

A Morphological Tree of Shapes for Color Images

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Abstract—In mathematical morphology the tree of shapes of a gray level image is a versatile representation that allows for multiple powerful applications. That structure is highly interesting because it is a self-dual representation invariant by contrast changes and since many authors state that object contours are well described by level lines. Such a representation has not yet been defined (thus used) on color images because *a priori* a total order on colors is required that really make sense on data. In this paper we propose a solution to obtain a tree of shapes on color images without resorting to an ordering of colors. To that aim we relax the definition of shapes and we show that relevant applications follow from our proposal.

Keywords—Mathematical morphology, color images, tree of shapes, connected filters.

I. INTRODUCTION

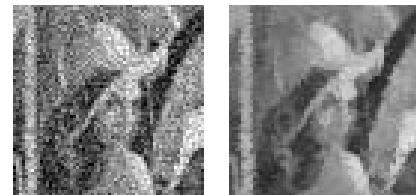
Mathematical morphology operators can be divided into two large classes: the most known operators make use of structuring elements, whereas connected operators [1] are based on neighborhoods and connected components. The prominent property of the latter is that they do not shift contours. Many connected filters on gray level images are dual, for instance, algebraic openings and closings [2], and can be defined from the min-tree and/or the max-tree. This couple of dual trees represent the image and encodes that connected components of respectively lower and upper thresholds cuts (also called *cuts* [3]) form a tree w.r.t. component inclusion. A self-dual tree has been defined by Monasse and Guichard [4], called *tree of shapes*. It describes the image contents in a unique way; such a tree can be understood as the result of merging the dual trees components. Some new connected operators can be derived from that tree, that make no assumption about the contrast of image components: the inclusion relationship can be due either to light objects surrounded by darker ones, or to the contrary. As a consequence self-dual operators process light and dark objects in the same way.

The tree of shapes is a morphological tool with a strong potential, that has been underexploited. Though its applications are numerous: image filtering [5, 6], simplification [7], and segmentation [8, 9, 10], and also texture indexing [11] and object recognition [12] (additional references about applications can be found in [13]). The reason why the tree of shapes is interesting is reported by several authors who claim that, in gray level images, object contours coincide with level lines [14, 15, 16]. Indeed, in [17] we have shown that object

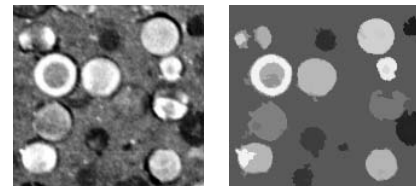
detection based on the tree of shapes can outperform the Chan and Vese method.

Yet obtaining a tree of shapes for color images is challenging. In a gray level image f , shape contours are level lines; they thus correspond to connected components of $\{x \mid f(x) = \lambda\}$, where λ is a gray level. Transposing that definition to a color image means that we expect every objects contained in this image to be surrounded by a curve which has a constant color λ . Of course that hardly happens in images, which makes this definition of shapes unusable in practice. Another idea to construct a tree of color shapes is to rely on the classical approaches of adapting a gray-level-based method or operator to color images, as performed in [10] and [18]. Unfortunately we will see that the trees obtained that way are not so well formed since *a posteriori* applications (noise removal and segmentation) do not lead to proper results.

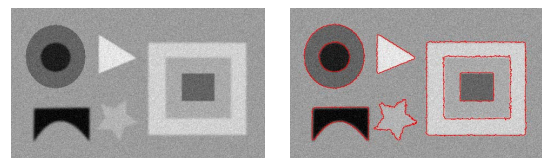
To the authors knowledge the present paper is the first



(a) Denoising (self-dual grain removal).



(b) Shape Filtering (keeping round objects) [6].



(c) Object Detection (energy-based method) [17].

Fig. 1. Sample uses of gray-level tree of shapes (left column: input images; right column: state-of-the-art results).

attempt to define a tree of shapes for color images *without* imposing some arbitrary or questionable ordering of colors. This paper is organized as follows. In section II we take a tour of the classical approaches for processing color images that can be used to construct trees of shapes. In section III we detail our proposal and we present, compare and discuss the results in section IV. Finally in section V we conclude and give some perspectives of our work.

II. STATE OF THE ART

When dealing with hierarchical representations of images, we can distinguish two classes of trees: hierarchies of segmentation and morphological trees. The first approach is directly linked to hierarchical clustering. It aims at either growing and merging regions in a bottom-up fashion or splitting regions in a top-down fashion. Among some well-known hierarchical segmentation trees [19], ultrametric watersheds [20], binary partition trees [21], or alpha-trees [22] have been successfully applied to segmentation, object detection, image matching [6, 23]... The force of those methods when dealing with colors lies in that they only require a *distance* function between values. Any norm on colors has sense (even if some colorspace or norms may be closer from human perception than others), whereas an ordering of the colorspace is less obvious. Unfortunately, a total ordering is required to compute the second class of trees. While hierarchical segmentation trees consider clustering of adjacent region, morphological trees focus on the inclusion relationship of components. The components result from thresholding the image at different levels, and a total order on the image levels ensure the level sets inclusion. In what follows, we present the classical approaches to deal with the absence of a natural total order.

A. Marginal Processing

A common and simple approach when dealing with multivariate data is split each component and process them independently. In our case, it would result in computing trees of shapes for the red, green, and blue channels of the original image and filter each of them. Red, blue and green images are reconstructed from the three trees of shapes and merged back to give the final result. Although this simple method gives visually good results when the objective remains filtering or image simplification, it suffers from two majors drawbacks. First, algebraic properties of grain filters are not preserved. Filtering red, blue and green images with a grain of size λ implies that extreme components have an area greater than λ . However, since these components do not share the same location, the merge procedure introduces component overlap and noise in large flat zones. Second, this method remains useless for computer vision purposes. Since it does not build a single hierarchical representation of the image but rather three trees, computer vision methods that relies on node identification in a single tree cannot be applied.

B. Imposing a Total (Pre-)Order

A classification of ordering relations can rely on their algebraic properties (totality, anti-symmetry...) or with respect to the way these relations are built. Barnett [24] proposed to classify them into four groups: marginal ordering (M-ordering), conditional ordering (C-ordering), partial ordering

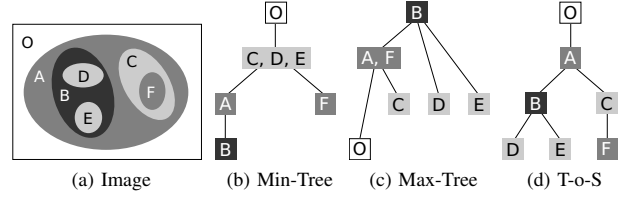


Fig. 2. An image (a), and its morphological component trees (b) to (d).

(P-ordering) and reduced ordering (R-ordering). M-ordering, is a component-wise ordering that deals separately with each channel. Since it is a partial order, it is out of scope of the current section.

In C-ordering, vectors are ordered by mean of one, several, or all of their marginal components. The most well-known C-ordering is the lexicographical ordering, that is a total order. If only some components participate in the comparison, it yields to a total-pre-order. For example let $v, w \in \mathbb{R}^n$, the lexicographical ordering \leq using only the first two components defined as $v \leq_L w$ iff. $(v_1 < w_1) \vee (v_1 = w_1 \wedge v_2 \leq w_2)$ is a total-preorder. Colors (1, 1, 2) and (1, 1, 3) are considered as *equivalent* by the above relation. The main pitfall of the C-ordering resides in the importance given to first components. In the case of the RGB space, it implies for example that the red component is more relevant than the others. Several workarounds have been proposed [25, 26] like changing the color space (LSH is commonly used) or sub-quantization of first components (known as α -lexicographical ordering). This enables to lower the importance given to the first dimension but still introduce some color artifacts.

In R-ordering, vectors are reduced to scalar values using a mapping $h : \mathbb{R}^n \rightarrow \mathbb{R}$. Then, for two vectors $v, w \in \mathbb{R}^n$ a new relation \leq_R is defined as $v \leq_R w$ iff. $h(v) \leq h(w)$. If h is injective then each index is mapped to a unique color and the relation is a total order, otherwise it is a total pre-order. Typical examples of R-ordering are distance-based orderings [27]. It consists in choosing a reference vector (or a reference set of vectors) v_{ref} and the order relation is built upon a distance to v_{ref} , i.e., $v \leq_R w$ iff $d(v, v_{ref}) \leq d(w, v_{ref})$. The main drawback of distance-based orders lies in the choice of a reference vector (or a set of reference vectors). More sophisticated methods can use statistical learning to rank colors [28, 29] but actually they are yet another kind of R-ordering.

C. Other Approaches

Since the natural order on colors is partial and forms a lattice, Passat and Naegel [30, 31] proposed an extension of min trees (resp. max-trees) to lattice representing inclusion of lower cuts (resp. upper cuts). Since this representation is not self-dual, it is out of scope of this topic and the study of a *self-dual lattice of shapes* is postponed as a further work.

III. PROPOSED METHOD

A. Basic Notions

Let an image $f : \Omega \rightarrow (E, \leq)$ and $\lambda \in E$. We note $[f \leq \lambda]$ (resp. $[f \geq \lambda]$) a lower cut (resp. upper cut) of f defined as $[f \leq \lambda] = \{x, f(x) \leq \lambda\}$. Let $CC(X), X \in \mathcal{P}(E)$,

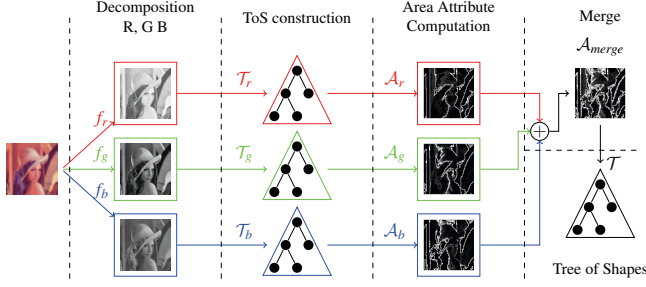


Fig. 3. Scheme of the proposed method.

denote the set of connected components of X . If \leq is a total relation, any two connected components $X, Y \in CC([f < \lambda])$ are either disjoint or nested, thus the set $CC([f < \lambda])$ endowed with the inclusion relation forms a tree called the *min-tree* (see figure 2b). Its dual tree, defined on upper cuts, is called the *max-tree* (see figure 2c). Let the saturation operator $Sat : \mathcal{P}(E) \rightarrow \mathcal{P}(E)$, defined as $Sat(X) = X \cup Y$ with Y hole of X , s.t. $Y \in CC((\Omega \setminus X)^c)$, $Y \subset X$. We call a *shape* any element of $Sat([f \leq \lambda]) \cup Sat([f \geq \lambda])$. If \leq is total, any two shapes are either disjoint or nested, hence form a tree called the *tree of shapes* (see figure 2d).

B. Method Description

The method relies on the simple observations that marginal filtering described in the previous section gives visually good results while methods introducing a total order introduce color artifacts as well. As, we require a unique hierarchical representation, we aim at merging the three trees of shapes from marginal processing in a single tree \mathcal{T} while preserving the following three algebraic properties of trees of shapes:

- a node is a connected component without holes (a shape);
- removing a node boils down to merging some flat zones;
- parent relationship of nodes reflects shape inclusion in f .

The proposed method consists in the following five steps, also depicted in figure 3:

- Step 1. Decompose the color image into its three (red, green, and blue) channels images.
- Step 2. Independently compute their respective trees of shapes $(\mathcal{T}_r, \mathcal{T}_g, \mathcal{T}_b)$.
- Step 3. Compute area attributes as images $(\mathcal{A}_r, \mathcal{A}_g, \mathcal{A}_b)$. To that aim, each node is valued with its area, e.g., the number of pixels of the shape it represents[32]; attribute images are reconstructed from trees, with each pixel getting its value from the corresponding node's area.
- Step 4. Merge attribute images using a point-wise operator; the result is \mathcal{A}_{merge} .
- Step 5. Compute the trees of shapes of \mathcal{A}_{merge} , that is, our tree of shapes of the color input image.

Before explaining the rationale behind this method, let us recall that any tree \mathcal{T} valued with a strictly increasing attribute a can be rebuilt by computing the min-tree of the attribute image \mathcal{A} . Indeed, any cut $CC([\mathcal{A} < \lambda])$ matches in the same time a single node in \mathcal{T} (because a is increasing) and a node of in the min-tree of \mathcal{A} (by lower cuts based definition

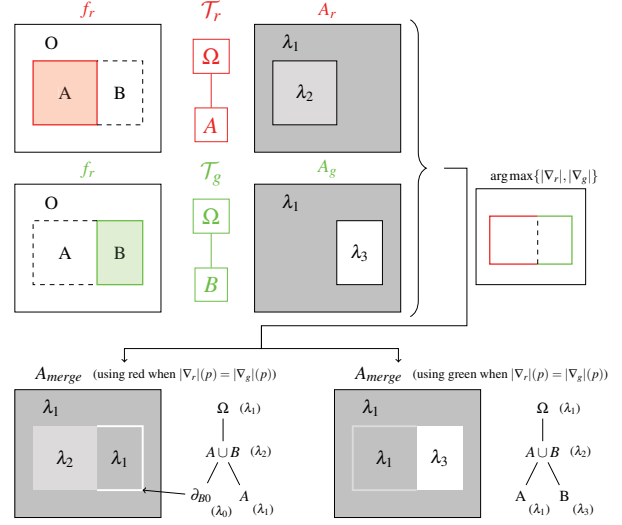


Fig. 4. Illustration of the proposed method on a multi-channel image where chrominance holds geometry.

of min-trees). As a result, instead of merging nodes in the original color space that would lead to overlapping components, merge takes place in an attribute space. The final tree of shapes is recovered from the tree of shapes of the attribute image. *Area* attribute is a simple algebraic criterion—hence totally uncorrelated with the color space—that preserves shape inclusion semantics, e.g., a shape s_1 can only be included in s_2 if $\mathcal{A}(s_1) < \mathcal{A}(s_2)$. Let us now explain, how attribute images are merged together. For each point p , we need to select a scalar value from the triplet $(\mathcal{A}_r(p), \mathcal{A}_g(p), \mathcal{A}_b(p))$. While being purely algebraic, an order on the area values is meaningless, since a large area is not necessarily a good criterion to render shape significance. Since most problems arise when choosing the most significant channel on contours, we propose to merge the trees with gradient guidance. The red, blue, green gradients of each original images are denoted ∇_r, ∇_g and ∇_b . The resulting attribute image is composed from the pixels of the attributes images with the highest gradient magnitude:

$$\mathcal{A}_{merge}(p) = \begin{cases} \mathcal{A}_r(p) & \text{if } |\nabla_r(p)| \geq \max(|\nabla_g(p)|, |\nabla_b(p)|) \\ \mathcal{A}_g(p) & \text{if } |\nabla_g(p)| \geq \max(|\nabla_r(p)|, |\nabla_b(p)|) \\ \mathcal{A}_b(p) & \text{if } |\nabla_b(p)| \geq \max(|\nabla_r(p)|, |\nabla_g(p)|) \end{cases}$$

In order to have the \mathcal{A}_{merge} image suitable for the final tree construction, we need a kind of spatial coherence between the shapes taken from the three trees. This is due to the fact that attribute values are set back in the original image space before the final tree computation. The red, green and blue channels being highly correlated, the marginal trees of shapes are similar in terms of shapes and the merged attribute values are thus spatially coherent. The marginal trees of shapes computed on the CIELAB space for example are very different, so merging their attribute values is not very appropriate.

Figure 4 shows a simple case where geometry information are held by the chrominance. It shows two regions A and B that belongs respectively to red and green channels. If we

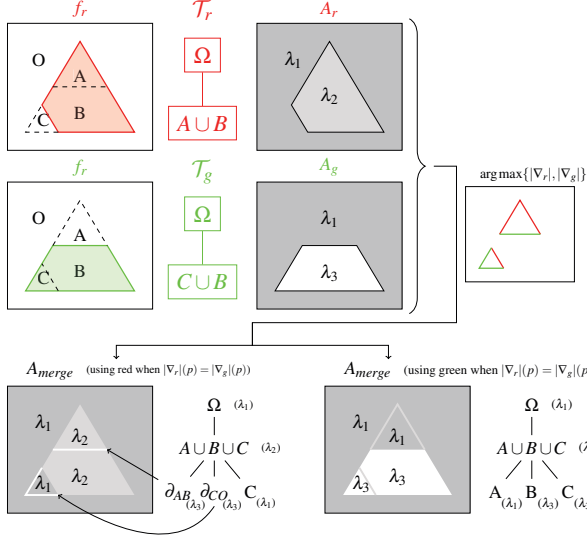


Fig. 5. Illustration of the proposed method on a multi-channel image where each channel taken independently holds incomplete information.

computed the tree of shapes using achromatic information only, it would detect $A \cup B$ as a single shape, but neither A nor B . Using nodes from the trees of shapes computed independently, we get three possible area values $\lambda_1, \lambda_2, \lambda_3$ that relate to regions Ω (the full domain), A and B (we assume $\lambda_1 > \lambda_2 > \lambda_3$). To merge the attribute images, we rely on the gradient which is null everywhere except on the regions' boundaries. The image $\arg \max\{|\nabla_r|, |\nabla_g|\}$ shows in red (resp. green) the location where the maximal gradient comes from the red (resp. green) channel and in white the location where red and green gradient are equals. Thus, pixels of the fusion attribute image A_{merge} in white regions can be computed in two ways: using area values coming either from red or green channel. Computing the tree of shapes of those images afterward results in two different trees but both trees are able to distinct A and B regions.

Figure 5 shows a more complex case where geometry information is split across the channels of the image. The main object, a triangle, is composed by the regions A, B and C . The red channel of the image misses the C region, while the green channel misses the A region. Thus, the trees of shapes computed on those channels independently are not able to fully detect the object. When computing the area of each node, we get three possible values λ_1, λ_2 and λ_3 ($\lambda_1 > \lambda_2 > \lambda_3$) that relates to regions Ω (the full domain), $A \cup B$ and $B \cup C$ respectively. The same reasoning then holds as for the previous example. Afterward, computing the tree of shapes A_{merge} results in a tree holding a node that fully matches the triangle (the level line λ_2).

IV. RESULTS

In the following, we compare an object detection method from Xu et al. [17] on both the tree of shapes of the luminance of the image and the tree of shapes of the color image computed with our proposed method. This method consists in computing a contextual energy composing three terms: the curvature that renders the regularity of a shape, a distance term

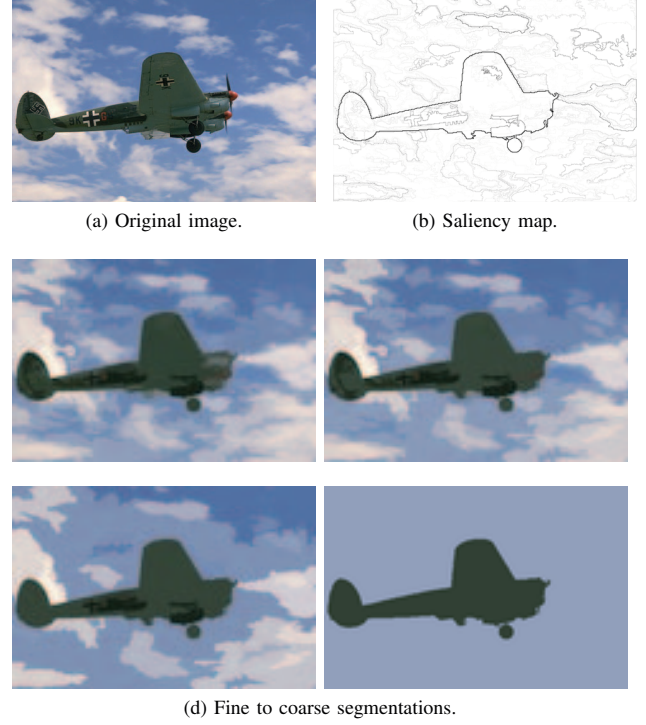


Fig. 6. Hierarchical image simplification using saliency maps built by the proposed method. Thresholding the saliency map with increasing levels yields fine to coarse segmentations.

between the internal and the external distribution around shape boundaries that measures how meaningful is a shape boundary and a constraint term that penalizes small components. The method parameters are the same for valuating nodes of the “gray level” and the color trees of shapes. Afterward, we select all shapes having local minimum energy value and we compute their extinction values. Since an extinction value reflects the persistence of the shape, we can then set this extinction value on shape boundaries to get a saliency map as shown in figures 6 and 7. By thresholding this image, we can filter out some shapes such that only the most significant objects remain and leads to a simplification of the original image. In figures 6 and 8, the remaining objects are filled with the mean of the component values.

Authors acknowledge that since most natural images have geometry information held by the luminance, images have been chosen such that luminance only does not allow to retrieve object shapes. Images are taken from the Berkley Segmentation Dataset [33] and from the Barnard’s Color Constancy Dataset [34].

The calibration image in figure 7 is a particularly good sample of images where chrominance matters, as it proves that using luminance only, some color squares cannot be distinguished from the background and merge rapidly with it. The corresponding saliency map does affirm this proposition. Also, our method tends to improve object detection in real case examples of natural images (see figure 8) where still, the luminance misses geometry information.

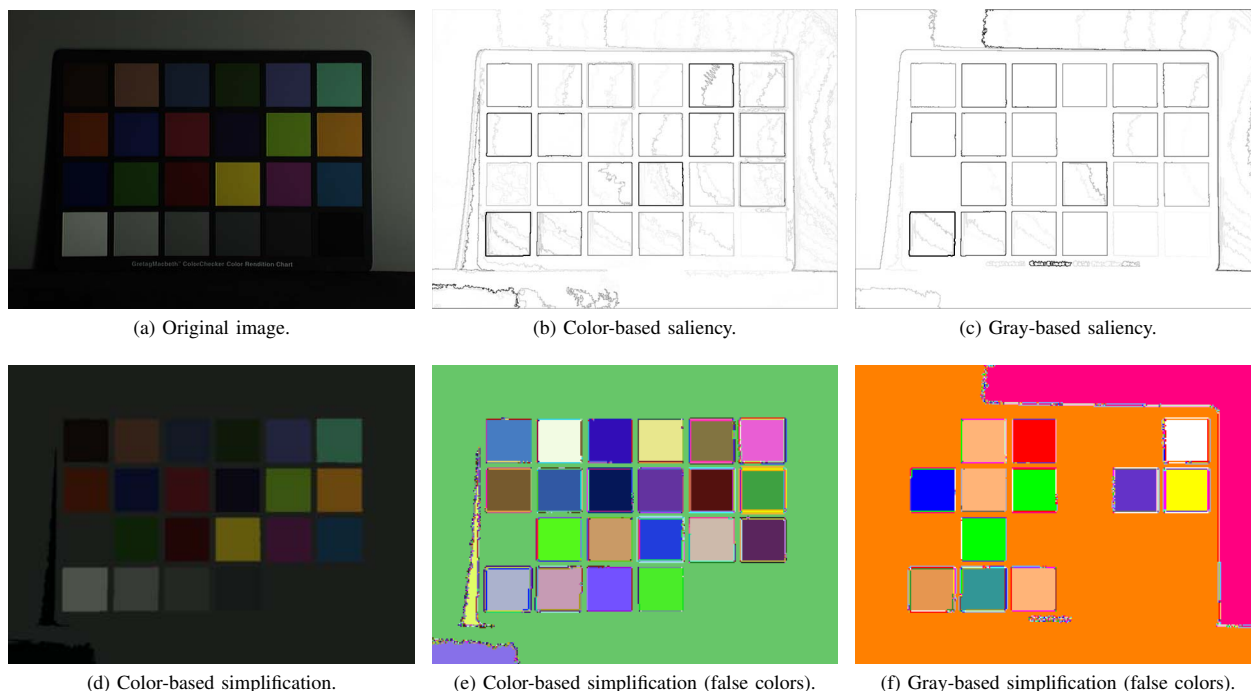


Fig. 7. Saliency-based image simplification: the first two columns rely on the color tree of shapes; the rightmost column relies on the gray-level tree of shapes.

V. CONCLUSION AND PERSPECTIVES

In this paper we have presented a first proposal to compute a tree of shapes for color images whereas the equivalent notion of level lines, iso-color lines, cannot drive anymore the tree definition. The key idea behind our proposal is first to maintain some algebraic properties of shapes and second, to work in the attribute space instead of the original value space.

To obtain the tree of color shapes, we need to compute the trees of shapes on every image channels (Step 2, described in section III-B); for that we can rely on a quasi-linear algorithm given in [3]. Eventually the proposed method has a quasi-linear complexity, which makes it very usable in practice. It has been implemented within our C++ image processing library Olena [35] and it will be available in the next release.

The perspectives of the work presented here are related to the many applications relying on the tree of shapes. The preliminary experiments that we have performed show that the color tree of shapes is well-suited to applications such as filtering, simplifying, and segmenting color images. Some quantitative evaluations of the relevancy of the proposed tree will be the subject of a forthcoming extended paper.

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Fig. 8. Results of simplification. First row: original images. Second row: simplification by thresholding the saliency map of the tree of shapes on luminance. Third row: simplification by thresholding (with the same value) the saliency map of the tree of shapes on colors.

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