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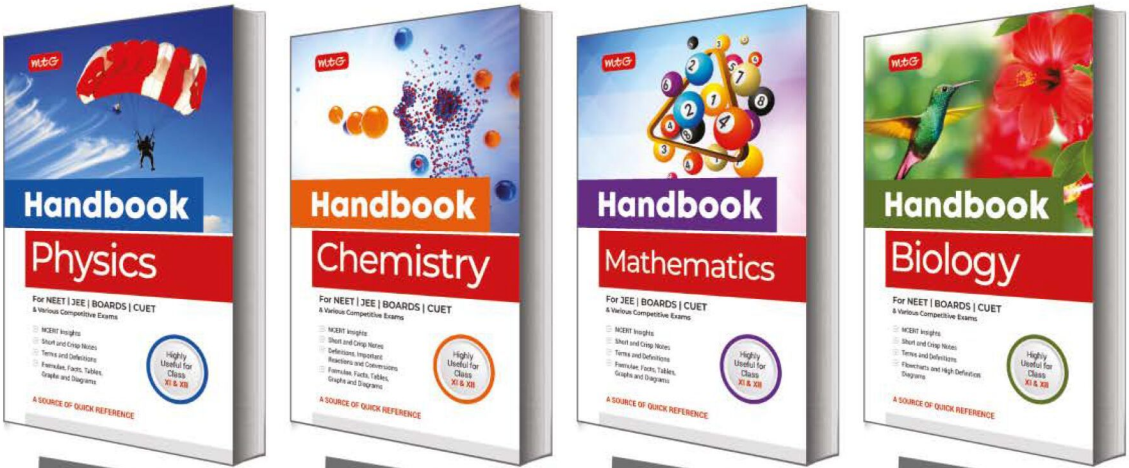


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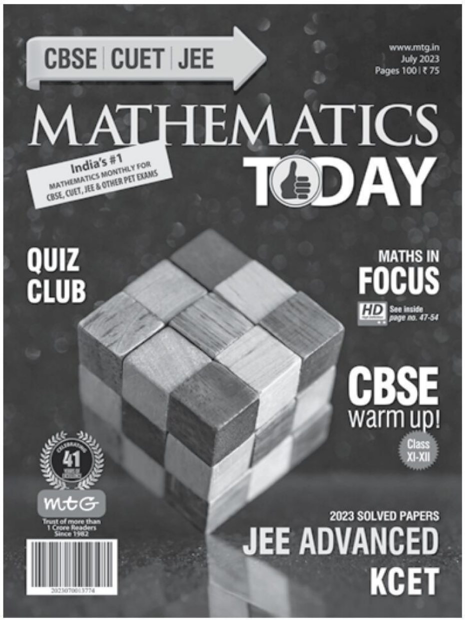


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## Contents





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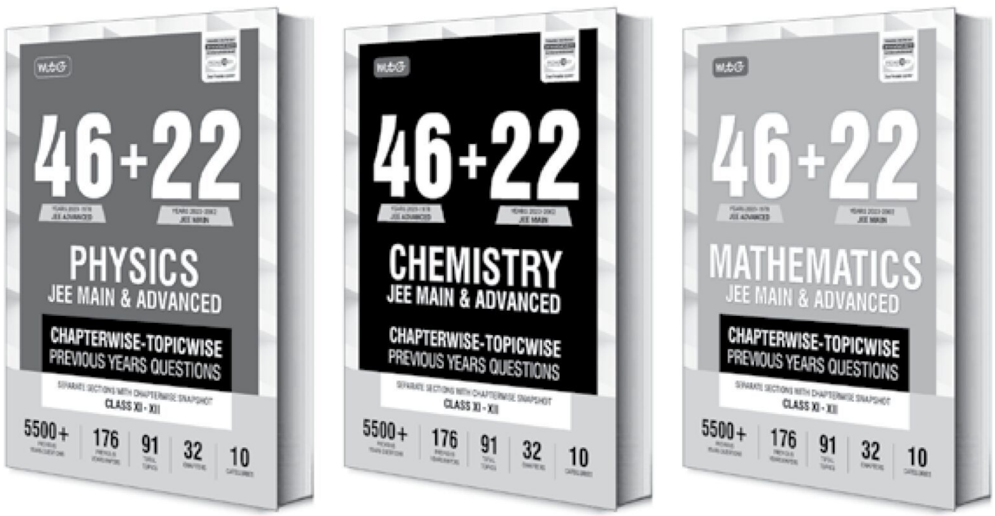
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# JEE ADVANCED 2023 SOLVED PAPER

Exam held on 4th June

*Alok Kumar

## PAPER-1

### SECTION 1 (Maximum Marks : 12)

- *This section contains THREE (03) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme :*

*Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;*
*Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;*
*Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;*
*Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;*
*Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*
*Negative Marks : –2 In all other cases.*

**1.** Let $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$. Then which of the following statements is(are) true?
(a) There are infinitely many functions from $S$ to $T$
(b) There are infinitely many strictly increasing functions from $S$ to $T$
(c) The number of continuous functions from $S$ to $T$ is at most 120
(d) Every continuous function from $S$ to $T$ is differentiable

**2.** Let $T_1$ and $T_2$ be two distinct common tangents to the ellipse $E: \frac{x^2}{6}+\frac{y^2}{3}=1$ and the parabola $P : y^2 = 12x$. Suppose that the tangent $T_1$ touches $P$ and $E$ at the points $A_1$ and $A_2$, respectively and the tangent $T_2$ touches $P$ and $E$ at the points $A_4$ and $A_3$, respectively. Then which of the following statements is (are) true?
(a) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units
(b) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units
(c) The tangents $T_1$ and $T_2$ meet the $x$-axis at the point $(-3, 0)$
(d) The tangents $T_1$ and $T_2$ meet the $x$-axis at the point $(-6, 0)$

**3.** Let $f : [0, 1] \to [0, 1]$ be the function defined by $f(x)=\frac{x^3}{3}-x^2+\frac{5}{9}x+\frac{17}{36}$. Consider the square region $S = [0, 1] \times [0, 1]$. Let $G = \{(x, y) \in S : y > f(x)\}$ be called the green region and $R = \{(x, y) \in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h) \in S : x \in [0, 1]\}$ be the horizontal line drawn at a height $h \in [0, 1]$. Then which of the following statements is (are) true?
(a) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line $L_h$ equals the area of the green region below the line $L_h$
(b) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line $L_h$ equals the area of the red region below the line $L_h$
(c) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line $L_h$ equals the area of the red region below the line $L_h$
(d) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line $L_h$ equals the area of the green region below the line $L_h$

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

## SECTION 2 (Maximum Marks : 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme :*
  *Full Marks : +3 If ONLY the correct option is chosen;*
  *Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*
  *Negative Marks : –1 In all other cases.*

**4.** Let $f: (0, 1) \to \mathbb{R}$ be the function defined as

$$f(x) = \sqrt{n} \text{ if } x \in \left[\frac{1}{n+1}, \frac{1}{n}\right) \text{where } n \in \mathbb{N}.$$

Let $g: (0, 1) \to \mathbb{R}$ be a function such that

$$\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\,dt < g(x) < 2\sqrt{x} \text{ for all } x \in (0, 1).$$

Then $\lim_{x\to 0} f(x)g(x)$

(a) does NOT exist (b) is equal to 1
(c) is equal to 2 (d) is equal to 3

**5.** Let $Q$ be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \in \{0, 1\}\}$. Let $F$ be the set of all twelve lines containing the diagonals of the six faces of the cube $Q$. Let $S$ be the set of all four lines containing the main diagonals of the cube $Q$; for instance, the line passing through the vertices $(0, 0, 0)$ and $(1,1,1)$ is in $S$. For lines $l_1$ and $l_2$, let $d(l_1, l_2)$ denote the shortest distance between them. Then the maximum value of $d(l_1, l_2)$, as $l_1$ varies over $F$ and $l_2$ varies over $S$, is

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{\sqrt{8}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{12}}$

**6.** Let $X = \left\{(x, y) \in \mathbb{Z}\times\mathbb{Z} : \frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x\right\}$.

Three distinct points $P$, $Q$ and $R$ are randomly chosen from $X$. Then the probability that $P$, $Q$ and $R$ form a triangle whose area is a positive integer, is

(a) $\frac{71}{220}$ (b) $\frac{73}{220}$ (c) $\frac{79}{220}$ (d) $\frac{83}{220}$

**7.** Let $P$ be a point on the parabola $y^2 = 4ax$, where $a > 0$. The normal to the parabola at $P$ meets the $x$-axis at a point $Q$. The area of the triangle $PFQ$, where $F$ is the focus of the parabola, is 120. If the slope $m$ of the normal and $a$ are both positive integers, then the pair $(a, m)$ is

(a) (2, 3) (b) (1, 3) (c) (2, 4) (d) (3, 4)

## SECTION 3 (Maximum Marks : 24)

- *This section contains SIX (06) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  *Full Marks : +4 If ONLY the correct integer is entered;*
  *Zero Marks : 0 In all other cases.*

**8.** Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in \mathbb{R}$.

Then the number of real solutions of the equation $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

**9.** Let $n \geq 2$ be a natural number and $f: [0,1] \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if } 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \leq x \leq 1 \end{cases}$$

If $n$ is such that the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$ and $y = f(x)$ is 4, then the maximum value of the function $f$ is

**10.** Let $\overbrace{75\cdots57}^{r}$ denote the $(r + 2)$ digit number where the first and the last digits are 7 and the remaining $r$ digits are 5. Consider the sum $S = 77 + 757 + 7557 + \ldots + \overbrace{75\cdots57}^{98}$. If $S = \frac{\overbrace{75\cdots57}^{99} + m}{n}$, where $m$ and $n$ are natural numbers less than 3000, then the value of $m + n$ is

**11.** Let $A = \left\{\frac{1967 + 1686i\sin\theta}{7 - 3i\cos\theta} : \theta \in \mathbb{R}\right\}$. If $A$ contains exactly one positive integer $n$, then the value of $n$ is

**12.** Let $P$ be the plane $\sqrt{3}x + 2y + 3z = 16$ and let

$$S = \left\{\begin{array}{l}\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1 \text{ and the} \\ \text{distance of } (\alpha, \beta, \gamma) \text{ from the plane } P \text{ is } \frac{7}{2}\end{array}\right\}.$$

Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in $S$ such that $|\vec{u}-\vec{v}| = |\vec{v}-\vec{w}| = |\vec{w}-\vec{u}|$. Let $V$ be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is

**13.** Let $a$ and $b$ be two nonzero real numbers. If the coefficient of $x^5$ in the expansion of $\left(ax^2+\frac{70}{27bx}\right)^4$ is equal to the coefficient of $x^{-5}$ in the expansion of $\left(ax-\frac{1}{bx^2}\right)^7$, then the value of $2b$ is

## SECTION 4 (Maximum Marks : 12)

- *This section contains FOUR (04) Matching List Sets.*
- *Each set has ONE Multiple Choice Question.*
- *Each set has TWO lists: List-I and List-II.*
- *List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).*
- *FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.*
- *Answer to each question will be evaluated according to the following marking scheme:*

  *Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;*

  *Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*

  *Negative Marks : –1 In all other cases.*

**14.** Let $\alpha$, $\beta$ and $\gamma$ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List-I** to the correct entries in **List-II**.

| List-I | | List-II | |
|---|---|---|---|
| (P) | If $\beta=\frac{1}{2}(7\alpha-3)$ and $\gamma = 28$, then the system has | (1) | a unique solution |
| (Q) | If $\beta=\frac{1}{2}(7\alpha-3)$ and $\gamma \neq 28$, then the system has | (2) | no solution |
| (R) | If $\beta\neq\frac{1}{2}(7\alpha-3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has | (3) | infinitely many solutions |
| (S) | If $\beta\neq\frac{1}{2}(7\alpha-3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has | (4) | $x = 11, y = -2$ and $z = 0$ as a solution |
| | | (5) | $x = -15, y = 4$ and $z = 0$ as a solution |

The correct option is:

(a) (P) → (3), (Q) → (2), (R) → (1), (S) → (4)
(b) (P) → (3), (Q) → (2), (R) → (5), (S) → (4)
(c) (P) → (2), (Q) → (1), (R) → (4), (S) → (5)
(d) (P) → (2), (Q) → (1), (R) → (1), (S) → (3)

**15.** Consider the given data with frequency distribution

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_i$ | 3 | 8 | 11 | 10 | 5 | 4 |
| $f_i$ | 5 | 2 | 3 | 2 | 4 | 4 |

Match each entry in **List-I** to the correct entries in **List-II**.

| List-I | | List-II | |
|---|---|---|---|
| (P) | The mean of the above data is | (1) | 2.5 |
| (Q) | The median of the above data is | (2) | 5 |
| (R) | The mean deviation about the mean of the above data is | (3) | 6 |
| (S) | The mean deviation about the median of the above data is | (4) | 2.7 |
| | | (5) | 2.4 |

The correct option is:

(a) (P) → (3), (Q) → (2), (R) → (4), (S) → (5)
(b) (P) → (3), (Q) → (2), (R) → (1), (S) → (5)
(c) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)
(d) (P) → (3), (Q) → (3), (R) → (5), (S) → (5)

**16.** Let $l_1$ and $l_2$ be the lines $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2 = (\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let $X$ be the set of all the planes $H$ that contain the line $l_1$. For a plane $H$, let $d(H)$ denote the smallest possible distance between the points of $l_2$ and $H$. Let $H_0$ be a plane in $X$ for which $d(H_0)$ is the maximum value of $d(H)$ as $H$ varies over all planes in $X$.

Match each entry in **List-I** to the correct entries in **List-II**.

| List-I | | List-II | |
|---|---|---|---|
| (P) | The value of $d(H_0)$ is | (1) | $\sqrt{3}$ |
| (Q) | The distance of the point (0, 1, 2) from $H_0$ is | (2) | $\frac{1}{\sqrt{3}}$ |

| | | | |
|---|---|---|---|
| (R) | The distance of origin from the $H_0$ is | (3) | 0 |
| (S) | The distance of origin from the point of intersection of planes $y = z$, $x = 1$ and $H_0$ is | (4) | $\sqrt{2}$ |
| | | (5) | $\frac{1}{\sqrt{2}}$ |

The correct option is:

(a) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)
(b) (P) → (5), (Q) → (4), (R) → (3), (S) → (1)
(c) (P) → (2), (Q) → (1), (R) → (3), (S) → (2)
(d) (P) → (5), (Q) → (1), (R) → (4), (S) → (2)

**17.** Let $z$ be a complex number satisfying $|z|^3+2z^2+4\bar{z}-8=0$, where $\bar{z}$ denotes the complex conjugate of $z$. Let the imaginary part of $z$ be nonzero.

Match each entry in **List-I** to the correct entries in **List-II**.

| | List-I | List-II | |
|---|---|---|---|
| (P) | $\lvert z\rvert^2$ is equal to | (1) | 12 |
| (Q) | $\lvert z-\bar{z}\rvert^2$ is equal to | (2) | 4 |
| (R) | $\lvert \bar{z}\rvert^2+\lvert z+\bar{z}\rvert^2$ is equal to | (3) | 8 |
| (S) | $\lvert z+1\rvert^2$ is equal to | (4) | 10 |
| | | (5) | 7 |

The correct option is:

(a) (P) → (1), (Q) → (3), (R) → (5), (S) → (4)
(b) (P) → (2), (Q) → (1), (R) → (3), (S) → (5)
(c) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)
(d) (P) → (2), (Q) → (3), (R) → (5), (S) → (4)

## PAPER-2

### SECTION 1 (MAXIMUM MARKS: 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  *Full Marks : +3 If ONLY the correct option is chosen;*
  *Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered.*
  *Negative Marks : –1 In all other cases.*

**1.** Let $f:[1,\infty)\to\mathbb{R}$ be a differentiable function such that $f(1)=\frac{1}{3}$ and $3\int_1^x f(t)dt = xf(x)-\frac{x^3}{3}, x\in[1,\infty)$.
Let $e$ denote the base of the natural logarithm. Then the value of $f(e)$ is

(a) $\frac{e^2+4}{3}$ (b) $\frac{\log_e 4+e}{3}$
(c) $\frac{4e^2}{3}$ (d) $\frac{e^2-4}{3}$

**2.** Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is

(a) $\frac{1}{3}$ (b) $\frac{5}{21}$ (c) $\frac{4}{21}$ (d) $\frac{2}{7}$

**3.** For any $y\in\mathbb{R}$, let $\cot^{-1}(y)\in(0,\pi)$ and $\tan^{-1}(y)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.
Then the sum of all the solutions of the equation
$$\tan^{-1}\left(\frac{6y}{9-y^2}\right)+\cot^{-1}\left(\frac{9-y^2}{6y}\right)=\frac{2\pi}{3} \text{ for } 0<|y|<3,$$
is equal to

(a) $2\sqrt{3}-3$ (b) $3-2\sqrt{3}$
(c) $4\sqrt{3}-6$ (d) $6-4\sqrt{3}$

**4.** Let the position vectors of the points $P$, $Q$, $R$ and $S$ be $\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{b}=3\hat{i}+6\hat{j}+3\hat{k}$, $\vec{c}=\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}$ and $\vec{d}=2\hat{i}+\hat{j}+\hat{k}$, respectively.
Then which of the following statements is true?

(a) The points $P$, $Q$, $R$ and $S$ are not coplanar
(b) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides $PR$ internally in the ratio 5 : 4
(c) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides $PR$ externally in the ratio 5 : 4

(d) The square of the magnitude of the vector $\vec{b} \times \vec{d}$ is 95

**SECTION 2 (Maximum Marks : 12)**

- *This section contains THREE (03) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

*Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;*
*Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;*
*Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;*
*Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;*
*Zero Marks : 0 If unanswered;*
*Negative Marks : –2 In all other cases.*

**5.** Let $M = (a_{ij})$, $i, j \in \{1, 2, 3\}$, be the $3 \times 3$ matrix such that $a_{ij} = 1$ if $j + 1$ is divisible by $i$, otherwise $a_{ij} = 0$. Then which of the following statements is(are) true?

(a) $M$ is invertible

(b) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(c) The set $\{X \in \mathbb{R}^3 : MX = \text{O}\} \neq \{\text{O}\}$, where $\text{O} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(d) The matrix $(M - 2I)$ is invertible, where $I$ is the $3 \times 3$ identity matrix

**6.** Let $f : (0, 1) \to \mathbb{R}$ be the function defined as $f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right)$, where $[x]$ denotes the greatest integer less than or equal to $x$. Then which of the following statements is(are) true?

(a) The function $f$ is discontinuous exactly at one point in $(0,1)$

(b) There is exactly one point in $(0,1)$ at which the function $f$ is continuous but NOT differentiable

(c) The function $f$ is NOT differentiable at more than three points in $(0,1)$

(d) The minimum value of the function $f$ is $-\frac{1}{512}$

**7.** Let $S$ be the set of all twice differentiable functions $f$ from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2 f}{dx^2}(x) > 0$ for all $x \in (-1,1)$. For $f \in S$, let $X_f$ be the number of points $x \in (-1, 1)$ for which $f(x) = x$. Then which of the following statements is(are) true?

(a) There exists a function $f \in S$ such that $X_f = 0$
(b) For every function $f \in S$, we have $X_f \leq 2$
(c) There exists a function $f \in S$ such that $X_f = 2$
(d) There does NOT exist any function $f$ in $S$ such that $X_f = 1$

## SECTION 3 (Maximum Marks : 24)

- *This section contains SIX (06) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  *Full Marks : +4 If ONLY the correct integer is entered;*
  *Zero Marks : 0 In all other cases.*

**8.** For $x \in \mathbb{R}$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}}dt$ is

**9.** For $x \in \mathbb{R}$, let $y(x)$ be a solution of the differential equation $(x^2-5)\frac{dy}{dx} - 2xy = -2x(x^2-5)^2$ such that $y(2) = 7$. Then the maximum value of the function $y(x)$ is

**10.** Let $X$ be the set of all five digit numbers formed using 1,2,2,2,4,4,0. For example, 22240 is in $X$ while 02244 and 44422 are not in $X$. Suppose that each element of $X$ has an equal chance of being chosen. Let $p$ be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of $38p$ is equal to

**11.** Let $A_1, A_2, A_3, \ldots\ldots, A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let $P$ be a point on the circle and let $PA_i$ denote the distance between the points $P$ and $A_i$ for $i = 1, 2, \ldots.., 8$. If $P$ varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2 \ldots\ldots PA_8$, is

**12.** Let $R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a,b,c,d \in \{0,3,5,7,11,13,17,19\} \right\}$

Then the number of invertible matrices in $R$ is

**13.** Let $C_1$ be the circle of radius 1 with center at the origin. Let $C_2$ be the circle of radius $r$ with center at the point $A = (4, 1)$, where $1 < r < 3$. Two distinct common tangents $PQ$ and $ST$ of $C_1$ and $C_2$ are drawn. The tangent $PQ$ touches $C_1$ at $P$ and $C_2$ at $Q$. The tangent $ST$ touches $C_1$ at $S$ and $C_2$ at $T$. Mid points of the line segments $PQ$ and $ST$ are joined to form a line which meets the $x$-axis at a point $B$. If $AB = \sqrt{5}$, then the value of $r^2$ is

## SECTION 4 (Maximum Marks : 12)

- *This section contains TWO (02) paragraphs.*
- *Based on each paragraph, there are TWO (02) questions.*
- *The answer to each question is a NUMERICAL VALUE.*
- *If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  *Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;*
  *Zero Marks : 0 In all other cases.*

### PARAGRAPH-I

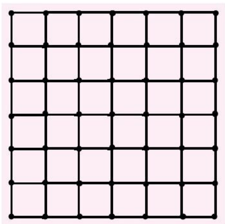
Consider an obtuse angled triangle $ABC$ in which the difference between the largest and the smallest angle is $\pi/2$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

Following are the two questions based on PARAGRAPH I.

**14.** Let $a$ be the area of the triangle $ABC$. Then the value of $(64a)^2$ is

**15.** Then the inradius of the triangle $ABC$ is

### PARAGRAPH-II



Consider the $6 \times 6$ square in the figure. Let $A_1, A_2, \ldots.., A_{49}$ be the points of intersections (dots in the picture) in some order. We say that $A_i$ and $A_j$ are friends if they are adjacent along a row or along a column. Assume that each point $A_i$ has an equal chance of being chosen.

Following are two questions based on PARAGRAPH II.

**16.** Let $p_i$ be the probability that a randomly chosen point has $i$ many friends, $i = 0,1,2,3,4$. Let $X$ be a random variable such that for $i = 0, 1, 2, 3, 4$, the probability $P(X = i) = P_i$. Then the value of $7E(X)$ is

**17.** Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots\ldots, A_{49}$. Let $p$ be the probability that they are friends. Then the value of $7p$ is

## SOLUTIONS

### PAPER-1

**1. (a, c, d) :** (a) True, As domain has infinite elements.
(b) False, As domain has infinite elements but codomain has only 4 elements. Hence no strictly increasing function is possible.
(c) True, Co-domain has finite number of elements.
(d) True, $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$.

Since $\lim_{h\to 0} f(a+h) = f(a)$ due to continuous.

Numerator will be a perfect 0 as output of function is discrete. Hence $f'(a) = 0$, Thus differentiable.

**[Rating : Easy]**

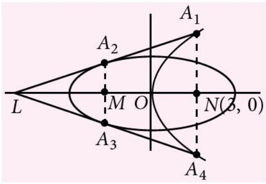
**2. (a, c) :** Equation of tangent of parabola is $y = mx + \frac{a}{m}$.

So, $y = mx + \frac{3}{m}$

$\Rightarrow c^2 = a^2m^2 + b^2$

$\Rightarrow \frac{9}{m^2} = 6m^2 + 3$

$\Rightarrow m^2 = 1$




Equation of tangents $T_1$ and $T_2$ are

$y = x + 3$, $y = -x - 3$, which cuts $x$-axis at $(-3, 0)$.

So, we have $A_1(3, 6)$, $A_2(-2, 1)$, $A_3(-2, -1)$ and $A_4(3, -6)$

$\therefore A_1A_4 = 12$, $A_2A_3 = 2$, $MN = 5$

Thus, area of quadrilateral $= \frac{1}{2}(12+2)\times 5 = 35$ sq.unit

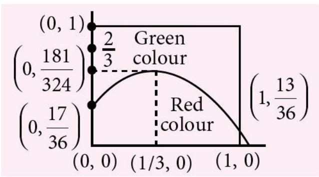
**[Rating : Moderate]**

**3. (b, c, d) :** $y = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$

$\Rightarrow y' = x^2 - 2x + \frac{5}{9}$

$\Rightarrow 9x^2 - 18x + 5 = 0$

$\Rightarrow x = \frac{1}{3}, \frac{5}{3}$




$\therefore y\left(\frac{1}{3}\right) = \frac{181}{324}$; $y(1) = \frac{13}{36}$

Area of green region $= 1 - \int_0^1 f(x)dx = \frac{1}{2}$

(a) $h = \frac{2}{3}$ area of green region above $L_h = \frac{1}{3} > \frac{1}{4}$.

Hence option (a) is wrong.

(b) For $L_h = \frac{1}{4}$ area of red region below $L_h = \frac{1}{4}$.

Hence option (b) is correct.

(c) For $h = \frac{181}{324}$, area of green region above $L_h = \frac{143}{324}$ and red region below $L_h = \frac{1}{2}$ and for $h = \frac{1}{3}$, area of green region above $L_h = \frac{1}{2}$ and red region below $L_h = \frac{13}{36}$

$\therefore$ By IMVT the areas are equal some where in between.

(d) Check at $h = \frac{181}{324}$ and $h = \frac{13}{36}$ and use IMVT.

**[Rating : Moderate]**

**4. (c) :** $f(x) = \sqrt{n}, x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$

and $\int_{x^2}^{x} \sqrt{\frac{1-t}{t}} < g(x) < 2\sqrt{x} \Rightarrow \lim_{x\to 0} g(x) = 0$

also $\lim_{x\to 0} f(x) = \lim_{n\to\infty} f(x) = \infty$; $\lim_{x\to 0} f(x)\cdot g(x) = 0\times\infty$

Since, $\frac{1}{n+1} \le x < \frac{1}{n}$ so, $n+1 \ge \frac{1}{x} > n$

$\Rightarrow \sqrt{\frac{1-x}{x}} < f(x) < \frac{1}{\sqrt{x}}$

$$\lim_{x\to 0^+}\left(\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt\right)\sqrt{\frac{1-x}{x}} < \lim_{x\to 0^+} f(x)g(x)$$

$$< \lim_{x\to 0^+} 2\sqrt{x}\frac{1}{\sqrt{x}} = 2$$

$$\left[\because \lim_{x\to 0^+}\left(\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt\right)\cdot\left(\sqrt{\frac{1-x}{x}}\right) = 2\right]$$

(Using Leibnitz rule and L′ Hospital rule]

**[Rating : Moderate]**

**5. (a) :** DR's of $OG = 1, 1, 1$

DR's of $AF = -1, 1, 1$

DR's of $CE = 1, 1, -1$

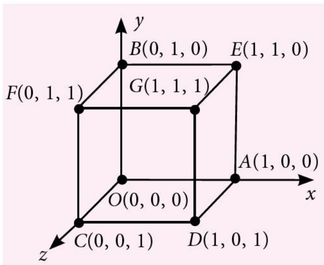
DR's of $BD = 1, -1, 1$

Equation of $OG$ is

$\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

Equation of $AB$ is

$\frac{x-1}{1} = \frac{y}{-1} = \frac{z}{0}$



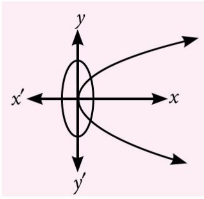
Normal to both the lines $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} - 2\hat{k}$

$\overrightarrow{OA} = \hat{i}$

$$\text{S.D.} = \frac{|\hat{i}\cdot(\hat{i}+\hat{j}-2\hat{k})|}{|\hat{i}+\hat{j}-2\hat{k}|} = \frac{1}{\sqrt{6}}$$

**[Rating : Moderate]**

**6. (b) :** The points inside are (1, 0), (1, ±1), (1, ±2), (2, 0), (2, ±1), (2, ±2) and (2, ±3)




Area = Integer if height is even

As area $= \frac{1}{2}\cdot B\cdot H = \frac{1}{2}\times 1\times H$

$H$ is even when ${}^5C_1({}^4C_2 + {}^3C_2) + {}^7C_4({}^3C_2 + {}^2C_2) = 73$

$\therefore$ Total number of ways $= {}^{12}C_3$ $\therefore$ $P = \frac{73}{220}$

**[Rating : Moderate]**

**7. (a) :** $2yy' = 4a$

Slope of normal to the parabola at point $P\,(at^2, 2at)$ is

$$m = -\frac{2at\times 2}{4a} = -t$$

Now equation of normal at $P(at^2, 2at)$is

$y - 2at = -t(x - at^2)$

So $Q(2a + at^2, 0)$

$$\text{Area of } \Delta PFQ = \frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ 2a+at^2 & 0 & 1 \\ a & 0 & 1 \end{vmatrix} = 120$$

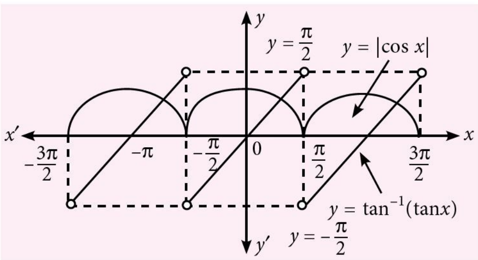
$$\Rightarrow \left|\frac{2at}{2}(a+at^2)\right| = 120$$

$\Rightarrow$ $a^2t(1 + t^2)| = 120 \Rightarrow |2^2 \times 3(1 + 3^2)| = 120$

So, $m = 3$, $a = 2$

**[Rating : Moderate]**

**8. (3) :** $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$



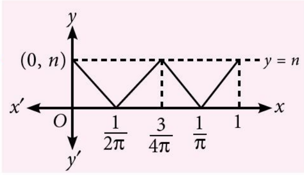

$\Rightarrow$ $\sqrt{2}\,|\cos x| = \sqrt{2}\tan^{-1}(\tan x)$

$\Rightarrow$ $|\cos x| = \tan^{-1}(\tan x)$

Number of solution = Number of intersection points = 3

**[Rating : Moderate]**

**9. (8) :**




$$\text{Area} = \frac{1}{2}\left(\frac{1}{2n} + \frac{1}{n} - \frac{1}{2n} + 1 - \frac{1}{n}\right)\times n = 4 \quad \Rightarrow n = 8$$

Now, maximum value of $f(x) = n = 8$

**[Rating : Moderate]**

**10. (1219) :** $S = 7(10 + 10^2 + 10^3 + \ldots.. + 10^{99}) + 50$ $(1 + 11 + 111 + \ldots.. + \underbrace{111\ldots.11}_{98}) + 99 \times 7$

$$= \frac{7\times 10^{100}}{9} - \frac{70}{9} + \frac{50}{9}[(10-1)+(10^2-1)+\ldots.+(10^{98}-1)] + 99\times 7$$

$$= \frac{7\times 10^{100}}{9} + \frac{50}{9}\left[\frac{10^{99}-10}{9} - 98\right] + 99\times 7 - \frac{70}{9}$$

$$= \frac{7\times 10^{100}}{9} + \frac{50}{9}\left[\frac{10^{99}-1}{9} - 99\right] + 693 - \frac{70}{9}$$

$$= \frac{\overbrace{755\ldots.57}^{99}}{9} + 693 - \frac{5020}{9} - \frac{7}{9} = \frac{\overbrace{755\ldots.57}^{99} + 1210}{9}$$

$\therefore$ $m + n = 1210 + 9 = 1219$ **[Rating : Difficult]**

**11. (281) :** $A = \dfrac{1967 + 1686\,i\sin\theta}{7 - 3i\cos\theta}$

$$= \frac{281(7+6i\sin\theta)}{7-3i\cos\theta} \times \frac{7+3i\cos\theta}{7+3i\cos\theta}$$

$$= \frac{281(49-18\sin\theta\cos\theta)}{49+9\cos^2\theta} + \frac{5901i(2\sin\theta+\cos\theta)}{49+9\cos^2\theta}$$

For it to be positive integer (*i.e.*, real number)

$2\sin\theta + \cos\theta = 0$

$$\Rightarrow \tan\theta = \frac{-1}{2}, \sin\theta = \frac{1}{\sqrt{5}} \text{ and } \cos\theta = \frac{-2}{\sqrt{5}}$$

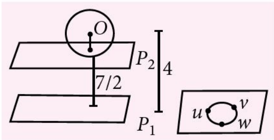
$$\therefore A = \frac{281\left(49+18\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)\right)}{49+9\left(\frac{4}{5}\right)} = 281$$

**[Rating : Moderate]**

**12. (45) :** $\perp$ from $O$ to $P_1 = 4$

Sphere with center (0, 0, 0) and $r = 1$ is $x^2 + y^2 + z^2 = 1$

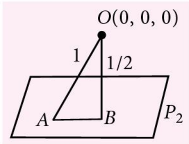
$$AB^2 = 1 - \frac{1}{4}$$




$$\therefore\ AB = \frac{\sqrt{3}}{2}$$

So, equilateral $\Delta$ is inscribed in this circle of radius $\frac{\sqrt{3}}{2}$.

$$\text{Base area} = 3 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 \sin 120°$$




$$= \frac{3}{2} \times \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{9\sqrt{3}}{16}$$

$\therefore$ Volume of pyramid, $V = \frac{1}{3} \times h \times \text{base area},$

$$\Rightarrow V = \frac{1}{6} \times \frac{9\sqrt{3}}{16} = \frac{1}{6}[abc]$$

$$\Rightarrow V = \frac{9\sqrt{3}}{16} = [abc] \Rightarrow \frac{80V}{\sqrt{3}} = 45$$

**[Rating : Difficult]**

**13. (3) :** Given, $\left(ax^2 + \frac{70}{27bx}\right)^4$

The general term is given by

$$T_{r+1} = {}^4C_r (ax^2)^{4-r} \left(\frac{70}{27bx}\right)^r \qquad \text{...(1)}$$

For coefficient of $x^5$, put $8 - 2r - r = 5 \Rightarrow r = 1$

$\therefore$ The coefficient $x^5$ in $\left(ax^2 + \frac{70}{27bx}\right) = {}^4C_1 \cdot \frac{a^3 \cdot (70)}{27 \cdot b}$

General term of $\left(ax - \frac{1}{bx^2}\right)^7$ is given by

$$T_{r+1} = {}^7C_r (ax)^{7-r} \left(-\frac{1}{bx^2}\right)^r$$

For coefficient of $x^{-5}$, put $7 - r - 2r = -5 \Rightarrow r = 4$

$\therefore$ The coefficient of $x^{-5}$ in $\left(ax - \frac{1}{bx^2}\right)^7 = {}^7C_4 \frac{a^3(-1)^4}{b^4}$

$$\text{Now, } {}^4C_1 \frac{a^3(70)}{27(b)} = {}^7C_4 \frac{a^3}{b^4}$$

$$\Rightarrow \frac{4 \times 70}{27(b)} = \frac{35}{b^4} \Rightarrow b^3 = \frac{27}{8} \Rightarrow b = \frac{3}{2} \quad \therefore\ 2b = 3$$

**[Rating : Moderate]**

**14. (a) :** $D = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 7\alpha - 2\beta - 3$

$$D_1 = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix} = -11(2\beta + 3) - \alpha(-21 - 2\gamma)$$

$= -22\beta - 33 + 21\alpha + 2\alpha\gamma = 21\alpha - 22\beta + 2\alpha\gamma - 33$

$$D_2 = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix} = (11\beta - \alpha\gamma) - 7(\beta - 2\alpha) + 1(\gamma - 22)$$

$= 14\alpha + 4\beta + \gamma - \alpha\gamma - 22$

$$D_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = -1(2\gamma + 21) - 11(-3 - 4)$$

$= -2\gamma - 21 + 77 = -2\gamma + 56$

(P) At $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$

$D = D_1 = D_2 = D_3 = 0$

Thus, the system has infinite many solutions.

At $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, $D = 0$

(Q) $D_1, D_2, D_3 \neq 0$

Thus, the system has no solution.

(R) At $\beta \neq \frac{1}{2}(7\alpha - 3), \alpha = 1, \gamma \neq 28.$

$D \neq 0; D_1 \neq 0; D_2 \neq 0; D_3 \neq 0$

Thus, the system has a unique solution.

(S) At $\beta \neq \frac{1}{2}(7\alpha - 3), \alpha = 1, \gamma = 28.$

$\therefore$ $D = 4 - 2\beta$, $D_1 = 44 - 22\beta$, $D_2 = 4\beta - 8$, $D_3 = 0$

$\therefore$ $x = 11, y = -2$ and $z = 0$ is the solution.

**[Rating : Easy]**

**15. (a) :**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_i$ | 3 | 4 | 5 | 8 | 10 | 11 |
| $f_i$ | 5 | 4 | 4 | 2 | 2 | 3 |
| $c.f.$ | 5 | 9 | 13 | 15 | 17 | 20 |

$$\text{Mean} = \frac{15 + 16 + 33 + 20 + 20 + 16}{20} = 6$$

Median = 5

Mean deviation (about mean)

$$= \frac{15 + 4 + 15 + 8 + 4 + 8}{20} = \frac{54}{20} = 2.7$$

Mean deviation (about median)

$$=\frac{10+6+18+10+0+4}{20}=\frac{48}{20}=2.4$$

Hence option (a) is correct. **[Rating : Easy]**

**16. (b) :** Equation of plane containing $l_1$ and parallel to $l_2$ is given by

$$\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}=0 \Rightarrow x-z=0$$

(P) $d(H_0)$ = distance of point (0, 1, –1) from $x - z = 0$

$$=\frac{1}{\sqrt{2}}$$

(Q) Distance of point (0, 1, 2) from $H_0 = \left|\frac{0-2}{\sqrt{2}}\right|=\sqrt{2}$

(R) The distance of origin from $H_0 = \left|\frac{0-0}{\sqrt{2}}\right|=0$

(S) Point of intersection of $y = z$, $x = 1$ and $x - z = 0$ is (1, 1, 1). Its distance from (0, 0, 0) $=\sqrt{3}$

**[Rating : Moderate]**

**17. (b) :** Let $Z = r(\cos\theta + i\sin\theta)$

$r^3 + 2r^2e^{2i\theta} + 4re^{-i\theta} - 8 = 0$

Comparing real and imaginary part, we get

$r^3 + 2r^2\cos2\theta + 4r\cos\theta = 8$ ...(i)

and $4r^2\sin\theta\cos\theta = 4r\sin\theta$ ...(ii)

$\Rightarrow r\cos\theta = 1$ put in (i)

$$\Rightarrow \cos 2\theta=2\cos^2\theta-1=\frac{2-r^2}{r^2}$$

$\Rightarrow r^3 + 2(2 - r^2) = 4 \Rightarrow r = 2$

(P) $|Z|^2 = 4$

(Q) $|Z-\bar{Z}|^2=|2ir\sin\theta|^2 = 4r^2(1-\cos^2\theta) = 16 - 4 = 12$

(R) $|Z|^2+|Z+\bar{Z}|^2=4+4=8$

(S) $|Z+1|^2=(Z+1)(\bar{Z}+1)=Z\bar{Z}+Z+\bar{Z}+1$

$= 4 + 2 + 1 = 7$ **[Rating : Moderate]**

## PAPER-2

**1. (c) :** Differentiate w.r.t. '$x$'; $3f(x) = f(x) + xf'(x) - x^2$

$$\frac{dy}{dx}-\left(\frac{2}{x}\right)y=x\ ;\ I.F.=e^{-2\ln x}=\frac{1}{x^2}$$

Now, $y\left(\frac{1}{x^2}\right)=\int x\cdot\frac{1}{x^2}dx+C \Rightarrow y = x^2\ln x + cx^2$

$$\therefore\ y(1)=\frac{1}{3}\Rightarrow c=\frac{1}{3}\qquad \therefore\ y(e)=\frac{4e^2}{3}$$

**[Rating : Moderate]**

**2. (b) :** HH + THH + HTHH + .... + THTHH + ....

$\therefore$ Probability that the experiment stops with head,

$$P=\left(\frac{1}{3}\right)^2+\frac{2}{3}\times\left(\frac{1}{3}\right)^2+\frac{2}{3}\times\frac{1}{3}\times\left(\frac{1}{3}\right)^2+.....\infty$$

$$=\left(\left(\frac{1}{3}\right)^2+\left(\frac{2}{9}\right)\left(\frac{1}{3}\right)^2+...\infty\right)+\frac{2}{3}\left(\frac{1}{3}\right)^2\left(1+\frac{2}{9}+\left(\frac{2}{9}\right)^2+...\infty\right)$$

$$=\frac{1}{9}\left(1+\frac{2}{9}+....\infty\right)+\frac{2}{3}\times\frac{1}{9}\left(1+\frac{2}{9}+....\infty\right)$$

$$=\frac{1}{9}\times\frac{1}{1-\frac{2}{9}}+\frac{2}{27}\times\frac{1}{1-\frac{2}{9}}=\frac{1}{9}\times\frac{9}{7}+\frac{2}{27}\times\frac{9}{7}$$

$$=\frac{9}{7}\left(\frac{1}{9}+\frac{2}{27}\right)=\frac{9}{7}\left(\frac{5}{27}\right)=\frac{5}{21}$$

**[Rating : Moderate]**

**3. (c) :** Case I : $y\in(-3, 0)$

$$\therefore\ \tan^{-1}\left(\frac{6y}{9-y^2}\right)+\pi+\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}$$

$$\left[\because\ \tan^{-1}\left(\frac{1}{x}\right)=-\pi+\cot^{-1}x,\ x<0\right]$$

$$\Rightarrow\ 2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=-\frac{\pi}{3}$$

$$\Rightarrow\ y^2-6\sqrt{3}y-9=0 \Rightarrow y=3\sqrt{3}-6\quad(\because\ y\in(-3,0))$$

Case II : $y\in(0, 3)$

$$\therefore\ 2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}\Rightarrow \sqrt{3}y^2+6y-9\sqrt{3}=0$$

$\Rightarrow\ y=\sqrt{3}$ or $y=-3\sqrt{3}$ (rejected)

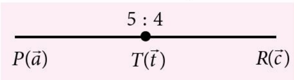
$\therefore$ Sum of solutions $=\sqrt{3}+3\sqrt{3}-6=4\sqrt{3}-6$

**[Rating : Moderate]**

**4. (b) :** $[\overrightarrow{PQ}\ \overrightarrow{PR}\ \overrightarrow{PS}]=\begin{vmatrix} 2 & 4 & 8 \\ \frac{12}{5} & \frac{6}{5} & 12 \\ 1 & -1 & 6 \end{vmatrix}$

$$=\frac{-12}{5}(32)+\frac{6}{5}(4)-12(-2-4)=0$$

$\therefore$ $P$, $Q$, $R$ and $S$ are coplanar.

Now, 


$$\vec{t}=\frac{5\vec{c}+4\vec{a}}{9}=\frac{7}{3}\hat{i}+\frac{8}{3}\hat{j}+\frac{5}{3}\hat{k}=\frac{\vec{b}+2\vec{d}}{3}$$

Also, $\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 6 & 3\\ 2 & 1 & 1\end{vmatrix}=3\hat{i}+3\hat{j}-9\hat{k}$

$\therefore\ |\vec{b}\times\vec{d}|=99$ **[Rating : Easy]**

**5. (b, c):** $M=\begin{vmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 0\end{vmatrix}$

$\because\ |M| = 0$, so matrix is non-invertible.
So, option (a) is incorrect.

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix}=\begin{bmatrix}a_1+a_2+a_3\\ a_1+a_3\\ a_2\end{bmatrix}=\begin{bmatrix}-a_1\\ -a_2\\ -a_3\end{bmatrix}$$

$2a_1 + a_2 + a_3 = 0$ ...(i) ; $a_2 + a_1 + a_3 = 0$ ...(ii);
$a_2 + a_3 = 0$ ...(iii)
Using (iii) in (ii), we get
$a_1 = 0$ and $a_2 = -a_3$ [Using (iii)]
So, (b) is correct.
$a_1 + a_2 + a_3 = 0,\ a_1 + a_3 = 0,\ a_2 = 0 \Rightarrow a_1 = -a_3$
So, (c) is correct.

$$|M-2I|=\begin{vmatrix}-1 & 1 & 1\\ 1 & -2 & 1\\ 0 & 1 & -2\end{vmatrix}=(-1)3-(-2)+1\times 1$$

$= -3 + 2 + 1 = 0$
So, $(M - 2I)$ is non-invertible.
**[Rating : Difficult]**

**6. (a, b) :** $f(0, 1) \to R,\ f(x)=[4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$
where $[\cdot]$ is G.I.F.

$$\begin{cases}0, & 0<x<\frac{1}{4}\\ \left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right), & \frac{1}{4}\le x<\frac{1}{2}\\ 2\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right), & \frac{1}{2}\le x<\frac{3}{4}\\ 3\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right), & \frac{3}{4}\le x<1\end{cases}$$

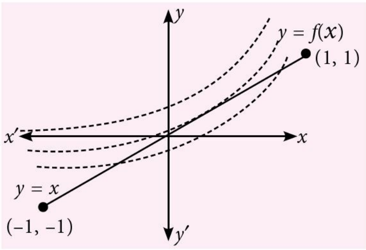
Clearly, $f(x)$ is continuous at $x=\frac{1}{4},\frac{1}{2}$ but not continuous at $x=\frac{3}{4}$.

$f(x)$ is continuous at $x=\frac{1}{2}$ but not differentiable at $x=\frac{1}{2}$.

Clearly, $\min f(x)=-\frac{1}{432}$ at $x=\frac{5}{12}$

**[Rating : Difficult]**

**7. (a, b, c) :**




$f(x)$ is convex function.
So, $y = f(x)$ and $y = x$ can intersect each other at atmost two points and minimum is zero.
**[Rating : Easy]**

**8. (0) :** Since, $x\tan^{-1}x \ge 0$
So, integral of a positive integrand will be minimum at $x\tan^{-1}x = 0$.
$\therefore\ f(x)_{\min} = 0$
**[Rating : Moderate]**

**9. (16) :** $(x^2-5)\frac{dy}{dx}-2xy=-2x(x^2-5)^2$

$\Rightarrow \frac{dy}{dx}-\frac{2x}{x^2-5}y=-2x(x^2-5)$

$\text{I.F.}=e^{\int\frac{-2x}{x^2-5}dx}=e^{-\ln(x^2-5)}=\frac{1}{x^2-5}$

Now, $y\cdot\frac{1}{x^2-5}=\int -2x\,dx+C \Rightarrow y\cdot\frac{1}{x^2-5}=-x^2+c$

$\Rightarrow\ -7 = -4 + c \Rightarrow c = -3$
$\therefore\ y = -(x^4 - 2x^2 - 15) \Rightarrow y = -((x^2 - 1)^2 - 16)$
So, maximum value = 16 **[Rating : Moderate]**

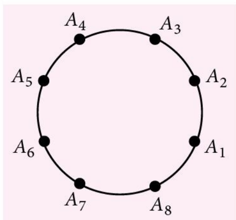
**10. (31) :** $p=\frac{P(\text{multiple of }20)}{P(\text{multiple of }5)}$

$$=\frac{P(\_\,\_\,\_\ \underline{2}\,\underline{0})+P(\_\,\_\,\_\ \underline{4}\,\underline{0})}{\left(2\times\frac{4!}{3!}+\frac{4!}{2!2!}+{}^2C_1\times\frac{4!}{2!}\right)}$$

$$=\frac{3!+{}^2C_1\times 2\times\frac{3!}{2!}+1+3!+2\times\frac{3!}{2!}}{8+6+24}=\frac{31}{38}$$

$\therefore\ 38p = 31$ **[Rating : Difficult]**

**11. (512) :**



$A_i$ are $8^{th}$ roots of $2^8$ and let $P$ be $2e^{i\alpha}$

So, $z^8 - 2^8 = (z - A_1)(z - A_2) \ldots. (z - A_8)$

Put $z = 2e^{i\alpha}$, we get

$$2^8 e^{i8\alpha} - 2^8 = (2e^{i\alpha} - A_1)(2e^{i\alpha} - A_2)(2e^{i\alpha} - A_3) \ldots. (2e^{i\alpha} - A_8)$$

$$\Rightarrow 2^8 |e^{i8\alpha} - 1| = |(2e^{i\alpha} - A_1)(2e^{i\alpha} - A_2)(2e^{i\alpha} - A_3) \ldots. (2e^{i\alpha} - A_8)|$$

$$\Rightarrow 2^8 |e^{i4\alpha} - e^{-i4\alpha}| = |(2e^{i\alpha} - A_1)(2e^{i\alpha} - A_2)(2e^{i\alpha} - A_3) \ldots. (2e^{i\alpha} - A_8)|$$

$$\Rightarrow 2^9 |\sin 4\alpha| = |(2e^{i\alpha} - A_1)(2e^{i\alpha} - A_2)(2e^{i\alpha} - A_3) \ldots. (2e^{i\alpha} - A_8)|$$

$\because$ Maximum value of $\sin 4\alpha = 1$

$\therefore$ Maximum value of the product $PA_1 \cdot PA_2 \ldots PA_8 = 2^9$

**[Rating : Difficult]**

**12. (3780) :** $\Delta = \begin{vmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{vmatrix} = -5(ad - bc)$

Number of invertible matrices = Total numer of matrices Total number of non invertible matrices ($\Delta = 0$)

Total number of matrices = $8 \times 8 \times 8 \times 8 = 4096$

Now, $\Delta = 0 \Rightarrow ad = bc$

**Case (1) :** (i) When all four $a, b, c, d$ are zero = 1

(ii) When three zero, one non-zero $= 7 \times \dfrac{4!}{3!} = 28$

(iii) When two zero and two non zero

$= {}^2C_1 \cdot {}^2C_1 \cdot 7 \cdot 7 = 196$

**Case (2) :** When all 4 are non zero

(i) All 4 are same = 7

(ii) 2same + 2other same $= {}^7C_2 \cdot {}^2C_1 \cdot {}^2C_1 = 21 \times 4 = 84$

Total non invertible matrices = 1 + 28 + 196 + 7 + 84 = 316

Number of invertible matrices = 4096 – 316 = 3780

**[Rating : Difficult]**

**13. (2) :** We have, $C_1 : x^2 + y^2 = 1$,

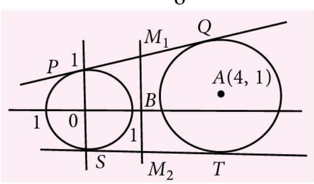
$C_2 : (x - 4)^2 + (y - 1)^2 = r^2$ $(1 < r < 3)$ with centre at $A(4, 1)$

Clearly, $M_1M_2$ is radical axis

$\therefore$ Equation of $M_1M_2$ is $S_1 - S_2 = 0$

$\Rightarrow 8x + 2y - 18 + r^2 = 0$ ...(i)

Putting $y = 0$, we get $x = \dfrac{18 - r^2}{8}$



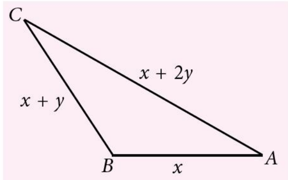

$\therefore$ The coordinates of $B$ are $\dfrac{18 - r^2}{8}, 0$

Now, we have $B\left(\dfrac{18 - r^2}{8}, 0\right), A(4, 1)$ and $AB = \sqrt{5}$

$$\therefore \left(\frac{18 - r^2}{8} - 4\right)^2 + (0 - 1)^2 = 5 \Rightarrow r^2 = 2$$

**[Rating : Difficult]**

**14. (1008) :** Let $B$ be the greatest angle and $C$ be the smallest angle with $AB = x$, $BC = x + y$ and $CA = x + 2y$




Now, $B - C = \dfrac{\pi}{2}$ (given)

$\Rightarrow B = \dfrac{\pi}{2} + C$

Since, $A + B + C = \pi$

$\Rightarrow A + \dfrac{\pi}{2} + C + C = \pi \Rightarrow A = \dfrac{\pi}{2} - 2C$

Since, $AB$, $BC$, $CA$ are in A.P.

$\therefore 2BC = AC + AB$

$\Rightarrow 4R\sin A = 2R\sin B + 2R\sin C$

$\Rightarrow 2\sin A = \sin B + \sin C$

$\Rightarrow 2\sin\left(\dfrac{\pi}{2} - 2C\right) = \sin\left(\dfrac{\pi}{2} + C\right) + \sin C$

$\Rightarrow 2\cos 2C = \cos C + \sin C$

$\Rightarrow 2(\cos^2 C - \sin^2 C) = \cos C + \sin C$

$$\Rightarrow \cos C - \sin C = \frac{1}{2}$$

$$\Rightarrow 1 - \sin 2C = \frac{1}{4} \Rightarrow \sin 2C = \frac{3}{4}$$

Area of $\Delta ABC = \dfrac{AB \cdot BC \cdot CA}{4R} = \dfrac{8\sin A \sin B \sin C}{4}$

$$= 2\sin\left(\frac{\pi}{2} - 2C\right)\sin\left(\frac{\pi}{2} + C\right)\sin C$$

$= 2\cos 2C \cos C \sin C = \cos 2C \sin 2C$

Area of triangle, $a = \sqrt{1 - \dfrac{9}{16}} \cdot \dfrac{3}{4} = \dfrac{3\sqrt{7}}{16}$

$$\Rightarrow 64a = 12\sqrt{7} \Rightarrow (64a)^2 = 1008$$

**15. (0.25) :** Inradius, $r = \dfrac{\Delta}{s}$

$$= \frac{2a}{2R\sin A + 2R\sin B + 2R\sin C} = \frac{a}{\sin A + \sin B + \sin C}$$

$$= \frac{a}{\sin\left(\frac{\pi}{2} - 2C\right) + \sin\left(\frac{\pi}{2} + C\right) + \sin C}$$

$$= \frac{a}{\cos 2C + \cos C + \sin C} = \frac{a}{\cos 2C + \sqrt{1 + \sin 2C}}$$

$$= \frac{\frac{3\sqrt{7}}{16}}{\sqrt{1 - \frac{9}{16}} + \sqrt{1 + \frac{3}{4}}} = \frac{\frac{3\sqrt{7}}{16}}{\frac{\sqrt{7}}{4} + \frac{\sqrt{7}}{2}} = \frac{3\sqrt{7}}{16} \times \frac{4}{3\sqrt{7}} = \frac{1}{4}$$

$\therefore \; r = 0.25$ **[Rating : Difficult]**

**16. (24) :** $P(x = 0) = P(x = 1) = 0$

$$P(x = 2) = \frac{4}{49}, \; P(x = 3) = \frac{20}{49}$$

and $P(x = 4) = 1 - \dfrac{24}{49} = \dfrac{25}{49}$

Now, $E(X) = \displaystyle\sum_{i=0}^{4} i \cdot P(x = i) = 2 \cdot \frac{4}{49} + 3 \cdot \frac{20}{49} + 4 \cdot \frac{25}{49}$

$$= \frac{8 + 60 + 100}{49} = \frac{168}{49} = \frac{24}{7}$$

$\therefore \; 7E(X) = 24$ **[Rating : Difficult]**

**17. (0.50) :** $p = \dfrac{6\times 7 + 6 \times 7}{{}^{49}C_2} = \dfrac{2 \times 6 \times 7 \times 2}{49 \times 48} = \dfrac{1}{14}$

$$7p = \frac{1}{2} = 0.5$$

**[Rating : Difficult]**

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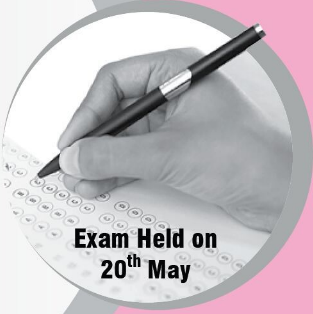
# SOLVED PAPER 2023

# Karnataka CET




**1.** The value of $\cot^{-1}\left[\dfrac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$, where $x \in \left(0, \dfrac{\pi}{4}\right)$ is

(a) $\pi - \dfrac{x}{3}$ (b) $\dfrac{x}{2}$ (c) $\pi - \dfrac{x}{2}$ (d) $\dfrac{x}{2} - \pi$

**2.** If $x\begin{bmatrix}3\\2\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix}$, then the value of $x$ and $y$ are

(a) $x = -4, y = -3$ (b) $x = 4, y = 3$
(c) $x = -4, y = 3$ (d) $x = 4, y = -3$

**3.** If $A$ and $B$ are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(a) $AB$ (b) $A + B$
(c) $2BA$ (d) $2AB$

**4.** If $A = \begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$ is singular matrix, then the value of $5k - k^2$ is equal to

(a) $-4$ (b) $4$ (c) $6$ (d) $-6$

**5.** The area of a triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 sq. units, the value of $k$ is

(a) 6 (b) 9 (c) 3 (d) $-9$

**6.** If $\Delta = \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$ and $\Delta_1 = \begin{vmatrix}1 & 1 & 1\\ bc & ca & ab\\ a & b & c\end{vmatrix}$, then

(a) $\Delta_1 \neq \Delta$ (b) $\Delta_1 = \Delta$
(c) $\Delta_1 = -\Delta$ (d) $\Delta_1 = 3\Delta$

**7.** If $\sin^{-1}\left(\dfrac{2a}{1+a^2}\right) + \cos^{-1}\left(\dfrac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ where $a, x \in (0, 1)$, then the value of $x$ is

(a) $\dfrac{2a}{1+a^2}$ (b) 0 (c) $\dfrac{2a}{1-a^2}$ (d) $\dfrac{a}{2}$

**8.** If $u = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, then $\dfrac{du}{dv}$ is

(a) $\dfrac{1-x^2}{1+x^2}$ (b) $\dfrac{1}{2}$ (c) 1 (d) 2

**9.** The function $f(x) = \cot x$ is discontinuous on every point of the set

(a) $\left\{x = (2n+1)\dfrac{\pi}{2}; n \in Z\right\}$

(b) $\{x = n\pi; n \in Z\}$

(c) $\left\{x = \dfrac{n\pi}{2}; n \in Z\right\}$

(d) $\{x = 2n\pi; n \in Z\}$

**10.** If the function is $f(x) = \dfrac{1}{x+2}$, then the point of discontinuity of the composite function $y = f(f(x))$ is

(a) $\dfrac{2}{5}$ (b) $\dfrac{-5}{2}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{5}{2}$

**11.** If $y = a\sin x + b\cos x$, then $y^2 + \left(\dfrac{dy}{dx}\right)^2$ is a

(a) function of $x$ and $y$ (b) function of $x$
(c) constant (d) function of $y$

**12.** If $f(x) = 1 + nx + \dfrac{n(n-1)}{2}x^2 + \dfrac{n(n-1)(n-2)}{6}x^3 + \ldots\ldots + x^n$, then $f''(1) =$

(a) $n(n-1)2^n$ (b) $(n-1)2^{n-1}$
(c) $2^{n-1}$ (d) $n(n-1)2^{n-2}$

**13.** If $A = \begin{bmatrix}1 & \tan\dfrac{\alpha}{2}\\ -\tan\dfrac{\alpha}{2} & 1\end{bmatrix}$ and $AB = I$, then $B =$

(a) $\cos^2\frac{\alpha}{2}\cdot I$
(b) $\cos^2\frac{\alpha}{2}\cdot A^T$
(c) $\sin^2\frac{\alpha}{2}\cdot A$
(d) $\cos^2\frac{\alpha}{2}\cdot A$

**14.** A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. The rate at which its area is increasing when the radius is 5.2 cm is

(a) 5.05 π cm$^2$/sec
(b) 5.2 π cm$^2$/sec
(c) 0.52 π cm$^2$/sec
(d) 27.4 π cm$^2$/sec

**15.** The distance '*s*' in meters travelled by a particle in '*t*' seconds is given by $s=\frac{2t^3}{3}-18t+\frac{5}{3}$. The acceleration when the particle comes to rest is

(a) 12 m$^2$/sec.
(b) 3 m$^2$/sec.
(c) 18 m$^2$/sec.
(d) 10 m$^2$/sec.

**16.** A particle moves along the curve $\frac{x^2}{16}+\frac{y^2}{4}=1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

(a) III or IV
(b) I or III
(c) II or III
(d) II or IV

**17.** An enemy fighter jet is flying along the curve given by $y = x^2 + 2$. A soldier is placed at (3, 2) wants to shoot down the jet when it is nearest to him. Then the nearest distance is

(a) 2 units
(b) $\sqrt{3}$ units
(c) $\sqrt{5}$ units
(d) $\sqrt{6}$ units

**18.** $\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}\,dx=$

(a) 4
(b) 5
(c) 3
(d) 6

**19.** $\int\sqrt{\operatorname{cosec} x-\sin x}\,dx=$

(a) $2\sqrt{\sin x}+C$
(b) $\sqrt{\sin x}+C$
(c) $\frac{2}{\sqrt{\sin x}}+C$
(d) $\frac{\sqrt{\sin x}}{2}+C$

**20.** If $f(x)$ and $g(x)$ are two functions with $g(x)=x-\frac{1}{x}$ and $fog(x)=x^3-\frac{1}{x^3}$, then $f'(x)$ =

(a) $x^2-\frac{1}{x^2}$
(b) $3x^2 + 3$
(c) $1-\frac{1}{x^2}$
(d) $3x^2+\frac{3}{x^4}$

**21.** $\int\frac{1}{1+3\sin^2 x+8\cos x^2}dx=$

(a) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$
(b) $\frac{1}{6}\tan^{-1}(2\tan x)+C$
(c) $6\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$
(d) $\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

**22.** $\int_{-2}^{0}(x^3+3x^2+3x+3+(x+1)\cos(x+1))dx=$

(a) 4
(b) 0
(c) 1
(d) 3

**23.** $\int_0^{\pi}\frac{x\tan x}{\sec x\cdot\operatorname{cosec} x}dx=$

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi^2}{2}$
(d) $\frac{\pi^2}{4}$

**24.** $\int\sqrt{5-2x+x^2}\,dx=$

(a) $\frac{x-1}{2}\sqrt{5+2x+x^2}+2\log\left|(x-1)+\sqrt{5+2x+x^2}\right|+C$
(b) $\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|(x+1)+\sqrt{x^2+2x+5}\right|+C$
(c) $\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|(x-1)+\sqrt{5-2x+x^2}\right|+C$
(d) $\frac{x}{2}\sqrt{5-2x+x^2}+4\log\left|(x+1)+\sqrt{x^2-2x+5}\right|+C$

**25.** The area of the region bounded by the line $y = x + 1$, and the lines $x = 3$ and $x = 5$ is

(a) $\frac{11}{2}$ sq. units
(b) 10 sq. units
(c) 7 sq. units
(d) $\frac{7}{2}$ sq. units

**26.** If a curve passes through the point (1, 1) and at any point $(x, y)$ on the curve,the product of the slope of its tangent and $x$ co-ordinate of the point is equal to the $y$ co-ordinate of the point, then the curve also passes through the point

(a) (–1, 2)
(b) (2, 2)
(c) $(\sqrt{3},0)$
(d) (3, 0)

**27.** The degree of the differential equation

$$1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1} \text{ is}$$

(a) 1 (b) 6 (c) 2 (d) 3

**28.** If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$, then

(a) $\vec{a}$ and $\vec{b}$ are coincident.

(b) $\vec{a}$ and $\vec{b}$ are perpendicular.

(c) Inclined to each other at 60°.

(d) $\vec{a}$ and $\vec{b}$ are parallel.

**29.** The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is

(a) $6\sqrt{6}$ (b) $\sqrt{6}$ (c) $\frac{\sqrt{6}}{6}$ (d) 6

**30.** In the interval $\left(0,\frac{\pi}{2}\right)$, area lying between the curves $y = \tan x$ and $y = \cot x$ and the $X$-axis is

(a) 4 log 2 sq. units (b) 3 log 2 sq. units

(c) log 2 sq. units (d) 2 log 2 sq. units

**31.** If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})$ $=\lambda(\vec{b}\times\vec{c})$ then the value of $\lambda$ is equal to

(a) 4 (b) 2 (c) 6 (d) 3

**32.** If a line makes an angle of $\pi/3$ with each $X$ and $Y$ axis then the acute angle made by $Z$-axis is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

**33.** The length of perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

(a) $\sqrt{33}$ (b) $\sqrt{66}$ (c) $\sqrt{53}$ (d) $\sqrt{29}$

**34.** The equation of the plane through the points (2, 1, 0), (3, 2, –2) and (3, 1, 7) is

(a) $6x - 3y + 2z - 7 = 0$ (b) $3x - 2y + 6z - 27 = 0$

(c) $7x - 9y - z - 5 = 0$ (d) $2x - 3y + 4z - 27 = 0$

**35.** The point of intersection of the line $x + 1 = \frac{y+3}{3}=\frac{-z+2}{2}$ with the plane $3x + 4y + 5z = 10$ is

(a) (2, 6, –4) (b) (–2, 6, –4)

(c) (2, 6, 4) (d) (2, –6, –4)

**36.** If (2, 3, –1) is the foot of the perpendicular from (4, 2, 1) to a plane, then the equation of the plane is

(a) $2x - y + 2z = 0$ (b) $2x - y + 2z + 1 = 0$

(c) $2x + y + 2z - 5 = 0$ (d) $2x + y + 2z - 1 = 0$

**37.** $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$ then $|\vec{b}|$ is equal to

(a) 8 (b) 12 (c) 4 (d) 3

**38.** If $A$ and $B$ are events such that $P(A)=\frac{1}{4}$, $P\left(\frac{A}{B}\right)=\frac{1}{2}$ and $P\left(\frac{B}{A}\right)=\frac{2}{3}$ then $P(B)$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

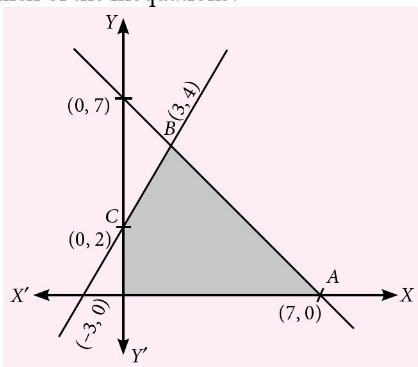
**39.** A bag contains $2n + 1$ coins. It is known that $n$ of these coins have head on both sides whereas the other $n + 1$ coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $\frac{31}{42}$, then the value of $n$ is

(a) 8 (b) 5 (c) 10 (d) 6

**40.** Let $A = \{x, y, z, u\}$ and $B = \{a, b\}$. A function $f : A \to B$ is selected randomly. The probability that the function is an onto function is

(a) $\frac{5}{8}$ (b) $\frac{7}{8}$ (c) $\frac{1}{35}$ (d) $\frac{1}{8}$

**41.** The shaded region in the figure given is the solution of which of the inequations?




(a) $x + y \geq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(b) $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(c) $x + y \leq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

(d) $x + y \geq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

**42.** If $f(x) = ax + b$, where $a$ and $b$ are integers, $f(-1) = -5$ and $f(3) = 3$ then $a$ and $b$ are respectively

(a) 0, 2 (b) –3, –1 (c) 2, 3 (d) 2, –3

**43.** The value of

$$e^{\log_{10}\tan 1^\circ+\log_{10}\tan 2^\circ+\log_{10}\tan 3^\circ+....+\log_{10}\tan 89^\circ} \text{ is}$$

(a) $\frac{1}{e}$ (b) 0 (c) 1 (d) 3

44. The value of $\begin{vmatrix} \sin^2 14° & \sin^2 66° & \tan 135° \\ \sin^2 66° & \tan 135° & \sin^2 14° \\ \tan 135° & \sin^2 14° & \sin^2 66° \end{vmatrix}$ is

(a) 1 (b) −1 (c) 2 (d) 0

45. The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{4}{\sqrt{2}}$ (c) $\frac{\sqrt{2}}{4}$ (d) $\frac{2}{\sqrt{2}}$

46. Given that $a$, $b$ and $x$ are real numbers and $a < b$, $x < 0$ then

(a) $\frac{a}{x} < \frac{b}{x}$ (b) $\frac{a}{x} > \frac{b}{x}$

(c) $\frac{a}{x} \le \frac{b}{x}$ (d) $\frac{a}{x} \ge \frac{b}{x}$

47. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is

(a) ${}^6C_3 \times {}^4P_2$ (b) ${}^6C_3 \times {}^4C_2$

(c) ${}^6P_3 \times {}^4C_2$ (d) ${}^6P_3 \times {}^4P_2$

48. Which of the following is an empty set?

(a) $\{x : x^2 - 9 = 0, x \in R\}$

(b) $\{x : x^2 - 1 = 0, x \in R\}$

(c) $\{x : x^2 = x + 2, x \in R\}$

(d) $\{x : x^2 + 1 = 0, x \in R\}$

49. $n^{\text{th}}$ term of the series $1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + .....$ is

(a) $\frac{2n-1}{7^n}$ (b) $\frac{2n-1}{7^{n-1}}$

(c) $\frac{2n+1}{7^{n-1}}$ (d) $\frac{2n+1}{7^n}$

50. If $p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in A.P., then $p, q, r$

(a) are in A.P. (b) are not in A.P.

(c) are not in G.P. (d) are in G.P.

51. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its $y$-intercept is

(a) 1 (b) 1/3

(c) 4/3 (d) 2/3

52. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(a) $2x^2 - 3y^2 = 7$ (b) $x^2 - y^2 = 32$

(c) $y^2 - x^2 = 32$ (d) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

53. If $\lim\limits_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = A\cos B$, then the values of $A$ and $B$ respectively are

(a) 2, 1 (b) 2, 2

(c) 1, 1 (d) 1, 2

54. If $n$ is even and the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924\, x^6$, then $n$ is equal to

(a) 12 (b) 10

(c) 8 (d) 14

55. The mean of 100 observations is 50 and their standard deviation is 5. Then the sum of squares of all observations is

(a) 250000 (b) 50000

(c) 255000 (d) 252500

56. $f : R \to R$ and $g : [0, \infty) \to R$ are defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true?

(a) $(fog)(2) = 2$ (b) $(gof)(4) = 4$

(c) $(gof)(-2) = 2$ (d) $(fog)(-4) = 4$

57. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$ then $gof$ is

(a) $\frac{3x^2}{x^4 + 2x^2 - 4}$ (b) $\frac{3x^2 - 5}{9x^4 - 30x^2 + 26}$

(c) $\frac{3x^2}{9x^4 + 30x^2 - 2}$ (d) $\frac{3x^2 - 5}{9x^4 - 6x^2 + 26}$

58. Let the relation $R$ be defined in $N$ by $aRb$ if $3a + 2b = 27$ then $R$ is

(a) {(1, 12), (3, 9), (5, 6), (7, 3), (9, 0)}

(b) {(1, 12), (3, 9), (5, 6), (7, 3)}

(c) {(2, 1), (9, 3), (6, 5), (3, 7)}

(d) $\left\{\left(0, \frac{27}{2}\right), (1, 12), (3, 9), (5, 6), (7, 3)\right\}$

59. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain

(a) $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

(c) $x \in \left[0, \frac{\pi}{4}\right]$ (d) $x \in \left[\frac{-\pi}{8}, \frac{\pi}{8}\right]$

**60.** The contrapositive of the statement

"If two lines do not intersect in the same plane then they are parallel." is

(a) If two lines are not parallel then they do not intersect in the same plane.

(b) If two lines are not parallel then they intersect in the same plane.

(c) If two lines are parallel then they do not intersect in the same plane.

(d) If two lines are parallel then they intersect in the same plane.

## SOLUTIONS

**1. (c) :** We have,

$$\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right], x \in \left(0, \frac{\pi}{4}\right)$$

$$= \cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}} \times \frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}+\sqrt{1+\sin x}}\right]$$

$$= \cot^{-1}\left[\frac{1-\sin x+1+\sin x+2\sqrt{1-\sin^2 x}}{1-\sin x-1-\sin x}\right]$$

$$= \cot^{-1}\left[\frac{2+2\cos x}{-2\sin x}\right] = \cot^{-1}\left(\frac{2(1+\cos x)}{-2\sin x}\right)$$

$$= \cot^{-1}\left(\frac{2\cos^2 x/2}{-2\sin x/2\cos x/2}\right)$$

[$\because$ $1 + \cos x = 2\cos^2 x/2$, $\sin x = 2\sin x/2 \cos x/2$]

$= \cot^{-1}(-\cot x/2) = \pi - \cot^{-1}(\cot x/2)$

[$\because$ $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$]

$= \pi - \dfrac{x}{2}$ [$\because$ $\cot^{-1}(\cot x) = x, x \in (0, \pi)$]

**2. (b) :** We have, $x\begin{bmatrix}3\\2\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}3x\\2x\end{bmatrix} + \begin{bmatrix}y\\-y\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix} \Rightarrow \begin{bmatrix}3x+y\\2x-y\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix}$$

$\Rightarrow$ $3x + y = 15$ ...(i)

$2x - y = 5$ ...(ii)

Solving equation (i) and (ii), we get $x = 4$ and $y = 3$

**3. (b) :** We have, $AB = B$ and $BA = A$

Now, $A^2 + B^2 = (BA)^2 + (AB)^2 = BA \cdot BA + AB \cdot AB$

$= B(AB)A + A(BA)B = B(BA) + A(AB)$

$= BA + AB = A + B$

**4. (b) :** $A = \begin{bmatrix}2-k & 2\\1 & 3-k\end{bmatrix}$ is a singular matrix

$\therefore$ $|A| = 0$

$\Rightarrow$ $(2 - k)(3 - k) - 2 = 0$

$\Rightarrow$ $6 - 2k - 3k + k^2 - 2 = 0$

$\Rightarrow$ $k^2 - 5k + 4 = 0 \Rightarrow 5k - k^2 = 4$

**5. (c) :** Vertices of triangle are $(-3, 0)$, $(3, 0)$ and $(0, k)$.

Area of the triangle = 9 sq. units (Given)

$$\Rightarrow \frac{1}{2}\left\|\begin{matrix}-3 & 0 & 1\\3 & 0 & 1\\0 & k & 1\end{matrix}\right\| = 9$$

$$\Rightarrow \frac{1}{2}|-3(0-k)-0+1(3k-0)| = 9 \Rightarrow \frac{1}{2}|3k+3k| = 9$$

$\Rightarrow$ $|3k| = 9 \Rightarrow 3k = 9$ or $3k = -9$

$\Rightarrow$ $k = 3$ or $k = -3$

# SAMURAI SUDOKU

## ANSWER - JUNE 2023


| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 4 | 3 | 9 | 7 | 2 | 5 | 6 | 1 | | | | 1 | 7 | 8 | 5 | 6 | 3 | 9 | 4 | 2 |
| 5 | 6 | 9 | 1 | 3 | 4 | 7 | 8 | 2 | | | | 3 | 5 | 2 | 8 | 9 | 4 | 6 | 7 | 1 |
| 2 | 1 | 7 | 8 | 6 | 5 | 4 | 3 | 9 | | | | 4 | 6 | 9 | 2 | 7 | 1 | 8 | 5 | 3 |
| 6 | 5 | 1 | 7 | 4 | 3 | 9 | 2 | 8 | | | | 9 | 8 | 4 | 3 | 5 | 2 | 7 | 1 | 6 |
| 7 | 3 | 4 | 2 | 9 | 8 | 6 | 1 | 5 | | | | 6 | 2 | 7 | 1 | 8 | 9 | 5 | 3 | 4 |
| 9 | 8 | 2 | 5 | 1 | 6 | 3 | 7 | 4 | | | | 5 | 1 | 3 | 7 | 4 | 6 | 2 | 9 | 8 |
| 1 | 9 | 5 | 6 | 2 | 7 | 8 | 4 | 3 | 6 | 5 | 2 | 7 | 9 | 1 | 6 | 3 | 8 | 4 | 2 | 5 |
| 4 | 2 | 6 | 3 | 8 | 9 | 1 | 5 | 7 | 9 | 8 | 3 | 2 | 4 | 6 | 9 | 1 | 5 | 3 | 8 | 7 |
| 3 | 7 | 8 | 4 | 5 | 1 | 2 | 9 | 6 | 4 | 1 | 7 | 8 | 3 | 5 | 4 | 2 | 7 | 1 | 6 | 9 |
| | | | | | | 7 | 6 | 4 | 1 | 3 | 9 | 5 | 8 | 2 | | | | | | |
| | | | | | | 3 | 2 | 8 | 5 | 4 | 6 | 9 | 1 | 7 | | | | | | |
| | | | | | | 5 | 1 | 9 | 7 | 2 | 8 | 3 | 6 | 4 | | | | | | |
| 8 | 5 | 1 | 9 | 7 | 6 | 4 | 3 | 2 | 8 | 7 | 1 | 6 | 5 | 9 | 7 | 8 | 4 | 2 | 1 | 3 |
| 7 | 4 | 9 | 3 | 2 | 5 | 6 | 8 | 1 | 2 | 9 | 5 | 4 | 7 | 3 | 1 | 2 | 6 | 9 | 5 | 8 |
| 6 | 3 | 2 | 1 | 8 | 4 | 9 | 7 | 5 | 3 | 6 | 4 | 1 | 2 | 8 | 5 | 3 | 9 | 7 | 4 | 6 |
| 4 | 8 | 5 | 7 | 3 | 2 | 1 | 6 | 9 | | | | 3 | 4 | 5 | 2 | 1 | 7 | 6 | 8 | 9 |
| 2 | 9 | 7 | 6 | 5 | 1 | 3 | 4 | 8 | | | | 8 | 9 | 6 | 3 | 4 | 5 | 1 | 2 | 7 |
| 1 | 6 | 3 | 8 | 4 | 9 | 2 | 5 | 7 | | | | 7 | 1 | 2 | 9 | 6 | 8 | 5 | 3 | 4 |
| 9 | 7 | 4 | 2 | 6 | 8 | 5 | 1 | 3 | | | | 9 | 3 | 1 | 4 | 7 | 2 | 8 | 6 | 5 |
| 3 | 2 | 6 | 5 | 1 | 7 | 8 | 9 | 4 | | | | 2 | 8 | 7 | 6 | 5 | 3 | 4 | 9 | 1 |
| 5 | 1 | 8 | 4 | 9 | 3 | 7 | 2 | 6 | | | | 5 | 6 | 4 | 8 | 9 | 1 | 3 | 7 | 2 |

***Winner :*** *Munish Sharma*

6. **(c) :** We have, $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & (b-a) & (b-a)(b+a) \\ 0 & (c-a) & (c+a)(c-a) \end{vmatrix}$$

Taking common $(b - a)$ and $(c - a)$ from $R_2$ and $R_3$ respectively, we get

$$\Delta = (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

Expanding along $C_1$, we get

$\Delta = (b - a)(c - a)\{1(c + a - b - a)\}$

$\Rightarrow \quad \Delta = (b - a)(c - a)(c - b) = (a - b)(b - c)(c - a)$

Also, given $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ bc & c(a-b) & b(a-c) \\ a & (b-a) & (c-a) \end{vmatrix}$$

Taking $(a - b)$ and $(c - a)$ common from $C_2$ and $C_3$, we get $\Delta_1 = (a-b)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ bc & c & -b \\ a & -1 & 1 \end{vmatrix}$

Expanding along $R_1$, we get $\Delta_1 = (a - b)(c - a)\{1(c - b)\}$

$\Rightarrow \quad \Delta_1 = -(a - b)(b - c)(c - a)$

$\therefore \quad \Delta_1 = -\Delta$

7. **(c) :** We have, $\sin^{-1}\left[\frac{2a}{1+a^2}\right] + \cos^{-1}\left[\frac{1-a^2}{1+a^2}\right]$

$$= \tan^{-1}\left[\frac{2x}{1-x^2}\right], a, x \in (0, 1)$$

$\Rightarrow \quad 2\tan^{-1} a + 2\tan^{-1} a = 2\tan^{-1} x$

$$\left[\because \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x, -1 \le x \le 1 \right.$$
$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x, 0 \le x < \infty$$
$$\left. \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x, -1 < x < 1 \right]$$

$\Rightarrow \quad 4\tan^{-1} a = 2\tan^{-1}x \Rightarrow 2\tan^{-1} a = \tan^{-1}x$

$\Rightarrow \quad \tan^{-1}\left(\frac{2a}{1-a^2}\right) = \tan^{-1} x \Rightarrow x = \frac{2a}{1-a^2}$

8. **(c) :** We have, $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right), v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\Rightarrow \quad u = 2\tan^{-1}x, v = 2\tan^{-1}x$

Differentiate $u$ and $v$ w.r.t. $x$, we get

$$\frac{du}{dx} = \frac{2}{1+x^2}, \frac{dv}{dx} = \frac{2}{1+x^2} \qquad \left[\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right]$$

Now, $\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = \frac{2}{1+x^2} \times \frac{1+x^2}{2} = 1$

9. **(b) :** We have, $f(x) = \cot x$

Domain of $f = R - n\pi, n \in Z$

$\cot x$ is continuous at all points except $\{x = n\pi; n \in Z\}$

10. **(b) :** We have, $f(x) = \frac{1}{x+2}$

$f(x)$ is not defined at $x = -2$.

$$y = f(f(x)) = f\left(\frac{1}{x+2}\right) = \frac{1}{\frac{1}{x+2}+2} = \frac{x+2}{2x+5}$$

$y$ is not defined at $x = -5/2$

$\therefore \quad y$ is not continuous at $x = -5/2$

11. **(c) :** We have, $y = a\sin x + b\cos x$

Differentiate $y$ w.r.t. $x$, we get

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now, $y^2 + \left(\frac{dy}{dx}\right)^2 = (a\sin x + b\cos x)^2 + (a\cos x - b\sin x)^2$

$$\Rightarrow \quad y^2 + \left(\frac{dy}{dx}\right)^2 = a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x + a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$$

$$\Rightarrow \quad y^2 + \left(\frac{dy}{dx}\right)^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$$

$= a^2 + b^2 =$ Constant

12. **(d) :** We have,

$$f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots + x^n$$

It can be written as $f(x) = (1 + x)^n$

Now, differentiate w.r.t. $x$, we get

$$f'(x) = n(1 + x)^{n-1}$$

Again, differentiate w.r.t. $x$, we get

$$f''(x) = n(n-1)(1 + x)^{n-2}$$

Put $x = 1$, we get $f''(1) = n(n-1)(1 + 1)^{n-2} = n(n-1)2^{n-2}$

**13. (b) :** We have, $A=\begin{bmatrix}1 & \tan\alpha/2\\ -\tan\alpha/2 & 1\end{bmatrix}$

$\therefore \quad |A| = 1 + \tan^2\alpha/2 = \sec^2\alpha/2$

Also, $AB = I$

Pre-multiplying by $A^{-1}$ on both sides, we get

$A^{-1}AB = A^{-1}$

$\Rightarrow \quad B = A \qquad [\because A^{-1}A = I]$

$A_{11} = 1, A_{12} = \tan\alpha/2, A_{21} = -\tan\alpha/2, A_{22} = 1$

Now, $\text{adj } A=\begin{bmatrix}1 & -\tan\alpha/2\\ \tan\alpha/2 & 1\end{bmatrix}$

and $A^{-1}=\frac{1}{|A|}(\text{adj } A)=\frac{1}{\sec^2\alpha/2}\begin{bmatrix}1 & -\tan\alpha/2\\ \tan\alpha/2 & 1\end{bmatrix}$

$\Rightarrow \quad A^{-1} = \cos^2\alpha/2 \cdot A^T$

**14. (c) :** We know that area of circle, $A = \pi r^2$

We have, $\frac{dr}{dt}=0.05$ cm/sec

Now, $\frac{dA}{dt}=2\pi r\frac{dr}{dt}=2\pi(5.2)\times 0.05 \qquad [\because r = 5.2 \text{ cm}]$

$\Rightarrow \quad \frac{dA}{dt}=0.52\pi \text{ cm}^2/\text{sec}$

**15. (a) :** Distance travelled by a particle in '$t$' seconds,

$s=\frac{2t^3}{3}-18t+\frac{5}{3}$

$\therefore \quad \frac{ds}{dt}=2t^2-18$

When particle is at rest, then $\frac{ds}{dt}=0$

$\therefore \quad 2t^2 - 18 = 0 \Rightarrow t^2 = 9$

$\Rightarrow \quad t = 3 \qquad (\because t = -3$ is not possible$)$

Now, $\frac{d^2s}{dt^2}=4t=a(t)$

$\therefore \quad$ Acceleration at $t = 3$

$a(3)=4(3)=12 \text{ m}^2/\text{sec}$

**16. (d) :** We have, $\frac{x^2}{16}+\frac{y^2}{4}=1 \qquad \text{...(i)}$

and $\frac{dx}{dt}=\frac{4dy}{dt}$

Differentiating (i) w.r.t. $t$, we get

$\frac{2x}{16}\frac{dx}{dt}+\frac{2}{4}y\frac{dy}{dt}=0 \Rightarrow \frac{2}{16}x\times\frac{4dy}{dt}+\frac{1}{2}y\frac{dy}{dt}=0$

$\Rightarrow \quad \frac{x}{2}\frac{dy}{dt}=-\frac{1}{2}y\frac{dy}{dt} \Rightarrow x=-y \text{ or } y=-x$

$\therefore \quad$ The particle lies in quadrant II or IV.

**17. (c) :** We have, $y = x^2 + 2 \Rightarrow y - 2 = x^2 \qquad \text{...(i)}$

Let $P(x, y)$ be the position of jet and soldier is placed at $Q(3, 2)$.

Now, $PQ=\sqrt{(x-3)^2+(y-2)^2}$

$\Rightarrow \quad PQ^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow \quad PQ^2 = (x-3)^2 + x^4 \qquad$ [Using (i)]

Let $D = (x-3)^2 + x^4$

Differentiating w.r.t. $x$, we get

$\frac{dD}{dx}=2(x-3)+4x^3$

Put $\frac{d\omega}{dx}=0$

$\Rightarrow \quad 2x - 6 + 4x^3 = 0 \Rightarrow x - 3 + 2x^3 = 0$

$\Rightarrow \quad (x-1)(2x^2 + 2x + 3) = 0 \Rightarrow x = 1$

[$\because$ Real roots of $2x^2 + 2x + 3 = 0$ does not exist]

Again differentiating w.r.t. $x$, we get

$\frac{d^2D}{dx^2}=2+12x^2=2+12(1)=14>0$

$\therefore \quad$ Distance is minimum at $x = 1$

At $x = 1, y = 1 + 2 = 3$

$\therefore \quad$ Nearest distance $=PQ=\sqrt{(1-3)^2+(3-2)^2}$

$=\sqrt{4+1}=\sqrt{5}$ units

**18. (c) :** Let $I=\int_2^8\frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}dx \qquad \text{...(i)}$

$\Rightarrow \quad I=\int_2^8\frac{5^{\sqrt{10-10+x}}}{5^{\sqrt{10-x}}+5^{\sqrt{x}}}dx$

$\Rightarrow \quad I=\int_2^8\frac{5^{\sqrt{x}}}{5^{\sqrt{10-x}}+5^{\sqrt{x}}}dx \qquad \text{...(ii)}$

Adding (i) and (ii), we get

$2I=\int_2^8\frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}dx+\int_2^8\frac{5^{\sqrt{x}}}{5^{\sqrt{10-x}}+5^{\sqrt{x}}}dx$

$\Rightarrow \quad 2I=\int_2^8\frac{5^{\sqrt{10-x}}+5^{\sqrt{x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}dx=\int_2^8 1\,dx$

$\Rightarrow \quad 2I=[x]_2^8=6 \Rightarrow I=3$

**19. (a) :** We have, $\int\sqrt{\text{cosec } x-\sin x}\,dx$

$=\int\sqrt{\frac{1}{\sin x}-\sin x}\,dx=\int\sqrt{\frac{1-\sin^2 x}{\sin x}}\,dx=\int\sqrt{\frac{\cos^2 x}{\sin x}}\,dx$

$=\int\frac{\cos x}{\sqrt{\sin x}}\,dx=2\sqrt{\sin x}+C$

[Putting $t=\sin x \Rightarrow dt=\cos x\,dx$

$\therefore \int\frac{dt}{\sqrt{t}}=2\sqrt{t}+C=2\sqrt{\sin x}+C$]

**20. (b) :** We have, $g(x)=x-\frac{1}{x}$

$$fog(x)=x^3-\frac{1}{x^3} \Rightarrow f(g(x))=x^3-\frac{1}{x^3}$$

$$\Rightarrow \quad f(g(x))=f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$$

$$\Rightarrow \quad f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left(\left(x-\frac{1}{x}\right)^2+2+1\right)$$

$$=\left(x-\frac{1}{x}\right)\left(\left(x-\frac{1}{x}\right)^2+3\right)$$

Now, replacing $\left(\frac{x-1}{x}\right)$ by $x$, we get

$$f(x) = x(x^2 + 3) = x^3 + 3x$$

Differentiating $f(x)$ w.r.t. $x$, we get

$$f'(x) = 3x^2 + 3$$

**21. (a) :** We have, $\int\frac{1}{1+3\sin^2 x+8\cos^2 x}dx$

$$=\int\frac{dx}{1+3\sin^2 x+3\cos^2 x+5\cos^2 x}$$

$$=\int\frac{dx}{4+5\cos^2 x}=\int\frac{\sec^2 x}{4\sec^2 x+5}dx$$

$$=\int\frac{\sec^2 x}{4(1+\tan^2 x)+5}dx=\int\frac{\sec^2 x}{4\tan^2 x+9}dx$$

Let $2\tan x = t$

$$\Rightarrow \quad 2\sec^2 x\, dx = dt \Rightarrow \quad \sec^2 x\, dx=\frac{1}{2}dt$$

$$\therefore \quad \int\frac{1}{1+3\sin^2 x+8\cos^2 x}dx=\frac{1}{2}\int\frac{dt}{t^2+(3)^2}$$

$$=\frac{1}{2}\times\frac{1}{3}\tan^{-1}\frac{t}{3}+C \;=\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$$

**22. (a) :** $\int_{-2}^{0}(x^3+3x^2+3x+3+(x+1)\cos(x+1))dx$

$$=\int_{-2}^{0}(x^3+3x^2+3x+3)dx+\int_{-2}^{0}(x+1)\cos(x+1)dx$$

$$=\left[\frac{x^4}{4}+x^3+\frac{3x^2}{2}+3x\right]_{-2}^{0}+[(x+1)\sin(x+1)]_{-2}^{0}-\int_{-2}^{0}\sin(x+1)dx$$

$$=-\frac{16}{4}+8-\frac{12}{2}+6+\sin(1)+\sin(-1)+[\cos(x+1)]\big|_{-2}^{0}$$

$$= -4 + 8 - 6 + 6 + \sin(1) - \sin(1) + \cos(1) - \cos(1)$$

$$= 4$$

**23. (d) :** We have, $\int_0^\pi\frac{x\tan x}{\sec x\cdot \operatorname{cosec} x}dx$

$$=\int_0^\pi\frac{x\sin x}{\frac{1}{\cos x}\cdot\cos x\cdot\frac{1}{\sin x}}dx=\int_0^\pi x\sin^2 x\,dx$$

$$=\int_0^\pi x\left(\frac{1-\cos 2x}{2}\right)dx=\frac{1}{2}\int_0^\pi x\,dx-\frac{1}{2}\int_0^\pi x\cos 2x\,dx$$

$$=\frac{1}{2}\left[\frac{x^2}{2}\right]_0^\pi-\frac{1}{2}\left[\frac{x\sin 2x}{2}\bigg|_0^\pi-\frac{1}{2}\int_0^\pi\sin 2x\,dx\right]$$

$$=\frac{1}{2}\times\frac{\pi^2}{2}-\frac{1}{8}[\cos 2x]_0^\pi=\frac{\pi^2}{4}-\frac{1}{8}(\cos 2\pi-\cos 0)=\frac{\pi^2}{4}$$

**24. (c) :** We have, $\int\sqrt{5-2x+x^2}\,dx=\int\sqrt{5+(x^2-2x)}\,dx$

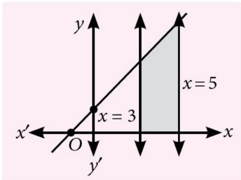
$$=\int\sqrt{5+(x^2-2x+1)-1}\,dx=\int\sqrt{2^2+(x-1)^2}\,dx$$

$$=\frac{x-1}{2}\sqrt{5-2x+x^2}+\frac{4}{2}\log|(x-1)+\sqrt{5-2x+x^2}|+C$$

$$\left[\because \int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+C\right]$$

$$=\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log|(x-1)+\sqrt{5-2x+x^2}|+C$$

**25. (b) :** We have, $y = x + 1$, $x = 3$ and $x = 5$




$\therefore$ The area of this region bounded by the line $y = x + 1$ and lines $x = 3$ and $x = 5$

$$=\int_3^5(x+1)dx=\left[\frac{x^2}{2}+x\right]_3^5$$

$$=\frac{25}{2}+5-\frac{9}{2}-3=8+2=10 \text{ sq. units}$$

**26. (b) :** According to question,

$$x\frac{dy}{dx}=y \quad \Rightarrow \quad \int\frac{dy}{y}=\int\frac{dx}{x}$$

$$\Rightarrow \quad \ln y = \ln x + \ln c \quad \Rightarrow \quad \ln y = \ln xc \quad \Rightarrow \quad y = cx$$

Put $x = 1$, $y = 1$, then we get $1 = c$

$\therefore$ $y = x$, which passes through the point (2, 2).

**27. (b) :** We have, $1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1}$

$$\Rightarrow \left(1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2\right)^3=\frac{d^2y}{dx^2}+1$$

$\therefore$ Degree of the given differential equation = 6

**28. (b) :** $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ (Given)

Squaring on both sides, we get

$$|\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}$$

$$\Rightarrow 4\,\vec{a}\cdot\vec{b}=0 \Rightarrow \vec{a}\cdot\vec{b}=0$$

$\therefore$ $\vec{a}$ and $\vec{b}$ are perpendicular

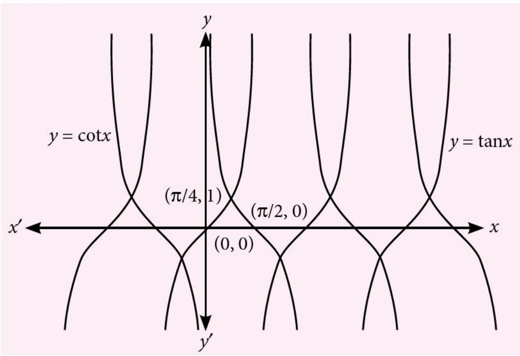
**29. (c) :** Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

$$|\vec{a}|=\sqrt{1^2+1^2+2^2}=\sqrt{6}$$

$$\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}=\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$$

$$\hat{i}\cdot\hat{a}=\hat{i}\cdot\left(\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}\right)=\frac{1}{\sqrt{6}}=\frac{\sqrt{6}}{6}$$

**30. (c) :** Given curves are $y = \tan x$ and $y = \cot x$




Now, the area lying between given curves

$$=\int_0^{\pi/4}\tan x\,dx+\int_{\pi/4}^{\pi/2}\cot x\,dx$$

$$=|\log\sec x|_0^{\pi/4}+\log|\sin x|_{\pi/4}^{\pi/2}$$

$$=\log\sec\frac{\pi}{4}-0+\log(1)-\log|\sin\pi/4|$$

$$=\log\sqrt{2}-\log\frac{1}{\sqrt{2}}=\frac{1}{2}\log 2+\frac{1}{2}\log 2=\log 2 \text{ sq. units}$$

**31. (c) :** We have, $\vec{a}+2\vec{b}+3\vec{c}=0$

$$\Rightarrow \vec{a}=-(2\vec{b}+3\vec{c})$$

$$\Rightarrow \vec{a}\times\vec{a}=-(2\vec{b}\times\vec{a}+3\vec{c}\times\vec{a})$$

$$\Rightarrow 2(\vec{b}\times\vec{a})+3(\vec{c}\times\vec{a})=0 \quad ...(1)$$

Again $2\vec{b}=-(\vec{a}+3\vec{c})$

$$\Rightarrow 2(\vec{b}\times\vec{b})=-(\vec{a}\times\vec{b}+3(\vec{c}\times\vec{b}))$$

$$\Rightarrow \vec{a}\times\vec{b}+3(\vec{c}\times\vec{b})=0$$

Given, $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\lambda(\vec{b}\times\vec{c})$

$$\Rightarrow -3(\vec{c}\times\vec{b})+(\vec{b}\times\vec{c})-\frac{2}{3}(\vec{b}\times\vec{a})=\lambda(\vec{b}\times\vec{c})$$

$$\Rightarrow 3(\vec{b}\times\vec{c})+(\vec{b}\times\vec{c})+\frac{2}{3}\times 3(\vec{b}\times\vec{c})=\lambda(\vec{b}\times\vec{c})$$

$$\Rightarrow 6(\vec{b}\times\vec{c})=\lambda(\vec{b}\times\vec{c}) \Rightarrow \lambda=6$$

**32. (c) :** Let a line makes angle $\alpha$, $\beta$ and $\gamma$ with $X$, $Y$ and $Z$-axes respectively.

$\therefore$ $\alpha=\beta=\pi/3$

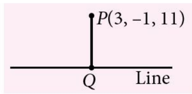
$\because$ $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$$\Rightarrow \cos^2\frac{\pi}{3}+\cos^2\frac{\pi}{3}+\cos^2\gamma=1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\cos^2\gamma=1 \Rightarrow \cos^2\gamma=\frac{1}{2}$$

$$\Rightarrow \cos\gamma=\pm\frac{1}{\sqrt{2}} \Rightarrow \gamma=\pm\frac{\pi}{4}$$

**33. (c) :** Equation of the line is




$$\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$$

$\Rightarrow$ $x = 2\lambda$, $y = 3\lambda + 2$, $z = 4\lambda + 3$

D.r.'s of line $PQ$ are

$\langle 3-2\lambda, -1-3\lambda-2, 11-4\lambda-3\rangle$

*i.e.*, $\langle 3-2\lambda, -3\lambda-3, -4\lambda+8\rangle$

Now, $2(3-2\lambda)+3(-3\lambda-3)+4(-4\lambda+8)=0$

$\Rightarrow$ $6-4\lambda-9\lambda-9-16\lambda+32=0$

$\Rightarrow$ $-29\lambda+29=0 \Rightarrow \lambda=1$

$\therefore$ $Q\equiv(2, 5, 7)$

Now, $PQ=\sqrt{(3-2)^2+(-1-5)^2+(11-7)^2}$

$$=\sqrt{1+36+16}=\sqrt{53}$$

**34. (c) :** Let $(x_1, y_1, z_1)\equiv(2, 1, 0)$,

$(x_2, y_2, z_2)\equiv(3, 2, -2)$, $(x_3, y_3, z_3)\equiv(3, 1, 7)$

The equation of the plane passing through three points, $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & 2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & 1 & -2 \\ 1 & 0 & 7 \end{vmatrix}=0$$

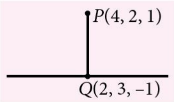
$\Rightarrow$ $(x-2)(7-0)-(y-1)(7+2)+z(0-1)=0$

$\Rightarrow$ $7x-14-9y+9-z=0$

$\Rightarrow$ $7x-9y-z=5$

**35. (a) :** Equation of line is $x+1=\frac{y+3}{3}=\frac{-z+2}{2}=\lambda$ (say). The general point on the line is
$(x = \lambda - 1, y = 3\lambda - 3, z = 2 - 2\lambda)$
This point will lie on the plane $3x + 4y + 5z = 10$
$\therefore \quad 3(\lambda - 1) + 4(3\lambda - 3) + 5(2 - 2\lambda) = 10$
$\Rightarrow \quad 3\lambda - 3 + 12\lambda - 12 + 10 - 10\lambda = 10$
$\Rightarrow \quad 5\lambda - 15 = 0 \Rightarrow \lambda = 3$
$\therefore \quad x = 2, y = 6, z = -4$
$\therefore$ The point of intersection of the line and plane is $(2, 6, -4)$.

**36. (b) :** Let $P(4, 2, 1)$ and $Q(2, 3, -1)$ be given points.




Let the required plane be
$ax + by + cz = d$
D.r.'s of line $PQ$ : $\langle 4 - 2, 2 - 3, 1 + 1\rangle$
*i.e.*, $\langle 2, -1, 2\rangle$
Thus, the normal vector to the plane is $2\hat{i}-\hat{j}+2\hat{k}$.
$\therefore \quad a = 2, b = -1, c = 2.$
Also, the point $Q(2, 3, -1)$ lies on the plane.
$\therefore \quad a(2) + b(3) + c(-1) = d$
$\Rightarrow \quad 2a + 3b - c = d$ ...(i)
Put $a = 2, b = -1, c = 2$ in (i), we get
$2 \times 2 + 3 \times (-1) - (2) = d \Rightarrow d = -1$
Hence, the equation of plane is $2x - y + 2z = -1$
$\Rightarrow \quad 2x - y + 2z + 1 = 0$

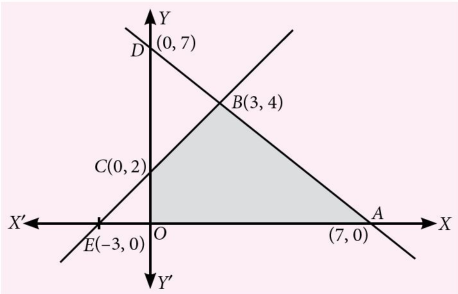
**37. (d) :** We have, $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$
$\Rightarrow \quad |\vec{a}|^2\cdot|\vec{b}|^2=144 \Rightarrow 4^2\cdot|\vec{b}|^2=144$
$\Rightarrow \quad |b|^2=\frac{144}{16}=9 \quad \Rightarrow \quad |b| = 3$

**38. (d) :** We have, $P(A)=\frac{1}{4}, P\left(\frac{A}{B}\right)=\frac{1}{2}$
and $P\left(\frac{B}{A}\right)=\frac{2}{3}$
Since, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$
$\Rightarrow \quad P(B)=\frac{P(A\cap B)}{P\left(\frac{A}{B}\right)}=\frac{P(A\cap B)}{\frac{1}{2}}$
$\Rightarrow \quad P(B) = 2P(A \cap B)$ ...(i)
Also, $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$
$\Rightarrow \quad \frac{2}{3}=\frac{P(A\cap B)}{\frac{1}{4}} \Rightarrow P(A\cap B)=\frac{2}{3}\times\frac{1}{4}=\frac{1}{6}$
$\therefore \quad P(B)=2\times\frac{1}{6}=\frac{1}{3}$

**39. (c) :** Total number of coins are $(2n + 1)$.
Let $E$ denotes the number of coins that have head on both sides and $F$ denotes the number of fair coins.
$\therefore \quad P(E)=\frac{n}{2n+1}, P(F)=\frac{n+1}{2n+1}$
Required probability $=\frac{n}{2n+1}\times1+\frac{n+1}{2n+1}\times\frac{1}{2}$
$\Rightarrow \quad \frac{31}{42}=\frac{n}{2n+1}+\frac{n+1}{2(2n+1)} \Rightarrow \frac{31}{42}=\frac{2n+n+1}{2(2n+1)}$
$\Rightarrow \quad 124n + 62 = 126n + 42$
$\Rightarrow \quad 2n = 20 \Rightarrow n = 10$

**41. (b) :** We have, $A = \{x, y, z, u\}, |A| = 4$
$B = \{a, b\}, |B| = 2$
Total number of functions $= 2^4 = 16$
Number of onto functions
$= 2^4 - {}^2C_1(2 - 1)^4 + {}^2C_2(2 - 2)^4$
$= 16 - 2 \times 1 + 0 = 16 - 2 = 14$
$\therefore \quad$ Required probability $=\frac{14}{16}=\frac{7}{8}$

**41. (b) :** Equation of $AD$ : $\frac{y-7}{x-0}=\frac{7-0}{0-7}=-1$
$\Rightarrow \quad y - 7 = -x$
$\Rightarrow \quad x + y - 7 = 0$ ...(i)




Equation of $BE$ : $\frac{y-0}{x+3}=\frac{4-0}{3+3}=\frac{4}{6}=\frac{2}{3}$
$\Rightarrow \quad 3y = 2x + 6$
$\Rightarrow \quad 2x - 3y + 6 = 0$ ...(ii)
$\therefore \quad$ The required inequalities are
$x + y - 7 \le 0, 2x - 3y + 6 \ge 0$
$x \ge 0, y \ge 0$

**42. (d) :** We have, $f(x) = ax + b$
Put $x = -1$
$f(-1) = a(-1) + b = -5$
$\Rightarrow \quad a - b = 5$ ...(i)
Put $x = 3$, we get $f(3) = 3a + b = 3$
Adding equations (i) and (ii), we get $4a = 8 \Rightarrow a = 2$
$\therefore \quad b = 2 - 5 = -3.$

**43. (c) :** Let $y = \log_{10}\tan 1° + \log_{10}\tan 2° + \log_{10}\tan 3° + ... + \log_{10}\tan 89°$

$\Rightarrow\ y = \log_{10}(\tan 1°\cdot\tan 89°) + \log_{10}(\tan 2°\cdot\tan 88°) + ... + \log_{10}\tan 45°$

$\Rightarrow\ y = \log_{10}\tan 1°\cdot\tan(90° - 1°) + \log_{10}\tan 2°\cdot\tan(90° - 2°) + ... + \log_{10}\tan 45°$

$= \log_{10}(\tan 1°\cdot\cot 1°) + \log_{10}(\tan 2°\cdot\cot 2°) + ... + \log_{10}(1)$

$= 0 + 0 + .... + 0$

$\therefore\ y = 0$

So, $e^y = e^0 = 1$

**44. (d) : Note : Given question is incorrect. In place of $\sin^2 14°$ there should be $\sin^2 24°$.**

$$\text{Let } \Delta = \begin{vmatrix} \sin^2 24° & \sin^2 66° & \tan 135° \\ \sin^2 66° & \tan 135° & \sin^2 24° \\ \tan 135° & \sin^2 24° & \sin^2 66° \end{vmatrix}$$

Since, $\sin(90° - \theta) = \cos\theta$

and $\tan(180° - \theta) = -\tan\theta$

$$\therefore\quad \Delta = \begin{vmatrix} \sin^2 24° & \sin^2(90° - 24°) & \tan(180° - 45°) \\ \sin^2(90° - 24°) & \tan(180° - 45°) & \sin^2 24° \\ \tan(180° - 45°) & \sin^2 24° & \sin^2(90° - 24°) \end{vmatrix}$$

$$= \begin{vmatrix} \sin^2 24° & \cos^2 24° & -\tan 45° \\ \cos^2 24° & -\tan 45° & \sin^2 24° \\ -\tan 45° & \sin^2 24° & \cos^2 24° \end{vmatrix}$$

$$= \begin{vmatrix} \sin^2 24° & \cos^2 24° & -1 \\ \cos^2 24° & -1 & \sin^2 24° \\ -1 & \sin^2 24° & \cos^2 24° \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 0 & \cos^2 24° & -1 \\ 0 & -1 & \sin^2 24° \\ 0 & \sin^2 24° & \cos^2 24° \end{vmatrix} = 0$$

**45. (c) :** Let, $z = \dfrac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$

$$\Rightarrow\quad z = \frac{(1+i)^3(1+3i)^2}{4(1-3i)(1+3i)(1-i)(1+i)}$$

$$= \frac{(2i-2)(6i-8)}{4\times(1+9)(1+1)} = \frac{2\times 2(i-1)(3i-4)}{4\times 10\times 2}$$

$$= \frac{1}{20}[-3 - 4i - 3i + 4] = \frac{1}{20}[1 - 7i]$$

$$|z| = \frac{1}{20}|1 - 7i| = \frac{\sqrt{1+49}}{20} = \frac{\sqrt{2}}{4}$$

**46. (b) :** Since $x < 0$ and $a < b$

Dividing by $x$ we get, $\dfrac{a}{x} > \dfrac{b}{x}$

**47. (d) :** Three women choose the chairs marked 1 to 6. The number of ways of selecting 3 chairs out of 6 will be ${}^6C_3$. But we have to arrange the women themselves in $\lfloor 3$ ways.

$\therefore$ So, no. of ways of occupying chair by women $= {}^6C_3 \times 3! = {}^6P_3$

Similarly, 2 men choose the remaining chairs.

$\therefore$ The no. of ways of selecting 2 chairs out of 4 chairs will be ${}^4C_2$ and arrangement of the men will be done by 2! ways.

$\therefore$ No. of ways of occupying chairs by men $= {}^4C_2 \times 2! = {}^4P_2$

$\therefore$ Total number of ways $= {}^6P_3 \times {}^4P_2$

**48. (d) :** Consider option (a),

$\{x : x^2 - 9 = 0, x \in R\} = \{-3, 3\} \neq \phi$

option (b),

$\{x : x^2 - 1 = 0, x \in R\} = \{-1, 1\} \neq \phi$

option (c),

$\{x : x^2 = x + 2, x \in R\} = \{-1, 2\} \neq \phi$

option (d),

$\{x : x^2 + 1 = 0, x \in R\}$

There does not exist any $x \in R$ for which $x^2 + 1 = 0$

Hence, this is an empty set.

**49. (b) :** The given series is

$$1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + .... \Rightarrow \frac{1}{7^0} + \frac{3}{7^1} + \frac{5}{7^2} + ....$$

The $n^{th}$ term will be $\dfrac{2n-1}{7^{n-1}}$.

**50. (a) :** We have,

$p\left(\dfrac{1}{q} + \dfrac{1}{r}\right), q\left(\dfrac{1}{r} + \dfrac{1}{p}\right), r\left(\dfrac{1}{p} + \dfrac{1}{q}\right)$ are in A.P.

**Recipe for Success**

"A little progress each day adds up to big results."

*– Satya Nani*

$\Rightarrow\ p\left(\frac{1}{q}+\frac{1}{r}\right)+1, q\left(\frac{1}{r}+\frac{1}{p}\right)+1, r\left(\frac{1}{p}+\frac{1}{q}\right)+1$ are in A.P.

$\Rightarrow\ p\left(\frac{1}{q}+\frac{1}{r}+\frac{1}{p}\right), q\left(\frac{1}{r}+\frac{1}{p}+\frac{1}{q}\right), r\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right)$ are in A.P.

$\Rightarrow\ p, q, r$ are in A.P.

**51. (c) :** Let the slope of required line is $m$.
Slope of line $3x + y = 3$ is $-3$.
Since, the required line is perpendicular to $3x + y = 3$.

$\therefore\ m = 1/3$

So, equation of required line is $\frac{y-2}{x-2}=\frac{1}{3}$

$\Rightarrow\ 3y - 6 = x - 2 \Rightarrow x - 3y + 4 = 0$

$\therefore\ Y\text{-intercept} = 4/3$

**52. (b) :** Let the equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Distance between foci is 16.

$\therefore\ 2ae = 16 \Rightarrow 2a\times\sqrt{2}=16 \Rightarrow a=\frac{8}{\sqrt{2}}=4\sqrt{2}$

Also, $e^2=\frac{a^2+b^2}{a^2}$

$\Rightarrow\ 2=\frac{32+b^2}{32} \Rightarrow b^2 = 64 - 32 \Rightarrow b^2 = 32$

$\therefore$ Equation of hyperbola is

$\frac{x^2}{32}-\frac{y^2}{32}=1 \Rightarrow x^2 - y^2 = 32$

**53. (b) :** Given, $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}=A\cos B$

$\Rightarrow\ \lim_{x\to 0}\frac{2\sin(x)\cdot\cos 2}{x}=A\cos B$

$\Rightarrow\ 2\cos 2 \times 1 = A \cos B$

On comparing , we get $A = 2, B = 2$

**54. (a) :** We have, $\left(x^2+\frac{1}{x}\right)^n$

Given, $n$ is even.

$\therefore\ r$ (middle term) $= n/2$

Middle term $= {}^nC_{n/2}(x^2)^{n-n/2}\left(\frac{1}{x}\right)^{n/2}$

$= {}^nC_{n/2}(x^2)^{n/2}\frac{1}{x^{n/2}} = {}^nC_{n/2}\, x^n x^{-n/2} = {}^nC_{n/2}x^{n/2}$

$\Rightarrow\ 924x^6 = {}^nC_{n/2}\, x^{n/2}$

On comparing, we get $x^6 = x^{n/2}$

$\Rightarrow\ 6=\frac{n}{2} \Rightarrow n=12$

**55. (d) :** Mean $(x) = 50$, $n = 100$

Standard deviation = 5

Variance $= 5^2 = \frac{\sum x_i^2}{n}-(\bar{x})^2$

$\Rightarrow\ 25=\frac{\Sigma x_i^2}{100}-(50)^2 \Rightarrow \Sigma x_i^2 = 252500$

**56. (d) :** $f: R\to R, g: [0,\infty)\to R$

$f(x)=x^2, g(x)=\sqrt{x}$

(a) $fog(2) = 2$

$f(g(2)) = f(\sqrt{2}) = 2 \in R$

$\therefore\ fog(2) = 2$

(b) $gof(4) = 4$

$g(f(4)) = g(16) = 4 \in R$

$\therefore\ gof(4) = 4$

(c) $gof(-2) = 2$

$g(f(-2)) = g(4) = 2 \in R$

$\therefore\ gof(-2) = 2$

(d) $f(g(-4))$ = not defined since, $-4 \notin [0, \infty)$

**57. (b) :** $f: R\to R, f(x) = 3x^2 - 5$

$g: R\to R, g(x)=\frac{x}{x^2+1}$

$gof = g(f(x)) = g(3x^2-5)=\frac{3x^2-5}{(3x^2-5)^2+1}$

$=\frac{3x^2-5}{9x^4+25-30x^2+1}=\frac{3x^2-5}{9x^4-30x^2+26}$

**58. (b) :** $aRb \Rightarrow 3a + 2b = 27 \Rightarrow b=\frac{27-3a}{2}$

If $a = 1, b = 12$

$a=2, b=\frac{21}{2}$ ; $a = 3, b = 9$

$a=4, b=\frac{15}{2}$; $a = 5, b = 6$

$a=6, b=\frac{9}{2}$ ; $a = 7, b = 3$

**59. (d) :** We have $f(x) = \sin 2x + \cos 2x$
and $g(x) = x^2 - 1$

$g(f(x)) = g[\sin 2x + \cos 2x] = (\sin 2x + \cos 2x)^2 - 1$

$\Rightarrow\ 1 + 2\sin 2x\cos 2x - 1 = \sin 4x$

Since, $\sin 4x$ will be invertible in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

*i.e.*, $4x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \Rightarrow x\in\left[-\frac{\pi}{8},\frac{\pi}{8}\right]$

**60. (b) :** The contrapositive of the statement
"If two lines do not intersect in the same plane, then they are parallel" is
"The two lines are not parallel, then they intersect in the same plane."

# Challenging PROBLEMS For JEE

ON

## Complex Numbers

### MULTIPLE CORRECT ANSWER TYPE

**1.** If $z_1$ and $z_2$ are two non-zero complex numbers such that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then

(a) $z_1\overline{z}_2$ is purely imaginary

(b) $\dfrac{z_1}{z_2}$ is purely imaginary

(c) $z_1\overline{z}_2 + \overline{z}_1 z_2 = 0$

(d) 0, $Z_1$, $Z_2$ are the vertices of a right angled triangle which is right angled at origin

**2.** If $\alpha$ is a variable complex number such that $|\alpha| > 1$ and $z = \alpha + \dfrac{1}{\alpha}$ lies on a conic, then

(a) Eccentricity of the conic is $\dfrac{2|\alpha|}{1+|\alpha|^2}$

(b) Distance between foci is 4

(c) Length of latusrectum is $\dfrac{2(|\alpha|^2-1)}{|\alpha|^2+1}$

(d) Distance between directrices is $\left(|\alpha|+\dfrac{1}{|\alpha|}\right)^2$

**3.** The equations of two lines making on angle 45° with a given line $\overline{a}z + a\overline{z} + b = 0$ (where '$a$' is a complex number and $b$ is real) and passing through a given point $C$ ($C$ is a complex number), is/are

(a) $\dfrac{z+c}{a} + i\dfrac{\overline{z}-\overline{c}}{\overline{a}} = 0$ (b) $\dfrac{z-c}{a} + i\dfrac{\overline{z}-\overline{c}}{\overline{a}} = 0$

(c) $\dfrac{z-c}{a} - i\dfrac{\overline{z}-\overline{c}}{\overline{a}} = 0$ (d) $\dfrac{z+c}{a} - i\dfrac{\overline{z}-\overline{c}}{\overline{a}} = 0$

**4.** Let $z_1, z_2, z_3, \ldots, z_n$ are the complex numbers such that $|z_1| = |z_2| = \ldots\ldots = |z_n| = 1$. If $z = \left(\sum_{k=1}^{n} z_n\right)\left(\sum_{k=1}^{n} \dfrac{1}{z_k}\right)$ then

(a) $z$ is purely imaginary

(b) $z$ is real

(c) $0 < z \le n^2$

(d) $z$ is a complex number of the form $a + ib$

**5.** If $a$, $b$, $c$ are non – zero complex numbers of equal moduli and satisfy $az^2 + bz^2 + c = 0$, then

(a) $\min|z| = \dfrac{\sqrt{5}-1}{2}$

(b) $\min|z| = 0$

(c) $\min|z|$ does not exist

(d) $\min|z| = \dfrac{\sqrt{5}+1}{2}$

**6.** Let $x_1$, $x_2$ are the roots of the quadratic equation $x^2 + ax + b = 0$ where $a$, $b$ are complex numbers and $y_1$, $y_2$ are the roots of the quadratic equation $y^2 + |a|y + |b| = 0$. If $|x_1| = |x_2| = 1$, then

(a) $|y_1| = 1$ (b) $|y_2| = 1$

(c) $|y_1| \ne |y_2|$ (d) $|y_1| = |y_2| = 2$

**7.** Let $a$, $b$, $c$ be distinct complex numbers with $|a| = |b| = |c| = 1$ and $z_1, z_2$ be the roots of the equation $az^2 + bz + c = 0$ with $|z_1| = 1$. Let $P$ and $Q$ represent the complex numbers $z_1$ and $z_2$ in the argand plane with $\angle POQ = \theta, 0° < \theta < 180°$, (where $O$ being the origin), then

(a) $b^2 = ac; \theta = \dfrac{2\pi}{3}$ (b) $\theta = \dfrac{2\pi}{3}; PQ = \sqrt{3}$

(c) $PQ = 2\sqrt{3}; b^2 = ac$ (d) $\theta = \dfrac{\pi}{3}; b^2 = ac$

**8.** If $A(z_1)$, $B(z_2)$ and $C(z_3)$ are three points in argand plane where $|z_1 + z_2| = |z_1| - |z_2|$ and $|(1 - i)z_1 + iz_3| = |z_1| - |z_3 - z_1|$, then

(a) $A$, $B$ and $C$ lie on a fixed circle with centre $\left(\dfrac{z_2+z_3}{2}\right)$

(b) $A$, $B$, $C$ form right angle triangle

(c) $A$, $B$, $C$ from an equilateral triangle

(d) $A$, $B$, $C$ form an obtuse angle triangle

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**9.** The complex numbers satisfying the equation $(3z + 1)(4z + 1)(6z + 1)(12z + 1) = 2$, is /are

(a) $\frac{\sqrt{33}-5}{24}$ (b) $\frac{\sqrt{33}+5}{24}$

(c) $\frac{-i\sqrt{23}-5}{24}$ (d) $\frac{-i\sqrt{23}+5}{24}$

**10.** Let $z_1$ and $z_2$ are non-zero (given) complex numbers and $k$ be any positive real number. Consider the system of equations $|3z - z_1 - 2z_2| = |z_1 - z_2|$ and $\arg\left(\frac{z_1-z_2}{z-kz_1-(1-k)z_2}\right)=\pm\frac{\pi}{2}$. Then

(a) The system of equations has no solution if $k\in\left(\frac{2}{3},\infty\right)$

(b) The system of equations have more than one solutions if $k\in\left(0,\frac{2}{3}\right)$

(c) The system of equations have no solution if $k\in\left(0,\frac{2}{3}\right)$

(d) The system of equations have more than one solution if $k\in\left(\frac{2}{3},\infty\right)$

**11.** If all the three roots of $az^3 + bz^2 + cz + d = 0$ have negative real parts ($a, b, c \in R$) then

(a) $ab > 0$ (b) $bc > 0$

(c) $ad > 0$ (d) $bc - ad > 0$

## NUMERICAL/INTEGER ANSWER TYPE

**12.** Let 1, $\omega$, $\omega^2$ be the cube root of unity. The least possible degree of a polynomial with real coefficients having roots $2\omega$, $(2 + 3\omega)$, $(2 + 3\omega^2)$, $(2 - \omega - \omega^2)$, is ________.

**13.** If a complex number $z$ satisfies $|z - 8 - 4i| + |z - 14 - 4i| = 10$, then the maximum value of $\arg(z)=\tan^{-1}\frac{11}{3k}$, then find $k$.

**14.** If '$a$' and '$b$' are complex numbers. One of the roots of the equation $x^2 + ax + b = 0$ is purely real and the other is purely imaginary, then $a^2 - \bar{a}^2 = kb$, then find $k$.

**15.** If $\sum_{j=1}^{n-1}\frac{1}{1-e^{\frac{2\pi ij}{n}}}=\frac{n-1}{k}$, then find $k$. $(i=-\sqrt{1})$

**16.** Let $A,B,C$ be equilateral triangle with $\frac{\sqrt{3}}{2}A=e^{i\pi/2}$, $\frac{\sqrt{3}}{2}B=e^{-i\pi/6}$ and $\frac{\sqrt{3}}{2}C=\overline{e}^{i5\pi/6}$.

Let $P$ be any point on the incircle of $\Delta ABC$. Find the value of $PA^2 + PB^2 + PC^2$.

**17.** Let $\lambda$, $z_0$ be two complex numbers. $A(z_1)$, $B(z_2)$, $C(z_3)$ be the vertices of a triangle such that

$z_1 = z_0 + \lambda$, $z_2=z_0+\lambda e^{i\pi/4}$, $z_3=z_0+\lambda e^{i7\pi/11}$

and $\angle ABC=\frac{3k\pi}{22}$, then the value of $k$ is_____.

**18.** The roots of the equation $z^5+z^6+.....+z^{10}=0$, where $z \neq 0, 1$, are represented by vertices of a pentagon having longest side length is equal $d$. Find $d^2$.

**19.** If $|z_1-z_2|=\sqrt{25-12\sqrt{3}}$, and $\frac{z_1-z_3}{z_2-z_3}=\frac{3}{4}e^{i\frac{\pi}{6}}$, then area of triangle (in square units) whose vertices are represented by $z_1$, $z_2$ and $z_3$, is______.

**20.** If $\alpha=e^{i2\pi/7}$ and $f(x)=A_0+\sum_{k=1}^{20}A_kx^k$ and the value of $f(x) + f(\alpha x) + f(\alpha^2x)+.....+ f(\alpha^6x)$ is $k(A_0+A_7x^7+A_{14}x^{14})$, then find the value of $k$.

**21.** Find the least positive integral value of '$a$' such that there is at least one complex number satisfying $|z+\sqrt{2}|<a^2-3a+2$ and $|z+i\sqrt{2}|<a^2$.

**22.** A triangle with vertices represented by $z_1$, $z_2$, $z_3$ has opposite sides of lengths in the ratio $2:\sqrt{19}:3$ respectively. Then the value of $4(z_1 - z_2)^2 + 6(z_1 - z_2)(z_3 - z_2) + 9(z_3 - z_2)^2$ is $k$. Find $k$.

**23.** If $2^7\cos^3\theta\cdot\sin^5\theta = a\sin8\theta - b\sin6\theta + c\sin4\theta + d\sin2\theta$ and $\theta$ is real then the value of $a + b + c + d$ must be equal to______.

## SOLUTIONS

**1. (a, b, c, d) :** $|z_1+z_2|^2=|z_1|^2+|z_2|^2$

$\Rightarrow (z_1+z_2)(\bar{z}_1+\bar{z}_2)=|z_1|^2+|z_2|^2$

$\Rightarrow \bar{z}_1z_2+\bar{z}_2z_1=0 \Rightarrow \bar{z}_1z_2=-z_1\bar{z}_2 \Rightarrow z_1\bar{z}_2$ is purely imaginary

$\Rightarrow \frac{z_1}{z_2}$ is purely imaginary $\Rightarrow \arg\left(\frac{z_1}{z_2}\right)=\frac{\pi}{2}$

thus 0, $z_1$, $z_2$ form a right triangle.

**2. (a, b, d) :** Let $|\alpha| = r > 1$ and $\alpha = r\text{cis}\theta$, then

$z = x + iy = \alpha+\frac{1}{\alpha}=r\,cis\theta+\frac{cis(-\theta)}{r}$

$\Rightarrow\ x = \left(r + \frac{1}{r}\right)\cos\theta$ and $y = \left(r - \frac{1}{r}\right)\sin\theta$

Eliminating $\theta$ gives $\dfrac{x^2}{\left(r+\frac{1}{r}\right)^2} + \dfrac{y^2}{\left(r-\frac{1}{r}\right)^2} = 1$,

Which is an ellipse

$a = r + \frac{1}{r},\ \ b = r - \frac{1}{r} (r = |\alpha| > 1 \Rightarrow a > b)$

$\therefore\ e = \sqrt{1 - \frac{b^2}{a^2}} = \dfrac{2}{r + \frac{1}{r}}$,

Distance between foci = $2ae = 4$

Distance between directrices $= \dfrac{2a}{e}$

Length of Latus rectum $= \dfrac{2(|\alpha|^2 - 1)^2}{|\alpha|(|\alpha|^2 + 1)}$

**3. (b, c) :** Let $z_1$, $z_2$ be two points on the given line, then $\dfrac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2} = -\dfrac{a}{\bar{a}}$ ...(1)

Also, $\dfrac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2} = \pm i \dfrac{z - c}{\bar{z} - \bar{c}}$ ...(2)

From (1) and (2), $\dfrac{z - c}{a} \pm i \dfrac{\bar{z} - \bar{c}}{\bar{a}} = 0$

**4. (b, c) :** $z = (z_1 + z_2 + \ldots + z_n)\left(\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right)$

$= \ |z_1 + z_2 + \ldots. + z_n|^2$, which is real

$\therefore |z_1 + z_2 + z_3 + \ldots + z_n| \le |z_1| + |z_2| + \ldots + |z_n|$

$\Rightarrow |z_1 + z_2 + z_3 + \ldots + z_n| \le n \Rightarrow |z_1 + z_2 + z_3 + \ldots + z_n|^2 \le n^2$

**5. (a, d) :** $|a| = |b| = |c| = r$

$|c| = |-az^2 - bz| \le r|z|^2 + r|z|$

$\Rightarrow |z|^2 + |z| - 1 \ge 0$ ...(1)

and $az^2 = -(bz + c)$

$|a||z|^2 \le r|z| + r$

$|z|^2 - |z| - 1 \le 0$ ...(2)

Solving (1) and (2), we get

$\min|z| = \dfrac{\sqrt{5} - 1}{2}$, $\max|z| = \dfrac{\sqrt{5} + 1}{2}$

**6. (a, b) :** $x^2 + ax + b = 0 \Rightarrow |a| \le 2$ and $|b| = 1$

$y = \dfrac{-|a| \pm \sqrt{|a|^2 - 4|b|}}{2} = \dfrac{-|a| \pm i\sqrt{4 - |a|^2}}{2}$

$\Rightarrow |y| = 1$

**7. (a, b) :** $|z_1 + z_2| = \left|\dfrac{-b}{a}\right|; z_1 z_2 = \left|\dfrac{c}{a}\right|$

$\therefore |z_1 + z_2|^2 = 1 \Rightarrow (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = 1$

$\Rightarrow\ 2 + z_1\bar{z}_2 + z_2\bar{z}_1 = 1 \Rightarrow \dfrac{(z_1 + z_2)^2}{z_1 z_2} = 1$

$\Rightarrow\ \dfrac{b^2}{a^2} = \dfrac{c}{a} \Rightarrow b^2 = ac$

Now, $z_2 = z_1 e^{i\theta}$, then

$|z_1 + z_2| = |z_1||1 + e^{i\theta}| \Rightarrow 2\cos\frac{\theta}{2} = 1\ \therefore\ \theta = \frac{2\pi}{3}$

$PQ = |z_2 - z_1| = \sqrt{3}$

**8. (a, b) :** $\arg\dfrac{z_1}{z_2} = \pm\pi$ and

$|z_1 + i(z_3 - z_1)| = |z_1| + |z_3 - z_1|$ iff $\arg\dfrac{z_1}{z_3 - z_1} = \dfrac{\pi}{2}$

So centre of circle = $\left(\dfrac{z_2 + z_3}{2}\right)$ and $ABC$ is right angle triangle.

**9. (a, c) :** $(3z + 1)(4z + 1)(6z + 1)(12z + 1) = 2$

$8(3z + 1)6(4z + 1)4(6z + 1)2(12z + 1) = 2 \times 8 \times 6 \times 4 \times 2$

$(24z + 8)(24z + 6)(24z + 4)(24z + 2) = 768$

Let $24z + 5 = U$

$(U + 3)(U + 1)(U - 1)(U - 3) = 768$

$\Rightarrow (U^2 - 9)(U^2 - 1) = 768$

$\Rightarrow U^4 - 10U^2 - 759 = 0 \Rightarrow U^2 = 33$ or $-23$

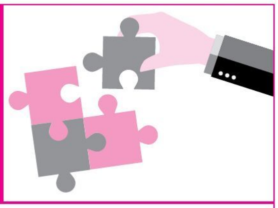
$\Rightarrow\ 24z + 5 = \pm\sqrt{33}$ or $\pm i\sqrt{33}$

$z = \dfrac{\pm\sqrt{33} - 5}{24}$ or $\dfrac{\pm i\sqrt{23} - 5}{24}$

**10. (a, b) :** $\left|z - \dfrac{z_1 + 2z_2}{3}\right| = \dfrac{1}{3}|z_1 - z_2|$

And $\operatorname{Arg}\left(\dfrac{z_1 - z_2}{z - (kz_1 + (1 - k)z_2)}\right) = \pm\dfrac{\pi}{2}$

# PUZZLE CORNER



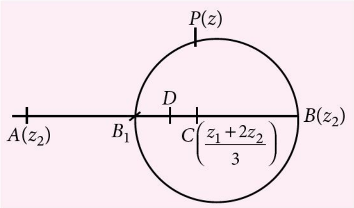
**ANSWER - JUNE 2023**

| | | | | | |
|---|---|---|---|---|---|
| 8+ 3 | 3÷ 6 | 2 | 20× 4 | 1 | 5 |
| 5 | 3– 1 | 4 | 12× 6 | 5+ 3 | 2 |
| 1– 6 | 5 | 1 | 2 | 1– 4 | 3 |
| 2÷ 4 | 2 | 30× 6 | 2– 3 | 5 | 4× 1 |
| 2÷ 2 | 1– 3 | 5 | 15+ 1 | 6 | 4 |
| 1 | 4 | 8+ 3 | 5 | 2 | 6 |

***Winner :*** *Rachit Verma*

Angle between the line segment joining $z_1$ and $z_2$; $z$ and $kz_1 + (1-k)z_2$ is $\frac{\pi}{2} \cdot \frac{z_1+2z_2}{3}$ is a point on segment $AB$ such that $AC : CB = 2:1$ and $D\,(kz_1 + (1-k)z_2)$ is a point on segment $AB$ such that $AD : DB = 1-k : k$

$BD = k|z_1 - z_2|$




(a) for no solution, $BD > BB_1 \Rightarrow k|z_1 - z_2| > \frac{2}{3}|z_1 - z_2|$

$\Rightarrow k > \frac{2}{3}$

(b) for more than one solution, $0 < BD < BB_1$

$$\Rightarrow 0 < k|z_1 - z_2| < \frac{2}{3}|z_1 - z_2|$$

$\Rightarrow 0 < k < 2/3.$

**11. (a, b, c, d) :** Let $z_1 = x_1,\ z_2, z_3 = x_2 \pm iy_2$

$$\Rightarrow z_1 + z_2 + z_3 = -\frac{b}{a} \Rightarrow x_1 + 2x_1 = -\frac{b}{a} < 0 \Rightarrow ab > 0$$

Also, $z_1 z_2 z_3 = x_1\left[x_2^2 + y_2^2\right] = -\frac{d}{a} \Rightarrow ad > 0$

Also $-\frac{bc}{a^2} < x_1(x_2^2 + y_2^2)$

$\Rightarrow bc > ad$

**12. (5) :** Roots are $2\omega$, $(2 + 3\omega)$, $(2 + 3\omega^2)$, $(2 - w - \omega^2)$. $(2 + 3\omega)$ and $2 + 3\omega^2$ are conjugate of each other, $2w$ is complex root, then other root must be $2\omega^2$ (as conjugate root occur in conjugate pair)

$2 - \omega - \omega^2 = 2 - (-1) = 3$, which is real.

Hence least degree of the polynomial is 5.

**13. (4) :** Locus of $z$ is an ellipse

$$\frac{(x-11)^2}{25} + \frac{(y-4)^2}{16} = 1$$

Equation of tangent is

$y - 4 = m(x - 11) + c = 11\text{m} - 4$

As $c^2 = a^2m^2 + b^2$ for standard ellipse

$\Rightarrow (11m - 4)^2 = 25m^2 + 16 \Rightarrow \text{m} = 0$ or $m = \frac{11}{12}$

$$\therefore \tan\theta = \frac{11}{12} \Rightarrow \theta = \tan^{-1}\frac{11}{12}$$

**14. (4) :** Let $\alpha$ and $i\beta$, $\alpha, \beta \in R$ are roots of $x^2 + ax + b = 0 \Rightarrow \alpha + i\beta = -a,\ i\alpha\beta = b$

$\alpha - i\beta = -\bar{a}$

$$\Rightarrow 2\alpha = -(a + \bar{a}) \text{ and } 2i\beta = -(a - \bar{a})$$

$$\therefore 4i\alpha\beta = a^2 - \bar{a}^2 \Rightarrow 4b = a^2 - \bar{a}^2$$

**15. (2) :** Let $e^{i\frac{2\pi}{n}} = \alpha$ then

$$\sum_{j=1}^{n-1} \frac{1}{1-\alpha^j} = \frac{1}{1-\alpha} + \frac{1}{1-\alpha^2} + \ldots\ldots + \frac{1}{1-\alpha^{n-1}},$$

where $\alpha$ is a $n^{\text{th}}$ root of unity.

$(\alpha, \alpha^2, \alpha^3, \ldots\ldots, \alpha^{n-1})$ are the roots of

$$\frac{x^n - 1}{x-1} = (x-\alpha)(x-\alpha^2)\ldots\ldots(x-\alpha^{n-1})$$

Taking log on both side

$$\log\frac{x^n-1}{x-1} = \log(x-\alpha) + \log(x-\alpha^2) + \ldots\ldots + \log(x-\alpha^{n-1})$$

Differentiate w.r.t. $x$ and use $\lim\limits_{x\to 1}$, we get

$$\frac{n-1}{2} = \frac{1}{1-\alpha} + \frac{1}{1-\alpha^2} + \ldots\ldots + \frac{1}{1-\alpha^{n-1}}$$

**16. (5) :** Given triangle is an equilateral triangle

$\therefore$ Incircle is $x^2 + y^2 = \frac{1}{3}$

Let point on the incircle is $(x, y)$

$\therefore PA^2 + PB^2 + PC^2$

$$= x^2 + \left(y - \frac{2}{\sqrt{3}}\right)^2 + (x-1)^2 + \left(y + \frac{1}{\sqrt{3}}\right)^2 + (x+1)^2 + \left(y + \frac{1}{\sqrt{3}}\right)^2$$

$= 3(x^2 + y^2) + 4 = 1 + 4 = 5$

**17. (5) :** $|z_1 - z_0| = |z_2 - z_0| = |z_3 - z_0| = |\lambda|$

$$\frac{z_3 - z_0}{z_2 - z_0} = \frac{e^{i7\pi/11}}{e^{i\pi/4}} = i^{i17/44}$$

$$\Rightarrow \angle BSC = 17\frac{\pi}{44} \Rightarrow \angle BAC = 17\frac{\pi}{88}$$

Similarly, $\frac{z_2 - z_0}{z_1 - z_0} = e^{i\pi/4} \Rightarrow \angle ACB = \frac{\pi}{8}$

$$\therefore \angle ABC = \pi - \frac{\pi}{8} - \frac{17\pi}{88} = \frac{15\pi}{22}.$$

**18. (3) :** Equation reduces to $z^6 = 1$

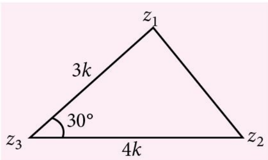
$$\Rightarrow\ z = \cos 2k\frac{\pi}{6} + i\sin 2k\frac{\pi}{6},\ k = 1, 2, 3, 4, 5$$

The longest side ($d$)

$$= \left|\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - \left(\cos 5\frac{\pi}{3} + i\sin 5\frac{\pi}{3}\right)\right| = \sqrt{3}.$$

$d^2 = 3.$

**19. (3) :** $\left|\dfrac{z_1 - z_3}{z_2 - z_3}\right| = \dfrac{3}{4}$




Let $|z_1 - z_3| = 3k$,

$|z_2 - z_3| = 4k$

angle at $Z_3 = \dfrac{\pi}{6}$

$$\cos 30° = \frac{16k^2 + 9k^2 - 25 + 12\sqrt{3}}{2\times 4k \times 3k} \Rightarrow k = 1$$

$$\text{Area} = \frac{1}{2}\cdot 3\cdot 4\sin 30° = 3$$

**20. (7) :** $f(x) + f(\alpha x) + f(\alpha^2 x) + \ldots + f(\alpha^6 x)$

$$= 7A_0 + \sum_{k=1}^{20} A_k x^k (1 + \alpha^k + \ldots + \alpha^{6k})$$

but when $k \neq 7$ and $k \neq 14$, then $1 + \alpha^k + \alpha^{2k} + \ldots + \alpha^{6k} = 0$

Hence,

$f(x) + f(\alpha x) + \ldots + f(\alpha^6 x) = 7A_0 + 7A_7 x^7 + 7A_{14}x^{14}$

$= 7(A_0 + A_7 x^7 + A_{14}x^{14})$

$\therefore\ k = 7$

**21. (3) :** ($a$ = 3) Atleast one complex number $z$ satisfy the required condition if the two circle intersect at two distinct points.

**22. (0) :** $\cos B = -\dfrac{1}{2} \Rightarrow B = \dfrac{2\pi}{3}$

By rotation, $\dfrac{z_1 - z_2}{z_3 - z_2} = \left|\dfrac{z_1 - z_2}{z_3 - z_2}\right| e^{i\frac{2\pi}{3}}$

$$\Rightarrow\ 2(z_1 - z_2) + \frac{3}{2}(z_3 - z_2) = (z_3 - z_2)\left(i\frac{3\sqrt{3}}{2}\right)$$

Squaring to get the required result.

**23. (7) :** Let $z = e^{i\theta} \Rightarrow 2\cos\theta \left(z + \dfrac{1}{z}\right)$ and $2i\sin\theta\left(z - \dfrac{1}{z}\right)$

Now, $(2\cos\theta)^3(2i\sin\theta)^5 = \left(z + \dfrac{1}{z}\right)^3\left(z - \dfrac{1}{z}\right)^5$

$$= \left(z^8 - \frac{1}{z^8}\right) - 2\left(z^6 - \frac{1}{z^6}\right) - 2\left(z^4 - \frac{1}{z^4}\right) + 6\left(z^2 - \frac{1}{z^2}\right)$$

Comparing $a = 1,\ b = 2,\ c = -2,\ d = 6$

$a + b + c + d = 1 + 2 - 2 + 6 = 7$

# Pi Approximation Day


**Pi Approximation Day** is a light hearted observance that takes place on July 22nd each year. The date was chosen because the fraction $\frac{22}{7}$ is a common approximation of the mathematical constant $\pi$(Pi). Pi approximation Day is an opportunity for maths enthusiasts and educators to celebrate the mathematical concept Pi and its significance in various fields, including Mathematics, Physics, Engineering, etc.

Though Pie is an infinite number, it is a fun challenge for students to remember the Pi number to a certain extent without repetition of any pattern. In US people celebrate Pi Approximation Day by baking a pie cake with a decoration at the top.

*Dear Mathematics today readers! This year, let us celebrate this amazing day by making a delicious Pie cake of your favourite flavour (Taking help from your mother) and decorate it by using toppings of your choice. Readers can share the pictures of their beautiful Pie cakes with us by sending them at editor@mtg.in.*

# beat the TIME TRAP

**Duration : 60 minutes**

## SECTION-I

### Single Option Correct Type

**1.** The domain of the function

$f(x)=\log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is

(a) (0, 1) (b) (0, 1] (c) [1, ∞) (d) (1, ∞)

**2.** If the shortest distance between the lines

$\frac{x-1}{\alpha}=\frac{y+1}{-1}=\frac{z}{1}, (\alpha\neq-1)$ and

$x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{1}{\sqrt{3}}$, then the value of $\alpha$ is

(a) $-\frac{16}{19}$ (b) $-\frac{19}{16}$ (c) $\frac{32}{19}$ (d) $\frac{19}{32}$

**3.** Let $\vec{u}, \vec{v}$ and $\vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other, then $|\vec{u}-\vec{v}+\vec{w}|$ is equal to

(a) 2 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 14

**4.** Find the points of local maxima and local minima respectively for the function

$f(x)=\sin 2x-x$, where $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$.

(a) $-\pi/6, \pi/6$ (b) $\pi/3, -\pi/3$
(c) $-\pi/3, \pi/3$ (d) $\pi/6, -\pi/6$

**5.** Let $A = R - \{3\}, B = R - \{1\}$. Let $f: A \to B$ be defined by $f(x)=\frac{x-2}{x-3}$. Then,

(a) $f$ is bijective
(b) $f$ is one-one but not onto
(c) $f$ is onto but not one-one
(d) None of these

**6.** Four candidates $A$, $B$, $C$ and $D$ have applied for the assignment to coach of a school cricket team. If $A$ is twice as likely to be selected as $B$ and $B$, $C$ are given about the same chance of being selected, while $C$ is twice as likely to be selected as $D$, what is the probability that $A$ will not be selected?

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{9}{4}$ (d) $\frac{9}{5}$

**7.** If $\lim_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right)=4$,

then the values of $a$, $b$, $c$ and $d$ is

(a) $a = 1, b = 2, c = 3$ and $d = 4$
(b) $a = 2, b = 3, c = 5$ and $d = 0$
(c) $a = 2, b = 4, c = 5$ and $d \in R$
(d) $a = 2, c = 5$ and $b, d \in R$

**8.** The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. Find the equation of the hyperbola if its eccentricity is 2.

(a) $\frac{x^2}{12}-\frac{y^2}{4}=1$ (b) $\frac{x^2}{4}-\frac{y^2}{12}=1$

(c) $\frac{x^2}{3}-\frac{y^2}{4}=1$ (d) $\frac{x^2}{4}-\frac{y^2}{3}=1$

**9.** Let $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then $S_n$ can take value

(a) 1056 (b) 1088 (c) 1120 (d) 1152

**10.** A house master in a vegetarian boarding school takes 3 children from his house to the nearby dhaba (road side hotel) for non-veg food at a time as often as he can, but he does not take the same three children more than once. He finds that he goes to the dhaba 84 times more than a particular child goes with him. Then the number of children taking non-veg. food in his hostel, is

(a) 15 (b) 5 (c) 20 (d) 10

## SECTION-II

### Numerical Answer Type

**11.** A function $f(x)$ is given by $f(x)=\dfrac{5^x}{5^x+5}$, then the sum of the series

$$2\left[f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+\ldots..+f\left(\frac{39}{20}\right)\right]$$ is equal to ________ .

**12.** If the solution of the differential equation $\sec^2 y\dfrac{dy}{dx}+2x\tan y=x^3$ is $2\tan y=\lambda(x^2-1)+ce^{-x^2}$, $c$ is arbitrary constant, then the numerical value of $\lambda$ must be _______.

**13.** If $\displaystyle\int\frac{xdx}{\sqrt{(7x-10-x^2)^3}}=\frac{\lambda\cdot(7x-20)}{\sqrt{(7x-10-x^2)}}+c$, then the value of 1800 $\lambda$ must be equal to _______.

**14.** If $f(x)=(\log_{\cot x}\tan x)(\log_{\tan x}\cot x)^{-1}+\tan^{-1}\left(\dfrac{4x}{4-x^2}\right)$, then $f'(2)$ is equal to ________.

**15.** If $A=\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}$ and $I$ is the unit matrix of order 3, then $A^2+2A^4+4A^6$ is equal to $kA^8$. The value of $k$ is ________.

## SOLUTIONS

**1. (a) :** We have $f(x)=\log_2\left(-\log_{1/2}\left(1+\dfrac{1}{\sqrt[4]{x}}\right)-1\right)$

$f(x)$ is defined if $-\log_{1/2}\left(1+\dfrac{1}{\sqrt[4]{x}}\right)-1>0$

or if $\log_{1/2}\left(1+\dfrac{1}{\sqrt[4]{x}}\right)<-1$ or if $\left(1+\dfrac{1}{\sqrt[4]{x}}\right)>(1/2)^{-1}$

or if $1+\dfrac{1}{\sqrt[4]{x}}>2$ or if $\dfrac{1}{\sqrt[4]{x}}>1$

or if $x^{1/4}<1$ or if $0<x<1$

$\therefore$ Domain of the function is (0, 1).

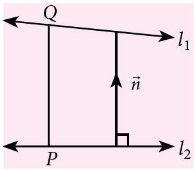
**2. (c) :** We have,

$x+y+z+1=0=2x-y+z+3$ ... (i)

Clearly, a point on the line of intersection of above planes is $P(0, 1, -2)$.

Given equation of line is $\dfrac{x-1}{\alpha}=\dfrac{y+1}{-1}=\dfrac{z}{1}$ ...(ii)

$\because$ Point $Q$ (1, –1, 0) lies on above line




$\therefore\ \overrightarrow{PQ}=\hat{i}-2\hat{j}+2\hat{k}$

Also, line (i) is parallel to

$$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\2&-1&1\end{vmatrix}=2\hat{i}+\hat{j}-3\hat{k}\quad\text{...(iii) (from (i))}$$

Now, $\vec{n}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\\alpha&-1&1\\2&1&-3\end{vmatrix}$ (from (ii) and (iii))

$=2\hat{i}+\hat{j}(3\alpha+2)+\hat{k}(\alpha+2)$

Shortest distance between lines

$$=|\overrightarrow{PQ}.\hat{n}|=\left|\frac{2-2(3\alpha+2)+2(\alpha+2)}{\sqrt{4+(3\alpha+2)^2+(\alpha+2)^2}}\right|=\frac{1}{\sqrt{3}}$$

$\Rightarrow 3(2-4\alpha)^2=10\alpha^2+(16\alpha+12)$

$\Rightarrow 19\alpha^2-32\alpha=0 \Rightarrow \alpha=0, \dfrac{32}{19}$

**3. (c) :** Given, $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$

The projection of $\vec{v}$ along $\vec{u}=\dfrac{\vec{v}\cdot\vec{u}}{|\vec{u}|}$ and the projection of $\vec{w}$ along $\vec{u}=\dfrac{\vec{w}\cdot\vec{u}}{|\vec{u}|}$

According to the question,

$\dfrac{\vec{v}\cdot\vec{u}}{|\vec{u}|}=\dfrac{\vec{w}\cdot\vec{u}}{|\vec{u}|} \Rightarrow \vec{v}\cdot\vec{u}=\vec{w}\cdot\vec{u}$

and $\vec{v}, \vec{w}$ are perpendicular to each other.

$\therefore\ \vec{v}\cdot\vec{w}=0$

Now, $|\vec{u}-\vec{v}+\vec{w}|^2=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2-2\vec{u}\cdot\vec{v}-2\vec{v}\cdot\vec{w}+2\vec{u}\cdot\vec{w}$
$=1+4+9-2\vec{u}\cdot\vec{v}+2\vec{v}\cdot\vec{u}$

$\Rightarrow |\vec{u}-\vec{v}+\vec{w}|^2=14 \Rightarrow |\vec{u}-\vec{v}+\vec{w}|=\sqrt{14}$

**4. (d) :** We have, $f(x)=\sin 2x-x \Rightarrow f'(x)=2\cos 2x-1$

For local maximum or minimum, we have

$f'(x)=0 \Rightarrow 2\cos 2x-1=0 \Rightarrow \cos 2x=\dfrac{1}{2}$

$\Rightarrow 2x=-\dfrac{\pi}{3}$ or $2x=\dfrac{\pi}{3}$

$\left[\because -\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}\quad\therefore -\pi\le 2x\le\pi\right]$

$\Rightarrow x=-\dfrac{\pi}{6}$ or $x=\dfrac{\pi}{6}$

Thus, $x=-\dfrac{\pi}{6}$ and $x=\dfrac{\pi}{6}$ are possible points of local maxima or minima.

Now, we test the function at each of these points.
We have, $f''(x) = -4\sin 2x$
At $x = -\pi/6$ : We have,

$$f''\left(-\frac{\pi}{6}\right)=-4\sin\left(-\frac{\pi}{3}\right)=-4\times\frac{-\sqrt{3}}{2}=2\sqrt{3}>0$$

So, $x=-\frac{\pi}{6}$ is a point of local minimum.

At $x=\frac{\pi}{6}$ : We have,

$$f''\left(\frac{\pi}{6}\right)=-4\sin\frac{\pi}{3}=-4\left(\frac{\sqrt{3}}{2}\right)=-2\sqrt{3}<0$$

So, $x=\frac{\pi}{6}$ is a point of local maximum.

**5. (a) :** Let $x$ and $y$ be two arbitrary elements in $A$.

Then, $f(x) = f(y) \Rightarrow \frac{x-2}{x-3}=\frac{y-2}{y-3}$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow x = y, \forall\ x, y \in A$

So, $f$ is an injective mapping.

Let $y$ be an arbitrary element in $B$, then $f(x) = y$

$$\Rightarrow \frac{x-2}{x-3}=y \Rightarrow x=\frac{3y-2}{y-1}$$

Clearly, $\forall\ y\in B$, $x=\frac{3y-2}{y-1}\in A$, thus of all $y\in B$, there exists $x\in A$ such that $f(x)=f\left(\frac{3y-2}{y-1}\right)=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}=y$

Thus, every element in the co-domain $B$ has its pre-image in $A$, so $f$ is a surjective. Hence, $f: A\to B$ is bijective.

**6. (b) :** Since, it is given that $A$ is twice as likely to be selected as $B$.

$\therefore\ P(A) = 2P(B) \Rightarrow \frac{P(A)}{2}=P(B)$ ...(i)

While $C$ is twice as likely to be selected as $D$.

$\therefore\ P(C) = 2P(D) \Rightarrow P(B) = 2P(D)$ [$\because P(B) = P(C)$]

$\Rightarrow \frac{P(A)}{2}=2P(D) \Rightarrow P(D)=\frac{P(A)}{4}$ ...(ii)

Now, $P(C)=P(B)=\frac{P(A)}{2}$ ...(iii)

As we know, sum of probabilities = 1

$\Rightarrow P(A) + P(B) + P(C) + P(D) = 1$

$\Rightarrow P(A)+\frac{P(A)}{2}+\frac{P(A)}{2}+\frac{P(A)}{4}=1$ [From (i), (ii) & (iii)]

$$\Rightarrow \frac{4P(A)+2P(A)+2P(A)+P(A)}{4}=1$$

$$\Rightarrow 9P(A)=4 \Rightarrow P(A)=\frac{4}{9}$$

$P(A$ will not be selected$) = P(A') = 1 - P(A) = 1-\frac{4}{9}=\frac{5}{9}$

**7. (d) :** Given that,

$$4=\lim_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right)$$

$$=\lim_{x\to\infty}\frac{(a-2)x^3+(3+c)x^2+(b-3)x+2+d}{\sqrt{x^4+ax^3+3x^2+bx+2}+\sqrt{x^4+2x^3-cx^2+3x-d}}$$

Since, the limit is finite, the degree of the numerator must be at the most 2 $\Rightarrow a - 2 = 0$, *i.e.*, $a = 2$

Hence,

$$4=\lim_{x\to\infty}\frac{(3+c)+\frac{b-3}{x}+\frac{2+d}{x^2}}{\sqrt{1+\frac{a}{x}+\frac{3}{x^2}+\frac{b}{x^3}+\frac{2}{x^4}}+\sqrt{1+\frac{2}{x}-\frac{c}{x^2}+\frac{3}{x^3}-\frac{d}{x^4}}}$$

$$\Rightarrow 4=\frac{3+c}{2} \Rightarrow c=5$$

Hence, $a = 2$, $c = 5$ and $b$, $d$ are any real numbers.

**8. (b) :** The equation of the ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$.

$\therefore\ a^2 = 25$ and $b^2 = 9$

Eccentricity of ellipse $=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

So the co-ordinates of the foci are $(\pm4, 0)$.

$\therefore$ The foci of hyperbola are $(\pm4, 0)$.

Let equation of required hyperbola be $\frac{x^2}{a'^2}-\frac{y^2}{b'^2}=1$

Let $e'$ be the eccentricity of required hyperbola then

$a'e' = 4 \Rightarrow 2a' = 4$ [$\because e' = 2$ given] $\Rightarrow a' = 2$

$\therefore\ b'^2 = a'^2(e'^2 - 1) \Rightarrow b'^2 = 4(4 - 1) = 12$

Thus equation of required hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1$.

**9. (a) :** $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$

$= -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 - \ldots$

$= (3^2 - 1^2) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2) + \ldots$

$= 2(4 + 6 + 12 + 14 + \ldots$ upto $2n$ terms$)$

$= 2[(4 + 12 + 20 + \ldots) + (6 + 14 + 22 + \ldots)]$

$$=2\left[\frac{n}{2}(8+8(n-1))+\frac{n}{2}(12+8(n-1))\right]$$

$= 2(4n^2 + 4n^2 + 2n) = 2(8n^2 + 2n) = 4n(4n + 1)$

As $1056 = 32 \times 33$, $1088 = 32 \times 34$,

$1120 = 32 \times 35$, $1152 = 32 \times 36$

We have only 1056 as the value that $S_n$ can take.

**10. (d):** Let $n$ be the number of children taking non-veg food. Then, the number of times the house master can go to dhaba is ${}^nC_3$.

Now, according to the question, ${}^nC_3 - {}^{n-1}C_2 = 84$

$\Rightarrow \dfrac{n(n-1)(n-2)}{6} - \dfrac{(n-1)(n-2)}{2} = 84$

$\Rightarrow (n-1)(n-2)(n-3) = 6 \times 6 \times 14 = 7 \times 8 \times 9$

$\Rightarrow (n-1) = 9 \Rightarrow n = 10.$

**11. (39):** $f(x) = \dfrac{5^x}{5^x+5}$

$$f(2-x) = \frac{5^{2-x}}{5^{2-x}+5} = \frac{25}{25+5\cdot 5^x} = \frac{5}{5^x+5}$$

$\therefore f(x) + f(2-x) = 1$

Now, $2\left[f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+....+f\left(\frac{39}{20}\right)\right]$

$$=2\left[\left(f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)\right)+\left(f\left(\frac{2}{20}\right)+f\left(\frac{38}{20}\right)\right)+...\right.$$

$$\left.+....+\left(f\left(\frac{19}{20}\right)+f\left(\frac{21}{20}\right)\right)+f\left(\frac{20}{20}\right)\right]$$

$$=2\left[1\times 19+\frac{1}{2}\right]=\frac{39}{2}\times 2=39$$

**12. (1):** The given differential equation is

$$\sec^2 y\frac{dy}{dx}+2x\tan y = x^3 \quad ...(i)$$

Let $\tan y = v \Rightarrow \sec^2 y\dfrac{dy}{dx}=\dfrac{dv}{dx}$

From eq. (i), we have $\dfrac{dv}{dx}+2vx=x^3$

$\therefore$ I.F. $= e^{\int 2x\,dx} = e^{x^2}$

$\therefore$ Solution is $\tan y\cdot e^{x^2} = \int x^3\cdot e^{x^2}dx + c'$

Put $x^2 = t \Rightarrow x\,dx = \dfrac{dt}{2}$

$\therefore \tan y\cdot e^{t^2} = \dfrac{1}{2}\int te^t dt + c' = \dfrac{1}{2}(te^t - e^t) + c'$

$\Rightarrow \tan y = \dfrac{1}{2}(t-1)+c'e^{-t^2}$

$\Rightarrow \tan y = \dfrac{1}{2}(x^2-1)+c'e^{-x^2}$

$\Rightarrow 2\tan y = (x^2-1) + 2c'e^{-x^2}$

$= (x^2-1) + ce^{-x^2}$ (Replacing $2c'$ by $c$)

Hence, $\lambda = 1$

**13. (400):** $\because 7x - 10 - x^2 = (x-2)(5-x)$

Let $x = 2\sin^2\theta + 5\cos^2\theta$

$\Rightarrow x - 2 = 3\cos^2\theta,\ 5 - x = 3\sin^2\theta$

$\therefore \sqrt{(7x-10-x^2)} = 3\sin\theta\cos\theta$ and

$dx = -6\sin\theta\cos\theta\, d\theta$

Now, $\displaystyle\int\frac{x\,dx}{\sqrt{(7x-10-x^2)^3}}$

$$=\int\frac{(2\sin^2\theta+5\cos^2\theta)}{27\sin^3\theta\cos^3\theta}(-6\sin\theta\cos\theta)\,d\theta$$

$$=-\frac{2}{9}\int(2\sec^2\theta+5\,\text{cosec}^2\theta)\,d\theta$$

$$=-\frac{2}{9}(2\tan\theta-5\cot\theta)+c=\frac{2}{9}(5\cot\theta-2\tan\theta)+c$$

$$=\frac{2}{9}\left\{5\sqrt{\left(\frac{x-2}{5-x}\right)}-2\sqrt{\left(\frac{5-x}{x-2}\right)}\right\}+c$$

$$=\frac{2}{9}\left\{\frac{7x-20}{\sqrt{(7x-10-x^2)}}\right\}+c \quad \therefore \lambda=\frac{2}{9}$$

Then, $1800\,\lambda = 1800 \times \dfrac{2}{9} = 400$

**14. (0.5):** $f(x) = (\log_{\cot x}\tan x)$

$$(\log_{\tan x}\cot x)^{-1}+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$=\frac{\log\tan x}{\log\cot x}\cdot\frac{\log\tan x}{\log\cot x}+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$=\frac{(\log\tan x)^2}{(-\log\tan x)^2}+\tan^{-1}\left(\frac{4x}{4-x^2}\right)=1+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

$$\therefore f'(x)=\frac{1}{1+\left(\frac{4x}{4-x^2}\right)^2}\cdot\frac{(4-x^2)4-4x(-2x)}{(4-x^2)^2}$$

$$=\frac{4(4+x^2)}{(4-x^2)^2+(4x)^2}$$

Hence, $f'(2)=\dfrac{4(4+4)}{0+(8)^2}=\dfrac{32}{64}=\dfrac{1}{2}=0.5$

**15. (7):** $A^2 = \begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$A^2 = A^4 = A^6 = A^8 = I_3$

$\therefore A^2 + 2A^4 + 4A^6$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}+\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$$

$$=\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}=7I_3=7A^8=kA^8 \quad \therefore k=7$$

# UCD Unique Career in Demand

Explore the available Unique Career Options!

## INTERIOR DESIGNING

Interior Designing is the concept of art and science making interior space more attractive and pleasing with structure, colour scheme, furnishing and decoration. Designing task is handled by Interior Designer, who provides initial design concept and space planning proposals, product and material specifications, as well as co-ordinate with intersecting trades to oversee a project from start to finish.

### Entrance Exams

Most colleges offer admission to the B.Sc. Interior Design Course based on the marks obtained in Class 12th Examination. However, some offer it through the following national level entrance exams.

| Name of the Exams | Registration Months (Tentative) |
|---|---|
| NPAT | December - May |
| CUET | April |
| CUCET | November - May |
| SET | December - April |

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### Courses offered

1. **Diploma Course :** Interested aspirants who are looking for diploma in Interior Designing can join this course. Its duration is 1 year and eligibility for the course is 10th or 12th passed with minimum 50% marks from any stream.
2. **Undergraduate Course :** There are several undergraduate programmes in Interior Designing. Some are mentioned below :
Bachelor of Interior Design (BID), B.Des. Interior Design, B.Sc. Interior Design and B.Arch Interior Design. The duration for these courses are 4 years, 4 years, 3 years and 5 years respectively and eligibility for the courses is 12th passed with 50% marks in any stream.
3. **Postgraduate Course :** It is a master degree course and degree provided in this course is M.A in Interior Design, M.Sc in Interior Design and MBA in Interior Design. The duration for the course is 2 years and eligibility is graduated from any stream with minimum 55% marks from a relevant discipline from a recognised university.

### Jobs and Career Prospects

The type of job that an interior designer gets depends upon several factors which include the industry in which he/she gets hired. There are some most popular job roles for interior designers in India.

- Healthcare Designer
- Set Designer
- Exhibition Designer
- Furniture Designer
- Product Designer
- Lighting Designer
- Industry Designer

## Top Interior Designing Colleges for Placements

**PRIVATE COLLEGES**

Here is a list of some of the top private colleges that provide Interior Design courses along with the highest placement package :

- Lovely Professional University, Jalandhar
- National Institute of Design, Ahmedabad
- Chandigarh University, Chandigarh
- National Institute of Fashion Technology, Delhi
- CEPT University, Ahmedabad
- SRM Institute of Science and Technology, Chennai
- Symbiosis Institute of Design, Pune
- Pearl Academy, Mumbai
- Vogue Institute of Art and Design, Bangalore
- JD Institute of Fashion Technology, Bangalore

**GOVERNMENT COLLEGES**

Below is a list of some of the top government/ public colleges that provide Interior Design courses along with the highest placement package :

- Sir JJ School of Art, Mumbai
- Garware Institute of Career Education, Mumbai
- Bangalore University, Bangalore
- Aligarh Muslim University, Aligarh
- Visva Bharati University, West Bengal
- MAKAUT, Kolkata
- YCMOU, Nashik
- Annamalai University, Tamil Nadu
- CSJM, Kanpur
- SNDT, Mumbai

Class 11 & 12

# *Maths in* FOCUS

TOPIC

Sets, Relations and Functions

*Detailed theory with High Definition images of the given topic is covered under this heading.*

## Sets

### A well defined collection of distinct objects.

#### Standard Notations

$N$ : A set of all natural numbers.
$W$ : A set of all whole numbers.
$Z$ : A set of all integers.
$Z^+/Z^-$ : A set of all positive/negative integers.
$Q$ : A set of all rational numbers.
$Q^+/Q^-$ : A set of all positive/negative rational numbers.
$R$ : A set of all real numbers.
$R^+/R^-$ : A set of all positive/negative real numbers.
$C$ : A set of all complex numbers.

#### Representation of Sets

1. **Roster form/Listing Method/Tabular form :** A set is described by listing the objects or elements, separated by commas and enclosed within braces.
2. **Set-builder form/ Rule Method :** We write a rule or property which gives all the elements of the set.

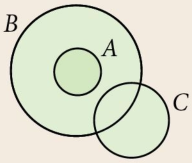
#### Types of Sets

1. **Empty/Null/Void Set :** A set which has no element in it. It is denoted by $\phi$ or { }.
2. **Singleton Set :** A set which contains only one element. It can be written as $\{a\}$.
3. **Finite/Infinite Set :** A set contains a finite number of elements called finite set otherwise it is infinite set.
   $A = \{1, 2, 3, 4, 5\}$ = Finite set
   $A = \{1, 2, 3, ....\}$ = Infinite set
4. **Equivalent Sets :** Two sets are said to be equivalent if they have equal number of elements in each.
   $A = \{a, b, c, d\}$, $B = \{1, 2, 4, 8\}$, $n(A) = n(B) = 4$.
   $\therefore$ $A$ and $B$ are equivalent sets.
5. **Equal Sets :** Two sets $A$ and $B$ are said to be equal if each element of $A$ is a element of $B$ and vice-versa.
   $A = \{a, b, c, d\}$, $B = \{c, d, b, a\} \Rightarrow A = B$

**Note :** The number of elements in a finite set is called order of that set.
If $A = \{9, 8, 3, 2, 1\}$, then order $= 5 = n(A)$

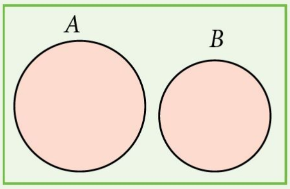
## Subset

$A$ is a subset of $B$ $\forall\, x \in A \Rightarrow x \in B$




$A \subseteq B$, $C \nsubseteq B$

## Disjoint sets

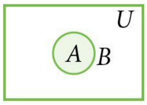



$A \cap B = \phi$

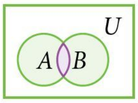
## Operations on sets

### Union of sets

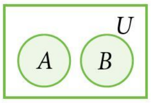
$A \cup B = \{x : x \in A \text{ or } x \in B\}$




$A \cup B$ when $A \subseteq B$



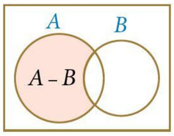

$A \cup B$ when neither $A \subseteq B$ nor $B \subseteq A$



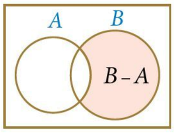

$A \cup B$ when $A$ and $B$ are disjoint sets

### Difference of two sets

$A - B = \{x : x \in A$ and $x \notin B\}$

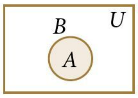



$B - A = \{x : x \in B$ and $x \notin A\}$

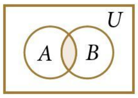



### Intersection of sets

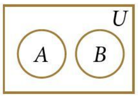
$A \cap B = \{x : x \in A \text{ and } x \in B\}$




$A \cap B$ when $A \subseteq B$
$A \cap B = A$



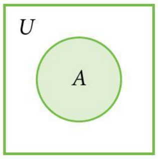

$A \cap B$ when neither $A \subseteq B$ nor $B \subseteq A$




$A \cap B = \phi$ (no shaded region)

### Complement of a set

$A' = U - A = \{x : x \in U \text{ and } x \notin A\}$




## Cartesian product

### Ordered pair

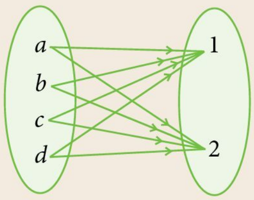
A pair of two elements or objects in a particular order. Let $a \in A$ and $b \in B$. Then $(a, b)$ is an ordered pair.

### Ordered Triplets

For $A$, $B$ and $C$,
$A \times B \times C = \{(a, b, c) : a \in A, b \in B \text{ and } c \in C\}$

### Cartesian product of two sets

For two sets $A$ and $B$, the cartesian product of $A$ and $B$ is defined as $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.

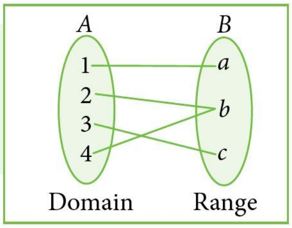


# Relations

A relation from $A$ to $B$ is a subset of $A \times B$ *i.e.* $R \subseteq A \times B$. If $R \subseteq A \times B$ and $(a, b) \in R$, then we say that $a$ is related to $b$ by the relation $R$, represented as $aRb$.

**Domain and Range of a relation.**

Domain = $\{x : x \in A \text{ and } (x, y) \in R \text{ for some } y \in B\}$
Range = $\{y : y \in B \text{ and } (x, y) \in R\ \forall\ x \in A\}$
Domain $\subseteq A$, Range $\subseteq$ B.




## Types of Relations

### Empty or void relations

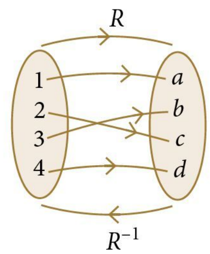
Every subset of $A \times A$ is a relation in A. Since $\phi \subseteq A \times A$, therefore the null set $\phi$ is also a relation.

### Universal relation

Since $A \times A \subseteq A \times A$, so $A \times A$ is the relation on $A$, called universal relation.

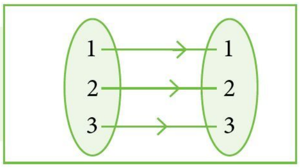
### Inverse relation

Let $R$ be relation from $A$ to $B$, then $R^{-1} = \{(y, x) : y \in B, x \in A, (x, y) \in R\}$.




### Identity relation

For any set $A$, the identity relation $(I_A)$ is defined as $I_A = \{(x, y) : x \in A, y \in A, x = y\}$




## Properties of relation in a set

1. **Reflexive relation :** A relation $R$ on a set $A$ is called reflexive, if $(a, a) \in R\ \forall\ a \in A$.
2. **Symmetric relation :** A relation $R$ on set $A$ is called symmetric relation if $(a, b) \in R \Rightarrow (b, a) \in R\ \forall\ a, b \in A$.
3. **Transitive relation :** A relation $R$ on a set $A$ is called transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R\ \forall\ a, b, c, \in$ A.
4. **Equivalence relation :** A relation $R$ on set $A$ is called an equivalence relation iff $R$ is reflexive, symmetric and transitive.

## Important Points to Remember

Total number of relations from a set consisting of $m$ elements to a set consisting of $n$ elements is $2^{mn}$.

- Number of reflexive relations on a set with $n$ number of elements is given by $2^{n(n-1)}$.
- Number of symmetric relations on a set with $n$ number of elements is given by $2^{\frac{n(n+1)}{2}}$.
- Number of relations on a set with $m$ number of elements which are both reflexive and symmetric is $2^{\frac{m(m-1)}{2}}$.

# CONCEPT MAP

# SETS, RELATIONS AND FUNCTIONS

**Class XI/XII**

## SETS

Collection of well defined distinct objects

### Representation of Sets

- **Set Builder Form :** A set is described by characterising property $P(x)$ of its element $x$.
- **Roster or Tabular Form :** A set is described by listing all the elements separated by commas and enclosed within braces.

### Types of Sets

- **Singleton set :** A set with only one element.
- **Empty set :** A set with no element.
- **Finite set :** A set contains finite number of elements.
- **Infinite set :** Set which is not finite.
- **Equal sets :** Two sets $A$ and $B$ are equal if they have exactly same elements.
- **Equivalent sets :** Two finite sets $A$ and $B$ are equivalent if they have same number of elements.
- **Universal set :** The set that contains all the sets in the given context.
- **Subset :** A set $A$ is said to be subset of set $B$ if each element of set $A$ is also an element of set $B$ *i.e.*, $A \subseteq B$.
- **Proper subset :** If set $A$ is a subset of set $B$ but $A \neq B$, then $A \subset B$.
- **Superset :** If set $A$ is a subset of set $B$ *i.e.*, $A \subseteq B$, then set $B$ is called superset of set $A$.
- **Power set :** Set of all subsets of a given set.

### Cardinal Properties

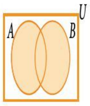
If $A$, $B$ and $C$ are finite sets, $U$ be the finite universal set, then

- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B)$, if $A$, $B$ are disjoint.
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
- $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
- $n(\text{exactly two}) = n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- $n(\text{exactly one}) = n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$

### Venn Diagrams

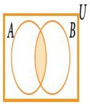
### Operations on Sets

**Union of Sets**



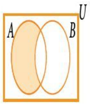

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

**Intersection of Sets**



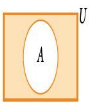

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

**Difference of Sets**



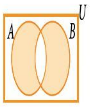

$A - B = \{x : x \in A,\ x \notin B\}$

**Complement of Sets**




$A' = U - A$.

**Symmetric Difference of Sets**




$A \Delta B = \{x : x \in A - B \text{ or } x \in B - A\}$

### Laws of Algebra of Sets

**Idempotent :** $A \cup A = A$; $A \cap A = A$

**Commutative :** $A \cup B = B \cup A$; $A \cap B = B \cap A$

**Associative :** $A \cup (B \cup C) = (A \cup B) \cup C$; $A \cap (B \cap C) = (A \cap B) \cap C$

**Identity :** $A \cup \phi = \phi \cup A = A$; $A \cap \phi = \phi$; $A \cap U = U \cap A = A$; $A \cup U = U$

**Distributive :** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

**Complement Law :** $A \cup A' = U$; $A \cap A' = \phi$

**De Morgan's Law :** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

**Law of Double Complement :** $(A')' = A$

**Law of Empty Set and Universal Set :** $\phi' = U$ and $U' = \phi$

### Properties of Cartesian Product

(i) $A \times B = \phi$, if either $A$ or $B$ is an empty set.

(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(iii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iv) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(v) If $A \subseteq B$ and $C \subseteq D$, then $(A \times C) \subseteq (B \times D)$

(vi) $A \times B = B \times A \Leftrightarrow A = B$

(vii) If $A$ and $B$ have $n$ elements in common, then the number of elements common to $A \times B$ and $B \times A$ is $n^2$.

### Cartesian Product of Sets

Cartesian product of two sets $A$ & $B$ is denoted and defined as, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

Cartesian product of two sets is not commutative.

Note : (i) If $n(A) = p$, $n(B) = q$, then $n(A \times B) = p \times q$.

(ii) $(a, b) = (p, q) \Leftrightarrow a = p \text{ \& } b = q$

## RELATIONS

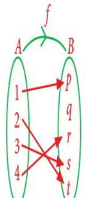
- $R$ is a relation from $A$ to $B$ (where $A, B \neq \phi$) if $R \subseteq A \times B \Leftrightarrow R \subseteq \{(a, b) : a \in A, b \in B\}$
- Domain $(R) = \{a : (a, b) \in R\}$
- Range $(R) = \{b : (a, b) \in R\ \forall\ a \in A\}$

### Types of Relations

- **Empty (Void) Relation :** $R = \phi \Rightarrow R$ is void.
- **Universal Relation :** $R = A \times B \Rightarrow R$ is universal.
- **Reflexive Relation :** Every element is related to itself. *i.e.*, $R$ is reflexive in $A \Leftrightarrow (a, a) \in R\ \forall\ a \in A$.
- **Symmetric Relation :** $R$ is symmetric in $A$ iff $(a, b) \in R \Rightarrow (b, a) \in R\ \forall\ a, b \in A$.
- **Transitive Relation :** $R$ is transitive in $A$ if $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R\ \forall\ a, b, c \in A$.
- **Equivalence Relation :** If $R$ is reflexive, symmetric and transitive then $R$ is equivalence.
- **Antisymmetric Relation :** $R$ is antisymmetric if $(a, b) \in R, (b, a) \in R \Rightarrow a = b$.
- **Identity Relation :** $R = \{(a, a)\ \forall\ a \in A\}$ is an identity relation in $A$.
- **Inverse Relation :** $R^{-1}$ is the inverse relation of $R$ if $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$.
- **Note :** Domain $(R)$ = Range $(R^{-1})$
  Range $(R)$ = Domain $(R^{-1})$

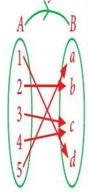
## TYPES OF FUNCTIONS

**One-One (Injective) Function**




- No two elements of $A$ have same image in $B$
  (i) $n(A) \leq n(B)$
  (ii) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

**Onto (Surjective) Function**



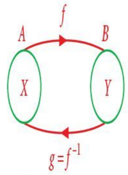

- All the elements of $B$ have atleast one pre-image in $A$.
  (i) $n(A) \geq n(B)$
  (ii) Range = Co-domain

**Bijective Function**

- A function which is both one-one and onto.
  (i) $n(A) = n(B)$
  (ii) Range = Co-domain
- If $n(A) = a = n(B)$, then number of bijections $= a!$

**Inverse of a Function**

- Let $f : A \to B$ is one-one and onto *i.e.*, bijective function. Then we can define a function $g : B \to A$, such that $f(x) = y \Rightarrow g(y) = x$, which is called inverse of $f$ and vice-versa. $g = f^{-1}$




(i) Domain $(f^{-1})$ = Range $(f)$

(ii) Range $(f^{-1})$ = Domain $(f)$

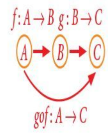
(iii) If $f(x) = y$, then $f^{-1}(y) = x$ and vice-versa.

## FUNCTIONS

- A relation $(f : A \to B)$ where every element of set $A$ has only one image in set $B$.
- Domain $(f) = \{a : (a, b) \in f\}$
- Range $(f) = \{b : (a, b) \in f\ \forall\ a \in A\}$

**Composition of Functions**

- A function $gof$, defined on the range of function $f$, is known as composition of functions.




**Binary operations**

A function $*$ from $A \times A$ to $A$ is called binary operation on non empty set $A$ *i.e.*, $* : A \times A \to A$ is a binary operation on set $A$.

**Existence of Identity**

$e \in X$ is an identity element if $a * e = a = e * a\ \forall a \in A$

**Commutative:** $a * b = b * a\ \forall\ a, b \in A$

**Existence of Inverse**

$a \in A$ is invertible for $*$ if $\exists\ b \in A$ s.t. $a * b = e = b * a\ \forall a, b \in A$

**Associative :** $(a * b) * c = a * (b * c)\ \forall\ a, b, c \in A$

**Algebra of Functions**

For $f : X \to R$ and $g : X \to R$

$(f + g)(x) = f(x) + g(x)$, $x \in X$

$(kf)(x) = kf(x)$, $x \in X$, $k$ is constant

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $x \in X, g(x) \neq 0$

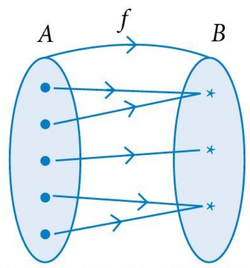
$(f \cdot g)(x) = f(x) \cdot g(x)$, $x \in X$

$(f - g)(x) = f(x) - g(x)$, $x \in X$

# Functions

An expression, rule or law that defines a relationship between one independent variable and another dependent variable.

**Definition :** Let $A$ and $B$ be two non-empty sets. Let there exists a correspondence, denoted by $f$, which associates to each element of $A$ with a unique element of $B$.

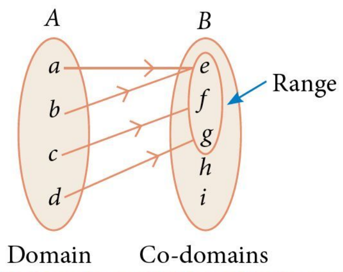



**Domain, Co-domain and Range of a function :**

Let $f : A \to B$

Domain = $\{x : x \in A\}$, Co-domain = $\{x : x \in B\}$

Range $f(A) = \{f(x) : x \in A\}$




## Equal functions

Two functions $f$ and $g$ are said to be equal iff

(i) domain of $f$ = domain of $g$.

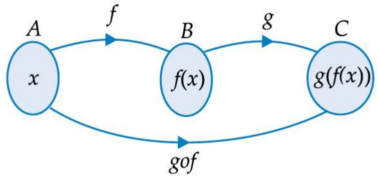
(ii) co-domain of $f$ = co-domain of $g$.

(iii) $f(x) = g(x)$ for every $x$ belonging to their common domain and then we write $f = g$

## Composition of two functions

Let $f : A \to B$ and $g : B \to C$ be two functions. Then the function $gof : A \to C$ defined by $(gof)(x) = g(f(x))$ $\forall\ x \in A$ is called the composition of the two functions $f$ and $g$.

Thus the image of every $x \in A$ under the function $gof$ is the $g$-image of the $f$-image of $x$.



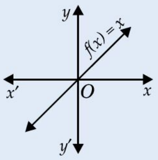

## Properties of Composite Functions

- The composite functions are not commutative *i.e.*, $gof \neq fog$.
- The composite functions are associative *i.e.*, if $f$, $g$ and $h$ are three functions such that $fo(goh)$ and $(fog)\ oh$ are defined, then $fo\ (goh) = (fog)\ oh$.
- The composition of two bijections is a bijection, *i.e.*, if $f$ and $g$ are two bijections such that $gof$ is defined, then $gof$ is also a bijection.

## Some Important Types of Functions

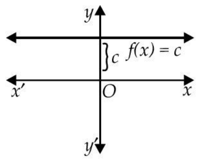
### Identity Functions

The function $f : R \to R$ defined by $y = f(x) = x$ for each $x \in R$. Domain of $f = R$. Range of $f = R$.




### Constant function

The function $f : R \to R$ defined by $y = f(x) = c$ $\forall\ x \in R$, where $c$ is a constant $\in R$.
Domain of $f = R$.
Range of $f = \{c\}$.




### Polynomial function

A real valued function $f : R \to R$ defined by $y = f(x)$ $= a_0 + a_1x + \ldots\ldots + a_nx^n$, where $n \in N$ and $a_0$, $a_1$, $a_2, \ldots, a_n \in R$ for each $x \in R$.

### Rational function

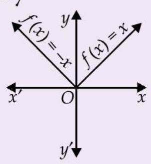
These are the real functions of the type $\dfrac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of $x$ defined in a domain, where $g(x) \neq 0$.

### Modulus function

The real function $f : R \to R$ defined by

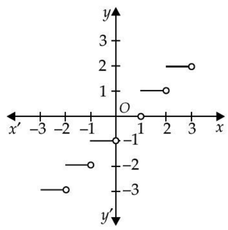
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \quad \forall x \in R.$$

Domain of $f = R$.
Range of $f = R^+ \cup \{0\}$.



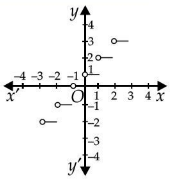

### Greatest Integer Function (Floor Function)

The real function $f : R \to R$ defined by $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer less than or equal to $x$.
Domain of $f = R$.
Range of $f = Z$.



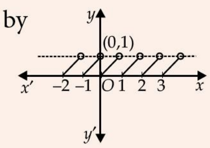

### Smallest Integer Function (Ceiling Function)

The real function $f : R \to R$ defined by $f(x) = \lceil x \rceil \forall x \in R$ assumes the value of the smallest integer greater than or equal to $x$.
Domain of $f = R$.
Range of $f = Z$.




### Fractional part function

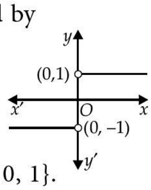
The function $f : R \to R$ defined by $f(x) = \{x\} = x - [x]\ \forall x \in R$.
Domain of $f = R$.
Range of $f = [0, 1)$.




### Signum function

The real function $f : R \to R$ defined by

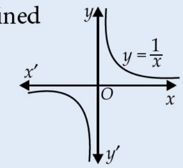
$$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$




Domain of $f = R$. Range of $f = \{-1, 0, 1\}$.

### Reciprocal function

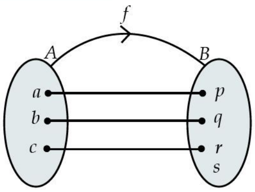
The real function $f : R - \{0\} \to R$ defined by $f(x) = \dfrac{1}{x}$
Domain of $f = R - \{0\}$.
Range of $f = R - \{0\}$.



# Different Types of Mapping

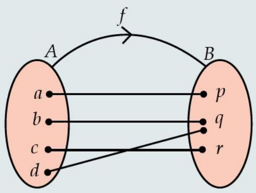
## One-One (Injective)

A function $f : A \to B$ is called one-one, if distinct elements of $A$ have distinct images in $B$ *i.e.*, for every $x_1, x_2 \in A$, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$ and if $f(x_1) = f(x_2)$, then $x_1 = x_2$.



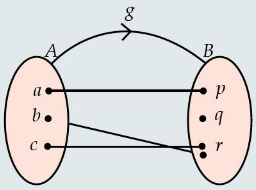

## Onto (Surjective)

$A \to B$ is called an onto or surjective function if each element of $B$ is the image of some element of $A$ under $f$ *i.e.*, for every $y \in B$, there exists an element $x$ in $A$ such that $f(x) = y$.



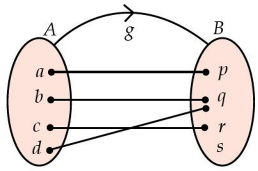

## Into

A function $f : A \to B$ is called an into function if there exists an element in $B$, which has no pre-image in $A$.




## Many-One

A function $f : A \to B$ is called many-one function if there exist atleast two distinct elements in $A$, whose images are same in $B$ *i.e.*, if there exist $x_1, x_2 \in A$ such that $x_1 \neq x_2$ but $f(x_1) = f(x_2)$.

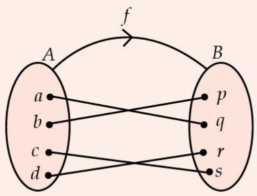



## One-One and Onto (Bijective)

A function $f : A \to B$ is called one-one and onto (or bijective) iff it is both one-one and onto *i.e.*, if

(i) distinct elements of $A$ have distinct images in $B$.

(ii) each element of $B$ is the image of some elements of $A$.



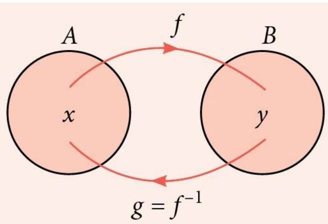

## Importants Points to Remember

Let $f : A \to B$. $A$ and $B$ be two non-empty sets such that $n(A) = p$ and $n(B) = q$, then

- Number of functions from $A$ to $B$ is $q^p$.
- Number of one-one functions from $A$ to $B$ $= \begin{cases} {}^qP_p; & p \leq q \\ 0; & p > q \end{cases}$
- Number of onto functions from $A$ to $B$ $= \begin{cases} \displaystyle\sum_{r=1}^{q} (-1)^{q-r}\, {}^qC_r\, r^p; & p \geq q \\ 0 & ; p < q \end{cases}$
- Number of bijections from $A$ to $B$ $= \begin{cases} p! & ; p = q \\ 0 & ; p \neq q \end{cases}$

## Inverse of a function

Let $f : A \to B$ is one-one and onto function, then we can define a function $g : B \to A$, such that $f(x) = y$
$\Rightarrow g(y) = x$, which is called inverse of $f$ and vice-versa.
A function whose inverse exists, is called an invertible or inversible.
(i) Domain $(f^{-1})$ = Range $(f)$ (ii) Range $(f^{-1})$ = Domain $(f)$
(iii) If $f(x) = y$, then $f^{-1}(y) = x$ and vice-versa.



# GK CORNER

Enhance Your General Knowledge with Current Updates!

## BOOKS AND AUTHORS

- **Collective Spirit, Concrete Action**

  On the occasion of the 100th episode of 'Mann Ki Baat', a book called 'Collective Spirit, Concrete Action' written by Shashi Shekhar Vempati, former CEO of Prasar Bharati (2017-2022), documenting various facets of PM Modi's ongoing conversations with the people that resonate with the nation, was released.

- **My life as a Comrade**

  KK Shailaja, the former health minister of Kerala, who gained worldwide recognition for her success in managing the Covid-19 pandemic, has released a memoir entitled "My Life as a Comrade".

- **The Golden Years: The Many Joys of Living a Good Long Life**

  Ruskin Bond, an Indian author of British descent, authored a new book titled "The Golden Years: The Many Joys of Living a Good Long Life", published by HarperCollins India, which released on 19th May 2023. The book is about the last 20 to 30 years of his life (60s, 70s and 80s).

- **Partitioned Freedom**

  Assam Governor Gulab Chand Kataria released a book titled 'Partitioned Freedom' written by Ram Madhav at an event in Guwahati. The book explores the untold history of India's partition in 1947 and the birth of Pakistan.

- **NTR-A Political Biography**

  A Journalist, editor and writer, Ramachandra Murthy Kondubhatla, has authored a new book titled "NTR-A Political Biography" which presents a realistic picture of Nandamuri Taraka Rama Rao (NTR). It is a comprehensive political biography that traces his journey from a remote Andhra village to the forefront of the national stage via a thriving career in Telugu films.

- **RINGSIDE: Up, Close and Personal on India and Beyond**

  A new book written by Chairman of the Editorial Board of Lokmat Media Group and former MP Dr. Vijay Darda, released by renowned author and Congress MP Dr Shashi Tharoor on May 30. This book is a reflection of contemporary political, social, cultural and other significant happenings and developments.

- **Just Aspire: Notes on Technology, Entrepreneurship and the Future**

  Padma Bhushan recipient, Ajai Chowdhry shares the inspiring story in his book 'Just Aspire: Notes on Technology, Entrepreneurship and the Future', how his dreams and aspirations propelled him from a small town to the forefront of pathbreaking revolutions that transformed India.

- **Cyber Encounters**

  On 7th May 2023, Uttarakhand Chief Minister Pushkar Singh Dhami released the Hindi version of the book 'Cyber Encounters' at St. Joseph's Academy, Dehradun. The author of this book, DGP Ashok Kumar, is a 1989 Indian Police Service (IPS) officer of the Uttarakhand cadre.

- **Guts Amidst Bloodbath : The Aunshuman Gaekwad Narrative**

  Anshuman Gaekwad, a former Indian Test cricketer, released his semi-autobiographical book titled "Guts Amidst Bloodbath" at the Cricket Club of India (CCI).

- ## Droupadi Murmu: From Tribal Hinterlands to Raisina Hills

  Journalist Kasturi Ray authored the book "Droupadi Murmu: From Tribal Hinterlands to Raisina Hills" chronicles the remarkable journey of a tribal girl who defied the odds to become an embodiment of resilience, determination, and perseverance.

- ## The Indian Metropolis: Deconstructing India's Urban Spaces

  Feroze Varun Gandhi is a third-term Member of Parliament, presented a monumental work that shows how economic vitality can go hand-in-hand with creating vibrant cities offering a haven for cultural and intellectual expression.

- ## Supreme Court On Commercial Arbitration

  A compendium of three volumes with Judgments spanning from the year 1988 till 2022 covering Arbitration Act 1940 and 1996, the book titled 'Supreme Court on Commercial Arbitration' by Dr.Manoj Kumar and fore worded by Shri.R.Venkataramani was released on 13$^{th}$ May, 2023 being the founders day of Hammurabi & Solomon Partners.

## Test Yourself!

1. The book 'Droupadi Murmu: From Tribal Hinterlands to Raisina Hills'is authored by
   (a) Arundhati Roy (b) Kasturi Ray
   (c) Jhumpa Lahiri (d) Aravind Adiga

2. Which of the following released the Hindi version of the book 'Cyber Encounters' at St. Joseph's Academy, Dehradun?
   (a) Bina Agarwal (b) Arjun Appadurai
   (c) Rajni Kothari (d) Pushkar Singh Dhami

3. Which book talks about Ajai Chowdhry's own life, who cofounded HCL and reflects on the journey of India's IT and hardware industry?
   (a) The Oxford Handbook of India
   (b) Cybersecurity For Dummies
   (c) Just Aspire: Notes on Technology, Entrepreneurship and the Future
   (d) As Long As Grass Grows

4. On the occasion of the 100th episode of 'Mann Ki Baat', which of the following book was released?
   (a) Madhushala
   (b) Hear Yourself
   (c) The People Who Report More Stress
   (d) Collective Spirit, Concrete Action

5. The book The Indian Metropolis: Deconstructing India's Urban Spaces is written by
   (a) Jeffrey Archer
   (b) Shashi Tharoor
   (c) Feroze Varun Gandhi
   (d) Alexandra Gajda

6. Anshuman Gaekwad, a former Indian Test cricketer, released his semi-autobiographical book, what is the name of his book?
   (a) Beyond a Boundary
   (b) Guts Amidst Bloodbath
   (c) The Art of Captaincy
   (d) The Unquiet Ones

7. The newly published book "The Golden Years: The Many Joys of Living a Good Long Life"is written by
   (a) Ruskin Bond (b) Khushwant Singh
   (c) Haruki Murakami (d) Stephen King

8. A book titled "NTR-A Political Biography" which presents a realistic picture of Nandamuri Taraka Rama Rao (NTR) is authored by
   (a) Vikram Seth
   (b) Kalki Krishnamurthy
   (c) Ramachandra Murthy Kondubhatla
   (d) R. K. Narayan

9. KK Shailaja, the former health minister of Kerala, who gained worldwide recognition for her success in managing the Covid-19 pandemic, has released a memoir entitled
   (a) Our Country Friends
   (b) India's Vaccine Growth Story
   (c) My life as a Comrade
   (d) The God of Small Things

10. Who wrote "RINGSIDE – Up, Close and Personal on India and Beyond"?
    (a) Dr. Vijay Darda (b) Arvind adiga
    (c) Chetan Bhagat (d) Anita nair

### Answer Key

**1.** (b) **2.** (d) **3.** (c) **4.** (d) **5.** (c)
**6.** (b) **7.** (a) **8.** (c) **9.** (c) **10.** (a)

# Brain Teaser

# LOGICAL REASONING
## for Various Competitive Exams

***Direction Q.(1 and 2) : Find the wrong term in the given series.***

**1.** 3, 4, 12, 45, 198, 1005, 6066
(a) 4 (b) 6066 (c) 45 (d) 196

**2.** A25Z, B23Y, C21X, D19W, E15V
(a) A25Z (b) B23Y (c) C21X (d) E15V

***Direction Q.(3 and 4) : The two words or numbers on the right of (: :) are related in the some way as the words on the left of (: :). Identify what will come in place of blank?***

**3.** 839 : 919 :: 1012 : ……………
(a) 1052 (b) 1072 (c) 1132 (d) 1093

**4.** PEN : SAS :: NIB : ……………
(a) MLY (b) SCI (c) QEG (d) RBR

***Direction Q.(5 and 6) : Three of the following four are alike in a certain way and so form a group. Which is the one that does not belong to that group?***

**5.** (a) Extend (b) Higher
(c) Upward (d) Rise

**6.** (a) ABDG (b) STVZ
(c) EFHK (d) HIKN

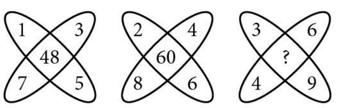
***Direction Q.(7 and 8): In each of the following figure, find the number/letter which replaces the sign of '?'.***

**7.**

| | | |
|---|---|---|
| 1 | 26 | 6 |
| 21 | 41 | 31 |
| 16 | ? | 11 |

(a) 27 (b) 36 (c) 34 (d) 32

**8.**




(a) 60 (b) 66 (c) 56 (d) 73

**9.** In a code language, 'SHOULDER' is coded as 'VPITQDCK', then 'MORNINGS' will be coded as
(a) OSPNRFMH (b) NPSORFMH
(c) OSPNHMFR (d) OSPNSFEM

**10.** In a code language, 'PROBLEM' is coded as 'MELAPRO', then 'SAVIOUR' will be coded as
(a) RUOHVAS (b) ROUHSAV
(c) RUOJSAV (d) RUOHSAV

***Direction Q.(11 and 12): Read the given information carefully and answer the following questions.***
*M* is the mother of *B*. *A* is husband of *M*. *N* is the only brother of *B*. *C* is married to *N*. *Q* is only child of *C*. *N* does not have sister. *J* is the father of *A*.

**11.** If *A* does not have any grandson, then how is *Q* related to *B*?
(a) Cannot be determined
(b) Sister-in-law
(c) Daughter-in-law
(d) Niece

**12.** How is *A* related to *C* ?
(a) Uncle (b) Cannot be determined
(c) Father-in-law (d) Nephew

**13.** Two buses start from opposite points on a main road 120 km apart. The first bus A runs for 30 km along the main road and takes a right turn and then runs for 10 km. It then turns left and run for another 30 km and takes the direction back to reach the main road. In the meantime due to minor breakdown the other bus B has run only 40 km along the main road. What would be the distance between two buses at this point?
(a) 10 km (b) 30 km (c) 20 km (d) 40 km

***Direction Q.(14 and 15): In the following questions there are given three statements followed by two conclusions numbered (I) and (II). You have to take the given three statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the three given statements, disregarding commonly known facts. Read both the statements and Mark answer as :***
(a) if only conclusion (I) follows.
(b) if only conclusion (II) follows.
(c) if both (I) and (II) follows.
(d) if neither (I) nor (II) follows.

**14. Statements :**
All gliders are parachutes.
No parachute is an airplane.
All airplanes are helicopters.
**Conclusions :**
(I) No glider is an airplane.
(II) All gliders being helicopters is a possibility.

**15. Statements:**
No stone is a metal.
Some metals are papers.
All papers are glass.
**Conclusions:**
(I) No glass is a metal.
(II) Atleast some glass is metal.

**16.** In a class of students, Raman is $10^{th}$ from top in terms of height and Lalit is $18^{th}$ from bottom in terms of height. There are 6 persons in between Raman and Lalit. What will be the minimum and maximum number of students in class?
(a) 26, 32 (b) 20, 34 (c) 18, 34 (d) 20, 32

***Direction Q.(17 to 20): Study the following information carefully and answer the questions given below:***
Each of the seven friends viz., P, Q, R, S, T, U and V joined different courses viz., MBA, MBBS, Law, Engineering, Arts, Science and Commerce, but not necessarily in the same order, on seven different days of the same week from Monday to Sunday.
Only three friends joined courses after S. Only two friends joined courses between S and one who joined Law. Only three friends joined courses between the one who joined Law and the one who joined Engineering. Only one friend joined between V and the one who joined Arts. V joined before that friend who joined Arts. V joined courses neither on Tuesday nor on Wednesday. V did not join Engineering. Only three friends joined courses between V and R. P joined course on the day immediately before the one who joined Commerce. Neither S nor T joined Commerce. Q Joined MBBS. P did not join Science.

**17.** Who amongst the following joined course before the day the one joined MBA?
(a) U (b) P
(c) The friend who joined MBBS
(d) The friend who joined Engineering

**18.** Who amongst the following joined course on Tuesday?
(a) The one who joined Engineering
(b) S
(c) R
(d) The one who joined MBBS

**19.** On which of the following days did one friend join Commerce?
(a) Friday (b) Saturday
(c) Sunday (d) Wednesday

**20.** The friend who joined Science, joined on which of the following days?
(a) Wednesday (b) Saturday
(c) Monday (d) Thursday

## SOLUTIONS

**1. (d) :** $3 \xrightarrow{\times 1+(1)^2} 4 \xrightarrow{\times 2+(2)^2} 12 \xrightarrow{\times 3+(3)^2}$
$45 \xrightarrow{\times 4+(4)^2} 198 \xrightarrow{\times 5+(5)^2} 1005 \xrightarrow{\times 6+(6)^2} 6066$

**2. (d) :** Place value of $Z$ – Place value of $A \Rightarrow 26 - 1 = 25$
Place value of Y – Place value of B $\Rightarrow 25 - 2 = 23$
Place value of X – Place value of C $\Rightarrow 24 - 3 = 21$
Place value of W – Place value of D $\Rightarrow 23 - 4 = 19$
Place value of V – Place value of E $\Rightarrow 22 - 5 = \boxed{15}$
Thus, wrong term is E15V.

**3. (a) :** 839 shall be read as 8 : 39 on the clock.
9 : 19 = 8 : 39 + 40 minutes.

**4. (c) :** P + 3 = S, E – 4 = A, N + 5 = S.
So, N + 3 = Q, I – 4 = E, B + 5 = G.

**5. (a) :** 'Extend' is different from the others words. Except the word Extend all other words indicate altitude.

**6. (b) :** (a) A + 1 = B; B + 2 = D; D + 3 = G
(b) S + 1 = T; T + 2 = V; V + 4 = Z
(c) E + 1 = F; F + 2 = H; H + 3 = K
(d) H + 1 = I; I + 2 = K; K + 3 = N

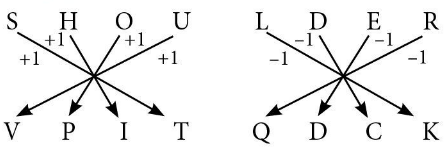
**7. (b) :**

| | | |
|---|---|---|
| $(5 \times 0) + 1$ | $(5 \times 5) + 1$ | $(5 \times 1) + 1$ |
| $(5 \times 4) + 1$ | $(5 \times 8) + 1$ | $(5 \times 6) + 1$ |
| $(5 \times 3) + 1$ | ? | $(5 \times 2) + 1$ |

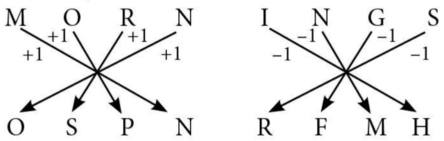
$\therefore$ ? = $(5 \times 7) + 1 = 36$

**8. (b) :** Figure 1 : $3 \times (1 + 3 + 7 + 5) = 48$
Figure 2 : $3 \times (4 + 6 + 8 + 2) = 60$
Figure 3 : ? = $3 \times (3 + 9 + 6 + 4) = 66$.

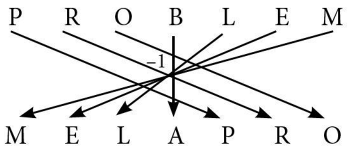
**9. (a) :**



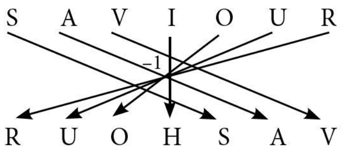
Similarly,



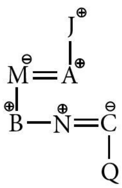

**10. (d) :**




Similarly,

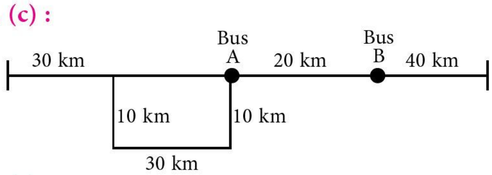



**(11 - 12) :**

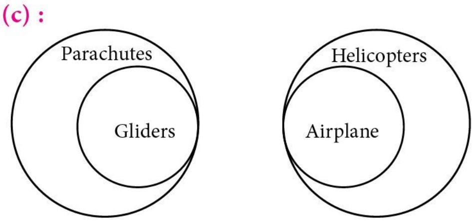



**11. (d)** **12. (c)**

**13. (c) :**

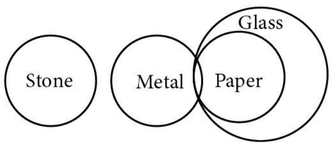



**14. (c) :**



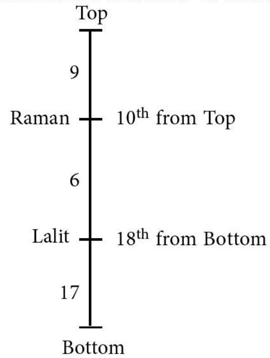

As no parachute is airplane, thus conclusion (I) is valid. Gliders may be or may not be helicopters, thus conclusion (II) is valid. Thus, conclusions (I) and (II) both follow.

**15. (b) :**



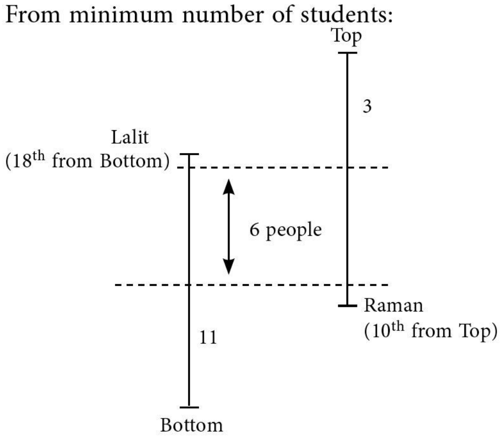

As metal and glass intersect thus conclusion (I) is not valid and conclusion (II) is valid. Thus, only conclusion (II) follows.

**16. (b) :** For maximum number of students:




Hence, total students (maximum) = 18 + 10 + 6 = 34

From minimum number of students:




Hence, total number of students = 11 + 3 + 6 = 20 here in top 3, Lalit is included and in bottom 11, Raman is included.

| Day | Friends | Courses |
|---|---|---|
| Monday | V | Law |
| Tuesday | Q | MBBS |
| Wednesday | T | Arts |
| Thursday | S | Science |
| Friday | R | Engineering |
| Saturday | P | MBA |
| Sunday | U | Commerce |

**17. (d) :** P joined MBA on Saturday. R joined Engineering on Friday.

**18. (d) :** Q joined MBBS on Tuesday.

**19. (c) :** U joined Commerce on Sunday.

**20. (d) :** S joined Science on Thursday.

# Quiz CLUB


Unlock Your Knowledge!

**1.** Find the angle between the lines whose direction cosines are given by the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

**2.** If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of $k$.

**3.** What is the integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} - xy = 1$?

**4.** Find the area of the region bounded by the curve $y = \sqrt{4-x^2}$ and $x$-axis.

**5.** Find the value of $\int\limits_{-\pi/4}^{\pi/4} \frac{e^x \cdot \sec^2 x\, dx}{e^{2x}-1}$.

**6.** It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$. Find the value of $a$.

**7.** Find the angle made by the tangent to the parabola $x^2 = 2y$ at the point $\left(1, \frac{1}{2}\right)$ with the $x$-axis.

**8.** Find the length of the longest interval, in which the function $3 \sin x - 4\sin^3 x$ is increasing.

**9.** If $p > q > 0$ and $pr < -1 < qr$, then find the value of $\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right)$.

**10.** If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on $[1, 3]$ with $c = \left(2 + \frac{1}{\sqrt{3}}\right)$, then find the value of $a$ and $b$.

**11.** If the system of equations $2x + 3y + 5 = 0, x + ky + 5 = 0$, $kx - 12y - 14 = 0$ has non-trivial solution, then find the value of $k$.

**12.** Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that only one of them will qualify the examination.

**13.** For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The find the variance of the combined data set.

**14.** Evaluate : $\lim\limits_{x\to\infty}\left(\frac{\sum\limits_{k=1}^{1000}(x+k)^m}{x^m + 10^{1000}}\right)$, for $m > 101$.

**15.** What is the length of foot of perpendicular drawn from the point $P(3, 4, 5)$ on $y$-axis?

**16.** If the tangent at $(1, 7)$ to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then find the value of $c$.

**17.** If length of common chord of two circles $x^2 + y^2 + 8x + 1 = 0$ and $x^2 + y^2 + 2\mu y - 1 = 0$ is $2\sqrt{6}$, then find the positive value of $\mu$.

**18.** If $x, y, z$ are positive integers, then find the value of expression $(x + y)(y + z)(z + x)$.

**19.** Find the coefficient of $x$ in the expansion of $(1 + x)(1 + 2x)(1 + 3x) \ldots (1 + 100x)$.

**20.** How many different nine digit numbers can be formed from the number 223355888 by re-arranging its digits so that the odd digits occupy even positions?

◆◆

*Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.*

# QUANTITATIVE APTITUDE

For Various Competitive Exams

**1.** Let $N = 55^3 + 17^3 - 72^3$. $N$ is divisible by
(a) both 7 and 13 (b) both 3 and 13
(c) both 16 and 7 (d) both 3 and 17

**2.** $\frac{2}{5}$ of the voters promise to vote for $P$ and the rest promised to vote for $Q$. Of these, on the last day 15% of the voters went back on their promise to vote for $P$ and 25% of voters went back on their promise to vote for $Q$, and $P$ lost by 2 votes. Then, the total number of voters is
(a) 100 (b) 110 (c) 90 (d) 95

**3.** Sanjay borrowed ₹ $X$ at the rate of 10% on a compound interest basis. He repaid ₹ 100 at the end of year 1, ₹ 200 at the end of year 2, ₹ 300 at the end of year 3 and ₹ 44 at the end of year 4 to repay his debt. What was $X$?
(a) ₹ 517.64 (b) ₹ 511.65
(c) ₹ 528.61 (d) ₹ 524.62

**4.** A balance of a dealer weighs 10% less than it should be. Still the trader marked - up his goods to get an overall profit of 20%. What is the markup done by the dealer on the cost price?
(a) 4% (b) 8% (c) 12% (d) 16%

**5.** A fruit seller buys some oranges at the rate of 4 for ₹ 10 and an equal number more at 5 for ₹ 10. He sells the whole lot at 9 for ₹ 20. What is his loss or gain per cent?
(a) Loss per cent $1\frac{19}{81}$ (b) Gain percent $1\frac{19}{81}$
(c) No loss or no profit (d) Loss per cent 2%

**6.** *A* contractor employed 25 labourers on a job. He was paid ₹ 27500 for the work. After retaining 20 percent of the sum, he distributed the remaining amount amongst the labourers. If the number of men to women labourers was in the ratio 2 : 3 and their wages in the ratio 5 : 4, what wages did each woman labourer get?
(a) ₹ 800 (b) ₹ 1000 (c) ₹ 1200 (d) ₹ 1500

**7.** Tarun and Vikram enter into a partnership with ₹ 50000 and ₹ 60000, respectively. Mahesh joins them after $n$ months contributing ₹ 70000 and Vikram leaves $n$ months before the end of the year. If they share the profit in the ratio of 20 : 18 : 21, then find the value of $n$.
(a) 3 (b) 6 (c) 8 (d) 9

**8.** In a class with a certain number of students, if one new student weighing 50 kg is added, then the average weight of the class increased by 1 kg. If one more student weighing 50 kg is added, then the average weight of the class increases by 1.5 kg over the original average. What is the original weight (in kg) of the class?
(a) 46 (b) 42 (c) 27 (d) 47

**9.** Three math classes: $X$, $Y$, and $Z$, take an algebra test.
The average score in class $X$ is 83.
The average score in class $Y$ is 76.
The average score in class $Z$ is 85.
The average score of all students in classes $X$ and $Y$ together is 79.
The average score of all students in classes $Y$ and $Z$ together is 81.
What is the average score for all the three classes, taken together?
(a) 81 (b) 81.5 (c) 82 (d) 84.5

**10.** In Nuts and Bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 minutes after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 minutes after production of every 1500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9000 pairs of nuts and bolts?
(a) 130 minutes (b) 135 minutes
(c) 170 minutes (d) 180 minutes

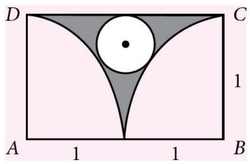
**11.** Two people $A$ and $B$ are separated by a distance of 120 km at 6 a.m. in the morning. After 2 hours, $A$ starts moving towards $B$ at a speed of 25 km/h while at 10 a.m. $B$ starts moving towards $A$ at a speed of 10 km/h. At what time will they meet?

(a) 12 noon (b) 12:30 p.m.
(c) 1 p.m. (d) 1:30 p.m.

**12.** Sunil reaches school everyday at 5 P.M. to pick up his children. On Friday, the school got over at 4 P.M. and the children started walking towards home. Sunil met them on the way and returned home 30 minutes early. For how many minutes did the children walk?
(a) 45 minutes (b) 60 minutes
(c) 30 minutes (d) None of these

**13.** A two-digit number is equal to four times the sum of its digits. The difference between the number obtained by interchanging the number and the original number is 27. Find the original number.
(a) 72 (b) 63 (c) 36 (d) 54

**14.** In the following figure, $ABCD$ is a rectangle with $AD$ and $DC$ equal to 1 and 2 units, respectively. Two quarter circle are drawn with centers at $B$ and $A$, respectively. Now a circle is drawn touching both the quarter circles and one of the sides of rectangle. Find the area of shaded region.




(a) $\frac{32}{115}$ sq. unit (b) $\frac{13}{56}$ sq. unit
(c) $\frac{16}{83}$ sq. unit (d) $\frac{7}{20}$ sq. unit

**15.** 2 Men, 6 women and 8 boys can do a work in 192 hours; 4 men and 16 boys can do it in 160 hours; and 4 men and 6 women can do it in 240 hours. In how many hours can it be done by 10 men and 24 boys?
(a) $91\frac{9}{11}$ (b) $92\frac{28}{31}$ (c) $94\frac{27}{29}$ (d) $87\frac{3}{11}$

***Directions for questions 16 to 20: Answer the questions on the basis of the information given below:***

**Annual Sales Achieved by Five Companies (₹ in lakh) from 2017-2022**

| Company | Year | | | | | |
|---|---|---|---|---|---|---|
| | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 |
| A | 864 | 900 | 875 | 1000 | 975 | 970 |
| B | 785 | 1040 | 980 | 1060 | 980 | 1200 |
| C | 620 | 780 | 820 | 910 | 940 | 1000 |
| D | 695 | 790 | 740 | 800 | 850 | 900 |
| E | 740 | 850 | 520 | 980 | 460 | 900 |

Percent profit

$$=\left(\frac{\text{Sales-Cost}}{\text{Cost}}\right)\times 100 \text{ or } \left(\frac{\text{Revenue-Expenditure}}{\text{Expenditure}}\right)\times 100$$

**16.** In 2021, the sales of company $B$ is what percentage of sales of company $E$?
(a) 47% (b) 150% (c) 300% (d) 213%

**17.** If the expenditure of company $D$ for 2020 is ₹ 600 lakhs, what is the profit percentage?
(a) 30% (b) 33.33% (c) 25% (d) 20%

**18.** If the five companies together have 50% market share in that industry in 2020, what is the market share of $B$ in 2020?
(a) 10.4% (b) 15.35% (c) 11.15% (d) 12.2%

**19.** If the industry in 2021 grows by 20%, what is the market share of $B$ in 2021? (use data from the previous question)
(a) 8.2% (b) 8.6% (c) 9.2% (d) 12.33%

**20.** If simple annual growth rate is 15%, then what should be the sales for company $C$ in 2024?
(a) ₹ 1150 lakh (b) ₹ 1300 lakh
(c) ₹ 1325 lakh (d) ₹ 1450 lakh

## SOLUTIONS

**1. (d) :** $N = 55^3 + 17^3 - 72^3$
(If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$) $= -3 \times 55 \times 17 \times 72$
So, it is divisible by 3 and 17.

**2. (a) :** Let there be 100 voters in all. So initially there were 40 of these who promised to vote for $P$, while 60 of them promised to vote for $Q$. On the last day, (15% of 40) = 6 voters shifted their interest from $P$ to $Q$ and (25% of 60) = 15 voters shifted their interest from $Q$ to $P$. So finally, $P$ would end up getting (40 – 6 + 15) = 49 votes and $Q$ would end up getting (60 – 15 + 6) = 51 votes. Hence, margin of victory for $Q$ = (51 – 49) = 2, which matches the data given in the question. Hence, there were 100 voters in all.

**3. (b) :** Amount borrowed = ₹ $x$
Amount remaining at the end of year 1 = $1.1x - 100$
Amount remaining at the end of year 2
$= (1.1x - 100) \times 1.1 - 200 = 1.21x - 310.$
Amount remaining at the end of year 3
$= (1.21x - 310) \times 1.1 - 300 = 1.331x - 641.$
Amount remaining at the end of year 4 = 0
$(1.331x - 641) \times 1.1 - 44 = 0$
$x$ = ₹ 511.65

**4. (b) :** Let the CP be ₹ 1 per g.
But he weights 900 g. for every 1000 g.
$\therefore$ Value of goods sold = 900
Now, let the markup be $x$%

$\therefore$ M.P. $= 1000 + \dfrac{1000x}{100} = (1000 + 10x)$

But since M.P. = SP, $\therefore$ SP = $(1000 + 10x)$

Hence, Profit (%) $= \dfrac{(1000 + 10x) - 900}{900} \times 100 = 20$

$\Rightarrow \; x = 8$

Thus the markup = 8%.

**5. (a) :** Let the no. of items bought be LCM of 4, 5, 9 =180
So, let the number of oranges who bought 180 of 1st variety and 180 of 2nd variety as it is given he bought the same number.
As 1st variety of ₹ 10 for 4 , For 180 = ₹ 450
As 2nd variety of ₹ 10 for 5 , For 180 = ₹ 360
Total CP = ₹ 810
As he is selling 360 oranges for 9 for ₹ 20

$\therefore \; \dfrac{360}{9} \times 20$ = ₹ 800

Total SP = ₹ 800, As CP > SP, Loss = 10

Loss % $= \dfrac{10}{810} \times 100 = 1\dfrac{19}{81}$%

**6. (a) :** Suppose the wages of each man = $5x$ and wages of each woman = $4x$

Number of men $= \dfrac{2}{5} \times 25 = 10$

Number of women $= \dfrac{3}{5} \times 25 = 15$

Now ₹ 22000 are to be divided among 10 men and 15 women.
So, $10 \times 5x + 15 \times 4x = 22000$
$110x = 22000 \Rightarrow x = 200$
So, each women get = $4x = 4 \times 200$ = ₹ 800

**7. (a) :** Tarun invests 50000(12)
Vikram invests $60000(12 - n)$
Mahesh invests $70000(12 - n)$
Ratio is $[60 : 72 - 6n : 84 - 7n]$ ...(i)
Ratio given is 20 : 18 : 21 ...(ii)
Dividing equation (i) by 3

which gives $20 : \dfrac{72 - 6n}{3} : \dfrac{84 - 7n}{3}$ ...(iii)

Now comparing (iii) and (ii),

$\dfrac{72 - 6n}{3} = 18 \Rightarrow 72 - 6n = 54$

$\Rightarrow 6n = 18 \Rightarrow n = 3$

**8. (d) :** Let number of students be $n$ and average weight $w$.
According to the given condition,

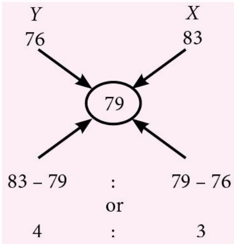
$\dfrac{nw + 50}{n + 1} = w + 1$ ...(1)

and $\dfrac{nw + 50 + 50}{n + 2} = w + 1.5$

$1.5n + 2w = 97$ ...(2)

On solving (1) and (2), we get
$w = 47$

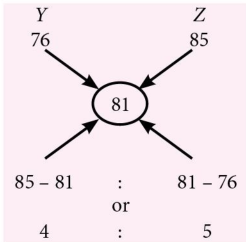
**9. (b) :** Average score of $X = 83$
Average score of $Y = 76$
Average score of $Z = 85$
Average of $X$ & $Y = 79$
Average of $Y$ & $Z = 81$
Using cross alligation for $X$ & $Y$




$\dfrac{X}{Y} = \dfrac{3}{4}$ ...(1)

Using cross alligation for $Y$ & $Z$

$\dfrac{Y}{Z} = \dfrac{4}{5}$ ...(2)




$\therefore X : Y : Z = 3 : 4 : 5$

So, average score of ($X$, $Y$, $Z$)

$= \dfrac{83 \times 3 + 76 \times 4 + 85 \times 5}{3 + 4 + 5}$

$= \dfrac{978}{12} = 81.5$

**10. (c) :** Machine I : Number of nuts produced in one minute = 100
To produce 1000 nuts time required = 10 min.
Cleaning time for nuts = 5 min.
Overall time to produce 1000 nuts = 15 min.
Overall time to produce 9000 nuts = 135 min – 5 min.
= 130 min ...(i)
Machine II: To produce 75 bolts time required = 1 min.
To produce 1500 bolts time required = 20 min.
Cleaning time for bolts = 10 min.
Overall time to produce 1500 bolts = 30 min.
Overall time to produce 9000 bolts = 30 × 6 – 10
= 170 min ...(ii)
From (i) and (ii),
Minimum time = 170 minutes

**11. (a) :** As per the problem: $A$ will cover a distance of $25 \times 2 = 50$ km in 2 hours time.

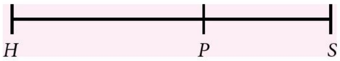
Distance separating the two people at 10 a.m. = 120 – 50 = 70 km

Now, both of them are moving towards each other so relative speed = 25 + 10 = 35 km/h

Time taken to cover a distance of 70 km $= \frac{70}{35} = 2$ hours after 10 a.m.

Required answer is 12 noon.

**12. (a) :**




Everyday Sunil reaches S at 5 p.m. On Friday, he saves 30 minutes. So, he must have met the children at point *P*, 15 minutes early i.e. at 4 : 45 p.m., to save 30 minutes (15 minutes of going from *P* to *S* and 15 minutes of returning from *S* to *P*).

Hence, the children were walking from 4 p.m. to 4 : 45 p.m. *i.e.* for 45 minutes.

Sunil has picked his children from the point *P* on that day.

**13. (c) :** Let the digit in the unit's place be '$x$' and the digit in the ten's place be '$y$'.

The original number = $10y + x$.

Number obtained by interchanging the digits = $10x + y$.

Now, $10y + x = 4(x + y)$

$3x - 6y = 0$ ...(i)

Also, $(10x + y) - (10y + x) = 27$.

$\therefore\ 10x + y - 10y - x = 27$

$\therefore\ 9x - 9y = 27$

$\therefore\ x - y = 3$ ...(ii)

Multiplying (ii) by 3,

$3x - 3y = 9$ ...(iii)

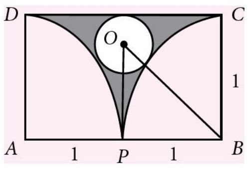
Subtracting (iii) from (i), $-3y = -9$

$\therefore\ y = 3$

Substituting the value of $y$ in equation (ii), $x - 3 = 3$

$\therefore\ x = 6$  $\therefore$ The number is $10 \times 3 + 6 = 36$.

**14. (b) :** Let the radius of the circle is $r$ units.




$OP = (1 - r)$, $OB = (1 + r)$

In $\Delta OPB : OP^2 + BP^2 = OB^2$

$(1 - r)^2 + 1^2 = (1 + r)^2$

So, $r = \frac{1}{4}$ units

Area of circle $= \pi r^2 = \pi \times \left(\frac{1}{4}\right)^2 = \frac{\pi}{16}$

Area of both quarter circle $= 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

$\therefore$ Shaded area $= 2 \times 1 - \frac{\pi}{16} - \frac{\pi}{2} = \frac{32 - \pi - 8\pi}{16}$

$= \frac{32 - 9\pi}{16} = \frac{13}{56}$ sq. unit

**15. (d) :** Let the work done be 960 units [LCM of (192, 160 and 240)].

$2m + 6w + 8b = 5$ units/hour

$4m + 16b = 6$ units/hour

$4m + 6w = 4$ units/hour

$3 + 6w = 5\ \therefore\ 6w = 2$ units/hour

$4m + 2 = 4 \Rightarrow 4m = 2 \Rightarrow 2m = 1 \Rightarrow 10m = 5$ units/hour

$24b = 6$ units/hour

$10m + 24b = 11$ units/hour

Time required $= \frac{960}{11} = 87\frac{3}{11}$ hours.

**16. (d) :** $\frac{\text{Sales of company } B \text{ in 2021}}{\text{Sales of company } E \text{ in 2021}} \times 100$

$= \frac{980}{460} \times 100 \approx 213\%$

**17. (b) :** Expenditure of company *D* for 2020 = ₹ 600 lakhs

Revenue of company *D* for 2020 = ₹ 800 lakhs

Profit percentage $= \left(\frac{\text{Revenue - Expenditure}}{\text{Expenditure}}\right) \times 100$

$= \left(\frac{800 - 600}{600}\right) \times 100 = 33.33\%$

**18. (c) :** Let $x$ be the total market share in that industry in 2021.

Then, $\frac{50}{100} \times x = 1000 + 1060 + 910 + 800 + 980$

$\Rightarrow\ \frac{x}{2} = 4750 \Rightarrow x = 9500$

So, Market share of *B* in 2021 $= \frac{1060}{9500} \times 100 = 11.15\%$

**19. (b) :** In 2021, the industry grows by 20%.

Hence, total market share in 2021

$= 9500 + \frac{20}{100} \times 9500 = 11400$

So, Market share of *B* in 2021 $= \frac{980}{11400} \times 100 \approx 8.6\%$

**20. (b) :** Sales of company *C* in 2022 = 1000

Simple annual growth rate for the period 2022 to 2024, $R = 15\%$

Hence, sales of company *C* in 2024

$= 1000 + \frac{1000 \times 15 \times 2}{100} =$ ₹ 1300 lakhs

# YOU ASK WE ANSWER

**Do you have a question that you just can't get answered?**
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

**1.** Let $I_n = \int_1^e x^{19}(\log|x|)^n\,dx$, where $n \in N$.
If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers $\alpha$ and $\beta$, then $\alpha - \beta$ is equal to ________. *(Sumit, Karnataka)*

**Ans.** We have, $I_n = \int_1^e x^{19}(\log|x|)^n\,dx$

$$= \left[(\log|x|)^n \cdot \frac{x^{20}}{20}\right]_1^e - \int_1^e \frac{n(\log|x|)^{n-1}}{x}\cdot\frac{x^{20}}{20}\,dx$$

$\Rightarrow \quad 20I_n = e^{20} - nI_{n-1} \Rightarrow 20I_{10} = e^{20} - 10I_9$ ...(i)
and $20I_9 = e^{20} - 9I_8$ ...(ii)
From (i) and (ii), we have
$20I_{10} = 10I_9 + 9I_8 \Rightarrow \alpha = 10, \beta = 9$
So, $\alpha - \beta = 1$

**2.** If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$

then find the value of $\int_{-\pi}^{\pi} f(x)\,dx$. *(Rajat, M.P.)*

**Ans.** $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$

Taking out $\cos x$ common from $C_2$, we get

$$f(x) = \cos x \begin{vmatrix} \sec x & 1 & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos x & \operatorname{cosec}^2 x \\ 1 & \cos x & \cos^2 x \end{vmatrix}$$

Applying $C_2 \to C_2 - \cos x\, C_1$, we get

$$f(x) = \cos x \begin{vmatrix} \sec x & 0 & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos x - \cos^3 x & \operatorname{cosec}^2 x \\ 1 & 0 & \cos^2 x \end{vmatrix}$$

$\Rightarrow \quad f(x) = \cos x\,(\cos x - \cos^3 x)\left(\cos x - \frac{1}{\cos^2 x} - \frac{\cos x}{\sin^2 x}\right)$
$\Rightarrow \quad f(x) = -\cos^5 x - \sin^2 x$

$\therefore \quad \int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{\pi} -\cos^5 x\,dx - \int_{-\pi}^{\pi} \sin^2 x\,dx$

$= -2\int_0^{\pi} \cos^5 x\,dx \; -2\int_0^{\pi} \sin^2 x\,dx$

$= 0 - 2\times 2\int_0^{\pi/2} \sin^2 x\,dx = -4\times\frac{1}{2}\times\frac{\pi}{2} = -\pi$

**3.** Let $z = \frac{1 - i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then find the value of

$$21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3.$$

*(Aryan, Haryana)*

**Ans.** Clearly,

$z = \frac{1 - i\sqrt{3}}{2} = \frac{-1 - i\sqrt{3}}{2} + 1 = 1 + \omega^2 = -\omega$

$[\because\ 1 + \omega + \omega^2 = 0]$

Now, $z + \frac{1}{z} = -\omega - \frac{1}{\omega} = 1$, $z^2 + \frac{1}{z^2} = \omega^2 + \frac{1}{\omega^2} = -1$,

$z^3 + \frac{1}{z^3} = -\omega^3 - \frac{1}{\omega^3} = -2$,

$z^4 + \frac{1}{z^4} = \omega^4 + \frac{1}{\omega^4} = \omega + \frac{1}{\omega} = -1$,

$z^5 + \frac{1}{z^5} = -\omega^5 - \frac{1}{\omega^5} = -\left(\omega^2 + \frac{1}{\omega^2}\right) = 1$,

$z^6 + \frac{1}{z^6} = \omega^6 + \frac{1}{\omega^6} = 2$

$z^7 + \frac{1}{z^7} = -\omega^7 - \frac{1}{\omega^7} = -\omega - \frac{1}{\omega} = 1$ and so on.

Now, $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots\ldots$
$\ldots\ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$

$= 21 + (1 - 1 - 8 - 1 + 1 + 8) \times 3 + 1 - 1 - 8$
$= 21 + [0 - 8] = 13$

# MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapter. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

## Series 1: Sets, Relations and Functions

Time Taken : 60 Min.

### Only One Option Correct Type

1. The value of $(A \cup B \cup C) \cap (A \cap B' \cap C') \cap C'$ is
   (a) $B \cap C'$ (b) $B' \cap C'$
   (c) $B \cap C$ (d) none of these

2. If $f(x) = \cos(\log x)$, then
   $$f(x^2)f(y^2) - \frac{1}{2}\left[f\left(\frac{x^2}{y^2}\right) + f(x^2y^2)\right] \text{ is equal to}$$
   (a) $-2$ (b) $-1$ (c) $0$ (d) $1/2$

3. The relation $R$ defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 16\}$ is given by
   (a) $\{(1, 1)\ (2, 1), (3, 1), (4, 1), (2, 3)\}$
   (b) $\{(2, 2), (3, 2), (4, 2), (2, 4)\}$
   (c) $\{(3, 3), (4, 3), (5, 4), (3, 4)\}$
   (d) None of these

4. Let $f(x) = \dfrac{\sin^{101} x}{\left[\dfrac{x}{\pi}\right] + \dfrac{1}{2}}$, where $[x]$ denotes the integral part of $x$, is
   (a) an odd function (b) an even function
   (c) neither odd nor even function
   (d) None of these

5. The domain of the function $\sqrt{1 - f(x)}$, where $f^3(x) = 1 - x^3 - 3xf(x)\ \forall\ x \in R$ is $(f(1) \neq -1)$
   (a) $R$ (b) $R - \{0\}$
   (c) $R^+ \cup \{0\}$ (d) $R^-$

6. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is
   (a) $(-\infty, 0)$ (b) $(-\infty, \infty) - \{0\}$
   (c) $(-\infty, \infty)$ (d) $(0, \infty)$

### One or More Than One Option(s) Correct Type

7. In a survey of population of 450 people, it is found that 205 can speak English ($E$), 210 can speak Hindi ($H$) and 120 people can speak Tamil (T). If 100 people can speak both $E$ and $H$, 80 can speak both $E$ and $T$, 35 can speak both $H$ and $T$, and 20 can speak all the three languages. Then
   (a) $n(E' \cap H' \cap T') = 110$
   (b) $n(E \cap H' \cap T') = 45$
   (c) $n(E' \cap H' \cap T') = 340$
   (d) None of these

8. If $U = \{x : x \in N \text{ and } 2 \leq x \leq 12\}$, $A = \{x : x \text{ is an even prime}\}$ and $B = \{x : x \text{ is a factor of } 12\}$, then which of the following is true?
   (a) $A - B$ is an empty set (b) $A - B = B \cap A'$
   (c) $A' - B' = B - A$ (d) $(A \cap B)' = A' \cup B'$

9. Let $f : R \to R$ be a function defined by $f(x+1) = \dfrac{f(x) - 5}{f(x) - 3}\ \forall\ x \in R$. Then which of the following statement(s) is/are true?
   (a) $f(2008) = f(2004)$ (b) $f(2006) = f(2010)$
   (c) $f(2006) = f(2002)$ (d) $f(2006) = f(2018)$

10. The domain of the function
    $$f(x) = \log_e\left\{\log_{|\sin x|}(x^2 - 8x + 23) - \frac{3}{\log_2|\sin x|}\right.$$
    contains which of the following interval(s)?
    (a) $(3, \pi)$ (b) $\left(\pi, \frac{3}{2}\pi\right)$
    (c) $\left(\frac{3\pi}{2}, 5\right)$ (d) None of these

**11.** If $f(x)=\sqrt{1-\sin^2 x}+\sqrt{1+\tan^2 x}$, then
(a) fundamental period of $f(x)$ is $\pi$
(b) range of $f(x)$ is $[2, \infty)$
(c) domain of $f(x)$ is $R$
(d) $f(x) = 2$ has 3 solutions in $[0, 2\pi]$

**12.** Let a function $f(x)$, $x \neq 0$ be such that
$$f(x)+f\left(\frac{1}{x}\right)=f(x)\cdot f\left(\frac{1}{x}\right),$$ then $f(x)$ can be
(a) $1 - x^{2013}$
(b) $\sqrt{|x|}+1$
(c) $\dfrac{\pi}{2\tan^{-1}|x|}$
(d) $\dfrac{2}{1+k\log_e|x|}$

**13.** If a function satisfies $(x-y)f(x+y)-(x+y)f(x-y) = 2(x^2y-y^3)\ \forall\, x, y \in R$ and $f(1) = 2$, then
(a) $f(x)$ must be polynomial function
(b) $f(3) = 12$
(c) $f(0) = 0$
(d) $f(2) = 4$

## Comprehension Type

### Paragraph for Q. No. 14 and 15

Let $f : R \to [-1, 1]$, $g : R \to [-1, 1]$ are defined by $f(x) = 1 - 2\sin^2 x$, $g(x) = \cos 2x$ and
$$F(x)=f(x)+g(x),\ G(x)=\frac{f(x)}{g(x)}$$

**14.** Domain and range of $G(x)$ are respectively
(a) $R$ and $\{1\}$
(b) $R$ and $\{0, 1\}$
(c) $R-\left\{(2n+1)\dfrac{\pi}{2}\right\}$ and $\{0,1\}, n\in I$
(d) $R-\left\{(2n+1)\dfrac{\pi}{4}\right\}$ and $\{1\}, n\in I$

**15.** Which of the following statements is correct ?
(a) periods of $f(x)$, $g(x)$ and $F(x)$ are in A.P with common difference $\pi/3$
(b) periods of $f(x)$, $g(x)$ and $F(x)$ are same and is equal to $2\pi$
(c) sum of the periods of $f(x)$, $g(x)$ and $F(x)$ is $3\pi$
(d) sum of the periods of $f(x)$, $g(x)$ and $F(x)$ is $6\pi$

## Matrix Match Type

**16.** Match Column-I with Column-II and select the correct answer using options given below.

| | Column-I | | Column-II |
|---|---|---|---|
| (P) | If $f(x)=\log_{10}\sin(x-3)+\sqrt{16-x^2}$, then | (1) | $x\in[-5,-1]\cup[1,2)\cup(2,3)$ |
| (Q) | If $f(x)=\sqrt{\dfrac{4-\lvert x\rvert}{7-\lvert x\rvert}}$, then | (2) | $x\in(-\infty,-7]\cup[7,\infty)$ |
| (R) | If $f(x)=\cos^{-1}\left(\dfrac{\lvert x\rvert-3}{2}\right)+[\log_{10}(3-x)]^{-1}$, then | (3) | $x\in(-2\pi+3,-\pi+3)\cup(3,4]$ |
| (S) | If $f(x)=\dfrac{1}{\sqrt{\lvert[\lvert x\rvert-1]\rvert-5}}$, then | (4) | $x\in(-\infty,-7)\cup[-4,4]\cup(7,\infty)$ |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 3 | 4 | 2 | 1 |
| (b) | 3 | 4 | 1 | 2 |
| (c) | 4 | 3 | 2 | 1 |
| (d) | 1 | 2 | 2 | 3 |

## Numerical Answer Type

**17.** If $A = \{x : x = n^2, n = 7, 8, 9\}$, then number of proper subsets is ______.

**18.** Let $f(x+y+1)=\left(\sqrt{f(x)}+\sqrt{f(y)}\right)^2$ for all $x, y \in R$ and $f(0) = 1$. Then $f(x) = (x+1)^m$. Then the value of $m$ is ______ .

**19.** If $f(x)=[x]+\sum_{k=1}^{2008}\dfrac{x+k-[x+k]}{2008}$, then the value of $f(3)$ is ________ .

**20.** Let $f(x)$ be a polynomial of degree 3 such that $f(k)=-\dfrac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10f(10)$ is equal to _______ .

*Keys are published in this issue. Search now! ☺*

**SELF CHECK**

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time. |
| < 60% | NOT SATISFACTORY! | Revise thoroughly and strengthen your concepts. |

# CBSE warm-up!

CLASS-XI

Chapterwise practice questions for CBSE Exams as per the latest pattern and rationalised syllabus by CBSE for the academic session 2023-24.

Series-3

## Trigonometric Functions and Complex Numbers

Time Allowed : 3 hours
Maximum Marks : 80

**General Instructions**

*(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
*(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
*(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
*(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
*(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
*(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

### SECTION - A (MULTIPLE CHOICE QUESTIONS)

**Each question carries 1 mark.**

**1.** The angles of a triangle are in A.P. and the number of degrees in the least to the number of radians in the greatest is $60 : \pi$, then the angles in degrees are.
(a) $30°, 60°, 90°$ (b) $40°, 60°, 90°$
(c) $30°, 30°, 120°$ (d) $20°, 130°, 30°$

**2.** The square root of $i$ is
(a) $\frac{1}{\sqrt{2}}(1-i)$ (b) $\frac{1}{\sqrt{2}}(1+i)$
(c) $1 - i$ (d) $1 + i$

**3.** The value of 6 radians into degree measure is
(a) $343°38'11''$ (b) $348°33'11''$
(c) $433°38'11''$ (d) $343°37'12''$

**4.** If $\tan A + \cot A = 4$, then $\tan^4 A + \cot^4 A$ is equal to
(a) 110 (b) 191 (c) 80 (d) 194

**5.** If $\cos x = \frac{4}{5}$, where $x \in [0, \pi/2]$, then the value of $\cos\left(\frac{x}{2}\right)$ is equal to
(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{3}{\sqrt{10}}$ (d) $-\frac{2}{5}$

**6.** The value of $3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10°$ equals
(a) 0 (b) $-1$
(c) 1 (d) None of these

**7.** If $1 + \cos x = k$, where $x$ is acute, then $\sin(x/2)$ is
(a) $\sqrt{\frac{1-k}{2}}$ (b) $\sqrt{2-k}$
(c) $\sqrt{\frac{2+k}{2}}$ (d) $\sqrt{\frac{2-k}{2}}$

**8.** If $\sin\theta = \sin 45° + \sin 15°$, where $0° < \theta < 180°$, then $\theta$ is equal to
(a) $45°$ (b) $150°$ (c) $60°$ (d) $75°$

**9.** The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is
(a) $\frac{1}{2}$ (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{1}{8}$

10. The real part of $\frac{(1+i)^2}{(3-i)}$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{5}$

11. If $x = 2 + 5i$, then value of the expression $x^3 - 5x^2 + 33x - 49$ equals

(a) $-20$ (b) $10$ (c) $20$ (d) $-29$

12. The value of $(1 + i)^6 + (1 - i)^3$ is

(a) $-2 - 10i$ (b) $2 - 10i$
(c) $-2 + 10i$ (d) $2 + 10i$

13. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is

(a) $\frac{8}{25}-\frac{31}{25}i$ (b) $-\frac{8}{25}-\frac{31}{25}i$
(c) $-\frac{8}{25}+\frac{31}{25}i$ (d) None of these

14. The modulus of $\frac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ is

(a) 2 (b) 4 (c) $3\sqrt{2}$ (d) $2\sqrt{2}$

15. The conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ is

(a) $\frac{63}{25}-\frac{16}{25}i$ (b) $\frac{63}{25}+\frac{16}{25}i$
(c) $-\frac{63}{25}+\frac{16}{25}i$ (d) None of these

16. If $a + ib = c + id$, then

(a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$
(c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

17. The value of $i^{n+100} + i^{n+50} + i^{n+48} + i^{n+46}$ is

(a) 0 (b) 1 (c) 2 (d) 3

18. If $\frac{5z_2}{11z_1}$ is purely imaginary, then the value of $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$ is

(a) 1 (b) 2 (c) 0 (d) $-1$

## ASSERTION-REASON BASED QUESTIONS

**In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.**

**(a) Both (A) and (R) are true and (R) is the correct explanation of (A).**
**(b) Both (A) and (R) are true but (R) is not the correct explanation of A.**
**(c) (A) is true but (R) is false.**
**(d) (A) is false but (R) is true.**

19. **Assertion (A) :** The value of $\sin(-690°)\cos(-300°) + \cos(-750°)\sin(-240°) = 1$

**Reason (R) :** The values of sin and cos is negative in third and fourth quadrant respectively.

20. **Assertion (A) :** $\frac{z_2}{z_1}$ is purely imaginary, then $\left|\frac{6z_1-8z_2}{4z_2+3z_1}\right| = 2$.

**Reason (R) :** If $z$ is purely imaginary, then $z+\overline{z}=0$.

## SECTION - B

**This section comprises of very short answer type questions (VSA) of 2 marks each.**

21. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then prove that $\cos2\alpha + \cos2\beta = -2\cos(\alpha + \beta)$.

**OR**

Find the value of $\sin20° (4 + \sec20°)$.

22. Find the distance from the eye at which a coin of diameter 2 cm should be held so as just to conceal the full moon, whose angular diameter is $31'$.

23. If $a+ib=\frac{(x+i)^2}{2x^2+1}$, then prove that

$$a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}.$$

**OR**

If $z_1 = 2 - i$, $z_2 = -2 + i$, then find $\text{Re}\left(\frac{z_1z_2}{\overline{z_1}}\right)$.

24. What are the real numbers '$x$' and '$y$', if $(x - iy)(3 + 5i)$ is the conjugate of $(-1 - 3i)$?

25. Find the value of $\tan\left(\frac{13\pi}{12}\right)$.

## SECTION - C

**This section comprises of short answer type questions (SA) of 3 marks each.**

26. If $\frac{\sin^4\theta}{a}+\frac{\cos^4\theta}{b}=\frac{1}{a+b}$, then find the value of $\frac{\sin^{12}\theta}{a^5}+\frac{\cos^{12}\theta}{b^5}$.

27. Prove that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$.

**OR**

Show that $4\sin\alpha \sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha+\frac{2\pi}{3}\right)=\sin 3\alpha$.

28. If $\alpha$ and $\beta$ lie between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$, given that $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$.

29. Find the least $x$ (> 0) for which $\tan(x^\circ + 100^\circ) = \tan(x^\circ + 50^\circ)\tan x^\circ \tan(x^\circ - 50^\circ)$.

30. If $(x+iy)^3 = u+iv$, then prove that $\frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$.

**OR**

If $|z+1| = z + 2(1+i)$, then find $z$.

31. Find the value of $m \sin x + n \cos x$, if $\tan\frac{x}{2} = \frac{m}{n}$.

## SECTION - D

**This section comprises of long answer type questions (LA) of 5 marks each.**

32. Find all non-zero complex numbers $z$ satisfying $\bar{z} = iz^2$.

33. Find the magnitude and conjugate of the number $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$.

**OR**

If for the complex numbers $z_1$ and $z_2$, $|1-\bar{z}_1 z_2|^2 - |z_1 - z_2|^2 = k(1-|z_1|^2)(1-|z_2|^2)$, then find the value of $k$.

34. Evaluate : $\sqrt{4+3\sqrt{-20}}+\sqrt{4-3\sqrt{-20}}$

35. If $A + B + C = 180^\circ$, then show that

$$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2} = 1-2\sin\frac{A}{2}\cdot\sin\frac{B}{2}\sin\frac{C}{2}.$$

**OR**

If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$, then prove that $\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) = 1 - 2a^2 - 2b^2$.

## SECTION - E

**This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. The first two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.**

36. Sudhir who is a student of class XI got a Maths assignment from his class teacher. He did all the questions except a few. If the value of $\sin x = \frac{3}{5}$ and $\cos y = -\frac{12}{13}$, where $x$ and $y$ both lie in third quadrant.

(i) What will be the value of $\cos x$ ?

(ii) What will be the value of $\sin y$ ?

(iii) Find the value of $\sin(x+y)$.

**OR**

Find the value of $\sin 75^\circ$.

37. A math teacher explained the students about the topic "Argand plane". He told that the plane having a complex number assigned to each of its points is called complex plane or argand plane. The $x$-axis and $y$-axis in the argand plane are called, respectively, the real axis and the imaginary axis. The representation of a complex number $z = x+iy$ and its conjugate $\bar{z} = x - iy$ in the argand plane are $(x, y)$ and $(x, -y)$ and modulus of the complex number $x+iy = \sqrt{x^2+y^2}$ is the distance between the point $(x, y)$ and the origin $O(0, 0)$.

Based on the above information, answer the following questions.

(i) Find the modulus of $z = \frac{1}{3+4i}$.

(ii) Find the conjugate of $z = \frac{(-2+3i)}{(1-i)}$.

(iii) Find the real part of $\left(\frac{3+i}{2-i}+\frac{3-i}{2+i}\right)$.

**OR**

If $z_1 = \sqrt{3}+i\sqrt{3}$ and $z_2 = \sqrt{3}+i$, then find the quadrant in which $\left(\frac{z_1}{z_2}\right)$ lies.

38. A teacher mentioned in his class that there is some $\theta \in R$ such that $\tan\theta = \frac{-8}{15}$, where $\theta$ lies in second quadrant. Then, he asks his students about some other values.

(i) Find the value of $\sin 2\theta$.

(ii) What is the value of $\tan 2\theta$?

## SOLUTIONS

**1. (a) :** Let the angles of the triangle be $(a-d)^\circ, a^\circ, (a+d)^\circ$, where $d > 0$ ...(i)

then $(a-d) + a + (a+d) = 180 \Rightarrow a = 60$

$\therefore$ From (i), the angles are $(60-d)^\circ, 60^\circ, (60+d)^\circ$

Now, the least angle $= (60-d)^\circ$

and the greatest angle $= (60+d)^\circ$

$= (60+d) \times \frac{\pi}{180}$ radian $\quad (\because 180^\circ = \pi \text{ radian})$

By the given condition, we have

$$\frac{60-d}{\frac{\pi}{180}(60+d)} = \frac{60}{\pi} \Rightarrow \frac{180(60-d)}{(60+d)} = 60$$

$\Rightarrow 180 - 3d = 60 + d \Rightarrow 4d = 120 \Rightarrow d = 30$

$\therefore$ From (i), the angles are $(60-30)^\circ, 60^\circ, (60+30)^\circ$ *i.e.*, $30^\circ, 60^\circ, 90^\circ$.

**2. (b) :** Let $z = i = \frac{1}{2} + \frac{1}{2}i^2 + 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}i$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}i\right)^2 + 2\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}i = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^2$$

$\therefore \quad \sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$

**3. (a) :** We know that $\pi$ radian $= 180^\circ$.

Hence, 6 radians $= \frac{180}{\pi} \times 6$ degree $= \frac{1080 \times 7}{22}$ degree

$= 343\frac{7}{11}$ degree $= 343^\circ + \frac{7 \times 60}{11}$ minute $\quad [\text{as } 1^\circ = 60']$

$= 343^\circ + 38' + \frac{2}{11} \times 60$ seconds $\quad [\text{as } 1' = 60'']$

$= 343^\circ + 38' + 10.9'' = 343^\circ 38' 11''$ approximately.

Hence, 6 radians $= 343^\circ 38' 11''$ approximately.

**4. (d) :** $\tan A + \cot A = 4$ ...(i)

Squaring (i) both sides, we get

$\tan^2 A + \cot^2 A = 14$ ...(ii)

Squaring (ii) both sides, we get

$\tan^4 A + \cot^4 A = 194$.

**5. (c) :** We have, $\cos x = \frac{4}{5}, x \in \left[0, \frac{\pi}{2}\right]$

$\cos x = 2\cos^2\frac{x}{2} - 1 \Rightarrow 2\cos^2\frac{x}{2} = \frac{9}{5}$

$\Rightarrow \cos^2\frac{x}{2} = \frac{9}{10} \Rightarrow \cos\frac{x}{2} = \pm\frac{3}{\sqrt{10}}$

$\cos\frac{x}{2}$ is positive in I quadrant $\left(0 < \frac{x}{2} < \frac{\pi}{4}\right)$

$\therefore \quad \cos\frac{x}{2} = \frac{3}{\sqrt{10}}$

**6. (c) :** $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$ ...(i)

Putting $A = 10^\circ$ in (i), we get

$\therefore \quad \frac{1}{\sqrt{3}}(1 - 3\tan^2 10^\circ) = \tan 10^\circ(3 - \tan^2 10^\circ)$ ...(ii)

Squaring (ii) both sides, we get

$(1 - 3\tan^2 10^\circ)^2 = 3\tan^2 10^\circ (3 - \tan^2 10^\circ)^2$

$\Rightarrow 3\tan^2 10^\circ (9 + \tan^4 10^\circ - 6\tan^2 10^\circ)$

$= 1 + 9\tan^4 10^\circ - 6\tan^2 10^\circ$

$\Rightarrow 3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ = 1$

**7. (d) :** $1 + \cos x = k \Rightarrow 2\cos^2\frac{x}{2} = k$

$\Rightarrow 1 - \sin^2\frac{x}{2} = \frac{k}{2} \Rightarrow \sin\frac{x}{2} = \sqrt{\frac{2-k}{2}}$

**8. (d) :** We have, $\sin\theta = \sin 45^\circ + \sin 15^\circ$

$\Rightarrow \sin\theta = 2\sin\left(\frac{45^\circ + 15^\circ}{2}\right)\cos\left(\frac{45^\circ - 15^\circ}{2}\right)$

$= 2\sin 30^\circ \cos 15^\circ = 2 \times \frac{1}{2}\sin(90^\circ - 15^\circ)$

$\Rightarrow \sin\theta = \sin 75^\circ \Rightarrow \theta = 75^\circ$.

**9. (c) :** $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$

$= \cos 156^\circ + \cos 84^\circ + \cos 132^\circ + \cos 12^\circ$

$$= 2\cos\left(\frac{156^\circ + 84^\circ}{2}\right)\cos\left(\frac{156^\circ - 84^\circ}{2}\right) + 2\cos\left(\frac{132^\circ + 12^\circ}{2}\right)\cos\left(\frac{132^\circ - 12^\circ}{2}\right)$$

$= 2\cos 120^\circ \cos 36^\circ + 2\cos 72^\circ \cos 60^\circ$

$= 2\left(-\frac{1}{2}\right)\cos 36^\circ + 2\cos 72^\circ \times \frac{1}{2} = -\cos 36^\circ + \cos 72^\circ$

$= -\frac{\sqrt{5}+1}{4} + \frac{\sqrt{5}-1}{4} = -\frac{1}{2}$

**10. (d) :** $(1+i)^2 = 1 + i^2 + 2i = 2i$

$\therefore \quad \frac{(1+i)^2}{3-i} = \frac{2i(3+i)}{3^2 - i^2} = \frac{6i-2}{10} = \frac{-1+3i}{5}$

$\therefore$ Real part $= \frac{-1}{5}$

**11. (a) :** Given $x = 2 + 5i \Rightarrow x - 2 = 5i$

$\Rightarrow x^2 - 4x + 29 = 0$

Now, $x^3 - 5x^2 + 33x - 49$

$= x(x^2 - 4x + 29) - 1(x^2 - 4x + 29) - 20 = -20$

**12. (a) :** $(1+i)^6 = \{(1+i)^2\}^3 = (2i)^3 = 8i^3 = -8i$

and $(1-i)^3 = 1 - i^3 - 3i + 3i^2 = 1 + i - 3i - 3 = -2 - 2i$

$\therefore \quad (1+i)^6 + (1-i)^3 = -8i - 2 - 2i = -2 - 10i$

**13. (b) :** Let $z = \frac{3+4i}{4-5i} \times \frac{4+5i}{4+5i} = -\frac{8}{41} + \frac{31}{41}i$

Then $\bar{z} = -\frac{8}{41} - \frac{31}{41}i$

and $|z| = \sqrt{\left(-\frac{8}{41}\right)^2 + \left(\frac{31}{41}\right)^2} = \frac{5}{\sqrt{41}}$

$\therefore$ Multiplicative inverse of $z$

$$= \frac{\bar{z}}{|z|^2} = \frac{-\frac{8}{41} - \frac{31}{41}i}{\frac{25}{41}} = -\frac{8}{25} - \frac{31}{25}i$$

**14. (d) :** $\left|\frac{(1+i\sqrt{3})(2+2i)}{\sqrt{3}-i}\right| = \frac{|1+i\sqrt{3}||2+2i|}{|\sqrt{3}-i|}$

$$= \frac{2 \times 2\sqrt{2}}{2} = 2\sqrt{2}$$

**15. (b) :** We have, $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

$$= \frac{6+9i-4i+6}{2-i+4i+2} = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{48-36i+20i+15}{16+9} = \frac{63-16i}{25} = \frac{63}{25} - \frac{16}{25}i$$

Therefore, conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ is $\frac{63}{25} + \frac{16}{25}i$.

**16. (d) :** We have, $a + ib = c + id$

$\Rightarrow \quad |a + ib| = |c + id|$

$\Rightarrow \quad \sqrt{a^2 + b^2} = \sqrt{c^2 + d^2} \Rightarrow a^2 + b^2 = c^2 + d^2$

**17. (a) :** Given expression is $i^{n+100} + i^{n+50} + i^{n+48} + i^{n+46}$

$$= i^n(i^{100} + i^{50} + i^{48} + i^{46})$$

$$= i^n[(i^2)^{50} + (i^2)^{25} + (i^2)^{24} + (i^2)^{23}]$$

$$= i^n[(-1)^{50} + (-1)^{25} + (-1)^{24} + (-1)^{23}]$$

$$= i^n[1-1+1-1] = i^n \cdot 0 = 0$$

**18. (a)**

**19. (c) :** $\sin(-690°) = -\sin 690° = -\sin(2 \times 360° - 30°)$

$$= -(-\sin 30°) = \frac{1}{2}$$

$\cos(-300°) = \cos 300° = \cos(360° - 60°) = \cos 60° = \frac{1}{2}$

$\cos(-750°) = \cos 750° = \cos(2 \times 360° + 30°)$

$= \cos 30° = \sqrt{3}/2$

$\sin(-240°) = -\sin 240° = -\sin(180° + 60°)$

$= -(-\sin 60°) = \sin 60° = \sqrt{3}/2$

$\therefore \quad \sin(-690°)\cos(-300°) + \cos(-750°)\sin(-240°)$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = 1.$$

**20. (c) :** $\frac{z_2}{z_1} = ik$, $k$ is real, $z_2 = ikz_1$

$\Rightarrow \quad \frac{6z_1 - 8z_2}{4z_2 + 3z_1} = \frac{6z_1 - 8ikz_1}{4ikz_1 + 3z_1} = \frac{6-8ki}{4ik+3}$

$\therefore \quad \left|\frac{6z_1 - 8z_2}{4z_2 + 3z_1}\right| = \frac{|6-8ik|}{|4ik+3|} = \frac{\sqrt{36+64k^2}}{\sqrt{16k^2+9}} = \sqrt{4} = 2$

**21.** We have, $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$

$\Rightarrow \quad (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$

$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta - \sin^2\alpha - \sin^2\beta - 2\sin\alpha\sin\beta = 0$

$\Rightarrow \quad \cos^2\alpha - \sin^2\alpha + \cos^2\beta - \sin^2\beta = 2(\sin\alpha\sin\beta - \cos\alpha\cos\beta)$

$\Rightarrow \quad \cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$

$[\because \quad \cos 2A = \cos^2 A - \sin^2 A]$

Hence proved.

**OR**

$$\sin 20°\left(4 + \frac{1}{\cos 20°}\right) = \frac{\sin 20°}{\cos 20°}(4\cos 20° + 1)$$

$$= \frac{1}{\cos 20°}(4\sin 20°\cos 20° + \sin 20°)$$

$$= \frac{1}{\cos 20°}(2\sin 40° + \sin 20°)$$

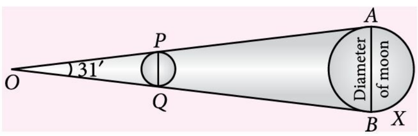
$$= \frac{1}{\cos 20°}(\sin 40° + (\sin 40° + \sin 20°))$$

$$= \frac{1}{\cos 20°}(\sin 40° + 2\sin 30°\cos 10°$$

$$= \frac{1}{\cos 20°}(\sin 40° + \cos 10°) = \frac{1}{\cos 20°}(\sin 40° + \sin 80°)$$

$$= \frac{2\sin 60° \cdot \cos 20°}{\cos 20°} = \sqrt{3}.$$

**22. (c) :** The coin will just hide the full moon if the lines joining the observer's eye $O$ to the ends $A$ and $B$ of moon's diameter touch the coin at the ends $P$ and $Q$ of its diameter.



Here $\angle POQ = \angle AOB = 31'$

$$= \left(\frac{31}{60}\right)^\circ = \frac{31}{60} \times \frac{\pi}{180} \text{ radian.}$$

Since, this angle is very small, the diameter $PQ$ of the coin can be regarded as an arc of a circle whose centre is $O$ and radius equal to the distance of the coin from $O$.

$$\therefore \quad \frac{31\pi}{60 \times 180} = \frac{l}{r} = \frac{2}{r} \qquad \left(\because \ \theta = \frac{l}{r}\right)$$

$$\Rightarrow \quad r = \frac{60 \times 180 \times 2}{31\pi} \Rightarrow r = \frac{60 \times 180 \times 7 \times 2}{31 \times 22} \approx 221.70 \text{ cm}$$

**23.** Consider, $a + ib = \dfrac{(x+i)^2}{2x^2+1} \Rightarrow |a+ib| = \left|\dfrac{(x+i)^2}{2x^2+1}\right|$

$$\Rightarrow |a+ib| = \frac{|x+i|^2}{|2x^2+1|} \qquad \left[\because |z^2| = |z|^2\right]$$

$$\Rightarrow \sqrt{a^2+b^2} = \frac{\left(\sqrt{x^2+1^2}\right)^2}{2x^2+1}$$

Squaring on both sides, we get $a^2 + b^2 = \dfrac{(x^2+1)^2}{(2x^2+1)^2}$

**OR**

We have, $z_1 = 2 - i$, $z_2 = -2 + i$

$z_1 z_2 = (2 - i)(-2 + i) = (-4 + 2i + 2i - i^2) = (-3 + 4i)$

$\bar{z}_1 = 2 + i\,;\ \dfrac{1}{\bar{z}_1} = \dfrac{1}{2+i} = \dfrac{2-i}{4-i^2} = \dfrac{2-i}{5}$

Now, $\dfrac{z_1 z_2}{\bar{z}_1} = (-3+4i)\left(\dfrac{2-i}{5}\right) = \dfrac{1}{5}(-6+3i+8i-4i^2)$

$$= \left(\frac{-2+11i}{5}\right)$$

$$\therefore \ \text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = -\frac{2}{5}$$

**24.** Given, $(x - iy)(3 + 5i) = \overline{-1-3i}$

$$\Rightarrow x - iy = \frac{-1+3i}{3+5i} = \frac{(-1+3i)(3-5i)}{9-25i^2}$$

$$\Rightarrow x - iy = \frac{-3+5i+9i-15i^2}{9+25} = \frac{12+14i}{34} = \frac{6}{17} + \frac{7}{17}i$$

$$\Rightarrow x = \frac{6}{17}, y = -\frac{7}{17}$$

**25.** $\tan\left(\dfrac{13\pi}{12}\right) = \tan\left(\pi + \dfrac{\pi}{12}\right) = \tan\left(\dfrac{\pi}{12}\right)$

$$= \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = (2 - \sqrt{3})$$

**26.** Given, $\dfrac{\sin^4\theta}{a} + \dfrac{\cos^4\theta}{b} = \dfrac{1}{a+b}$

$$\Rightarrow \frac{(\sin^2\theta)^2}{a} + \frac{(1-\sin^2\theta)^2}{b} = \frac{1}{a+b}$$

$\Rightarrow \lambda^2 b(a + b) + (a + b)\, a\, (1 - \lambda)^2 = ab$, where $\lambda = \sin^2\theta$

$\Rightarrow \lambda^2 ab + \lambda^2 b^2 + a^2 + a^2\lambda^2 - 2a^2\lambda + ab + ab\lambda^2 - 2ab\lambda = ab$

$\Rightarrow \lambda^2\{a^2 + b^2 + 2ab\} + a^2 - 2\lambda a\,(a + b) = 0$

$\Rightarrow \{\lambda(a + b)\}^2 + a^2 - 2\lambda a\,(a + b) = 0$

$\Rightarrow \{\lambda(a + b) - a\}^2 = 0$

$\Rightarrow \lambda = \dfrac{a}{a+b} \Rightarrow \sin^2\theta = \dfrac{a}{a+b}$ ...(i)

From (i), $\cos^2\theta = 1 - \sin^2\theta = 1 - \dfrac{a}{a+b} = \dfrac{b}{a+b}$ ...(ii)

Using (i) and (ii), we get

$$\frac{\sin^{12}\theta}{a^5} + \frac{\cos^{12}\theta}{b^5} = \frac{(\sin^2\theta)^6}{a^5} + \frac{(\cos^2\theta)^6}{b^5}$$

$$= \frac{\left(\frac{a}{a+b}\right)^6}{a^5} + \frac{\left(\frac{b}{a+b}\right)^6}{b^5} = \frac{a+b}{(a+b)^6} = \frac{1}{(a+b)^5}$$

**27.** L.H.S. $= \cos 20° \cos 40° \cos 80°$

$= \cos 20° \cos(60° - 20°) \cos(60° + 20°)$

$= \cos 20° (\cos 60° \cos 20° + \sin 60° \sin 20°)$
$(\cos 60° \cos 20° - \sin 60° \sin 20°)$

$$= \cos 20°\left(\frac{\cos 20°}{2} + \frac{\sqrt{3}\sin 20°}{2}\right)\left(\frac{\cos 20°}{2} - \frac{\sqrt{3}}{2}\sin 20°\right)$$

$$= \cos 20°\left(\frac{\cos^2 20°}{4} - \frac{3}{4}\sin^2 20°\right)$$

$$= \cos 20°\left(\frac{\cos^2 20°}{4} - \frac{3}{4}(1 - \cos^2 20°)\right)$$

$$= \cos 20°\left(\frac{\cos^2 20°}{4} + \frac{3\cos^2 20°}{4} - \frac{3}{4}\right)$$

$$= \cos 20°\left(\cos^2 20° - \frac{3}{4}\right)$$

$$= \frac{4\cos^3 20° - 3\cos 20°}{4} = \frac{\cos 60°}{4} = \frac{1}{2 \cdot 4} = \frac{1}{8} = \text{R.H.S.}$$

**OR**

We have, $4\sin\alpha \sin\left(\alpha + \dfrac{\pi}{3}\right)\sin\left(\alpha + \dfrac{2\pi}{3}\right)$

$$= 2\sin\alpha\left\{2\sin\left(\alpha + \frac{2\pi}{3}\right)\sin\left(\alpha + \frac{\pi}{3}\right)\right\}$$

$= 2\sin\alpha[2\sin(\alpha + 120°)\sin(\alpha + 60°)]$

$= 2\sin\alpha[\cos(\alpha + 120° - \alpha - 60°) -$
$\cos(\alpha + 120° + \alpha + 60°)]$

$= 2\sin\alpha[\cos 60° - \cos(180° + 2\alpha)]$

$= 2\sin\alpha \cdot \frac{1}{2} - 2\sin\alpha(-\cos 2\alpha)$

$= \sin\alpha + 2\cos 2\alpha \sin\alpha = \sin\alpha + \sin(2\alpha + \alpha) - \sin(2\alpha - \alpha)$

$= \sin\alpha + \sin 3\alpha - \sin\alpha = \sin 3\alpha$

**28.** Since $\alpha$ and $\beta$ lies between 0 and $\pi/4$ and $\sin(\alpha - \beta) > 0$, therefore, both $\alpha + \beta$ and $\alpha - \beta$ are positive acute angles.

Now, $\sin(\alpha - \beta) = 5/13$

$\Rightarrow \cos(\alpha - \beta) = \sqrt{1-\sin^2(\alpha-\beta)} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$

and $\cos(\alpha + \beta) = 4/5$

$\Rightarrow \sin(\alpha + \beta) = \sqrt{1-\cos^2(\alpha+\beta)} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{5}$.

Hence, $\tan(\alpha + \beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{3}{4}$

and $\tan(\alpha - \beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{5}{12}$.

Now, $\tan(2\alpha) = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{56}{33}.$$

**29.** Given,

$\tan(x° + 100°) = \tan(x° + 50°) \tan x° \tan(x° - 50°)$

$\Rightarrow \frac{\tan(x°+100°)}{\tan x°} = \frac{\tan(x°+50°)}{\cot(x°-50°)}$

Applying componendo and dividendo rule, we get

$$\frac{\tan(x°+100°)+\tan x°}{\tan(x°+100°)-\tan x°} = \frac{\tan(x°+50°)+\cot(x°-50°)}{\tan(x°+50°)-\cot(x°-50°)}$$

$\Rightarrow \frac{\sin(x°+100°+x°)}{\sin(x°+100°-x°)} = \frac{\cos(x°+50°-x°+50°)}{-\cos(x°+50°+x°-50°)}$

$\Rightarrow \sin(2x° + 100°) \cos 2x° = -\sin 100° \cos 100°$

$\Rightarrow 2\sin(2x° + 100°) \cos 2x° = -2\sin 100° \cos 100°$

$\Rightarrow \sin(2x° + 100° + 2x°) + \sin(2x° + 100° - 2x°) = -\sin 200°$

$\Rightarrow \sin(100° + 4x°) = -(\sin 200° + \sin 100°)$

$\Rightarrow \sin(100° + 4x°) = -2\sin 150° \cos 50°$

$\Rightarrow \sin(100° + 4x°) = -\cos 50° = \sin(270° - 50°)$

(note that $100° + 4x° > 100°$)

$\Rightarrow 100° + 4x° = 220° \Rightarrow x° = 30° \Rightarrow x = 30$.

**30.** We have, $(x + iy)^3 = u + iv$

$(x + iy)^3 = x^3 + (iy)^3 + 3x^2(iy) + 3x(iy)^2$

$= x^3 - y^3 i + 3x^2 yi - 3xy^2$

$= (x^3 - 3xy^2) + i(3x^2 y - y^3)$

$\Rightarrow (x^3 - 3xy^2) + i(3x^2 y - y^3) = u + iv$

Comparing the real and imaginary parts, we get

$u = x^3 - 3xy^2, v = 3x^2 y - y^3$

$u = x(x^2 - 3y^2), v = y(3x^2 - y^2)$

$\Rightarrow \frac{u}{x} = (x^2 - 3y^2), \frac{v}{y} = 3x^2 - y^2$

$\therefore \frac{u}{x} + \frac{v}{y} = 4x^2 - 4y^2 = 4(x^2 - y^2)$.

**OR**

We have, $|z + 1| = z + 2(1 + i)$

Putting $z = x + iy$, we get

$\Rightarrow |x + iy + 1| = x + iy + 2(1 + i)$

$\Rightarrow |x + 1 + iy| = (x + 2) + i(y + 2)$

$\Rightarrow \sqrt{(x+1)^2 + y^2} = (x+2) + i(y+2)$

$\left[\because |z| = \sqrt{x^2+y^2}\right]$

Comparing real and imaginary parts, we get

$\sqrt{(x+1)^2 + y^2} = x+2 \Rightarrow (x+1)^2 + y^2 = (x+2)^2$ ...(i)

and $0 = y + 2 \Rightarrow y = -2$ ...(ii)

Substituting the value of $y = -2$ in (i), we get

$(x + 1)^2 + (-2)^2 = (x + 2)^2 \Rightarrow x^2 + 2x + 1 + 4 = x^2 + 4x + 4$

$\Rightarrow 2x + 1 = 4x \Rightarrow x = 1/2$

Hence, $z = x + iy = \frac{1}{2} - 2i$

**31.** We have, $\tan\frac{x}{2} = \frac{m}{n}$

and, $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2\frac{m}{n}}{1+\frac{m^2}{n^2}} = \frac{2mn}{m^2+n^2}$

$$\cos x = \sqrt{1-\left(\frac{2mn}{m^2+n^2}\right)^2} = \sqrt{\frac{(m^2+n^2)^2-4m^2n^2}{(m^2+n^2)^2}}$$

$= \frac{m^2-n^2}{m^2+n^2}$

$\therefore m\sin x + n\cos x = m\left(\frac{2mn}{m^2+n^2}\right) + n\left(\frac{m^2-n^2}{m^2+n^2}\right)$

$$= \frac{2m^2n}{m^2+n^2} + \frac{nm^2}{m^2+n^2} - \frac{n^3}{m^2+n^2} = \frac{3m^2n-n^3}{m^2+n^2}$$

**32.** Let $z = x + iy$. Then,

$\bar{z} = iz^2 \Rightarrow \overline{(x+iy)} = i(x+iy)^2$

$\Rightarrow x - iy = i(x^2 - y^2 + 2ixy) \Rightarrow x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow (x + 2xy) - i(x^2 - y^2 + y) = 0$

$\Rightarrow x + 2xy = 0$ ... (i) and $x^2 - y^2 + y = 0$ ...(ii)

Now, $x + 2xy = 0 \Rightarrow x(1 + 2y) = 0$

$\Rightarrow x = 0$ or $1 + 2y = 0 \Rightarrow x = 0$ or $y = -\frac{1}{2}$

**Case I :** When $x = 0$

From (ii), we have

$\Rightarrow \quad -y^2 + y = 0 \Rightarrow -y(y-1) = 0 \Rightarrow y = 0, y = 1$

Thus, we have the following pairs of values of $x$ and $y$ :

$x = 0, y = 0; x = 0\ y = 1$

$\therefore \quad z = 0 + i\,0 = 0,\ z = 0 + 1i = i$

**Case II :** When $y = -\dfrac{1}{2}$

From (ii), we get

$$x^2 - y^2 + y = 0 \Rightarrow x^2 - \frac{1}{4} - \frac{1}{2} = 0$$

$$\Rightarrow \quad x^2 - \frac{3}{4} = 0 \Rightarrow x = \pm\frac{\sqrt{3}}{2}$$

Thus, we have the following pairs of values of $x$ and $y$ :

$$x = \frac{\sqrt{3}}{2}, y = \frac{-1}{2}; x = \frac{-\sqrt{3}}{2}, y = \frac{-1}{2}$$

$$\therefore \quad z = \frac{\sqrt{3}}{2} - \frac{1}{2}i,\ z = \frac{-\sqrt{3}}{2} - \frac{1}{2}i$$

Hence, $z = 0, i, \dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i, \dfrac{-\sqrt{3}}{2} - \dfrac{1}{2}i$

**33.** Consider, $\left(\dfrac{1}{1-4i} - \dfrac{2}{1+i}\right)\left(\dfrac{3-4i}{5+i}\right)$

$$= \left\{\frac{1+i-2+8i}{(1-4i)(1+i)}\right\}\left\{\frac{(3-4i)(5-i)}{(5+i)(5-i)}\right\}$$

$$= \left\{\frac{-1+9i}{1+i-4i-4i^2}\right\}\left\{\frac{15-3i-20i+4i^2}{25-i^2}\right\}$$

$$= \left\{\frac{-1+9i}{5-3i}\right\}\left\{\frac{11-23i}{26}\right\} = \left\{\frac{(-1+9i)(5+3i)}{25-9i^2}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \left\{\frac{-5-3i+45i+27i^2}{25+9}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \left\{\frac{-32+42i}{34}\right\}\left\{\frac{11-23i}{26}\right\} = \left\{\frac{-16+21i}{17}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \frac{-176+368i+231i-483i^2}{442} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599}{442}i$$

$$\text{Magnitude} = \sqrt{\left(\frac{307}{442}\right)^2 + \left(\frac{599}{442}\right)^2}$$

$$= \sqrt{\frac{94249+358801}{(442)^2}} = \sqrt{\frac{453050}{195364}} = \sqrt{\frac{1025}{442}}$$

$$\text{Conjugate} = \frac{307}{442} - \frac{599}{442}i$$

**OR**

$$\text{L.H.S.} = \left|1-\bar{z}_1 z_2\right|^2 - \left|z_1 - z_2\right|^2$$

$$= (1-\bar{z}_1 z_2)\overline{(1-\bar{z}_1 z_2)} - (z_1 - z_2)\overline{(z_1 - z_2)}$$

$$= (1-\bar{z}_1 z_2)(1-z_1\bar{z}_2) - (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$= 1 + z_1\bar{z}_1\ z_2\bar{z}_2 - z_1\bar{z}_1 - z_2\bar{z}_2$$

$$= 1 + |z_1|^2 \cdot |z_2|^2 - |z_1|^2 - |z_2|^2 = \left(1-|z_1|^2\right)\left(1-|z_2|^2\right)$$

$$\text{R.H.S.} = k\left(1-|z_1|^2\right)\left(1-|z_2|^2\right)$$

Hence, equating L.H.S. and R.H.S., we get $k = 1$.

**34.** We have, $\sqrt{4+3\sqrt{-20}} = \sqrt{4+6\sqrt{5}i}$

and $\sqrt{4-3\sqrt{-20}} = \sqrt{4-6\sqrt{5}i}$

Let $\sqrt{4+6\sqrt{5}i} = (x+iy)$ ...(i)

On squaring both sides of (i), we get

$(4+6\sqrt{5}i) = (x^2 - y^2) + i(2xy)$ ...(ii)

On comparing real and imaginary parts on both sides of (ii), we get

$(x^2 - y^2) = 4$ and $2xy = 6\sqrt{5}$

$$\therefore \quad (x^2+y^2) = \sqrt{(x^2-y^2)^2 + 4x^2y^2} = \sqrt{(4)^2 + (6\sqrt{5})^2}$$

$$= \sqrt{16+180} = \sqrt{196} = 14$$

Thus, $x^2 - y^2 = 4$ ...(iii) and $x^2 + y^2 = 14$ ...(iv)

On solving (iii) and (iv), we get $x^2 = 9$ and $y^2 = 5$

$\therefore \quad x = \pm 3$ and $y = \pm\sqrt{5}$

Since $xy > 0$, so $x$ and $y$ are of the same sign.

$\therefore \quad x = 3, y = \sqrt{5}$ or $x = -3, y = -\sqrt{5}$

$$\therefore \quad \sqrt{4+3\sqrt{-20}} = \pm(3+\sqrt{5}i)$$

Similarly, $\sqrt{4-3\sqrt{-20}} = \pm(3-\sqrt{5}i)$

Hence, $\sqrt{4+3\sqrt{-20}} + \sqrt{4-3\sqrt{-20}} = \pm 6$

**35.** L.H.S. $= \sin^2\dfrac{A}{2} + \sin^2\dfrac{B}{2} + \sin^2\dfrac{C}{2}$

$$= \frac{1-\cos A}{2} + \frac{1-\cos B}{2} + \frac{1-\cos C}{2}$$

$$= \frac{3-(\cos A+\cos B+\cos C)}{2} = \frac{3-S}{2} \quad \text{...(i)}$$

where $S = \cos A + \cos B + \cos C = (\cos A + \cos B) + \cos C$

$$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos\left(2\cdot\frac{C}{2}\right)$$

$$= 2\cos\left(90^\circ - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos\left(2\cdot\frac{C}{2}\right)$$

$$= 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + 1 - 2\sin^2\frac{C}{2}$$

$$= 1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right\}$$

$$= 1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\left(90^\circ - \frac{A+B}{2}\right)\right\}$$

$$= 1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}$$

$$= 1 + 2\sin\frac{C}{2}\left\{-2\sin\frac{A}{2}\sin\left\{-\frac{B}{2}\right\}\right\}$$

$$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

Then from (i), we get

$$\text{L.H.S.} = \frac{3-\left(1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)}{2}$$

$$= 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

**OR**

Given, $\sin(\theta + \alpha) = a$ ...(i)

and $\sin(\theta + \beta) = b$ ...(ii)

$\therefore\quad \cos(\theta+\alpha) = \sqrt{1-a^2}$ and $\cos(\theta+\beta) = \sqrt{1-b^2}$

Since, $\cos(\alpha - \beta) = \cos[(\theta + \alpha) - (\theta + \beta)]$

$= \cos(\theta + \alpha)\cos(\theta + \beta) + \sin(\theta + \alpha)\sin(\theta + \beta)$

$[\because\ \cos(A - B) = \cos A\cos B + \sin A\sin B]$

$$= \sqrt{1-a^2}\sqrt{1-b^2} + a\cdot b = ab + \sqrt{(1-a^2)(1-b^2)}$$

$$\Rightarrow\quad \cos(\alpha-\beta) = ab + \sqrt{1-a^2-b^2+a^2b^2}$$

Now, L.H.S. $= \cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$

$= 2\cos^2(\alpha - \beta) - 1 - 4ab\cos(\alpha - \beta)$

$$= 2\left[ab+\sqrt{1-a^2-b^2+a^2b^2}\right]^2 - 1 - 4ab\left[ab+\sqrt{1-a^2-b^2+a^2b^2}\right]$$

$$= 2\left[(ab)^2+1-a^2-b^2+a^2b^2+2ab\sqrt{1-a^2-b^2+a^2b^2}\right] - 1 - 4a^2b^2 - 4ab\sqrt{1-a^2-b^2+a^2b^2}$$

$$= 2a^2b^2+2-2a^2-2b^2+2a^2b^2+4ab\sqrt{1-a^2-b^2+a^2b^2} - 1 - 4a^2b^2 - 4ab\sqrt{1-a^2-b^2+a^2b^2}$$

$= 1 - 2a^2 - 2b^2 =$ R.H.S.

**36. (i)** $\because \cos^2 x = 1 - \sin^2 x = 1 - \dfrac{9}{25} = \dfrac{16}{25}$

Thus $\cos x = \pm\dfrac{4}{5}$. Since $x$ lies in third quadrant

$\therefore\quad \cos x$ is negative

$\therefore\quad \cos x = -4/5$

**(ii)** $\sin^2 y = 1 - \cos^2 y = 1 - \dfrac{144}{169} = \dfrac{25}{169} = \pm\dfrac{5}{13}$

Since $y$ lies in third quadrant

$\therefore\quad \sin y$ is negative $\therefore\ \sin y = -5/13$

**(iii)** $\sin(x+y) = \left(\dfrac{3}{5}\right)\times\left(-\dfrac{12}{13}\right) + \left(-\dfrac{4}{5}\right)\times\left(\dfrac{-5}{13}\right)$

$$= -\frac{36}{65} + \frac{20}{65} = -\frac{16}{65}$$

**OR**

$\sin 75^\circ = \sin(30^\circ + 45^\circ) = \sin 30^\circ\cdot\cos 45^\circ + \cos 30^\circ\cdot\sin 45^\circ$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

**37. (i)** $z = \dfrac{1}{3+4i}$ ; $|z| = \left|\dfrac{1}{3+4i}\right| = \dfrac{1}{\sqrt{3^2+4^2}} = \dfrac{1}{5}$

**(ii)** $z = \dfrac{-2+3i}{1-i}\times\dfrac{1+i}{1+i}$

$$= \frac{(-2+3i)(1+i)}{1^2-(i)^2} = \frac{-2-2i+3i+3i^2}{1+1} = \frac{-5+i}{2} = \frac{-5}{2}+\frac{i}{2}$$

Conjugate of $z$ is $\dfrac{-5}{2} - \dfrac{i}{2}$

**(iii)** $z = \dfrac{3+i}{2-i} + \dfrac{3-i}{2+i} = \dfrac{(3+i)(2+i)+(3-i)(2-i)}{2^2-i^2}$

$$= \frac{6+5i-1+6-5i-1}{4+1} = \frac{10}{5} = 2\ \therefore\ \text{Real part} = 2$$

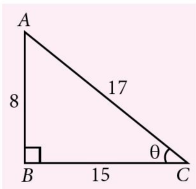
**OR**

$$\frac{z_1}{z_2} = \frac{\sqrt{3}+i\sqrt{3}}{\sqrt{3}+i}\times\frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}$$

$$= \frac{3+3i-\sqrt{3}i+\sqrt{3}}{(\sqrt{3})^2-(i)^2} = \left(\frac{3+\sqrt{3}}{4}\right) + \left(\frac{3-\sqrt{3}}{4}\right)i$$

which is represents by a point in first quadrant.

**38.** Given, $\tan\theta = \dfrac{-8}{15}$




In $\Delta ABC$,

$AC^2 = AB^2 + BC^2$

$\Rightarrow\quad AC^2 = 8^2 + 15^2$

$\Rightarrow\quad AC = \sqrt{289} = 17$

$\therefore\ \cos\theta = \dfrac{-15}{17}$ $[\because \theta$ lies in second quadrant$]$

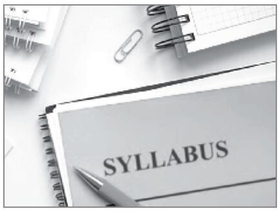
**(i)** $\sin 2\theta = 2\sin\theta.\cos\theta = 2\times\dfrac{8}{17}\times\left(\dfrac{-15}{17}\right), = \dfrac{-240}{289}$

**(ii)** $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\times\left(\frac{-8}{15}\right)}{1-\left(\frac{-8}{15}\right)^2} = \dfrac{\frac{-16}{15}}{1-\frac{64}{225}} = \dfrac{-240}{161}$

# *How to crack* **IIT JEE** *in first attempt?*

1. Manage your time carefully. The better your planning, the better your preparation. Draw a clear, comprehensive, and realistic timetable.

2. Know the syllabus thoroughly so that nothing is left from the preparation.

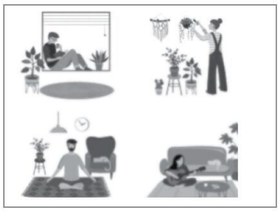



3. Divide each topic of the syllabus into 3 catagories : Easy, Medium and Difficult. This will help you to manage your preparation time, according to the level of difficulty.

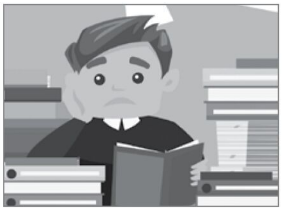

4. Use reliable resources for your preparation. Trust good books for gaining additional knowledge. Invest in good study material and past year papers.

5. If you have friends or neighbours, who are also studying for the JEE, consider forming study group with them. It can help in understanding concepts and getting motivation from fellow aspirants.

6. Studying is important but doing it continuously can make it monotonous. Spend the time in activities that relax you by taking break and give rest to your mind to function more efficiently.



7. Study each chapter of each topic until you have understood the concept clearly. You can also makes notes for better recalling.



8. Depending on the layout of your home, see if it is possible to have secluded space to study without disturbance. Also keep the space neat, clean and organized as this will help to keep your mind calm and composed.

9. It is very important to practice solving JEE previous year question papers and mock test papers. This will help you to get an idea of the question pattern and marking scheme of each topic.


10. Positive attitude is a must. Always remind yourself what your goal is. You can listen to motivational speeches and also put such posters in your room.

# MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapter. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

## Series 1: Relations and Functions

Time Taken : 60 Min.

### Only One Option Correct Type

1. Let $f : X \to Y$, $A \subseteq X$ and $B \subseteq Y$. If $f(A) = \{y : y = f(x), x \in A\}$ and $f^{-1}(B) = \{x : f(x) = y, x \in B\}$, then
   (a) $f^{-1}(f(A)) = A$
   (b) $f(f^{-1}(B)) = B$
   (c) $f^{-1}(f(A)) = A$, if $f(X) = Y$
   (d) $f(f^{-1}(B)) = B$, if $f(X) = Y$

2. A function $f : A \to B$, where $A = \{x : -1 \le x \le 1\}$ and $B = \{y : 1 \le y \le 2\}$ is defined by the rule $y = f(x) = 1 + x^2$. Which of the following statement is true?
   (a) $f$ is injective but not surjective
   (b) $f$ is surjective but not injective
   (c) $f$ is both injective and surjective
   (d) $f$ is neither injective nor surjective

3. The number of onto functions from $A = \{1, 2, 3, 4, 5\}$ to $B = \{6, 7, 8\}$ is
   (a) 60 (b) 72 (c) 90 (d) 150

4. If $f : [1, \infty) \to [2, \infty)$, $f(x) = x + \frac{1}{x}$, then $f^{-1}(x) =$
   (a) $\frac{x+\sqrt{x^2-4}}{2}$ (b) $\frac{x}{1+x^2}$
   (c) $\frac{x-\sqrt{x^2-4}}{2}$ (d) $1+\sqrt{x^2-4}$

5. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation $R$ is
   (a) function (b) reflexive
   (c) not symmetric (d) transitive

6. Consider the following relations :
   $R = \{(x, y) : x, y$ are real and $x = wy$ for some non-zero rational number $w\}$ $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) : m, n, p, q\right.$ are integers, such that $\left. n, q \neq 0, mq = np\right\}$, then
   (a) $R$ is an equivalence relation, but $S$ is not
   (b) $S$ is an equivalence relation, but $R$ is not
   (c) both $R$ and $S$ are equivalence relations
   (d) neither $R$ nor $S$ is an equivalence relation.

### One or More Than One Option(s) Correct Type

7. If $f(x) = \sqrt{1-\sin^2 x} + \sqrt{1+\tan^2 x}$, then
   (a) fundamental period of $f(x)$ is $\pi$
   (b) range of $f(x)$ is $[2, \infty)$
   (c) domain of $f(x)$ is $R$
   (d) $f(x) = 2$ has 3 solutions in $[0, 2\pi]$

8. Let $f : R \to R$ be defined as $f(x) = \frac{\sin \pi\{x\}}{x^2 - x + 1}$ $\forall\, x \in R$, where $\{x\}$ is fractional part function. Then
   (a) $f$ is neither even nor odd function
   (b) $f$ is a zero function
   (c) $f$ is many-one and non-constant function
   (d) $f$ is one-one function

9. Let $X$ be a non-empty set and $P(X)$ be the set of all subsets of $X$. For $A, B \in P(X)$, $ARB$ if and only if $A \cap B = \phi$, then the relation
   (a) $R$ is not reflexive (b) $R$ is symmetric
   (c) $R$ is not transitive
   (d) $R$ is an equivalence relation

10. Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = [x]$ and $g(x) = \frac{3-2x}{4}$, then
    (a) $f$ is neither one-one nor onto.

(b) $g$ is one-one but $f$ is not one-one
(c) $f$ is one-one and $g$ is onto
(d) neither $f$ nor $g$ is onto

**11.** A relation $R$ on the set of non-zero complex numbers is defined by $z_1 \, R \, z_2$ if and only if $\frac{z_1 - z_2}{z_1 + z_2}$ is real. Then $R$ is
(a) equivalence (b) reflexive
(c) symmetric (d) none of these

**12.** The inverse of the function $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is
(a) $\log_{10}(2 - x)$ (b) $\frac{1}{2}\log_{10}\frac{1+x}{1-x}$
(c) $\frac{1}{2}\log_{10}(2x-1)$ (d) $\log_{10}\left(\frac{1-x}{1+x}\right)^{-1/2}$

**13.** Let $f(x) = 1 + \sqrt{x}$ and $g(x) = \frac{2x}{x^2+1}$, then
(a) dom $(f + g) = (-1, \infty)$
(b) dom $(f + g) = [0, \infty)$
(c) range $f \cap$ range $g = \{1\}$
(d) range $f \cup$ range $g = [-1, \infty)$

## Comprehension Type

### Paragraph for Q. No. 14 and 15

Let $f(x) = \begin{cases} 2x + a : x \geq -1 \\ bx^2 + 3 : x < -1 \end{cases}$ and

$g(x) = \begin{cases} x + 4 \quad : 0 \leq x \leq 4 \\ -3x - 2 : -2 < x < 0 \end{cases}$ functions

**14.** $g(f(x))$ is not defined, if
(a) $a \in (10, \infty), b \in (5, \infty)$
(b) $a \in (4, 10), b \in (5, \infty)$
(c) $a \in (10, \infty), b \in (0, 1)$
(d) $a \in (4, 10), b \in (1, 5)$

**15.** If $a = 2$ and $b = 3$, then range of $g(f(x))$ is
(a) $(-2, 8]$ (b) $(0, 8]$
(c) $[4, 8]$ (d) $[-1, 8]$

## Matrix Match Type

**16.** Match the functions given in Column-I with the nature of functions given in column-II.

| | Column - I | | Column - II |
|---|---|---|---|
| (P) | $f: [0, \infty) \to [0, \infty), f(x) = \frac{x}{1+x}$ | (1) | one-one onto |
| (Q) | $f: R - \{0\} \to R, f(x) = x - \frac{1}{x}$ | (2) | one-one but not onto |
| (R) | $f: R - \{0\} \to R, f(x) = x + \frac{1}{x}$ | (3) | onto but not one-one |
| (S) | $f: R \to R, f(x) = 2x + \sin x$ | (4) | neither one-one nor onto |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 2 | 4 | 1 | 2 |
| (b) | 4 | 1 | 2 | 3 |
| (c) | 2 | 3 | 4 | 1 |

(d) None of these.

## Numerical Answer Type

**17.** If $[x]^2 + [x - 2] < 0$ and $\{x\} = 1/2$, then the number of possible values of $x$, is ($[x]$ and $\{x\}$ denote greatest integer less than or equal to $x$ and fractional part of $x$, respectively) ________.

**18.** A non-zero function $f(x)$ is symmetrical about the line $y = x$, then the value of $\lambda$ (constant) such that $f^2(x) = (f^{-1}(x))^2 - \lambda x f(x) f^{-1}(x) + 3x^2 f(x) \ \forall \ x \in R^+$ is _______.

**19.** Let $f$ be a real-valued function defined on $R$ such that one root of the quadratic equation $x^2 - 3tx + 2f(t) + f(2 - t) = 0$ is double the other root, then the value of $[f(2)]$ is (where $[\cdot]$ denotes the greatest integer function) _______.

**20.** If $f(x) = \left(100^5 - x^{10}\right)^{\frac{1}{10}}$, then find the value of $\frac{1}{2^{10}} f(f(1024))$.

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time. |
| < 60% | NOT SATISFACTORY! | Revise thoroughly and strengthen your concepts. |

# CBSE warm-up!

CLASS-XII

**Chapterwise practice questions for CBSE Exams as per the latest pattern and rationalised syllabus by CBSE for the academic session 2023-24.**

## Series-3 Matrices and Determinants

Time Allowed : 3 hours
Maximum Marks : 80

***General Instructions***

*(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
*(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
*(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
*(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
*(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
*(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

### SECTION - A (MULTIPLE CHOICE QUESTIONS)

**Each question carries 1 mark.**

**1.** If $A=\begin{bmatrix}-2 & 4\\ -1 & 2\end{bmatrix}$, then $A^2$ is

(a) null matrix (b) unit matrix
(c) $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ (d) $\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

**2.** The number of all the possible matrices of order $3 \times 3$ with each entry 2 or 0 is

(a) 9 (b) 27 (c) 81 (d) 512

**3.** If $A=[a_{ij}]=\begin{bmatrix}2 & -1\\ -3 & 4\\ 1 & 2\end{bmatrix}$ and

$B=[b_{ij}]=\begin{bmatrix}2 & 3 & -5\\ 1 & 4 & 9\\ 0 & 7 & -2\end{bmatrix}$, then value of $a_{11}b_{11}+a_{22}b_{22}$ is

(a) 8 (b) 20 (c) 16 (d) 24

**4.** If matrix $A=\begin{bmatrix}3 & -3\\ -3 & 3\end{bmatrix}$ and $A^2 = kA$, then write the value of $k$.

(a) 6 (b) 3 (c) 12 (d) 18

**5.** If $A$ is a square matrix such that $A^2 = A$, then $(I-A)^3 + A$ is equal to

(a) $I$ (b) 0 (c) $I - A$ (d) $I + A$

**6.** If $A$ and $B$ are symmetric matrices of the same order, then

(a) $AB$ is a symmetric matrix
(b) $A - B$ is a skew-symmetric matrix
(c) $AB + BA$ is a symmetric matrix
(d) $AB - BA$ is a symmetric matrix

**7.** If for the non-singular matrix $A$, $A^2 = I$, then find $A^{-1}$.

(a) $A$ (b) $I$
(c) $O$ (d) None of these

**8.** If $A^2 - A + I = O$, then the inverse of $A$ is

(a) $I - A$ (b) $A - I$
(c) $A$ (d) $A + I$

9. If $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = \frac{(i+2j)^2}{2}$, then $A$ is equal to

(a) $\begin{bmatrix} 9 & 25 \\ 8 & 18 \end{bmatrix}$ (b) $\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$

(c) $\begin{bmatrix} 9 & 25 \\ 4 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 9/2 & 15/2 \\ 4 & 9 \end{bmatrix}$

10. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$, then $A^{-1}$ exists if

(a) $\lambda \neq 1$ (b) $\lambda \neq 2$

(c) $\lambda \neq -2$ (d) None of these

11. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2, \beta = ab$

(b) $\alpha = a^2 + b^2, \beta = 2ab$

(c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$

(d) $\alpha = 2ab, \beta = a^2 + b^2$

12. The value of $\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$

is independent of

(a) $\alpha$ (b) $\beta$

(c) $\alpha, \beta$ (d) None of these

13. If $x$ is a complex root of the equation

$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} + \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0,$$

then $x^{2007} + x^{-2007} =$

(a) 1 (b) $-1$ (c) $-2$ (d) 2

14. The value of $\begin{vmatrix} \sin 10° & -\cos 10° \\ \sin 80° & \cos 80° \end{vmatrix}$ is

(a) 0 (b) 1 (c) $-1$ (d) 2

15. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then $x$ is

(a) $\frac{13}{25}$ (b) $-\frac{25}{13}$ (c) $\frac{5}{13}$ (d) $\frac{25}{13}$

16. Suppose that system of equations $x = cy + bz$, $y = az + cx$, $z = bx + ay$ has non-trivial solution. Then $a^2 + b^2 + c^2 + 2abc =$

(a) 2 (b) $-1$ (c) 0 (d) 1

17. If a square matrix $A = [a_{ij}]$, $a_{ij} = i^2 - j^2$ is of even order, then $A$ is

(a) Symmetric matrix

(b) Skew-symmetric matrix

(c) Identity matrix

(d) None of these

18. If $c < 1$ and the system of equations $x + y - z = 0$, $2x - y - cz = 0$ and $-bx + 3by - cz = 0$ has non-trivial solution, then the possible real values of $b$ are

(a) $b \in \left(-3, \frac{3}{4}\right)$ (b) $b \in \left(-\frac{3}{4}, 4\right)$

(c) $b \in \left(-\frac{3}{4}, 3\right)$ (d) None of these

## ASSERTION-REASON BASED QUESTIONS

**In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.**

**(a) Both (A) and (R) are true and (R) is the correct explanation of (A).**

**(b) Both (A) and (R) are true but (R) is not the correct explanation of A.**

**(c) (A) is true but (R) is false.**

**(d) (A) is false but (R) is true.**

19. **Assertion (A):** The inverse of the matrix $A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 5 & 2 \\ 12 & -4 & 5 \end{bmatrix}$ certainly exists.

**Reason (R) :** Every non-singular matrix has unique inverse.

20. **Assertion (A) :** $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix.

**Reason (R) :** $A = [a_{ij}]$ is a square matrix such that $a_{ij} = 0, \forall\ i \neq j$, then $A$ is called diagonal matrix.

## MONTHLY TEST DRIVE CLASS XI ANSWER KEY

| | | | | |
|---|---|---|---|---|
| 1. (a) | 2. (c) | 3. (d) | 4. (b) | 5. (c) |
| 6. (a) | 7. (a,b) | 8. (a,c,d) | 9. (a,b,c,d) | 10. (a,b,c) |
| 11. (a,b,d) | 12. (a,b,c,d) | 13. (a,b,c) | 14. (d) | 15. (c) |
| 16. (b) | 17. (7) | 18. (2) | 19. (3) | 20. (26) |

## SECTION - B

**This section comprises of very short answer type questions (VSA) of 2 marks each.**

**21.** If $A=\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$, then find the value of $\alpha$ for which $A^2 = B$.

**OR**

If $A^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, then find $A^T - B^T$.

**22.** Find the value of $x$ for which

$$[1\ \ x\ \ 1]=\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=O$$

**23.** If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, then find $A^{-1}$.

**OR**

Find the values of $x$, $y$ and $z$ if $\begin{bmatrix}-2x+y\\ x+y+z\\ x+y\end{bmatrix}=\begin{bmatrix}-3\\ 3\\ 3\end{bmatrix}$.

**24.** Find the values of $x$, $y$, $z$ respectively, if the matrix

$A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfy the equation $A^TA = I_3$.

**25.** If $\begin{bmatrix}x+3 & z+4 & 2y-7\\ 4x+6 & a-1 & 0\\ b-3 & 3b & z+2c\end{bmatrix}=\begin{bmatrix}0 & 6 & 3y-2\\ 2x & -3 & 2c+2\\ 2b+4 & -21 & 0\end{bmatrix}$,

then find the values of $a$, $b$, $c$, $x$, $y$ and $z$ respectively.

## SECTION - C

**This section comprises of short answer type questions (SA) of 3 marks each.**

**26.** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of an equilateral triangle whose each side is equal to $a$, then show that $\begin{vmatrix}x_1 & y_1 & 4\\ x_2 & y_2 & 4\\ x_3 & y_3 & 4\end{vmatrix}^2 = 12a^4$.

**27.** If $A=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}$, then find the value of $A^4 - 2^4(A - I)$.

**28.** If matrix $A=\begin{bmatrix}a & b & c\\ b & c & a\\ c & a & b\end{bmatrix}$, where $a$, $b$ and $c$ are real positive numbers, $abc = 1$ and $A^TA = I$, then find the value of $a^3 + b^3 + c^3$.

**29.** Let $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, then show that $A^2-4A-5I_3=O$

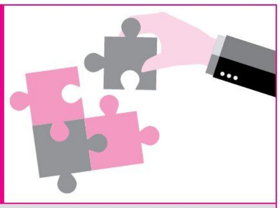
and $A^{-1}=\frac{1}{5}(A-4I_3)$.

**OR**

If $A$ and $B$ are square matrices of same order such that $AB = BA$, then show that

$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$.

# PUZZLE CORNER



### MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

| | | | | | |
|---|---|---|---|---|---|
| 2− | 3÷ | | 4− | | 32× |
| | 5− | | 72× | | |
| 3+ | 60× | | | | 10+ |
| | | 3− | | 2− | |
| 2− | 1− | 6+ | | | |
| | | | 60× | | |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

**30.** Find a $2 \times 2$ matrix $B$, such that

$$\begin{bmatrix} 2 & 5 \\ -3 & 7 \end{bmatrix} B = \begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}.$$

**OR**

Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where $I$ is $2 \times 2$ matrix and $O$ is $2 \times 2$ zero matrix. Using the equation, find $A^{-1}$.

**31.** Solve the system of linear equations using matrix method $5x + 2y = 4$ and $7x + 3y = 5$.

## SECTION - D

**This section comprises of long answer type questions (LA) of 5 marks each.**

**32.** Solve the following system of equations $x - y + z = 4$, $x - 2y + 2z = 9$ and $2x + y + 3z = 1$.

**33.** A diet is to contain 30 units of vitamin A, 40 units of vitamin B and 20 units of vitamin C. Three types of foods $F_1$, $F_2$ and $F_3$ are available. 1 unit of food $F_1$ contains 3 units of vitamin A, 2 units of vitamin B, 1 unit of vitamin C. 1 unit of food $F_2$ contains 1 unit of vitamin A, 2 units of vitamin B and 1 unit of vitamin C. 1unit of food $F_3$ contains 5 units of vitamin A, 3 units of vitamin B and 2 units of vitamin C. Represent the above situation algebraically and find the diet contains each types of food by using matrix method.

**OR**

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find adj$A$ and verify that $A\,(\text{adj}\,A) = (\text{adj}\,A)A = |A|\,I_3$.

**34.** If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $6A^{-1} = A^2 + cA + dI$, then find the value of $(c, d)$.

**35.** Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

**OR**

Three shopkeeper $A$, $B$ and $C$ go to a store to buy stationery. $A$ purchased 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. $B$ purchase 10 dozen notebooks, 6 dozen pens and 7 dozens pencils. $C$ purchase 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹ 40, a pen costs ₹ 8.50 and a pencil costs ₹ 3.50. Use matrix multiplication to calculate each individuals bill.

## SECTION - E

**This section comprises of 3 case-study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.**

**36.** In a city there are two factories $A$ and $B$. Each factory produces Winter clothes for men and women. There are three types of clothes produced in both the factories, type I, II and III. For men the number of units of types I, II and III respectively are 70, 80 and 90 in factory $A$ and 80, 60 and 70 are in factory $B$. For women the number of units of types I, II and III respectively are 90, 60, 80 in factory $A$ and 40, 65, 70 are in factory $B$.

Based on the above information, answer the following questions.

**(i)** Represent '$R$' as matrix of number of units of each type produced by factory $A$ for both men and women.

**(ii)** Represents as a matrix of number of units of each type produced by factory $B$ for both men and women.

**(iii)** Write the total production of winter clothes of each type for men in matrix from.

**OR**

Write the total production of winter clothes of each type for women in the matrix from.

**37.** A maths teacher mentioned in a class, area of a triangle Whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$? is given by the determinant

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since, area is a positive quantity, so we always take the absolute value of the determinant $\Delta$. Also, the area of the triangle formed by three collinear points is zero.

Based on the above information, answer the following questions.

**(i)** Find the area of triangle whose vertices are $(-6, 2)$, $(4, -6)$ and $(2, 6)$.

**(ii)** If the points $(4, -6)$, $(k, -2)$ and $(0, 8)$ are collinear, then find the value of $7k$.

**(iii)** If the area of a triangle $PQR$, with vertices $P(2, 3)$, $Q(3, 4)$ and $R(k, 0)$ is 4 sq. units, then find the value of $k$.

**OR**

Using determinants, find the equation of the line joining the points $A(2, 4)$ and $B(4, 8)$.

**38.** Let $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ and $X_1$, $X_2$ are first and second columns of a $2 \times 2$ matrix $x$, also let the column matrices $X_1$ and $X_2$ satisfying

$$AX_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{and } AX_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Based on the above information, answer the following questions.

**(i)** Find the value of $X_1 + X_2$.

**(ii)** If $Z = [2 \ \ 3] \times \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then find the value of $|Z|$.

## SOLUTIONS

**1.** **(a) :** Given that, $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$

$\therefore \quad A^2 = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \quad A^2$ is a null matrix.

**2.** **(d) :** As, there are 9 elements in $3 \times 3$ order matrix and each element is filled by either 2 or 0 in 2 ways.

$\therefore$ Total number of all the possible matrices of order $3 \times 3$ with each entry 2 or 0 is $2^9$ *i.e.*, 512.

**3.** **(b) :** We have, $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$

Here, $a_{11} = 2$, $a_{22} = 4$, $b_{11} = 2$, $b_{22} = 4$

$\therefore \quad a_{11}b_{11} + a_{22}b_{22} = 2(2) + 4(4) = 4 + 16 = 20$

**4.** **(a) :** We have, $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = kA$

$$\Rightarrow \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = k\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+9 & -9-9 \\ -9-9 & 9+9 \end{bmatrix} = k\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = k\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$

# SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 7 | | | 6 | | | 4 | | | | | | 1 | | | 3 | | | 8 | |
| 8 | | | | | | | | 7 | | | | 9 | | | | | | | | 4 |
| | | 9 | 3 | | 4 | 8 | | | | | | | | 7 | 2 | | 5 | 3 | | |
| | | 5 | | | | 9 | | | | | | | | 5 | | | | 6 | | |
| 1 | | | | | | | | 5 | | | | 3 | | | | | | | | 2 |
| | | 4 | | | | 7 | | | | | | | | 2 | | | | 4 | | |
| | | 8 | 6 | | 9 | 1 | | | | | | | | 8 | 3 | | 7 | 1 | | |
| 3 | | | | | | | | 6 | 4 | | 8 | 1 | | | | | | | | 7 |
| | 5 | | | 1 | | | 8 | | | 1 | | | 7 | | | 1 | | | 2 | |
| | | | | | | | 6 | | | | | | 9 | | | | | | | |
| | | | | | | | | 7 | | | | 6 | | | | | | | | |
| | | | | | | | 1 | | | | | | 2 | | | | | | | |
| | 3 | | | 5 | | | 4 | | | 9 | | | 1 | | | 7 | | | 6 | |
| 6 | | | | | | | | 1 | 3 | | 2 | 9 | | | | | | | | 3 |
| | | 4 | 6 | | 2 | 8 | | | | | | | | 2 | 3 | | 8 | 1 | | |
| | | 8 | | | | 3 | | | | | | | | 9 | | | | 7 | | |
| 3 | | | | | | | | 4 | | | | 2 | | | | | | | | 5 |
| | | 6 | | | | 9 | | | | | | | | 1 | | | | 2 | | |
| | | 2 | 3 | | 9 | 1 | | | | | | | | 5 | 7 | | 1 | 3 | | |
| 1 | | | | | | | | 9 | | | | 3 | | | | | | | | 4 |
| | 9 | | | 8 | | | 6 | | | | | | 2 | | | 8 | | | 5 | |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

$\Rightarrow 6\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = k\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \Rightarrow k = 6$

**5. (a) :** We have, $A^2 = A$
Now, $(I - A)^3 + A = (I - A)(I - A)(I - A) + A$
$= (I \cdot I - I \cdot A - A \cdot I + A \cdot A)(I - A) + A$
$= (I - A - A + A)(I - A) + A$
$[\because I \cdot A = A \cdot I = A \text{ and } A^2 = A]$
$= (I - A)(I - A) + A = (I \cdot I - I \cdot A - A \cdot I + A^2) + A$
$= (I - A - A + A) + A = (I - A) + A = I.$

**6. (c) :** $(AB + BA)^T = (AB)^T + (BA)^T$
$= B^T A^T + A^T B^T = BA + AB = AB + BA$
$(\because A^T = A \text{ and } B^T = B)$
Hence, $AB + BA$ is a symmetric matrix.

**7. (a) :** Given, $A^2 = I$ ...(i)
Since, $A$ is non-singular matrix
$\therefore\ |A| \neq 0$, So, $A^{-1}$ exists.
Multiplying (i) by $A^{-1}$ on both sides, we get
$A^{-1}(A^2) = A^{-1}(I)$
$\Rightarrow (A^{-1}A)A = A^{-1} \Rightarrow IA = A^{-1}$ $(\because A^{-1}A = I)$
$\therefore A^{-1} = A$ $(\because IA = A)$

**8. (a) :** If $A$ is any square matrix, then
$AA^{-1} = I$ and $A^{-1}I = A^{-1}$
Since, $A^2 - A + I = O$
$\Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = O$
$\Rightarrow (A^{-1}A)A - (A^{-1}A) + A^{-1} = O$
$\Rightarrow IA - I + A^{-1} = O \Rightarrow A - I + A^{-1} = O$
$\Rightarrow A^{-1} = I - A$

**9. (b)**

**10. (d) :** $A^{-1}$ exists if $|A| \neq 0$

*i.e.*, $\begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} \neq 0 \Rightarrow 2(6 - 5) + 1(5\lambda + 6) \neq 0$

$\Rightarrow 2 + 5\lambda + 6 \neq 0 \Rightarrow 5\lambda \neq -8$ *i.e.*, $\lambda \neq -8/5$

**11. (b) :** Given, $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$, $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$

$\Rightarrow \alpha = a^2 + b^2, \beta = 2ab$

**12. (a) :** Expanding along $R_1$, we get
$\Delta = \cos^2(\alpha + \beta) + \sin^2(\alpha + \beta) + \cos 2\beta = 1 + \cos 2\beta.$
Hence, independent of $\alpha$.

**13. (c) :** Expanding the two determinants, we get
$(1 - 3x^2 + 2x^3) + (3x^2 - x^3) = 0$
$\Rightarrow x^3 + 1 = 0 \Rightarrow x = -\omega, -\omega^2, -1$
$\therefore x^{2007} + x^{-2007} = -1 - 1 = -2.$

**14. (b) :** We have, $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix}$

$= \sin 10^\circ \cos 80^\circ + \cos 10^\circ \sin 80^\circ = \sin(10^\circ + 80^\circ)$
$= \sin 90^\circ = 1$

**15. (b) :** Since the matrix is singular.

$\therefore \begin{vmatrix} 2 + x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$

$\Rightarrow (2 + x)(5 - 2) - 3(-5 - 2x) + 4(1 + x) = 0$

$\Rightarrow 13x = -25 \Rightarrow x = -\dfrac{25}{13}$

**16. (d) :** $\begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

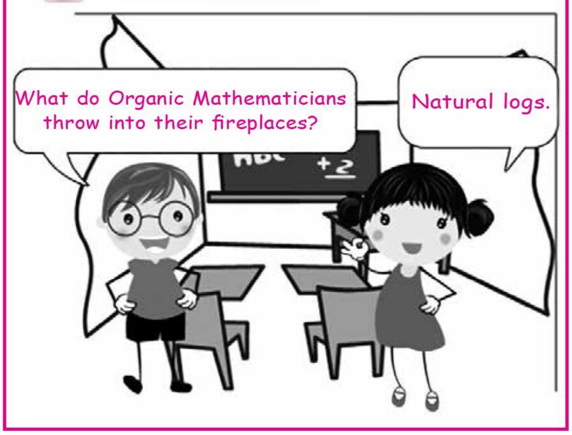
Expanding the determinant, we get
$-1(1 - a^2) - c(-c - ab) + b(ca + b) = 0$
$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1.$

**17. (b) :** We have, $a_{11} = 1^2 - 1^2 = 0$
$a_{12} = 1^2 - 2^2 = -3$, $a_{21} = 2^2 - 1^2 = 3$, $a_{22} = 2^2 - 2^2 = 0$

Now, $A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$ and $A^T = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$\therefore A^T = -A$, therefore $A$ is a skew-symmetric matrix.

## COMIC CAPSULE



**18. (c) :** Since system of equations has non-trivial solution.

$$\therefore \quad \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0$$

$\Rightarrow \quad c + 3bc + 2c + bc - 6b + b = 0$

$\Rightarrow \quad 3c + 4bc - 5b = 0 \Rightarrow c = \dfrac{5b}{3+4b}$

But $c < 1 \Rightarrow \dfrac{5b}{3+4b} < 1$

$\Rightarrow \quad \dfrac{b-3}{3+4b} < 0 \Rightarrow b \in \left(-\dfrac{3}{4}, 3\right)$

**19. (b) :** We have, $A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 5 & 2 \\ 12 & -4 & 5 \end{bmatrix}$

$|A| = 4(25 + 8) - 2(40 - 24) + 3(-32 - 60) = -176 \neq 0$

$\therefore$ $A$ is non singular matrix.

Hence, $A^{-1}$ exists.

**20. (a) :** If $A = [a_{ij}]_{n \times n}$ is a square matrix such that $a_{ij} = 0$ for $i \neq j$; then $A$ is called diagonal matrix.

Thus, the given statement is true and $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix.

**21.** Given that, $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 + 0 & 0+0 \\ \alpha + 1 & 0+1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$$

Also, $B = A^2 \Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

Clearly, this is not satisfied by any real value of $\alpha$.

**OR**

$$B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

So, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

**22.** $[1 \;\; x \;\; 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$

$$\Rightarrow [1 \; x \; 1] \begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix} = O$$

$\Rightarrow 7 + 2x + 12x + x^2 + 21 + 2x = 0$

$\Rightarrow \quad x^2 + 16x + 28 = 0 \Rightarrow x = -2, -14$

**23.** $|A| = 1(1 - 4) - 2(2 - 4) + 2(4 - 2)$ (Expanding along $R_1$)

$= -3 + 4 + 4 = 5 \neq 0$ $\therefore A^{-1}$ exists.

So, $\text{adj}(A) = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

Now, $A^{-1} = \dfrac{1}{|A|} \cdot \text{adj}(A) = \dfrac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

**OR**

We have, $\begin{bmatrix} -2x + y \\ x + y + z \\ x + y \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$

Equating the corresponding elements of two matrices, we get

$-2x + y = -3$ ... (i)

$x + y + z = 3$ ... (ii)

$x + y = 3$ ... (iii)

From (ii) and (iii), we get $3 + z = 3 \Rightarrow z = 0$

Solving (i) and (iii), we get $-3x = -6 \Rightarrow x = 2$

From (iii), $2 + y = 3 \Rightarrow y = 1$

$\therefore \quad x = 2, y = 1, z = 0$

**24.** We have,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$\because \quad A^T A = I_3$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad 2x^2 = 1, 6y^2 = 1, 3z^2 = 1$

$\Rightarrow \quad x = \pm\dfrac{1}{\sqrt{2}}, y = \pm\dfrac{1}{\sqrt{6}}, z = \pm\dfrac{1}{\sqrt{3}}$

**25.** Since,

$$\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$

$\therefore \quad x + 3 = 0 \Rightarrow x = -3,$
$b - 3 = 2b + 4 \Rightarrow b = -7,$
$z + 4 = 6 \Rightarrow z = 2,$
$a - 1 = -3 \Rightarrow a = -2,$
$2y - 7 = 3y - 2 \Rightarrow y = -5,$
$2c + z = 0 \Rightarrow 2c + 2 = 0 \Rightarrow c = -1,$
$\therefore \quad x = -3, y = -5, z = 2, a = -2, b = -7, c = -1$

**26.** Let $\Delta$ be the area of triangle $ABC$. Then,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow 2\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow \ 8\Delta = 4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}$$

$$\Rightarrow \ 64\Delta^2 = \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2 \qquad \text{...(i)}$$

But, the area of an equilateral triangle with each side $a$ is $\frac{\sqrt{3}}{4}a^2$.

$$\therefore \quad \Delta = \frac{\sqrt{3}}{4}a^2 \Rightarrow 16\Delta^2 = 3a^4 \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2 = 12a^4$$

**27.** We have, $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 0 & 8 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 7 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 0 & 16 \end{bmatrix}$$

Now, $A^4 - 2^4(A - I) = \begin{bmatrix} 1 & 15 \\ 0 & 16 \end{bmatrix} - 16\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 15 \\ 0 & 16 \end{bmatrix} - \begin{bmatrix} 0 & 16 \\ 0 & 16 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Also, $2I - A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

$\therefore \quad A^4 - 2^4(A - I) = 2I - A$

**28.** Since, $A^{\mathrm{T}}A = I$

$$\Rightarrow \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad a^2 + b^2 + c^2 = 1$ and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$= 1 + 2 \cdot 0 = 1$

$\Rightarrow \quad a + b + c = 1 \qquad \text{...(i)}$

Now, $(a^3 + b^3 + c^3)$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$\Rightarrow \quad a^3 + b^3 + c^3 = 1 + 3 = 4 \qquad$ [Using (i)]

**29.** $A^2 - 4A - 5I_3$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$\therefore \quad A^2 - 4A - 5I_3 = O$

$\Rightarrow \quad A^{-1}A^2 - 4A^{-1}A - 5A^{-1}I_3 = O$

$\Rightarrow \quad (A^{-1}A)A - 4I_3 - 5A^{-1} = O$

$\Rightarrow \quad IA - 4I_3 - 5A^{-1} = O \ \Rightarrow \ A^{-1} = \frac{1}{5}(A - 4I_3)$

**OR**

Consider, $(A+B)^3 = (A+B)^2 (A+B)$

$= [(A+B)(A+B)](A+B)$

$= (A^2 + AB + BA + B^2)(A+B)$

$= (A^2 + 2AB + B^2)(A+B)$ [$\because$ Given $AB = BA$]

$= A^2(A+B) + 2AB(A+B) + B^2(A+B)$

$= A^3 + A^2B + 2(AB)A + 2(AB)B + B^2A + B^3$

$= A^3 + A^2B + 2A^2B + 2AB^2 + AB^2 + B^3$

[$\because$ $(AB)A = A(BA) = A(AB) = A^2B$

and $B^2A = B(BA) = B(AB) = (BA)B = (AB)B = AB^2$]

$= A^3 + 3A^2B + 3AB^2 + B^3$.

**30.** Let $AB = \begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$, ...(i)

where $A = \begin{bmatrix} 2 & 5 \\ -3 & 7 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & 5 \\ -3 & 7 \end{vmatrix} = 14 + 15 = 29 \neq 0$

$\therefore$ $A^{-1}$ exists.

Pre-multiplying both sides of (i) by $A^{-1}$, we get

$$A^{-1}(AB) = A^{-1}\begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$$

$\Rightarrow \quad B = A^{-1}\begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$ ...(ii)

Also, $A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{1}{29}\begin{bmatrix} 7 & 3 \\ -5 & 2 \end{bmatrix}' = \dfrac{1}{29}\begin{bmatrix} 7 & -5 \\ 3 & 2 \end{bmatrix}$

From (ii), we have

$$B = \left(\frac{1}{29}\begin{bmatrix} 7 & -5 \\ 3 & 2 \end{bmatrix}\right)\begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$$

$$= \frac{1}{29}\begin{bmatrix} 119-235 & -7+65 \\ 51+94 & -3-26 \end{bmatrix} = \frac{1}{29}\begin{bmatrix} -116 & 58 \\ 145 & -29 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 \\ 5 & -1 \end{bmatrix}$$

**OR**

We have,

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

Hence, $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Since, $|A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1 \neq 0$

$\therefore$ $A^{-1}$ exists.

Now, $A^2 - 4A + I = O$

$\Rightarrow$ $A \cdot A - 4A = -I$

$\Rightarrow$ $A \cdot (AA^{-1}) - 4AA^{-1} = -IA^{-1}$ [Post multiply by $A^{-1}$]

$\Rightarrow$ $AI - 4I = -A^{-1}$

$\Rightarrow$ $A^{-1} = 4I - A = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.

**31.** The given system of equations can be written as $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ *i.e.*, $AX = B$,

where $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1 \neq 0$

$\therefore$ $A^{-1}$ exists.

$\therefore$ The system of equation has unique solution which is given by $X = A^{-1}B$.

To find $A^{-1}$ : $A_{11} = 3$, $A_{12} = -7$, $A_{21} = -2$, $A_{22} = 5$

$\therefore$ adj $A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$\therefore$ Solution is given by, $X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$\therefore$ $x = 2, y = -3$

**32.** Given system of equations is $x - y + z = 4$, $x - 2y + 2z = 9$ and $2x + y + 3z = 1$, which is written in the matrix form as

**MONTHLY TEST DRIVE CLASS XII ANSWER KEY**

| | | | | |
|---|---|---|---|---|
| **1.** (d) | **2.** (b) | **3.** (d) | **4.** (a) | **5.** (c) |
| **6.** (c) | **7.** (a,b,d) | **8.** (a,c) | **9.** (a,b,c) | **10.** (a,b) |
| **11.** (a,b,c) | **12.** (b,d) | **13.** (b,c,d) | **14.** (a) | **15.** (c) |
| **16.** (c) | **17.** (2) | **18.** (3) | **19.** (5) | **20.** (1) |

$$\begin{bmatrix}1 & -1 & 1\\1 & -2 & 2\\2 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\9\\1\end{bmatrix} \text{ or} AX = B$$

where, $A=\begin{bmatrix}1 & -1 & 1\\1 & -2 & 2\\2 & 1 & 3\end{bmatrix}, B=\begin{bmatrix}4\\9\\1\end{bmatrix}$ and $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

Now, $|A| = \begin{vmatrix}1 & -1 & 1\\1 & -2 & 2\\2 & 1 & 3\end{vmatrix}$

$= 1(-6-2) + 1(3-4) + 1(1+4)$

$= -8 - 1 + 5 = -4 \neq 0 \quad \therefore \; A$ is invertible.

So, there exists a unique solution $X = A^{-1}B$.

Now, cofactor matrix of $A = \begin{bmatrix}-8 & 1 & 5\\4 & 1 & -3\\0 & -1 & -1\end{bmatrix}$

$$\therefore \quad \text{adj } A=\begin{bmatrix}-8 & 1 & 5\\4 & 1 & -3\\0 & -1 & -1\end{bmatrix}^T=\begin{bmatrix}-8 & 4 & 0\\1 & 1 & -1\\5 & -3 & -1\end{bmatrix}$$

So, $A^{-1}=\frac{1}{|A|}(\text{adj } A)=\frac{-1}{4}\begin{bmatrix}-8 & 4 & 0\\1 & 1 & -1\\5 & -3 & -1\end{bmatrix}$

Now, $X = A^{-1}B = \frac{-1}{4}\begin{bmatrix}-8 & 4 & 0\\1 & 1 & -1\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{-1}{4}\begin{bmatrix}-32+36+0\\4+9-1\\20-27-1\end{bmatrix}=\frac{-1}{4}\begin{bmatrix}4\\12\\-8\end{bmatrix}=\begin{bmatrix}-1\\-3\\2\end{bmatrix}$$

On comparing, we get $x = -1, y = -3$ and $z = 2$

**33.** Let the diet contains $x$ units of food $F_1$, $y$ units of food $F_2$ and $z$ units of food $F_3$. According to the given condition, we make the following equations

$3x + y + 5z = 30$, $2x + 2y + 3z = 40$ and $x + y + 2z = 20$

These algebraic equations can be written in matrix form as

$$\begin{bmatrix}3 & 1 & 5\\2 & 2 & 3\\1 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}30\\40\\20\end{bmatrix} \; i.e., AX = B \quad \text{... (i)}$$

Now, $|A|=\begin{vmatrix}3 & 1 & 5\\2 & 2 & 3\\1 & 1 & 2\end{vmatrix}=3(4-3)-1(4-3)+5(2-2)$

$= 3 - 1 + 0 = 2 \neq 0$

$\therefore$ There exists a unique solution $X = A^{-1}B$.

$$\therefore \quad \text{adj}(A)=\begin{bmatrix}1 & 3 & -7\\-1 & 1 & 1\\0 & -2 & 4\end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{\text{adj}(A)}{|A|}=\frac{1}{2}\begin{bmatrix}1 & 3 & -7\\-1 & 1 & 1\\0 & -2 & 4\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 3 & -7\\-1 & 1 & 1\\0 & -2 & 4\end{bmatrix}\begin{bmatrix}30\\40\\20\end{bmatrix}=\frac{1}{2}\begin{bmatrix}30+120-140\\-30+40+20\\0-80+80\end{bmatrix}$$

On solving, we get $x = 5$, $y = 15$ and $z = 0$.

**OR**

Given, $A = \begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{bmatrix}$

Now, $A_{11} = \cos\alpha$, $A_{12} = -\sin\alpha$, $A_{13} = 0$,

$A_{21} = \sin\alpha$, $A_{22} = \cos\alpha$,

$A_{23} = 0$, $A_{31} = 0$, $A_{32} = 0$, $A_{33} = 1$

$$\therefore \text{adj}(A)=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\-\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{bmatrix}$$

$$A\cdot\text{adj}(A)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\-\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \quad \text{...(i)}$$

$$\text{adj}(A)\cdot A=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\-\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \quad \text{...(ii)}$$

$$|A|=\begin{vmatrix}\cos\alpha & -\sin\alpha & 0\\\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{vmatrix}$$

$= \cos\alpha(\cos\alpha - 0) + \sin\alpha(\sin\alpha - 0) + 0 = 1 \quad$ ...(iii)

From (i), (ii) and (iii), we get

$A(\text{adj } A) = (\text{adj } A)A = |A| I_3$.

**34.** Given, $6A^{-1} = A^2 + cA + dI$

$\Rightarrow \quad 6A^{-1}\cdot A = A^2A + cA\cdot A + dI\cdot A \Rightarrow 6I = A^3 + cA^2 + dA$

$\Rightarrow \quad A^3 + cA^2 + dA - 6I = O \qquad \ldots(i)$

$$\text{Now, } A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -11 & 19 \\ 0 & -38 & 46 \end{bmatrix}$$

$\therefore$ From equation (i), we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -11 & 19 \\ 0 & -38 & 46 \end{bmatrix} + c\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} + d\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} - 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 1+c+d-6 & 0 & 0 \\ 0 & -11-c+d-6 & 19+5c+d \\ 0 & -38-10c-2d & 46+14c+4d-6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad 1 + c + d - 6 = 0, -11 - c + d - 6 = 0$

$\Rightarrow \quad c + d = 5, -c + d = 17$

$\Rightarrow \quad c = -6, d = 11$

**35.** We have, $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2}\left\{\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{bmatrix} = \begin{bmatrix} 2 & \frac{11}{2} & \frac{-5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 4 \end{bmatrix},$$

which is a symmetric matrix.

$$\text{Also, } Q = \frac{1}{2}(A - A') = \frac{1}{2}\left\{\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & \frac{-7}{2} & 0 \end{bmatrix},$$

which is a skew-symmetric matrix.

$$\text{Now, } A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = P + Q$$

$$= \begin{bmatrix} 2 & \frac{11}{2} & \frac{-5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 4 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & \frac{-7}{2} & 0 \end{bmatrix}$$

Hence, $A$ is represented as sum of symmetric and skew symmetric matrix.

**OR**

$A$ purchases 144 notebooks, 60 pens and 72 pencils, $B$ purchases 120 notebooks, 72 pens and 84 pencils, $C$ purchases 132 notebooks, 156 pens and 96 pencils.

Let $D = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix}$ be the matrix of purchases.

Let $E$ be the price matrix.

$$\therefore \quad E = \begin{bmatrix} 40 \\ 8.50 \\ 3.50 \end{bmatrix}$$

$$\therefore \quad DE = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix}\begin{bmatrix} 40 \\ 8.50 \\ 3.50 \end{bmatrix} = \begin{bmatrix} 5760+510+252 \\ 4800+612+294 \\ 5280+1326+336 \end{bmatrix} = \begin{bmatrix} 6522 \\ 5706 \\ 6942 \end{bmatrix}$$

$\therefore$ Bill of $A$, $B$ and $C$ are ₹ 6522, ₹ 5706 and ₹ 6942 respectively.

**36. (i)** In factory $A$, number of units of types I, II and III for men are 70, 80, 90 respectively and for women number of units of types I, II and III are 90, 60, 80 respectively.

$$\therefore \quad R = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{array}{c} \text{men} \quad \text{women} \\ \begin{bmatrix} 70 & 90 \\ 80 & 60 \\ 90 & 80 \end{bmatrix} \end{array}$$

**(ii)** In factory $B$, number of units of types I, II and III for men are 80, 60, 70 respectively and for women number of units of types I, II and III are 40, 65, 70 respectively.

$$\therefore\ U = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{array}{c} \begin{array}{cc}\text{men} & \text{women}\end{array} \\ \begin{bmatrix} 80 & 40 \\ 60 & 65 \\ 70 & 70 \end{bmatrix}\end{array}$$

**(iii)** Let $X$ be the matrix that represents the number of units of each type produced by factory $A$ for men, and $Y$ be the matrix that represents the number of units of each type produced by factory $B$ for men.

Then, $X = \begin{array}{c}\begin{array}{ccc}\text{I} & \text{II} & \text{III}\end{array} \\ [70 \quad 80 \quad 90]\end{array}$ and $Y = \begin{array}{c}\begin{array}{ccc}\text{I} & \text{II} & \text{III}\end{array} \\ [80 \quad 60 \quad 70]\end{array}$

Now, required matrix $= X + Y = [70\ \ 80\ \ 90] + [80\ \ 60\ \ 70]$
$= [150\ \ 140\ \ 160]$

**OR**

Required matrix $= [90\ \ 60\ \ 80] + [40\ \ 65\ \ 70]$
$= [130\ \ 125\ \ 150]$

**37. (i)** $\Delta = \frac{1}{2}\begin{vmatrix} -6 & 2 & 1 \\ 4 & -6 & 1 \\ 2 & 6 & 1 \end{vmatrix}$

$= \frac{1}{2}|-6(-6-6)-2(4-2)+1(24+12)|$

$= \frac{1}{2}|-6(-12)-2(2)+36| = \frac{1}{2}|72-4+36|$

$= \frac{1}{2}|104|$ sq. units $= 52$ sq. units

**(ii)** Given points are collinear.

$\therefore\ \frac{1}{2}\begin{vmatrix} 4 & -6 & 1 \\ k & -2 & 1 \\ 0 & 8 & 1 \end{vmatrix} = 0 \Rightarrow -8(4-k) + 1(-8+6k) = 0$

$\Rightarrow\ -32 + 8k - 8 + 6k = 0 \Rightarrow 14k = 40$

$\Rightarrow\ k = \frac{40}{14} = \frac{20}{7},\ \therefore\ 7k = \frac{20}{7}\times 7 = 20$

**(iii)** Area of $\Delta PQR = 4$ sq. units

$\Rightarrow\ \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 4$

$\Rightarrow\ 2(4-0) - 3(3-k) + 1(0-4k) = \pm 8$

$\Rightarrow\ 8 - 9 + 3k - 4k = \pm 8$

$\Rightarrow\ -1 - k = \pm 8 \Rightarrow k = -9, 7$

**OR**

Let $P(x, y)$ be any points on the line joining $A(2, 4)$ and $B(4, 8)$.

Then area of $\Delta APB = 0$

$\Rightarrow\ \frac{1}{2}\begin{vmatrix} 2 & 4 & 1 \\ 4 & 8 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow\ 2(8-y) - 4(4-x) + 1(4y-8x) = 0$

$\Rightarrow\ 16 - 2y - 16 + 4x + 4y - 8x = 0$

$\Rightarrow\ 2y - 4x = 0, \qquad \therefore\ y = 2x$

**38. (i)** We have, $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ Let $X_1 = \begin{bmatrix} a \\ b \end{bmatrix}$

Given, $AX_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\Rightarrow\ \begin{bmatrix} 2a \\ a+2b \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

On comparing $a = 1, b = 0$

Let, $X_2 = \begin{bmatrix} c \\ d \end{bmatrix}$

Also, $AX_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2c \\ c+2d \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

On comparing, $c = 1, d = 1$

$\therefore\ X_1 + X_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

**(ii)** $X = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$Z = \begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} = [19]$

$\therefore\ |Z| = 19$

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## SOLUTIONS TO JUNE 2023 QUIZ CLUB

| | |
|---|---|
| **1.** ω | **11.** 5/3 |
| **2.** –6/5 | **12.** 5.4 |
| **3.** 20/21 | **13.** $\frac{\sqrt{5}-1}{2}$ |
| **4.** 1/66 | **14.** –10/7 |
| **5.** 201 | **15.** 5 |
| **6.** 3 | **16.** π/4 |
| **7.** 0 | **17.** 14/3 |
| **8.** $2e^2$ | **18.** 1/4 |
| **9.** $\left(-\infty, \frac{1}{2}\right]$ | **19.** 12 |
| **10.** 3 | **20.** ϕ |

***Winner** : Ashish Chowdhary*

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