CS273A Midterm Exam
Introduction to Machine Learning: Fall 2023
Monday November 6th, 2023

Your name:

Row/Seat Number:

Your ID #(e.g., 123456789)

UCINetID (e.g.ucinetid@uci.edu)

- Please put your name and ID **on every page**.

- Total time is 80 minutes. READ THE EXAM FIRST and organize your time; don't spend too long on any one problem.

- Please **write clearly** and **show all your work**.

- If you need clarification on a problem, please raise your hand and wait for the instructor or TA to come over.

- You may use **one** sheet containing handwritten notes for reference, and a (basic) calculator.

- Turn in your notes and any scratch paper with your exam.

Problems

Total, *(75 points.)*
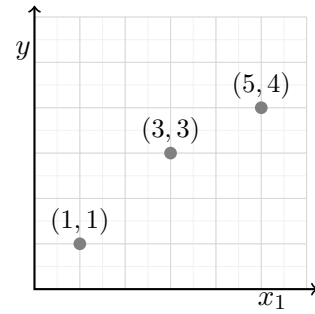
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Problem 1 Cross-Validation, *(15 points.)*

For a regression problem to predict real-valued y given a single real-valued feature x_1, we observe training data (pictured at right):

x_1	y
1.0	1.0
3.0	3.0
5.0	4.0

(1) Compute the **leave-one-out** cross-validation MSE of a 1-nearest neighbor predictor. (In case of ties, prefer to use the data listed earlier in the table.) *(3 points.)*

(2) Compute the **leave-one-out** cross-validation MSE of a 2-nearest neighbor predictor. (In case of ties, prefer to use the data listed earlier in the table.) *(3 points.)*

(3) Compute the **leave-one-out** cross-validation MSE of a constant predictor, $f(x) = \theta_0$. *(3 points.)*

(4) Compute the **leave-one-out** cross-validation MSE of a linear predictor, $f(x) = \theta_1 x_1 + \theta_0$. *(3 points.)*

(5) If choosing among these models, which would you select (based on cross-validation)? *(3 points.)*

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Problem 2   True/False, *(10 points.)*

Here, assume that we have m data points $y^{(i)}$, $x^{(i)}$, $i = 1 \ldots m$, each with n features, $x^{(i)} = [x_1^{(i)} \ \ldots \ x_n^{(i)}]$. For each of the scenarios below, circle one of "true" or "false" to indicate whether you agree with the statement.

True or **false**: Using "batch" gradient descent (all m data per step) is less prone to getting stuck in local optima than stochastic gradient descent.

True or **false**: Optimizing a linear classifier using the hinge loss and L_2 regularization will always converge to a global optimum of the loss.

True or **false**: Increasing the number of features available to a perceptron model will increase its bias.

True or **false**: The perceptron algorithm is always guaranteed to converge.

True or **false**: With sufficiently many data, an SVM with polynomial kernel $K(x, x') = (1+x\cdot x'^T)^2$ can approximate any decision function.

True or **false**: The SVM optimization problem is an example of a linear program (a linear objective with linear constraints).

True or **false**: Using a soft-margin SVM in place of a hard-margin SVM will typically reduce the training error.

True or **false**: A 1-nearest neighbor model is an example of a learner with no inductive bias.

True or **false**: Feature selection (electing to ignore certain features of the data in our model) can be used to reduce model variance.

True or **false**: Given sufficiently many layers, a neural network with one hidden node per layer can approximate any function.
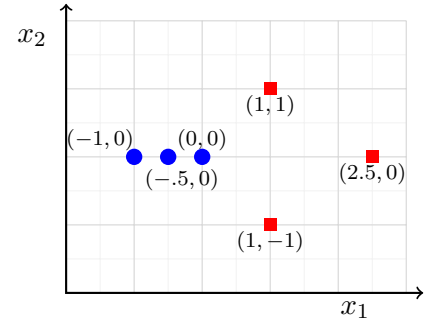
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Problem 3   Support Vector Machines, *(12 points.)*

For a classification problem to predict binary y given two real-valued features x_1, x_2, we observe training data (pictured at right):

x_1	x_2	y
0.0	0.0	-1
-0.5	0.0	-1
-1.0	0.0	-1
1.0	-1.0	1
1.0	1.0	1
2.5	0.0	1

Our linear classifier takes the form,

$$f(x; w_1, w_2, b) = \text{sign}(w_1 x_1 + w_2 x_2 + b).$$

(1) Consider the optimal linear SVM classifier for the data, i.e., the one that separates the data and has the largest margin. **Sketch** its decision boundary in the above figure, and **list** the support vectors here.*(2 points.)*

(2) Derive the parameter values w_1, w_2, b of this $f(x)$ using these support vectors. *(4 points.)*

(3) What is the *training error rate* of a linear SVM on these data?*(3 points.)*

(4) What is the the *leave-one-out cross validation error rate* for a linear SVM trained on these data?*(3 points.)*

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Problem 4   Naïve Bayes Classifiers, *(12 points.)*

x_1	x_2	y
b	0	0
c	1	0
a	0	0
a	1	0
c	0	1
c	1	1
c	1	1
a	0	1
c	0	1
c	1	1

Consider the table of measured data given at right. We will use the two observed features x_1 , x_2 to predict the class y. Feature x_1 can take on one of three values, $x_1 \in \{a, b, c\}$; feature x_2 is binary, $x_2 \in \{0, 1\}$. In the case of a tie, we will prefer to predict class $y = 0$.

(1) Write down the probabilities learned by a naïve Bayes classifier: *(4 points.)*

$p(y = 0)$: $p(y = 1)$:

$p(x_1 = a \,|\, y = 0)$: $p(x_1 = a \,|\, y = 1)$:

$p(x_1 = b \,|\, y = 0)$: $p(x_1 = b \,|\, y = 1)$:

$p(x_1 = c \,|\, y = 0)$: $p(x_1 = c \,|\, y = 1)$:

$p(x_2 = 0 \,|\, y = 0)$: $p(x_2 = 0 \,|\, y = 1)$:

$p(x_2 = 1 \,|\, y = 0)$: $p(x_2 = 1 \,|\, y = 1)$:

(2) Using your naïve Bayes model, compute the probability $p(y = 1 \,|\, x_1 = c, x_2 = 0)$: *(4 points.)*

(3) Using your naïve Bayes model, compute the probability $p(y = 1 \,|\, x_1 = b, x_2 = 1)$: *(4 points.)*

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Problem 5 Separability, *(16 points.)*

Consider each of the following pictured data sets. Select **all** sets of features that could be used by a perceptron (linear classifier) to separate the data. (Note: check each feature list carefully; they may differ by subproblem.)
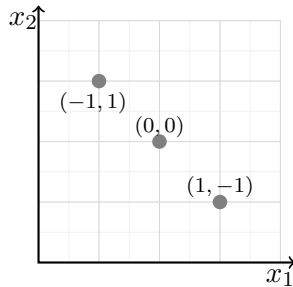

☐ $[1, x_1]$

☐ $[1, x_2]$
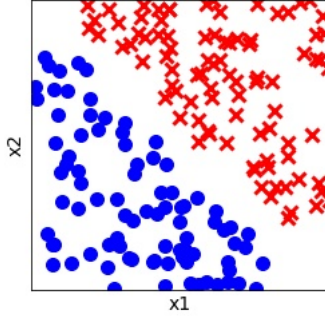
☐ $[1, x_1, x_2, (x_1)^2]$

☐ $[1, x_1, x_2, (x_2)^2]$
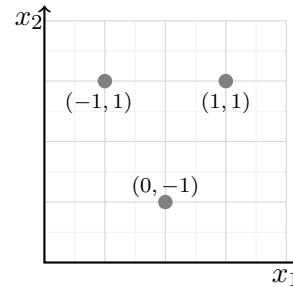
☐ $[1, x_1, x_2, (x_1)^2, (x_2)^2, (x_1 x_2)]$

☐ $[1, x_1]$

☐ $[1, x_2]$
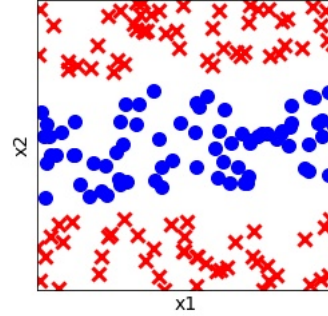
☐ $[1, x_1, x_2, (x_1)^2]$

☐ $[1, x_1, x_2, (x_2)^2]$

☐ $[1, x_1, x_2, (x_1)^2, (x_2)^2, (x_1 x_2)]$

Now consider the following sets of feature vectors, with no target labels. Select **all** the sets of features that could be used by a perceptron (linear classifier) to **shatter** the data, i.e., correctly separate any setting of the target values $y^{(i)}$:

☐ $[1, x_1]$

☐ $[1, x_2]$

☐ $[1, x_1, x_2]$

☐ $[1, x_1, (x_1)^2]$

☐ $[1, x_2, (x_2)^2]$

☐ $[1, x_1]$

☐ $[1, x_2]$

☐ $[1, x_1, x_2]$

☐ $[1, x_1, (x_1)^2]$

☐ $[1, x_2, (x_2)^2]$

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Problem 6   Gradient Descent, *(10 points.)*

Suppose that we have training data $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$, where $x^{(i)}$ is a scalar feature and $y^{(i)} \in \{-1, +1\}$, and we wish to train a linear classifier, $\hat{y} = \text{sign}[a + bx]$, with two parameters a, b. In order to train the model, we use stochastic gradient descent on the *exponential loss* surrogate, whose loss for data point i is given by:

$$J^{(i)}(X, Y) = \exp\left[\; -y^{(i)}(a + bx^{(i)}) \;\right].$$

(1) Write down the gradient of the single-data surrogate loss $J^{(i)}$ *(4 points.)*:

(2) Give one advantage of stochastic gradient descent over batch gradient *(2 points.)*:

(3) Give pseudocode for a stochastic gradient descent function `theta = train(X,Y)`, including all necessary elements for it to work. *(4 points.)*

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