Competition 04

Due: September 21

GOOD LUCK, EVERYBODY!

Problem A. Farmer John has N cows that need to be milked ($1 \leq N \leq 10,000$), each of which takes only one unit of time to milk.

Being impatient animals, some cows will refuse to be milked if Farmer John waits too long to milk them. More specifically, cow i produces g_i gallons of milk ($1 \leq g_i \leq 1000$), but only if she is milked before a deadline at time d_i ($1 \leq d_i \leq 10,000$). Time starts at $t = 0$, so at most x total cows can be milked prior to a deadline at time $t = x$.

Please help Farmer John determine the maximum amount of milk that he can obtain if he milks the cows optimally.

input format: The input consists of $\rho \geq 1$ *challenges*, each in the format:

- Line 1: The value of N.
- Lines $2 \ldots 1 + N$: Line $i + 1$ contains the integers g_i and d_i.

No extra lines intervene between two successive challenges. The last challenge is followed by a line containing a single copy of the character `#`.

sample challenge:

```
4
10 3
7 5
8 1
2 1
```

sample details: There are 4 cows. The first produces 10 gallons of milk if milked by time 3, and so on.

output format: The output should consist of ρ *responses*, each in the format:

- Line 1: The maximum number of gallons of milk Farmer John can obtain.

No extra lines should intervene between two successive responses.

sample response:

```
25
```

sample details: Farmer John milks cow 3 first, giving up on cow 4 since she cannot be milked by her deadline due to the conflict with cow 3. Farmer John then milks cows 1 and 2.

Problem B. The cross-country skiing course at the winter Moolympics is described by an $M \times N$ grid of elevations ($1 \leq M, N \leq 500$), each elevation being in the range $0 \ldots 1,000,000,000$.

Some of the cells in this grid are designated as waypoints for the course. The organizers of the Moolympics want to assign a difficulty rating D to the entire course so that a cow can reach any waypoint from any other waypoint by repeatedly skiing from a cell to an adjacent cell with absolute elevation difference at most D. Two cells are adjacent if one is directly north, south, east, or west of the other. The difficulty rating of the course is the minimum value of D such that all waypoints are mutually reachable in this fashion.

input format: The input consists of $\rho \geq 1$ *challenges*, each in the format:

- Line 1: The integers M and N.
- Lines $2 \ldots 1 + M$: Each of these M lines contains N integer elevations.
- Lines $2 + M \ldots 1 + 2M$: Each of these M lines contains N values that are either 0 or 1, with 1 indicating a cell that is a waypoint.

No extra lines intervene between two successive challenges. The last challenge is followed by a line containing a single copy of the character `#`.

sample challenge:

```
3 5
20 21 18 99 5
19 22 20 16 26
18 17 40 60 80
1 0 0 0 1
0 0 0 0 0
0 0 0 0 1
```

sample details: The ski course is described by a 3×5 grid of elevations. The upper-left, upper-right, and lower-right cells are designated as waypoints.

output format: The output should consist of ρ *responses*, each in the format:

- Line 1: The difficulty rating for the course (the minimum value of D such that all waypoints are still reachable from each-other).

No extra lines should intervene between two successive responses.

sample response:

```
21
```

sample details: If $D = 21$, the three waypoints are reachable from each-other. If $D < 21$, then the upper-right waypoint cannot be reached from the other two.

Problem C. The cows have decided on a daring plan to escape from the clutches of Farmer John. They have managed to procure a small inflatable raft, and during the cover of night, a group of cows will board the raft and row across the river bordering the farm. The plan seems perfect, until the cows realize that their small inflatable raft may not be able to hold much weight!

The N cows ($1 \leq N \leq 20$) have weights $w_1, \ldots, w_N$. To figure out if a group of cows is light enough to avoid sinking the raft, the cows add up all of the weights in the group. Unfortunately, cows are notoriously bad at arithmetic, and if the addition of the weights of the cows in a group causes any carries to occur (using standard base 10 addition), then the cows give up and conclude that group must weigh too much to use the raft. Any group whose weights can be added without any carries is assumed to be light enough to fit on the raft.

Please help the cows determine the size of the largest group that they believe can fit on the raft (that is, the largest group whose weights can be added together with no carries).

input format: The input consists of $\rho \geq 1$ *challenges*, each in the format:

- Line 1: The number of cows, N.
- Lines $2 \ldots N+1$: Each line contains the weight of one cow, an integer in the range $1 \ldots 100,000,000$.

No extra lines intervene between two successive challenges. The last challenge is followed by a line containing a single copy of the character `#`.

sample challenge:

```
5
522
6
84
7311
19
```

sample details: There are 5 cows, with weights 522, 6, 84, 7311, and 19.

output format: The output should consist of ρ *responses*, each in the format:

- Line 1: The number of cows in the largest group whose weights can be added together with no carries.

No extra lines should intervene between two successive responses.

sample response:

```
3
```

sample details: The three weights 522, 6, and 7311, can be added together with no carries:

```
522
6
+ 7311
------
7839
```