Question 1 [8 points]

BioAgri Solutions, a company specializing in bio-fertilizer production, sources raw material from farms across the United States. The raw material is first sent to intermediate processing facilities before the resulting fertilizer is distributed to home centers nationwide. BioAgri Solutions aims to optimize its network in order to minimize total costs while adhering to the capacity limits of the farms and the processing facilities. To facilitate this, they have collected three data sets:

***farms.csv*:** A list of farms, the amount of raw material that can be collected at that farm, a quality indicator (1 is the worst, 4 is the best) of the material, the cost per ton of raw material, and the cost per ton of transporting the raw material from the farm to a processing facility.

***processing.csv*:** A list of processing plants, the region of the US they are located in (North, East, South, West), the plant capacity, the processing cost per ton, and the cost per ton of transporting the fertilizer from the processing facilities to the home centers.

***centers.csv*:** A list of home centers, the region of the US the retailer is located in (North, East, South, West), and the requested demand of fertilizer (tons).

Using the data sets that have been provided, create a transportation and procurement plan that minimizes the total cost while satisfying the demand of each of the home centers.

(a) How many sources of costs must be considered? How many decision variables are there?

(b) Using Gurobi, what is the minimum cost of the transportation and procurement plan?

(c) If the processing plants of the raw material are restricted to only send fertilizer to home centers within the same region of the US, what is the optimal cost?

(d) If only the highest quality raw material (i.e., levels 3 and 4) is sourced from farms to make fertilizer, what is the optimal cost?

(e) If each facility is limited to processing no more than 3% of all raw material sourced from farms (as a sourcing risk mitigation measure), what is the optimal cost? Alternatively, if a production facility is limited to supplying no more than 50% of all fertilizer to a single home center (as a supply risk mitigation measure), what is the optimal cost?

(f) Four options were evaluated to understand how changes to the supply chain impacted cost, i.e., see parts (c) through (e). Which of these options (or multiple) are financially defensible, and why? What is the optimal cost when you implement *all of the defensible options* together?

(g) While implementing all of the defensible options together incurs a higher cost as compared to the original system, it may still represent a strong business decision. How would you concisely defend the implementation of *all of the defensible options* to management?

(h) The supply chain network has a limited capacity for risk mitigation. To see this, when implementing all of the defensible options from part (f), at what value (to the nearest tenth of a percent) does the model become infeasible when reducing the sourcing risk mitigation percentage from the value given in part (e) of 3%? What is the managerial interpretation of this result, and what are the implications for managing supply chain risk?

Question 2 [8 points]

As part of a shift towards data-driven planning, RP Strength aims to use data analytics to design an optimized weight lifting program that achieves multiple fitness goals while balancing time and recovery. A comprehensive dataset of 2,637 weightlifting exercises has been compiled, covering beginner, intermediate, and advanced levels. This dataset includes details on muscles targeted, required equipment, estimated time (in minutes) to complete 4 sets of 10 reps (the most common rep range), stimulus-to-fatigue ratio (SFR), and each exercise's hypertrophy rating (HR). The stimulus-to-fatigue ratio ranges from 0 to 1, with a value of 1 indicating an excellent hypertrophic exercise that causes minimal fatigue. The hypertrophy rating also ranges from 0 to 1, where a rating of 1 signifies an outstanding muscle-building exercise when performed correctly. The objective is to allocate a proportion of the workout program to each exercise in order to maximize the overall hypertrophy rating, while satisfying several constraints that can be customized to each lifter. Common constraints include:

1. No single exercise should account for more than 5% of the total workout program.
2. Each body part should be included in at least 2.5% of the program, except for Traps, Neck, and Forearms, which require at least 0.5%, and Abdominals, which should make up at least 4%.
3. Leg muscles (i.e., Adductors, Abductors, Calves, Glutes, Hamstrings, Quadriceps) must receive at least 2.6 times the allocation than all upper body exercises combined.
4. The proportion of the workout program devoted to Biceps and Triceps training should be equal, as should the proportion of the program devoted to Chest and All Back exercises combined.
5. The overall stimulus-to-fatigue (SFR) ratio of the program should not exceed 0.55.
6. The proportion of Beginner exercises should be at least 1.4× that of Intermediate exercises, and the proportion of Intermediate exercises should be at least 1.1× that of Advanced exercises.
7. Strongman exercises should consist of less than 8% of the workout program, Powerlifting exercises should exceed 9%, and Olympic Weightlifting exercises should exceed 10%.
8. The proportion of the workout program utilizing Barbells, Dumbbells, Machines, Cables, the E-Z Curl bar, and Bands should exceed 60% of the workout program.

Formulate a linear program to determine the proportion of the workout program that should be allocated to each exercise listed in the *updated_gym_data.csv* data set.

(a) How many decision variables are in the optimization problem and what is their range?

(b) The objective is to *"allocate a proportion of the workout program to each exercise."* Explain why this approach is more practical than specifying exact exercises for each session.

(c) Using Gurobi, what is the optimal hypertrophy rating using all constraints?

(d) If the SFR requirement (i.e., constraint 5) were relaxed by 0.001, how much would the hypertrophy rating of the workout program improve by? Is this estimate valid?

(e) Is there value in increasing the minium proportions for Traps, Neck, Forearms, or Abdominals?

(f) Barbell Back Squats (my favorite exercise) are currently not included in the workout program. By how much would their hypertrophy rating need to increase for them to be included?

(g) Examine the optimal workout program that is suggested. You will notice that it includes few exercises commonly found in standard weightlifting routines. Discuss two possible reasons for this.

(h) Suppose that all of the *common constraints* are removed except $\{1, 2, 8\}$ from the list above. What is the optimal hypertrophy rating, and why is it higher than in the original solution?

(i) Formulate and solve the dual linear program for model in part (h) demonstrating that the model you create is, indeed, the correct dual problem of the primal formulation.

(j) Which formulation, the primal or the dual model, do you think is easier to solve?