Competition 03

Due: September 14

GOOD LUCK, EVERYBODY!

Problem A. One of Farmer John's fields is particularly hilly, and he wants to purchase a new tractor to drive around on it. The field is described by an $N \times N$ grid of non-negative integer elevations ($1 \leq N \leq 500$). A tractor capable of moving from one grid cell to an adjacent cell (one step north, east, south, or west) of height difference D costs exactly D units of money.

FJ would like to pay enough for his tractor so that, starting from some grid cell in his field, he can successfully drive the tractor around to visit at least half the grid cells in the field (if the number of total cells in the field is odd, he wants to visit at least half the cells rounded up). Please help him compute the minimum cost necessary for buying a tractor capable of this task.

input format: The input consists of $\rho \geq 1$ *challenges*, each in the format:

- Line 1: The value of N.
- Lines $2 \ldots 1 + N$: Each line contains N space-separated non-negative integers (each at most 1 million) specifying a row of FJ's field.

No extra lines intervene between two successive challenges. The last challenge is followed by a line containing a single copy of the character `#`.

sample challenge:

```
5
0 0 0 3 3
0 0 0 0 3
0 9 9 3 3
9 9 9 3 3
9 9 9 9 3
```

sample details: FJ's farm is a 5×5 grid. The elevations in the first row are 0, 0, 0, 3, and 3, and so on.

output format: The output should consist of ρ *responses*, each in the format:

- Line 1: The minimum cost of a tractor that is capable of driving around at least half of FJ's field.

No extra lines should intervene between two successive responses.

sample response:

```
3
```

sample details: A tractor of cost 3 is capable of moving between elevation 0 and elevation 3, so it can visit the block of cells at zero elevation as well as the block of cells at elevation 3. Together, these represent at least half of FJ's farm.

Problem B. Farmer John wants to give gifts to his N ($1 \leq N \leq 1000$) cows, using his total budget of B ($1 \leq B \leq 1,000,000,000$) units of money.

Cow i requests a gift with a price of $P(i)$ units, and a shipping cost of $S(i)$ units (so the total cost would be $P(i) + S(i)$ for FJ to order this gift). FJ has a special coupon that he can use to order one gift of his choosing at only half its normal price. If FJ uses the coupon for cow i, he therefore would only need to pay $P(i)/2 + S(i)$ for that cow's gift. Conveniently, the $P(i)$'s are all even numbers.

Please help FJ determine the maximum number of cows to whom he can afford to give gifts.

input format: The input consists of $\rho \geq 1$ *challenges*, each in the format:

- Line 1: Two space-separated integers, N and B.
- Lines $2 \ldots 1{+}N$: Line $i+1$ contains two space-separated integers, $P(i)$ and $S(i)$. ($0 \leq P(i), S(i) \leq 1,000,000,000$, with $P(i)$ even)

No extra lines intervene between two successive challenges. The last challenge is followed by a line containing a single copy of the character `#`.

sample challenge:

```
5 24
4 2
2 0
8 1
6 3
12 5
```

sample details: There are 5 cows, and FJ has a budget of 24. Cow 1 desires a gift with price 4 and shipping cost 2, etc.

output format: The output should consist of ρ *responses*, each in the format:

- Line 1: The maximum number of cows for whom FJ can purchase gifts.

No extra lines should intervene between two successive responses.

sample response:

```
4
```

sample details: FJ can purchase gifts for cows 1 through 4, if he uses the coupon for cow 3. His total cost is $(4+2) + (2+0) + (4+1) + (6+3) = 22$. Note that FJ could have used the coupon instead on cow 1 or 4 and still have met his budget.

Problem C. Farmer John has N $(1 \leq N \leq 50,000)$ pastures, conveniently numbered $1 \ldots N$, connected by M $(1 \leq M \leq 100,000)$ bidirectional paths. Path i connects pasture A_i $(1 \leq A_i \leq N)$ to pasture B_i $(1 \leq B_i \leq N)$ with $A_i \neq B_i$. It is possible for two paths to connect between the same pair of pastures.

Bessie has decided to decorate the pastures for FJ's birthday. She wants to put a large sign in each pasture containing either the letter 'F' or 'J', but in order not to confuse FJ, she wants to be sure that two pastures are decorated by different letters if they are connected by a path.

The sign company insists on charging Bessie more money for an 'F' sign than a 'J' sign, so Bessie wants to maximize the number of 'J' signs that she uses. Please determine this number, or output -1 if there is no valid way to arrange the signs.

input format: The input consists of $\rho \geq 1$ *challenges*, each in the format:

- Line 1: Two integers N and M.
- Lines $2 \ldots M+1$: Two integers, A_i and B_i indicating that there is a bidirectional path from A_i to B_i.

No extra lines intervene between two successive challenges. The last challenge is followed by a line containing a single copy of the character `#`.

sample challenge:

```
4 4
1 2
2 3
3 4
4 1
```

sample details: The pastures and paths form the vertices and edges of a square.

output format: The output should consist of ρ *responses*, each in the format:

- Line 1: A single integer indicating the maximum number of 'J' signs that Bessie can use. If there is no valid solution for arranging the signs, output -1.

No extra lines should intervene between two successive responses.

sample response:

```
2
```

sample details: Bessie can either choose to label pastures 1 and 3 with 'J' signs, or alternatively pastures 2 and 4.