CS273a Midterm Exam

Introduction to Machine Learning: Fall 2016

Tuesday November 1st, 2016

Your name:

Your ID # and UCINetID (e.g., 123456789, myname@uci.edu):

Your seat (row and number):

- Total time is 80 minutes. READ THE EXAM FIRST and organize your time; don't spend too long on any one problem.
- Please **write clearly** and **show all your work**.
- If you need clarification on a problem, please raise your hand and wait for the instructor or TA to come over.
- Turn in any scratch paper with your exam

(This page intentionally left blank)

Problem 1: (10 points) Bayes Classifiers

In this problem you will use Bayes Rule: $p(y|x) = p(x|y)p(y)/p(x)$ to perform classification. Suppose we observe some training data with two binary features x_1 , x_2 and a binary class y. After learning the model, you are also given some validation data.

Table 1: Training Data

x_1	x_2	y
0	0	0
0	1	0
0	1	1
0	1	1
1	0	1
1	0	1
1	1	0
1	1	0

Table 2: Validation Data

x_1	x_2	y
0	0	1
0	1	0
1	0	1
1	1	0

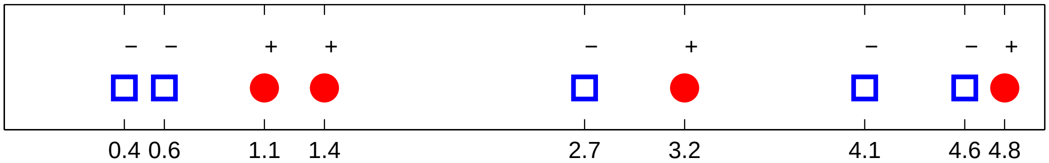
In the case of any ties, we will prefer to predict class 0.

(a) Give the predictions of a joint Bayes classifier on the validation data. What is the validation error rate?

(b) Give the predictions of a naïve Bayes classifier on the validation data. What is the validation error rate?

(c) **True** or **False** : In a naïve Bayes model, the features x_i are independent, i.e., $p(x_1, x_2) = p(x_1)\, p(x_2)$.

Problem 2: (9 points) Nearest Neighbor Classification



Given the above data with one scalar feature x (whose values are given below each data point) and a class variable $y \in \{-1, +1\}$, with filled circles indicating $y = +1$ and squares $y = -1$ (the sign is also shown above each data point for redundancy), we use a k-nearest neighbor classifier to perform prediction; in the case of ties, we prefer to predict class -1. Answer the following:

(a) Compute the training error rate of a 1-Nearest-Neighbor classifier trained on these data.

(b) Compute the leave-one-out cross-validation error rate of a 1-Nearest-Neighbor classifier on these data.

(c) Compute the training error for a 3-Nearest-Neighbor classifier on these data.

Problem 3: (10 points) Gradient Descent

Suppose that we have training data $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$, where $x^{(i)}$ is a scalar feature and $y^{(i)} \in \{-1, +1\}$, and we wish to train a linear classifier, $\hat{y} = \text{sign}[\, a + bx \,]$, with two parameters a, b. In order to train the model, we use gradient descent on a smooth surrogate loss called the *exponential loss*:

$$J(X, Y) = \frac{1}{m} \sum_i \exp\left(y^{(i)}(a + bx^{(i)}) \right)$$

(a) Write down the gradient of our surrogate loss function.

(b) Give one advantage of batch gradient descent over stochastic gradient.

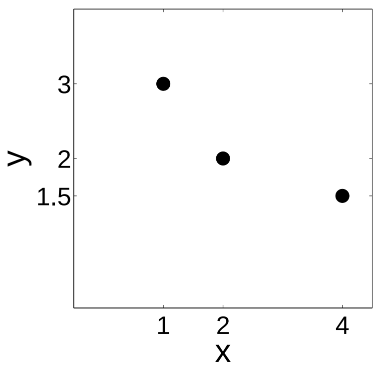
(c) Give pseudocode for a (batch) gradient descent function `theta = train(X,Y)`, including all necessary elements for it to work.

Problem 4: (10 points) Linear Regression, Cross-validation

Consider the following data points, copied in each part. We wish to perform linear regression to minimize the mean squared error (MSE) of our predictions.

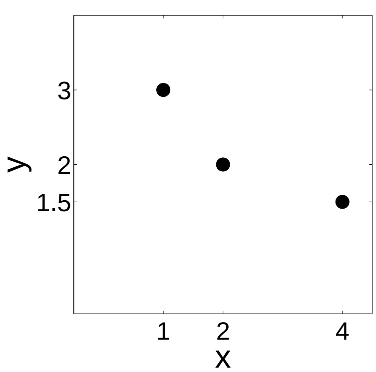
(a) Compute the **leave-one-out** cross-validation error of a zero-order (constant) predictor,

$$\hat{y}(x) = \theta_0$$



(b) Compute the **leave-one-out** cross-validation error of a first-order (linear) predictor,

$$\hat{y}(x) = \theta_0 + \theta_1 \, x$$



Problem 5: (20 points) Multiple Choice

For the following questions, assume that we have m data points $y^{(i)}$, $x^{(i)}$, $i = 1 \ldots m$, each with n features, $x^{(i)} = [x_1^{(i)} \; \ldots \; x_n^{(i)}]$.

Circle one answer for each:

Suppose that we are training a linear classifier (perceptron). Before training, we decide to remove (throw away) 10% of our features (selected at random). This is most likely to make it **more** **equally** **less** likely to overfit the data.

When training a k-nearest neighbor model, we decide to increase the value of k. This will most likely make our model **more** **equally** **less** likely to overfit the data.

Again, training a k-nearest neighbor model, we double the amount of data available to the model. We then re-train the model, including re-optimizing k.
This is likely to **increase** **not change** **decrease** the bias.

Suppose that, when training a linear regressor, we double the amount of data available for training. This is most likely to decrease the **bias** **variance** **both** **neither** of our learned model.

Still training a linear regressor, instead of providing more real data, we instead include m additional points of "fake" data, $(x^{(i)}, y^{(i)}) = (0, 0)$.
This will most likely **increase** **not change** **decrease** the bias.
It will most likely **increase** **not change** **decrease** the variance.

True or **false**: if the VC dimension of a model is H, then the model can shatter any set of H training points.

True or **false**: Linear regression can be solved using either matrix algebra or gradient descent.

True or **false**: Increasing the regularization of a linear regression model will decrease the variance.

Before training a linear classifier, we transform one of our features by taking its logarithm, i.e., `X[:,1] = np.log(X[:,1]);`. This is likely to **increase** **not change** **decrease** the model's VC dimension.

We train a Gaussian Bayes classifier, but then decide to re-train it, forcing the two classes' covariance matrices to be equal, i.e., $\Sigma_{(y=+1)} = \Sigma_{(y=-1)}$. This is likely to **increase** **not change** **decrease** the variance of our model.

Problem 6: (9 points) Short answer

Consider the two possible decision boundaries (indicated by Line 1 and Line 2) for the binary classification problem shown in Figure 1. For each algorithm below, will it possibly produce boundary 1, boundary 2, or both? Please give a concise explanation of your choice.

Perceptron Algorithm :

Logistic Regression :

Support Vector Machine (hard-margin) :

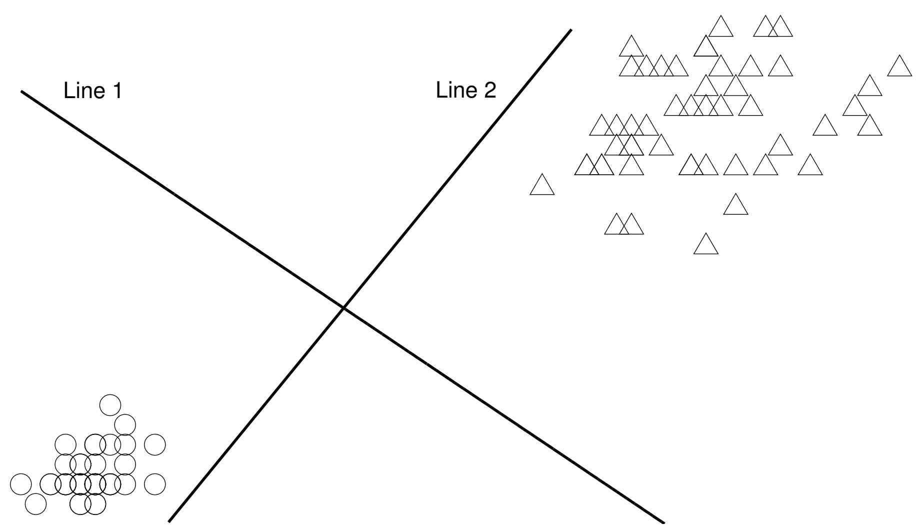


Figure 1: Possible linear decision boundaries.

Problem 7: (10 points) Support Vector Machines

Suppose we are learning a linear support vector machine with a single scalar feature x and binary target $y \in \{-1, +1\}$. We observe training data:

$$D = \{(x^{(i)}, y^{(i)})\} = \{(0, +1), (-3, +1), (1, -1)\}$$

Our linear classifier takes the form $f(x; a, b) = \text{sign}(ax + b)$.

(a) Write down the primal optimization problem for a support vector machine on these data.

(b) Sketch (graph) the constraint set on the parameters a, b, and give the values of a, b at the solution.

(c) Identify the support vectors.

(d) Give **two** possible advantages of the *dual* form of the SVM over the primal.

Problem 8: (10 points) VC Dimension

Consider the following classifier, parameterized by a single scalar parameter a and operating on a scalar feature x:

$$f(x\ ;\ a) = \begin{cases} +1 & x \le a \text{ or } a+1 < x \le a+2 \\ -1 & \text{otherwise} \end{cases}$$

In this problem, we will show the VC dimension of $f(x\ ;\ a)$ is 3.

(a) Show by example that $f(x\ ;\ a)$ can shatter three points. Hint: place your points at $x^{(1)} = 0$, $x^{(2)} = 0.75$, $x^{(3)} = 1.5$.

(b) Argue that $f(x\ ;\ a)$ cannot shatter four points. (Which target pattern cannot be reproduced?)