Approximation Algorithm Notes (Vertex Cover)

1. Problem Overview

A vertex cover of a graph G(V, E) is a set of vertices such that every edge in the graph has at least one endpoint in the set.

2. Greedy Maximal-Matching Approximation

- Pick any uncovered edge.
- Add both its endpoints to the cover.
- Remove all edges incident to these vertices.
- Repeat until no edges remain.

This yields a 2-approximation.

3. Why It Works

Every time we pick an edge, the optimal solution must include at least one of its endpoints. We include both, so we never perform worse than a factor of 2.

4. Time Complexity

The greedy approach runs in O(V + E) if implemented efficiently.

5. Behavior in Practice

In sparse graphs, the approximation often performs close to optimal. In dense graphs, the gap increases slightly but remains within the theoretical bound.