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Locational Instruments for Efficient Power Generation Investment under Zonal Pricing

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There is currently an intense debate in Europe on the best way of allocating transmission capacity in the internal electricity market. The debate revolves around the fundamental distinction between nodal and zonal pricing and many discussions tend to treat the question in a dichotomic way. In this paper, we instead take zonal pricing as a starting point and investigate whether the long run efficiency could be restored by means of additional market-based instruments. Three types of instruments are considered: capacity, energy and re-dispatch markets. We formulate the long run economic equilibrium under these three policies under a unifying modeling framework and compare their performance both from a theoretical and empirical perspective. We find that theoretical conditions under which efficiency would be restored exist for all three policies, but that the strictness of these conditions render their practical implementation difficult and thereby, a full recovery of the efficiency unlikely.

Key words: Zonal pricing, capacity expansion, congestion management

History:

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1. Introduction

1.1. Motivation

The debate between nodal and zonal pricing is by all means an old story in the US evolution of the power system. In their three phases review of the history of the process, Hobbs and Oren

(2019) note, almost in passing, that the subject had been settled in the first phase, that is after the Californian experience: the zonal system had fundamental flaws that the nodal system avoided. The Texas experience came a bit later but confirmed the observation (Triolo and Wolak 2021). The situation is quite different in Europe: a recent report (ACER 2022) issued in response to criticisms against the high prices observed in the power market in the end of 2021 (that is well before the perturbations on the gas market induced by Ukraine events) explains that the current European (zonal) system is functioning properly but that one might consider some improvements, among which a reduction of the size of the zones. In particular, the nodal system is mentioned as an example of this reduction (page 27). This difference between the US and EU points of view after so many years of theory and experience on a subject that looks purely technical is strange and must have some fundamental explanation. It cannot be a misunderstanding of the nodal system, which had been described in the French literature as early as Boiteux and Stasi (1952). But institutions suggest a possible rationale. The restructuring of the power system in Europe was part of the integration of sectoral markets that created the internal market in 1992, with delays expected in "difficult sectors", electricity being one of them. Suffice it to note here, that a nodal model in an integrated European market would have merged national systems and interconnections into a new supranational model where all components of national networks were put on the same footing. Conversely, the coupling of the nodal system keeps the individual national systems (which became zones) and developed a framework facilitating exchanges between them through interconnections. In other words, the zonal system in the internal EU market would have had a supranational flavor that the zonal system was thought able to bypass.

In contrast with what is sometimes thought (both in the public in Europe and the US), Europe is a very weak supranational entity with resistance against any further integration beyond what was acquired in the early days. The worst of this is the real backward step against the supremacy of European law advocated by some member states. The supranational dimension implicit in nodal pricing certainly did not help. This paper is an attempt to deal with this issue: it takes the existing day-ahead zonal system as given but tries to complete it by addressing the needed integration at another level. This is done by inserting features with a nodal flavor in market processes on which stakeholders are still working. Needless to say, the economic and physics requirements remain and one could argue that our proposals look like a covering up of the nodal system (Molière's Tartuffe famous "Cachez ce sein que je ne saurais voir" adapted here into "Cover up that nodal system, which I can't endure to look on"). But it is just the opposite: we do not try to cover up but to make clearly visible. The requirements of basic physics and economics embedded in the nodal system namely the need to internalize the externalities created by Kirchoff's laws come up here or there depending on the remedies considered and they are always clearly referred to. One of our contribution is that they are fully inserted in the zonal system.

Our main interest is to understand the thoretical conditions under which additional market-based instruments could restore the efficiency of nodal pricing in the long run. In order to identify candidate market-based instruments for restoring the efficiency of zonal, we base ourselves on the thorough review of existing locational instruments provided in Eicke et al. (2020). To the list of Eicke et al. (2020), we add market-based re-dispatch as a market-based way to introduce a locational component in the price. In total, we consider three main classes of cadidates: capacity-based instruments, energy-based instruments and market-based re-dispatch. The present paper analyses the combination of zonal pricing and these three classes of locational instruments under a unifying modeling framework in order to study their efficiency, first from a theoretical point of view and then, based on simulations in a realistic case-study.

1.2. Related literature

There is an emerging stream of literature on the study of the impact of locational instruments in zonal markets from a quantitative perspective. The first paper that tackled this problem quantitatively is Grimm et al. (2019). The authors propose a tri-level model of the long run equilibrium of zonal pricing with cost-based re-dispatch. The tri-level structure aims at representing the sequential nature of the problem: grid investment by the TSO, capacity investment by private firms and re-dispatching in the short-term by the TSO. A simplified spatially differentiated capacity signal based on average nodal prices is then added to the model. The authors find that although it influences the location of investment, the introduction of a well-calibrated capacity signal only slightly improves the welfare as the signal fails to also impact the operational efficiency of the system. Schmidt and Zinke (2020) studies the impact of uniform pricing in Germany on the siting of wind generation. They find that nodal pricing, by incentivizing the siting of investment closer to load centers and therby reducing wind curtailment, has a positive effect on welfare. The paper also investigates the restoration of a locational component to the uniform price through capacity-based latitude-dependent connection charges but find that these simplified locational instruments are not adequate to mitigate the inefficiencies associated to unfirm pricing. Finally, Eicke (2021) proposes a model to quantify the optimal capacity-based locational signal for restoring the efficiency of investment in zonal pricing with cost-based re-dispatch. Unlike the two papers previsouly mentioned, this paper is the first to consider a technology-differentiated signal as it is recognized that differentiating by technology is a necessary condition for restoring the efficiency of nodal. The model of Eicke (2021) differs in its structure from the model that we propose in the present paper: it is a two-stage model where the regulator first decides on the capacity signal and then, private firms react to the regulator's decision by investing in generation capacities taking into account the capacity signal. The author finds that although locational capacity signals have a significant cost-saving potential, they do not lead to a full recovery of the efficiency of nodal.

Our paper also contributes to the literature on the quantitative analysis of zonal pricing with market-based re-dispatch. In the early days of the discussions on market-based congestion management in Europe, De Vries and Hakvoort (2002) analyzed five different designs among which uniform pricing followed market-based re-dispatch, so called *countertrading*. The authors proposed a stylized two-node one-zone example and conclude that in theory, countertrading is efficient in the short-term. The model does not account for inc-dec gaming, which is an important aspect in market-based re-dispatch. Potential inefficiencies stemming from the deviation of the idealized setting analyzed in the paper are qualitatively discussed. One of the potential inefficiencies that could be associated to market-based re-dispatch in the short-term is analyzed in Grimm et al. (2018), still under the assumption of absence of inc-dec gaming. The authors show that the outcome of the market-based re-disaptch could be inefficient if the TSO manipulates the market in order to minimize its re-dispatch costs. An important drawback of re-dispatch markets is that they offer the possibility to market participants to engage in arbitrage between the zonal and re-dispatch price, the so-called inc-dec game, which leads to a distortion of the long run incentives. This situation is analyzed quantitatively in Holmberg and Lazarczyk (2015). The authors show that inc-dec gaming is an arbitrage strategy that also occurs under perfect competition. They conclude that under some restrictive assumptions, zonal pricing followed by market based re-dispatch is efficient in the short-term and the outcomes only differ by a redistribution of the welfare. More recently, Hirth and Schlecht (2020) proposed a simple model on a two-node one-zone example to analyze inc-dec gaming. The authors emphasize that the design leads to undue arbitrage opportunities for market participants even when they cannot exercise market power and further discuss the conditions under which this arbitrage is emphasized or can be mitigated.

1.3. Contributions

Our paper contributes to the existing literature in the following ways:

- Our main contribution is to propose a unifying modeling framework for analyzing the long run equilibrium resulting from different market-based congestion management designs, including capacity and energy-based locational instruments, market-based re-dispatch and several of their variations.
- Using this common framework, we establish the theoretical conditions under which the long run efficiency of nodal pricing can be recovered in a zonal pricing market.
- Regarding market-based re-dispatch, the present paper is the first to propose a model that internalizes the investment decisions of private firms in generating capacity while accounting for inc-dec gaming. From a mathematical point of view, we show how the loss of efficiency of that design originates from the property of generalized Nash equilibirum of the associated game and formally establish the existence of such an equilibrium.

- We provide a comparison of the different designs on a relistic-size instance of the Central Western Europe network area and show how the generalized Nash equilibrium problem associated to market-based re-dispatch can be solved by a splitting-based algorithm that levarages its specific structure.

1.4. Organization of the paper

The paper is organized as follows: we start, in section 2, by providing a roadmap to the study. The terminology used in the paper is described, the different designs that we analyze are listed and the main policy messages of the paper are summarized. Section 3 is the main section of the paper: it presents our models and our theoretical results. Then, in section 4, we propose a case study using a realistic instance of the Central Western Europe network area on which we compare the efficiency of the different designs considered in the paper. Section 5 concludes the paper.

2. Roadmap to the study

2.1. Terminology

Markets Our focus in this paper is on the restoration of the efficiency of nodal in a market-based way. For this reason, to any locational instrument considered in the paper, a market will be associated. We distinguish in total four different markets: (i) the *electricity market* which is the basic market, nodal or zonal on which electricity is traded, (ii) a *capacity market*, which is the market associated to capacity-based locational instruments, (iii) an *energy market*, associated to energy-based locational instruments and (iv) a *re-dispatch market* which is a nodal market subsequent to the zonal electricity market that arise under market-based re-dispatch.

Prices To every market, a price is associated. The price in the electricity market will be referred to as the electricity price, the price in the capacity market as the capacity price and so on.

Rights We refer to as capacity (energy) rights the commodity that is traded on the capacity (energy) market. These commodities should be interpreted as follows. Let us consider the case of the capacity market. The TSO has a certain demand on this market, which is its targeted capacity.

Any producer that wishes to connect a certain generation capacity to the transmission network should acquire capacity rights in the capacity market which are sold by the TSO to the highest offering. Note that in general, the price of these capacity rights could be negative, in which case the TSO is willing to pay the producers in exchange of them acquiring the rights as well as an obligation to connect the associated capacity.

Two-sided vs one-sided markets The capacity and energy markets can either be two-sided or one-sided. The two-sided case is the general case where the TSO and producers can be both buyers and sellers on these markets. The case of one-sided capacity or energy markets refer to the situation in which the TSO is only a buyer or a seller on these markets. For instance, if the TSO is only a buyer on the capacity market, it will only be willing to pay candidate investors but not getting paid in exchange of capacity rights.

Positive or negative rights In the case of one-sided capacity or energy markets, there are two situations: either the TSO is a buyer and the producers are sellers or the other way around. In the former situation, the price is positive and the situation will thus be referred to as *one-sided capacity (energy) market with positive capacity (energy) price*. The same holds for negative prices.

2.2. Studied designs

We study three main classes of locational instruments combined with zonal pricing: (i) locational capacity markets, (ii) locational energy markets and (iii) market-based re-dispatch. In the two former classes, we assume that the re-dispatch is cost-based. These designs are compared against two benchmarks: (i) nodal pricing, which corresponds to the most efficient design possible under our assumptions and (ii) pure zonal pricing with cost-based re-dispatch, which is the closest to the currently implemented design in Europe under our framework. For each of the three main classes of instruments, we study a certain number of variations.

For locational capacity markets, we consider four variations: (a) two-sided capacity markets, (b) one-sided with negative capacity price, (c) one-sided with positive capacity price and (d) two-sided without technology differentiation. For the three former variations, we assume that the capacity price is differentiated among both locations and technologies whereas the latter is only differentiated among locations.

For locational energy markets, we simply consider two variations: (a) two-sided and (b) one-sided energy markets. In both these markets, the price is differentiated among locations and time periods.

For market-based re-dispatch, we consider three variations: (a) the pure zonal pricing with market based re-dispatch design, where there is no additional market, (b) market-based re-dispatch with a two-sided capacity market and (c) market-based re-dispatch with a one-sided capacity market. Note that these two latter variations corresponds therefore to the only two designs among all studied that are made of three markets: a zonal electricity market, a re-dispatch market and a capacity market. All other designs are made either of one single market (for the benchmarks) or two markets.

2.3. Preview of the policy messages

From our analysis, we conclude that seven of the studied designs are theoretically able to recover the efficiency of the nodal benchmark: two-sided capacity markets with technology differentiation, one-sided capacity markets with a negative capacity price and technology differentiation, all three energy markets (whether two-sided or one-sided), market-based re-dispatch with two-sided capacity market and market-based re-dispatch with one-sided capacity market and positive capacity price. These conclusions only holds under a set of strict assumptions. In particular, the efficiency with capacity and energy markets only hold under specific conditions on the feasible set of zonal net positions which are not respected in the current European methodology of capacity calculation. We find that, although it restores a locational component in the electricity price, the least efficient solution is obtained in the case of zonal pricing with market-based re-dispatch. This comes from the distortion of long run incentives due to arbitrage in the combination of the two markets and confirms results of the literature obtained with short-term models.

These results will be revisited from a theoretical point of view in section 3 and from an empirical point of view in section 4.

3. Analysis

3.1. Modeling framework and main assumptions

The goal of our paper is to develop a modeling framework for comparing the different designs of market-based congestion management in the long run. We model the long run economic equilibrium on the electricity market as a Nash equilibrium between three types of agents: a single TSO, the producers and a Walrasian auctioneer that clears the market. In order to focus on the difference between the designs regarding congestion management, we will make a set of simplifying assumptions that will allow us to isolate the effects related to congestion management. In particular, we assume that the market is perfectly competitive and model the agents as price takers. This implies that we ignore market power. Although real markets deviate from the situation of perfect competition, our view is that it remains essential to understand the performance of market designs under perfect competition as it is unlikely that identified inefficiencies will disappear in imperfectly competitive markets. In addition, models with market power require assumptions on strategic behavior of the firms and lead to significantly more complex models that are often intractable on realistic instances. The assumption of perfect competition allows us to keep the models transparent and tractable.

We assume that the profit-maximizing problems of all agents are convex and that there is no inter-temporal operating constraints: in the short run, all periods are independent. We consider that producers can invest in generation capacity in a continuous way and we ignore transmission capacity expansion for which the continuous assumption would be unrealistic. The demand is assumed to be known and inelastic.

The investment problem is a two-stage problem in which producers first decide on their investment and then decide on their production at each period in the short run market given their investment decision. Under the assumption of perfect competition, however, the two-stage formulation is equivalent to a single-stage formulation where the agents decide at the same time their investment and their production in the short run market (Gurkan et al. 2013). For this reason, we will consider throughout the paper only single-stage formulations.

In the simplest situation, i.e. when there is no additional market instrument, the agents compete on a single market, the electricity market, with locationally differentiated prices (with nodal granularity for nodal pricing and zonal granularity for zonal pricing). We will add throughout the paper different market instruments (capacity or energy-based locational instruments, market-based re-dispatch). These market instruments will be represented as additional markets on which the agents compete. For the same reason as the one mentioned in the previous paragraph, we will use a single-stage formulation in which agents compete at the same time in the electricity market and markets of additional instruments.

For zonal pricing models, we will consider both models with cost-based and models with market-based re-dispatch. In the case of cost-based re-dispatch, we will assume that the variable cost of production is perfectly known to the TSO when compensating the producers. Moreover, all resources are available for re-dispatch and there is no irrevocable decision made based on the outcome of the zonal pricing auction. In particular, there is no unit commitment decision made based on the zonal auction and we assume that the zonal net positions cleared in the zonal auction are not firmed and can be freely modified during the re-dispatch phase.

We will now describe the models for the different market designs that we consider based on this common modeling framework and set of assumptions.

3.2. Nodal and zonal pricing benchmarks

3.2.1. Nodal pricing Let us describe the game between the three types of agents (TSO, producers and auctioneer) when they compete on an electricity market with nodal pricing. We describe the profit maximizing problem of each agent successively.

Producers The producers with technology $i \in I$ at node $n \in N$ of the transmission network maximize their profit from the selling of electricity given the prices at their node ρ_{nt} for each period $t \in T$ of the capacity expansion horizon. They incur a linear cost with marginal cost MC_i and investment cost IC_{in} and face capacity limits on their production. Their problem can be written as follows:

$$\begin{aligned}
&\max_{y_{int},x_{in}} \sum_t \Big(\rho_{nt}y_{int} - MC_i y_{int}\Big) - IC_{in}x_{in} \\
&X_{in} + x_{in} - y_{int} \geq 0 \\
&x_{in}, y_{int} \geq 0
\end{aligned} \tag{1}$$

where y_{int} is the production in period t, x_{in} is the capacity invested and X_{in} is the existing capacity.

TSO The TSO maximizes its profit from the transmission of electricity given the nodal prices by controlling the injection and withdrawal of power in every node while making sure that the resulting power flows on the transmission network respect the thermal limits of the lines. Under the DC approximation of the power flow equations, the feasible set $\mathcal{R}$ of nodal net injections r reads as:

$$\begin{aligned}
\mathcal{R} = \Big\{ r \in \mathbb{R}^{|N|} \;\Big|\; &\exists f \in \mathbb{R}^{|K|} \; : \\
&f_k = \sum_n PTDF_{kn} \cdot r_n, k \in K \\
&\sum_{n \in N} r_n = 0, -TC_k \leq f_k \leq TC_k, k \in K \Big\}
\end{aligned} \tag{2}$$

where f_k is the power flow on line $k \in K$, r_n is the net injection at node $n \in N$, $PTDF_{kn}$ is the power transfer distribution factor of line k and node n, and TC_k is the thermal limit of line k. Based on this feasible set of nodal net injections, the profit-maximizing problem of the TSO can be written in the following way:

$$\begin{aligned}
&\max_{r_{nt}} - \sum_{nt} r_{nt}\rho_{nt} \\
&\text{s.t. } r_{:t} \in \mathcal{R}
\end{aligned} \tag{3}$$

Auctioneer The auctioneer determines the nodal prices while making sure that the market clears in every node of the network. Its representative profit-maximizing problem for node n and period t can written as:

$$\max_{\rho_{nt}} \; \rho_{nt}(r_{nt} + D_{nt} - \sum_i y_{int}) \tag{4}$$

Equivalent optimization problem It can be shown that the Nash equilibrium between these three types of agents is equivalent to the solution of the centralized capacity expansion problem (Gurkan et al. 2013):

$$\begin{aligned}
&\min_{x_{in}, y_{int}} \sum_{int} MC_i y_{int} + \sum_{in} IC_{in} x_{in} \\
&\text{s.t. } X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \\
&r_{nt} - \sum_{in} y_{int} + D_{nt} = 0, n \in N, t \in T \\
&r_{:t} \in \mathcal{R}, t \in T
\end{aligned} \tag{5}$$

Within our set of assumptions, the nodal pricing market design achieves the best efficiency possible while managing congestion in a market-based way, in the sense that it achieves the lowest possible total operating and investment cost in the long run. Therefore, the results of this nodal pricing problem will be used as a benchmark throughout the paper. In particular, we will say that a market design is efficient if it achieves the same total cost as the nodal pricing design.

3.2.2. Zonal pricing The zonal pricing market design will be a building block for the different designs that we will compare in the paper, which will correspond to market-based instruments added on top of the pure zonal pricing design. For this reason, pure zonal pricing is also an important benchmark in order to evaluate the impact of each instrument on efficiency. The formulation of the Nash equilibrium in pure zonal pricing is a direct extension of its nodal pricing version. The profit-maximizing problems are as follows:

Producers

$$\begin{aligned}
&\max_{y_{int}, x_{in}} \sum_{t} \left(\rho_{Z(n)t} y_{int} - MC_i y_{int} \right) - IC_{in} x_{in} \\
&X_{in} + x_{in} - y_{int} \geq 0 \\
&x_{in}, y_{int} \geq 0
\end{aligned} \tag{6}$$

where ρ_{zt} is now the zonal price of zone z and $Z(n)$ is to zone to which node n belong.

TSO

$$\begin{aligned}
&\max_{p_{zt}} - \sum_{zt} p_{zt} \rho_{zt} \\
&\text{s.t. } p_{:t} \in \mathcal{P}
\end{aligned} \tag{7}$$

where p_{zt} is now the zonal net position of zone z in period t and $\mathcal{P}$ is the feasible set of zonal net positions. There exists different ways to define the set $\mathcal{P}$ but we keep it general at this stage of the paper and only assume that it depends solely on grid quantities. The reader is referred to section 3.6 for a discussion of the possible formulations of $\mathcal{P}$.

Auctioneer

$$\max_{\rho_{zt}} \ \rho_{zt}(p_{zt} + D_{zt} - \sum_{i,n \in N(z)} y_{int}) \tag{8}$$

where $N(z)$ denotes the set of nodes that belong to zone z.

Equivalent optimization problem The result of equivalence between the centralized optimization problem and the Nash equilibrium easily extends under our set of assumptions to the case of zonal pricing (Lété et al. 2022). The equivalent optimization problem becomes the following:

$$\begin{aligned}
&\min_{x_{in}, y_{int}} \sum_{int} MC_i y_{int} + \sum_{in} IC_{in} x_{in} \\
&\text{s.t. } X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \\
&\quad p_{zt} - \sum_{i,n \in N(z)} y_{int} + D_{zt} = 0, z \in Z, t \in T \\
&\quad p_{:t} \in \mathcal{P}, t \in T
\end{aligned} \tag{9}$$

Cost-based re-dispatch Problem (9) correctly represents the investment and producing decisions in the zonal auction that would result from zonal pricing followed by cost-based re-dispatch within our set of assumptions. Indeed, as cost-based re-dispatch does not modify the revenues of the producers, their investment decision will be based solely on the results from the zonal auction itself. Problem (9), however, is not sufficient to understand the efficiency of zonal pricing in the long run as the dispatch y_{int} obtained does not respect the real nodal transmission constraints of the grid. The TSO must resort to re-dispatch and its cost must be accounted for in the efficiency. As stated in section 3.1, we assume in this paper that the TSO has full flexibility when performing re-dispatch. The goal of the TSO is to minimize re-dispatch cost, which is equivalent to solving a nodal economic dispatch problem at each period given the capacity investment $\bar{x}_{in}$:

$$\begin{aligned} \min_{y_{int}\geq 0} \sum_{in} & MC_i y_{int} \\ \text{s.t. } & X_{in} + \bar{x}_{in} - y_{int} \geq 0, i \in I, n \in N \\ & r_n - \sum_i y_{int} + D_n = 0, n \in N \\ & r_{:t} \in \mathcal{R} \end{aligned} \tag{10}$$

TSO coordination for re-dispatch As we mentioned in section 3.1, this model of re-dispatch assumes that there is no constraint on the final net positions obtained after re-dispatch. This implies that the TSOs coordinate perfectly including in resorting to cross-border re-dispatch. This assumption is important as it implies that zonal pricing is efficient in the short run. Indeed, in that case, the re-dispatch problem of the TSO is equivalent to the nodal economic dispatch problem and the final dispatch obtained is thus the same as the dispatch obtained in nodal pricing. The two designs would thus differ only by the allocation of revenues between the different agents. For this reason, as we try to understand the designs that can restore the efficiency of zonal pricing, this assumption can also be viewed in this paper as a necessary condition without which the recovery of efficiency cannot take place in zonal pricing.

Although this condition is not fully respected in the current European practices, we point out that this is more and more the case. Indeed, the Guideline on Capacity Allocation and Congestion Management (CACM) of the European Commission states that TSOs should develop a common methodology for coordinated re-dispatch and countertrading (Regulation (EC) 2015/1222, Art. 35, §1). Moreover, it is now stated explicitly that cross-border re-dispatch must be considered for the calculation of available cross-border capacity in the recent recast of the electricity regulation (Regulation (EU) 2019/943, Art.16, §4). This is the reason why we do not penalize the deviation from day-ahead net positions in the re-dispatch problem, unlike in some previous work of the authors (Aravena and Papavasiliou 2017, Aravena et al. 2021).

3.3. Locational capacity markets

In this section, we investigate the potential of locational capacity markets in improving the efficiency of investment under zonal pricing. In particular, we would like to understand the conditions under which such an instrument could lead to an equivalent efficiency between nodal and zonal. We assume that the producers and the TSO compete in a capacity market in addition to the electricity market. The TSO has a certain inflexible demand of capacity in the capacity market that is based on the solution of a prospective resource adequacy study. This can be assimilated in current practices to ENTSO-E's Mid-term Adequacy Forecast (ENTSO-E 2020).

Resource adequacy We assume that the resource adequacy problem solved by the TSO is a nodal capacity expansion problem with perfect information on the state of the system, including on marginal and investment costs of the producers. This is a strong assumption that can also be viewed in the context of this paper as a condition without which efficiency of nodal pricing cannot be recovered. The TSO solves therefore a problem exactly equivalent to problem (5) prior to its participation to the capacity market. We denote by $\bar{x}_{in}$ the value of the investment in the solution of problem (5).

3.3.1. Two-sided capacity market with location and technology differentiation We start by considering the case of a capacity market that has full flexibility: it is two-sided and depends on the location n and technology i of the investment. We describe successively the profit-maximizing problem of each agent in the electricity and capacity markets in this case.

Producers

$$
\begin{aligned}
&\max_{y_{int}, x_{in}} \sum_t \left(\rho_{Z(n)t} y_{int} - MC_i y_{int}\right) - IC_{in} x_{in} - \pi_{in} x_{in} \\
&X_{in} + x_{in} - y_{int} \geq 0 \\
&x_{in}, y_{int} \geq 0
\end{aligned}
\tag{11}
$$

where π_{in} is the capacity price. If it is positive (negative), it corresponds to an additional cost (revenue) to the producers.

TSO As the demand of the TSO in the capacity market is assumed to be inflexible, the problem of the TSO is unchanged and corresponds to problem (3.2.2).

Auctioneer of the electricity market The problem of the auctioneer of the zonal market does not change (see Problem (3.2.2)).

Auctioneer of the capacity market In the capacity market, the auctioneer determines the price that leads to a matching between the capacity invested by the producers and the inflexible demand of the TSO:

$$\max_{\pi_{in}} \pi_{in}(\bar{x}_{in} - x_{in}) \tag{12}$$

Equivalent optimization problem It can be easily checked that in this case, the Nash equilibrium can be obtained by solving the following equivalent optimization problem:

$$\begin{aligned} \min_{x_{in}, y_{int}} & \sum_{int} MC_i y_{int} + \sum_{in} IC_{in} x_{in} \\ \text{s.t. } & X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \\ & p_{zt} - \sum_{i,n \in N(z)} y_{int} + D_{zt} = 0, z \in Z, t \in T \\ & p \in \mathcal{P} \\ & \bar{x}_{in} - x_{in} = 0, i \in I, n \in N \end{aligned} \tag{13}$$

We are interested in the conditions under which Problem (13), which corresponds to the equilibrium in zonal pricing with a two-sided capacity market, is efficient in the long run. As stated in section 3.2.2, we already know that zonal pricing followed by cost-based re-dispatch is efficient in the short run under our assumptions. Moreover, any solution to Problem (13), if it exists, leads to the same investment decisions as the nodal pricing benchmark. Therefore, any equilibrium in zonal pricing with a two-sided capacity market will be efficient if it exists and it only remains to determine the conditions on $\mathcal{P}$ under which existence can be guaranteed. In the following definition, we propose a condition, that we call *nodal consistency*, which is both natural to require for a well-defined set of feasible net positions and sufficient for the existence.

Definition 1. Let r^* be a vector of values of the nodal injections in one solution of the nodal capacity expansion problem and let p^* be the corresponding vector of zonal net positions, i.e.

$$p_z^* = \sum_{n \in N(z)} r_n^* \ \ \forall z \in Z$$

The feasible set of zonal net position $\mathcal{P}$ is said to be **nodal consistent** if $p^* \in \mathcal{P}$.

Proposition 1. *If $\mathcal{P}$ is nodal consistent, then any equilibrium in zonal pricing with a two-sided capacity market is efficient.*

Proof of Proposition 1 As $\mathcal{P}$ is nodal consistent, existence of a solution to Problem (13) follows from the existence of a solution to the nodal capacity expansion problem. The efficiency follows from the short run efficiency of zonal pricing followed by cost-based re-dispatch. □

3.3.2. One-sided capacity markets From the analysis of the previous section, one may wonder whether it is possible to obtain the equivalence between nodal and zonal investment with a one-sided capacity market. There are two situations to distinguish in function of whether the capacity price is negative or positive.

Negative capacity price This is capacity market where investment in a certain technology and in a certain node is subsidized, but no investment is penalized. This corresponds more closely to current practices regarding capacity markets in Europe, for which the goal is indeed to remunerate investment, not penalize them. In this case, the optimization problem of the auctioneer of the capacity market will be modified in order to add a nonpositivity constraint on the capacity price:

$$\max_{\pi_{in} \leq 0} \ \pi_{in}(\bar{x}_{in} - x_{in}) \tag{14}$$

This implies that in the corresponding optimization problem, the capacity invested can only be forced to be greater than the nodal investment:

$$\begin{aligned}
\min_{x_{in}, y_{int}} & \sum_{int} MC_i y_{int} + \sum_{in} IC_{in} x_{in} \\
\text{s.t. } & X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \\
& p_{zt} - \sum_{i, n \in N(z)} y_{int} + D_{zt} = 0, z \in Z, t \in T \\
& p \in \mathcal{P} \\
& x_{in} - \bar{x}_{in} \geq 0, i \in I, n \in N
\end{aligned} \tag{15}$$

It turns out that in this case, efficiency can also be guaranteed under some conditions that are likely to be met in practice, as we show with Proposition 2. In order to determine a sufficient condition for which the efficiency holds in the one-sided case, we define some quantities related to capacity factors.

Definition 2. The **nodal capacity factor** of a technology i in a node n, that we denote by γ_{in}, is the proportion of the time for which the technology produces a positive quantity in a solution of the nodal capacity expansion problem, i.e.

$$\gamma_{in} = \frac{\sum_{t \in T} (\bar{y}_{int} > 0)}{|T|} \tag{16}$$

where $\bar{y}_{int}$ is the production in an optimal solution of the capacity expansion problem.

Definition 3. The **zonal capacity factor** of a technology i in a node n, that we denote by ζ_{in}, is the proportion of the time for which the technology produces a positive quantity in a solution of the zonal dispatch with capacities fixed to the optimal nodal investment, i.e.

$$\zeta_{in} = \frac{\sum_{t \in T} (\hat{y}_{int} > 0)}{|T|} \tag{17}$$

where $\hat{y}_{int}$ is the production in an optimal solution of the zonal economic dispatch problem with nodal optimal investment.

PROPOSITION 2. *If $\mathcal{P}$ is nodal consistent and if $\forall i,j \in I, n \in N, i \neq j$,*

$$IC_{in} < \zeta_{jn}(MC_j - MC_i) \tag{18}$$

implies that

$$IC_{in} - IC_{jn} < \gamma_{jn}(MC_j - MC_i) \tag{19}$$

then any equilibrium in zonal pricing and a one-sided capacity market with a nonpositive capacity price is efficient.

Proof of Proposition 2 Let us denote with the symbol $(\hat{\cdot})$ a solution to Problem (15) and $(\tilde{\cdot})$ a solution to Problem (13) and denote by $\Delta x_{in} = \hat{x}_{in} - \tilde{x}_{in}$. By contradiction, assume that

$$\sum_{in} IC_{in}\Delta x_{in} + \sum_{int} MC_i \hat{y}_{int} - \sum_{int} MC_i \tilde{y}_{int} < 0 \tag{20}$$

For equation (20) to hold, it is necessary that there exists i, j, n such that

$$\begin{aligned} &IC_{in}\Delta x_{in} + MC_i\zeta_{jn}\Delta x_{in} - MC_j\zeta_{jn}\Delta x_{in} < 0 \\ \Leftrightarrow &IC_{in} + \zeta_{jn}(MC_i - MC_j) < 0 \end{aligned}$$

Then, by our assumption, this implies that

$$IC_{in} + \gamma_{jn}MC_i < IC_{jn} + \gamma_{jn}MC_j$$

Note that ζ_{jn} cannot be zero (as it would lead to $IC_{in} < 0$ by equation (18)), which means that $\tilde{x}_{jn} > 0$. But then, this implies that we can find $\epsilon > 0$ such that in the nodal solution, decreasing x_{jn} by ϵ and increasing x_{in} by ϵ decreases the cost by at least $(IC_{jn} + \gamma_{jn}MC_j - IC_{in} - \gamma_{jn}MC_i)\epsilon$, which is positive. This is a contradiction of the optimality of the nodal investment $\bar{x}_{in}$. □

The additional condition on the costs based on the capacity factors is needed in order to avoid some extreme cases, but will in general be respected in practice. In order to understand better this condition, assume a very simple two-node one-zone network with a single fictive transmission line with zero capacity. The demand in the first node (node 1) is equal to D during the first half of the capacity expansion horizon and 0 in the second half. The opposite holds for node 2, with 0 demand in the first half and D in the second. There are two candidate technologies, A and B with costs respectively IC_A, MC_A and IC_B, MC_B. Assume that

$$IC_A + \frac{1}{2}MC_A < IC_B + \frac{1}{2}MC_B \tag{21}$$

This implies that the optimal nodal solution is to invest two units of technology A in the two nodes, with capacities D. Is it possible that the zonal solution with a lower bound $x_A \leq D$ and $x_B \leq 0$ in the two nodes deviate from the nodal optimal investment ? In that case, technology B of capacity D would be invested in one node to cover the zonal demand, constant at level D during the whole horizon. It would be optimal if

$$IC_B + MC_B < MC_A \tag{22}$$

It is possible to find values for the costs such that both (21) and (22) are respected. Take for instance $IC_A = 0, MC_A = 10, IC_B = 3, MC_B = 3$. But this situation is quite extreme, both in the congestion and the specific choice of costs. In this case, $\zeta_A = 1$ and $\gamma_A = \frac{1}{2}$ and our condition thus prevent such cases from happening.

Positive capacity price In the same vein, one can easily, under this framework, model the situation in which a nonnegative capacity price is imposed. This corresponds to a market design with locational and technology differentiated connection charges. In this case, the equivalent optimization problem to the Nash equilibrium becomes as follows:

$$\begin{aligned}
\min_{x_{in}, y_{int}} \; & \sum_{int} MC_i y_{int} + \sum_{in} IC_{in} x_{in} \\
\text{s.t. } & X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \\
& p_{zt} - \sum_{i, n \in N(z)} y_{int} + D_{zt} = 0, z \in Z, t \in T \\
& p_{:t} \in \mathcal{P} \\
& \bar{x}_{in} - x_{in} \geq 0, i \in I, n \in N
\end{aligned} \tag{23}$$

where the capacity of the nodal benchmark now implies an upper bound on the investment. In this case, however, the efficiency is in general not recovered. Here, the situation is quite different than with a negative price and one cannot find a simple condition for which efficiency would be obtained. Indeed, let us consider again our simple two-node one-zone example. Assume that there is only a single technology, such that the condition for efficiency of the negative price is trivially respected. In this case, the nodal solution is to invest D capacity of the only technology available. The upper bounds, however, are not constraining in the zonal solution which will be optimal with a single unit of capacity D invested.

3.3.3. Technology-independent capacity markets The previous results have all been obtained under the assumption that the capacity markets are differentiated between all technologies. Although this is a necessary condition for a capacity-based market to be efficient (Eicke 2021), it is usually not the case in existing market-based price signals (Eicke et al. 2020). Once again, this assumption can easily be lifted within our modeling framework. In that case, the dual variables of the capacity market clearing constraints depend only on the node and so do the constraints. The equivalent optimization problem remains the same except for the capacity market clearing constraints which now read as:

$$\sum_{i\in I} x_{in} - \sum_{i\in I} \bar{x}_{in} \geq 0, n \in N \tag{24}$$

3.4. Locational energy markets

The second class of instruments that we consider in this paper are locational energy markets. These additional markets imply that on top of the electricity price a location differentiated energy price is added, which can be either positive or negative.

3.4.1. Two-sided markets Let us start with the case of a two-sided energy market by describing successively the profit-maximizing problems of each type of agent.

Producers The objective of the producers is modified by the addition of the locational energy price ν_{nt} to their marginal cost.

$$\begin{aligned} \max_{x_{in},y_{int}} & \sum_t \Big((\rho_{Z(n)t} - \nu_{nt} - MC_i) y_{int} \Big) - IC_{in} x_{in} \\ \text{s.t. } & X_{in} + x_{in} - y_{int} \geq 0 \\ & x_{in} \geq 0, y_{int} \geq 0 \end{aligned} \tag{25}$$

TSO The TSO collects the zonal congestion rent as well as the revenues (or costs) from its participation to the energy market so that the total production at each node n and time t is feasible with the nodal constraints of the grid.

$$\begin{aligned} \max_{p_{zt},\tilde{y}_{nt}} & - \sum_{zt} p_{zt}\rho_{zt} + \sum_{nt} \tilde{y}_{nt}\nu_{nt} \\ \text{s.t. } & p \in \mathcal{P} \\ & - r_{nt} + \tilde{y}_{nt} - D_{nt} = 0 \\ & r \in \mathcal{R} \end{aligned} \tag{26}$$

Auctioneer of the electricity market The problem of the auctioneer of the zonal market does not change (see Problem (3.2.2)).

Auctioneer of the energy market In the locational energy market, the auctioneer determines the energy price so that the total power determined at node n and time t by the TSO ($\tilde{y}_{nt}$) is equal to the total power produced at node n and time t ($\sum_i y_{int}$).

$$\max_{\nu_{nt}} \; \nu_{nt}\Big(\tilde{y}_{nt} - \sum_i y_{int}\Big) \tag{27}$$

Equivalent optimization problem Using the same reasoning as in the previous section, a Nash equilibrium can be obtained by solving the following equivalent optimization problem:

$$\begin{aligned}
\min_{x_{in}, y_{int}} & \sum_{int} MC_i y_{int} + \sum_{in} IC_{in} x_{in} \\
\text{s.t. } & X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \\
& -p_{zt} + \sum_{i \in I, n \in N(z)} y_{int} - \sum_{n \in N(z)} D_{nt} = 0, z \in Z, t \in T \\
& p_{:t} \in \mathcal{P}, t \in T \\
& \tilde{y}_{nt} - \sum_i y_{int} = 0, n \in N, t \in T \\
& -r_{nt} + \tilde{y}_{nt} - D_{nt} = 0, n \in N, t \in T \\
& r_{:t} \in \mathcal{R}, t \in T
\end{aligned} \tag{28}$$

Using the definition of nodal consistency that we introduced in section 3.3.1 (Definition 1), it is quite straightforward to observe that (28) is equivalent to the nodal problem and therefore leads to the same efficiency as the nodal capacity expansion problem.

PROPOSITION 3. *If $\mathcal{P}$ is nodal consistent, then zonal pricing with an energy-based tariff is efficient.*

Proof of Proposition 3 It suffices to observe that the feasible set of (28) is included in the feasible set of (5) and that any solution of (5) is feasible for (28) by definition of nodal consistency.

3.4.2. One-sided markets One may wonder whether the equivalence holds if we restrict ourselves to one-sided markets, i.e. when the energy price is restricted to be nonnegative or non-positive. If we consider a nonnegative price, which is probably to most realistic case in practice as it could be assimilated to an energy-based tariff, the equivalent optimization problem is modified by transforming the market clearing equality constraint in an inequality:

$$\sum_i y_{int} - \tilde{y}_{nt} \geq 0 \tag{29}$$

One can observe that this does not impact the equivalence with the nodal benchmark, as this inequality is always tight in any solutions of Problem (28). Indeed, the following equivalences hold successively:

$$\begin{aligned}
&\sum_{nt} D_{nt} = \sum_{int} y_{int} = \sum_{nt} \tilde{y}_{nt} \\
\Leftrightarrow &\sum_{int} y_{int} - \sum_{nt} \tilde{y}_{nt} = 0 \\
\Leftrightarrow &\sum_{nt} \Big(\sum_{i} y_{int} - \tilde{y}_{nt} \Big) = 0
\end{aligned}$$

which, using the fact that each $\sum_i y_{int} - \tilde{y}_{nt}$ must be nonnegative, leads to

$$\sum_{i} y_{int} - \tilde{y}_{nt} = 0$$

The reasoning holds also in the case of a nonpositive price.

3.5. Market-based re-dispatch

The last class of models that we consider in this paper are zonal pricing models with market-based re-dispatch. Unlike in cost-based re-dispatch that works with mandatory participation and does not influence the payoff of the agents, in market-based re-dispatch, the re-dispatch step is organized as a market with voluntary participation in which participants can bid freely and make profit. As the re-dispatch market is nodal, it is a natural candidate for restoring locational signals in zonal markets. Our goal is to understand the conditions under which zonal pricing followed by market-based re-dispatch can recover the efficiency of nodal. We formulate the long run equilibrium in the same unifying modeling framework as the one that we developed for zonal pricing with cost-based re-dispatch in the previous sections. We therefore use the same assumptions as introduced in section 3.1. In addition, we will assume as in Hirth and Schlecht (2020) that the re-dispatch market is organized with marginal pricing as opposed to pay-as-bid pricing which is the rule in most existing re-dispatch markets. The reason is that under the assumption of perfect competition, the outcome of the two types of auctions are identical (Holmberg and Lazarczyk 2015). We also follow the assumption of Hirth and Schlecht (2020) by preventing pure financial arbitrage. Using the terminology of Hirth and Schlecht (2020), our model is based on *asset-backed arbitrage* which implies that participants can only bid quantities that they would be able to physically produce in both the zonal and the re-dispatch markets.

We now proceed to the sequential description of the profit-maximizing problems of each agents under our unifying framework.

Producers The producers have the opportunity to participate to two different markets: the zonal market and the re-dispatch market. We denote by y_{int} the quantity cleared in the zonal market and $\tilde{y}_{int}$ the re-dispatch quantity cleared in the re-dispatch market, which is positive (negative) in case of upward (downward) re-dispatch.

$$\begin{aligned}
\max_{y_{int},\tilde{y}_{int},x_{in}} \quad & \sum_t \Big(\rho_{Z(n)t} y_{int} + \tilde{\rho}_{nt}\tilde{y}_{int} - MC_i(y_{int} + \tilde{y}_{int})\Big) - IC_{in}x_{in} \\
& X_{in} + x_{in} - y_{int} \geq 0 \\
& X_{in} + x_{in} - y_{int} - \tilde{y}_{int} \geq 0 \\
& y_{int} + \tilde{y}_{int} \geq 0 \\
& x_{in}, y_{int} \geq 0
\end{aligned} \tag{30}$$

where ρ_{zt} is the zonal price and $\tilde{\rho}_{nt}$ is the nodal re-dispatch price.

TSO in the zonal market We split the description of the TSO problems in the zonal and re-dispatch markets in the interest of clarity. As the two problems are completely independent, this can be done without loss of generality.

$$\begin{aligned}
\max_{p_{zt}} & - \sum_{zt} p_{zt}\rho_{zt} \\
\text{s.t. } & p_{:t} \in \mathcal{P}
\end{aligned} \tag{31}$$

TSO in RDM In the RDM, the TSO has to buy re-dispatch resources ($\tilde{r}_{nt}$, negative or positive) in order to recover a nodal dispatch (r_{nt}) that is feasible for the constraints of the DC approximation of the power flow equations.

$$\begin{aligned} & \max -\sum_n \tilde{r}_{nt}\tilde{\rho}_{nt} \\ & \text{s.t. } r_n - \sum_i y_{int} + D_{nt} - \tilde{r}_{nt} = 0 \\ & \quad r_{:t} \in \mathcal{R} \end{aligned} \tag{32}$$

Auctioneer in the zonal market

$$\max_{\rho_{zt}} \ \rho_{zt}(p_{zt} - \sum_{i,n\in N(z)} y_{int} + D_{zt}) \tag{33}$$

Auctioneer in the RDM

$$\max_{\tilde{\rho}_{nt}} \ \tilde{\rho}_{nt}(\tilde{r}_{nt} - \sum_i \tilde{y}_{int}) \tag{34}$$

It is important to note that the right solution concept for this game is that of a generalized Nash equilibrium (GNE). Indeed, the feasible set of the TSO in the RDM, as described in Problem (32), is based on the nodal net injections in the physical dispatch that depend on the production in the zonal market. This has consequences in terms of the properties of the equilibrium: on the one hand, the equilibrium is not equivalent to a single optimization problem. On the other hand, the question of existence and uniqueness of equilibria is not as straightforward as in the case of Nash equilibria that we encountered in the other designs considered in this paper. Although the equilibirum is not equivalent to a single optimization problem, it can still be formulated as a single problem in the form of a mixed complementarity problem (MCP).

Equivalent MCP The equivalent MCP can be obtained by aggregating the KKT optimality conditions (which are necessary and sufficient for linear programs) of the profit-maximizing problems of every agent. Using greek letters for denoting dual variables for each constraint, we obtain the following MCP:

$$0 \le x_{in} \perp IC_{in} - \sum_{t\in T} \mu_{int} - \sum_{t\in T} \tilde{\mu}_{int} \ge 0 \tag{35a}$$

$$0 \le y_{int} \perp MC_i + \mu_{int} + \tilde{\mu}_{int} - \rho_{Z(n)t} - \delta_{int} \ge 0 \tag{35b}$$

$$\tilde{y}_{int} \text{ free } \perp MC_i + \tilde{\mu}_{int} - \tilde{\rho}_{nt} - \delta_{int} = 0 \tag{35c}$$

$$0 \le \mu_{int} \perp X_{in} + x_{in} - y_{int} \ge 0 \tag{35d}$$

$$0 \le \tilde{\mu}_{int} \perp X_{in} + x_{in} - y_{int} - \tilde{y}_{int} \ge 0 \tag{35e}$$

$$0 \le \delta_{int} \perp y_{int} + \tilde{y}_{int} \ge 0 \tag{35f}$$

$$p_{zt} \text{ free } \perp \rho_{zt} + \sum_m V_{mz}\gamma_{mt} = 0 \tag{35g}$$

$$0 \le \gamma_{mt} \perp W_m - \sum_z V_{mz} p_{zt} \ge 0 \tag{35h}$$

$$\tilde{r}_{nt} \text{ free } \perp \tilde{\rho}_{nt} - \nu_{nt} = 0 \tag{35i}$$

$$r_{nt} \text{ free } \perp \nu_{nt} + \sum_m \tilde{V}_{mn}\tilde{\gamma}_{mt} = 0 \tag{35j}$$

$$0 \le \nu_{nt} \perp -r_{nt} + \sum_i y_{int} - D_{nt} + \tilde{r}_{nt} \ge 0 \tag{35k}$$

$$0 \le \tilde{\gamma}_{mt} \perp \tilde{W}_m - \sum_n \tilde{V}_{mn} r_{nt} \ge 0 \tag{35l}$$

$$\rho_{zt} \text{ free } \perp -p_{zt} + \sum_{i,n \in N(z)} y_{int} - D_{zt} = 0 \tag{35m}$$

$$\tilde{\rho}_{nt} \text{ free } \perp -\tilde{r}_{nt} + \sum_{in} \tilde{y}_{int} = 0 \tag{35n}$$

where we define $V \in \mathbb{R}^{|M| \times |Z|}$, $W \in \mathbb{R}^{|M|}$ and $M \in \mathbb{N}$ such that

$$p \in \mathcal{P} \Leftrightarrow W_m - \sum_z V_{mz} p_z \ge 0, m \in M$$

and $\tilde{V} \in \mathbb{R}^{|\tilde{M}| \times |N|}$, $\tilde{W} \in \mathbb{R}^{|\tilde{M}|}$ and $\tilde{M} \in \mathbb{N}$ such that

$$r \in \mathcal{R} \Leftrightarrow \tilde{W}_m - \sum_n \tilde{V}_{mn} r_n \ge 0, m \in \tilde{M}$$

which are well-defined as both $\mathcal{P}$ and $\mathcal{R}$ are polytopes.

Existence of soltuions It turns out that the existence of solutions is guaranteed for this problem, as we show in the following proposition.

PROPOSITION 4. *If the marginal costs, the investment costs and the demand in all nodes are non-negative, then model* (35) *has a solution.*

Proof of Proposition 4 We first note that if $MC, IC, D \geq 0$, then the prices are always nonnegative and the balance constraints can be transformed into inequalities without affecting the solution. Equation (35k) can thus be transformed into

$$0 \leq \nu_{nt} \perp -r_{nt} + \sum_i y_{int} - D_{nt} + \tilde{r}_{nt} \geq 0$$

Let us denote by M and q respectively the matrix and the vector of independent terms associated to our MCP. By Theorem 3.8.6 of Cottle et al. (2009), if M is copositive and if for all solutions v^* of the homogeneous MCP, it holds that $q^\top v^* \geq 0$, then there exists a solution to $MCP(q, M)$. We will use this theorem to prove existence.

Let us first show that M is copositive. To do this, we note that M is the sum of two matrices:

$$M = \tilde{M} + \tilde{N}$$

where $\tilde{M}$ is skew-symmetric (it is the matrix associated to the equivalent MCP of the centralized problem) and where $\tilde{N}$ is of the following form (in block formulation):

$$\nu_{nt} \tag{36}$$

$$\tilde{N} = \begin{array}{c} y_{int} \\ \\ \\ \\ \end{array} \begin{pmatrix} 0 & \dots & I & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix} \tag{37}$$

Here, I is the rectangular identity matrix, i.e. a matrix with 1 in the entries associated to line y_{int} and column ν_{nt}, and 0 otherwise. This implies that

$$v^\top M v = v^\top \tilde{M} v + v^\top \tilde{N} v = v^\top \tilde{N} v = \sum_{int} y_{int} \nu_{nt}$$

where v is the full vector of variables, which is indeed nonnegative if each y_{int} and ν_{nt} are nonnegative.

Now, let $v^* = (x^*_{in}, y^*_{int}, \dots, \rho^*_{zt})$ be a solution to the homogeneous version of MCP. We have

$$q^\top v^* = \sum_{in} IC_{in} x^*_{in} + \sum_{int} MC_i y^*_{int} + \sum_{int} X_{in} \mu^*_{int} + \sum_{int} X_{in} \tilde{\mu}^*_{int} + \sum_{kt} TC_k (\lambda_k^{+*} + \lambda_k^{-*}) + \sum_{kt} TC_k (\tilde{\lambda}_k^{+*} + \tilde{\lambda}_k^{-*}) +$$
$$\sum_{int} MC_i \tilde{y}^*_{int} - \sum_{nt} D_{nt} (\rho^*_{Z(n)t} + \nu^*_{nt})$$

All the terms of the first line are nonnegative, as they correspond to the product of nonnegative quantities. From equation (35m), we deduce that $y^*_{int} = 0$. This implies from equation (35f) that $y^*_{int} \geq 0$ and thus that the first term of the second line is also nonnegative. From equations (35f), (35i) and (35k), we get that $0 \leq \nu^*_{nt} = \tilde{\rho}^*_{nt} = -\delta^*_{nt} \leq 0$, which implies that these three quantities must be equal to 0. We then deduce from (35b) that $\rho^*_{zt} \leq 0$, which, together with $\nu^*_{nt} = 0$ yields the non-negativity of the second term of the second line and concludes the proof. $\square$

3.5.1. Market-based re-dispatch with locational capacity market As the long run equilibrium of zonal pricing followed by market-based re-dispatch is not efficient, one may wonder whether its efficiency can theoretically be recovered by adding locational capacity markets. This is what we analyze in the present section.

Two-sided markets We start by considering the case of a two-sided capacity market with location and technology differentiation, as in section 3.3.1 for cost-based re-dispatch. An auctioneer for the capacity market is added, which leads to the following additional complementarity condition:

$$\pi_{in} \text{ free } \perp \bar{x}_{in} - x_{in} = 0 \tag{38}$$

The investment condition becomes

$$0 \leq x_{in} \perp IC_{in} - \sum_{t \in T} \mu_{int} - \sum_{t \in T} \tilde{\mu}_{int} + \pi_{in} \geq 0 \tag{39}$$

Observe that the question of the efficiency of this design in the long run reduces to the question of the efficiency of pure zonal pricing followed by market-based re-dispatch in the short run. Indeed, equation (38) fixes the investment to $\bar{x}_{in}$, the nodal solution, while equation (39) becomes trivial with the addition of the free π_{in} variable. In the next proposition, we establish the short run efficiency.

Proposition 5. *Under the set of assumptions described in section 3.1, zonal pricing followed by market-based re-dispatch is efficient in the short run.*

Proof of Proposition 5 The short run problem is the MCP (35) where equation (35a) is removed, x_{in} is fixed to $\bar{x}_{in}$ and the time period index t is fixed. Let us consider the MCP obtained by isolating the equations related to the re-dispatch problem only, i.e. the MCP made of equations (35c), (35e), (35f), (35i), (35k), (35l) and (35n). If we denote by $\bar{y}_{int} = y_{int} + \tilde{y}_{int}$ the physical dispatch of technology i in node n and period t, one can simplify this MCP by eliminating variables $\delta_{int}, \nu_{nt}, \tilde{r}_{nt}$. We get:

$$\begin{aligned}
&0 \leq \bar{y}_{int} \perp MC_i + \tilde{\mu}_{int} - \tilde{\rho}_{nt} \geq 0 \\
&0 \leq \tilde{\mu}_{int} \perp X_{in} + \bar{x}_{in} - \bar{y}_{int} \geq 0 \\
&r_{nt} \text{ free } \perp \tilde{\rho}_{nt} + \sum_m \tilde{V}_{mn} \tilde{\gamma}_{mt} = 0 \\
&\tilde{\rho}_{nt} \text{ free } \perp r_{nt} - \sum_i \bar{y}_{int} + D_{nt} = 0 \\
&0 \leq \tilde{\gamma}_{mt} \perp \tilde{W}_m - \sum_n \tilde{V}_{mn} r_{nt} \geq 0
\end{aligned} \tag{40}$$

MCP (40) is exactly the set of KKT conditions of the nodal economic dispatch problem:

$$\begin{aligned}
\min \ & \sum_{in} MC_i \bar{y}_{int} \\
\text{s.t. } & X_{in} - \bar{y}_{int} \geq 0, i \in I, n \in N \\
& r_{nt} - \sum_{in} \bar{y}_{int} + D_{nt} = 0, n \in N \\
& r_{:t} \in \mathcal{R}
\end{aligned} \tag{41}$$

which shows that any solution of zonal pricing followed by market-based re-dispatch has the same operating cost as nodal pricing in the short run. □

Proposition 5 should be seen as closely related to Proposition 4 of Holmberg and Lazarczyk (2015) which also states that zonal pricing with market-based re-dispatch is efficient in the short run under slightly different framework and assumptions. In particular, Holmberg and Lazarczyk (2015) assume that the TSO sets the inter-zonal flows to their level in the efficient dispatch. This assumption is related to our assumption of perfect TSO coordination in the re-dispatch stage: both assumptions imply that the efficient dispatch can be recovered during the re-dispatch stage.

COROLLARY 1. *Zonal pricing followed by market-based re-dispatch augmented with a two-sided capacity market with locational and technology differentiation is efficient.*

One-sided capacity market One may wonder whether the same result can be obtained with a one-sided capacity market. As arbitrage result in general in an excess of profit for producers, the best candidate is a one-sided market with a positive capacity price $\pi_{in} \geq 0$. The capacity market clearing condition now becomes:

$$0 \leq \pi_{in} \perp \bar{x}_{in} - x_{in} \geq 0 \tag{42}$$

PROPOSITION 6. *There exists a location and technology differentiated capacity price that restores the efficiency of zonal pricing followed by market-based re-dispatch.*

Proof of Proposition 6 With the capacity price added to the market, the KKT condition associated to the investment becomes:

$$0 \leq x_{in} \perp IC_i - \sum_{t \in T} \mu_{int} - \sum_{t \in T} \tilde{\mu}_{int} + \pi_{in} \geq 0 \tag{43}$$

We need to show that there exists a solution to the MCP that consists of equations (43), (35b) - (35n), (38) with $\pi^*_{in} \geq 0$ for all $i \in I, n \in N$. This solution can be obtained by solving sequentially two distinct optimization problems, one nodal with investment variables and one zonal with the investment fixed. The nodal problem is the following:

$$\begin{aligned}
\min \ & \sum_{in} IC_{in} x_{in} + \sum_{int} MC_i \bar{y}_{int} \\
\text{s.t. } & X_{in} + x_{in} - \bar{y}_{int} \geq 0, i \in I, n \in N, t \in T \ [\tilde{\mu}_{int}] \\
& r_{nt} - \sum_{int} \bar{y}_{int} + D_{nt} = 0, n \in N, t \in T \ [\tilde{\rho}_{nt}] \\
& \tilde{W}_m - \sum_{n} \tilde{V}_{mn} r_{nt}, m \in \tilde{M}, t \in T \ [\tilde{\gamma}_{mt}] \\
& \bar{y}_{int} \geq 0, i \in I, n \in N, t \in T \ [\delta_{int}]
\end{aligned} \tag{44}$$

We use the notation $(\cdot)^*$ to denote the value of a primal or dual variable in this problem. The zonal problem is the following:

$$\begin{aligned}
\min\ & \sum_{int} \tilde{\rho}^*_{nt} y_{int} \\
\text{s.t. }\ & X_{in} + x^*_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T\ \ [\mu_{int}] \\
& p_{zt} - \sum_{i,n \in N(z)} y_{int} + D_{zt} = 0, z \in Z, t \in T\ \ [\rho_{zt}] \\
& W_m - \sum_z V_{mz} p_{zt} \geq 0, z \in Z, t \in T\ \ [\gamma_{mt}]
\end{aligned} \tag{45}$$

We now let

$$\begin{aligned}
& \tilde{y}^*_{int} = \bar{y}^*_{int} - y^*_{int} \\
& \nu^*_{nt} = -\tilde{\rho}^*_{nt} \\
& \tilde{r}^*_{nt} = r^*_{nt} - \sum y^*_{int} + D_{nt} \\
& \pi_{in} = \sum_{t \in T} \mu^*_{int}
\end{aligned}$$

Clearly, π^*_{in} is positive and it can be checked that $\left(x^*_{in}, y^*_{int}, \tilde{y}^*_{int}, \mu^*_{int}, \tilde{\mu}^*_{int}, \delta^*_{int}, p^*_{zt}, \gamma^*_{mt}, \tilde{r}^*_{nt}, r^*_{nt}, \nu^*_{nt}, \tilde{\gamma}^*_{mt}, \rho^*_{zt}, \tilde{\rho}^*_{nt}\right)$ is a solution of the MCP (43), (35b) - (35n), (38). $\square$

This proof is a constructive proof that shows that it is theoretically possible to define a price π_{in} that will restore the efficiency of zonal with market-based re-dispatch. The idea is to simply set this charge to the arbitrage profit that the producers are expected to be able to make on the market.

Interestingly, this is a symmetrical situation compared to zonal followed by cost-based re-dispatch. With cost-based re-dispatch, there is missing money for investment, and investment must be subsidized if we want to recover the efficiency of zonal. In zonal followed by market-based re-dispatch, on the contrary, there is excess profits to the producers that must be taxed in order to restore the efficiency.

One can go even further by noticing that the arbitrage rent $\sum_i \mu_{int}$ does not depend on the technology i and will be the same for each technology at a specific node. Indeed, the arbitrage rent is defined by the following equation:

$$0 \leq y_{int} \perp MC_i + \mu_{int} + \tilde{\mu}_{int} - \rho_{Z(n)t} - \delta_{int} \geq 0 \tag{46}$$

From the analysis of the complementarity conditions associated to the re-dispatch, i.e. MCP (40), we get that at equilibirum, the following must hold:

$$\tilde{\mu}_{int} - \delta_{int} = \rho_{nt} - MC_i \tag{47}$$

The complementarity condition on the zonal dispatch can thus be rewritten as:

$$0 \leq y_{int} \perp \tilde{\rho}_{nt} + \mu_{int} - \rho_{Z(n)t} \geq 0 \tag{48}$$

This equation implies that

$$\mu_{int} = \begin{cases} \rho_{Z(n)t} - \tilde{\rho}_{nt} \text{ if } \rho_{Z(n)t} > \tilde{\rho}_{nt} \\ 0 \text{ otherwise} \end{cases} \tag{49}$$

which shows that arbitrage rent does not depend on technology i. Therefore, the result of Proposition 6 can be strengthened in the following way:

Proposition 7. *There exists a location and technology differentiated capacity price that restores the efficiency of zonal pricing followed by market-based re-dispatch.*

Proof of Proposition 7 This can be proved in the same way as Proposition 6 with the additional observation that the arbitrage rent does not depend on the technology. □

As it stands, the proposition only states that there exists a capacity price that recovers the efficiency of zonal, i.e. there exists an equilibrium that is efficient. This means that the efficiency of the design would be guaranteed to be recovered if an external entity is able to compute and impose the price to market participants. One may wonder whether the result still holds if the charge is computed in a decentralized way, i.e. on a market.

Mathematically, the question of whether the mechanism remains efficient in this case translates into the question of whehter all solutions to the MCP augmented with the locational capacity market are efficient. In the case of a locational capacity market, the investment conditions of producers become

$$0 \leq x_{in} \perp IC_i - \sum_{t \in T} \mu_{int} - \sum_{t \in T} \tilde{\mu}_{int} + \pi_n \geq 0 \tag{50}$$

where π_n is the capacity price. The capacity market clearing condition becomes

$$0 \leq \pi_n \perp \sum_{i \in I} \bar{x}_{in} - \sum_{i \in I} x_{in} \geq 0 \tag{51}$$

PROPOSITION 8. *Zonal pricing followed by market-based re-dispatch augmented with a location differentiated capacity market recovers the efficiency of nodal pricing.*

Proof of Proposition 8 We need to show that all solutions of MCP (50), (35b) - (35n), (51) are as efficient as the nodal solution. We start by showing that in all solutions of the MCP, $\sum_{i \in I} x_{in} = \sum_{i \in I} \bar{x}_{in}$. Let $(\hat{\cdot})$ be an arbitrary solution of the MCP and let us denote by ν_n the arbitrage rent in this solution, i.e. $\nu_n = \sum_{i \in I} \hat{\mu}_{int}$. Assume by contradiction that there exists a nonempty set of indices $Q \subset N$ such that

$$\sum_{i \in I} \hat{x}_{in} < \sum_{i \in I} \bar{x}_{in} \ \ \forall n \in Q \tag{52}$$

and denote by $\bar{Q}$ the maximum of such sets. Now, observe that the corresponding variables of the solution $(\hat{\cdot})$ must also be a solution of the following optimization problem:

$$\begin{aligned}
\min \ & \sum_{in} (IC_i - \nu_n + \hat{\pi}_n) x_{in} + \sum_{int} MC_i y_{int} \\
\text{s.t. } & X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \ \ [\tilde{\mu}_{int}] \\
& r_{nt} - \sum_{int} y_{int} + D_{nt} = 0, n \in N, t \in T \ \ [\tilde{\rho}_{nt}] \\
& \tilde{W}_m - \sum_{n} \tilde{V}_{mn} r_{nt}, m \in \tilde{M}, t \in T \ \ [\tilde{\gamma}_{mt}] \\
& \bar{y}_{int} \geq 0, i \in I, n \in N, t \in T \ \ [\delta_{int}]
\end{aligned} \tag{53}$$

By definition, the solution $(\bar{\cdot})$ is the solution of the non-perturbed nodal capacity expansion problem, i.e. it is a solution of:

$$\begin{aligned}
\min\ & \sum_{in} IC_{in}x_{in} + \sum_{int} MC_i y_{int} \\
\text{s.t. } & X_{in} + x_{in} - y_{int} \geq 0, i \in I, n \in N, t \in T \ [\tilde{\mu}_{int}] \\
& r_{nt} - \sum_{int} y_{int} + D_{nt} = 0, n \in N, t \in T \ [\tilde{\rho}_{nt}] \\
& \tilde{W}_m - \sum_n \tilde{V}_{mn} r_{nt}, m \in \tilde{M}, t \in T \ [\tilde{\gamma}_{mt}] \\
& \bar{y}_{int} \geq 0, i \in I, n \in N, t \in T \ [\delta_{int}]
\end{aligned} \tag{54}$$

We deduce, as the feasible sets of both problems are the same, that the following should hold:

$$\sum_{in}(IC_i - \nu_n + \hat{\pi}_n)\bar{x}_{in} + \sum_{int} MC_i \bar{y}_{int} \geq \sum_{in}(IC_i - \nu_n + \hat{\pi}_n)\hat{x}_{in} + \sum_{int} MC_i \hat{y}_{int} \tag{55}$$

By reorganising the terms, we get

$$\sum_{in} IC_i \bar{x}_{in} + \sum_{int} MC_i \bar{y}_{int} - \sum_{in} IC_i \hat{x}_{in} - \sum_{int} MC_i \hat{y}_{int} \geq \tag{56}$$

$$\sum_{n \in \bar{Q}} (-\nu_n + \hat{\pi}_n)\big(\sum_{i \in I}(\bar{x}_{in} - \hat{x}_{in})\big) + \sum_{n \in N \setminus \bar{Q}} (-\nu_n + \hat{\pi}_n)\big(\sum_{i \in I}(\bar{x}_{in} - \hat{x}_{in})\big) \tag{57}$$

Note that as $\bar{Q}$ is the maximum of all the sets for which (52) holds, we have that

$$\sum_{n \in N \setminus \bar{Q}} \sum_{i \in I} (\bar{x}_{in} - \hat{x}_{in}) = 0$$

Moreover, for all $n \in \bar{Q}$, we have $\hat{\pi}_n = 0$ by the complementarity condition (51). Therefore, equation (56) simplifies to

$$\sum_{in} IC_i \bar{x}_{in} + \sum_{int} MC_i \bar{y}_{int} - \sum_{in} IC_i \hat{x}_{in} - \sum_{int} MC_i \hat{y}_{int} \geq \tag{58}$$

$$\sum_{n \in \bar{Q}} (-\nu_n)\big(\sum_{i \in I}(\hat{x}_{in} - \bar{x}_{in})\big) \tag{59}$$

The first term is strictly negative as $(\bar{\cdot})$ is the solution of the non-perturbed nodal capacity expansion problem (assuming it is unique in the investment) whereas the second term is nonnegative, which leads to a contradiciton. This implies that in any solution $(\hat{\cdot})$ of the MCP (43), (35b) - (35n), (51), $\sum_{i\in I}\hat{x}_{in} = \sum_{i\in I}\bar{x}_{in}$ for all $n \in N$.

Now, by the optimalitity of $(\hat{\cdot})$ for the perturbed problem, we have

$$\sum_{in} IC_i\hat{x}_{in} + \sum_{n\in N}(-\nu_n + \hat{\pi}_n)\sum_{i\in I}\hat{x}_{in} + \sum_{int} MC_i\hat{y}_{int} \le \sum_{in} IC_i\bar{x}_{in} + \sum_{n\in N}(-\nu_n + \bar{\pi}_n)\sum_{i\in I}\hat{x}_{in} + \sum_{int} MC_i\bar{y}_{int}$$

which implies

$$\sum_{in} IC_i\hat{x}_{in} + \sum_{int} MC_i\hat{y}_{int} \le \sum_{in} IC_i\bar{x}_{in} + \sum_{int} MC_i\bar{y}_{int}$$

By the optimalitity of $(\bar{\cdot})$ for the non-perturbed problem, we have

$$\sum_{in} IC_i\hat{x}_{in} + \sum_{int} MC_i\hat{y}_{int} \ge \sum_{in} IC_i\bar{x}_{in} + \sum_{int} MC_i\bar{y}_{int}$$

We conclude by observing that both solutions should have the same efficiency, i.e.

$$\sum_{in} IC_i\hat{x}_{in} + \sum_{int} MC_i\hat{y}_{int} = \sum_{in} IC_i\bar{x}_{in} + \sum_{int} MC_i\bar{y}_{int}$$

□

3.6. Feasible set of zonal net positions

In the previous sections, we kept the discussion abstract of considerations regarding the specific definition of the feasible set of zonal net positions $\mathcal{P}$. We have seen, however, that our results depend on the choice of $\mathcal{P}$. In capacity and energy markets, our results of efficiency hold when $\mathcal{P}$ satisfies nodal consistency. The shape of $\mathcal{P}$ can also influence the value of the instruments needed to recover efficiency. In this section, we describe some of the possible definitions for $\mathcal{P}$ and discuss their implication in the context of the present study.

3.6.1. Price aggregation The feasible set of zonal net positions with price aggregation, that we denote by $\mathcal{P}^{\text{PA}}$, was introduced in Lété et al. (2022). It consists in a very natural way of extending the uniquely defined feasible set of net injections $\mathcal{R}$ to the zonal setting. Using our notation, it is defined as follows:

$$\begin{aligned}\mathcal{P}^{\text{PA}} = \Bigg\{ p \in \mathbb{R}^{|Z|} \,\Bigg|\, & \exists (f,r) \in \mathbb{R}^{|K|} \times \mathbb{R}^{|N|} : p_z = \sum_{n\in N(z)} r_n \;\; \forall z \in Z, \\ & f_k = \sum_n PTDF_{kn}\cdot r_n \;\; \forall k \in K, \sum_n r_n = 0, -TC_k \le f_k \le TC_k \;\; \forall k \in K \Bigg\}\end{aligned} \tag{60}$$

It can be readily checked that $\mathcal{P}^{\text{PA}}$ is nodal consistent.

3.6.2. Flow-based market coupling Although interesting from a theoretical point of view, the set $\mathcal{P}^{\mathrm{PA}}$ is not implemented in practice. Instead, the dominant methodology in the European market is called flow-based market coupling (FBMC). Aravena et al. (2021) propose a definition of the feasible set of net positions in FBMC that, in our notation, can be written as follows:

$$\begin{aligned}\mathcal{P}^{\mathrm{FBMC}}(x) = \Bigg\{ p \in \mathbb{R}^{|Z|} \,\Bigg|\, & \exists (f, r, \tilde{y}) \in \mathbb{R}^{|K|} \times \mathbb{R}^{|N|} \times \mathbb{R}^{|I||N|} : p_z = \sum_{n \in N(z)} r_n \;\; \forall z \in Z, \\ & r_n = \tilde{y}_{int} - D_{nt} \;\; \forall n \in N, \, 0 \le \tilde{y}_{int} \le X_{in} + x_{in} \;\; \forall i \in I, n \in N, \\ & f_k = \sum_n PTDF_{kn} \cdot r_n \;\; \forall k \in K, \, \sum_n r_n = 0, -TC_k \le f_k \le TC_k, \;\; \forall k \in k \Bigg\}\end{aligned} \tag{61}$$

The specificity of FBMC is that it depends on the installed capacity x_{in}, which implies some inefficiency in the context of capacity expansion (Lété et al. 2022). In the present paper, it also implies that the definition of nodal consistency must be adapated to this situation.

Definition 4. Let r^* be a vector of values of the nodal injections in one solution of the nodal capacity expansion problem, x^* the associated investment and let p^* be the corresponding vector of zonal net positions, i.e.

$$p_z^* = \sum_{n \in N(z)} r_n^* \;\; \forall z \in Z$$

The feasible set of zonal net position $\mathcal{P}$ is said to be **nodal consistent** if $p^* \in \mathcal{P}(x^*)$.

It can be shown that our results of efficiency remain true with this new definition and it can be easily checked that $\mathcal{P}^{\mathrm{FBMC}}$ is nodal consistent.

3.6.3. Min-RAM By definition, a zonal market ignores the flows associated to intra-zonal trade which implies that re-dispatch is needed in general. As discussed in Meeus (2020), TSOs tend to limit cross-border trade in order to decrease re-dispatch. As a consequence, the European Commission introduced a minimum requirement of 70% for the capacity that should be made available for cross-border trade by TSOs. This requirement, sometimes called a min-RAM requirement, can easily be modeled within our framework. Indeed, if we start from the $\mathcal{P}^{\mathrm{PA}}$ that we defined in equation (60), we observe that the flow variables f_k represent purely flows due to inter-zonal trade.

In zonal pricing with the 70% min-RAM, these flows on each lines should be allowed to reach 70% of the line capacity. Let us denote by η the min-RAM requirement. The feasible set of zonal net positions under the min-RAM rule can be written as:

$$\mathcal{P}_{\eta}^{\mathrm{MR}} = \Bigg\{ p \in \mathbb{R}^{|Z|} \,\Bigg|\, \exists (f,r) \in \mathbb{R}^{|K|} \times \mathbb{R}^{|N|} : p_z = \sum_{n \in N(z)} r_n \;\; \forall z \in Z, \\ f_k = \sum_n PTDF_{kn} \cdot r_n \;\; \forall k \in K, \sum_n r_n = 0, -\eta \cdot TC_k \leq f_k \leq \eta \cdot TC_k \;\; \forall k \in K \Bigg\} \tag{62}$$

Note that $\mathcal{P}_{\eta}^{\mathrm{MR}}$ is not guaranteed to be nodal consistent, which implies that our results of efficiency in capacity and energy markets do not hold in this case.

4. Simulation results

In this section, we present simulation results for a reduced instance of the Central Western Europe network area.

4.1. Data

We use the same dataset as in Lété et al. (2022). The area consists of 6 countries (Austria, Belgium, Germany, France, Luxembourg and the Netherlands) grouped into 5 bidding zones, Germany and Luxembourg forming one single zone. Our network data is based on the European grid model of Hutcheon and Bialek (2013). Our time series data (hourly demand, solar and wind production in each country) are obtained from the ENSTO-E Transparency Platform for the year 2018.

The network and time series data have been reduced in order to obtain tractable capacity expansion problems. We fixed *a priori* the number of representative time periods and network buses that we wished to obtain to 20 periods and 100 buses. The 20 periods are obtained such that the corresponding aggregate net load duration curve is the best approximation in the sense of the Euclidean norm of the hourly aggregate net load duration curve. We used the dynamic programming algorithm presented in Konno and Kuno (1988) to solve for the best piecewise constant approximation. In order to obtain the reduced transmission network, we started by clustering the nodes into 100 representative nodes. The nodes were clustered using the Euclidean Commute Time

Type	Number of units	Total installed capacity [GW]	FC [k€/MW yr]	MC [€/MWh]
Nuclear	73	77.67	92	9.1
Natural gas	403	56.38	9.33	93.42-121.37
Coal	93	30.7	46.29	44.5-58
Lignite	59	20.82	101.5	36.7-42.12
Oil	75	6.37	9.33	116-210
Other	189	6.08	113.16	38.64

Table 1 Total installed capacity and cost data of conventional units in the database per type of fuel. The costs are annualized fixed cost and marginal cost range of existing open-cycle generators.

(ECT) distance (Yen et al. 2005) on the graph corresponding to the network, with the edge weights set to the difference in nodal prices between each pair of nodes. The cross-zonal lines were removed from the network for the clustering in order to obtain only clusters of nodes from the same bidding zone. The ECT distance was chosen for the clustering in order to favor clusters that are strongly connected. The PTDF matrix of the reduced transmission network has been obtained using the injection-independent method described in Fortenbacher et al. (2018). We then computed the thermal capacities of the lines of the reduced network in a way that minimizes the Euclidean norm of the difference between the average nodal price of the nodes in each cluster and the new nodal price of the cluster.

The generation data is obtained from Open Power System Data (2020). Table 1 presents the total installed capacity per generator type for the entire CWE region. The models that we use for the case study are generalized versions of the models presented in section 3. In the models of the CWE case study, we also consider revenues from reserve provision, where reserve is assumed to be cleared simulatenously with energy. For market-based re-dispatch, we limit the amount of arbitrage that can take place. As we discussed in section 3.5, even in the absence of market power, producers and incentivized to deviate from bidding at marginal cost. This deviation could caught the attention of the regulator if it is too important and it is therefore unrealistic to assume that it takes place to its full possible extent. Therefore, we assume that the profit made through the exercise of arbitrage by the producers is limited by the profit that they would make if they cannot

Type	IC [k€/MW yr]	FC [k€/MW yr]	MC [€/MWh]
CCGT	80.1	16.5	61.29
OCGT	56.33	9.33	100.4
CCGT&CHP	94.39	16.5	41.37

Table 2 Annualized investment cost, annualized fixed operating and maintenance cost and marginal cost for the three investment technologies considered for investment in the CWE case study.

bid 30% more (less) than the marginal cost of the peak-load (base-load) technology in the zonal market.

We also consider fixed operating and maintenance costs. Units that cannot cover their fixed costs are decommissioned. We assume that investment is possible in 3 different technologies, similarly to Ambrosius et al. (2020): CCGT units, OCGT units and Combined Heat and Power CCGT units. We use the same cost data as Ambrosius et al. (2020), which are presented in Table 2.

Wind and solar expansion are accounted for in an exogenous way. The fixed and marginal cost of existing capacity are also sourced from Ambrosius et al. (2020) and completed from Open Energy Information (2021) when missing (see Table 1). We separate each existing unit into open-cycle and combined-cycle generators. For combined-cycle units, we increase the fixed cost by 77% and reduce the marginal cost range by 39%, similarly to what is assumed for natural gas units in investment. We also distinguish between CHP and non-CHP generators. The marginal cost of CHP generators is reduced by 20 €/MWh in order to represent the additional revenues from the sale of heat. Finally, we assume a capacity expansion horizon of 2035. Consequently, we remove from the dataset the generators that will be shut down by then, based on the information available in the OPSD dataset (Open Power System Data 2020). We also remove all nuclear units from Belgium and Germany and integrate the planned closure of 14 nuclear reactors by EDF by 2035 for France (International Atomic Energy Agency 2020).

4.2. Algorithmic remarks

Most models that we have presented in section 3 can be formulated as linear programs (LP) and can thus be readily solved by state-of-the-art LP solvers. The only one that cannot be formulated as an

LP is the long run equilibrium of zonal pricing with market based re-dispatch which, in contrast, is equivalent to the MCP (35). As mentioned in 3.5, this equilibrium corresponds in fact to a GNE. The GNE property originates in the imperfect nature of the market as the impacts of the production decisions in the zonal market on the feasibility set of the TSO are not internalized by the producers. From a mathematical point of view, this results in the breaking of the equivalence between the MCP that aggregates the KKT optimality conditions at equilibrium of each type of agent and the LP that maximizes welfare. However, although the equivalence does not hold, this reasoning suggests a way to leverage this structure algorithmically by resorting to a splitting algorithm. We describe below a basic splitting algorithm for solving the $LCP(q,M)$ that corresponds to our MCP (Cottle et al. 2009):

Step 0. *Initialization.* Let z_0 be an arbitrary nonnegative vector, set $\nu = 0$.

Step 1. *General iteration.* Given $z^\nu \geq 0$, solve the $LCP(q^\nu, B)$ where

$$q^\nu = q + Cz^\nu$$

and let $z^{\nu+1}$ be an arbitrary solution.

Step 2. *Test for termination.* If $z^{\nu+1}$ satisfies a prescribed stopping rule, terminate. Otherwise, return to Step 1 with ν replaced by $\nu + 1$.

In our case, a natural choice for matrix B is that associated to the perfect version of our market, i.e. to the welfare-maximizing LP. The C matrix captures the imperfectness and the term Cz^ν has thus the same analytical formulation as the Pigouvian tax that would restore Pareto optimality, which in our case corresponds to the lack of impact of the re-dispatch price on the zonal production decisions. This is the methodology that we used for solving for the GNE corresponding to MCP (35). We note however that we have no guarantee of convergence for this algorithm on our problem and that we observed empirically that convergence is sensitive to the starting point.

4.3. Relative performance of policies

Table 3 presents the performance in terms of total operational and investment costs of the different policies. Two policies are able to reproduce the efficiency of the nodal pricing benchmark: the

Policy	Op. cost	Inv. cost [M€/yr]	Total cost
Benchmark			
Nodal	15,810	10,433	26,243
Zonal with PA	16,835	10,909	27,744
Capacity market			
Two-sided (TS)	16,041	10,839	26,880
Negative one-sided	16,041	10,839	26,880
Positive one-sided (POS)	16,899	10,795	27,694
TS no technology differentiation	16,705	10,619	27,324
TS with 70% rule	15,929	11,088	27,016
Energy market			
Two-sided	15,809	10,433	26,242
TS with low granularity	15,911	10,493	26,404
Market-based re-dispatch			
Market-based re-dispatch (MBR)	15,860	11,646	27,506
MBR with POS capacity market	15,810	10,433	26,243

Table 3 Performance comparison of the different policies.

energy signal policy with full temporal granularity and market-based re-dispatch with locational connection charges.

Regarding capacity markets, our simulation results confirm the theoretical results of Proposition 2 which states that the efficiency of the two-sided capacity market instrument can be obtained with its one-sided version with negative capacity price: the two policies obtain indeed the same efficiency in our simulations. Our results also highlight the importance of differentiating technologies in the capacity market as well as the importance of the design of the zonal transmission constraints for the efficiency of the instruments. The loss of efficiency of the capacity market policy when it is not differentiated by technologies is evaluated at 2.4% of the total cost. The loss of efficiency associated to the use of the feasible set of zonal net positions based on the 70% rule, that is not nodal-consistent, is estimated at 1.3%.

The energy market must have full temporal granularity in order to recover the efficiency of the nodal benchmark. In practice, it implies that in the case of a hourly day-ahead auction, the energy price that adds on top of the zonal day-ahead electricity price must also be updated on an hourly basis. This renders the policy complicated to implement in practice and one may wonder what happens if the energy market is implemented with a lower granularity. The energy market policy with low granularity that we have simulated has 4 times less temporal granularity than the full energy market. Its loss of efficiency is estimated in our simulations at 161 M€/year, which amounts to 0.6% of the total cost.

We observe that in the long run, zonal pricing followed by market-based re-dispatch is significantly more costly than all nodal and zonal pricing benchmarks. We obtain a loss of efficiency compared to nodal pricing of 4.8% of the total cost. This rise in the total cost is due almost exclusively to the investment cost, which is to be expected in regards of the analysis of section 3. Indeed, the arbitrage between the zonal electricity market and nodal re-dispatch market allows producers to extract a rent that translates to significantly more investment in the long run that does not improve the operational cost. The operational costs between the nodal policy and the MBR policy are indeed comparable. Although the MBR policy does not seem to be a good candidate to obtain an efficient design, it should be highlighted that this policy is probably the best candidate when appropriate locational instruments are used to steer the right investment on top of market prices. Indeed, in our simplified framework, the efficiency of the nodal benchmark is restored with a locational capacity market, which is probably the easiest instrument to implement among all that we have discussed in this study.

4.4. Value of the instruments and capacity mix

We now turn to the comparison of the different policies based on the value of the instruments and the capacity mix.

Figure 1 shows the different nature of the instruments. The capacicty market is differentiated among technologies, the two-sided energy market has both positive and negative prices and the

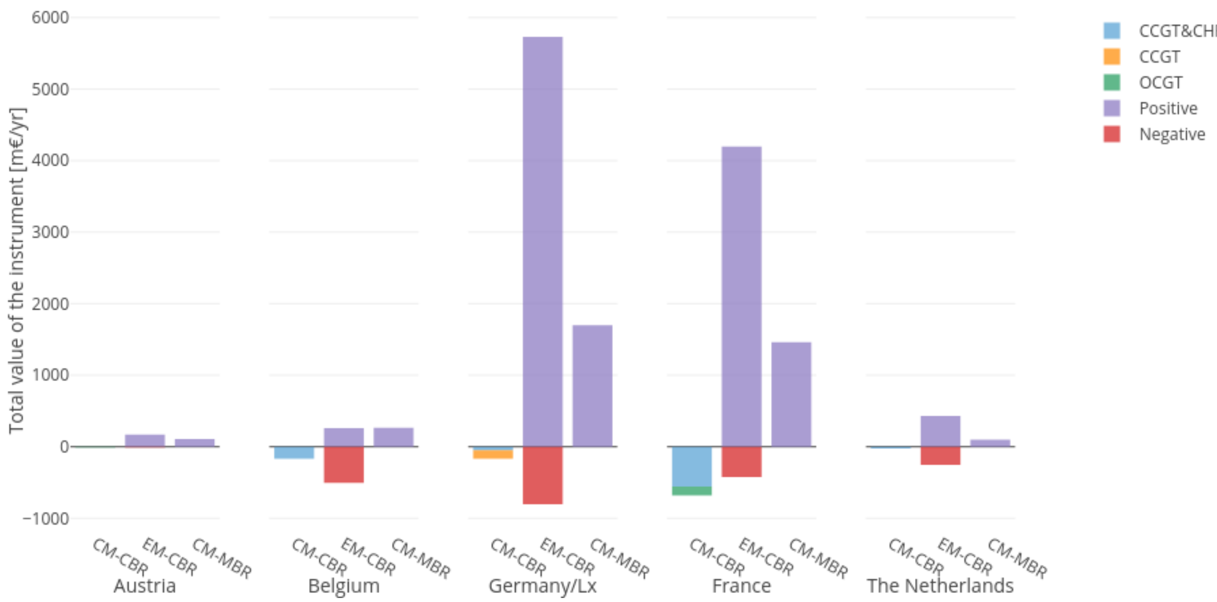

Figure 1 Total value of the different type of instrument in function of the bidding zone. CM-CBR stands for capacity market with cost-based re-dispatch. EM-CBR is the energy market policy with cost-based re-dispatch. CM-MBR is the combination of a capacity market and market-based re-dispatch. Different colors are used to differentiate technologies in the CM-CBR policy. The two different colors for EM-CBR differentiate the positive and negative prices in the to-sided version of the market.

market-based re-dispatch with one-sided capacity market has only postive prices. We observe that among the two policies that are able to recover the efficiency of nodal, the one with market-based re-dispatch requires a significantly smaller total value for the instrument. This is in part due to the fact that the loss of efficiency in market-based re-dispatch is moderated by the limit on the bidding behavior of market participants. In comparison, the capacity market policy with cost-based re-dispatch, which in this situation does not lead to a full recovery of the nodal efficiency, leads to even lower total value for the instruments. It is interesting to observe that in general in energy markets, the total value of positive prices is more important than for negative prices. This means that the electricity price must in general be penalized by a positive energy price. However, the opposite is true for Belgium where the zonal electricity price must more frequently be decreased by negative energy prices in order to recover the efficiency. This can be related to an analysis of the

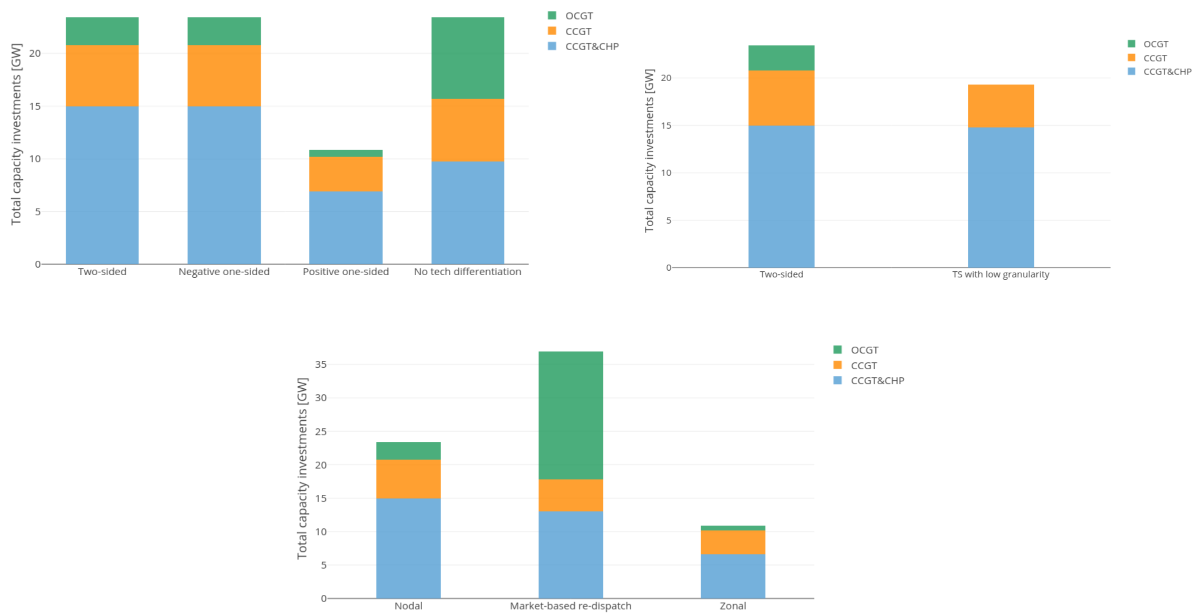

Figure 2 Total capacity investments for the different policies. The top-left subfigure compares the capacity-based instruments, the top-right subfigures compares the energy-based instruments and the bottom subfigure compares the market-based re-dispatch policies with the nodal and zonal benchmarks.

Belgium regulator (CREG 2019) which observed that the Belgian day-ahead price that results from market coupling is distorted. The prices are affected negatively in Belgium due to high North-South loop-flows that originate from congestion in Germany (CREG 2019).

In Figure 2, we compare the investment in new generation capacity between the different designs, separated in the three main classes of instruments. The top-left subfigure corresponds to zonal with locational capacity markets. As theoretically expected (see Proposition 2), the two-sided capacity market and one-sided capacity market with negative price lead to the same investment. This is not the case, however, for the one-sided version with positive price. This highlights the fact the zonal pricing with cost-based re-dispatch lead in general to a lack of investment that must be corrected with additional revenues to the investors, not additional costs. There is also an inefficiency associated to zonal pricing with locational capacity markets when the capacity prices are not differentiated among technologies, as already observed in Grimm et al. (2019). In this case, as one can observe in Figure 2, the policy will tend to favor peak technologies that exhibit

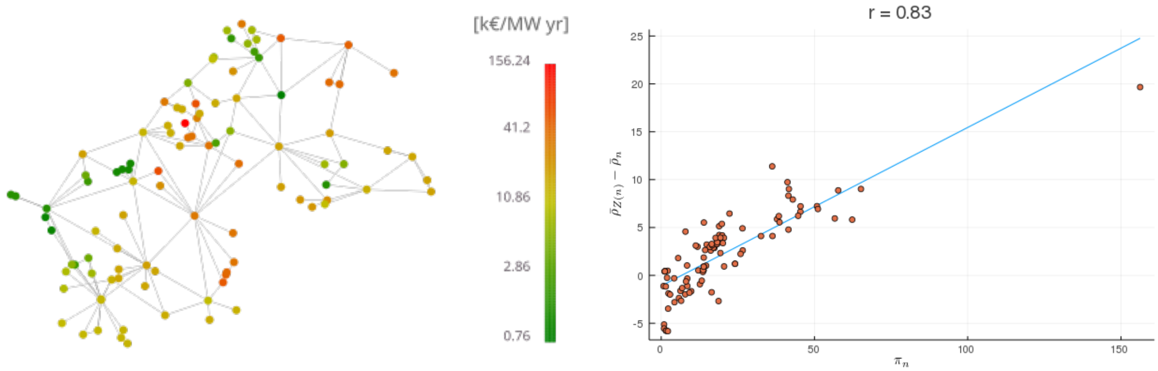

Figure 3 Left: value of the optimal connection charge in zonal pricing followed by MBR. Right: difference between the zonal and the nodal price in function of the value of the optimal connection charge instrument in MBR.

low investment costs. The top-right subfigures relates to zonal with locational energy markets. Efficiency can be recovered in this case under the condition that the energy price has full temporal granularity. When the temporal granularity is not complete, the situation is somewhat opposite as the one observed in capacity markets: peak-load technologies, that exhibit high marginal costs tend to be penalized. The situation with market-based re-dispatch is analyzed in the bottom subfigure. Interestingly, MBR leads to a large increase in peak-load technology investments, way above the optimal level, which confirms the intuition of Hirth and Schlecht (2020). In order to profit from arbitrage rents, large amounts of capacity with limited investment costs are build. The investment, however, will not benefit the system as most of this capacity will not be used for actually producing electricity. The investments in the two other capacities with larger investments costs, in contrast, are close to their optimal level.

To end with, we would like to analyze with slightly more details the market-based re-dispatch policy. In Figure 3, we represent on the left panel the value of the optimal capacity price under that policy for each node of the network. On the right panel, we plot the difference between zonal and nodal prices in function of the optimal capacity price. We observe large differences between the different values of the optimal capacity price. In particular, although the large majority of the charges are below 50k€/MW, one node at the border between Belgium and France gets a value of more than 150k€/MW. This large value can be explained by the fact that it is only connected to nodes in France while it belongs to the Belgian bidding zone. For this reason, it exhibits a large difference between its zonal and nodal price which makes it particularly prone to arbitrage. As displayed on the right panel, there is a high correlation between the difference of nodal and zonal prices in a location and the value of the optimal capacity price needed to restore efficiency of MBR, which suggests that this difference is the main driver for the inefficiency.

5. Conclusions

The question of the best market-based way of allocating transmission capacity remains at the center of intense discussions among European stakeholders. In the academic literature, it was mainly treated in a dichotomic way in the form of the nodal vs zonal pricing debate. In this paper, we approach the question from a different perspective and, instead, take zonal pricing as a given for the European electricity market. From that starting point, we investigate the potential of additional market-based instruments in restoring the efficiency of zonal. We consider three main classes of instruments: additional capacity-based and energy-based markets as well as re-dispatch markets. The paper aims at comparing the efficiency of the three different classes of instruments both theoretically and empirically on the basis of a unifying modeling framework.

We conclude that theoretically, the efficiency of the nodal design can be recovered in zonal with additional markets, that can be of each of the three classes that we considered. This, however, holds only under strong conditions that are unlikely to be met in practice. For locational capacity markets, efficiency can be recovered in the long run only if the price is differentiated among each type of producing unit. For energy markets, the drawback is that the price should have full temporal and locational granularity which would be difficult to implement. The best candidate that we identified for restoring long run efficiency under the zonal paradigm is the market-based re-dispatch policy. Although it is subject to inefficiencies due to arbitrage between the zonal and re-dispatch markets, it can be corrected by means of an additional capacity market that does not need to be differentiated by technologies. The main drawback of this policy lies in its complexity as it is made of two additional instruments. Additionally, our analysis highlights that these theoretical results are subject to conditions on the cooperation of TSOs and coherence of the zonal capacity calculation methodology, that are currently not observed in practice. For these reasons, it seems unlikely that the long run efficiency could be restored in electricity markets with zonal pricing by means of a practical additional market-based instrument.

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