Handout for sampling and probability - solutions

Try:

Figure out the probability that a random poker hand is "four-of-a-kind". This is a five card hand drawn at random without replacement with four card of one face value, and one card of another face value.

We know that the total number of possible sets of 5 card is

$$\binom{52}{5} = \frac{52 * 51 * 50 * 49 * 48}{5 * 4 * 3 * 2 * 1} = 2598960.$$

There are 13 possible face values for the quadruple, 1 way to take all 4 of that face value. There are then 12 ways to pick the face value for the non matching card, and 4 suits from which to select it. Therefore the number of possible "four-of-a-kind" hands is

$$13 * 12 * 4 = 624$$

Because all possible sets of 5 cards are equally likely this means the probability of 4 of a kind is

$$\frac{13 * 12 * 4}{\binom{52}{5}} = \frac{624}{2598960} = 0.00024$$

In [1]:
```
(52*51*50*49*48)/(5*4*3*2*1), 13*12*4, 624/2598960
```

Out[1]:
```
(2598960.0, 624, 0.00024009603841536616)
```

Try:

Suppose we wanted to simulate coin tossing using python. How could we do it using the pandas sample function? Contruct an appropriate data frame and show how to sample n coin tosses and report the number of tails.

In [2]:
```
import pandas as pd
```

In [3]:
```
coin = pd.DataFrame({'side': ['H', 'T']})
coin
```

Out[3]:

	side
0	H
1	T

In [4]:
```
n = 10
result = coin.sample(n, replace=True)
n_tails = (result['side']=='T').sum()
display(result, 'Number of Tails:', n_tails)
```

	side
1	T
1	T
0	H
0	H
1	T
1	T
1	T
0	H
0	H
1	T

```
'Number of Tails:'
```

```
6
```

Try:

Instead of coin tossing, suppose we roll a 6-sided die with sides number 1,2, ... ,6.

a) If we roll it twice, what is the probability of "snake eyes", i.e., rolling two 1's? Hint: consider how many possible outcomes there are.

Rolling twice is equivalent to drawing two numbers at random with replacement from {1,2,3,4,5,6}. There are $6^2 = 36$ ways to do this. out of these, there is only one way to get 1 both times. therefore the probability is $\frac{1}{36}$.

b) If we roll it 10 times, what is the probability of getting exactly two 1's?

For each of the 10 rolls, there are 6 possible values. So the number of possible sequences of 10 rolls is 6^{10}.

Out of these the number of sequences with exactly two 1's among the 10 rolls is the number of ways to pick 2 positions for the 1's multiplied by the number of ways to fill in the remaining slots with other numbers, so

$$\#\text{ ways to pick sequence with two 1's} = \binom{10}{2} * 5^8.$$

Therefore the probability is

$$\frac{\binom{10}{2} * 5^8}{6^{10}} = \binom{10}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8 = 0.2907$$

```
In [5]: (10*9/2)*(5**8)/(6**10)
```

```
Out[5]: 0.2907100492017223
```