Report

August 1, 2024

Set $C_0 = [0,1]$ and

$$C_n := \frac{C_{n-1}}{3} \cup \left(\frac{2}{3} + \frac{C_{n-1}}{3} \right).$$

Let $\mathcal{C} = \lim_{n\to\infty} C_n$ denote the Cantor set.

Let $\Omega = [-1,2] \times [-1,2] \setminus (\mathcal{C} \times \{0\})$ and $\Omega_n = [-1,2] \times [-1,2] \setminus (C_n \times \{0\})$.

Define u as the solution to

$$\begin{aligned} \Delta u &= 0 \text{ in } \Omega, \\ u &= 0 \text{ on } \Omega_{int}, \\ u &= 1 \text{ on } \Omega_{ext}. \end{aligned}$$

and u_n as the solution to

$$\begin{aligned} \Delta u_n &= 0 \text{ in } \Omega_n, \\ u_n &= 0 \text{ on } \Omega_{n,int}, \\ u_n &= 1 \text{ on } \Omega_{n,ext}. \end{aligned}$$

The following may be true or not:

$$\frac{\log u_n(x, y_n)}{\log y_n} = -\frac{\log u_n(x, 3^{-n})}{n \log 3} \to ?\alpha(x,0), \quad x \in \mathcal{C}$$

(a) initial mesh

(b) solution

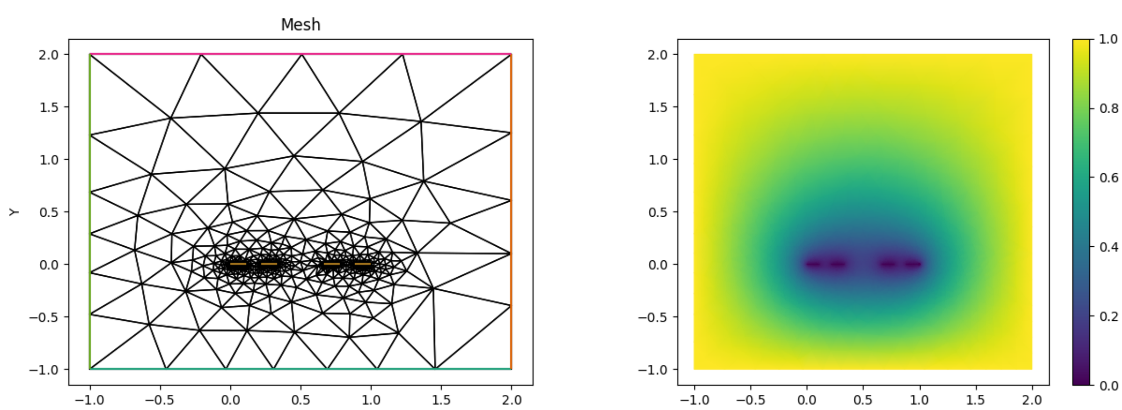

Figure 1: Ω_2, deg=5

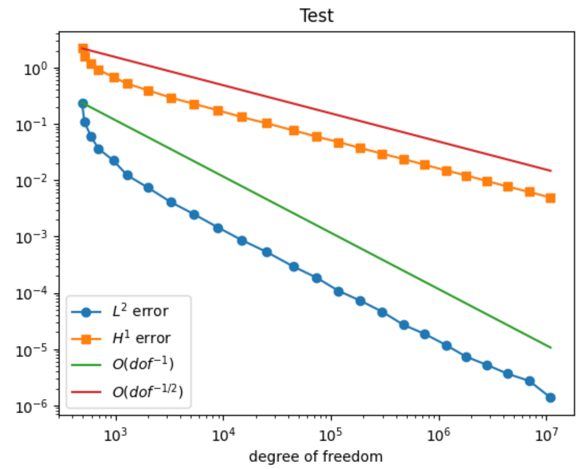

Figure 2: test on Ω_2: $u = 2 + x^2 + 3xy$. deg=1.