7-1) $\int x\cos(5x)\,dx$

$= x\cdot\sin\left(\frac{5x}{5}\right) - \int \sin\frac{5x}{5}\,dx$

$= \frac{x\sin 5x}{5} + \frac{\cos(5x)}{25} + C$

7-2) $\int t e^{-3t}\,dt$

$= -\frac{1}{3}\cdot e^{-3t}\cdot t - \int -\frac{1}{3}e^{-3t}\,dt$

$= \frac{1}{3}e^{-3t} + \int \frac{1}{3}e^{-3t}\,dt$

$= \frac{1}{3}te^{-3t} + \frac{1}{3}e^{-3t}\cdot\left(-\frac{1}{3}\right)$

$= -\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t}$

7-3) $\int \frac{1}{x^2(1+x^2)}$, $\rightarrow \sqrt{a^2+x^2}$

$x = \tan\theta,\ dx = \sec^2\theta$

$\frac{1}{\tan^2\theta(1+\tan^2(\theta))} = \int \frac{1}{\tan^2\theta\sec^2\theta}\cdot\sec^2\theta$

$= \int \frac{1}{\tan^2\theta} = \int \cot^2\theta\,d\theta$

$= \int -\csc^2(\theta) - 1\,d\theta$

$= -\cot\theta - \theta + C$

7-4 $\int \frac{x^2}{\sqrt{9-x^2}}$ $\quad x = 3\sin\theta$, $dx = 3\cos\theta\,d\theta$

$= \frac{(3\sin\theta)^2}{\sqrt{9-(3\sin\theta)^2}}\cdot 3\cos\theta\,d\theta$

$= \frac{9\sin^2\theta}{9(1-\sin^2(\theta))}\cdot 3\cos\theta\,d\theta$

$= \int 9\sin^2\theta$

$= 9\int \frac{1-\cos 2\theta}{2}\,d\theta$

$= \frac{9}{2}\int 1-\cos 2\theta$

$= \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$

7-5) $\int \frac{\sin(\tan x)}{\cos^2 x}$ $\quad u = \tan x,\ du = \sec^2 x$

$\int \sin(u) = -\cos(\tan x) + C$

dx/conv?

7-6) $\int \frac{1}{4\sqrt{1+x}}\,dx = \lim_{b\to\infty} \frac{1}{4\sqrt{1+x}}\,dx$

$u = 1+x,\ du = dx$

$\lim_{b\to\infty}\int_1^{1+b} \frac{1}{4\sqrt{u}}\,du$

$= \lim_{b\to\infty}\int_1^{1+b} u^{-\frac{1}{4}}\,du$

$= \lim_{b\to\infty}\left[\frac{u^{\left(-\frac{1}{4}+1\right)}}{-\frac{1}{4}+1}\right]_1^{1+b}$

$= \lim_{b\to\infty}\left[\frac{u^{\frac{3}{4}}}{\frac{3}{4}}\right]_1^{1+b}$

$= \lim \frac{4}{3}(1+b)^{\frac{3}{4}} - 1^{\frac{3}{4}}$

$= \infty$

div

8-1) length of curve $x = y^2 - 2y$, $0 \le y \le 2$

$f'(y) = 2y - 2$

$L = \int_0^2 \sqrt{1 + (2y-2)^2}\, dy$

$= 2.958$

Calc

8-6) $f(x) = \frac{1}{4}x^4 - \frac{1}{8}x^{-2}$, $x \in [1,2]$

$f'(x)^2 = (x^3 - \frac{1}{4}x^{-3})^2$

$x^6 - \frac{1}{2} + \frac{1}{16}x^{-6}$

$\int_1^2 \sqrt{1 + x^6 - \frac{1}{2} + \frac{1}{16}x^{-6}}\, dx$

$\sqrt{(x^3 + \frac{1}{4}x^{-3})^2}$

$\int_1^2 x^3 + \frac{1}{4}x^{-3}\, dx$

$\frac{1}{4}x^4 - \frac{1}{8}x^{-2} \Big|_1^2 = 3.84$

8-2) $y = \ln(\sec x)$, $0 \le x \le \frac{\pi}{4}$

$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \frac{d}{dx}(\ln(\sec x))^2}$

$= \int \sqrt{1 + (\frac{1}{\sec x} \cdot \sec x \tan x)^2}$

$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x}\, dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x}$

$= \int \sec x = \ln|\sec x + \tan x| \Big|_0^{\frac{\pi}{4}} = \ln(\sqrt{2} + 1)$

8-3) $y = 1 + 6x^{\frac{3}{2}}$

$y' = 0 + 9x^{\frac{1}{2}}$

$L = \int_0^1 \sqrt{1 + (9x^{\frac{1}{2}})^2}$

$= \int \sqrt{1 + 81x}\, dx$ $\quad u = 1 + 81x$, $du = 81\, dx$

$= \frac{1}{81}\int \sqrt{u} = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \Big|_1^{82} = 6.103$

Calc

8-4) $y = xe^{-x}$, $0 \le x \le 2$

$y' = e^{-x} - xe^{-x}$

$L = \int_0^2 \sqrt{1 + (e^{-x} - xe^{-x})^2}\, dx$

$= \sqrt{1 + (x-1)^2 \cdot e^{-2x}}\, dx = 2.104$

8.5 $f(x) = 4x^{\frac{3}{2}}$ $\quad x \in [2, 6]$

$f'(x)^2 = (2x^{\frac{1}{2}})^2 = 4x$

$\int_2^6 \sqrt{1 + 4x}\, dx$

$u = 1 + 4x$, $du = 4\, dx$

$\frac{1}{4}\int u^{\frac{1}{2}}\, du \to \frac{1}{4} \cdot \frac{2}{3}u^{\frac{3}{2}} \to \frac{1}{6}(1 + 4x)^{\frac{3}{2}} \Big|_2^6 = 16.33$

9-1) $\frac{dy}{dx} = x\sqrt{y},$

$\frac{dy}{dx} \frac{1}{\sqrt{y}} = x, \quad \frac{dy}{\sqrt{y}} = x\,dx$

$\int \frac{dy}{\sqrt{y}} = \int x\,dx$

$= \frac{y^{\frac{1}{2}}}{\frac{1}{2}} = \frac{x^2}{2} + C$

$= 2\sqrt{y} = \frac{x^2}{2} + 4, \quad \sqrt{y} = \frac{x^2}{4} + C$

$y = \left(\frac{x^2}{4} + C\right)^2$

9-2) $y' + x e^{y} = 0.$

$y' = -x e^{y},$

$\frac{dy}{dx} = -x e^{y},$

$\int e^{-y}\,dy = \int -x\,dx$

$-e^{-y} = \frac{-x^2}{-2} + C$

$\hookrightarrow \ln \; e^{-y} = \ln\left(\frac{x^2}{2} + C,\right)$

$-y = \ln\left(\frac{x^2}{2} + C\right)$

$y = \ln\left(\frac{x^2}{2} + C\right)$

9-3) $\frac{du}{dt} = \frac{1+t^4}{(u+u^4)\,t^2}$

$(u+u^4)\,du = \frac{1+t^4}{t^2}\,dt$

$\int u+u^4 = \int \frac{1}{t^2} + t^2$

$\frac{u^2}{2} + \frac{u^5}{5} = -\frac{1}{t} + \frac{t^3}{3} + C$

9-4) $\frac{dH}{dR} = \frac{RH^2\sqrt{1+R^2}}{\ln H}$

$\int \frac{\ln H}{H^2}\,dH = \int R\sqrt{1+R^2}\,dR$

$\hookrightarrow = \ln H \cdot \frac{-1}{H} - \int \frac{-1}{H} \cdot \frac{dH}{H} + C$

$= \frac{-\ln H}{H} - \frac{1}{H}$

$\hookrightarrow \int R\sqrt{1+R^2}, \quad u = 1+R^2$

$= \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\cdot\frac{2}{3}\,u^{\frac{3}{2}}$

$= \frac{1}{3}\,u^{\frac{3}{2}} + C = \frac{1}{3}(1+R^2)^{\frac{3}{2}}$

$= \frac{-\ln H}{H} - \frac{1}{H} = \frac{1}{3}(1+R^2)^{\frac{3}{2}} + C$

9-5) $\frac{dy}{dx} = \frac{x\sin x}{y}, \quad y(0) = -1$

$y\,dy = x\sin x\,dx$

$\int y\,dy = \int x\sin x\,dx,$

$\frac{1}{2}y^2 = -x\cos x - \int -\cos x\,dx$

$\frac{1}{2}y^2 = -x\cos x + \sin x + C$

$y^2 = -2x\cos x + 2\sin x + C$

$y = \pm\sqrt{-2x\cos x + 2\sin x + C}$

$-1 = -2(0)\cdot 1 + 2(0) + C$

$-1 = -\sqrt{C}, \; C = 1$

9-6) $\frac{dP}{dt} = \sqrt{Pt}, \quad P(1) = 2$

$\frac{dP}{dt} = P^{\frac{1}{2}} \cdot t^{\frac{1}{2}},$

$\frac{dP}{P^{\frac{1}{2}}} = t^{\frac{1}{2}}\,dt$

$\int P^{-\frac{1}{2}}\,dP = \int t^{\frac{1}{2}}\,dt$

$2P^{\frac{1}{2}} = \frac{2}{3}t^{\frac{3}{2}} + C$

$P = \left(\frac{1}{3}t^{\frac{3}{2}} + C\right)^2,$

$2 = \left(\frac{1}{3}(1)^{\frac{3}{2}} + C\right)^2, \; C = 1.08$

10-1) $x = te^t$, $y = t + \sin(t)$

$x' = e^t + te^t = e^t(1+t)$

$y' = 1 + \cos(t)$

$\frac{dy}{dx} = \frac{e^t(1+t)}{1+\cos(t)}$

10-2) $x = \sqrt{t}$ $y = t^2 - 2t$, $t = 4$

$x' = \frac{1}{2\sqrt{t}} = \frac{1}{4}$,

$y' = 2t - 2$, $= 6$, $\frac{dy}{dx} = 24$

$y - 8 = 24(x - 2)$

10-3) $x = t^3 + 1$, $y = t^2 - t$

$x' = 3t^2$, $y' = 2t - 1$ $\frac{dy}{dx} = \frac{2t-1}{3t^2}$

$\frac{d^2y}{dx^2} = \frac{2(3t^2) - (2t-1)(6t)}{(3t^2)^2} = \frac{6t(t - 2t + 1)}{3t^2(3t^2)^2} = \frac{2}{9}\frac{(1-t)}{t^5}$

10-4) $(4, 4\pi/3)$

$x = r\cos\theta = 4\cos(\frac{4\pi}{3}) = 4 \cdot (-\frac{1}{2}) = -2$

$y = r\sin\theta = 4\sin(\frac{4\pi}{3}) = 4 \cdot (\frac{\sqrt{3}}{2}) = -2\sqrt{3}$, $(-2, 2\sqrt{3})$

10-5, $r = \frac{1}{\theta}$, $\frac{\pi}{2} \leq \theta \leq 2\pi$

$A = \frac{1}{2}\int_{\frac{\pi}{2}}^{2\pi} (\frac{1}{\theta})^2 = \frac{1}{2}\int \frac{1}{\theta^2}$

$= \frac{1}{2}(-\frac{1}{\theta})\Big|_{\pi/2}^{2\pi} = \frac{3}{4\pi}$

10-6) $r = 3\sin\theta$, $r = 2 - \sin\theta$

$\to 3\sin\theta = 2 - \sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$, $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$r_1^2 - r_2^2 = 9\sin^2\theta - (4 - 4\sin\theta + \sin^2\theta)$

$= 8\sin^2\theta + 4\sin\theta - 4$

$= 8\frac{1}{2}(1 - \cos(2\theta)) + 4\sin\theta - 4$

$= 4(\sin\theta - \cos(2\theta))$

$A = \frac{1}{2}\int_a^b (4(\sin\theta - \cos 2\theta))\, d\theta$

$= 2(-\cos\theta - \frac{1}{2}\sin 2(\theta))\Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = 2\sqrt{3}$

11-1) $a_n = \frac{n^2-1}{n^2+1} = 0, \frac{3}{5}, \frac{4}{5}, \frac{15}{17}, \frac{12}{13}$

11-2) $(-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27})$ =

$\times\frac{-2}{3}$ $\times\frac{-2}{3}$ $\times\frac{-2}{3}$

$a_n = -3 \cdot \left(\frac{-2}{3}\right)^{n-1}$

11-3) $(1, 0, -1, 0, 1, 0, -1, 0 \ldots)$

$a_1 = \sin\left(\frac{\pi}{2}\right)$

$a_2 = \sin\left(\frac{2\pi}{2}\right) = 1$

$a_n = \sin\left(\frac{n\pi}{2}\right)$

11-4) $\sum_{n=1}^{\infty} \left(\frac{1}{3n-1}\right)^4$

$= \lim_{t\to\infty} \int_1^t (3x-1)^{-4}\, dx = \lim_{t\to\infty} \left[\frac{(3x-1)^{-3}}{-4}\right]\Big|_1^t$

$= 0 + \frac{1}{72} = \frac{1}{72}$, conv

11-5) $\sum_{n=3}^{\infty} n^{-.4444}$

$= \frac{1}{n^{.4444}}$ = p series, $p < 1$ = divergent

11-6) $\frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} \ldots$

$a_n = \frac{1}{2x+3}$

$\int_1^{\infty} \frac{1}{2x+3}\, dx = \lim_{t\to\infty} \int_1^t \frac{1}{2x+3}\, dx = \lim_{t\to\infty} \left(\frac{1}{2} \ln(2x+3)\right)\Big|_1^t$

$= \lim \frac{1}{2}\ln(2t+3) - \frac{1}{2}\ln 5 = \infty$, diverges