Black-Box Optimization for Design of Concentrating Solar Power and Photovoltaic Hybrid Systems with Optimal Dispatch Decisions

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Abstract. The hybridization of concentrating solar power (CSP) and photovoltaics (PV) can enable dispatchable renewable electricity generation at a lower price than current stand-alone CSP systems. However, designing a CSP-PV hybrid system can be challenging because of the many degrees of freedom in design that affect the internal and external system interactions and trade-offs. We develop a methodology to determine optimal designs for CSP-PV hybrids by implementing NLopt's derivative-free, or "black-box," algorithms around pre-existing CSP-PV hybrid simulation software that utilizes the National Renewable Energy Laboratory's System Advisor Model (SAM); we then employ a dispatch optimization model to determine operational decisions that maximize a plant's profits. We present optimal designs for CSP-PV hybrid systems dispatching against four time-of-delivery (ToD) pricing structures. NLopt's algorithms can improve the base case design's power purchase agreement (PPA) price by 15% to 21%, depending on the ToD pricing structure. In addition, we present the resulting optimal CSP-PV hybrid design's annual performance metrics, which tend to have capacity factors between 50% and 62%, but are able to generate electricity during the year's highest-valued periods about 90% of the time. Lastly, we investigate the trade-offs between capacity factor and PPA price using Pareto fronts and demonstrate that, for some ToD pricing structures, the system capacity factor can increase by 20% but at the expense of a 2% increase in PPA price.

INTRODUCTION

Due to their design flexibility and dispatchability, concentrating solar power (CSP) systems with thermal energy storage (TES) are becoming attractive solutions for renewable portfolios around the world. The design flexibility of CSP can fulfill many different market needs by providing either peaking, intermediate, or baseload power generation, depending on the system's solar multiple and hours of available TES. Utility companies and governments use time-of-delivery (ToD) price schedules to incentivize power generation during times of peak demand and/or low supply, which can vary depending on the utility's current generation portfolio and end-use customer demand. Due to the complexity of CSP system sizing and various constraints imposed on these systems, e.g., ToD pricing, design optimization using dispatch is required to ensure that CSP is competitive in electricity markets.

Locations with high direct normal irradiance (DNI) solar resource inherently have high penetration of photovoltaic (PV) systems on the grid. As a result, to stay competitive, CSP systems must dispatch power generation around PV generation. The PV system within a CSP-PV hybrid could be co-located with the CSP system, located on the same transmission line as the CSP system but miles apart, or be a conglomerate of small rooftop systems within the municipality to which the CSP system is supplying electricity.

In this paper, we develop and implement a methodology to determine optimal designs for CSP-PV hybrids using derivative-free, or "black-box," algorithms. We provide a description of our methodology to evaluate operational decisions. The contribution of this paper is to present the optimal designs for CSP-PV hybrids dispatching against four ToD pricing structures. Furthermore, we provide trade-off analysis using Pareto fronts to demonstrate the relationship between system capacity factor and PPA price for the four ToD pricing structures under investigation.

APPROACH

We simulate the performance of CSP-PV hybrid systems and evaluate their financial feasibility using the System Advisor Model (SAM)[1], which is an open-source simulation software package developed by the National Renewable Energy Laboratory (NREL) that enables users to analyze the financial feasibility and performance of renewable energy technologies. However, SAM's user interface only allows for the evaluation of a system design utilizing a single renewable energy technology and thereby precludes analysis of hybrid systems. In previous work, we developed a methodology to simulate the hybridization of the two commercial-scale solar technologies, power tower CSP with TES and a PV system with and without the inclusion of electrical energy storage (EES),[2] shown in Fig. 1. Our methodology utilizes SAM's Python application programming interface (API) to build a wrapper to evaluate the performance of the hybrid system as a whole.

Co-located CSP and PV system design can be particularly challenging if the design requires the two sub-systems to behave as a single power generation system, which could be applicable for the case in which two systems cover a *baseload*, as a result of, e.g., an island grid with a consistent load requirement, or a utility agreement to schedule CSP generation around PV generation. However, these scenarios require an intelligent simulation control strategy of the dispatchable energy storage technologies. To this end, previous work focuses on the development of an optimization model to schedule or dispatch each sub-system within the overall system.[2]

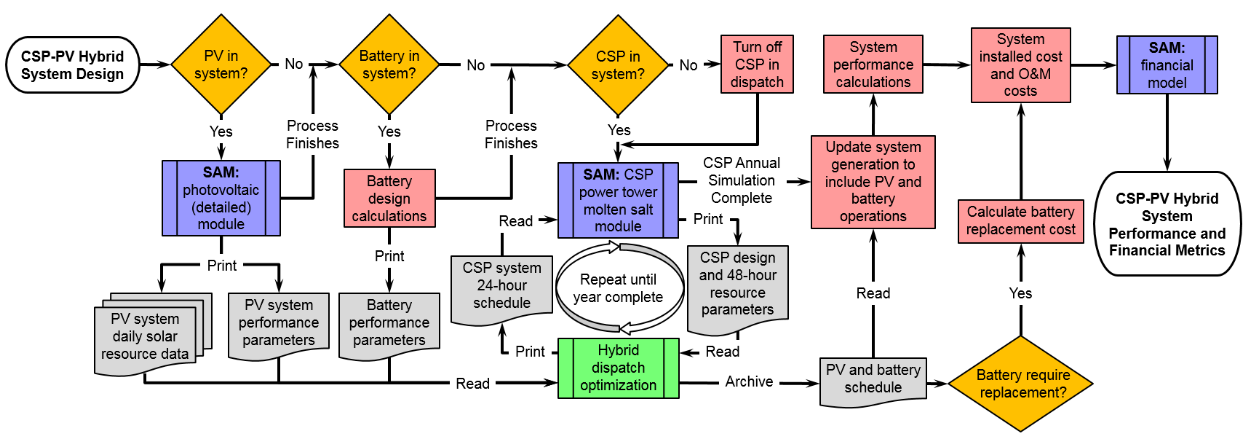

FIGURE 1. Flow diagram of the software architecture implemented around the hybrid dispatch optimization, modified from previous work.[2]

Hybrid Dispatch Optimization

Dispatch optimization enables optimal scheduling of sub-system operations, i.e., receiver, power cycle, and EES. With an uncertain solar resource and limited energy storage, optimal operational decisions are complex and multifaceted. Our hybrid dispatch optimization model is a mixed-integer linear program (MILP), written in A Mathematical Programming Language (AMPL),[3] and solved using IBM CPLEX.[4] A MILP enables the use of binary logic related to on-off decisions; furthermore, a linear model is often more tractable than its nonlinear equivalent.

To determine a solution for an annual horizon, we solve our dispatch optimization model using a rolling time horizon with a 48-hour look-ahead, every 24-hours, depicted in the lower center of Fig. 1. Relative to solving the monolith consisting of a year-long time horizon, this approach has the benefits of: (i) providing the dispatch optimization model with a limited view into the future to ensure that short-term decisions are uninfluenced by unrealistic future knowledge, such as solar resource or market pricing three to five days ahead, and (ii) increasing tractability and thereby faster solution times.

For brevity, we do not describe the dispatch optimization formulation in full detail; instead, we present the objective function, which maximizes system profits, i.e., revenue less operation costs. We calculate the following parameters: (i) revenue as the product of net system generation, a power purchase agreement (PPA) price multiplier (dictated by market structure), and an assumed base PPA price of $100/MWh; and (ii) fixed and variable operation costs for different system operations, the former of which are associated with binary decisions (e.g., power cycle start-up), and the latter of which pertain to continuous variables (e.g., power cycle output). We provide a detailed description of costs, as well as the mathematical formulation of the objective function and all constraints, in previous work.[2]

System Design Optimization

We explore the system design space using different combinations of solar and energy storage technologies, i.e., CSP with TES, PV, and EES, without the onerous development of a tailored heuristic to determine system operations for each design. We implement derivative-free, "black-box" optimization algorithms to intelligently explore and narrow the search.

Variables

High-level design variables dictate overall system performance and financial feasibility. Table 1 presents the design variables, their corresponding lower and upper bounds, and the base case values for the two sub-systems, i.e., CSP with TES, and PV.

TABLE 1. Sub-system design variables with upper bound, lower bound, and base case values

Design Variable	Units	Lower Bound	Upper Bound	Base Case
Concentrating Solar Power (CSP) with Thermal Energy Storage (TES)				
Receiver Diameter	[m]	10	30	17
Receiver Height	[m]	10	30	22
Tower Height	[m]	100	300	195
Power Cycle Capacity	[MW_e]	150	200	175
Solar Multiple	[-]	0.5	3.0	2.5
Hours of TES	[hours]	2	20	10
Photovoltaics (PV)				
Field DC Capacity	[MW_{dc}]	50	300	100
DC-to-AC ratio	[-]	1.0	1.6	1.2

Objective Function

The objective function minimizes the sum of PPA price and the penalized deviation of the design's resulting maximum solar flux from an imposed flux limitation, shown below.

$$\min\left\{PPA\left(\mathcal{S}(\mathcal{D})\right) + P^{FLUX} * \left|FLUX^{MAX}(\mathcal{RF}) - FLUX^{LIM}\right|\right\} \tag{1}$$

where PPA price [US¢/kWh] is a function of the system simulation (denoted by $\mathcal{S}$), which is a function of our dispatch optimization model (denoted by $\mathcal{D}$), P^{FLUX} is the penalty to scale the deviation of maximum flux to an appropriate magnitude to match changes in PPA price, $FLUX^{MAX}$ is the maximum solar flux experienced by the receiver and is a function of the receiver and heliostat field design (denoted by $\mathcal{RF}$), and $FLUX^{LIM}$ is the maximum flux limit. By minimizing PPA price, we can investigate the effect of ToD pricing on optimal system design. We calculate PPA price by specifying a project's target internal rate of return of 11% at 20 years with a PPA price escalation rate of 1%/year (default values for PPA price calculation in SAM's PPA single owner, "utility," financial model).[1] We use a value of 0.001 for P^{FLUX} to scale maximum flux deviations above and below the maximum flux limit. Our maximum flux limit, $FLUX^{LIM}$, is 1000 kW/m^2. Above this value, the receiver's design would require expensive materials, which our cost models do not represent. We penalize deviations below $FLUX^{LIM}$ to avoid low-flux receiver designs, which would result in an undersized heliostat field for the corresponding receiver.

Solution Technique

Our design optimization problem is challenging, "black-box," and requires a derivative-free algorithm because there is no explicit mathematical formula for PPA price as a function of system design variables. We implement NLopt, an open-source library for nonlinear optimization,[5] that contains algorithms for either global or local searches utilizing derivative-free or gradient-based, approaches. NLopt enables us to quickly implement and test multiple algorithms simultaneously through the Python API.

We employ a two-phase method utilizing both global and local search algorithms to reach an optimal system design, shown in Fig. 2. In the first phase, we employ the following NLopt global-search, derivative-free algorithms to explore the design variable space and provide the "best" system design: DIRECT,[6] DIRECT-L,[7] CRS2-LM,[8] ISRES,[9] and ESCH,[10] where the "best" corresponds to the lowest objective function value. In the second phase, we employ NLopt's local-search, derivative-free algorithms, with the "best" design as an initial condition, to "polish" the optimal system design. We utilize the following local-search algorithms in the "polishing" phase: COBYLA,[11] BOBYQA,[12] NEWUOA,[13] PRAXIS,[14] NELDERMEAD,[15] and SBPLX.[16] We reduce computational time through parallelization. After the second phase, we select the best solution found across all the local algorithms as the "optimal" system design. Because of the nature of "black-box" optimization, there is no guarantee that a solution found by this approach is a global optimum. However, in this paper, we refer to the best solution found by the two-phase approach as the "optimal system design." For the first phase, we set the stopping criterion to the minimum of an objective function absolute tolerance of 1×10^{-4} and a five-day computational solve time limit. For the second phase, we change the limit on computation time to two days.

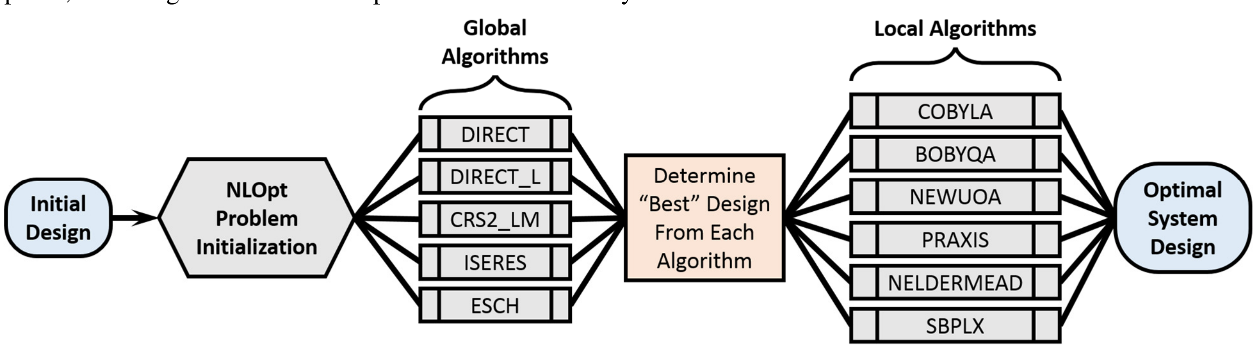

FIGURE 2. Schematic of the system design optimization methodology utilizing global and local search optimization algorithms within the open-source NLopt library.

Solar Hybrid Design Case Study

To investigate the effects that ToD schedules have on optimal system design, we conduct design optimization on a CSP-PV hybrid system operating within four different ToD pricing schedules at a fixed location (Rice, CA in the present analysis) using identical weather. Rice possesses high solar resource and abundant weather data with one-minute time fidelity for the years 1998 to 2017. For the system design, we assume that the PV sub-system configuration consists of single-axis-tracking PV modules, that the hybrid system lacks EES, and that the combined power output is constrained to an arbitrary interconnection capacity limit of 165 MW_{ac}.

The CSP-PV hybrid system operates within the following four ToD pricing schedules: Pacific Gas and Electric (PG&E) 2016 full capacity deliverability,[1] pool-price tariff,[17] two-tier tariff,[17] and California Independent System Operator (CAISO) price node data for Rice, CA in 2017,[18,19] shown in Fig. 3. The PG&E 2016 ToD schedule has three seasonal variations in pricing profile with the highest-valued periods occurring from July through September during a six-hour block between 3pm and 9pm. The pool-price ToD schedule is divided into two seasons, winter and the rest of the year, and includes a 3-hour morning and a 2-hour evening price peak. In addition, pool-price distinguishes between weekdays, Saturday, and Sunday pricing profiles. Figure 3b depicts the weekday profiles. Saturday's profile has a 5-hour morning and a 2-hour evening peak equal to the weekday "mid-day" price, i.e., 10am to 6pm; outside of peak pricing, the PPA multiplier value is equal to lowest price in the weekday profile. Sunday's profile is a constant value equal to the weekday's lowest price. The two-tier ToD schedule has no seasonal variation in pricing. The daily pricing schedule throughout the year is identical to that shown in Fig. 3c and is constructed of a base price (PPA multiplier equal to one) for the hours of 5am to 5pm and 9pm to 10pm, and a peak price (270% of the base price) occurring between 5pm to 9pm. Outside the base- and peak-price windows, the two-tier ToD schedule has a PPA multiplier equal to zero. Figure 3d depicts the CAISO-node hourly pricing data for every day in 2017 and the annual hourly average. The CAISO-node data has attributes of the three other ToD schedules, i.e., morning and evening peak prices and low prices in the middle of the day.

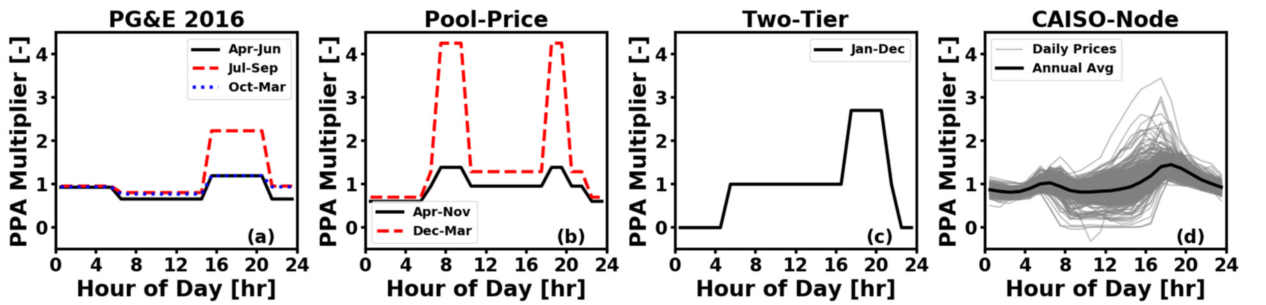

FIGURE 3. ToD pricing schedules for a) Pacific Gas and Electric (PG&E) 2016 full capacity deliverability, b) pool-price tariff, c) two-tier tariff, and d) CAISO-node price data for Rice, CA in 2017.

TABLE 2. The base case and optimal designs resulting PPA price, maximum flux, and objective function value for the four ToD pricing schedules.

ToD Pricing Schedules		PPA [$/MWh]	Maximum Flux [kW/m²]	Objective Value [US¢//kWh]
PG&E 2016	Base Case	97.31	1141	9.872
	Optimal	83.13	1001	8.313
	% Change	-14.6%	-12.3%	-15.8%
Pool-Price	Base Case	84.51	1141	8.592
	Optimal	66.71	1000	6.671
	% Change	-21.1%	-12.4%	-22.4%
Two-Tier	Base Case	76.04	1141	7.745
	Optimal	59.75	999	5.976
	% Change	-21.4%	-12.4%	-22.8%
CAISO-Node	Base Case	97.33	1141	9.874
	Optimal	82.30	1006	8.236
	% Change	-15.4	-11.8%	-16.6%

RESULTS

For the four ToD pricing schedules, Table 2 presents the PPA prices, maximum flux, and objective function values for the base case and optimal design. The base case's maximum flux exceeds the imposed limit of 1000 kW/m^2, making it an infeasible receiver and heliostat field combination. The 141 kW/m^2 flux over the maximum limit results in a 0.141 penalty to the objective function value. Due to the two terms in the objective function, NLopt's algorithms reduce the system's PPA price and converge the maximum flux to the imposed limit, simultaneously. Our methodology results in an improvement in the objective function value relative to the base case design of between 15.8% and 22.8%, depending on the ToD pricing schedule. Approximately 90% of this improvement is derived from decreasing the system's PPA price while the remainder results from reducing the penalized maximum flux deviation. Of the four ToD pricing schedules, two-tier and PG&E 2016 result in the lowest and highest optimal PPA prices at $59.8/MWh and $83.1/MWh, respectively.

Table 3 presents the optimal system design variable values for the four ToD pricing schedules. The optimal receiver aspect ratio, defined as the quotient of height and diameter, is about one for all ToD pricing schedules except for the pool-price schedule. The receiver and tower design variable values, i.e., tower height, receiver diameter, and receiver height, scale with the system's solar multiple. For all the ToD pricing schedules, cycle capacity is approximately at its imposed lower bound of 150 MW_e because the addition of the CSP sub-system increases the overall system's PPA price compared to a PV-only system, making it challenging to optimize CSP-PV hybrid designs using PPA price as the objective function without constraints on minimum power cycle sizing. The PG&E 2016 and CAISO-node schedules result in similar optimal designs and optimal solar multiples and hours of thermal energy storage of approximately 1.1 and 8.2, respectively. The pool-price and two-tier ToD schedules produce smaller optimal CSP designs, with solar multiples below unity, and approximately 5 hours of thermal energy storage. For the PV sub-system, the optimal design results in a PV capacity of approximately 255 MW_{dc} with a DC-to-AC ratio of 1.4. The PG&E 2016 pricing schedule yields a lower DC-to-AC ratio compared to the results of the other pricing schedules.

TABLE 3. Optimal system design variable values for the four ToD pricing schedules.

ToD Pricing Schedules	**Receiver Diameter** [m]	**Receiver Height** [m]	**Tower Height** [m]	**Cycle Capacity** [MW_e]	**Solar Multiple** [-]	**Hours of Storage** [hours]	**PV Capacity** [MW_{dc}]	**DC-to-AC Ratio** [-]
PG&E 2016	15.5	15.0	144	150.0	1.17	8.2	250	1.16
Pool-Price	10.2	12.1	108	150.1	0.58	4.9	266	1.49
Two-Tier	12.5	12.7	121	150.7	0.81	5.1	244	1.45
CAISO-Node	14.7	15.3	142	150.6	1.08	8.2	261	1.39

Table 4 presents the optimal system design's annual simulation metrics for the four ToD pricing schedules. Installed cost is the sum of direct and indirect costs of the proposed design and excludes any financing costs. We calculate installed cost using the default cost functions and parameter values within SAM's Photovoltaic (detailed) and CSP power tower molten salt modules.[1] The optimal designs corresponding to the PG&E 2016 and CAISO-node pricing result in higher installed cost than pool-price and two-tier because their design requires a larger CSP sub-system which, in turn, enables a higher system capacity factor due to higher annual energy generation. Because we constrain total system generation to remain below an interconnection capacity limit, we calculate capacity factor as the quotient of total energy generation and the product of the capacity limit and number of hours in a year. We define a 10% reliability level as the system's capacity factor for the top 10% highest-valued periods of the year, i.e., the highest-valued 876 hours. The 10% reliability level provides insight on how much energy generation is occurring during the highest-valued periods. All of the optimal designs have a 10% reliability level greater than 91%. We calculate the PV curtailment and clipping as the percentage of total PV DC electricity generation. The former is a result of the dispatch optimization model "taking" less DC power than available and the latter is a result of the PV DC generation exceeding the inverter's maximum DC input limit. By allowing for PV curtailment, we enable the NLopt optimization algorithms to evaluate the trade-off between increasing PV sub-system capacity, i.e., increasing PV annual generation, and curtailment during periods of high solar resource, i.e., spring and summer. The optimal design corresponding to the PG&E 2016 pricing produces no PV clipping because of the design's low DC-to-AC ratio. PV curtailment ranges from 1.2% to 4.6%, depending on the ToD pricing. Almost all of the PV curtailment is due to "shaving the peak" during the spring and summer months.

TABLE 4. Optimal system design's annual simulation metrics for the four ToD pricing schedules.

ToD Pricing Schedules	**Installed Cost** [$MM]	**Capacity Factor** [%]	**Total Generation** [GWh_e]	**10% Reliability** [%]	**PV Curtailment** [%]	**PV Clipping** [%]
PG&E 2016	861.6	62.4	901.4	95.3	4.60	0.00
Pool-Price	716.6	50.3	726.5	92.0	3.26	3.94
Two-Tier	745.7	52.4	757.1	97.1	1.23	2.94
CAISO-Node	855.5	61.8	892.5	91.0	4.57	1.62

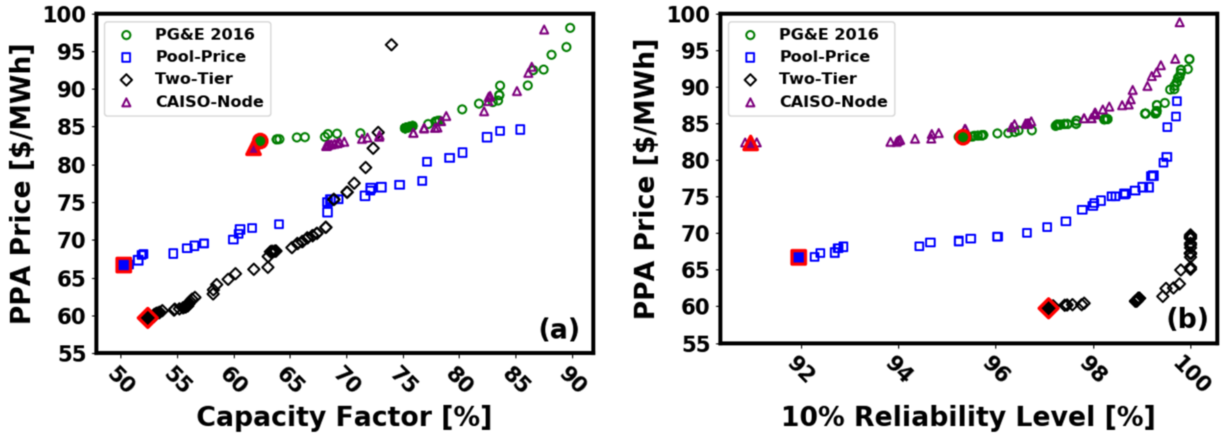

FIGURE 4. The "best-known" Pareto fronts of PPA price versus (a) capacity factor and (b) 10% reliability for the four ToD pricing structures. A red-outlined symbol distinguishes the optimal design presented in Table 3.

Figure 4 depicts the "best-known" Pareto fronts of PPA price versus (a) capacity factor and (b) 10% reliability for the four ToD pricing structures. A red-outlined symbol distinguishes the optimal design presented in Table 3. We generate these Pareto fronts by compiling a set of all explored designs and their performance during the two phases and across all algorithms. Then, we disregard any designs that exceed the imposed maximum flux limit. From this subset, we determine all the non-dominated points for the metrics of interest, shown in Fig. 4.

From these Pareto fronts, we quantify the trade-offs between CSP-PV hybrid system's PPA price versus capacity factor and 10% reliability level for the four ToD pricing structures. From Fig. 4a, the capacity factor Pareto fronts for PG&E 2016 and CAISO-node pricing have the smallest slope near the optimal designs, meaning that the system's capacity factor can increase with a small impact on PPA price. For example, a CSP-PV hybrid system in the PG&E 2016 pricing can achieve a capacity factor of 75% (20% increase) with an adverse effect on PPA price of $1.66/MWh (2% decrease) compared to the optimal design. We observe a similar result for CAISO-node pricing. However, this asset is not present in the pool-price and two-tier structures, in which the slope of the capacity factor Pareto front near the optimal design is greater. The two-tier pricing results in the most dramatic trade-off between capacity factor and PPA price. Due to the zero-valued PPA multiplier from 10pm to 5am, the two-tier pricing results in a steep increase in PPA price for capacity factors above 68% because it requires power generation during time periods with a zero PPA multiplier, i.e., resulting in no revenue. Figure 4b shows that the 10% reliability level Pareto fronts have a slope close to zero near the optimal design, meaning that CSP-PV hybrids can achieve higher reliabilities, up to about 97%, with a small increase in PPA price. For example, a system within the CAISO-node pricing can increase its 10% reliability level from 91% to 95% with a $1.88/MWh increase in PPA price.

CONCLUSION

This paper develops and implements a methodology to determine optimal designs for CSP-PV hybrids using derivative-free algorithms within the NLopt library. We evaluate CSP-PV hybrid designs using NREL's SAM utilizing dispatch optimization for operational decisions. This paper investigates optimal CSP-PV hybrid designs dispatching against four ToD pricing structures. The results indicate that NLopt's derivative-free algorithms can improve the base case PPA price by 15% to 21%, depending on the ToD pricing structure. We present the optimal CSP-PV hybrid designs, which tend to have small CSP sub-systems with large PV sub-systems. In addition, we present the optimal CSP-PV hybrid design's annual performance metrics, which tend to have low capacity factors -- from 50% to 62%, but high 10% reliability levels of greater than 90%. Lastly, we investigate the trade-offs between capacity factor and PPA price using Pareto fronts and demonstrate that, for some ToD pricing structures, system capacity factor can increase by 20% with less than a 2% increase in PPA price.

ACKNOWLEDGMENTS

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REFERENCES

1. National Renewable Energy Laboratory (NREL), "System Advisor Model (SAM) Version 2018.11.11," Golden, CO, 2018.
2. W. T. Hamilton, M. A. Husted, A. M. Newman, R. J. Braun and M. J. Wagner, "Dispatch Optimization of Concentrating Solar Power with Utility-Scale Photovoltaics," Optimization and Engineering, 2019. (Accepted)
3. AMPL Optimization LLC, "A Mathematical Programming Language (AMPL) Version 10.6.16," 2009.
4. IBM, "IBM ILOG CPLEX Optimization Studio: CPLEX User's Manual," 2016.
5. S. G. Johnson, "The NLopt nonlinear-optimization package," [Online]. Available: http://github.com/stevengj/nlopt. [Accessed 14 May 2019].
6. D. R. Jones, C. D. Perttunen and B. E. Stuckman, "Lipschitzian optimization without the Lipschitz constant," Journal of Optimization Theory and Applications, vol. 79, no. 1, pp. 157-181, 1993.
7. J. M. Gablonsky and C. T. Kelley, "A Locally-Biased form of the DIRECT Algorithm," Journal of Global Optimization, vol. 21, no. 1, pp. 27-37, 2001.
8. P. Kaelo and M. M. Ali, "Some Variants of the Controlled Random Search Algorithm for Global Optimization," Journal of Optimization Theory and Applications, vol. 130, no. 2, pp. 253-264, 2006.
9. T. P. Runarsson and X. Yao, "Search Biases in Constrained Evolutionary Optimization," IEEE Trans. on Systems, Man, and Cybernetics Part C: Applications and Reveiws, vol. 35, no. 2, p. 233–243, 2005.
10. C. H. da Silva Santos, M. S. Goncalves and H. E. Hernandez-Figueroa, "Designing Novel Photonic Devices by Bio-Inspired Computing," IEEE Photonics Technology Letters , vol. 22, no. 15, pp. 1177-1179, 2010.
11. M. J. D. Powell, "A Direct Search Optimization Method That Models the Objective and Constraint Functions by Linear Interpolation," in Advances in Optimization and Numerical Analysis, Dordrecht, Netherlands, Springer, 1994, pp. 51-67.
12. M. J. D. Powell, "The BOBYQA algorithm for bound constrained optimization without derivatives," Department of Applied Mathematics and Theoretical Physics, Cambridge England, 2009.
13. M. J. D. Powell, "The NEWUOA software for unconstrained optimization without derivatives,," in Proc. 40th Workshop on Large Scale Nonlinear Optimization , Erice, Italy, 2004.
14. R. Brent, Algorithms for Minimization without Derivatives, Prentice-Hall, 1972.
15. J. A. Nelder and R. L. Mead, "A Simplex Method for Function Minimization," The Computer Journal, vol. 7, pp. 308-313, 1965.
16. T. Rowman, Functional Stability Analysis of Numerical Algorithms, Ph.D. thesis, Department of Computer Sciences, University of Texas at Austin, 1990.
17. R. Guedez, M. Topel, I. C. Buezas, F. Ferragut, I. Callaba, J. Spelling, Z. Hassar, C. D. Perez-Segarra and B. Laumert, "A Methodology for Determining Optimum Solar Tower Plant Configurations and Operating Strategies to Maximize Profits Based on Hourly Electricity Market Prices and Tariffs," in ASME 2015 9th International Conference on Energy Sustainability, San Diego, California, 2015.
18. M. J. Wagner, W. T. Hamilton, A. M. Newman, J. Dent, C. Diep and R. J. Braun, "Optimizing Dispatch for a Concentrated Solar Power Tower," Solar Energy 174, pp. 1198-1211, 2018.
19. California ISO, "Today's Outlook," California Independent System Operator, 2019. [Online]. Available: http://www.caiso.com/TodaysOutlook/Pages/prices.aspx.