\equiv A_{nm}(\omega)$$

(see similar calculations for the solution of the Heisenberg S-spin chain for more detail [14, 15]. In terms of the hole distribution $\tilde{\rho}(\lambda)$ Eq. (1) can be rewritten in the universal form:

$$\tilde{\rho}(\lambda)-\int_0^{\infty}\mathcal{F}(\lambda-\lambda')\,\tilde{\rho}(\lambda')\,d\lambda'=f_{n,n}\left(\lambda-\frac{1}{\pi}\ln\frac{HL}{\mathcal{N}}\right) + \frac{1}{\mathcal{N}}f_{n,2S}\left(\lambda-\frac{1}{\pi}\ln\frac{H}{T_H}\right) \quad (16)$$

where

$$1-\mathcal{F}=A_{nn}^{-1}; \quad f_{n,m}(\lambda)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}e^{-2i\omega\lambda}\cdot\frac{A_{nm}(\omega)}{A_{nn}(\omega)}; \quad (17)$$

and

$$M_{\text{imp}}(H)+\frac{L}{2\pi\mathcal{N}}H=\frac{1}{2}\int_0^{\infty}\tilde{\rho}(\lambda)\,d\lambda. \quad (18)$$

The solution of the Wiener-Hopf Eq. (16) yields Expression (5).

8.

Let us come back to the Anderson model and (10), (11). Omitting the proof, we describe the structure of the ground state (the proof will be published elsewhere).

i) All λ's are real.
ii) There are $2S^z$ real k's. The other k's are complex. They form the bound states with λ's:

$$k^{(\pm)}=\lambda\mp i/2$$

iii) There are $((n-j)/n)2S^z$ real $\mu^{(j)}$ and $M(n-j)/2n$ 2-strings

$$\mu^{(j)}=\Lambda^{(j)}\pm i/2$$

λ's fill the interval (Q,∞) (we shall use the hole distribution $\tilde{\sigma}(\lambda)$ defined in the $\emptyset$ interval $(-\infty,Q)$, real $\mu^{(j)}$ and $\Lambda^{(j)}$ entirely fill the real axis and real k's fill the interval $(-\infty,-B)$ with the distribution $\rho(k)$.
Equations for the distribution function $\rho(k)$, $\zeta_j^{(2)}(1)$, $\zeta_j^{(1)}(\mu)$, $\sigma(\lambda)$ for real $k,\Lambda^{(j)},\mu^{(j)},\lambda$ respectively and their holes $\tilde{\rho},\tilde{\zeta}_j^{(2)},\tilde{\zeta}_j^{(1)},\tilde{\sigma}$ are

$$\frac{2\Gamma}{n\pi}+\frac{1}{L}a_2(\lambda-1/\mathcal{J}) = \tilde{\sigma}+(1+a_2)*\sigma+a_1*\rho-\sum_{p=1,2}s*A_{2p}*\zeta_1^{(p)} \quad (19a)$$

$$\frac{1}{L}a_1(\lambda-1/\mathcal{J})+\frac{\Gamma}{n\pi}=\tilde{\rho}+\rho+a_1*\sigma-\sum_{p=1,2}s*A_{1p}*\zeta_1^{(p)} \quad (19b)$$

$$\tilde{\zeta}_j^{(p)}+A^{pq}C_{jk}\zeta_k^{(q)}=\delta_{j1}s*(A_{p1}*(A_{p1}*\rho+A_{p2}*\sigma) \quad (19c)$$

We use the convolution symbol: $a*b=\int_{-\infty}^{\infty}a(\lambda)b(\lambda-\lambda')d\lambda'$. The kernels in (19) are:

$$a_m(\lambda)=\frac{2}{\pi}\frac{m}{4\lambda^2+m^2}; \quad s(\lambda)=\frac{1}{2\cosh\lambda};$$

$$C_{jk}=\delta_{jk}\,\delta(\lambda)-s(\lambda)\cdot(\delta_{jk-1}+\delta_{jk+1});$$

★ Unfortunately we could not find a more intelligent proof of the equivalence of the models other than comparing their Exact Solutions

Let us assert without proof that at $T=0$, $\zeta_j^{(p)}$ (the holes in $\mu^{(j)}$ and $\Lambda^{(j)}$ are absent). Then (19c) is solved by the Fourier method

$$\zeta_j^{(p)}=s_j*(\delta_{p1}\,\rho+\delta_{p2}\,\sigma); \tag{20}$$

where $s_j=\int\frac{d\omega}{2\pi}\,e^{i\omega\lambda}\,\frac{\sinh(n-j)\,\omega/2}{\sinh n\omega/2}$.

Insert (20) into (19a) and (19b):

$$\frac{2\Gamma}{\pi n}+\frac{1}{L}\,a_2(\lambda-1/\mathcal{J})$$
$$=\tilde{\sigma}+(1+a_2)*(1-a_1*s_1)*(\sigma+s*\rho) \tag{21a}$$

$$\frac{\Gamma}{\pi n}+\frac{1}{L}\,a_1(\lambda-1/\mathcal{J})=\tilde{\rho}+(1-a_1*s_1)*(\rho+a_1*\sigma). \tag{21b}$$

Converting the kernel in (21a), let us express σ in terms of $\tilde{\sigma}$ and ρ. Inserting it into (21b), we get:

$$\tilde{\rho}+\rho-\mathcal{F}*\rho=s*\tilde{\sigma}+\frac{1}{L}\,s(\lambda-1/\mathcal{J}). \tag{22}$$

In the universal limit we study only low magnetic fields $H\ll\Gamma$. Therefore in (22) it is necessary to restrict oneself to the asymptotics of $s*\tilde{\sigma}$ at $\lambda\gg 1$:

$$s*\tilde{\sigma}\simeq e^{-\pi\lambda}\,e^{\pi Q}\int_Q^\infty\tilde{\sigma}(\lambda+Q)\,e^{-\pi\lambda}\,d\lambda.$$

By means of the substitution of the variables

$$\rho\to\tilde{\rho};\qquad \lambda\to\lambda+\frac{1}{\pi}\ln\frac{a}{H}+Q$$

where

$$a=\int_0^\infty\tilde{\sigma}(\lambda+Q)\,e^{-\pi\lambda}\,d\lambda\cdot\sqrt{2\pi/e}.$$

Equation (22) is reduced to the form of (16) at $n=2S$. The temperature T_H is defined as

$$T_H=a\,e^{\pi Q}\,e^{-1/\mathcal{J}\rho(\varepsilon_F)}.$$

9.

Let us discuss the behaviour of the impurity magnetization (5) at $n\to\infty$, keeping $(n/2S)\geqq 1$ as fixed. In the region of the largest energy when $(\ln H/T_H)/n$ is kept fixed but much more then unity, we have the standart perturbation theory expansion. In this region the effective coupling has its smallest value: $z\ll 1/n$ (z/n is kept fixed) and the well known two-loop approximation gives [1]:

$$\beta(z)=z^2-nz^3+O(z^4).$$

Using the universal part of the function Φ (see Eq. (2))

$$1/z-(n/2)\ln z=\ln H/T_H \tag{23}$$

one obtains the perturbation series from the Exact Solution

$$M_{\text{imp}}=S\left(1-z+\sum_{K=2}^{\infty}a_k(n,S)\,z^k\right).$$

At $n\to\infty$ the coefficients $a_k(n,S)\,n^{-k+2}$ have the finite limit. Setting $nz=Z$ we have

$$M_{\text{imp}}(Z,S/n)=-in/4\pi\int_{-\infty}^{\infty}dy\,e^{+2iy}(iy+0)^{yZi}$$
$$\cdot\sinh(ZSy/n)/(\Gamma(1+iyZ)\sinh(yZ)/(y-i0)). \tag{24}$$

It means that in the $1/n$ expansion it is impossible to neglect high order terms in (22) or use any ladder diagramms at $zn\infty 1$. The class of diagramms which give the same contribution in the limit $n\to\infty$, and they can hardly be summed without the Exact Solution*.

Next consider the limit $n=2S\to\infty$, $\ln H/T=$fixed, $z=$fixed. The impurity magnetization has the finite limit

$$M_{\text{imp}}(H/T_H)=-\frac{\sqrt{n}}{8\pi^2}\int_{-\infty}^{\infty}\frac{dy}{(iy+0)^{3/2}}$$
$$\cdot\,e^{2iy\ln H/T_H}\,\Gamma(1+iy)\,\Gamma(1/2-iy). \tag{25}$$

At $H\gg T_H$ ($1/n\ll z\ll 1$) we obtain a logarithmic expansion

$$M_{\text{imp}}(H/T_H)=\frac{1}{\pi}\sqrt{n\ln(H/T_H)}$$
$$\cdot\left(1+\frac{\pi\ln 2}{2\ln H/T_H}+\dots\right). \tag{26}$$

At $H\ll T_H$ the Hamiltonian has the Fermi-Liquid-like fixed point at $z\to\infty$

$$M_{\text{imp}}(H/T_H)=\sum_{k=1}^{\infty}(H/T_H)^k(-1)^k\frac{k+1/2}{k!\sqrt{k}}. \tag{27}$$

The behaviour of the magnetization at small magnetic field for $n>2S$ is quite different. In the region $\ln H/T_H=$fixed the magnetization does not change at $n\to\infty$, $n/2S=$fixed. However it changes rapidly in the vicinity of the fixed point where the scaling stops at $z\to z^*\sim 1/n$. In this region $(\ln H/T_H)/n=$fixed $\ln T_H/H$

* We encountered such a situation when considered the degenerate Anderson model [16, 17]

$\gg 1/n$. Using (25) one can obtain the scaling law

$$M_{\text{imp}}(H/T_H)=\sum_{k=1}^{\infty}\left(\frac{H}{T_H}\right)^{2k/n}\frac{k^k}{k!}(-1)^k \sin\left(\frac{\pi k S}{n}\right);$$

predicted by Nozieres and Blandin [1].

10.

In the end we note that there is an intermediate case between the strong coupling fixed point at $n=2S$ and the pure scaling behaviour at $n=2S$. It is the case $n=2$, $S=1/2$. At $H \ll T_H$

$$M_{\text{imp}}=\sum_{k=0}^{\infty}(H/T_H)^{2k+1} A_k \ln(H/T_H)$$

where A_k, B_k are numerical coefficients. This case was treated numerically [2].

We are grateful to V.A. Fateev and S. Pokrovskii for valuable discussions.

References

1. Nozieres, P., Blandin, A.: J. Phys. (Paris) **41**, 193 (1980)
2. Cragg, D.M., Lloyd, P. Nozieres, P.: J. Phys. C **13**, 803 (1979)
3. Abrikosov, A.A., Migdal, A.A.: J. Low Temp. Phys. **3**, 519 (1970)
4. Fowler, M., Zawadowskii, A.: Solid State Commun. **9**, 471 (1971)
5. Schrieffer, J.R.: J. Appl. Phys. **38**, 1143 (1967)
6. Fateev, V.A., Wiegmann, P.B.: Phys. Lett. **81** A, 179 (1980)
7. Andrei, N., Furuya, K., Lowenstein, J.H.: Rev. Mod. Phys. **55**, 331 (1983)
8. Wiegmann, P.B.: In: Quantum theory of solids. Lifshitz, I.M. (ed.), p. 238. Mir publishers 1982; Tsvelick, A.M., Wiegmann, P.B.: Adv. Phys. (to be published)
9. Furuya, K., Lowenstein, J.H.: Phys. Rev. B **25**, 5935 (1982)
10. Wiegmann, P.B.: Phys. Lett. **80** A, 163 (1980)
11. Schlottmann, P.A.: Z. Phys. B – Condensed Matter **49**, 109 (1982)
12. Fateev, V.A.: (unpublished)
13. Kulish, P.P., Reshetichin, N., Sklyanin, E.: Lett. Math. Phys. **5**, 393 (1981)
14. Takhtajan, L.A.: Phys. Lett. **87** A, 479 (1982)
15. Babujan, H.M.: Nucl. Phys. B **215**, 317 (1982)
16. Schlottmann, P.A.: Phys. Rev. Lett. **50**, 1697 (1983)
17. Ogievetskii, E.I., Tsvelick, A.M., Wiegmann, P.B.: J. Phys. C **16**, L797 (1983)

A.M. Tsvelick
P.B. Wiegmann
Landau Institute
for Theoretical Physics
ul. Kosygina 2
11733 Moscow
USSR