CS273A Midterm Exam

Introduction to Machine Learning: Fall 2023

Monday November 6th, 2023

Your name: Peter Anteater

Row/Seat Number:

Your ID #(e.g., 123456789) 314159265

UCINetID (e.g.ucinetid@uci.edu) panteat@uci.edu

- Please put your name and ID **on every page**.
- Total time is 80 minutes. READ THE EXAM FIRST and organize your time; don't spend too long on any one problem.
- Please **write clearly** and **show all your work**.
- If you need clarification on a problem, please raise your hand and wait for the instructor or TA to come over.
- You may use **one** sheet containing handwritten notes for reference, and a (basic) calculator.
- Turn in your notes and any scratch paper with your exam.

Problems

Total, *(75 points.)*

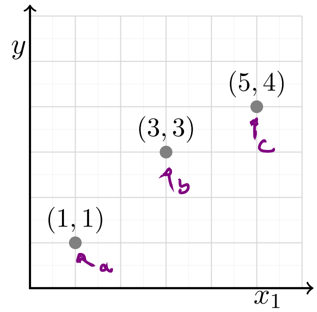
This page is intentionally blank, use as you wish.

Name: ______ ID#: ______

Problem 1 Cross-Validation, *(15 points.)*

For a regression problem to predict real-valued y given a single real-valued feature x_1, we observe training data (pictured at right):

x_1	y
1.0	1.0
3.0	3.0
5.0	4.0



(1) Compute the **leave-one-out** cross-validation MSE of a 1-nearest neighbor predictor. (In case of ties, prefer to use the data listed earlier in the table.) *(3 points.)*

leave out	predict	err
a	3	2
b	1	2
c	3	1

$MSE = \frac{1}{3}(2^2 + 2^2 + 1^2)$

$= 3.$

(2) Compute the **leave-one-out** cross-validation MSE of a 2-nearest neighbor predictor. (In case of ties, prefer to use the data listed earlier in the table.) *(3 points.)*

out	pred	err
a	3.5	2.5
b	2.5	0.5
c	2	2

$MSE = \frac{1}{3}\left((5/2)^2 + (1/2)^2 + (4/2)^2 \right)$

$= \frac{42}{12} = 3.5$

(3) Compute the **leave-one-out** cross-validation MSE of a constant predictor, $f(x) = \theta_0$. *(3 points.)*

out	pred	err
a	3.5	2.5
b	2.5	0.5
c	2	2

$MSE = 3.5$

(This happens to be the same as the 2-NN model, since the avg of the other 2 points is the same as the mean of all the other (2) points.)

(4) Compute the **leave-one-out** cross-validation MSE of a linear predictor, $f(x) = \theta_1 x_1 + \theta_0$. *(3 points.)*

out	pred	err
a	2	1
b	2.5	0.5
c	5	1

$MSE = \frac{1}{3}\left(1^2 + (\frac{1}{2})^2 + 1^2 \right)$

$= 9/12 = 3/4.$

(5) If choosing among these models, which would you select (based on cross-validation)? *(3 points.)*

We should choose the linear model (#4), since it has the smallest LOOXV MSE.

This page is intentionally blank, use as you wish.

Problem 2 True/False, *(10 points.)*

Here, assume that we have m data points $y^{(i)}, x^{(i)}$, $i = 1 \ldots m$, each with n features, $x^{(i)} = [x_1^{(i)} \; \ldots \; x_n^{(i)}]$. For each of the scenarios below, circle one of "true" or "false" to indicate whether you agree with the statement.

True or **false**: Using "batch" gradient descent (all m data per step) is less prone to getting stuck in local optima than stochastic gradient descent.

True or **false**: Optimizing a linear classifier using the hinge loss and L_2 regularization will always converge to a global optimum of the loss.

True or **false**: Increasing the number of features available to a perceptron model will increase its bias.

True or **false**: The perceptron algorithm is always guaranteed to converge.

True or **false**: With sufficiently many data, an SVM with polynomial kernel $K(x, x') = (1+x \cdot x'^T)^2$ can approximate any decision function.

True or **false**: The SVM optimization problem is an example of a linear program (a linear objective with linear constraints).

True or **false**: Using a soft-margin SVM in place of a hard-margin SVM will typically reduce the training error.

True or **false**: A 1-nearest neighbor model is an example of a learner with no inductive bias.

True or **false**: Feature selection (electing to ignore certain features of the data in our model) can be used to reduce model variance.

True or **false**: Given sufficiently many layers, a neural network with one hidden node per layer can approximate any function.

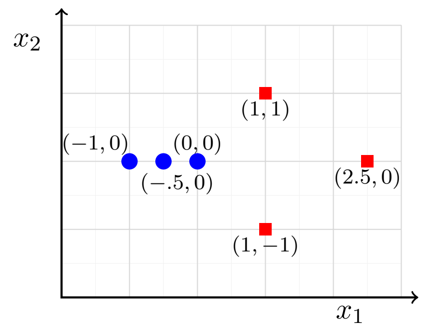
This page is intentionally blank, use as you wish.

Name: ID#:

Problem 3 Support Vector Machines, *(12 points.)*

For a classification problem to predict binary y given two real-valued features x_1, x_2, we observe training data (pictured at right):

x_1	x_2	y
0.0	0.0	-1
-0.5	0.0	-1
-1.0	0.0	-1
1.0	-1.0	1
1.0	1.0	1
2.5	0.0	1



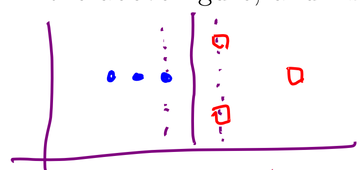
Our linear classifier takes the form,

$$f(x; w_1, w_2, b) = \text{sign}(w_1 x_1 + w_2 x_2 + b).$$

(1) Consider the optimal linear SVM classifier for the data, i.e., the one that separates the data and has the largest margin. **Sketch** its decision boundary in the above figure, and **list** the support vectors here. *(2 points.)*

SVs: (0,0), (1,1), (1,-1)

Bdry



(2) Derive the parameter values w_1, w_2, b of this $f(x)$ using these support vectors. *(4 points.)*

$$w_1 \cdot \varnothing + w_2 \cdot \varnothing + b = -1 \qquad b = -1$$
$$w_1 \cdot 1 + w_2 \cdot 1 + b = +1 \quad \Rightarrow \quad w_2 = \varnothing$$
$$w_1 \cdot 1 + w_2 \cdot (-1) + b = +1 \qquad w_1 = 2$$

(3) What is the *training error rate* of a linear SVM on these data? *(3 points.)*

$\varnothing$

(4) What is the the *leave-one-out cross validation error rate* for a linear SVM trained on these data? *(3 points.)*

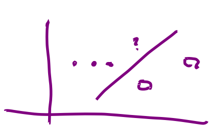
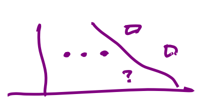
Non-SVs - all predict correctly ✓

(0,0) - ok : new boundary is at $x_1 = 1/2$

(1,1) - wrong:

(1,-1) - wrong:

$\Rightarrow$ Err $= 2/6 = 1/3$

This page is intentionally blank, use as you wish.

Name: ID#:

Problem 4 Naïve Bayes Classifiers, *(12 points.)*

x_1	x_2	y
b	0	0
c	1	0
a	0	0
a	1	0
c	0	1
c	1	1
c	1	1
a	0	1
c	0	1
c	1	1

Consider the table of measured data given at right. We will use the two observed features x_1 , x_2 to predict the class y. Feature x_1 can take on one of three values, $x_1 \in \{a, b, c\}$; feature x_2 is binary, $x_2 \in \{0, 1\}$. In the case of a tie, we will prefer to predict class $y = 0$.

(1) Write down the probabilities learned by a naïve Bayes classifier: *(4 points.)*

$p(y = 0)$: $4/10$ $p(y = 1)$: $6/10$

$p(x_1 = a \,|\, y = 0)$: $2/4$ $p(x_1 = a \,|\, y = 1)$: $1/6$

$p(x_1 = b \,|\, y = 0)$: $1/4$ $p(x_1 = b \,|\, y = 1)$: $\emptyset$

$p(x_1 = c \,|\, y = 0)$: $1/4$ $p(x_1 = c \,|\, y = 1)$: $5/6$

$p(x_2 = 0 \,|\, y = 0)$: $1/2$ $p(x_2 = 0 \,|\, y = 1)$: $1/2$

$p(x_2 = 1 \,|\, y = 0)$: $1/2$ $p(x_2 = 1 \,|\, y = 1)$: $1/2$

(2) Using your naïve Bayes model, compute the probability $p(y = 1 \,|\, x_1 = c, x_2 = 0)$: *(4 points.)*

$P_0 = p(y=0, x_1=c, x_2=0) = 4/10 \cdot 1/4 \cdot 1/2 = 1/20$

$P_1 = p(y=1, x_1=c, x_2=0) = 6/10 \cdot 5/6 \cdot 1/2 = 5/20$

$\Rightarrow p(y=1 \,|\, x_1=c, x_2=0) = \frac{5}{5+1} = 5/6$

(3) Using your naïve Bayes model, compute the probability $p(y = 1 \,|\, x_1 = b, x_2 = 1)$: *(4 points.)*

$P_0 = p(y=0, x_1=b, x_2=1) = 4/10 \cdot 1/4 \cdot 1/2 = 1/20$

$P_1 = p(y=1, x_1=b, x_2=1) = 6/10 \cdot \emptyset \cdot 1/2 = \emptyset$

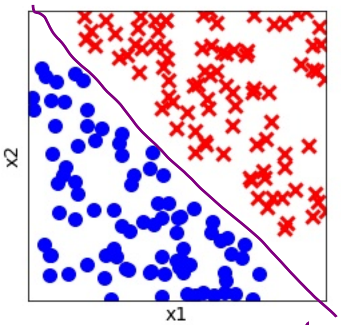
$\Rightarrow p(y=1 / x_1=b, x_2=1) = \emptyset.$

This page is intentionally blank, use as you wish.

Name: ______ ID#: ______

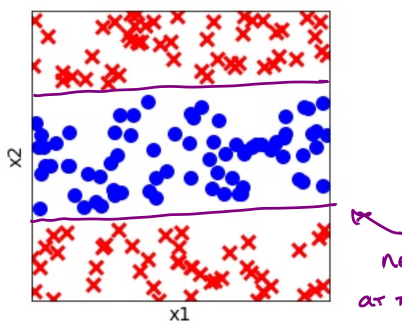
Problem 5 Separability, *(16 points.)*

Consider each of the following pictured data sets. Select **all** sets of features that could be used by a perceptron (linear classifier) to separate the data. (Note: check each feature list carefully; they may differ by subproblem.)



needs x_1 & x_2, anything else is extra.

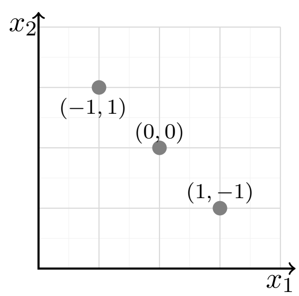
- ☐ $[1, x_1]$
- ☐ $[1, x_2]$
- ☑ $[1, x_1, x_2, (x_1)^2]$
- ☑ $[1, x_1, x_2, (x_2)^2]$
- ☑ $[1, x_1, x_2, (x_1)^2, (x_2)^2, (x_1 x_2)]$



needs decision boundary at two different values of x_2, but doesn't need to depend on x_1

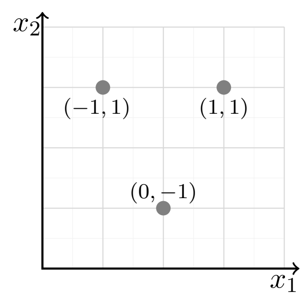
- ☐ $[1, x_1]$
- ☐ $[1, x_2]$
- ☐ $[1, x_1, x_2, (x_1)^2]$
- ☑ $[1, x_1, x_2, (x_2)^2]$
- ☑ $[1, x_1, x_2, (x_1)^2, (x_2)^2, (x_1 x_2)]$

Now consider the following sets of feature vectors, with no target labels. Select **all** the sets of features that could be used by a perceptron (linear classifier) to **shatter** the data, i.e., correctly separate any setting of the target values $y^{(i)}$:



colinear placement ⇒ can't shatter with $[1, x_1, x_2]$, but quad. function of either x_1 or x_2 can do it.

- ☐ $[1, x_1]$
- ☐ $[1, x_2]$
- ☐ $[1, x_1, x_2]$
- ☑ $[1, x_1, (x_1)^2]$
- ☑ $[1, x_2, (x_2)^2]$



placement ok for shattering with $[1, x_1, x_2]$, but two points with identical x_2 values means we can't do it without x_1.

- ☐ $[1, x_1]$
- ☐ $[1, x_2]$
- ☑ $[1, x_1, x_2]$
- ☑ $[1, x_1, (x_1)^2]$
- ☐ $[1, x_2, (x_2)^2]$

This page is intentionally blank, use as you wish.

Name: ID#:

Problem 6 Gradient Descent, *(10 points.)*

Suppose that we have training data $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$, where $x^{(i)}$ is a scalar feature and $y^{(i)} \in \{-1, +1\}$, and we wish to train a linear classifier, $\hat{y} = \text{sign}[\, a + bx \,]$, with two parameters a, b. In order to train the model, we use stochastic gradient descent on the *exponential loss* surrogate, whose loss for data point i is given by:

$$J^{(i)}(X, Y) = \exp\left[\ -y^{(i)}(a + bx^{(i)})\ \right].$$

(1) Write down the gradient of the single-data surrogate loss $J^{(i)}$ *(4 points.)*:

$$\frac{\partial J^i}{\partial a} = \exp\left[-y^i(a+bx^i)\right] \cdot (-y^i)$$

$$\frac{\partial J^i}{\partial b} = \exp\left[-y^i(a+bx^i)\right] \cdot (-y^i)(x^i)$$

$$\nabla J^i = \left[\frac{\partial J^i}{\partial a} \quad \frac{\partial J^i}{\partial b}\right].$$

(2) Give one advantage of stochastic gradient descent over batch gradient *(2 points.)*:

SGD is much faster at "early" optimization,

since it performs many more parameter updates per epoch.

(3) Give pseudocode for a stochastic gradient descent function `theta = train(X,Y)`, including all necessary elements for it to work. *(4 points.)*

Init $\theta = \underline{\emptyset}$, or at random

Set step size $\alpha = .001$ (or something)

for epoch in 1... max:

  for each data point i, in some order (eg. random)

    $\theta \leftarrow \theta - \alpha \nabla J^i$

– We can use fancier stopping criteria, or alter the step size as we go, but this is enough to function.

This page is intentionally blank, use as you wish.

This page is intentionally blank, use as you wish.

This page is intentionally blank, use as you wish.

This page is intentionally blank, use as you wish.

This page is intentionally blank, use as you wish.