TCA_genetic_correlation_check

September 2, 2019

```
In [1]: library(glmnet)
        library(matrixStats)
        library(abind)
        library(pracma)
        library(matrixcalc)
        library(TCA)
        library(MCMCpack)
        library(ggplot2)
Loading required package: Matrix
Loading required package: foreach
Loaded glmnet 2.0-18
Attaching package: pracma
The following objects are masked from package:Matrix:
    expm, lu, tril, triu
Loading required package: coda
Loading required package: MASS
## Markov Chain Monte Carlo Package (MCMCpack)
## Copyright (C) 2003-2019 Andrew D. Martin, Kevin M. Quinn, and Jong Hee Park
## Support provided by the U.S. National Science Foundation
## (Grants SES-0350646 and SES-0350613)
Attaching package: MCMCpack
The following object is masked from package:matrixcalc:
    vech
```

The following object is masked from package:pracma:

```
In [246]: summary_statistics <- function(mdl,train_X,test_X,train_c1,test_c1,</pre>
                                          train_G,test_G,train_Z,test_Z,beta,
                                          gamma_c1,gamma_c2,model=1){
              # TCA estimation & parameters
              Z_hat_tca = tensor(train_G,mdl,log_file=NULL,verbose=FALSE,debug=FALSE)
              rmse_tca = t(do.call(rbind,lapply(1:length(Z_hat_tca),
                                               function(x) sqrt(sum((Z_hat_tca[[x]]-train_Z[,x])^
              Z_hat_cor_tca = sapply(1:length(Z_hat_tca), function(x) cor(unlist(Z_hat_tca[x]),t
              cell_type = ncol(mdl$W)
              if(model==1){
                  tca_beta = t(matrix(mdl$gammas_hat[,grepl("_SNP",colnames(mdl$gammas_hat))],nc
                  beta_hat_cor_tca = lapply(1:cell_type,function(x)
                       cor(beta[x,],mdl$gammas_hat[,grepl(paste("Cell_type",x,".G",sep=''),colnam
              }else{ # model 2 does not predict beta for snps
                  tca_beta = NULL
                  beta_hat_cor_tca = NULL
              gamma_hat_cor_tca = cor(array(gamma_c1),
                                   mdl$gammas_hat[,grepl('male|smok|age|nerd|self', colnames(mdl$
              if(model==1){
                  pred = cbind(train_X,train_c1)
                  test_pred = cbind(test_X,test_c1)
              }else{ # model 2 does not use train_X/test_X as C1
                  pred = train_c1
                  test\_pred = test\_c1
              }
              # TCA parameter direct estimation <---> C1 & X
              Z_hat_train_second_eq_tca = sapply(1:cell_type,function(x) pred %*% mdl$gammas_hat
              Z_hat_train_second_eq_cor_tca = diag(cor(train_Z,Z_hat_train_second_eq_tca))
              Z_hat_test_second_eq_tca = sapply(1:cell_type,function(x) test_pred %*% mdl$gammas
              Z_hat_test_second_eq_cor_tca = diag(cor(test_Z,Z_hat_test_second_eq_tca))
              # Lasso bulk data regression <---> X only
              glmnet.mdl.X.cv <- cv.glmnet(x=train_X,y=t(train_G),nfolds=5)</pre>
              glmnet.mdl.X <- glmnet(x=train_X,y=t(train_G),lambda=glmnet.mdl.X.cv$lambda.min)</pre>
              beta_full_X_bulk <- as.numeric(glmnet.mdl.X$beta)</pre>
              # extract non zero predictors and recorrelate
              predictors.X <- colnames(train_X)[which(beta_full_X_bulk!=0)]</pre>
              beta_X <- as.matrix(c(glmnet.mdl.X$a0,as.matrix(glmnet.mdl.X$beta[predictors.X,]))</pre>
              bias_one <- numeric(nrow(train_X))+1</pre>
              G_hat_train_lasso <- cbind(bias_one,train_X[,predictors.X]) %*% beta_X</pre>
              G_hat_train_cor_bulk <- cor(t(train_G),G_hat_train_lasso)</pre>
              G_hat_test_lasso <- cbind(numeric(nrow(test_X))+1,test_X[,predictors.X]) %*% beta_</pre>
```

```
G_hat_test_cor_bulk <- cor(t(test_G),G_hat_test_lasso)</pre>
# cell type specific lasso
corrs = numeric(ncol(mdl$W))
corrs.real = numeric(ncol(mdl$W))
corrs.beta = numeric(ncol(mdl$W))
corrs.test.real = numeric(ncol(mdl$W))
dev_ratio = numeric(ncol(mdl$W))
rmse_lasso = numeric(ncol(mdl$W))
beta_full_cell = matrix(0,nrow=ncol(mdl$W),ncol=dim(train_X)[2])
Z_hat_test_lasso = matrix(0,nrow=nrow(test_Z),ncol=ncol(test_Z))
for (h in 1:ncol(mdl$W)){
    glmnet.mdl.cv <- cv.glmnet(x=train_X,y=Z_hat_tca[[h]],standardize=FALSE,alpha=
    rmse_lasso[h] = sqrt(glmnet.mdl.cv$cvm[glmnet.mdl.cv$lambda == glmnet.mdl.cv$l
    glmnet.mdl <- glmnet(x=train_X,y=Z_hat_tca[[h]],standardize=FALSE,alpha=1,lamb</pre>
    dev_ratio[h] <- glmnet.mdl$dev.ratio</pre>
    beta.full <- as.numeric(glmnet.mdl$beta)</pre>
    beta_full_cell[h,] <- beta.full</pre>
    predictors <- colnames(train_X)[which(beta.full != 0)]</pre>
    beta_lasso <- as.matrix(c(glmnet.mdl$a0,as.matrix(glmnet.mdl$beta[predictors,]</pre>
    Z_hat_train_lasso <- cbind(numeric(nrow(train_X))+1,train_X[,predictors]) %*%</pre>
    Z_hat_test_lasso[,h] <- cbind(numeric(nrow(test_X))+1,test_X[,predictors]) %*%</pre>
    Z_hat_test_lasso[,h] = signif(Z_hat_test_lasso[,h], digits = 5)
    if(sum(beta_lasso)==0 | sd(Z_hat_train_lasso)==0){
        # model 2 lasso forces SNPs effect to become zero.
        # If there were no predictors, then cor would be NaN. Which also means
        # no snps is correlated with Z. cor just set to 0.
        corrs[h] = 0
        corrs.real[h] = 0
        corrs.beta[h] = 0
        corrs.test.real[h] = 0
    }else{
        corrs[h] <- cor(t(Z_hat_tca[[h]]),Z_hat_train_lasso)</pre>
        corrs.real[h] <- cor(train_Z[,h],Z_hat_train_lasso)</pre>
        corrs.beta[h] <- cor(beta[h,],beta.full)</pre>
        corrs.test.real[h] <- cor(test_Z[,h],Z_hat_test_lasso[,h])</pre>
    }
}
# Precision & Recall for lasso
Binary_True=(beta!=0)
Binary_Pred_Lasso=(beta_full_cell!=0)
TP = do.call(rbind,lapply(1:cell_type,function(x) sum(as.integer(Binary_True[x,] &B
FP = do.call(rbind,lapply(1:cell_type,function(x) sum(as.integer((!Binary_True[x,]
FN = do.call(rbind, lapply(1:cell_type, function(x) sum(as.integer(Binary_True[x,]&(
precision_lasso = TP/(TP+FP)
recall_{lasso} = TP/(TP+FN)
```

```
return(list('Z_hat_test_lasso'=Z_hat_test_lasso,'Z_hat_tca'=Z_hat_tca,'Z_hat_cor_t
                         'Z_hat_train_second_eq_cor_tca'=Z_hat_train_second_eq_cor_tca,'Z_hat_te
                         'G_hat_train_cor_bulk'=G_hat_train_cor_bulk,'G_hat_test_cor_bulk'=G_hat
                         'cor_lasso_tca'=corrs,'cor_lasso_real_train'=corrs.real,'cor_lasso_real
                         'cor_beta_lasso_real'=corrs.beta,'beta_hat_lasso'=beta_full_cell,'dev_r
                         'precision_lasso'=precision_lasso,'recall_lasso'=recall_lasso,'rmse_las
          }
In [247]: generate_params <- function(cell_her=TRUE,gene_cor=FALSE,bulk_her=FALSE,seed=1,N=5000,
              set.seed(1)
              if(length(herr_arr)!=0){
                  M = length(herr_arr)
              }else{
                  M = M
              K = 4
              N = N
              pc_num = 2
              # cis_snps_nums = floor(runif(M, min = 150, max = 400))
              cis_snps_nums = array(D,M)
              sigma_g = 0.01
              sigma_z = 0.1
              if(gene_cor){ # varing genetic correlation across genes
                  corr_seq = seq(from=0,to=pslab,length.out=M)
              }else{
                  corr_seq = seq(from=0,to=0,length.out=M)
              pslab = matrix(pslab,nrow=M,ncol=K)
              corr_matrix = lapply(1:M,function(x) matrix(corr_seq[x],nrow=K,ncol=K))
              for(i in 1:M){
                  diag(corr_matrix[[i]])=1
              }
              if(cell_her){ # varing heribility across genes
                  heritibility_cell_specific = do.call(rbind,lapply(1:K,function(x) herr_arr))
              }else{
                  heritibility_cell_specific = matrix(her,nrow=K,ncol=M)
              }
              if(bulk_her){  # varing bulk heritibility across genes
                  heritibility_bulk = seq(from=0.1,to=her_bulk,length.out=M)
              }else{
                  heritibility_bulk = array(her_bulk,M)
              MAF = lapply(1:length(cis_snps_nums),function(x) runif(cis_snps_nums[x],min=0.1,magental)
```

```
id_name = sapply(1:N, function(x) paste('SAMPLE',toString(x),sep = ''))
cell_type_name=sapply(1:K, function(x) paste('Cell_type',x,sep = ''))
### generate C1 and C2
male = matrix(rbinom(N, 1, 0.5), nrow=N)
smoking = matrix(rbinom(N, 2, 0.2),nrow=N)
smoking = (smoking - min(smoking))/(max(smoking))
age = matrix(sapply(sapply(rnorm(N,50,20), function(x) floor(x)), function(x) if
age = (age-min(age))/(max(age)-min(age))
nerdiness = matrix(rbinom(N,2,0.2),nrow=N)
nerdiness = (nerdiness-min(nerdiness))/max(nerdiness)
selfishness = matrix(rbinom(N,2,0.2),nrow=N)
selfishness = (selfishness-min(selfishness))/max(selfishness)
c1 = scale(cbind(male, smoking, age,nerdiness,selfishness))
c2 = scale(matrix(rnorm(N*pc_num), nrow=N))
rownames(c1) = id_name
colnames(c1) = c('male', 'smoking', 'age', 'nerdiness', 'selfishness')
rownames(c2) = id_name
colnames(c2) = sapply(1:pc_num, function(x) paste('PC',toString(x),sep = ''))
p1 = \dim(c1)[2]
sigma_gamma = sqrt((1-heritibility_cell_specific[1,]-sigma_z^2)/p1)
var_beta = heritibility_cell_specific[1,]*(p1*sigma_gamma^2+sigma_z^2)/(1-heritibi
# cell type proportion, from the real data estimated
\#W\_alpha = 50.4500920752719
if(K <= length(W_alpha)){</pre>
   W = rdirichlet(N, W_alpha[1:K])
}else{ #more cell type prop
   W = rdirichlet(N, runif(K, 0, 1))
colnames(W) = cell_type_name
rownames(W) = id_name
alpha = W_alpha
alpha_0 = sum(alpha)
alpha_tilde = alpha/alpha_0
m2_alpha = alpha_tilde %*% t(alpha_tilde)*alpha_0/(alpha_0+1)
diag(m2_alpha) = alpha_tilde*(1-alpha_tilde)/(alpha_0+1)+alpha_tilde^2
return(list(
    'M'=M,'K'=K,'N'=N,'pc_num'=pc_num,'D'=D,'id_name'=id_name,'cell_name'=cell_typ
    'her'=heritibility_cell_specific, 'her_bulk'=heritibility_bulk, 'pslab'=pslab,
    'MAF'=MAF,'c1'=c1,'c2'=c2,'W'=W,'sigma_gamma'=sigma_gamma,'sigma_g'=sigma_g,'m
    'sigma_z'=sigma_z,'gene_cor'=corr_seq,'beta_cor'=corr_matrix,'var_beta'=var_be
```

```
}
In [248]: one_gene <- function(par,g,seed=1){</pre>
              set.seed(seed)
              cell_type = par$K
              n_snps = par$D
              her = par$her[,g]
              cor = par$beta_cor[[g]]
              pslab = par$pslab[g,]
              var_beta = array(par$var_beta[g],par$K)
              maf = par$MAF[[g]]
              dummy_var = 10
              var_matrix=sqrt(var_beta%*%t(var_beta))
              Sigma_beta = cor*var_matrix/(1-pslab)^2
              diag(Sigma_beta) = diag(Sigma_beta) *(1-pslab)
              beta_ = mvrnorm(n_snps,mu=rep(0,par$K),Sigma=Sigma_beta,tol=1e-4,empirical=TRUE)
              rmask = do.call(rbind,lapply(1:cell_type,function(x) rbinom(n_snps,1,1-pslab[x])))
              beta = t(beta_)*rmask
              X = t(do.call(rbind,lapply(1:n_snps,function(x) rbinom(par$N,2,maf[x]))))
              colnames(X) = sapply(1:n_snps, function(x) paste('G',g,'_SNP',toString(x),sep = ''
              rownames(X) = par$id_name
              X = scale(X)
              # enforce variance of samples by adjusting sd of beta
              xbeta_var = n_snps*var_beta[1]
              beta = do.call(rbind,
                                lapply(1:cell_type,function(x) beta[x,]*(sqrt(xbeta_var)/sd(X %*
              mu_z = X\%*\%t(beta)
              epsilon_z = array(rnorm(par$N*cell_type,mean=0,sd=par$sigma_z), c(par$N,cell_type)
              epsilon_z = apply(epsilon_z,2,function(x) x*par$sigma_z/sd(x))
              Sigma_gamma_c1 = matrix(0,nrow=cell_type,ncol=cell_type)
              diag(Sigma_gamma_c1) = par$sigma_gamma[g]^2
              gamma_c1 = mvrnorm(dim(par$c1)[2],mu=rep(0,cell_type),Sigma=Sigma_gamma_c1,tol=1e-
              \# gamma_c1 = do.call(rbind, lapply(1:cell_type, function(x) rnorm(dim(par$c1)[2], median)
              # enforce variance of samples by adjusting sd of gamma_c1
              # enforce the c1_gamma variance
              c1gamma_var = dim(par$c1)[2]*(par$sigma_gamma[g])^2
              gamma_c1 = do.call(rbind,
                                 lapply(1:cell_type,function(x) gamma_c1[,x]*(sqrt(c1gamma_var)/
              c1_gamma = par$c1 %*% t(gamma_c1)
              Z = mu_z + epsilon_z + c1_gamma#
              rownames(Z) = par$id_name
              colnames(Z) = par$cell_name
```

```
epsilon_G = epsilon_G*par$sigma_g/sd(epsilon_G)
              # use bulk level heritability to calculate sigma_gamma
              bulk_her_nom = sum(hadamard.prod(par$m2_alpha,var(mu_z)))
              bulk_her_c1_gamma = sum(hadamard.prod(par$m2_alpha,var(c1_gamma)))
              bulk_her_epsilon_z = sum(hadamard.prod(par$m2_alpha,var(epsilon_z)))
              bulk_her_z = sum(hadamard.prod(par$m2_alpha,var(Z)))
              bulk_her_denom = sum(hadamard.prod(par$m2_alpha,var(Z)))+var(epsilon_G)
              bulk_her_val = par$her_bulk[g]*her[1]
              sd_gamma_c2 = sqrt((bulk_her_nom/bulk_her_val-bulk_her_denom)/par$pc_num)
              gamma_c2 = matrix(rnorm(par$pc_num,mean=0,sd=sd_gamma_c2),nrow=par$pc_num,ncol=1)
              c2gamma_var = par$pc_num * (sd_gamma_c2)^2
              gamma_c2 = gamma_c2*(sqrt(c2gamma_var)/sd(par$c2\%*\%gamma_c2))
              c2_{gamma} = par$c2 %*% gamma_c2
              G = t(G + epsilon_G) # + c2_gamma)
              real_denom = var(t(G))
              colnames(G) <- par$id_name</pre>
              rownames(G) <- paste('gene',g,sep='_')
              G = as.data.frame(G)
              real_bulk_her = bulk_her_nom/(bulk_her_denom+var(c2_gamma))
              return(list('X'=X,'beta'=beta,'gamma_c1'=gamma_c1,'bulk_her'=real_bulk_her,
                         'gamma_c2'=gamma_c2,'Z'=Z,'G'=G,'epsilon_z'=epsilon_z))
          #data = one_gene(par=params,1)
0.0.1 Plotting Begins for Genetic Correlation Check
In [249]: params = generate_params(cell_her=FALSE,gene_cor=TRUE,bulk_her=FALSE,
                                    seed=1, N=5000, M=10,
                                   D=250,pslab=0.5,her=0.95,
                                   her_bulk=0.6)
          data = lapply(1:params$M,function(x) one_gene(par=params,x))
In [250]: params$gene_cor = t(do.call(rbind,lapply(1:length(data),function(x) params$beta_cor[[x
          temp = array(0,length(data))
          for(i in 1:length(data)){
              cor_act = cor(data[[i]]$X %*% t(data[[i]]$beta))
              diag(cor_act) = params$gene_cor[i]
              temp[i] = signif(mean(cor_act),3)
              print(paste(temp[i], signif(params$gene_cor[i],3)))
```

Generate Gene expressions

epsilon_G = rnorm(par\$N,mean=0,sd=par\$sigma_g)

G = rowSums((par\$W)*Z)

```
}
    params$gene_cor_real = temp

[1] "0.00328 0"
[1] "0.0759 0.0556"
[1] "0.119 0.111"
[1] "0.19 0.167"
[1] "0.228 0.222"
[1] "0.301 0.278"
[1] "0.34 0.333"
[1] "0.404 0.389"
[1] "0.452 0.444"
[1] "0.498 0.5"
```

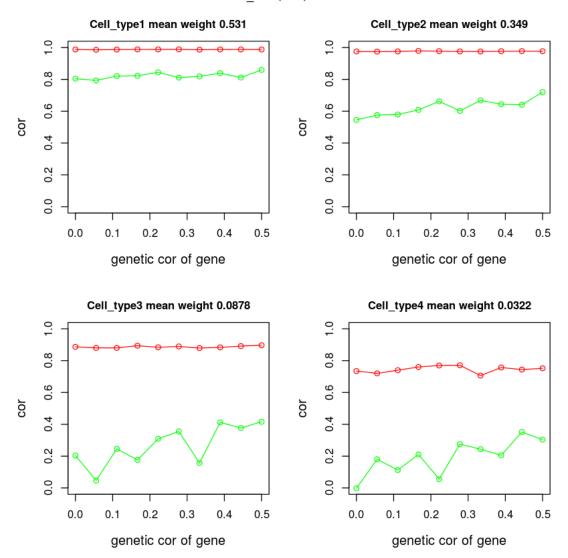
		Cell_type1	Cell_type	e2 Cell_t	ype3 Cell_type	4دِ
1. A matrix: 4 Œ 4 of type dbl	Cell_type1	1.0000000000	0 -0.000331	7917 -0.002	13270 0.0121354	15
	Cell_type2	-0.000331791	1.0000000	0.0295	66018 0.0345388	30
	Cell_type3	-0.002132700	0.0295601	1.0000	-0.068383	77
	Cell_type4	0.012135448	0.0345387	7986 -0.068	38377 1.0000000)0
		l				
_		Cell_type1	Cell_type2	Cell_type3	Cell_type4	
2. A matrix: 4 Œ 4 of type dbl	Cell_type1	1.00000000	0.04009888	0.04259027	0.08975178	
	Cell_type2	0.04009888	1.00000000	0.02878084	0.14921212	
	Cell_type3	0.04259027	0.02878084	1.00000000	0.13229550	
	Cell_type4	0.08975178	0.14921212	0.13229550	1.00000000	
			G 11	G 11	G 11	
3. A matrix: 4 Œ 4 of type dbl		Cell_type1	Cell_type2	Cell_type3	Cell_type4	
	Cell_type1	1.00000000	0.08992166	0.08861244	0.1286508	
	Cell_type2	0.08992166	1.00000000	0.10006931	0.1446577	
	Cell_type3	0.08861244	0.10006931	1.00000000	0.1419878	
	Cell_type4	0.12865083	0.14465772	0.14198778	1.0000000	
			C 11 . 2	C 11 . 2	C 11 4 4	
4. A matrix: 4 Œ 4 of type dbl	O 11	Cell_type1	Cell_type2	Cell_type3	Cell_type4	
	Cell_type1	1.0000000	0.1302058	0.1561839	0.2015018	
	Cell_type2	0.1302058	1.0000000	0.1700036	0.2387678	
	Cell_type3	0.1561839	0.1700036	1.0000000	0.2306276	
	Cell_type4	0.2015018	0.2387678	0.2306276	1.0000000	
		Coll tymo1	Coll trmo?	Call tyma?	Call trans	
5. A matrix: 4 Œ 4 of type dbl	Call true al	Cell_type1	Cell_type2	Cell_type3	Cell_type4	
	Cell_type1	1.0000000	0.1700356	0.2334695	0.2257214	
	Cell_type2	0.1700356	1.0000000	0.2349993	0.2146365	
	Cell_type3	0.2334695	0.2349993	1.0000000	0.2207478	
	Cell_type4	0.2257214	0.2146365	0.2207478	1.0000000	

		Cell_type1	Cell_type2	Cell_type3	Cell_type4
6. A matrix: 4 Œ 4 of type dbl	Cell_type1	1.0000000	0.1921008	0.2631345	0.3168799
	Cell_type2	0.1921008	1.0000000	0.2844620	0.3489239
	Cell_type3	0.2631345	0.2844620	1.0000000	0.3522017
	Cell_type4	0.3168799	0.3489239	0.3522017	1.0000000
	71				
_		Cell_type1	Cell_type2	Cell_type3	Cell_type4
	Cell_type1	1.0000000	0.2264679	0.2873016	0.3773279
	Cell_type2	0.2264679	1.0000000	0.2952850	0.3971018
	Cell_type3	0.2873016	0.2952850	1.0000000	0.3729353
	Cell_type4	0.3773279	0.3971018	0.3729353	1.0000000
	ı	Cell_type1	Cell_type2	Cell_type3	Cell_type4
_	Cell_type1	1.0000000	0.2853901	0.3513460	0.4337017
		0.2853901	1.0000000	0.3862238	0.4324173
8. A matrix: 4 Œ 4 of type dbl	Cell_type2	0.2653901	0.3862238	1.0000000	0.4324173
	Cell_type3 Cell_type4	0.3313400	0.3802238	0.4355035	1.0000000
	Cen_type4	0.433/01/	0.4324173	0.4333033	1.0000000
		Cell_type1	Cell_type2	Cell_type3	Cell_type4
	Cell_type1	1.0000000	0.3183749	0.3596084	0.5074710
9. A matrix: 4 Œ 4 of type dbl	Cell_type2	0.3183749	1.0000000	0.4544970	0.4856864
	Cell_type3	0.3596084	0.4544970	1.0000000	0.4722043
	Cell_type4	0.5074710	0.4856864	0.4722043	1.0000000
		Cell_type1	Cell_type2	Cell_type3	Cell_type4
_	Cell_type1	1.0000000	0.3515752	0.4180559	0.5512974
	Cell_type2	0.3515752	1.0000000	0.4787724	0.5215446
5 1	Cell_type3	0.4180559	0.4787724	1.0000000	0.5029956
	Cell_type4	0.5512974	0.5215446	0.5029956	1.0000000
	cen_type1	0.0012)/1	0.0210110	0.002	1.000000
	1.000000000	0.0022898	342 -0.00186	4556 0.0221	1653
1 A martinian A CE A of terms of del	0.002289842	2 1.0000000	000 0.039548	3699 0.0397	8192
1. A matrix: 4 Œ 4 of type dbl	-0.00186455	66 0.0395486	599 1.000000	0000 -0.075	63841
	0.022116530	0.0397819	920 -0.07563	8410 1.0000	00000
	1.00000000	0.04113077	0.03897940	0.1021882	
	0.04113077	1.00000000	0.03077740	0.1021002	
2. A matrix: 4 Œ 4 of type dbl	0.03897940	0.02489338	1.00000000	0.1300722	
	0.10218820	0.02407330	0.13220403	1.0000000	
	0.10210020	0.13007217	0.13220403	1.0000000	
	1.00000000	0.1027956	0.09144727	0.1288701	
3. A matrix: 4 Œ 4 of type dbl	0.10279565	1.0000000	0.10778292	0.1477207	
5. A matrix. 4 th 4 or type dor	0.09144727	0.1077829	1.00000000	0.1499510	
	0.12887007	0.1477207	0.14995103	1.0000000	
	1.0000000	0.1426765	0.1626129 0	.2087614	
	0.1426765			.2502022	
4. A matrix: 4 Œ 4 of type dbl	0.1420703			.2408941	
	0.1020123			.0000000	
	0.200/014	0.2002022	0. <u>~</u> 100/T1 1	.000000	

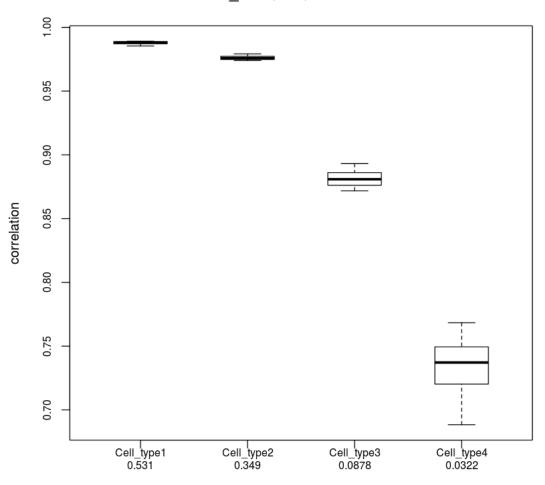
```
1.0000000
                                           0.1738589
                                                       0.2494111
                                                                  0.2449476
                                0.1738589
                                           1.0000000
                                                       0.2505405
                                                                  0.2241523
  5. A matrix: 4 Œ 4 of type dbl
                                0.2494111
                                            0.2505405
                                                       1.0000000
                                                                  0.2368979
                                0.2449476
                                           0.2241523
                                                       0.2368979
                                                                  1.0000000
                                1.0000000
                                           0.2078771
                                                       0.2763432
                                                                  0.3295500
                                            1.0000000
                                0.2078771
                                                       0.3038295
                                                                  0.3624918
  6. A matrix: 4 Œ 4 of type dbl
                                0.2763432
                                           0.3038295
                                                       1.0000000
                                                                  0.3697754
                                0.3295500
                                           0.3624918
                                                       0.3697754
                                                                  1.0000000
                                1.0000000
                                           0.2402361
                                                       0.2977960
                                                                  0.4027493
                                0.2402361
                                            1.0000000
                                                       0.3076504
                                                                  0.4202813
  7. A matrix: 4 Œ 4 of type dbl
                                0.2977960
                                           0.3076504
                                                       1.0000000
                                                                  0.3850410
                                0.4027493
                                           0.4202813
                                                       0.3850410
                                                                  1.0000000
                                1.0000000
                                           0.3049494
                                                       0.3721249
                                                                  0.4518753
                                0.3049494
                                           1.0000000
                                                       0.4089703
                                                                  0.4544544
  8. A matrix: 4 Œ 4 of type dbl
                                0.3721249
                                           0.4089703
                                                       1.0000000
                                                                  0.4583280
                                                       0.4583280
                                0.4518753
                                           0.4544544
                                                                  1.0000000
                                1.0000000
                                           0.3335088
                                                       0.3717337
                                                                  0.5350763
                                0.3335088
                                           1.0000000
                                                       0.4771118
                                                                  0.5148450
  9. A matrix: 4 Œ 4 of type dbl
                                0.3717337
                                           0.4771118
                                                       1.0000000
                                                                  0.4939253
                                0.5350763
                                           0.5148450
                                                       0.4939253
                                                                  1.0000000
                                1.0000000
                                            0.3711668
                                                       0.4469332
                                                                  0.5756336
                                0.3711668
                                            1.0000000
                                                       0.5082334
                                                                  0.5446554
 10. A matrix: 4 Œ 4 of type dbl
                                0.4469332
                                           0.5082334
                                                       1.0000000
                                                                  0.5336249
                                0.5756336
                                           0.5446554
                                                       0.5336249
                                                                  1.0000000
In [252]: # do TCA-TWAS for every gene
          prop = 0.7
           t_prop = floor(prop*params$N)
          li = list()
           for(g in 1:length(data)){
               print(paste('gene',g,sep='_'))
               train_X = (data[[g]] X)[1:t_prop,]
               test_X = data[[g]]$X[(t_prop+1):params$N,]
               train_W = params$W[1:t_prop,]
               test_W = params$W[(t_prop+1):params$N,]
               train_G = data[[g]]$G[,1:t_prop]
               test_G = data[[g]]$G[,(t_prop+1):params$N]
               train_c1 = params$c1[1:t_prop,]
               test_c1 = params$c1[(t_prop+1):params$N,]
               train_c2 = params$c2[1:t_prop,]
               test_c2 = params$c2[(t_prop+1):params$N,]
               train_Z = data[[g]]$Z[1:t_prop,]
               test_Z = data[[g]]$Z[(t_prop+1):params$N,]
               beta = data[[g]]$beta
               gamma_c1 = data[[g]]$gamma_c1
```

```
gamma_c2 = data[[g]]$gamma_c2
              tca.mdl1 = tca(X=train_G,W=train_W,C1=cbind(train_X,train_c1),verbose=FALSE)
              tca.mdl2 = tca(X=train_G,W=train_W,C1=train_c1,verbose=FALSE)
              tca.mdl1.summary = summary_statistics(mdl=tca.mdl1,train_X=train_X,test_X=test_X,
                                                     train_c1=train_c1,test_c1=test_c1,train_G=tr
                                                     test_G=test_G,train_Z=train_Z,test_Z=test_Z,
                                                     beta=beta,gamma_c1=gamma_c1,gamma_c2=gamma_c
              tca.mdl2.summary = summary_statistics(mdl=tca.mdl2,train_X=train_X,test_X=test_X,
                                                     train_c1=train_c1, test_c1=test_c1, train_G=tr
                                                     test_G=test_G,train_Z=train_Z,test_Z=test_Z,
                                                     beta=beta,gamma_c1=gamma_c1,gamma_c2=gamma_c
              li[[g]] = list('s1'=tca.mdl1.summary,'s2'=tca.mdl2.summary)
          }
[1] "gene_1"
[1] "gene_2"
[1] "gene_3"
[1] "gene_4"
[1] "gene_5"
[1] "gene_6"
[1] "gene_7"
[1] "gene_8"
[1] "gene_9"
[1] "gene_10"
In [253]: mean_weight = signif(apply(params$W,2,mean),digits=3)
          Z_hat_cor_tca1 = do.call(rbind,lapply(1:length(data),function(x) li[[x]]$s1$Z_hat_cor_
          Z_hat_cor_tca2 = do.call(rbind,lapply(1:length(data),function(x) li[[x]]$s2$Z_hat_cor_
          par(mfrow=c(2,2), mar=c(4, 3.8, 4, 2) + 0.1)
          opts=params$cell_name
          for(i in 1:length(params$cell_name)){
              heading = paste(opts[i], 'mean weight', mean_weight[i], sep=' ')
              plot(params$gene_cor,Z_hat_cor_tca1[,i],xlab='genetic cor of gene',
                   ylab='cor',cex.lab=1.2,ylim=c(0,1),yaxs='r',type='o',col='red')
              title(main=heading,font.main=2,cex.main=1,line=1,cex.lab=1)
              lines(params$gene_cor,Z_hat_cor_tca2[,i],col='green',type='o')
          mtext("TCA's Z_hat <Cor> Ground truth train Z", side=3, adj=6, cex=0.9, line=24, font=2)
          par(mfrow=c(2,1),mar=c(0,0,0,0))
          plot(1,type='n',axes=FALSE,xlab='',ylab='')
          legend(x='top',inset=0,legend=c('mdl1:SNPs+C1','mdl2:C1'),
                 col=c('red', 'green'), lwd=2, cex=0.7, horiz=TRUE)
```

TCA's Z_hat <Cor> Ground truth train Z

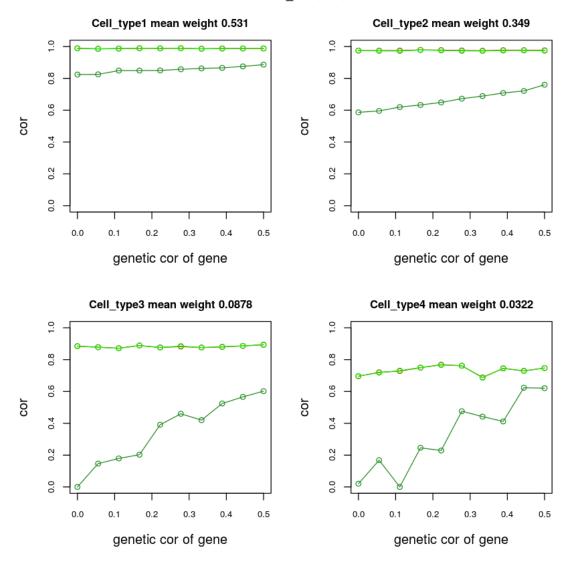


TCA's beta_hat <Cor> Ground truth beta

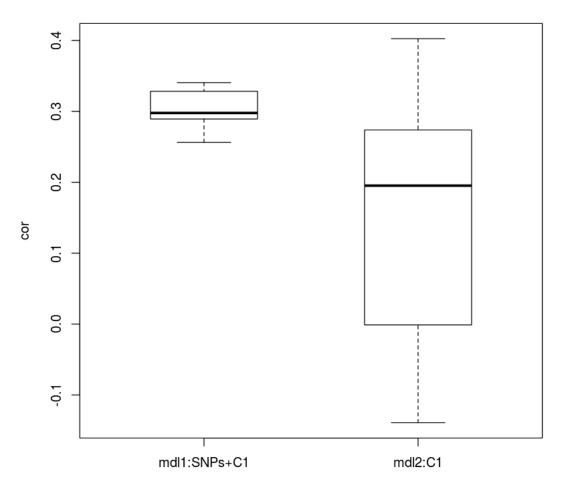


Each dot being correlation of real/estimated beta of length n*1

Estimated beta_hat <Cor> Ground truth beta hat



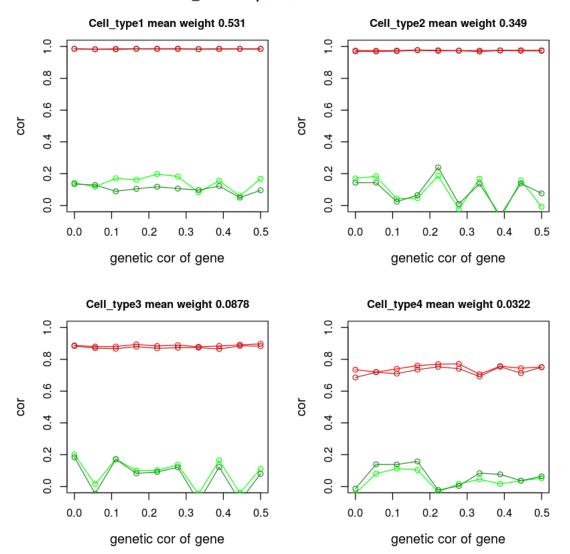
TCA's gamma_hat <Cor> Ground truth gamma_hat



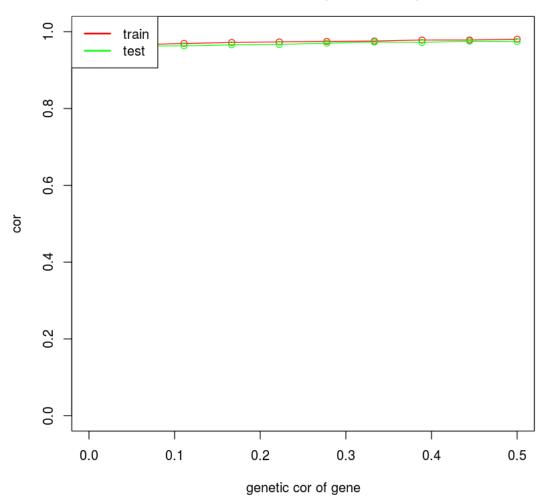
correlation of effect size of smoking/age/gender

```
lines(params$gene_cor,Z_hat_test_seq_eq_cor_tca1[,i],col='firebrick',type='o')
lines(params$gene_cor,Z_hat_test_seq_eq_cor_tca2[,i],col='forestgreen',type='o')
}
mtext("TCA's Z_hat 2nd eq <Cor> Ground truth train Z",side=3,adj=3,cex=0.9,line=24,for
par(mfrow=c(2,1),mar=c(0,0,0,0))
plot(1,type='n',axes=FALSE,xlab='',ylab='')
legend(x='top',inset=0,legend=c('mdl1 train:SNPs+C1','mdl2 train:C1','mdl1 test:SNPs+C1','mdl2 train:C1','mdl1 test:SNPs+C1','mdl2 train:C1','mdl1 test:SNPs+C1','mdl2 train:C1','mdl2 train:C1','m
```

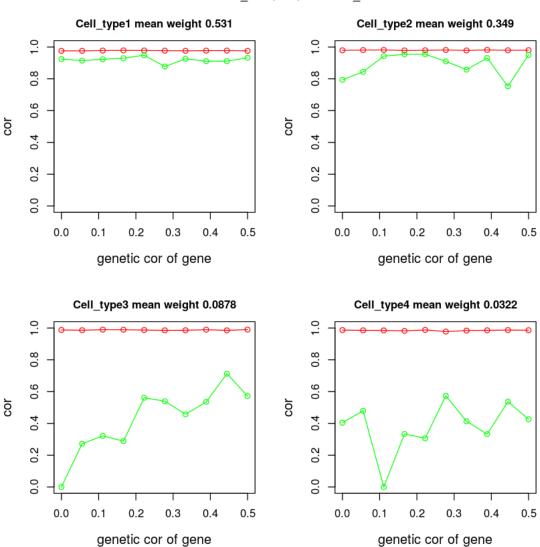
TCA's Z_hat 2nd eq <Cor> Ground truth train Z



Bulk level GE, lasso by SNPs directly

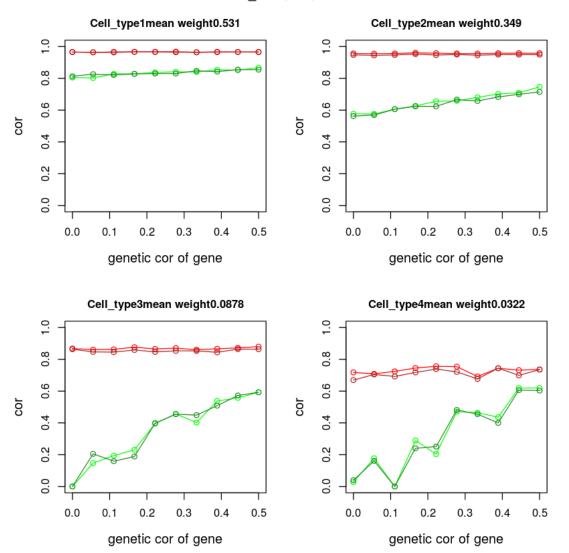


TCA's Z_hat <Cor> Lasso Z_hat



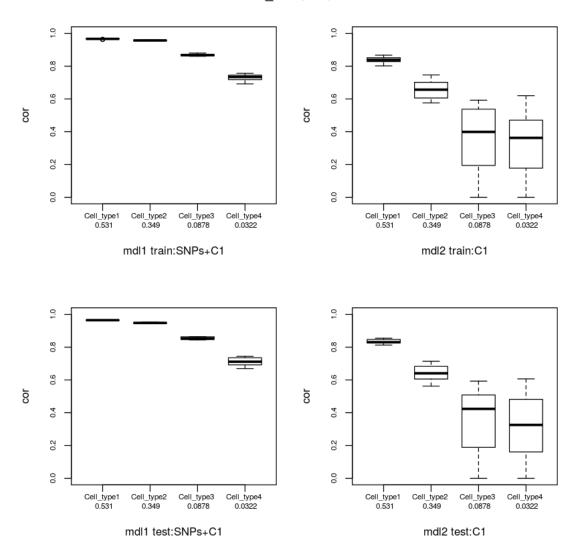
```
ylab='cor',cex.lab=1.2,ylim=c(0,1),yaxs='r',type='o',col='red')
title(main=heading,font.main=2,cex.main=1,line=1,cex.lab=1)
lines(params$gene_cor,cor_lasso_real_train2[,i],col='green',type='o')
lines(params$gene_cor,cor_lasso_real_test1[,i],col='firebrick',type='o')
lines(params$gene_cor,cor_lasso_real_test2[,i],col='forestgreen',type='o')
}
mtext("Lasso Z_hat <Cor> Ground Truth Z",side=3,adj=7,cex=1,line=24,font=2)
par(mfrow=c(2,1),mar=c(0,0,0,0))
plot(1,type='n',axes=FALSE,xlab='',ylab='')
legend(x='top',inset=0,legend=c('mdl1 train:SNPs+C1','mdl2 train:C1','mdl1 test:SNPs+C1',col=c('red','green','firebrick','forestgreen'),lwd=2,cex=0.7,horiz=TRUE)
```

Lasso Z_hat <Cor> Ground Truth Z



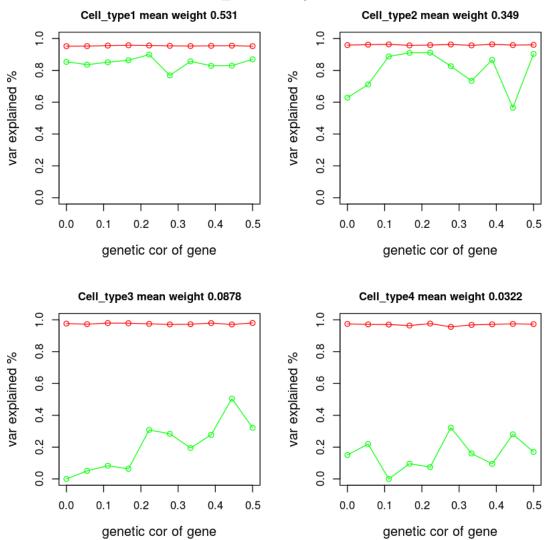
```
In [260]: # zhat_cor = data.frame(heritability=params$gene_cor,cor=cor_lasso_real_test1)
    # colnames(zhat_cor) = c('heritability','cor_celltype1','cor_celltype2','cor_celltype3'
# ggplot(zhat_cor,aes(heritability))+geom_line(
# aes(y=cor_celltype1,color=hue_pal()(4)[1]))+geom_line(
# aes(y=cor_celltype2,color=hue_pal()(4)[2]))+geom_line(
# aes(y=cor_celltype3,color=hue_pal()(4)[3]))+geom_line(
# aes(y=cor_celltype4,color=hue_pal()(4)[4]))+ggtitle(
# "Correlation: TWAS Z_hat and real Z")+theme(
# axis.text=element_text(size=18),
# axis.title=element_text(size=18,face="bold"),
# plot.title = element_text(size=15,face="bold",hjust=0.5))+labs(
# y="Correlation")+scale_color_discrete(name="Cell Type Weights",labels=c(0.08,0.3)
# # ggsave("/home/elessar/Documents/activities/hackathon/urop/UCLA/csst/tca-twas/pipe1
```

Lasso Z_hat <Cor> Ground Truth Z



```
In [262]: lasso_dev_rat1 = do.call(rbind,lapply(1:length(data),function(x) li[[x]]$s1$dev_rat))
          lasso_dev_rat2 = do.call(rbind,lapply(1:length(data),function(x) li[[x]]$s2$dev_rat))
          colnames(lasso_dev_rat1) = paste(params$cell_name,'\n',mean_weight)
          colnames(lasso_dev_rat2) = paste(params$cell_name,'\n',mean_weight)
          par(mfrow=c(2,2), mar=c(4, 3.8, 4, 2) + 0.1)
          opts=params$cell_name
          for(i in 1:params$K){
              heading = paste(opts[i], 'mean weight', mean_weight[i], sep=' ')
              plot(params$gene_cor,lasso_dev_rat1[,i],xlab='genetic cor of gene',
                   ylab='var explained %',cex.lab=1.2,ylim=c(0,1),yaxs='r',type='o',col='red')
              title(main=heading,font.main=2,cex.main=1,line=1,cex.lab=1)
              lines(params$gene_cor,lasso_dev_rat2[,i],col='green',type='o')
          }
          mtext("Lasso Z_hat <dev exp> Ground Truth Z",side=3,adj=3.5,cex=1,line=23.5,font=2)
          par(mfrow=c(2,1),mar=c(0,0,0,0))
          plot(1,type='n',axes=FALSE,xlab='',ylab='')
          legend(x='top',inset=0,legend=c('mdl1 train:SNPs+C1','mdl2 train:C1'),
                 col=c('red','green'),lwd=2,cex=0.7,horiz=TRUE)
```

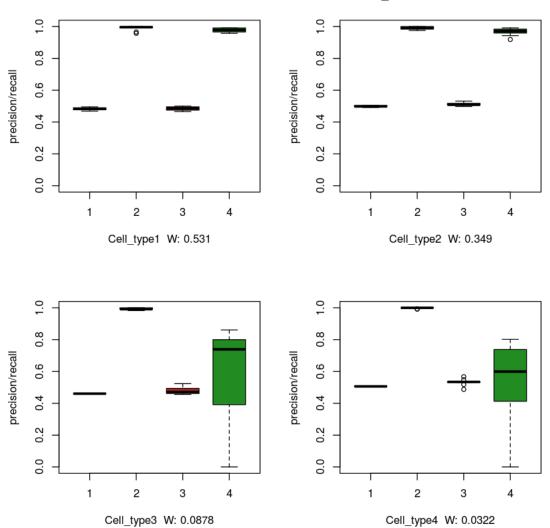
Lasso Z_hat <dev exp> Ground Truth Z



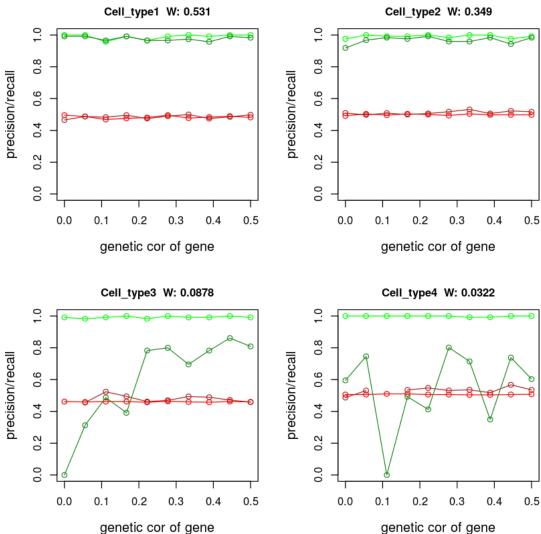
```
# # ggsave("/home/elessar/Documents/activities/hackathon/urop/UCLA/csst/tca-twas/pipel
                     plot = last_plot(), dpi = 300, limitsize = TRUE)
In [264]: beta_precision_lasso1 =matrix(do.call(rbind,lapply(1:length(data),function(x) li[[x]]$
          beta_precision_lasso2 = matrix(do.call(rbind,lapply(1:length(data),function(x) li[[x]]
          beta_recall_lasso1 =matrix(do.call(rbind,lapply(1:length(data),function(x) li[[x]]$s1$
          beta_recall_lasso2 = matrix(do.call(rbind,lapply(1:length(data),function(x) li[[x]]$s2
          prec_rec_frame = lapply(1:params$K,function(x) t(rbind(beta_precision_lasso1[x,],beta_
          par(mfrow=c(2,2), mar=c(4, 3.8, 4, 2) + 0.1)
          opts=paste(params$cell_name,' W:',mean_weight)
          for(i in 1:params$K){
              boxplot(prec_rec_frame[[i]],ylab='precision/recall',
                  xlab=opts[i],boxwex=0.7,ylim=c(0,1),col=c('red','green','firebrick','forestgreen')
          }
          mtext("Precision/Recall for Lasso's beta_hat", side=3, adj=12.5, cex=0.9, line=22.5, font=2
          par(mfrow=c(2,1),mar=c(0,0,0,0))
          plot(1,type='n',axes=FALSE,xlab='',ylab='')
          legend(x='top',inset=0,legend=c('precision mdl1:SNPs+C1','recall mdl1:SNPs+C1','precis
                 col=c('red','green','firebrick','forestgreen'),lwd=1,cex=0.65,horiz=TRUE)
```

y="Dev Explained %")+scale_color_discrete(name="Cell Type Weights",labels=c(0.08

Precision/Recall for Lasso's beta_hat

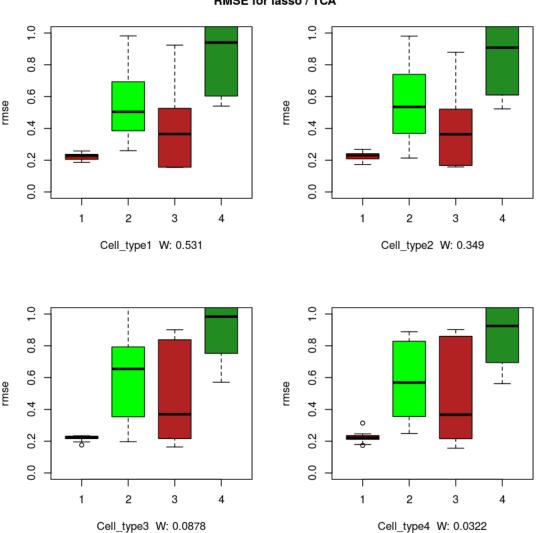


Precision/Recall for Lasso's beta_hat



```
par(mfrow=c(2,1),mar=c(0,0,0,0))
plot(1,type='n',axes=FALSE,xlab='',ylab='')
legend(x='top',inset=0,legend=c('lasso mdl1:SNPs+C1','lasso mdl2:C1','tca mdl1:SNPs+C1
col=c('red','green','firebrick','forestgreen'),lwd=1,cex=0.65,horiz=TRUE)
```

RMSE for lasso / TCA



- lasso mdl1:SNPs+C1 - lasso mdl2:C1 - tca mdl1:SNPs+C1 - tca mdl2:C1