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Real-Time Dispatch Optimization for Concentrating Solar Power Plants with Thermal Energy Storage

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Abstract **Keywords**

1 Introduction

Wagner et al. (2017) develop a model that seeks a concentrating solar power (CSP) central receiver plant design with thermal energy storage, in which the sum of capital, operations, and maintenance costs are minimized. Hamilton et al. (2019) extend the model to include photovoltaics and batteries in the plant design; further, they include additional ramping constraints and solve instances of the model in which decisions are made every 10 minutes for the first day, and hourly thereafter. As part of the current project, our objective is to develop a model in which decisions may be made at the same frequency that operators currently do, using the weather forecasting data they have available as input. The model we develop will capture the transient effects of off-design operations on the plant's productivity and reliability in the short term. Further, we will include constraints on minimum uptime and downtime, and further refine the ramping constraints proposed in Hamilton et al. (2019), so that physical limits previously not required in an hourly model are adhered to. Finally, we adapt the model so that the clock time between consecutive decisions may vary over time; we note that this is a generalization of the work previously done in Hamilton et al. (2019) to include two different step lengths.

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2 Dispatch Optimization Model

This section describes the optimization model, which is largely taken from **?**, but for removing the PV and battery systems from consideration. Parameters and variables that contain the subscript of period t indicate their time-varying nature. In general, upper-case letters denote parameters while lower-case letters are reserved for variables. We also use lower-case letters for indices and upper-case script letters for sets.

2.1 Notation

The following MILP, $(\mathcal{H})$, requires the initial operational state of the system, the PV field and receiver energy generation forecasts, the expected cycle conversion efficiency profile as a function of ambient temperature and thermal input, and the energy price or tariff profile (Table 1).

Table 1: Hybrid dispatch model, $(\mathcal{H})$, sets and parameters.

Indices and Sets

$t \in \mathcal{T}$	Set of all time periods in the time horizon

Time-indexed Parameters

Δ_t	Duration of period t [h]
Δ_t^e	Cumulative time elapsed at end of period t; i.e., $\Delta_t^e = \sum_{t'=1}^{t} \Delta_{t'}$ [h]
Δ_t^{rs}	Estimated fraction of period t required for receiver start-up [-]
η_t^{amb}	Cycle efficiency ambient temperature adjustment factor in period t [-]
η_t^c	Normalized condenser parasitic loss in period t [-]
P_t	Electricity sales price in period t [$/kWh$_\mathsf{e}$]
Q_t^{in}	Available thermal power generated by the CSP heliostat field in period t [kW$_\mathsf{t}$]
Q_t^c	Allowable power per period for cycle start-up in period t [kW$_\mathsf{t}$]
W_t^{net}	Net grid transmission upper limit in period t [kW$_\mathsf{e}$]
D_t	Time-weighted discount factor in period t [-]
$W_t^{u^+}$	Maximum power production when starting generation in period t [kW$_\mathsf{e}$]
$W_t^{u^-}$	Maximum power production in period t when stopping generation in period $t+1$ [kW$_\mathsf{e}$]

Cost Parameters

C^{rec}	Operating cost of heliostat field and receiver [$/kWh$_\mathsf{t}$]
C^{rsu}	Penalty for receiver cold start-up [$/start]
C^{rhsp}	Penalty for receiver hot start-up [$/start]
C^{pc}	Operating cost of power cycle [$/kWh$_\mathsf{e}$]
C^{csu}	Penalty for power cycle cold start-up [$/start]
C^{chsp}	Penalty for power cycle hot start-up [$/start]
$C^{\delta W}$	Penalty for change in power cycle production [$/$\Delta$kW$_\mathsf{e}$]
C^{vW}	Penalty for change in power cycle production beyond designed limits [$/$\Delta$kW$_\mathsf{e}$]
C^{csb}	Operating cost of power cycle standby operation [$/kWh$_\mathsf{t}$]

CSP Field and Receiver Parameters

Δ^l	Minimum time to start the receiver [hr]
E^{hs}	Heliostat field startup or shut down parasitic loss [$\mathrm{kWh_e}$]
E^r	Required energy expended to start receiver [$\mathrm{kWh_t}$]
E^u	Thermal energy storage capacity [$\mathrm{kWh_t}$]
L^r	Receiver pumping power per unit power produced [$\mathrm{kW_e/kW_t}$]
Q^{rl}	Minimum operational thermal power delivered by receiver [$\mathrm{kWh_t}$]
Q^{rsb}	Required thermal power for receiver standby [$\mathrm{kWh_t}$]
Q^{rsd}	Required thermal power for receiver shut down [$\mathrm{kWh_t}$]
Q^{ru}	Allowable power per period for receiver start-up [$\mathrm{kWh_t}$]
W^h	Heliostat field tracking parasitic loss [$\mathrm{kW_e}$]
W^{ht}	Tower piping heat trace parasitic loss [$\mathrm{kW_e}$]

Power Cycle Parameters

E^c	Required energy expended to start cycle [$\mathrm{kWh_t}$]
η^{des}	Cycle nominal efficiency [-]
η^p	Slope of linear approximation of power cycle performance curve [$\mathrm{kW_e/kW_t}$]
L^c	Cycle heat transfer fluid pumping power per unit energy expended [$\mathrm{kW_e/kW_t}$]
Q^b	Cycle standby thermal power consumption [$\mathrm{kW_t}$]
Q^l	Minimum operational thermal power input to cycle [$\mathrm{kW_t}$]
Q^u	Cycle thermal power capacity [$\mathrm{kW_t}$]
W^b	Power cycle standby operation parasitic load [$\mathrm{kW_e}$]
$\dot{W}^l$	Minimum cycle electric power output [$\mathrm{kW_e}$]
$\dot{W}^u$	Cycle electric power rated capacity [$\mathrm{kW_e}$]
$\dot{W}^{\delta^+}$	Power cycle ramp-up designed limit [$\mathrm{kW_e/h}$]
$\dot{W}^{\delta^-}$	Power cycle ramp-down designed limit [$\mathrm{kW_e/h}$]
$\dot{W}^{v^+}$	Power cycle ramp-up violation limit [$\mathrm{kW_e/h}$]
$\dot{W}^{v^-}$	Power cycle ramp-down violation limit [$\mathrm{kW_e/h}$]
Y^u	Minimum required power cycle uptime [h]
Y^d	Minimum required power cycle downtime [h]

Initial Condition Parameters

s_0	Initial TES reserve quantity [$\mathrm{kWh_t}$]
u_0^{csu}	Initial cycle start-up energy inventory [$\mathrm{kWh_t}$]
u_0^{rsu}	Initial receiver start-up energy inventory [$\mathrm{kWh_t}$]
$\dot{w}_0$	Initial power cycle electricity generation [$\mathrm{kW_e}$]
y_0^r	1 if receiver is generating "usable" thermal power initially; 0 otherwise
y_0^{rsb}	1 if receiver is in standby mode initially; 0 otherwise
y_0^{rsu}	1 if receiver is starting up initially; 0 otherwise
y_0	1 if cycle is generating electric power initially; 0 otherwise
y_0^{csb}	1 if cycle is in standby mode initially; 0 otherwise
y_0^{csu}	1 if cycle is starting up initially; 0 otherwise
$\hat{y}_0^{cgb}$	duration that cycle has been generating electric power [h]
$\hat{y}_0^{cge}$	duration that cycle has not been generating power (i.e., shut down or in standby mode) [h]

Miscellaneous Parameters

α	Conversion factor between unitless and monetary values [$]
$\mathbb{M}$	Sufficiently large number [-]
ϵ	Sufficiently small number [-]

The variables (see Table 2) describe energy (thermal $\mathrm{kWh_t}$ or electric $\mathrm{kWh_e}$) states and power flows (thermal $\mathrm{kW_t}$ or electric $\mathrm{kW_e}$) in the system. Continuous variables "x," "$\dot{w}$," "u," and "s" representing power and energy related to the receiver, power cycle, PV field, battery, and TES. Binary variables "y" enforce operational modes and sequencing such that start-up must occur before normal operation, for example.

Table 2: Variables used in ($\mathcal{H}$).

Continuous

s_t	TES reserve quantity at period t [$\mathrm{kWh_t}$]
u_t^{csu}	Cycle start-up energy inventory at period t [$\mathrm{kWh_t}$]
u_t^{rsu}	Receiver start-up energy inventory at period t [$\mathrm{kWh_t}$]
$\dot{w}_t$	Power cycle electricity generation at period t [$\mathrm{kW_e}$]
$\dot{w}_t^{\delta+}$	Power cycle ramp-up in period t [$\mathrm{kW_e}$]
$\dot{w}_t^{\delta-}$	Power cycle ramp-down in period t [$\mathrm{kW_e}$]
$\dot{w}_t^{v+}$	Power cycle ramp-up beyond designed limit in period t [$\mathrm{kW_e}$]
$\dot{w}_t^{v-}$	Power cycle ramp-down beyond designed limit in period t [$\mathrm{kW_e}$]
$\dot{w}_t^{s}$	Electrical power sold to the grid at period t [$\mathrm{kW_e}$]
$\dot{w}_t^{p}$	Electrical power purchased from the grid at period t [$\mathrm{kW_e}$]
x_t	Cycle thermal power utilization at period t [$\mathrm{kW_t}$]
x_t^r	Thermal power delivered by the receiver at period t [$\mathrm{kW_t}$]
x_t^{rsu}	Receiver start-up power consumption at period t [$\mathrm{kW_t}$]

Binary

y_t^r	1 if receiver is generating "usable" thermal power at period t; 0 otherwise
y_t^{rhsp}	1 if receiver hot start-up penalty is incurred at period t (from standby); 0 otherwise
y_t^{rsb}	1 if receiver is in standby mode at period t; 0 otherwise
y_t^{rsd}	1 if receiver is shut down at period t; 0 otherwise
y_t^{rsu}	1 if receiver is starting up at period t; 0 otherwise
y_t^{rsup}	1 if receiver cold start-up penalty is incurred at period t (from off); 0 otherwise
y_t	1 if cycle is generating electric power at period t; 0 otherwise
y_t^{chsp}	1 if cycle hot start-up penalty is incurred at period t (from standby); 0 otherwise
y_t^{csb}	1 if cycle is in standby mode at period t; 0 otherwise
y_t^{csd}	1 if cycle is shutting down at period t; 0 otherwise
y_t^{csu}	1 if cycle is starting up at period t; 0 otherwise
y_t^{csup}	1 if cycle cold start-up penalty is incurred at period t (from off); 0 otherwise
y_t^{cgb}	1 if cycle begins electric power generation at period t; 0 otherwise

y_t^{cge} 1 if cycle stops electric power generation at period t; 0 otherwise

2.2 Objective Function and Constraints

We maximize the sale of electricity given as the revenue based on sales minus the cost of grid purchase throughout the time horizon in question. We decrement the revenue by penalties incurred for start-ups and shut-downs, changes in production between successive time periods, and operating costs related to the power cycle and the receiver. Lesser penalties are introduced to enforce the logic associated with the receiver and power cycle shut down. The costs and revenues in each period are weighted according to a discount factor as a function of time elapsed; specifically, we adopt the exponential factor used in Hamilton et al. (2019).

$$
\begin{aligned}
(\mathcal{H}) \text{ maximize } \sum_{t\in\mathcal{T}} \Big[& \Delta_t \cdot P_t(\dot{w}_t^s - \dot{w}_t^p) \\
& - D_t(C^{csu} y_t^{csup} + C^{chsp} y_t^{chsp} + \alpha y_t^{csd}) \\
& - D_t \cdot \Delta_t (C^{\delta W}(\dot{w}_t^{\delta^+} + \dot{w}_t^{\delta^-}) + C^{vW}(\dot{w}_t^{v^+} + \dot{w}_t^{v^-})) \\
& - D_t \big(C^{rsu} y_t^{rsup} + C^{rhsp} y_t^{rhsp} + \alpha y_t^{rsd}\big) \\
& - D_t \cdot \Delta_t \big(C^{pc} \dot{w}_t + C^{csb} Q^b y_t^{csb} + C^{rec} x_t^r\big)\Big]
\end{aligned} \tag{1}
$$

2.2.1 Receiver Operations

We include the following constraints that govern receiver operations:

Receiver Start-up

$$u_t^{rsu} \le u_{t-1}^{rsu} + \Delta_t x_t^{rsu} \quad \forall\, t \in \mathcal{T} : t \ge 2 \tag{2a}$$
$$u_t^{rsu} \le E^r y_t^{rsu} \quad \forall\, t \in \mathcal{T} \tag{2b}$$
$$y_t^r \le \frac{u_t^{rsu}}{E^r} + y_{t-1}^r + y_{t-1}^{rsb} \quad \forall\, t \in \mathcal{T} : t \ge 2 \tag{2c}$$
$$y_t^{rsu} + y_{t-1}^r \le 1 \quad \forall\, t \in \mathcal{T} : t \ge 2 \tag{2d}$$
$$x_t^{rsu} \le Q^{ru} y_t^{rsu} \quad \forall\, t \in \mathcal{T} \tag{2e}$$
$$y_t^{rsu} \le \frac{Q_t^{in}}{Q^{rl}} \quad \forall\, t \in \mathcal{T} \tag{2f}$$

Receiver Supply and Demand

$$x_t^r + x_t^{rsu} + Q^{rsd} y_t^{rsd} \le Q_t^{in} \quad \forall\, t \in \mathcal{T} \tag{3a}$$
$$x_t^r \le Q_t^{in} y_t^r \quad \forall\, t \in \mathcal{T} \tag{3b}$$
$$x_t^r \ge Q^{rl} y_t^r \quad \forall\, t \in \mathcal{T} \tag{3c}$$
$$y_t^r \le \frac{Q_t^{in}}{Q^{rl}} \quad \forall\, t \in \mathcal{T} \tag{3d}$$

Logic Associated with Receiver Modes

$$y_t^{rsu} + y_t^{rsb} \le 1 \quad \forall\, t \in \mathcal{T} \tag{4a}$$

$$y_t^r + y_t^{rsb} \leq 1 \quad \forall\, t \in \mathcal{T} \tag{4b}$$
$$y_t^{rsb} \leq y_{t-1}^r + y_{t-1}^{rsb} \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{4c}$$
$$y_t^{rsup} \geq y_t^{rsu} - y_{t-1}^{rsu} \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{4d}$$
$$y_t^{rhsp} \geq y_t^r - (1 - y_{t-1}^{rsb}) \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{4e}$$
$$y_t^{rsd} \geq (y_{t-1}^r - y_t^r) + (y_{t-1}^{rsb} - y_t^{rsb}) \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{4f}$$

In order for the system to generate power, we impose Constraint (2a) which accounts for receiver start-up energy "inventory"; we employ an inequality such that inventory can revert to a level of zero in time periods after which start-up has completed. Inventory is allowed to assume a positive value during time periods of receiver start-up (Constraint (2b)). Power production is positive only upon completion of a start-up or if the receiver also operates in the time period prior (Constraint (2c)). If the receiver is producing thermal power in time $t-1$, it cannot be starting up in the following time period t (Constraint (2d)). Ramp-rate limits must be honored during the start-up procedure (Constraint (2e)). Trivial solar resource prevents receiver start-up (Constraint (2f)).

The parameter Q_t^{in} provides an upper bound on the thermal power produced by the receiver, from which any energy used for start-up or shutdown detracts (Constraint (3a)). Constraint (3b) permits the receiver to generate thermal power only while in power-producing mode. Receiver thermal power generation is subject to a lower bound by Constraint (3c) owing to the minimum turndown of molten salt pumps. In the absence of thermal power, the receiver is not able to operate (Constraint (3d)).

Standby mode allows molten salt to circulate between the TES tanks and the receiver such that a restart can occur quickly; such a restart incurs a smaller financial penalty. Constraints (4a) and (4b) preclude (i) standby and start-up modes and (ii) standby and power-producing modes from coinciding. Standby mode can only occur in time periods directly after which the receiver was either in standby or power-producing mode (Constraint (4c)). Specifically through the use of the start-up variables y_t^{rsup} and y_t^{rhsp} (as opposed to y_t^{rsu} which affords multiple time periods of start-up but therefore does not enforce penalty logic), Constraints (4d) and (4e) ensure that penalties for receiver start-up from an off or from a standby state are incurred, respectively. Constraint (4f) implements shut-down logic related to power-producing or standby states. Constraints (11a)-(11f) and (11g) enforce domain requirements on the variables except for x_t^r whose non-negativity is ensured via Constraints (3c).

2.2.2 Power Cycle Operations

Power cycle operation constraints are similar to those of receiver operations:

Cycle Start-up

$$u_t^{csu} \leq u_{t-1}^{csu} + \Delta_t Q_t^c y_t^{csu} \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{5a}$$
$$u_t^{csu} \leq E^c y_t^{csu} \quad \forall\, t \in \mathcal{T} \tag{5b}$$
$$y_t \leq \frac{u_{t-1}^{csu}}{E^c} + y_{t-1} + y_{t-1}^{csb} \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{5c}$$

$$x_t \leq Q^u y_t \quad \forall\, t \in \mathcal{T} \tag{5d}$$
$$x_t \geq Q^l y_t \quad \forall\, t \in \mathcal{T} \tag{5e}$$

Cycle Supply and Demand

$$\dot{w}_t = \frac{\eta_t^{amb}}{\eta^{des}}[\eta^p x_t + (\dot{W}^u - \eta^p Q^u) y_t] \quad \forall\, t \in \mathcal{T} \tag{6a}$$
$$\dot{w}_t \leq \dot{W}^u \left(\frac{\eta_t^{amb}}{\eta^{des}}\right) y_t \quad \forall\, t \in \mathcal{T} \tag{6b}$$
$$\dot{w}_t \geq \dot{W}^l \left(\frac{\eta_t^{amb}}{\eta^{des}}\right) y_t \quad \forall\, t \in \mathcal{T} \tag{6c}$$
$$\dot{w}_t^{\delta^+} \geq \dot{w}_t - \dot{w}_{t-1} \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{6d}$$
$$\dot{w}_t^{\delta^-} \geq \dot{w}_{t-1} - \dot{w}_t \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{6e}$$
$$\begin{aligned} \dot{w}_t^{\delta^+} - \dot{w}_t^{v^+} &\leq \Delta_t \dot{W}^{\delta^+} \\ &+ \left(\frac{\eta_t^{amb}}{\eta^{des}} W_t^{u^+} - \Delta_t \dot{W}^{\delta^+}\right) y_t^{cgb} \quad \forall\, t \in \mathcal{T} \end{aligned} \tag{6f}$$
$$\begin{aligned} \dot{w}_t^{\delta^-} - \dot{w}_t^{v^-} &\leq \Delta_t \dot{W}^{\delta^-} \\ &+ \left(\frac{\eta_t^{amb}}{\eta^{des}} W_t^{u^-} - \Delta_t \dot{W}^{\delta^-}\right) y_t^{cge} \quad \forall\, t \in \mathcal{T} \end{aligned} \tag{6g}$$
$$\dot{w}_t^s \leq W_t^{net} \quad \forall\, t \in \mathcal{T} \tag{6h}$$
$$\begin{aligned} \dot{w}_t^s - \dot{w}_t^p = (1 - \eta_t^c)\dot{w}_t &- L^r(x_t^r + x_t^{rsu} + Q^{rl} y_t^{rsb}) \\ &- L^c x_t - W^h y_t^r - W^b y_t^{csb} - W^{ht}(y_t^{rsu} + y_t^{rsb}) \\ &- \frac{E^{hs}}{\Delta_t}\left(y_t^{rsu} + y_t^{rsb} + y_t^{rsd}\right) \qquad \forall\, t \in \mathcal{T} \end{aligned} \tag{6i}$$

Minimum Cycle Uptime and Downtime

$$\sum_{t' \in \mathcal{T} : 0 \leq (\Delta_t^e - \Delta_{t'}^e) < Y^u} y_{t'}^{cgb} \leq y_t \quad \forall\, t \in \mathcal{T} : \Delta_t^e \geq Y^u \tag{7a}$$
$$\sum_{t' \in \mathcal{T} : 0 \leq (\Delta_t^e - \Delta_{t'}^e) < Y^d} y_{t'}^{cge} \leq (1 - y_t) \quad \forall\, t \in \mathcal{T} : \Delta_t^e \geq Y^d \tag{7b}$$
$$y_t^{cgb} - y_t^{cge} = y_t - y_{t-1} \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{7c}$$

Logic Governing Cycle Modes

$$y_t^{csu} + y_{t-1} \leq 1 \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{8a}$$
$$y_t + y_t^{csu} \leq 1 \quad \forall\, t \in \mathcal{T} \tag{8b}$$
$$y_t^{csb} \leq y_{t-1} + y_{t-1}^{csb} \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{8c}$$
$$y_t^{csu} + y_t^{csb} \leq 1 \quad \forall\, t \in \mathcal{T} \tag{8d}$$
$$y_t + y_t^{csb} \leq 1 \quad \forall\, t \in \mathcal{T} \tag{8e}$$
$$y_t^{chsp} \geq y_t - (1 - y_{t-1}^{csb}) \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{8f}$$

$$y_t^{csd} \geq y_{t-1} - y_t + y_{t-1}^{csb} - y_t^{csb} \quad \forall\ t \in \mathcal{T} : t \geq 2 \tag{8g}$$

$$y_t^{csup} \geq y_t^{csu} - y_{t-1}^{csu} \quad \forall\ t \in \mathcal{T} : t \geq 2 \tag{8h}$$

Constraint (5a) accounts for start-up energy "inventory," which can only be positive during time periods in which the cycle is starting up (Constraint (5b)). Normal cycle operation can occur upon completion of start-up energy requirements, if the cycle is operating normally, or directly after stand-by mode (Constraint (5c)). Constraint (5d) and Constraint (5e) form the upper and lower bounds on the heat input to the power cycle, respectively.

The relationship between electrical power and cycle heat input is modeled as a linear function with corrections for ambient temperature effects (Constraint (6a)). Constraints (6b)-(6c) restrict the power cycle's electric power output to within its designed lower and upper bounds, respectively, during periods in which it is operating, and zero otherwise. Constraints (6d) through (6g) limit the change in the production of electrical power over time, or ramp rate, to designed bounds, subject to a penalty for excessive ramping. Separate limits are imposed during periods in which the power cycle is starting or stopping power generation, given by the parameters $W_t^{u^+}$ and $W_t^{u^-}$, respectively. Excessive ramping up and down of the power cycle in each period t is reconciled via the variables $\dot{w}_t^{v^+}$ and $\dot{w}_t^{v^+}$, respectively. Constraint (6h) limits the grid transmission for net power production. Positive and negative power flow (corresponding to sold and purchased electricity, respectively) is determined by a power balance on the AC bus of the hybrid system (Constraint (6i)). The right-hand side of Constraint (6i) consists of the following terms, in the order in which they appear: (i) power cycle generation less condenser parasitic power, (ii) TES pumping power requirements for receiver operations, (iii) TES pumping power requirements for cycle operations, (iv) heliostat tracking power, (v) power cycle standby parasitic power, (vi) tower piping heat trace for receiver start-up, and (vii) heliostat field stow power for different receiver operations.

Constraints (7a)-(7b) enforce minimum uptime and downtime for the cycle. These constraints are used commonly in unit commitment problems (see, e.g., Kim et al., 2018; Morales-España et al., 2012), and Rajan et al. (2005) and Ostrowski et al. (2011) show that these constraints are computationally efficient when compared to alternative formulations. Constraint (7c) tracks the start-up and shutdown of plants, first developed by Garver (1962); the variables y_t^{cgb} and y_t^{cge} may be continuous when the operating variable, y_t, is binary, $t \in \mathcal{T}$.

Constraint (8a) precludes power cycle start-up in consecutive time periods of power-producing operation. Constraint (8b) precludes power cycle start-up and operation from coinciding. The cycle-standby-mode constraint (Constraint (8c)) is analogous to that for the receiver. Standby and start-up modes cannot simultaneously occur (Constraint (8d)); this also holds for standby and power-producing modes (Constraint (8e)). Constraints (8f) and (8g) implement the following logic, respectively: (i) starting up after standby and (ii) shutting down after producing power or standing by. Penalties incurred from a cycle cold start are incurred via Constraint (8h). Constraints (11a)-(11f) and (11h) enforce domain requirements on the variables except for x_t whose non-negativity is ensured via Constraint (5e).

2.2.3 TES Energy Balance

The system's energetic state implies power terms that can assume either sign; the thermal storage charge state (s_t) reconciles their difference. We therefore impose some additional constraints with respect to TES state of charge:

TES State of Charge

$$s_t - s_{t-1} = \Delta_t[x^r_t - (Q^c_t y^{csu}_t + Q^b y^{csb}_t + x_t + Q^{rsb} y^{rsb}_t)] \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{9a}$$

$$s_t \leq E^u \quad \forall\, t \in \mathcal{T} \tag{9b}$$

$$s_{t-1} \geq \Delta_t \cdot \Delta^{rs}_t [(Q^u + Q^b) \cdot (-3 + y^{rsu}_t + y_{t-1} + y_t + y^{csb}_{t-1} + y^{csb}_t) + x_t + Q^b y^{csb}_t] \quad \forall\, t \in \mathcal{T} : t \geq 2 \tag{9c}$$

Constraint (9a) balances energy to and from TES with the charge; a time-scaling parameter, Δ, reconciles power and energy. Constraint (9b) imposes the upper bound to TES charge state. If the power cycle is operating or standing by in time period $t-1$ or t, and if the receiver is starting up in period t, then there must be a sufficient charge level in the TES in time $t-1$ to ensure that the receiver can operate through its start-up period (Constraint (9c)). The expected fraction of a time period used for receiver start-up is given by (10), if applicable.

$$\Delta^{rs}_t = \min\left\{1, \max\left\{\Delta^l, \frac{E^c}{\max\left\{\epsilon, Q^{in}_{t+1}\Delta_t\right\}}\right\}\right\} \tag{10}$$

Constraints (9a)-(9c) measure TES state of charge via energy flow. (Introducing energy quality as a function of the molten salt temperature yields a non-linearity that, at the time of this writing, is a level of detail not worth the extra computational effort.)

2.2.4 Variable Bounds

Variable bounds are enforced in (11a)-(11h):

$$u^{csu}_t,\ u^{rsu}_t,\ \dot{w}_t,\ \dot{w}^p_t,\ \dot{w}^s_t,\ x^r_t,\ x^{rsu}_t \geq 0 \ \ \forall\, t \in \mathcal{T} \tag{11a}$$

$$0 \leq \dot{w}^{v^+}_t \leq \dot{W}^{v^+} \quad \forall\, t \in \mathcal{T} \tag{11b}$$

$$0 \leq \dot{w}^{v^-}_t \leq \dot{W}^{v^-} \quad \forall\, t \in \mathcal{T} \tag{11c}$$

$$0 \leq y^{cgb}_t \leq 1 \ \ \forall t \in \mathcal{T} \tag{11d}$$

$$0 \leq y^{cge}_t \leq 1 \ \ \forall t \in \mathcal{T} \tag{11e}$$

$$s_t, \dot{w}^{\delta^+}_t, \dot{w}^{\delta^-}_t, \dot{w}^{v^+}_t, \dot{w}^{v^-}_t, x_t \geq 0 \ \ \forall\, t \in \mathcal{T} \tag{11f}$$

$$y^r_t,\ y^{rhsp}_t,\ y^{rsb}_t,\ y^{rsd}_t,\ y^{rsu}_t,\ y^{rsup}_t \in \{0,1\} \ \ \forall\, t \in \mathcal{T} \tag{11g}$$

$$y_t,\ y^{chsp}_t,\ y^{csb}_t,\ y^{csd}_t, y^{csu}_t,\ y^{csup}_t \ \in \{0,1\} \ \ \forall\, t \in \mathcal{T}. \tag{11h}$$

2.2.5 Initial Conditions

Receiver startup constraints (12a), (12b), and (12c) below correspond to constraints (2a), (2c), and (2d), respectively:

$$u_t^{rsu} \leq u_0^{rsu} + \Delta_t x_t^{rsu} \quad \forall\, t \in \mathcal{T} : t = 1 \tag{12a}$$
$$y_t^r \leq \frac{u_t^{rsu}}{E^r} + y_0^r + y_0^{rsb} \quad \forall\, t \in \mathcal{T} : t = 1 \tag{12b}$$
$$y_t^{rsu} + y_0^r \leq 1 \quad \forall\, t \in \mathcal{T} : t = 1. \tag{12c}$$

Receiver logic constraints (12d)-(12g) below correspond to constraints (4c)-(4f):

$$y_t^{rsb} \leq y_0^r + y_0^{rsb} \quad \forall\, t \in \mathcal{T} : t = 1 \tag{12d}$$
$$y_t^{rsup} \geq y_t^{rsu} - y_0^{rsu} \quad \forall\, t \in \mathcal{T} : t = 1 \tag{12e}$$
$$y_t^{rhsp} \geq y_t^r - (1 - y_0^{rsb}) \quad \forall\, t \in \mathcal{T} : t = 1 \tag{12f}$$
$$y_t^{rsd} \geq (y_0^r - y_t^r) + (y_0^{rsb} - y_t^{rsb}) \quad \forall\, t \in \mathcal{T} : t = 1. \tag{12g}$$

Cycle startup constraints (12h) and (12i) correspond to constraints (5a) and (5c), respectively:

$$u_t^{csu} \leq u_0^{csu} + \Delta_t Q_t^c y_t^{csu} \quad \forall t \in \mathcal{T} : t = 1 \tag{12h}$$
$$y_t \leq \frac{u_0^{csu}}{E^c} + y_0 + y_0^{csb} \quad \forall t \in \mathcal{T} : t = 1. \tag{12i}$$

Cycle supply and demand constraints (12j) and (12k) correspond to constraints (**??**) and (**??**), respectively:

$$\dot{w}_t^{\delta^+} \geq \dot{w}_t - \dot{w}_0 \quad \forall t \in \mathcal{T} : t = 1 \tag{12j}$$
$$\dot{w}_t^{\delta^-} \geq \dot{w}_0 - \dot{w}_t \quad \forall t \in \mathcal{T} : t = 1. \tag{12k}$$

Constraints (12l)-(12n) are equivalent to constraints (**??**)-(**??**) for periods that require knowledge of the power cycle's initial conditions:

$$y_t^{cge} = 0 \quad \forall\, t \in \mathcal{T} : \Delta_t^e \leq Y^u - \hat{y}^{cgb} \tag{12l}$$
$$y_t^{cgb} = 0 \quad \forall\, t \in \mathcal{T} : \Delta_t^e \leq Y^d - \hat{y}^{cge} \quad \forall\, t \in \mathcal{T} : \Delta_t^e \geq Y^d \tag{12m}$$
$$y_t^{cgb} - y_t^{cge} = y_t - y_0 \quad \forall\, t \in \mathcal{T} : t = 1. \tag{12n}$$

Cycle logic constraints (12o), (12p), and (12q)-(12s) correspond to constraints (8a), (8c) and (8f)-(8h), respectively:

$$y_t^{csu} + y_0 \leq 1 \quad \forall t \in \mathcal{T} : t = 1 \tag{12o}$$
$$y_t^{csb} \leq y_0 + y_0^{csb} \quad \forall t \in \mathcal{T} : t = 1 \tag{12p}$$
$$y_t^{chsp} \geq y_t - (1 - y_0^{csb}) \quad \forall\, t \in \mathcal{T} : t = 1 \tag{12q}$$
$$y_t^{csd} \geq y_0 - y_t + y_0^{csb} - y_t^{csb} \quad \forall\, t \in \mathcal{T} : t = 1 \tag{12r}$$
$$y_t^{csup} \geq y_t^{csu} - y_0^{csu} \quad \forall t \in \mathcal{T} : t = 1. \tag{12s}$$

TES energy balance constraints (12t) and (12u) correspond to constraints (9a) and (9c), respectively:

$$s_t - s_0 = \Delta_t[x_t^r - (Q_t^c y_t^{csu} + Q^b y_t^{csb} + x_t + Q^{rsb} y_t^{rsb}]\forall t \in \mathcal{T} : t = 1 \tag{12t}$$

$$s_0 \geq \Delta_t \cdot \Delta_t^{rs}[(Q^u + Q^b) \cdot (-3 + y_t^{rsu} + y_0 + y_t + y_0^{csb} + y_t^{csb}) + x_t + Q^b y_t^{csb}] \quad \forall\, t \in \mathcal{T} : t = 1. \tag{12u}$$

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