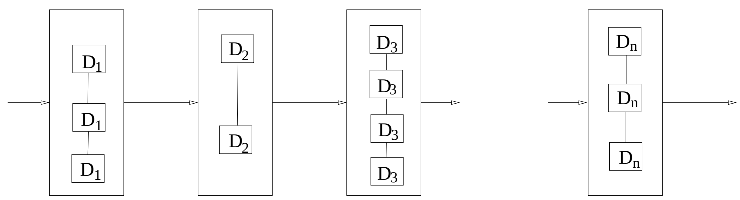
This take home is worth 20 points out of 30 of the Final Exam
You are required to work on the problem without any assistance from ANY OTHER SOURSE
You can't work as a team – if you need any form of assistance, come to me.

Due date/time: no later than 5:00 PM on December 10, 2012.

The Problem:

A system, consisting of n modules connected in a series, where each module consists of similar devices in parrellel, is shown in the following figure.



Here, for illustration purposes, there are two identical devices in module 2, and three in module n. Let r_i be the probability that device D_i will function properly. It can be verified that

$$\text{P (the system will function properly)} = \prod_{i=1}^{n} \left(1 - (1 - r_i)^{m_i}\right),$$

where m_i is the number of D_i's in the ith module.

Suppose that the cost of a D_i is c_i. Our goal is to maximise the system performance using m_i copies of D_i in the ith module; subject to the cost constraint C. That is:

$$\text{maximize}_{m_i, 1 \le i \le n} \quad \prod_{1 \le i \le n} (1 - (1 - r_i)^{m_i})$$

$$\text{subject to} \quad \sum_{1 \le i \le n} c_i m_i \le C; \;\; m_i \ge 1, \;\; 1 \le i \le n.$$

Note that subject to the specified cost considerations, the range of m_i satisfies $1 \le m_i \le u_i$, where

$$u_i = \lfloor \frac{C + c_i - \sum_{j=1}^{n} c_j}{c_i} \rfloor.$$

1. Find a dynamic programming solution of the problem. Clearly specify the decomposition equation and explain why would it work correctly. Also explain how would you apply the bottom up approach to solve the problem. Finally, derive the time complexity of the algorithm.

2. Solve the following numerical problem:

$$n = 4; c_1 = 3, c_2 = 5, c_3 = 2, c_4 = 4,$$

$$r_1 = 0.9, r_2 = 0.8, r_3 = 0.6, r_4 = 0.85; \text{ and } C = 30.$$

Optional problem for 10 extra points — Write a computer program, preferably using C, to solve the (general) problem.
Hint — Is it a variation of the knapsack problem?