Handout for random variables and population parameters

Try: For a discrete random variable X:

$$\mu = E[X] = \sum_x xp(x)$$
$$\sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 p(x)$$

where the summation is over all possible values for X.

Suppose $P(X=-1) = 0.25$, $P(X=0) = 0.50$ and $P(X=1) = 0.25$

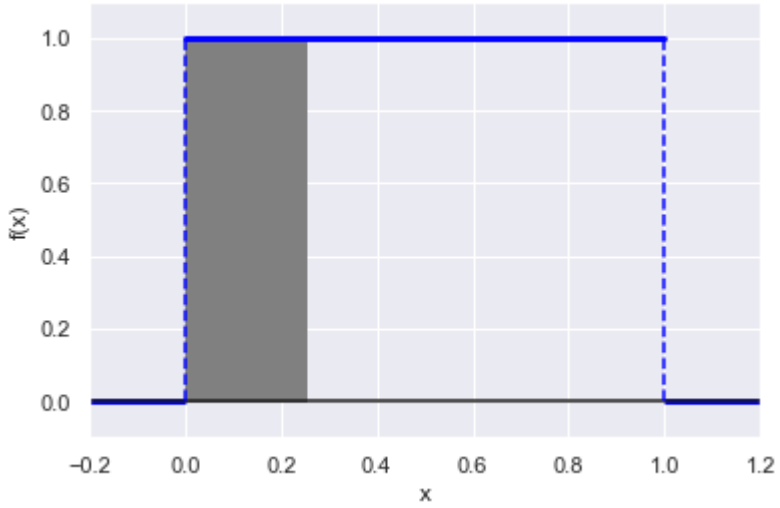
Calculate μ, σ^2 and σ for X.

$$\mu = (-1)(0.25) + (0)(0.50) + (1)(0.25) = 0$$
$$\sigma^2 = (0.25)(-1-0)^2 + (0.50)(0-0)^2 + (0.25)(1-0)^2 = 0.25 + 0.25 = 0.5$$
$$\sigma = \sqrt{0.5} = 0.707$$

Try: If X is a random number between 0 and 1, then it is a continuous random variable that can take on any value in the interval with equal likelihood, that is, its probability density function is constant over the interval. Because the area under a pdf curve must be 1, its pdf is given by

$$f(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

a) Find $P(X \le 0.25)$ (Hint: try sketching the pdf and remember that probabilities are areas under the curve).



$$P(X \le 0.25) = \text{Shaded area} = 1 * 0.25 = 0.25$$

b) Find an expression for $F(x) = P(X \leq x)$ as a function of x.

Similar to the figure above, if $0 < x < 1$, then $P(X \leq x) = x$. if $x \leq 0$, then $P(X \leq x) = 0$. If $x \geq 1$, then $P(X \leq x) = P(X \leq 1) = 1$ Summarizing,

$$F(x) = P(X \leq x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } 0 \leq x \leq 1 \\ 1, & \text{if } x > 1 \end{cases}$$

c) Find the 75th percentile of the distrbution of X, i.e., find the value q such that $P(X \leq q) = 0.75$.

$$P(X \leq q) = F(q) = q, \text{ therefore } q = 0.75.$$

Extra: code for making the graph

In [ ]:
```
import matplotlib.pyplot as plt
import seaborn as sns; sns.set()
import numpy as np
from scipy.stats import uniform
```

In [ ]:
```
x = np.linspace(0,1,100)
x25 = np.linspace(0,0.25, 100)
plt.plot(x, uniform.pdf(x), lw=3, color='blue')
plt.hlines(y=0, xmin=-0.2, xmax=0, lw=3, color='blue')
plt.hlines(y=0, xmin=1.0, xmax=1.2, lw=3, color='blue')
plt.hlines(y=0, xmin=-0.2, xmax=1.2)
plt.vlines(x=[0,1], ymin=0, ymax=1, linestyle='--', colors='blue')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.fill_between(x25, 0, uniform.pdf(x25), color='gray')
plt.xlim([-0.2, 1.2])
plt.ylim([-0.1,1.1])
plt.show()
```