L2S4 Séance 5 27 28 31 33 34 35

27 x : employé y tech. z cadres

$$\begin{cases} x + y + z = 60 \\ 15x + 26y + 42z = 1140 \\ 0,064 \times 1500x + 0,045 \times 2600y + 0,045 \times 4200z = 0.056 \times 114000 \end{cases}$$

$$\Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 15 & 26 & 42 & 1140 \\ 96 & 117 & 189 & 6384 \end{array}\right] L_3 \leftarrow \tfrac{1}{3} L_3 \Leftrightarrow \left[\begin{array}{ccc|c} & & & \\ & & & \\ 32 & 39 & 63 & 2128 \end{array}\right] L_3 \leftarrow L_3 - 2L_2$$

$$\Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 15 & 26 & 42 & 1140 \\ 2 & -13 & -21 & -152 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 - 15L_1 \\ L_3 \leftarrow L_3 - 2L_1 \end{array} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 11 & 27 & 240 \\ 0 & -15 & -23 & -272 \end{array}\right] 11L_3 + 15L_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 11 & 27 & 240 \\ 0 & 0 & 152 & 608 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 11 & 0 & 132 \\ 0 & 0 & 1 & 4 \end{array}\right] L_2 \leftarrow L_2 - 27L_3$$

$$\boxed{\begin{array}{l} x = 44 \\ y = 12 \\ z = 4 \end{array}}$$

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$$\begin{cases} ax + y + z = 1 \\ x + ay + z = a \\ x + y + az = a^2 \end{cases}$$

$$\left[\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & a^2 \end{array}\right] L_1 \leftrightarrow L_2 \left[\begin{array}{ccc|c} 1 & a & 1 & a \\ a & 1 & 1 & 1 \\ 1 & 1 & a & a^2 \end{array}\right]$$

$L_2 \leftarrow L_2 - aL_1 \quad L_3 \leftarrow L_3 - L_1$

Si $1 - a \neq 0 \Leftrightarrow a \neq 1$

$1 - a^2 = (1-a)(1+a)$

$a^2 - a = a(a-1)$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & a \\ 0 & 1-a^2 & 1-a & 1-a^2 \\ 0 & 1-a & a-1 & a^2-a \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & a & 1 & a \\ 0 & 1+a & 1 & 1+a \\ 0 & 1 & -1 & -a \end{array}\right]$$

$L_3 \leftarrow L_3 + L_2$

Si $a \neq -2$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & a \\ 0 & 1+a & 1 & 1+a \\ 0 & 2+a & 0 & 1-a \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & a & 1 & a \\ 0 & 1+a & 1 & 1+a \\ 0 & 1 & 0 & \frac{1-a}{2+a} \end{array}\right]$$

$$\begin{cases} x = a - ay - z \\ z = 1 - (1+a)y = \frac{2+2a-1+a^2}{2+a} \\ y = \frac{1-a}{2+a} \end{cases}$$

$$\begin{cases} x = a - \frac{a(1-a)}{2+a} - \frac{a^2+2a-1}{2+a} = \frac{2a+a^2-a+a^2-a^2-2a+1}{2+a} = \frac{a^2-a+1}{2+a} \\ y = \frac{1-a}{2+a} \\ z = \frac{a^2+2a-1}{2+a} \end{cases}$$

Si $\underline{a=1}$

$$\begin{cases} x+y+z=1 \\ x+y+z=1 \\ x+y+z=1 \end{cases} \iff x+y+z=1$$

Plan dans l'espace

$\infty^{té}$ sol$^{\text{ns}}$

Si $\underline{a=-2}$

$$\begin{cases} -2x+y+z=1 \\ x-2y+z=-2 \\ x+y-2z=4 \end{cases} \qquad L_1 \leftarrow L_1+L_2+L_3$$

$$\begin{cases} 0x+0y+0z=3 \\ \underline{\qquad} \\ \underline{\qquad} \end{cases}$$

__impossible__ $\varnothing$ sol.

5 1

1) $\left[\begin{array}{ccc|c} 1 & 2 & -4 & a \\ -1 & -1 & 5 & b \\ 2 & 7 & -3 & c \end{array}\right]$ $\begin{array}{l} L_2 \leftarrow L_2 + L_1 \\ L_3 \leftarrow L_3 - 2L_1 \end{array}$ $\left[\begin{array}{ccc|c} 1 & 2 & -4 & a \\ 0 & 1 & 1 & a+b \\ 0 & 3 & 5 & -2a+c \end{array}\right]$ $L_3 \leftarrow L_3 - 3L_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -4 & a \\ 0 & 1 & 1 & a+b \\ 0 & 0 & 2 & -5a-3b+c \end{array}\right]$ $L_1 \leftarrow L_1 - 2L_2$ $\left[\begin{array}{ccc|c} 1 & 0 & -6 & -3a-2b \\ 0 & 1 & 1 & a+b \\ 0 & 0 & 2 & -5a-3b+c \end{array}\right]$

$L_1 \leftarrow L_1 + 3L_3$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -18a-8b+3c \\ 0 & 1 & 1 & a+b \\ 0 & 0 & 2 & -5a-3b+c \end{array}\right]$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -18a-8b+3c \\ 0 & 1 & 0 & \frac{7}{2}a+\frac{5}{2}b-\frac{c}{2} \\ 0 & 0 & 1 & -\frac{5}{2}a-\frac{3}{2}b+\frac{c}{2} \end{array}\right]$

$L_2 \leftarrow L_2 - L_3$

Solution

2) $AX = Y \iff Y = BX$ où $B = \begin{pmatrix} -18 & -8 & 3 \\ \frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ -\frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{pmatrix} = A^{-1}$.

$\iff A$ inversible et $A^{-1} = B$

$$\det : \mathcal{M}_n(\mathbb{R}) \longrightarrow \mathbb{R}$$

$$\det A \neq 0 \iff A \text{ inversible}$$

- échanger 2 lignes (colonnes) $\longrightarrow$ change le signe
- $\underbrace{L_i \leftarrow L_i + \text{CL des autres}}_{\text{ne change pas le dét.}}$

Ex $\det I_n = 1$

$$\det \begin{bmatrix} \lambda_1 & & \times \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} = \lambda_1 \times \dots \times \lambda_n.$$

3)

$$\det I_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$\det M_1 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \underset{L_1 \leftrightarrow L_2}{=} - \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1.$$

$$\det M_2 = \begin{vmatrix} 2 & & \\ & 1 & \\ & & 1 \end{vmatrix} = 2 \times 1 \times 1 = 2$$

$$\det(2 I_3) = \begin{vmatrix} 2 & & \\ & 2 & \\ & & 2 \end{vmatrix} = 2^3 = 8$$

$$\det \begin{vmatrix} 9 & 4 & 0 \\ 1 & 3 & 0 \\ 6 & 2 & 0 \end{vmatrix} = 0$$

col de 0

$\operatorname{rg} M_3 \neq 3 \implies M_3$ non inversible

$\implies \det M_3 = 0$

$$\det N_3 = \begin{vmatrix} 9 & 4 & 18 \\ 1 & 3 & 2 \\ 6 & 2 & 12 \end{vmatrix}$$

$C_3 = 2\,C_1$ les colonnes sont liées

$\Rightarrow N_3$ non inversible

$\det N_3 = 0$

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$$\det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 3 & -1 & -9 & 13 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -1 & 0 \end{vmatrix} = 0 \begin{vmatrix} X \end{vmatrix} + 0 \begin{vmatrix} X \end{vmatrix} - (-1) \begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 13 \\ 0 & 0 & -2 \end{vmatrix} + 0 \begin{vmatrix} X \end{vmatrix}$$

DVLP % L_4

$$= \begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 13 \\ 0 & 0 & -2 \end{vmatrix} = -2 \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -2(-1-6) = 14$$

DVLP % L_3

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

2.

$$\det B = \begin{vmatrix} 1^{+} & -2 & 3 \\ 2^{-} & 4 & -1 \\ 1^{+} & 5 & -2 \end{vmatrix} = 1 \begin{vmatrix} 4 & -1 \\ 5 & -2 \end{vmatrix} - 2 \begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix} + 1 \begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix}$$

DVLP % C_1

$$= -8 + 5 - 2(4 - 15) + (2 - 12)$$

$$= -3 + 22 - 10 = 9.$$

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$$\Delta = \begin{vmatrix} 1/2 & -1 & -1/3 \\ 3/4 & 1/2 & -1 \\ 1 & -4 & 1 \end{vmatrix} \quad C_2 \leftarrow C_2 + 2C_1 + 2C_3 \quad \begin{vmatrix} \frac{1}{2}^{+} & -2/3^{-} & -\frac{1}{3} \\ 3/4 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix}$$

DVLP % C_2

$$\Delta = -\frac{2}{3} \begin{vmatrix} 3/4 & -1 \\ 1 & 1 \end{vmatrix} = +\frac{2}{3}\left(3/4 + 1\right) = \frac{2}{3} \times \frac{7}{4} = \frac{7}{6}.$$

$$\det B = \begin{vmatrix} 2 & 1 & -3 \\ 1 & 0 & 2 \\ -1 & 1 & 1 \end{vmatrix} \underset{=}{L_1 \leftarrow L_1 - L_3} \begin{vmatrix} 3^{+} & 0^{-} & -4 \\ 1 & 0^{+} & 2 \\ -1 & 1^{-} & 1 \end{vmatrix} \quad \text{DVLP \% } C2$$

$$\det B = -1 \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = -(6+4) = -10$$

$$\det C = \begin{vmatrix} -2 & 4 & -8 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -2^{+} & 6 & -6 \\ 2^{-} & -1 & 1 \\ 1^{+} & 0 & 0 \end{vmatrix} = 1 \begin{vmatrix} 6 & -6 \\ -1 & 1 \end{vmatrix} = 0.$$

$C_2 \leftarrow C_2 - C_1$ DVLP % L_3

$C_3 \leftarrow C_3 - C_1$