Lens Models in QLens

In the following lens models, ξ refers to the elliptical radius defined by $\xi = \sqrt{qx^2 + y^2/q}$. With the exception of the multipole models, all the formulas are written such that x is the major axis in the frame of the lens model. However, in accordance with custom, the default orientation of the major axis points along the y-axis in the lens plane ("observer's coordinates"), and then is rotated counter-clockwise by a specified angle θ. (If you prefer the orientation angle to be with respect to the x-axis of the lens plane instead, you can do this with the command "major_axis_along_y off").

ptmass: (point mass)

$$\kappa = \pi b^2 \delta(r) \tag{1}$$

alpha:

$$\kappa = \frac{2-\alpha}{2} \cdot \frac{b^\alpha}{(\xi^2 + s^2)^{\alpha/2}} \tag{2}$$

The normalization is chosen so the b parameter approximates the average Einstein radius, and b is the exact Einstein radius if $s = 0$, $q = 1$. The isothermal ellipsoid is the special case $\alpha = 1$. In the special cases where either $q = 1$, $\alpha = 1$ or $s = 0$, analytic formulas are automatically used for the lensing calculations; otherwise, for more general values (the case where $q \neq 1$, $\alpha \neq 1$ and $s \neq 0$) numerical integration must be used and the calculations are slower.

Also note that you *must* choose $\alpha < 2$ or the mass profile will diverge at small radii; in fact, the lens becomes a point mass in the limit $\alpha \to 2$ (assuming the core size s is set to zero). On the other hand, you must always choose $\alpha \geq 0$ or the density will diverge at large radii; in the limit $\alpha \to 0$, the lens becomes a uniform mass sheet. Thus we are confined to the range $0 \leq \alpha < 2$. I have found that for $\alpha \gtrsim 1.5$ and $q \neq 1$, $s \neq 0$, the numerical integrals become inaccurate by more than a few percent (at least, using the default number of integration points (20); however this can be increased for greater accuracy, but then it runs slower). Equivalently, if Romberg integration is used instead of Gaussian quadrature, it takes an increasingly long time for the integrals to converge. On the other hand, the integrals are accurate to within much smaller than a percent for $\alpha \lesssim 1$.

shear: (quadrupole external shear term)

$$\phi_\gamma = -\frac{1}{2}\gamma r^2 \cos\left[2\left(\theta - \theta_\gamma - \frac{\pi}{2}\right)\right] \tag{3}$$

For this and all the other multipole models, if "major_axis_along_y" is turned off, there is no $\frac{\pi}{2}$ phase shift in the sinusoidal term.

mpole: (general potential multipole term)

$$\phi_m = -\frac{1}{m} A_m r^n \cos\left[m\left(\theta - \theta_m - \frac{\pi}{2}\right)\right], \text{ or} \tag{4}$$

$$\phi_m = -\frac{1}{m} B_m r^n \sin\left[m\left(\theta - \theta_m - \frac{\pi}{2}\right)\right] \tag{5}$$

The user can specify whether to use a cosine or sine term. Note, the normalization is chosen so that $m = 2$, $n = 2$ corresponds to an external shear term with $A_2 = \gamma_{ext}$. If $m = 0$ is chosen, the $1/m$ factor is suppressed to avoid infinities. One can show that the monopole ($m = 0$) case is equivalent to the *alpha* model with $s = 0$, $q = 1$, where $n = 2 - \alpha$, $A_0 = -b^\alpha/(2 - \alpha)$.

kmpole: (kappa multipole term)

$$\kappa_m = A_m r^{-\beta} \cos\left[m\left(\theta - \theta_m - \frac{\pi}{2}\right)\right], \text{ or} \tag{6}$$

$$\kappa_m = B_m r^{-\beta} \sin\left[m\left(\theta - \theta_m - \frac{\pi}{2}\right)\right] \tag{7}$$

An important caveat is that we require $\beta \neq 2 - m$ or else deflections become infinite. Also note that the monopole ($m = 0$) case is equivalent to the *alpha* model with $s = 0, q = 1$, where $\beta = \alpha$, $A_0 = \frac{1}{2}(2 - \alpha)b^\alpha$.

pjaffe: (Pseudo-Jaffe profile, i.e. truncated isothermal profile)

$$\kappa = \frac{b}{2}\left[\frac{1}{(\xi^2 + s^2)^{1/2}} - \frac{1}{(\xi^2 + a^2)^{1/2}}\right] \tag{8}$$

The special case $s = 0$ (no core) and $q = 1$ (axially symmetric) is often used for substructure modeling, and takes the form:

$$\kappa = \frac{b}{2}\left[\frac{1}{r} - \frac{1}{(r^2 + a^2)^{1/2}}\right] \tag{9}$$

expdisk: (Exponential disk)

$$\kappa = \kappa_0 \exp\left[-\xi/R_d\right] \tag{10}$$

For the kappa profiles corresponding to the NFW and Hernquist (*nfw*, *hern*) models (if you really want to see them), please see Keeton (2002).