OMIS 6000

Week 1:

- Linear Programming (LP) Models
- Graphical Solution Method
- Corner Point Property

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Quantitative Models

- What is a model?
  - It is a *formal description* of the simplified representation of a real-world process, phenomenon, or situation. Once formulated, we can develop various techniques to analyze the model and have it *answer questions.*

- Benefits of a model:
  - Allow us to **focus on the important details** of an application while ignoring unimportant complications.
  - A way to **understand, share, and talk about** the parts of a process that are of vital interest to us.
  - We can use **computational tools** to efficiently solve mathematical problems to gain further insight.
  - It can be used to **make predictions and gain intuition** on the effect of our actions before they are taken.

Constrained Optimization

- A **mathematical program** is a model that attempts to find a set of **decision variables** that **optimize** (either *maximize* or *minimize*) some objective (usually a profit or cost function). This **objective** must be expressed as a mathematical function of the variables and is formally called the **objective function**.

- The presence of restrictions, or **constraints**, limits the degree to which we can pursue our objective. The constraints must also be represented by mathematical functions.

Constrained Optimization

- A **feasible solution** represents a setting of the **decision variables** that satisfy all the problem's constraints but may not be the best we can do. An **optimal solution** is a feasible solution that results in the best possible **objective function** value.
  - For **maximization** problems, the optimal solution is the **largest** value of the objective function provided the solution is also feasible.
  - For **minimization** problems, the optimal solutions is the **smallest** value of the objective function provided the solution is also feasible.

Constrained Optimization

- Given a budget constraint, solve an optimization problem to determine an investment portfolio that maximizes return for a specified level of risk.

- Schedule shift work at a manufacturing facility or a retail operation so that desired staffing levels are met while minimizing the expected cost of overtime.

- A medical school must schedule its students to clinical rotations so that all training requirements are met, idiosyncratic student preferences are accounted for, and internal medical school restrictions are adhered to such that yearly placement costs are minimized (link).

Constrained Optimization

- If both the objective function and the constraints are **linear** functions, the problem is referred to as a **linear program (LP).**
  - Linear functions have **decision variables** (e.g., x_i) that appear in the addition or subtraction of terms raised to the zeroth (i.e., 0) or first (i.e., 1) power; each can be multiplied by a number.

$$f(x, y, z) = 5x + 6y + 8z$$

$$f(\boldsymbol{x}) = \sum_{i=1}^{100} x_i$$

Constrained Optimization

1) Problem Comprehension
   - Understand what the business problem is.
2) Mathematical Modeling
   - Create an optimization (prescriptive) model.
3) Solution Techniques
   - Solve the problem with a computational tool.
4) Interpretation of Results
   - Use the results for decision making.

Linear Programming (LP) Models

Problem Comprehension

Mathematical Modeling

LP Formulation: Problem Comprehension

Qualitative/Quantitative Descriptions → Logical Statements

- Understand what the business problem is.

- Formulate the mathematical model:

  1. Write down the problem's business decision.

  2. Determine what the overall problem objective is.

  3. Verbally describe each constraint or restriction.

Comprehension must be done with language before modeling the problem in mathematical terms.

LP Formulation: Mathematical Modeling

There are three components to an LP:

1. Decision variables:
   - What business decisions have to be made?

2. Objective Function:
   - A statement about the overall business goal.
   - Do we maximize/minimize to achieve this goal?

3. Constraints:
   - Logical statements that represent resource requirements and/or operational restrictions.

LP Formulation: Mathematical Modeling

There are three components to an LP:

1. Decision variables:
   - A variable (e.g., x, y, or the collection $x_1, x_2, \ldots$).

2. Objective Function:
   - A linear function, representing our business objective, that we wish to maximize/minimize.

3. Constraints:
   - Linear functions, "$\leq$", "$=$", or "$\geq$", and a number.

Manufacturing Example

Manufacturing Example

- A company produces **smartphones** and **tablets.**
- Each **smartphone** costs $200 to produce and will sell for $800.
- Each **tablet** costs $350 to produce and will sell for $1000.
- The company has a production budget of $1,000,000 for this month.
- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

We assume that expenses are incurred now, and revenues are received in the future!

We are interested in modelling this problem mathematically and will solve it later.

Manufacturing Example

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*1. What are the **quantitative decisions** to be made?*

Manufacturing Example

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- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

*1. What are the **quantitative decisions** to be made?*

- *Number of **smartphones** to produce*
- *Number of **tablets** to produce*

These are your decision variables!

Manufacturing Example

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2. What is the goal (or objective) of the problem?

Can you write that mathematically?

Manufacturing Example

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- The company has a production budget of \$1,000,000 for this month.
- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

2. What is the goal (or objective) of the problem?

- *Maximize revenue*

Can you write that mathematically?

Maximize \$800 (# of smartphones) + \$1000 (# of tablets)

Manufacturing Example

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- The company has a production budget of \$1,000,000 for this month.
- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

3. Are there any constraints limiting my decisions?

Can you write that mathematically?

Manufacturing Example

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- The company has a production budget of $1,000,000 for this month.
- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

3. Are there any constraints limiting my decisions?

- *The production costs must be within the budget*

Can you write that mathematically?

$$\$200\,(\#\text{ of smartphones}) + \$350\,(\#\text{ of tablets}) \leq \$1{,}000{,}000$$

Manufacturing Example

- A company produces **smartphones** and **tablets.**
- Each **smartphone** costs $200 to produce and will sell for $800.
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- The company has a production budget of $1,000,000 for this month.
- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

Mathematical Model (put it all together):

Maximize *$800 (# of smartphones) + $1000 (# of tablets)*
subject to:

$200 (# of smartphones) + $350 (# of tablets) ≤ $1,000,000

Manufacturing Example

- A company produces **smartphones** and **tablets.**
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- The company has a production budget of $1,000,000 for this month.
- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

Mathematical Model (put it all together):

***Maximize** $800 (# of smartphones) + $1000 (# of tablets)*
subject to:

$200 (# of smartphones) + $350 (# of tablets) ≤ $1,000,000

The number of smartphones and tablets that can be manufactured must not be a negative number!

Manufacturing Example

- A company produces **smartphones** and **tablets.**
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- The company has a production budget of \$1,000,000 for this month.
- Question: **How many units of each product should the company manufacture this month to maximize future revenue?** Assume any unit produced is sold (that is, there is always sufficient demand).

Mathematical Model (put it all together):

Maximize $\$800x + \$1000y$

subject to:

$$\$200x + \$350y \leq \$1,000,000$$

$$x, y \geq 0$$

where:

x *: # of smartphones to produce*

y *: # of tablets to produce*

Use letters (x,y,…) because it is easier to write and read (typically problems have thousands of constraints)

Optimal Solution: *x=5000, y=0*

Manufacturing Example

We optimize (**maximize** or minimize) an **objective function** (a profit, **revenue**, or cost):

- It is a linear program (LP) because the **constraints** and **objective function** are linear functions of the **decision variables**.
- There are **3** constraints that limit the degree to which we can **maximize** the **objective function**.
- Any solution that satisfies all constraints are **feasible solutions** to the problem. The **optimal solution** is the **feasible solution** that results in the **largest** value of the **objective function**.

LP Formulation: Mathematical Modeling

- **Constraints (≥, ≤, =): <u>Linear</u> in variables**

  $3x_1 + 4x_2 - 3x_4 \leq 14$ **is ok**

  $4\ x_1\ x_2 - \log x_3 < 6$ **is <u>not ok</u>**

- **Objective function: <u>Linear</u> in variables**

  Maximize $6x_1 + 3x_2 - 10x_3$ **is ok**

  Minimize $3x_1x_2 - 2x_3$ **is <u>not ok</u>**

Linear Programming (LP) Models

Solution Techniques

Interpretation of Results

LP Formulation: Solution Techniques

- Large-scale LPs in industry (e.g., Canadian Tire, DHL) include thousands of **decision variables** and tens of thousands of **constraints**.
- We can get a sense of how these large-scale problems are *solved* by graphing a *two-dimensional* LP and finding the optimal solution.
- For a *two-dimensional* LP, we wish to find the optimal value of two business decisions. Define the decision variables for this LP as:

$$x_1, x_2$$

JFE Steel Example

JFE Steel Example

The ***JFE Steel*** company produces 2 types of plastic pipes. Three resources are crucial to the output of each pipe: extrusion hours, packaging hours, and a special additive to the plastic raw material. Below is next week's situation. Each unit of type 1 yields $34 in profit and each unit of type 2 yields $40 in profit. How many of each type of product should be produced to **maximize profit**?

Product			
Resource	Type 1	Type 2	Resource Availability
Extrusion	4 hr	6 hr	48 hr
Packaging	2 hr	2 hr	18 hr
Additive mix	2 lb	1 lb	16 lb

JFE Steel Example

Step 1: What are the decision variables?

Step 2: What is the objective?

Write the mathematical objective function as follows:

JFE Steel Example

Step 1: What are the decision variables?

x_1 = amount of type 1 pipe produced next week

x_2 = amount of type 2 pipe produced next week

Step 2: What is the objective?

Maximize total profit (Z)

Write the mathematical objective function as follows:

JFE Steel Example

Step 1: What are the decision variables?

x_1 = amount of type 1 pipe produced next week

x_2 = amount of type 2 pipe produced next week

Step 2: What is the objective?

Maximize total profit (Z)

Write the mathematical objective function as follows:

$$Maximize \quad Z = 34x_1 + 40x_2$$

JFE Steel Example

Step 3: Formulate the constraints

Product			
Resource	Type 1	Type 2	Resource Availability
Extrusion	4 hr	6 hr	48 hr
Packaging	2 hr	2 hr	18 hr
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JFE Steel Example

Step 3: Formulate the constraints

Product			
Resource	Type 1	Type 2	Resource Availability
Extrusion	4 hr	6 hr	48 hr
Packaging	2 hr	2 hr	18 hr
Additive mix	2 lb	1 lb	16 lb

Extrusion $4x_1 + 6x_2 \leq 48$

Packaging $2x_1 + 2x_2 \leq 18$

Additive mix $2x_1 + x_2 \leq 16$

JFE Steel Example

Summary of the final LP formulation.

$$\boldsymbol{Maximize} \quad Z = 34x_1 + 40x_2$$

$\boldsymbol{Subject\ To}$:

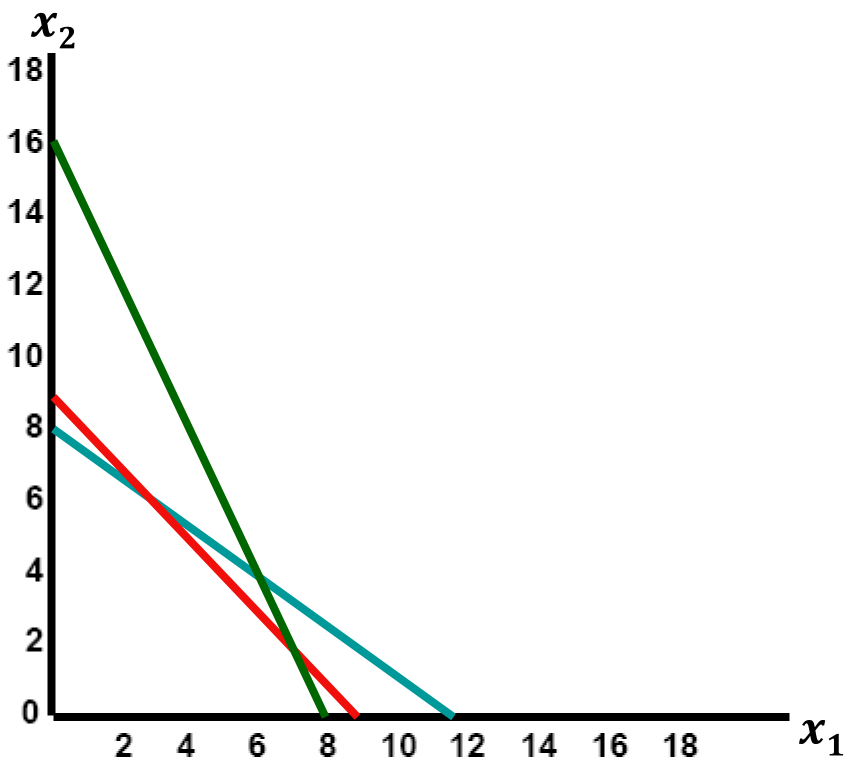
$$\begin{aligned}
4x_1 + 6x_2 &\leq 48 \quad (\textit{Extrusion constraint}) \\
2x_1 + 2x_2 &\leq 18 \quad (\textit{Packaging constraint}) \\
2x_1 + x_2 &\leq 16 \quad (\textit{Additive mix constraint}) \\
x_1, x_2 &\geq 0 \quad (\textit{non-negativity constraints})
\end{aligned}$$

Equations on the left side only

Values of constraints on the right side only

JFE Steel Example

- We can plot each of the constraints on a graph.



$4x_1 + 6x_2 \leq 48$ (extrusion)

(0,8)

(12,0)

$2x_1 + 2x_2 \leq 18$ (packaging)

(0,9)

(9,0)

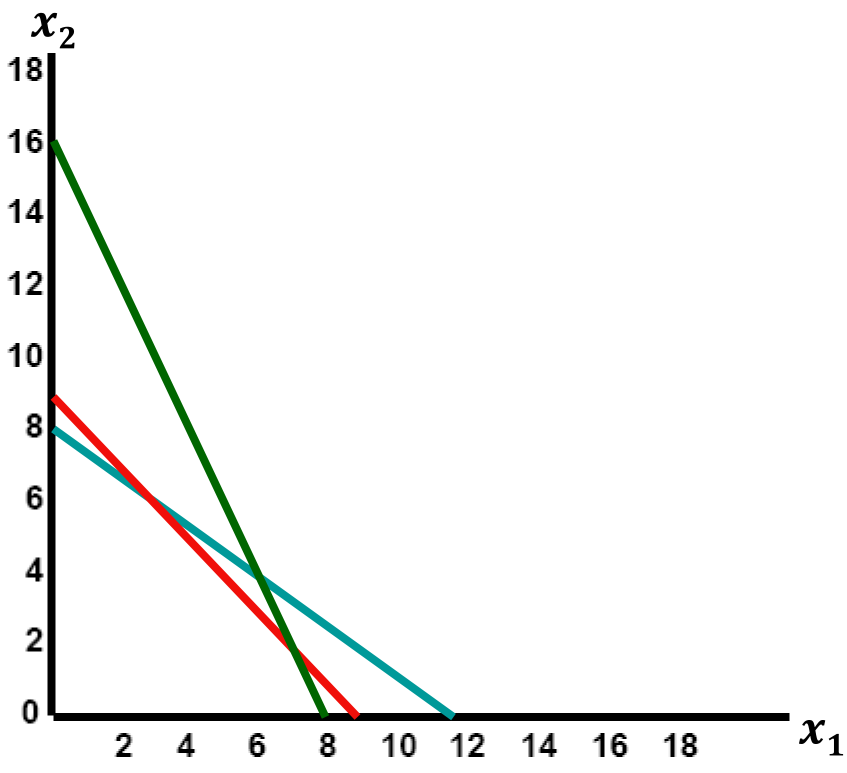
$2x_1 + x_2 \leq 16$ (additive mix)

(0,16)

(8,0)

JFE Steel Example

- We can plot each of the constraints on a graph.



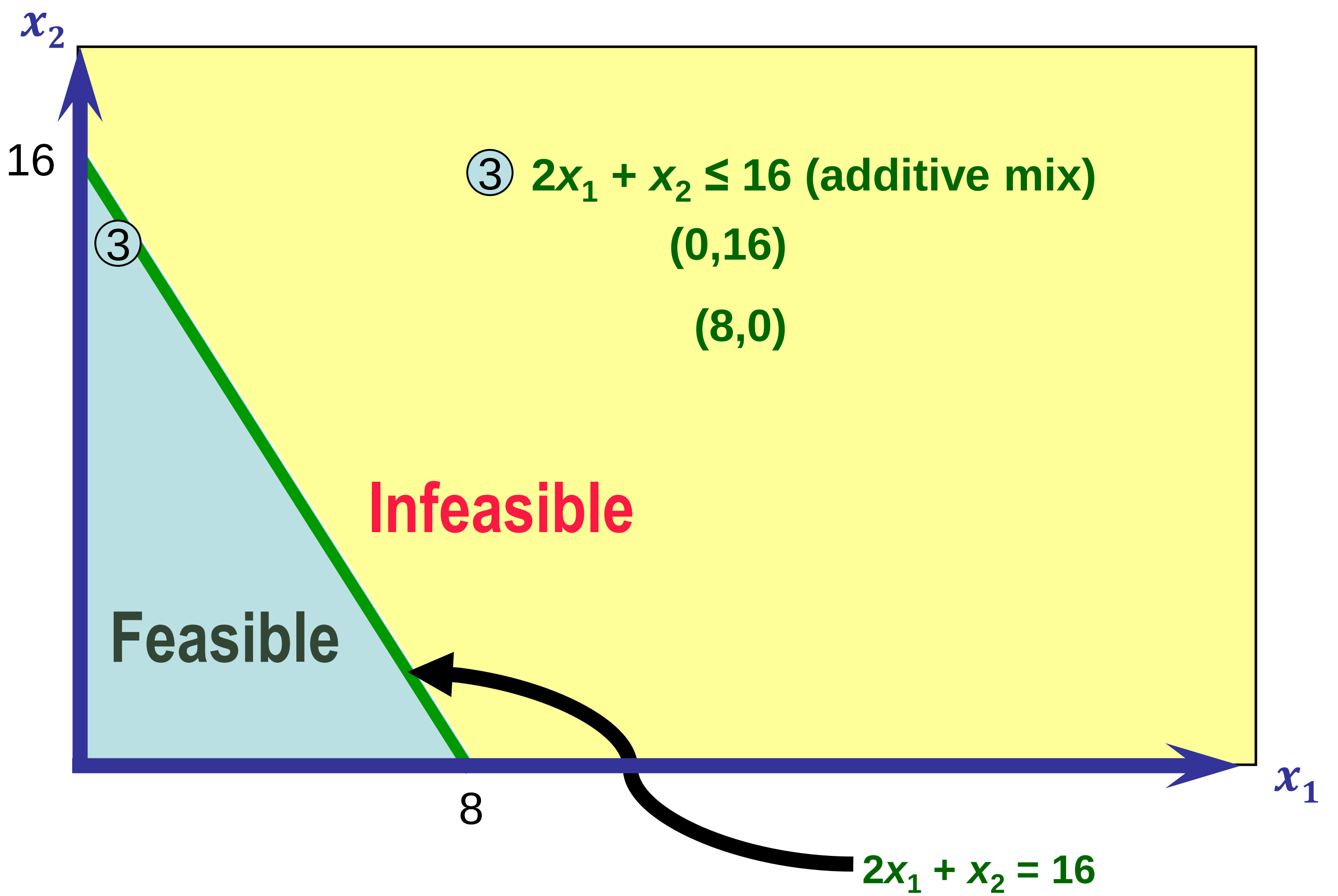
1. **For all = constraints**: Only points that lie exactly on the constraint line are **feasible solutions**.

2. **For all ≤ constraints:** The points on the line and below or to the left of the line are **feasible solutions**.

3. **For all ≥ constraints:** The points on the line and above or to the right of the line are **feasible solutions**.

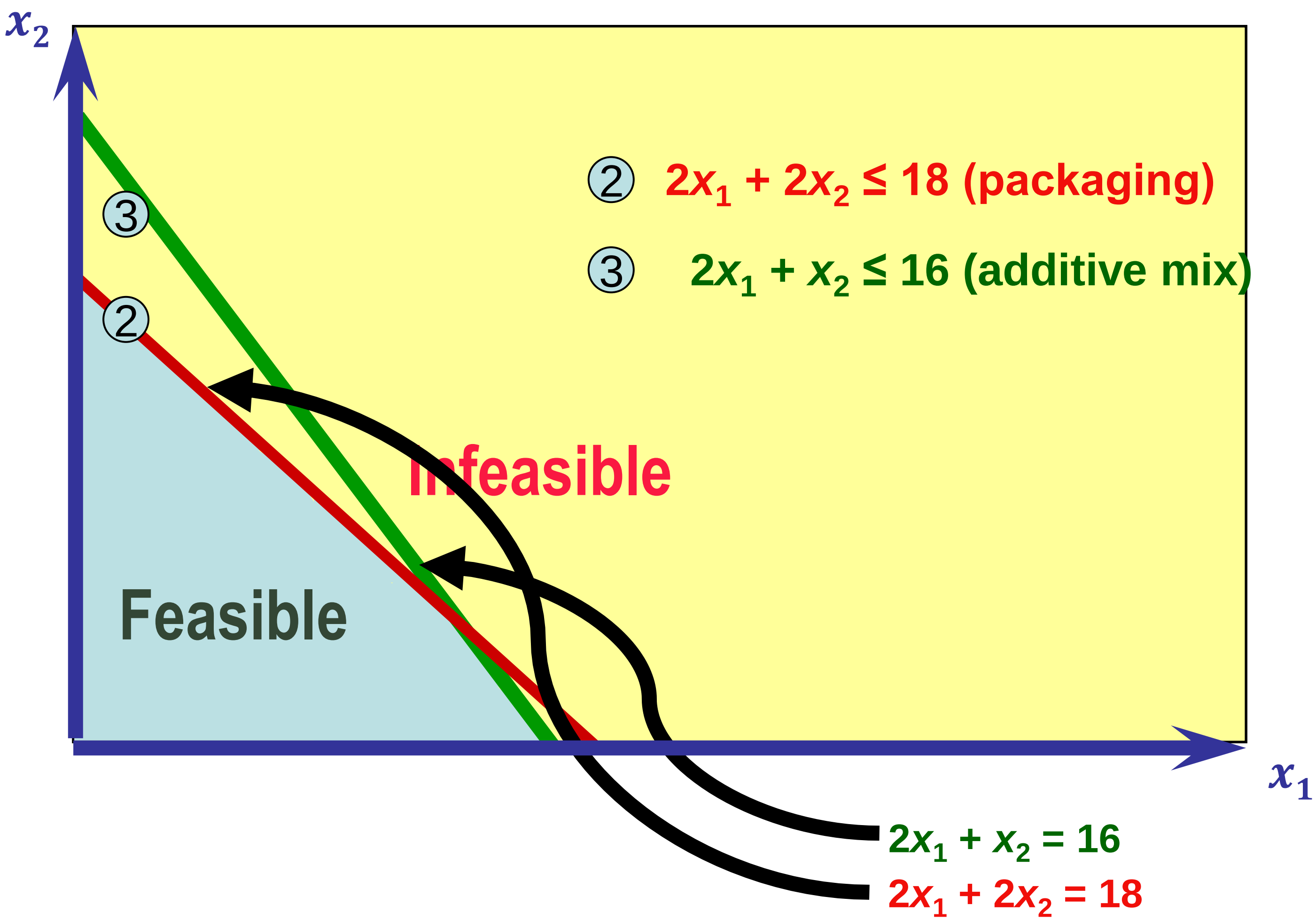
JFE Steel Example

- We can plot each of the constraints on a graph.



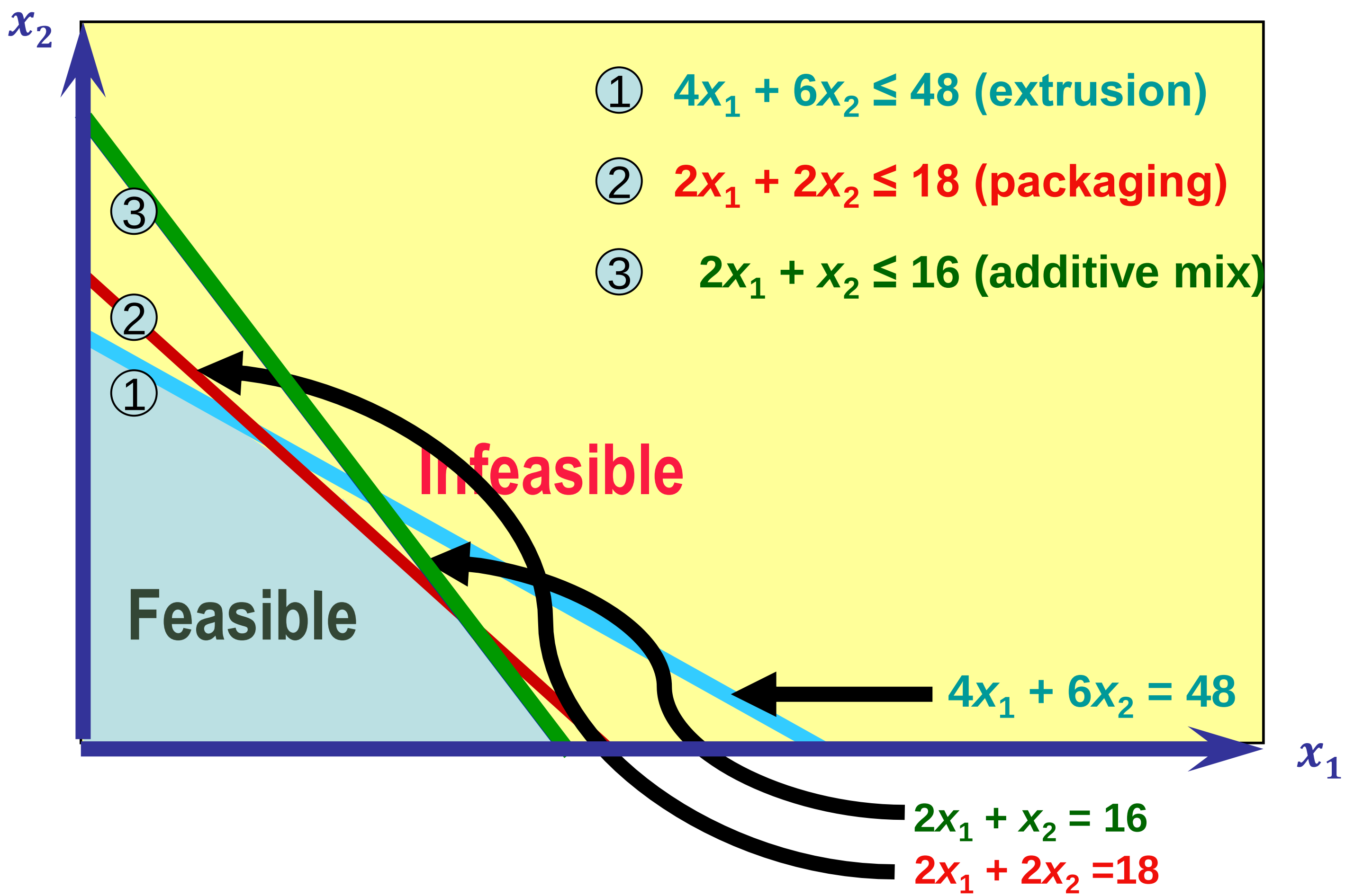
JFE Steel Example

- We can plot each of the constraints on a graph.



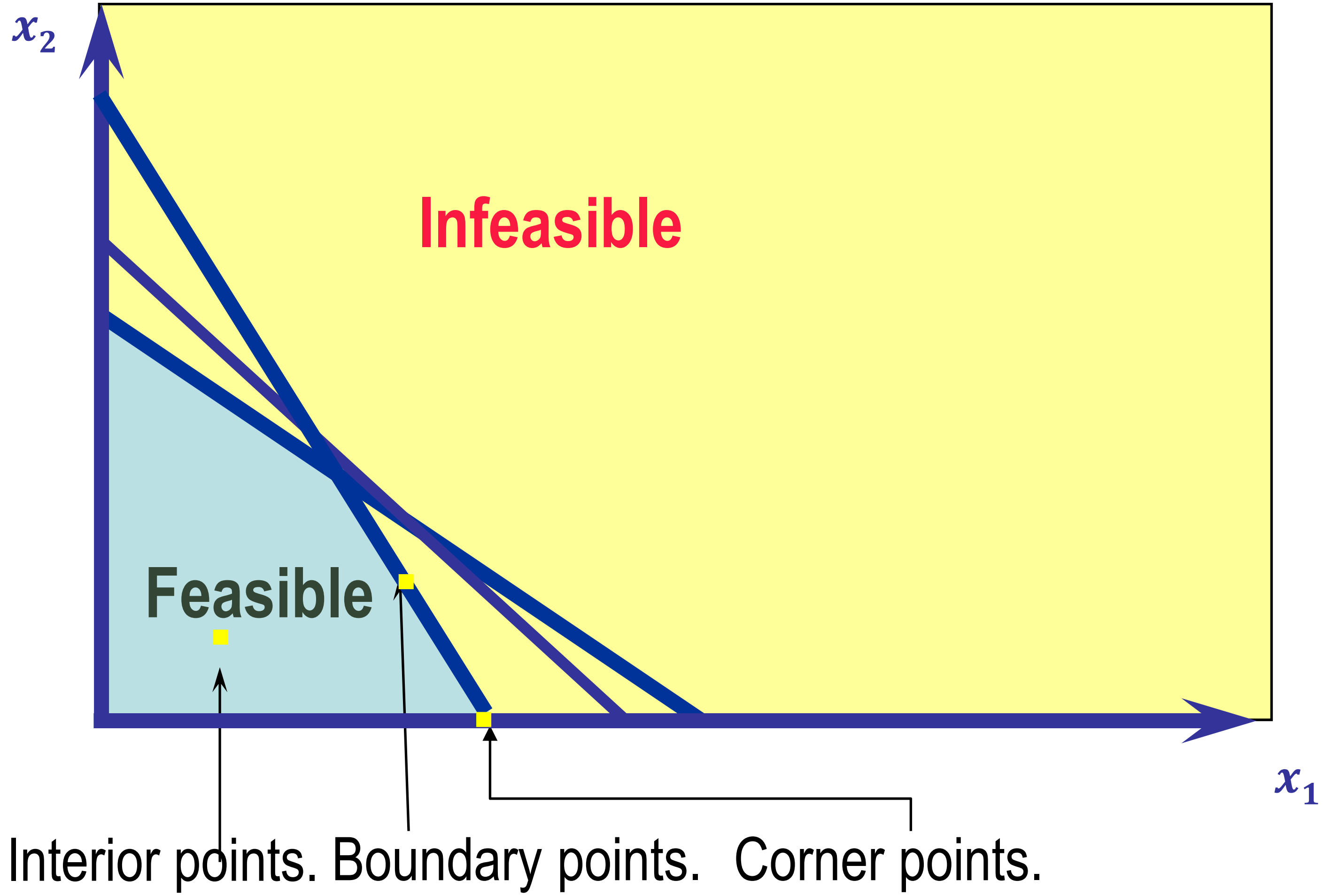
JFE Steel Example

- We can plot each of the constraints on a graph.



JFE Steel Example

- We can plot each of the constraints on a graph.



LP Formulation: Solution Techniques

- **Constraints:** They are line segments that partition the coordinate space into two regions: (1) ***feasible*** and (2) ***infeasible***

- **Feasible Region:** Points that simultaneously satisfy ***all*** the constraints in the problem.

- **Corner Points:** Solutions to the optimization problem that lie on the intersection of at least two *linearly independent* constraint lines.
  - Lie on the boundary of the feasible region.
  - There are a ***finite number*** of these points.

LP Formulation: Solution Techniques

Fundamental Theorem of Linear Programs:
An optimal solution to a linear programming problem, if one exists, occurs at a corner point.

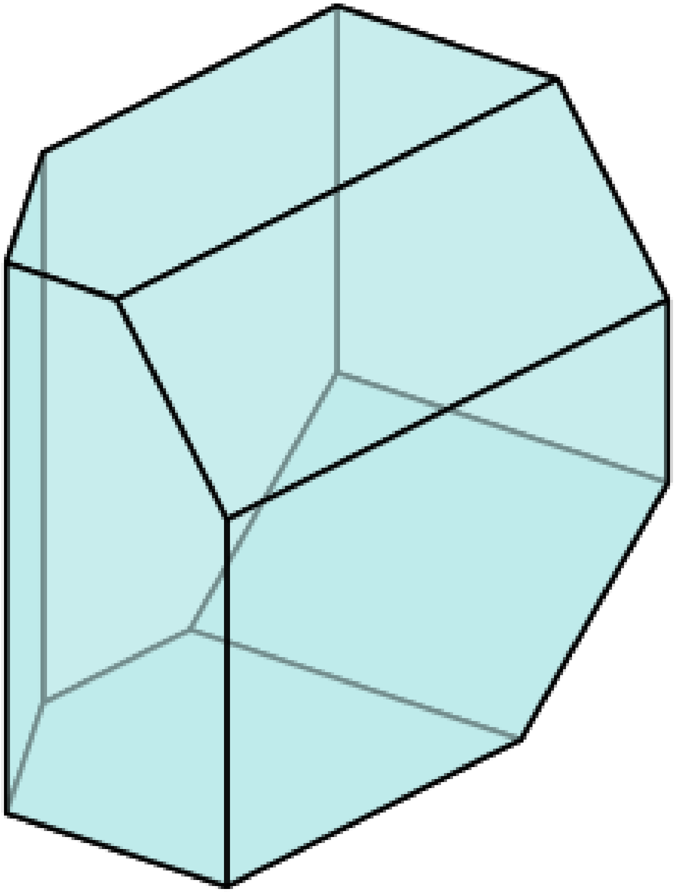
Consider the matrix representation of an LP:

$$z = \boldsymbol{Min}\ \boldsymbol{c}^{\boldsymbol{T}}\boldsymbol{x}$$
$$\boldsymbol{Ax} \leq \boldsymbol{b}$$
$$\boldsymbol{x} \geq \boldsymbol{0}$$

where $\boldsymbol{c}$ and $\boldsymbol{b}$ are parameter vectors, $\boldsymbol{A}$ is a matrix, and $\boldsymbol{x}$ is the vector of decision variables.

LP Formulation: Solution Techniques

Visualizing the Linear Polytope $(Ax \leq b)$.



LP Formulation: Solution Techniques

Theorem: Suppose that $\boldsymbol{Ax \leq b}$ describes a linear polytope (a bounded and convex feasible region). Then the optimal solution $\boldsymbol{x}^*$ to the linear program

$$z = \boldsymbol{Min\ c^T x}$$
$$\boldsymbol{Ax \leq b}$$
$$\boldsymbol{x \geq 0}$$

is located at a corner point (vertex) of the polytope.

Proof: In-Class.

LP Formulation: Solution Techniques

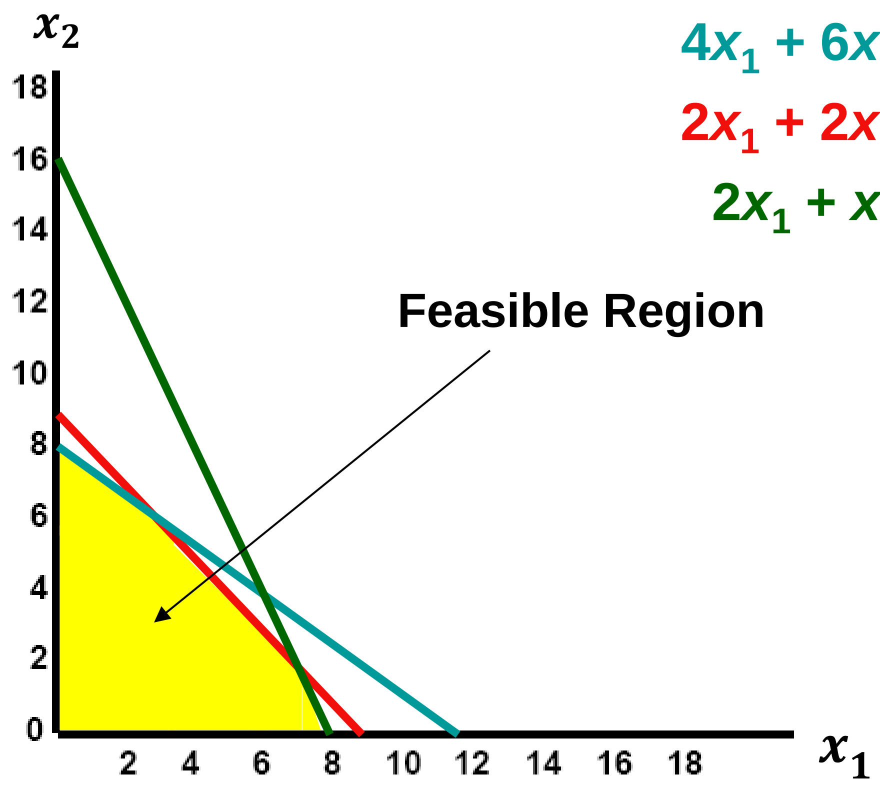
Optimal Solution: Based on the fundamental theorem of linear programs, the <u>corner point</u> that yields the ***best*** objective function value is the ***optimal solution*** to the linear program.

↑ The largest value in a maximization problem.

↓ The smallest value in a minimization problem.

Algorithm: Find all the corner points and check which one gives the ***best*** objective function value.

JFE Steel Example

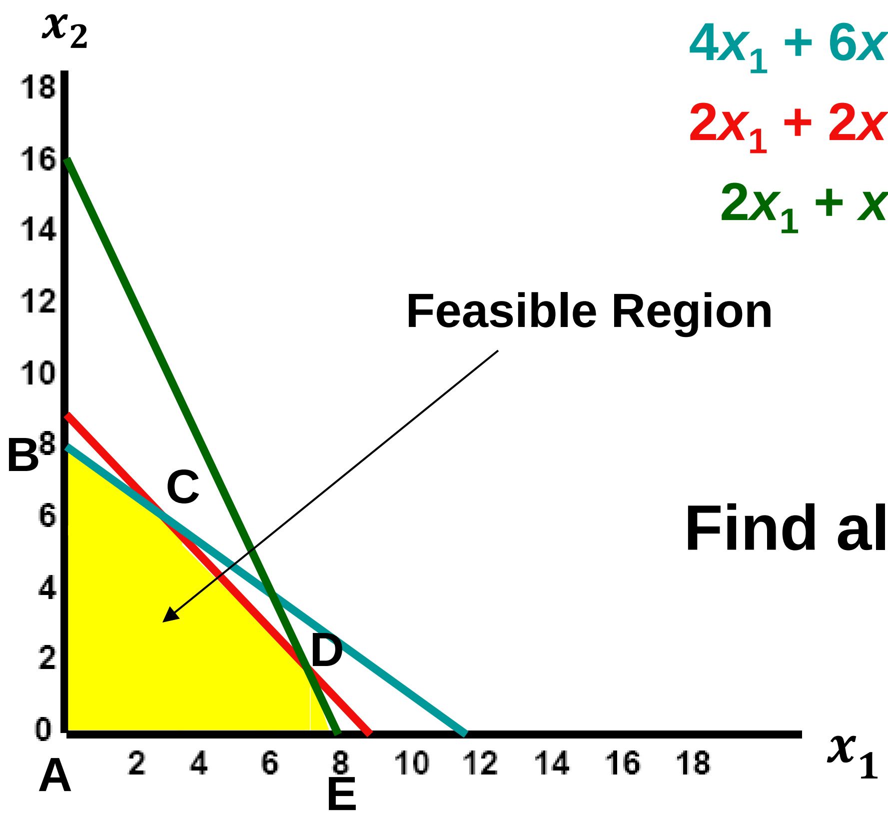


$4x_1 + 6x_2 \leq 48$ (extrusion)

$2x_1 + 2x_2 \leq 18$ (packaging)

$2x_1 + x_2 \leq 16$ (additive mix)

JFE Steel Example



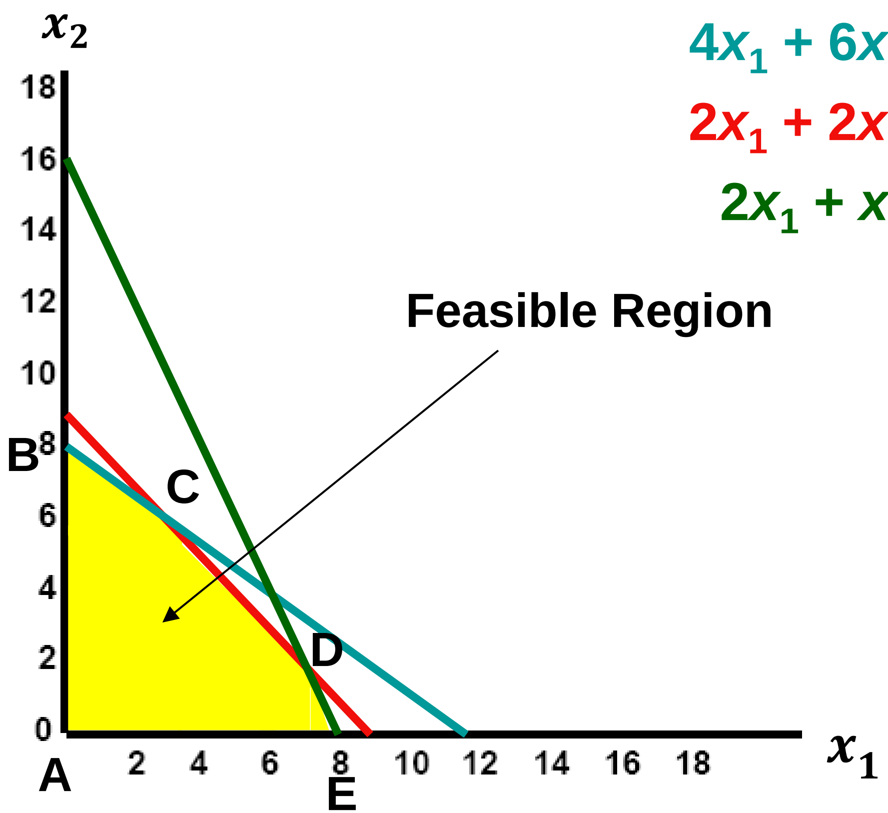
$4x_1 + 6x_2 \leq 48$ (extrusion)

$2x_1 + 2x_2 \leq 18$ (packaging)

$2x_1 + x_2 \leq 16$ (additive mix)

Find all the corner points.

JFE Steel Example



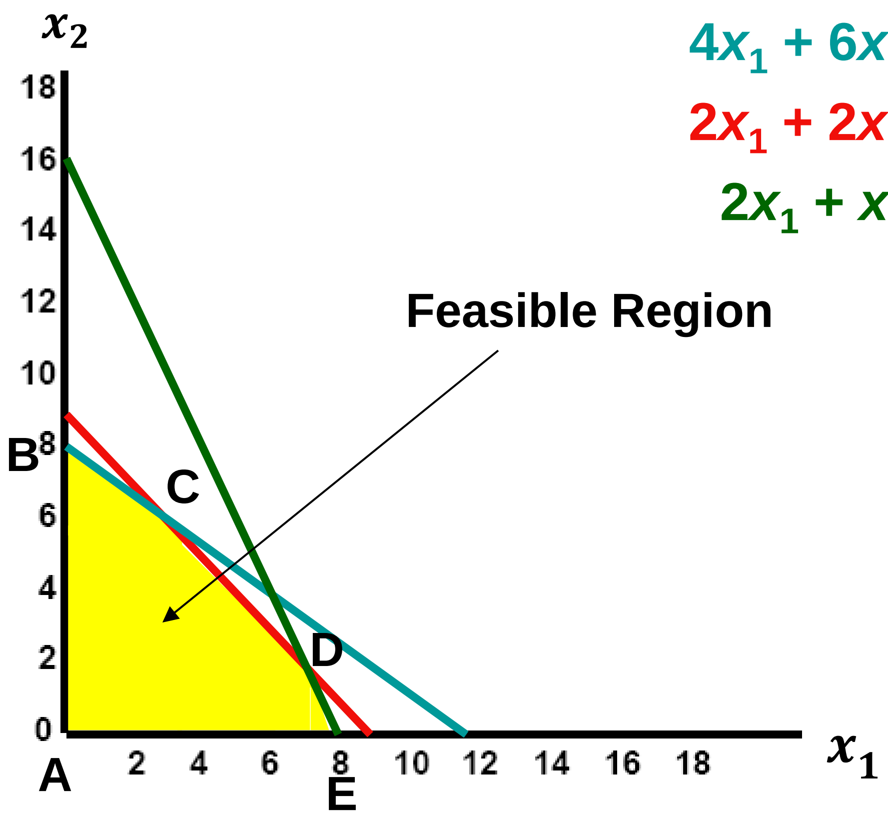
$4x_1 + 6x_2 \leq 48$ (extrusion)

$2x_1 + 2x_2 \leq 18$ (packaging)

$2x_1 + x_2 \leq 16$ (additive mix)

Point A: $(\mathbf{0}, \mathbf{0})$

JFE Steel Example



$4x_1 + 6x_2 \leq 48$ (extrusion)

$2x_1 + 2x_2 \leq 18$ (packaging)

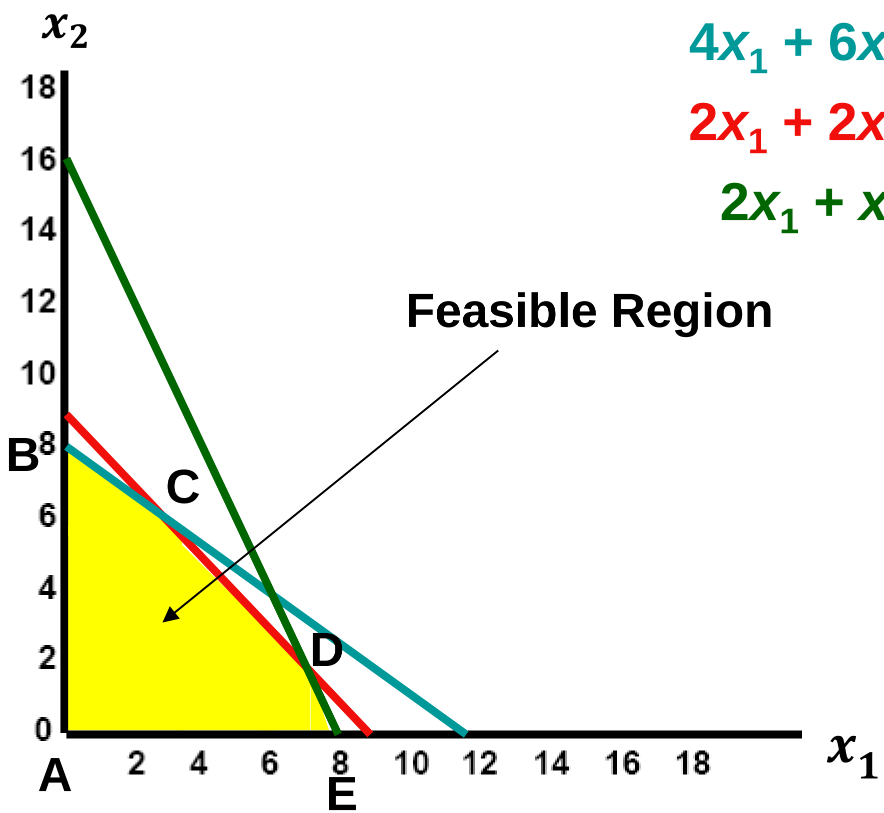
$2x_1 + x_2 \leq 16$ (additive mix)

Point B: $(0, 8)$

$$4x_1 + 6x_2 = 48$$
$$x_1 = 0$$

Solve a linear system: 2 equations and 2 unknowns.

JFE Steel Example



$4x_1 + 6x_2 \leq 48$ (extrusion)

$2x_1 + 2x_2 \leq 18$ (packaging)

$2x_1 + x_2 \leq 16$ (additive mix)

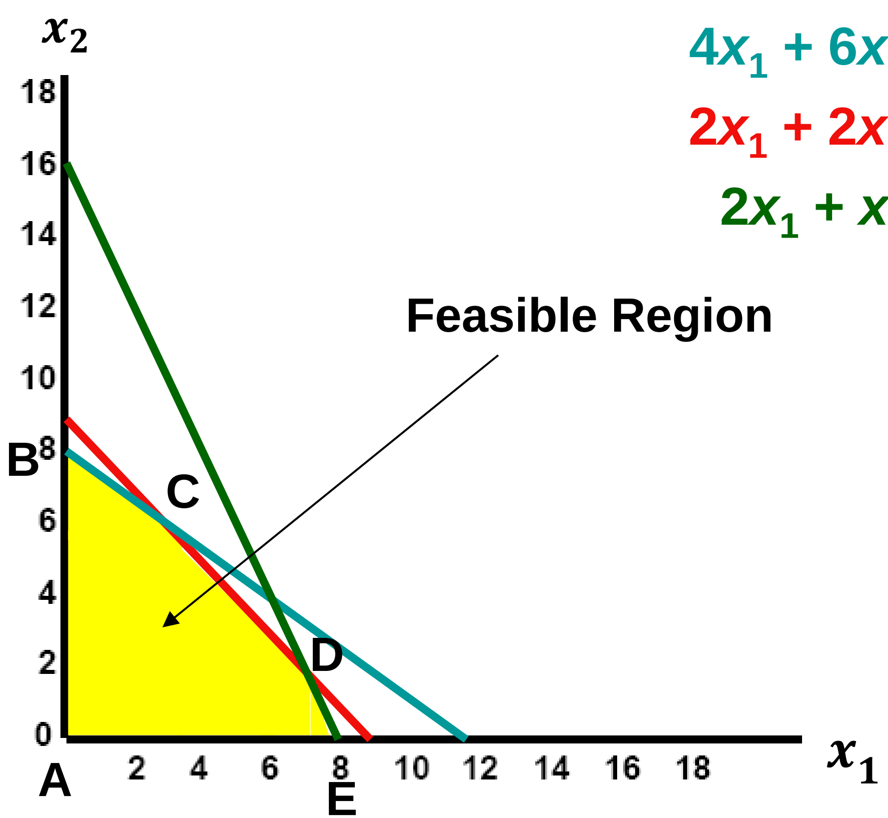
Point C: $(3, 6)$

$4x_1 + 6x_2 = 48$

$2x_1 + 2x_2 = 18$

Solve a linear system: 2 equations and 2 unknowns.

JFE Steel Example



$4x_1 + 6x_2 \leq 48$ (extrusion)

$2x_1 + 2x_2 \leq 18$ (packaging)

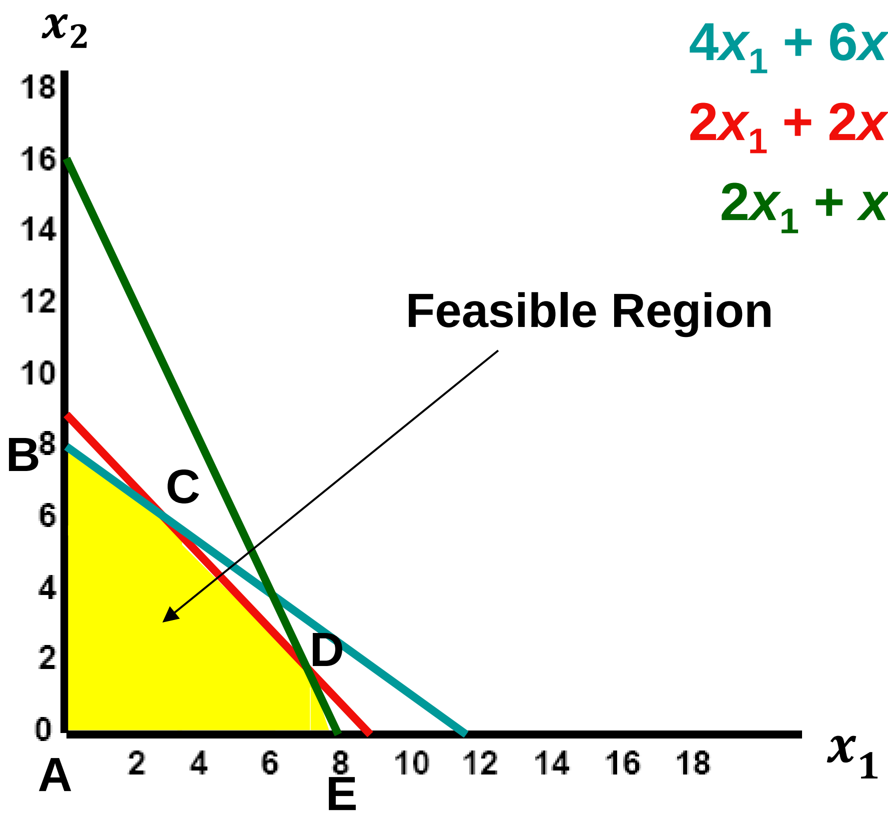
$2x_1 + x_2 \leq 16$ (additive mix)

Point D: $(7, 2)$

$$2x_1 + x_2 = 16$$
$$2x_1 + 2x_2 = 18$$

Solve a linear system: 2 equations and 2 unknowns.

JFE Steel Example



$4x_1 + 6x_2 \leq 48$ (extrusion)

$2x_1 + 2x_2 \leq 18$ (packaging)

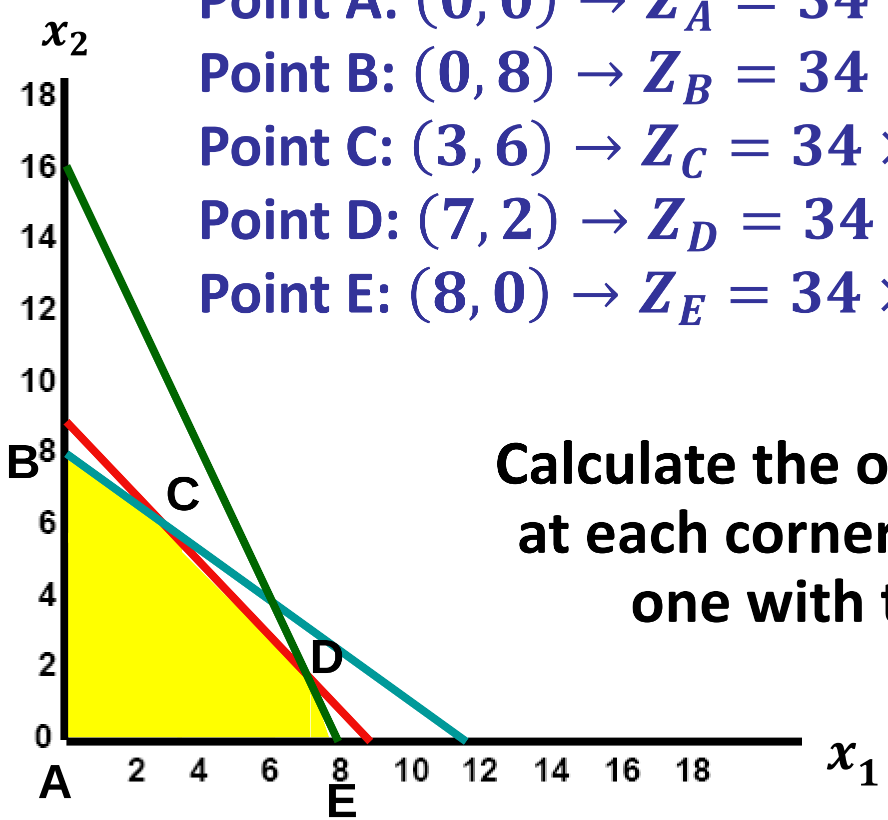
$2x_1 + x_2 \leq 16$ (additive mix)

Point E: $(8, 0)$

$$2x_1 + x_2 = 16$$
$$x_2 = 0$$

Solve a linear system: 2 equations and 2 unknowns.

JFE Steel Example



Point A: $(0,0) \rightarrow Z_A = 34 \times 0 + 40 \times 0 = \$0$

Point B: $(0,8) \rightarrow Z_B = 34 \times 0 + 40 \times 8 = \$320$

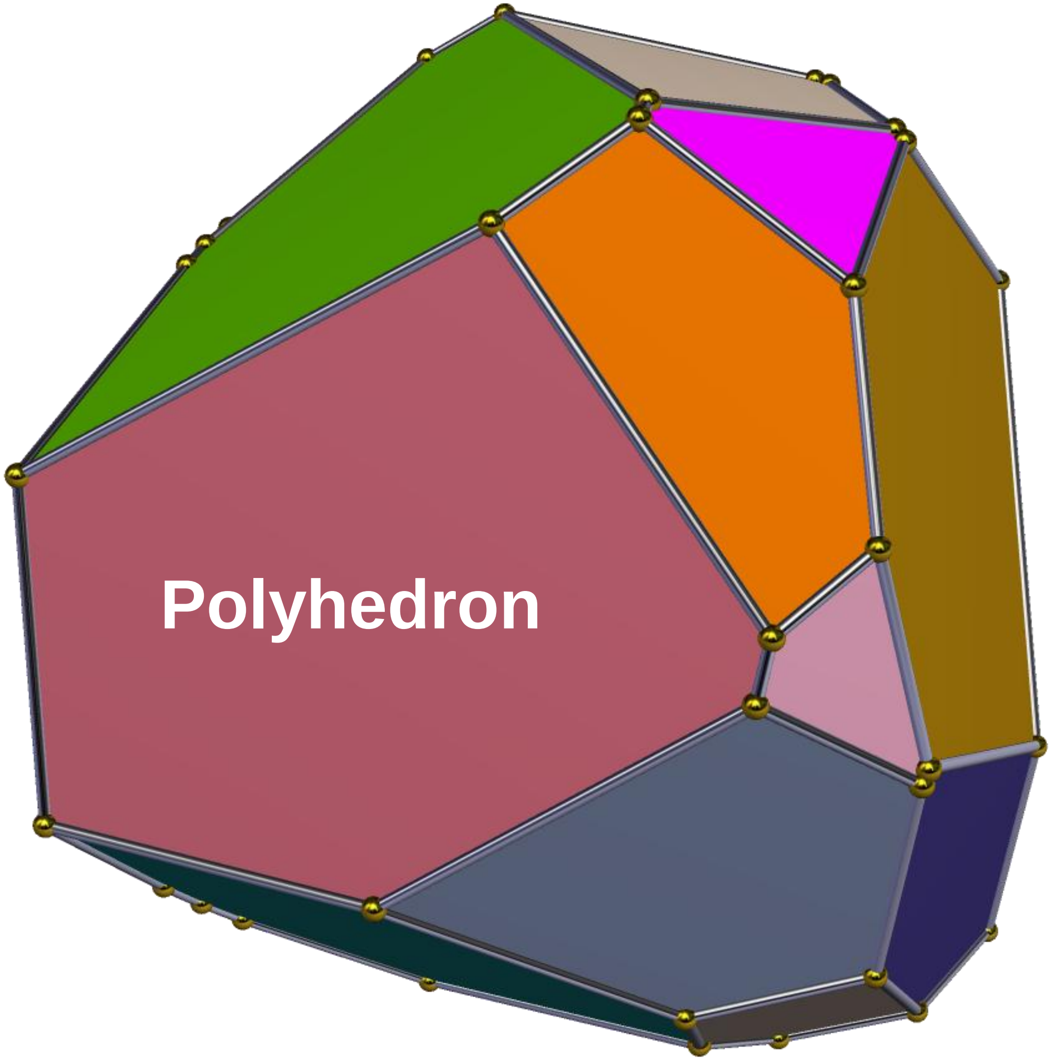
Point C: $(3,6) \rightarrow Z_C = 34 \times 3 + 40 \times 6 = \boxed{\$342}$

Point D: $(7,2) \rightarrow Z_D = 34 \times 7 + 40 \times 2 = \$318$

Point E: $(8,0) \rightarrow Z_E = 34 \times 8 + 40 \times 0 = \$272$

Calculate the objective function value at each corner point and choose the one with the highest profit.

Multidimensional Intuition?



LP Formulation: Interpreting the Results

LP Formulation: Interpreting the Results

- Does the model accurately represent the real-world problem? What are its strengths and limitations?

- What is the model advising you to do?
  - Does the solution make sense given your expertise?
  - Is the scale of the objective function value reasonable?

- Even problems with no **feasible** solutions tell you something about the *real* problem you are modeling.

- This is the ***art*** of modeling. It is about translating a real-world problem into a mathematical model, then using the results to ***inform*** decision-making.

LP Formulation: Interpreting the Results

- For each constraint, put terms with **decision variables** on the left-hand side (LHS) and the **constant** term on the right-hand side (RHS).
- **LHS Interpretation:** The amount of a particular resource that each decision consumes.
- **RHS Interpretation:** The amount of a particular resource that we have available. For $\leq$ constraints, we must consume less than that amount, for $=$ constraints we must consume exactly that amount, and for $\geq$ constraints, we must consume more.

LP Formulation: Interpreting the Results

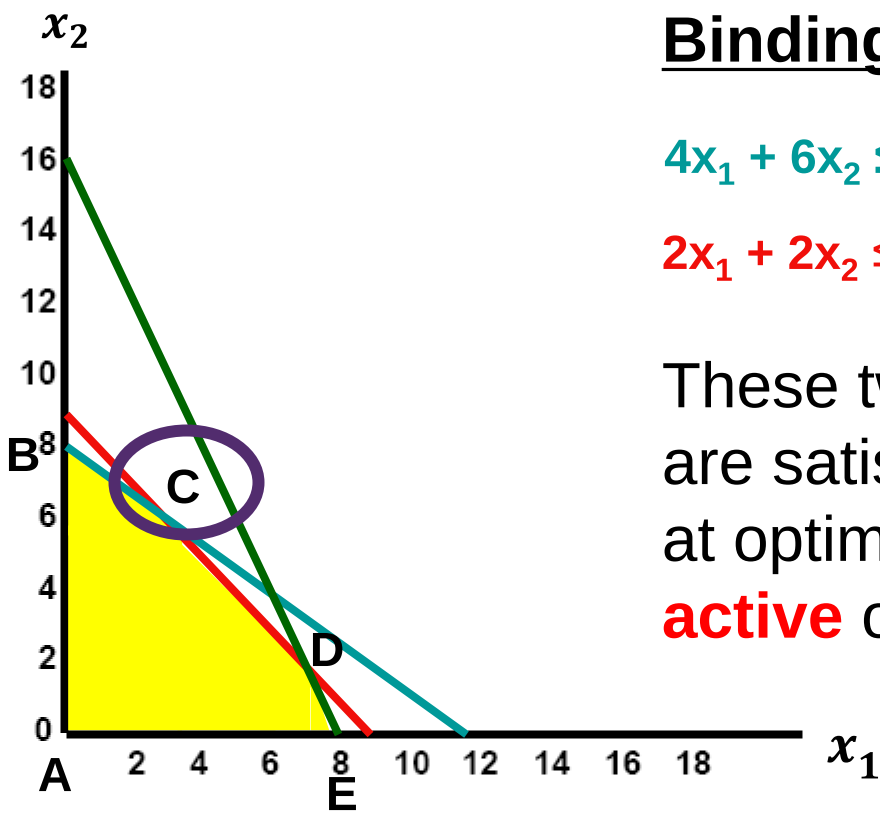
- **Active or binding constraints:**
  - For these constraints: $\boldsymbol{LHS = RHS}$
  - The optimal solution, *i.e., the optimal corner point*, is on this line segment.

- **Inactive or non-binding constraints:**
  - For these constraints: $\boldsymbol{LHS \neq RHS}$
  - The optimal solution, *i.e., the optimal corner point*, is **not** on this line segment.

LP Formulation: Interpreting the Results

- For each constraint, put terms with **decision variables** on the left-hand side (LHS) and the **constant** term on the right-hand side (RHS).
- Consider the difference between the LHS and RHS of **non-binding constraints**:
  - **Slack (≤):** We do not consume all the resource. It may be prudent to decrease the amount of this resource to save the company money.
  - **Surplus (≥):** The value of the resource is too low. It may be prudent to increase the amount of this resource to save the company money.

JFE Steel Example



Binding Constraints

$4x_1 + 6x_2 \leq 48$ **(extrusion)**

$2x_1 + 2x_2 \leq 18$ **(packaging)**

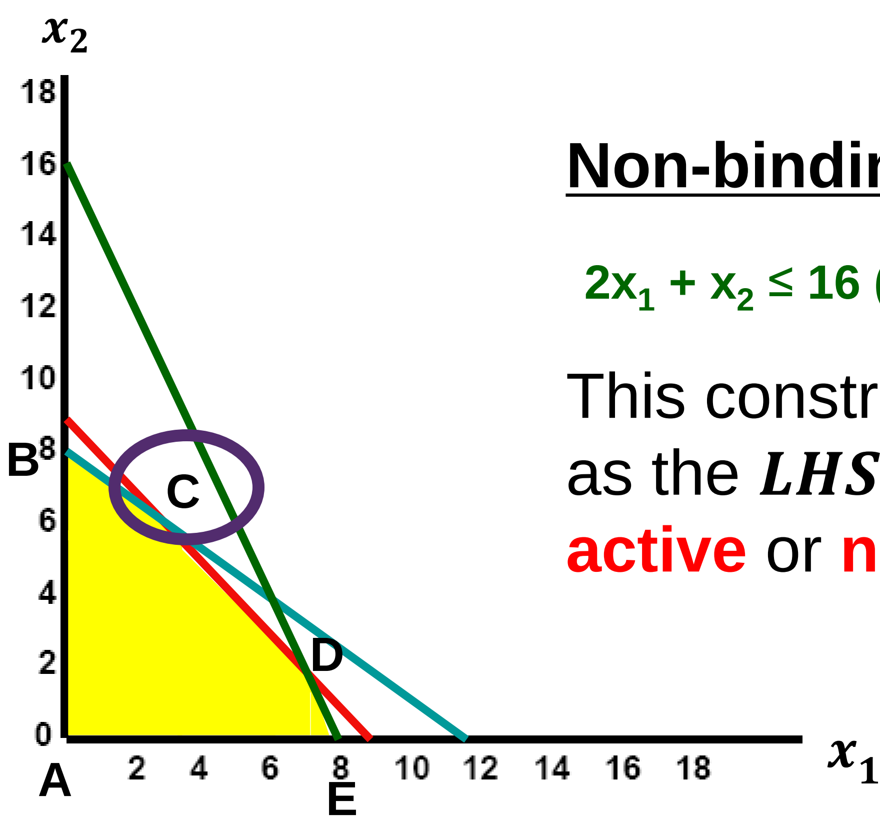
These two constraints are satisfied as equalities at optimality. They are **active** or **binding**.

Substitute (3,6) into these constraints:

4*3 + 6*6 = 48 (extrusion)

2*3 + 2*6 = 18 (packaging)

JFE Steel Example



Non-binding Constraints

$2x_1 + x_2 \leq 16$ (additive mix)

This constraint has **slack** as the $LHS < RHS$. It is in **active** or **nonbinding**.

Substitute (3,6) into this constraint: $\mathbf{2 \times 3 + 1 \times 6 = 12 < 16}$

LP Formulation: Interpreting the Results

Dissecting Problems:

- Redundant Constraints
- Multiple Optimal Solutions
- Infeasible Solution
- Unbounded Solutions

LP Formulation: Interpreting the Results

Dissecting Problems:

- Redundant Constraints
- Multiple Optimal Solutions
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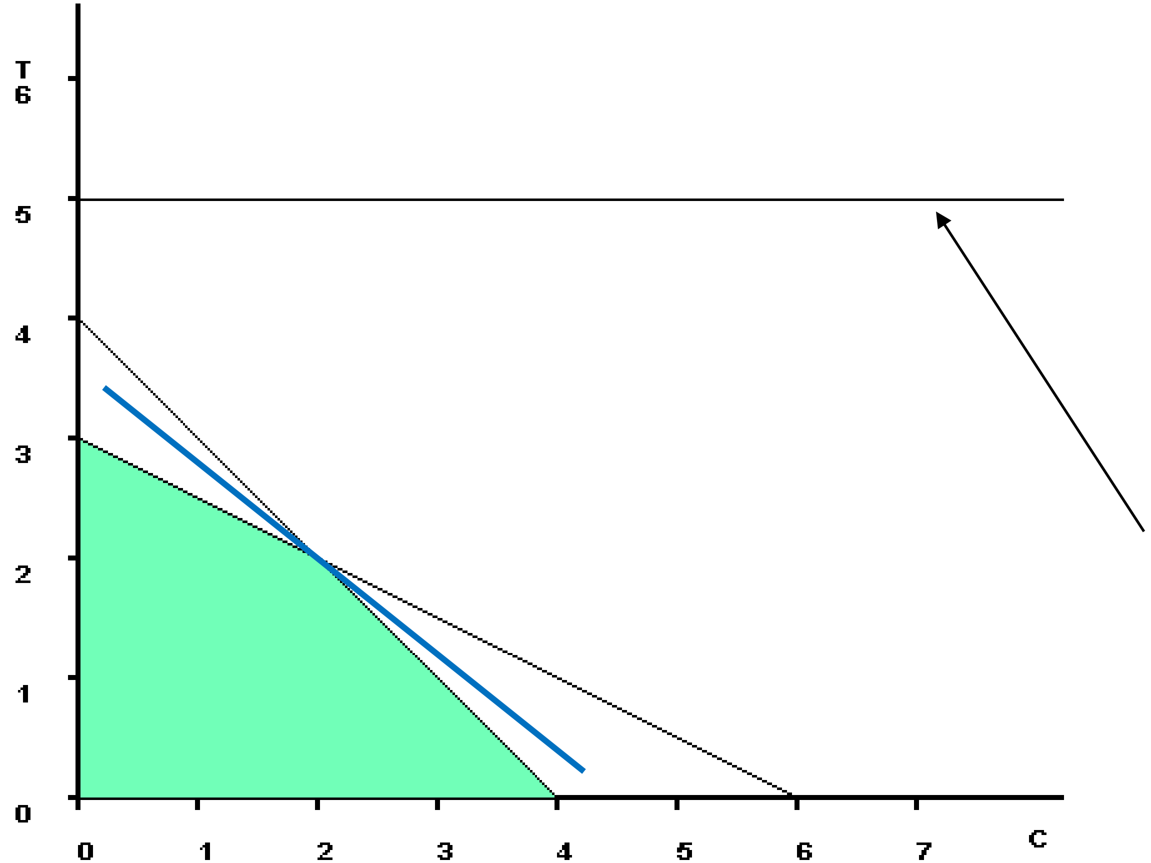
An optimal solution still exists.

LP Formulation: Interpreting the Results

- **Redundant Constraints**



The line does not define any part of the feasible region. It is redundant and can be removed from the formulation without affecting feasibility or the optimal solution.

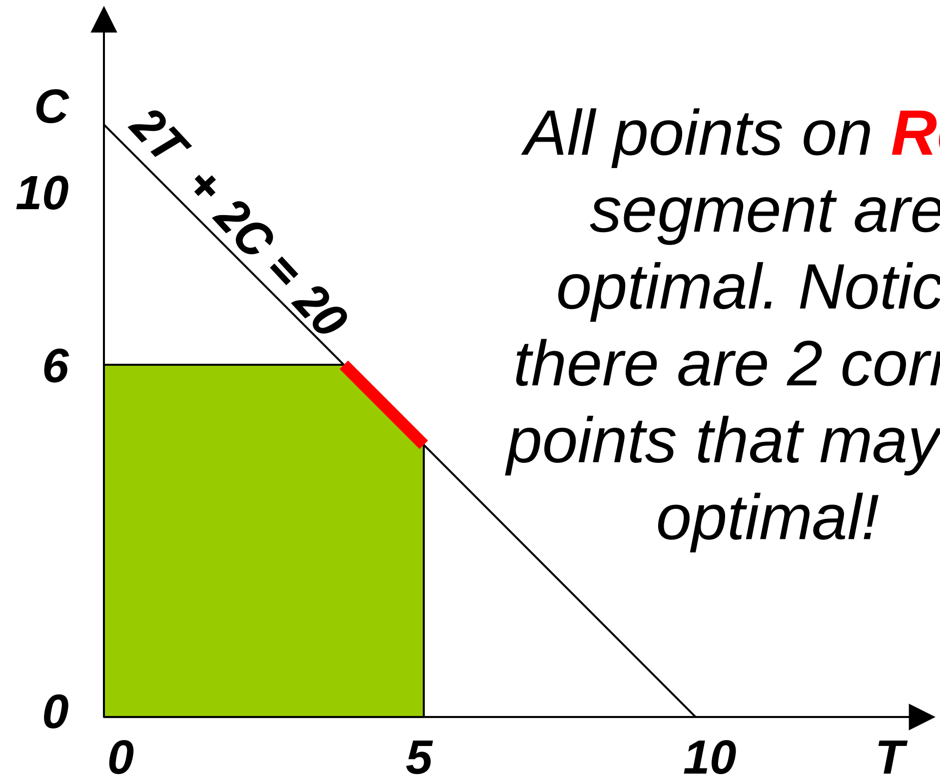
LP Formulation: Interpreting the Results

- **Multiple optimal solutions exist:**
  - If there are multiple optima, there must be at least one optimal corner point solution.

Maximize 2T + 2C

Subject to:

$$T + C \le 10$$
$$T \le 5$$
$$C \le 6$$
$$T, C \ge 0$$



All points on ***Red*** *segment are optimal. Notice there are 2 corner points that may be optimal!*

LP Formulation: Interpreting the Results

Dissecting Problems:

- – Redundant Constraints
- – Multiple Optimal Solutions
- – Infeasible Solution
- – Unbounded Solutions

A feasible solution does not exist!

LP Formulation: Interpreting the Results

- **Infeasible Solutions:**
  - No point satisfies all the constraints.

Example:

$$x \leq 10$$
$$x \geq 15$$
$$x \geq 0$$

  - The LP cannot be solved to optimality.
  - To obtain even a **feasible solution** to the LP, some of the constraints associated with the infeasibility must be removed.

LP Formulation: Interpreting the Results

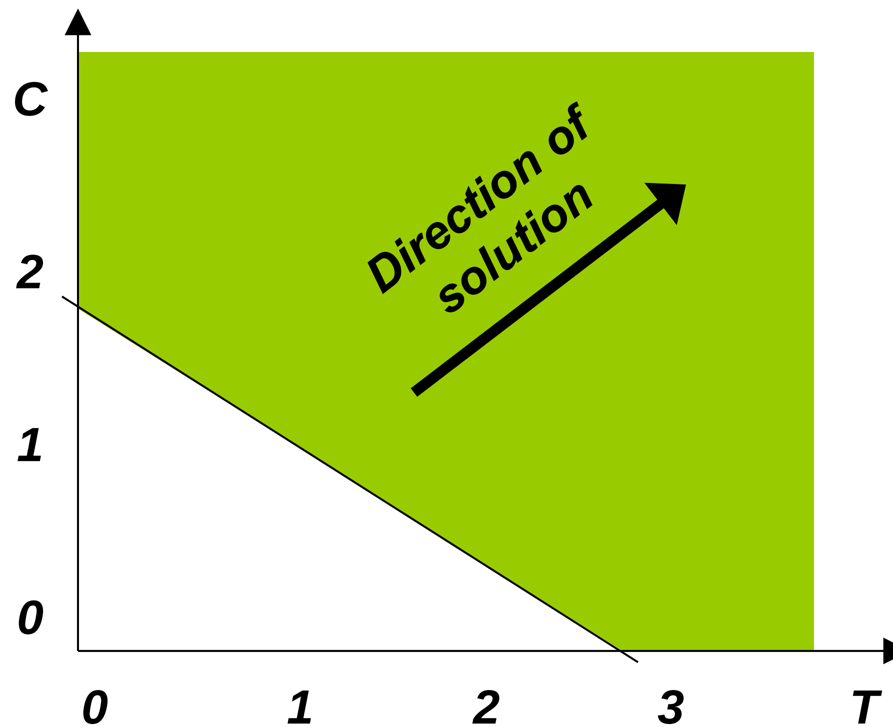
- **Unbounded Solutions:**
  - Nothing prevents the solution from becoming infinitely large (e.g., the objective is infinite).
  - Solution can be improved without limit!

Maximize 2T + 2C

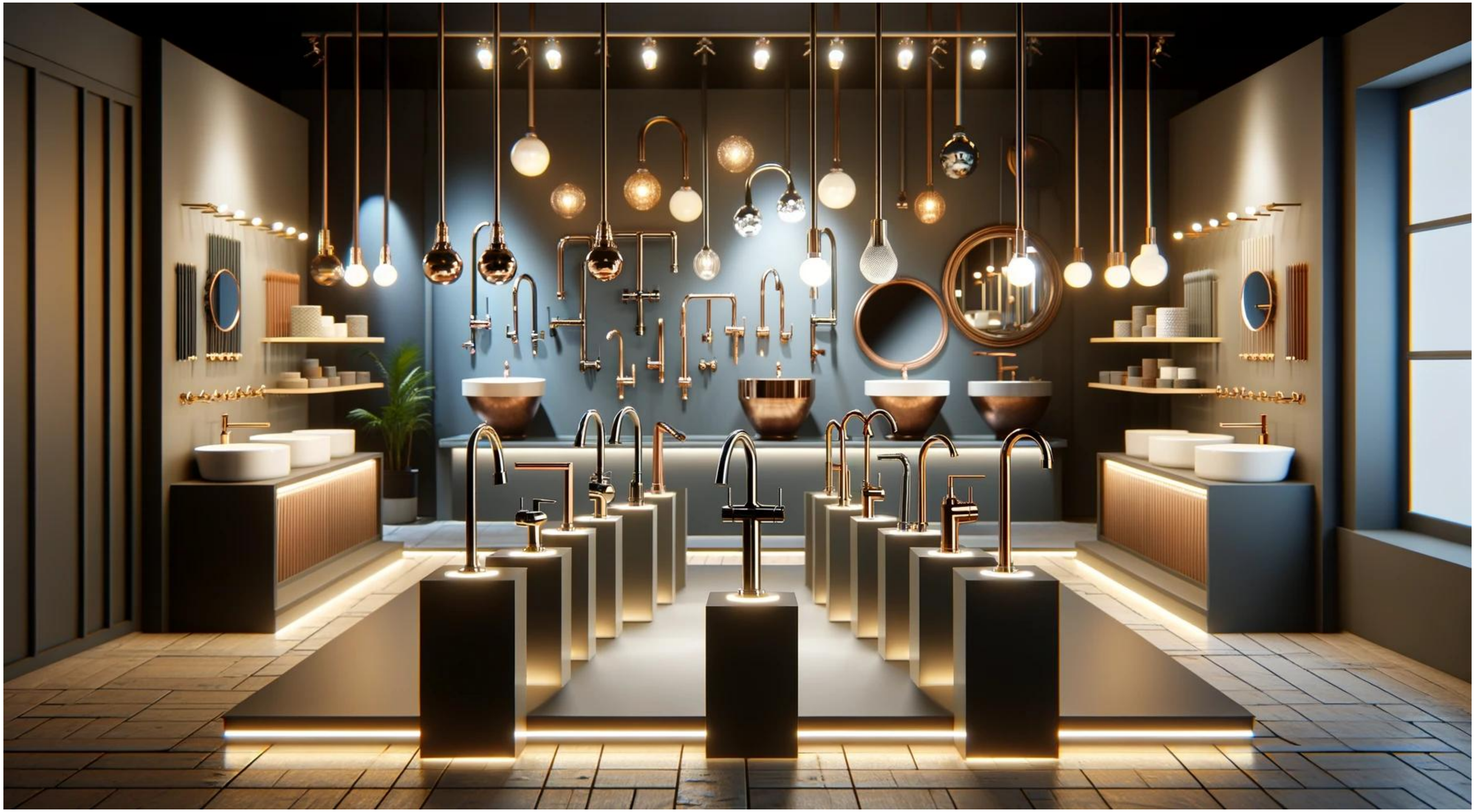
Subject to:

$2T + 3C \geq 6$

$T, C \geq 0$



Delton Company Example



Delton Company Example

The Delton company manufactures and sells two types of eco-friendly faucets: **AS & HL**. The operations department needs to decide how many of each faucet type to produce per day to **maximize daily profit** based on the available resources.

	AS	HL
Filters	1	1
Labour	9 hours	6 hours
Bolts	12	16
Unit Profit	$350	$300

There are 200 filters, 1566 person-hours of labour, and 2880 bolts are available.

Delton Company Example

Define the objective

Maximize daily profit

Define the decision variables

Write the mathematical objective function

Delton Company Example

Define the objective

Maximize daily profit

Define the decision variables

$x_1 =$ ***the number of AS to produce***
$x_2 =$ ***the number of HL to produce***

Write the mathematical objective function

Maximize $Z =$

Delton Company Example

Define the objective

Maximize daily profit

Define the decision variables

$x_1 =$ ***the number of AS to produce***

$x_2 =$ ***the number of HL to produce***

Write the mathematical objective function

Maximize $Z = 350x_1 + 300x_2$

Delton Company Example

Formulating the constraints

There are four constraints:

1. Filter constraint
2. Labour constraint
3. Bolts constraint
4. Non-negativity constraints

Delton Company Example

Formulating the constraints

Ensure that we do not exceed any of the resources.

(filter constraint)
(labour constraint)
(bolts constraint)

	AS	HL
Filters	1	1
Labour	9 hours	6 hours
Bolts	12	16
Unit Profit	$350	$300

Delton Company Example

Formulating the constraints

Ensure that we do not exceed any of the resources.

$$x_1 + x_2 \leq 200$$

(filter constraint)
(labour constraint)
(bolts constraint)

	AS	HL
Filters	1	1
Labour	9 hours	6 hours
Bolts	12	16
Unit Profit	$350	$300

Delton Company Example

Formulating the constraints

Ensure that we do not exceed any of the resources.

$$\boldsymbol{x_1 + x_2 \leq 200} \quad \text{(filter constraint)}$$
$$\boldsymbol{9x_1 + 6x_2 \leq 1566} \quad \text{(labour constraint)}$$
(bolts constraint)

	AS	HL
Filters	1	1
Labour	9 hours	6 hours
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Delton Company Example

Formulating the constraints

Ensure that we do not exceed any of the resources.

$$\boldsymbol{x_1 + x_2 \leq 200} \quad \text{(filter constraint)}$$
$$\boldsymbol{9x_1 + 6x_2 \leq 1566} \quad \text{(labour constraint)}$$
$$\boldsymbol{12x_1 + 16x_2 \leq 2880} \quad \text{(bolts constraint)}$$

	AS	**HL**
Filters	1	1
Labour	9 hours	6 hours
Bolts	12	16
Unit Profit	$350	$300

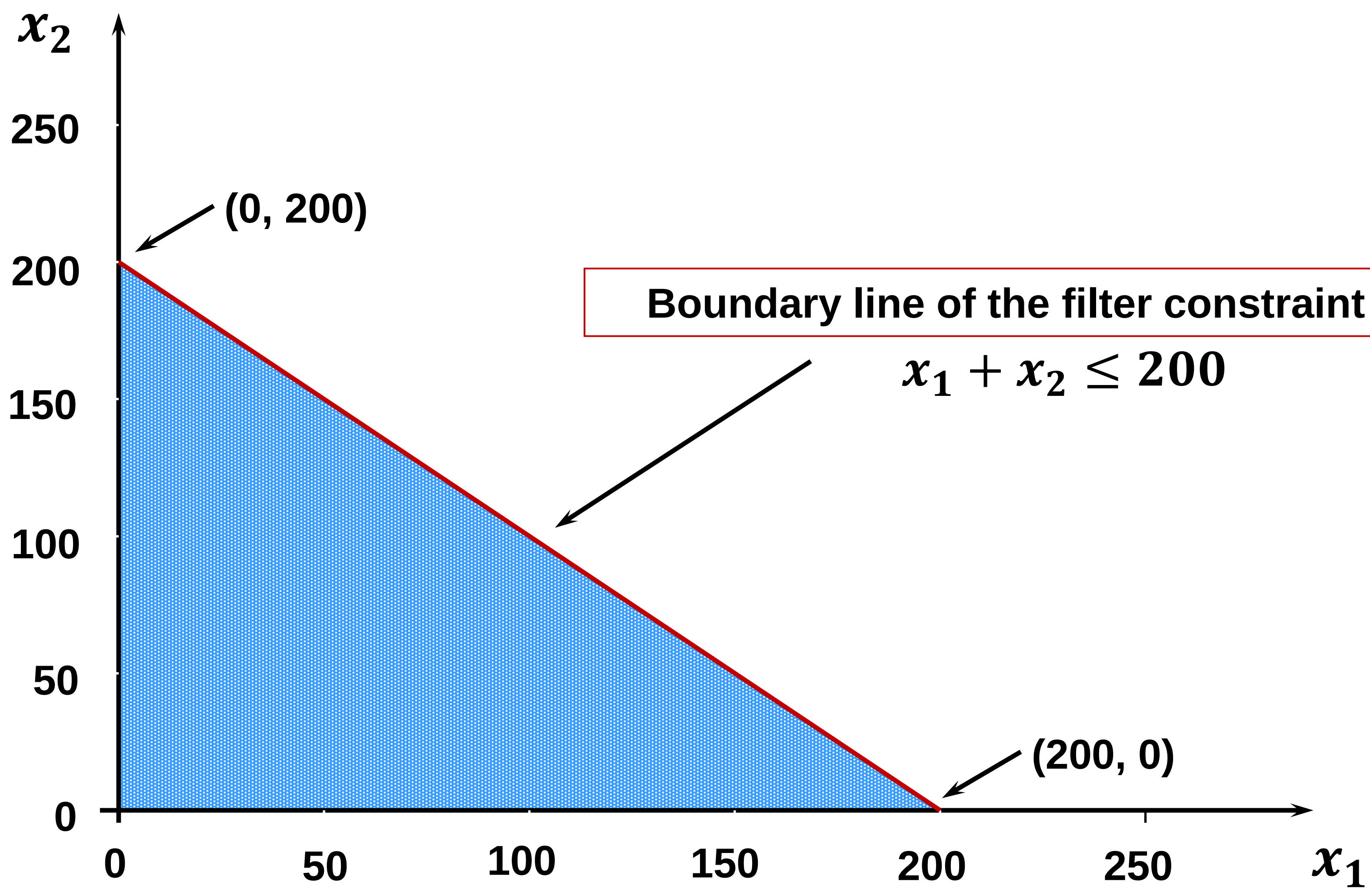
Delton Company Example

Maximize $Z = 350x_1 + 300x_2$

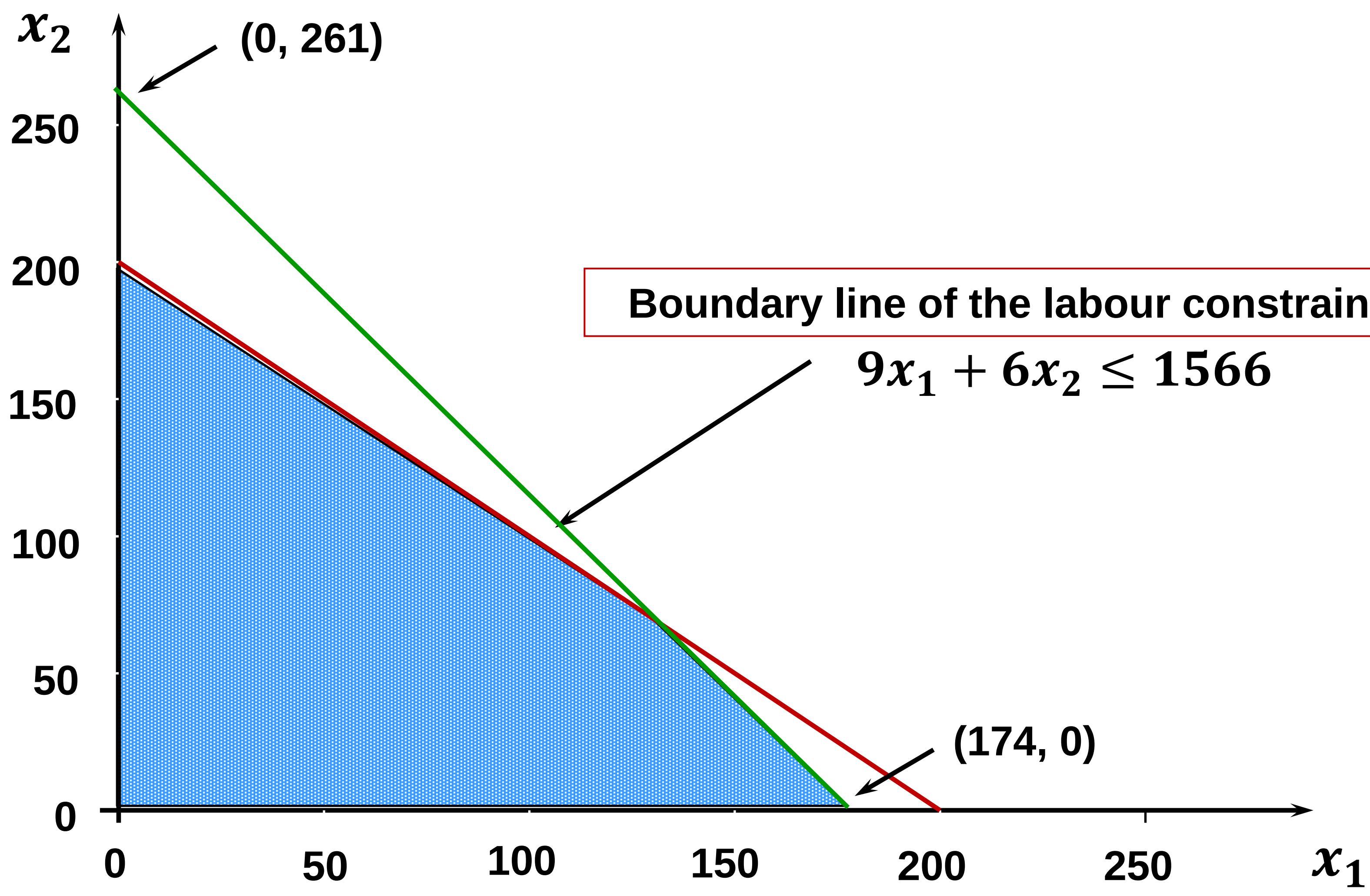
Subject to:

$x_1 + x_2 \leq 200$	(filter constraint)
$9x_1 + 6x_2 \leq 1566$	(labour constraint)
$12x_1 + 16x_2 \leq 2880$	(bolts constraint)
$x_1 \geq 0$	(non-negativity)
$x_2 \geq 0$	(non-negativity)

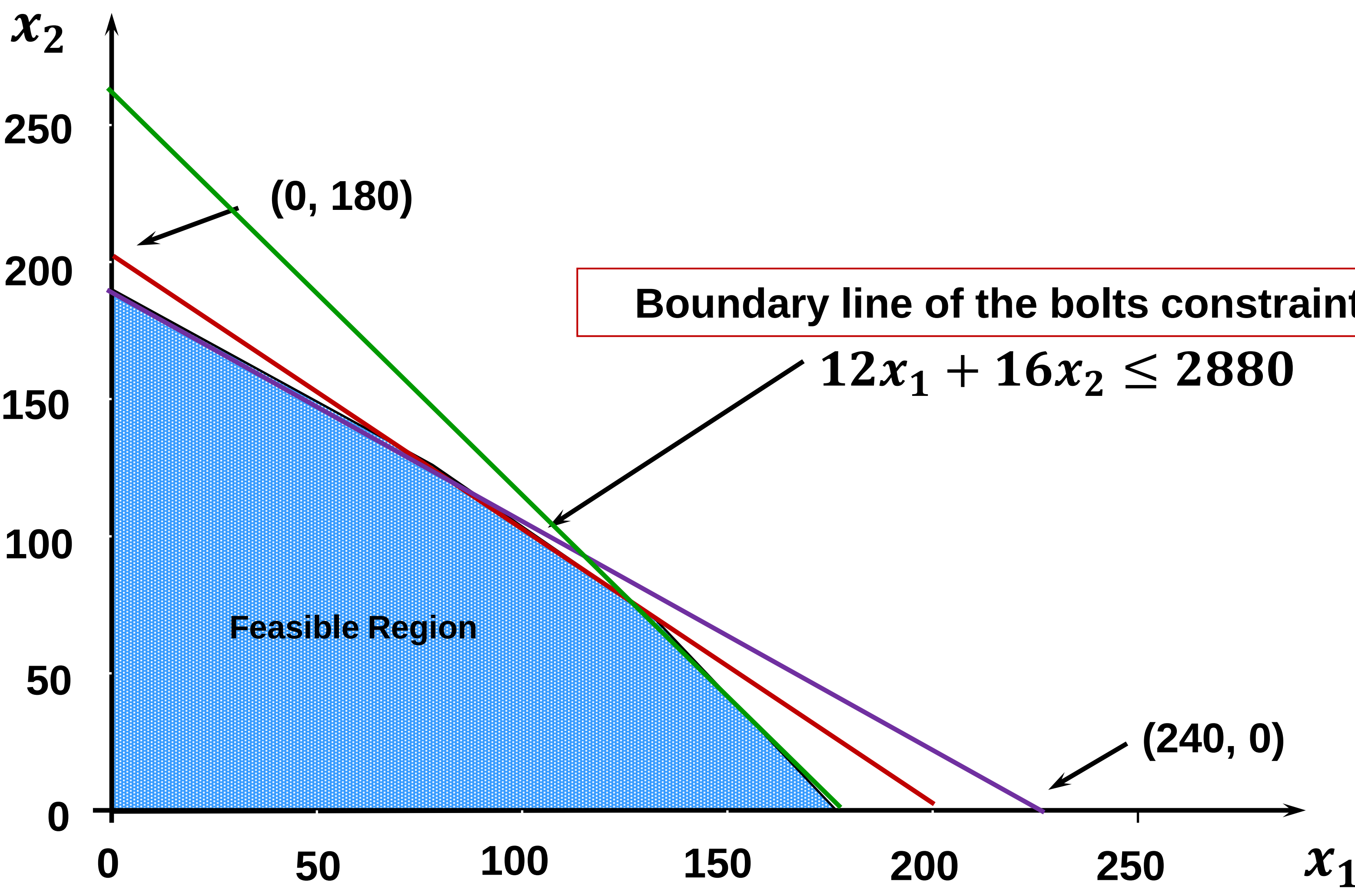
Delton Company Example



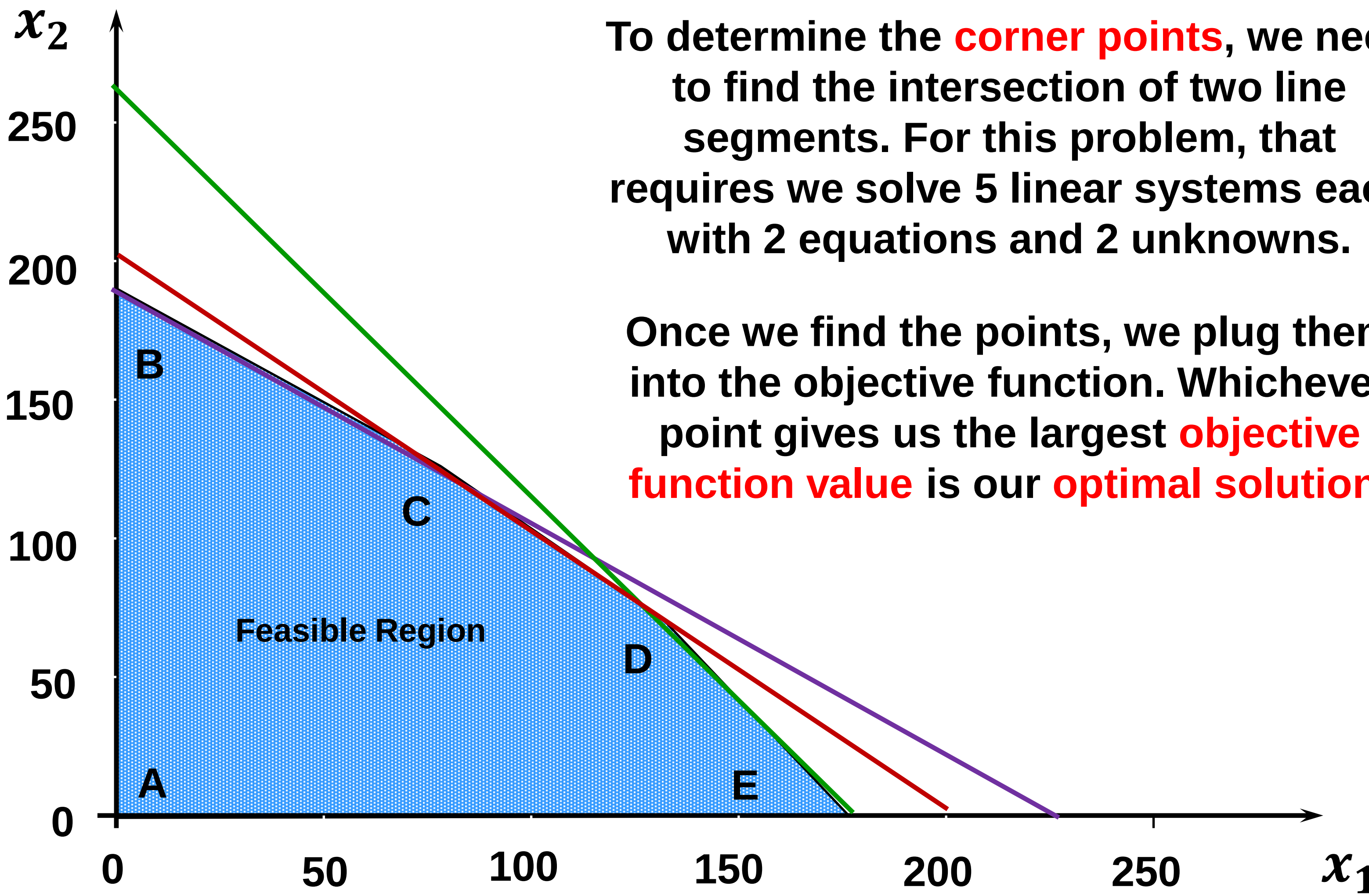
Delton Company Example



Delton Company Example



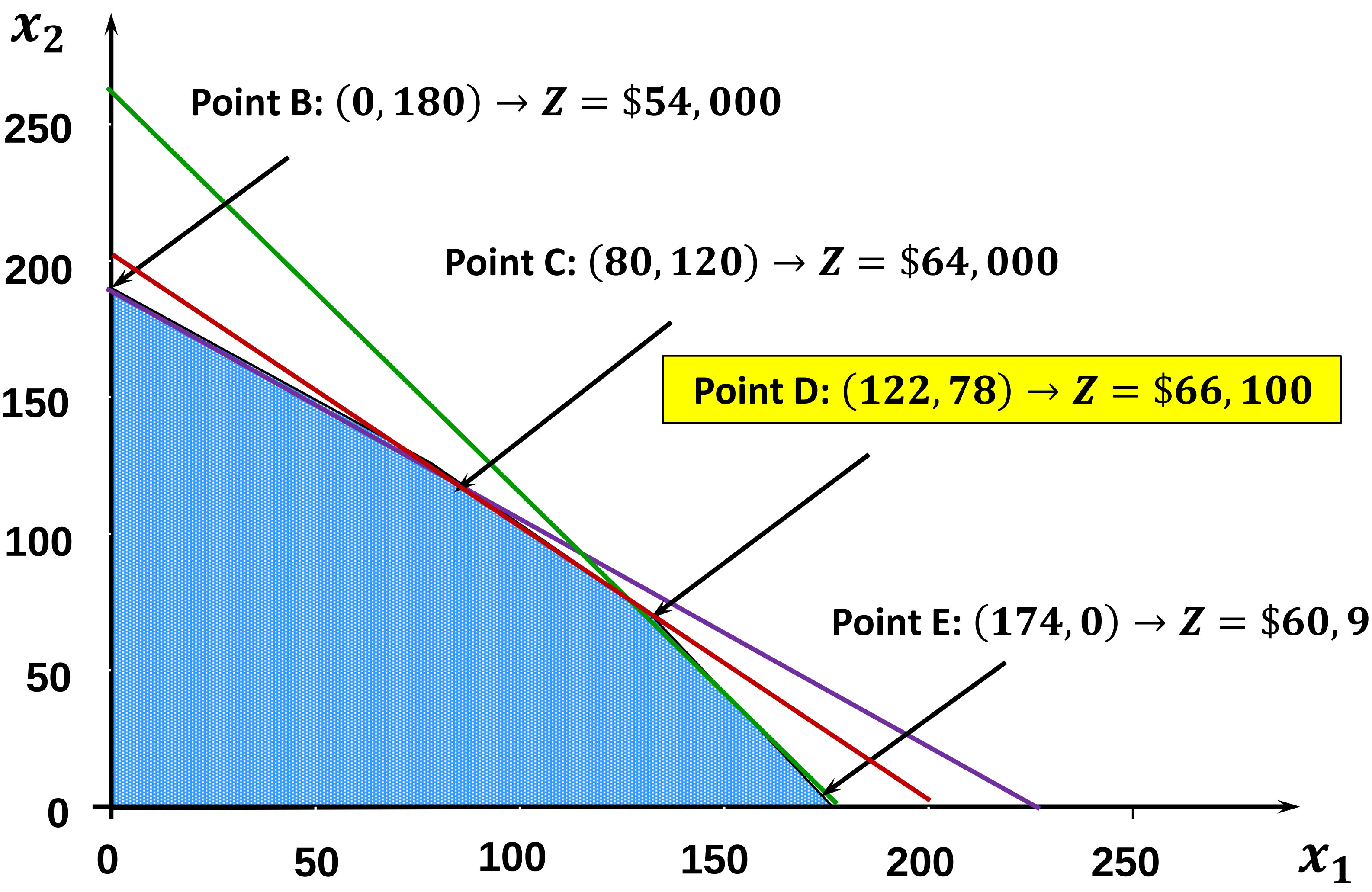
Delton Company Example



To determine the **corner points**, we need to find the intersection of two line segments. For this problem, that requires we solve 5 linear systems each with 2 equations and 2 unknowns.

Once we find the points, we plug them into the objective function. Whichever point gives us the largest **objective function value** is our **optimal solution**.

Delton Company Example



Next Class: Python and Gurobi

- Introducing the Gurobi Library:
  - Creating a new model and add decision variables.
  - Define the objective function and its "*sense*".
  - The `quicksum` function. How to add constraints to the model by taking advantage of loops in Python.
  - Solve the model to optimality and return the objective function and components of the optimal solution.
- Formulate linear programs with Gurobi and use ChatGPT as a personal assistant (or copilot).
- Finite convergence of the Simplex algorithm, a computational approach to solving LPs.