CS 232 Computer and Communication Networks
Assignment 3

November 14, 2024

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1. Consider a system with arrival rate $\lambda = 2$ pkt/s and service rate $\mu = 4$ pkt/s. (Hint: The average packet in the system can be calculated by $\mathbb{E}[N] = \frac{\rho}{1-\rho}$, where $\rho =$ traffic load)
   (a) Compute the expected time a packet needs to go through the system.

Ans:

(a) The average number of packets in the system $\mathbb{E}[N]$ can be calculated by:

$$\mathbb{E}[N] = \frac{\rho}{1-\rho}$$

Substituting $\rho = \frac{\lambda}{\mu} = \frac{2}{4} = 0.5$:

$$\mathbb{E}[N] = \frac{0.5}{1-0.5} = \frac{0.5}{0.5} = 1 \text{ packet}$$

Using Little's Law, $\mathbb{E}[N] = \lambda \cdot \mathbb{E}[T]$, where $\mathbb{E}[T]$ is the expected time a packet spends in the system, we solve for $\mathbb{E}[T]$:

$$\mathbb{E}[T] = \frac{\mathbb{E}[N]}{\lambda} = \frac{1}{2} = 0.5 \text{ seconds}$$

2. Consider a system with arrival rate $\lambda = 2$ pkt/s and service rate $\mu = 4$ pkt/s.
   (a) What is the average inter-arrival time?
   (b) Compute the probability that the next inter-arrival is larger than 3.
   (c) Compute the probability that 4 packets will arrive in the next 2 seconds.

Ans:

(a) The average inter-arrival time is the reciprocal of the arrival rate:

$$\text{Average Inter-Arrival Time} = \frac{1}{\lambda} = \frac{1}{2} = 0.5 \text{ seconds}$$

(b) For a Poisson process, the time between arrivals follows an exponential distribution with parameter $\lambda = 2$. The probability that the next inter-arrival time T is greater than 3 seconds is given by:

$$P(T > 3) = e^{-\lambda \cdot 3} = e^{-2 \cdot 3} = e^{-6}$$

(c) The number of arrivals in a given time interval t for a Poisson process with rate λ follows a Poisson distribution with parameter λt. Here, $\lambda t = 2 \times 2 = 4$.

The probability of observing $k = 4$ arrivals in 2 seconds is:

$$P(N = 4) = \frac{(\lambda t)^4 e^{-\lambda t}}{4!} = \frac{4^4 e^{-4}}{24} = \frac{256 e^{-4}}{24} = \frac{32 e^{-4}}{3}$$

3. Consider a system with arrival rate $\lambda = 2$ pkt/s and service rate $\mu = 4$ pkt/s.
   (a) What is the average time a packet spends in service?

   Now assume that the system has an infinite number of servers and the service rate for each server i is 4 pkt/s.
   (b) What is the average time a packet spends in service?
   (c) What is the average number of packets that exist in the system at any point in time?
   (d) Assume that the probability of a packet to be dropped is $P_b = 0.75$, what is the average number of packets that exist in the system at any point in time?

Ans:

(a) The average time a packet spends in service is the reciprocal of the service rate μ:

$$\text{Average Service Time} = \frac{1}{\mu} = \frac{1}{4} = 0.25 \text{ seconds}$$

(b) In a system with an infinite number of servers, each packet is served immediately upon arrival, so the service time remains the same for each packet:

$$\text{Average Service Time} = \frac{1}{\mu} = \frac{1}{4} = 0.25 \text{ seconds}$$

(c) For an $M/M/\infty$ queue (infinite servers), the number of packets in the system follows a Poisson distribution with mean $\frac{\lambda}{\mu}$. Therefore, the average number of packets in the system is:

$$\mathbb{E}[N] = \frac{\lambda}{\mu} = \frac{2}{4} = 0.5 \text{ packets}$$

(d) Since the probability of a packet being dropped is $P_b = 0.75$, only $1 - P_b = 0.25$ of packets actually enter the system.

Therefore, the effective arrival rate is:

$$\lambda_{\text{eff}} = \lambda \times (1 - P_b) = 2 \times 0.25 = 0.5\,\text{pkt/s}$$

Thus, the average number of packets in the system at any point in time is :

$$\mathbb{E}[N] = \frac{\lambda_{\text{eff}}}{\mu} = \frac{0.5}{4} = 0.125 \text{ packets}$$

4. Consider a system where user 1 is making a phone call using VoIP. During the transit of user 1's call packets are being received by a router according to a Poisson processes $\{N_1(t) = t \geq 0\}$ with rate $\lambda_1 = 5$ packets/second.

   (a) What is the expected number of packets that the router must receive from user 1's call after 2 minutes? I.e. compute $\mathbb{E}[N(120)]$.

   Now assume that user 1 begins a new call and suppose that two other users (2 and 3) begin making independent calls at the same time using VoIP that must pass through the same router according to respective poisson processes $\{N_2(t) = t \geq 0\}$ with rate $\lambda_2 = 20$ packets/second and $\{N_3(t) = t \geq 0\}$ with rate $\lambda_3 = 40$ packets/second.

   (b) What is the probability that the router will receive 1000 packets in the next 15 seconds?

   (c) If the router's queue is only large enough to hold 1300 packets and if the three users continue to send at the same rate, will we expect see a buffer overflow in the router after 25 seconds?

Ans:

(a) The expected number of packets from a Poisson process over time t is given by $\mathbb{E}[N(t)] = \lambda t$.

Here, $t = 2$ minutes $= 120$ seconds, and $\lambda_1 = 5$ packets/second, so:

$$\mathbb{E}[N_1(120)] = \lambda_1 \cdot 120 = 5 \times 120 = 600 \text{ packets}$$

(b) With three independent Poisson processes, the combined arrival rate at the router is:

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 5 + 20 + 40 = 65 \text{ packets/second}$$

The number of packets received in the next 15 seconds, $N(15)$, follows a Poisson distribution with parameter $\lambda \cdot t = 65 \cdot 15 = 975$.

Thus, the probability of receiving exactly 1000 packets in 15 seconds is:

$$P(N(15) = 1000) = \frac{(975)^{1000} e^{-975}}{1000!}$$

(c) The expected number of packets in 25 seconds is:

$$\mathbb{E}[N(25)] = 65 \times 25 = 1625 \text{ packets}$$

Since the router's buffer can only hold 1300 packets, the expected number of packets (1625) exceeds the buffer capacity. Therefore, we would expect a buffer overflow to occur after 25 seconds.

5. Assume that some large organization is using a single switch to route all traffic between the two halves of the organization's 2 LANs. Suppose that at time $t = 0$ the switch is empty and that at time $t = 2$ it is the case that 100 packets have arrived, 40 have departed, and 7 have been blocked.

   (a) What is the number of packets that are at the switch at time t = 2? (i.e. compute N (t))

   (b) Now suppose some clever engineer has discovered that the number of arrivals at the switch in the interval from time 0 to time t can be described by the function:

   $$A(t) = 11t^2 \sin \frac{1}{t} \tag{1}$$

   What is the long-term arrival rate at the switch?

   (c) Now suppose that same clever engineer has discovered that the number of departures from the switch in the same interval from time 0 to time t can be described by the function:

   $$D(t) = (10t \times e)(1 - \frac{1}{t})^t \tag{2}$$

What is the throughput of the switch?

Ans:

(a) The number of packets at the switch at time $t = 2$, $N(t)$, can be computed as:

$$N(t) = (\text{Total Arrivals}) - (\text{Departures}) - (\text{Blocked})$$

$$N(2) = 100 - 40 - 7 = 53 \text{ packets}$$

(b) To find the long-term arrival rate, we evaluate the average rate as $t \to \infty$:

$$\lambda = \lim_{t\to\infty} \frac{A(t)}{t} = \lim_{t\to\infty} \frac{11t^2 \sin\frac{1}{t}}{t} = \lim_{t\to\infty} 11t \sin\frac{1}{t}$$

Now, applying L'Hôpital's Rule:

$$\lambda = \lim_{t\to\infty} 11 \frac{\cos\frac{1}{t} \cdot \left(-\frac{1}{t^2}\right)}{-\frac{1}{t^2}} = \lim_{t\to\infty} 11 \cos\frac{1}{t} = 11 \times 1 = 11$$

Thus, the long-term arrival rate at the switch is 11 packets/second.

(c) To find the throughput, we evaluate the average departure rate as $t \to \infty$:

$$\mu = \lim_{t\to\infty} \frac{D(t)}{t} = \lim_{t\to\infty} \frac{10t \times e\left(1 - \frac{1}{t}\right)^t}{t} = 10e \lim_{t\to\infty} \left(1 - \frac{1}{t}\right)^t$$

Since $\lim_{t\to\infty} \left(1 - \frac{1}{t}\right)^t = \frac{1}{e}$:

$$\mu = 10e \cdot \frac{1}{e} = 10$$

Thus, the throughput of the switch is 10 packets/second.