

LINMA 2120 - Seminars in Applied Mathematics

High dimensional time-series prediction with missing values

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1 Introduction

Let us first give a definition of time series to explain the formal context in which we work.

Definition 1. Time series A time series is a sequence of data indexed by the time.

A time series is thus a realization of a stochastic process which can be seen as a family of random variables indexed by the time.

Classical methods

In this section, we present classical methods used for time series forecasting. These classical methods include AR (autoregressive) models and DLM (dynamic linear models) and are used for prediction. Let us explain sequentially these models.

Autoregressive models

As explained in [1], the idea of this model is to represent each observation as a noisy linear combination of previous observations. Formally, if X_t is the measurement at time t , the AR(p) model parametrized by the lag p and the coefficient vector $\alpha \in \mathbb{R}^p$ can be written as

$$X_t = \sum_{k=1}^p \alpha_k X_{t-k} + \epsilon_t$$

where $\{\epsilon_t\}_{t \in \mathbb{Z}}$ is a white noise.

This model is motivated by a theorem due to Wold that states that a stationary process can be represented by a MA(∞) model, that is

$$X_t = \sum_{i=1}^{\infty} \beta_i \epsilon_{t-i} + \epsilon_t$$

where $\sum_{i=1}^{\infty} \beta_i < \infty$ and $\{\epsilon_t\}_{t \in \mathbb{Z}}$ have zero mean and equal variance. And if $\{X_t\}_t^{\infty}$ is invertible, we also have that it can be represented by an AR(∞) model, that is

$$X_t = \sum_{i=1}^{\infty} \alpha_i X_{t-i} + \epsilon_t$$

With this theorem, it seems natural to use AR(p) models for prediction.

References

- [1] O. Anava, E. Hazan, and A. Zeevi, “Online time series prediction with missing data,” in *Proceedings of the 32nd International Conference on Machine Learning*, ser. Proceedings of Machine Learning Research, F. Bach and D. Blei, Eds., vol. 37. Lille, France: PMLR, 07–09 Jul 2015, pp. 2191–2199. [Online]. Available: <http://proceedings.mlr.press/v37/anava15.html>