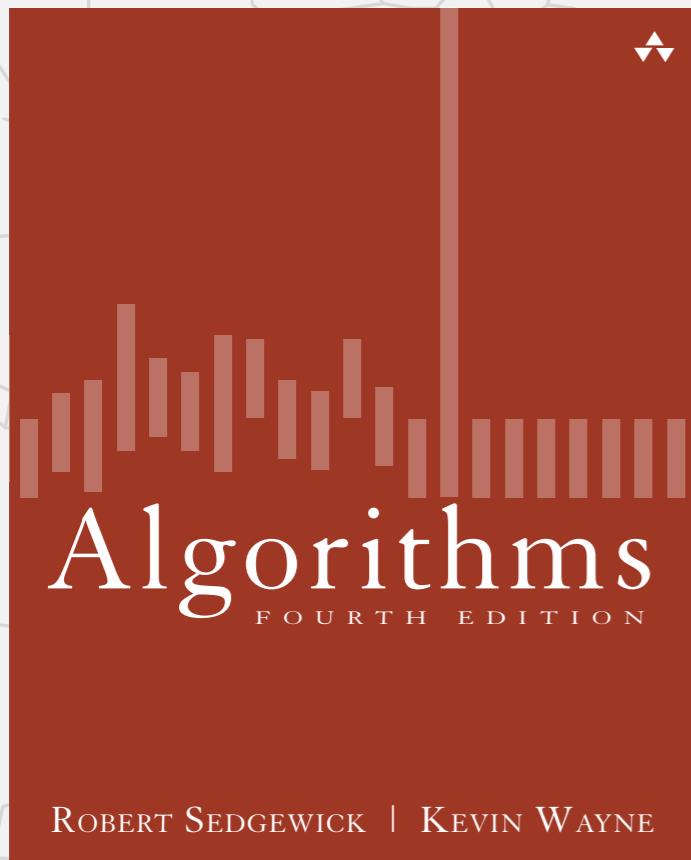


# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE



<https://algs4.cs.princeton.edu>

## 2.1 ELEMENTARY SORTS

---

- ▶ *rules of the game*
- ▶ *selection sort*
- ▶ *insertion sort*
- ▶ *shuffling*
- ▶ *comparators*
- ▶ *stability*

# Algorithms

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- ▶ *stability*

# Sorting problem

---

Ex. Student records in a university.

Chen	3	A	(991) 878-4944	308 Blair
Rohde	2	A	(232) 343-5555	343 Forbes
Gazsi	4	B	(800) 867-5309	101 Brown
<b>item</b> → Furia	1	A	(766) 093-9873	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Andrews	3	A	(664) 480-0023	097 Little
<b>key</b> → Battle	4	C	(874) 088-1212	121 Whitman

Sort. Rearrange array of  $n$  items in ascending order by key.

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

# Total order

Sorting is a well-defined problem if and only if there is a **total order**.

A **total order** is a binary relation  $\leq$  that satisfies:

- Totality: either  $v \leq w$  or  $w \leq v$  or both.
- Transitivity: if both  $v \leq w$  and  $w \leq x$ , then  $v \leq x$ .
- Antisymmetry: if both  $v \leq w$  and  $w \leq v$ , then  $v = w$ .

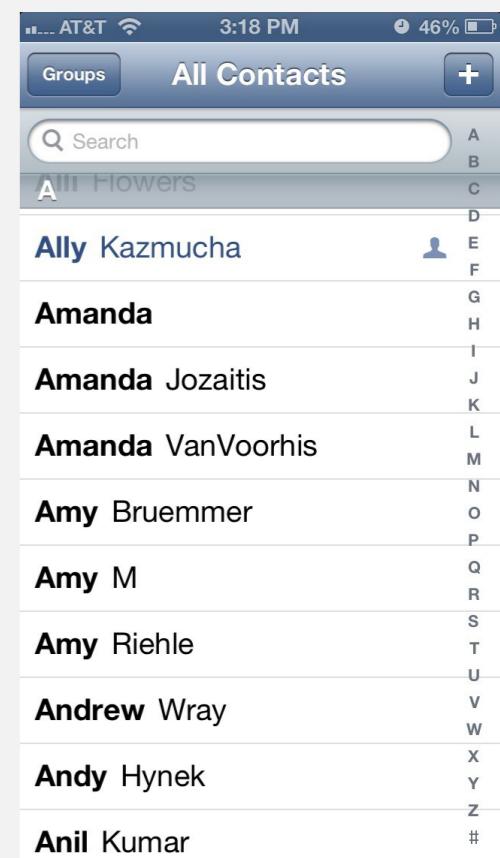
## Examples.

Video name	Views*
"Despacito" <sup>[6]</sup>	2,993,700,000
"See You Again" <sup>[11]</sup>	2,894,000,000
"Gangnam Style" <sup>[17]</sup>	803,700,000
"Baby" <sup>[41]</sup>	245,400,000
"Bad Romance" <sup>[146]</sup>	178,400,000
"Charlie Bit My Finger" <sup>[136]</sup>	128,900,000
"Evolution of Dance" <sup>[131]</sup>	118,900,000

numerical order (descending)

International Departures					
Flight No	Destination	Time	Gate	Remarks	
CX7183	Berlin	7:50	A-11	Gate closing	
QF3474	London	7:50	A-12	Gate closing	
BA372	Paris	7:55	B-10	Boarding	
AY6554	New York	8:00	C-33	Boarding	
KL3160	San Francisco	8:00	F-15	Boarding	
BA8903	Manchester	8:05	B-12	Gate lounge open	
BA710	Los Angeles	8:10	C-12	Check-in open	
QF3371	Hong Kong	8:15	F-10	Check-in open	
MA4866	Barcelona	8:15	F-12	Check-in at kiosks	
CX7221	Copenhagen	8:20	G-32	Check-in at kiosks	

chronological order



lexicographic order

# Total order

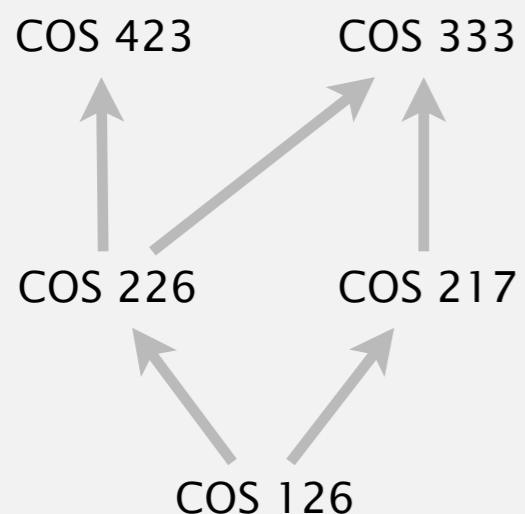
---

Sorting is a well-defined problem if and only if there is a **total order**.

A **total order** is a binary relation  $\leq$  that satisfies:

- Totality: either  $v \leq w$  or  $w \leq v$  or both.
- Transitivity: if both  $v \leq w$  and  $w \leq x$ , then  $v \leq x$ .
- Antisymmetry: if both  $v \leq w$  and  $w \leq v$ , then  $v = w$ .

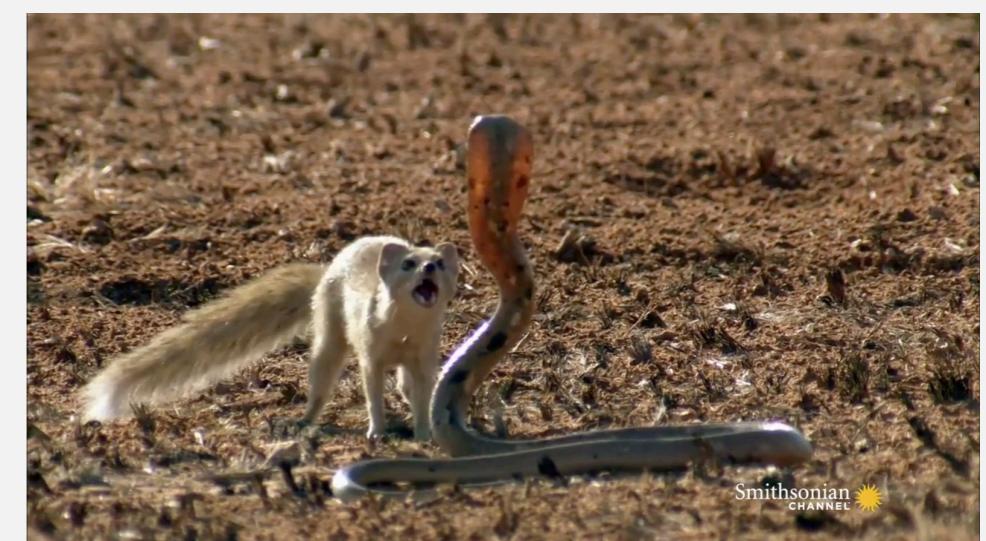
## Non-examples.



course prerequisites  
(violates totality)



Ro-sham-bo order  
(violates transitivity)



predator-prey  
(violates antisymmetry)

## Sample sort clients

---

Goal. Single function that sorts **any** type of data (that has a total order).

Ex 1. Sort strings in alphabetical order.

```
public class StringSorter
{
    public static void main(String[] args)
    {
        String[] a = StdIn.readAllStrings();
        Insertion.sort(a);
        for (int i = 0; i < a.length; i++)
            StdOut.println(a[i]);
    }
}
```

```
% more words3.txt
bed bug dad yet zoo ... all bad yes
```

```
% java StringSorter < words3.txt
all bad bed bug dad ... yes yet zoo
[suppressing newlines]
```

## Sample sort clients

Goal. Single function that sorts **any** type of data (that has a total order).

Ex 2. Sort random real numbers in ascending order.

seems artificial (stay tuned for an application)

```
public class Experiment
{
    public static void main(String[] args)
    {
        int n = Integer.parseInt(args[0]);
        Double[] a = new Double[n];
        for (int i = 0; i < n; i++)
            a[i] = StdRandom.uniform();
        Insertion.sort(a);
        for (int i = 0; i < n; i++)
            StdOut.println(a[i]);
    }
}
```

```
% java Experiment 10
0.08614716385210452
0.09054270895414829
0.10708746304898642
0.21166190071646818
0.363292849257276
0.460954145685913
0.5340026311350087
0.7216129793703496
0.9003500354411443
0.9293994908845686
```

## Sample sort clients

---

Goal. Single function that sorts **any** type of data (that has a total order).

Ex 3. Sort the files in a given directory by filename.

```
import java.io.File;

public class FileSorter
{
    public static void main(String[] args)
    {
        File directory = new File(args[0]);
        File[] files = directory.listFiles();
        Insertion.sort(files);
        for (int i = 0; i < files.length; i++)
            StdOut.println(files[i].getName());
    }
}
```

```
% java FileSorter .
Insertion.class
Insertion.java
InsertionX.class
InsertionX.java
Selection.class
Selection.java
Shell.class
Shell.java
ShellX.class
ShellX.java
```

# How can a single function sort any type of data?

Goal. Single function that sorts **any** type of data (that has a total order).

Solution. **Callback** = reference to executable code.

*Please sort these Japanese names for me:  
あゆみ, アユミ, Ayumi, 歩美, ....*



*But I don't speak Japanese and I  
don't know how words are ordered.*



*No problem. Whenever you need to  
compare two words, give me a call back.*



*オーケー. Just make sure  
to use a total order.*



# Callbacks

---

**Goal.** Single function that sorts **any** type of data (that has a total order).

**Solution.** **Callback** = reference to executable code.

- Client passes array of objects to sort() function.
- The sort() function calls object's compareTo() function as needed.

**Implementing callbacks.**

- Java: interfaces.
- C: function pointers.
- C++: class-type functors.
- C#: delegates.
- Python, Perl, ML, Javascript: first-class functions.

# Java interfaces

**Interface.** A type that defines a set of methods that a class can provide.

```
public interface Comparable<Item>
{
    public int compareTo(Item that);
}
```

contract: one method with this signature  
(and prescribed behavior)

**Class that implements interface.** Must implement all interface methods.

```
public class String implements Comparable<String>
{
    ...
    public int compareTo(String that)
    {
        ...
    }
}
```

class promises to  
honor the contract

class abides by  
the contract

**Impact.**

- You can treat any String object as an object of type Comparable.
- On a Comparable object, you can invoke (only) the compareTo() method.
- Enables callbacks.

“polymorphism”

# Callbacks in Java: roadmap

## client (StringSorter.java)

```
public class StringSorter
{
    public static void main(String[] args)
    {
        String[] a = StdIn.readAllStrings();
        Insertion.sort(a);
        for (int i = 0; i < a.length; i++)
            StdOut.println(a[i]);
    }
}
```

## java.lang.Comparable interface

```
public interface Comparable<Item>
{
    public int compareTo(Item that);
}
```

## sort implementation (Insertion.java)

```
public static void sort(Comparable[] a)
{
    int n = a.length;
    for (int i = 0; i < n; i++)
        for (int j = i; j > 0; j--)
            if (a[j].compareTo(a[j-1]) < 0)
                exch(a, j, j-1);
            else break;
}
```

## data type implementation (String.java)

```
public class String
implements Comparable<String>
{
    ...
    public int compareTo(String that)
    {
        ...
    }
}
```

key point: client code does not  
depend upon type of data to be sorted



Suppose that the Java architects left out `implements Comparable<String>` in the class declaration for `String`. Which would be the effect?

- A. `String.java` won't compile.
- B. `StringSorter.java` won't compile.
- C. `Insertion.java` won't compile.
- D. `Insertion.java` will throw an exception.

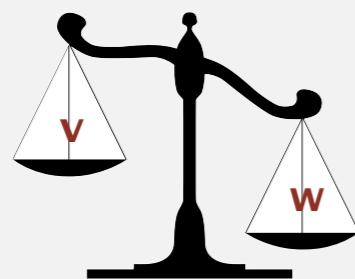
# Comparable API

Implement `compareTo()` so that `v.compareTo(w)`

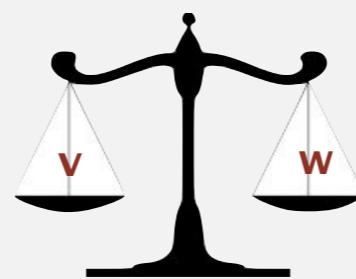
- Returns a
  - negative integer if `v` is less than `w`
  - positive integer if `v` is greater than `w`
  - zero if `v` is equal to `w`
- Defines a total order.
- Throws an exception if incompatible types (or either is `null`).



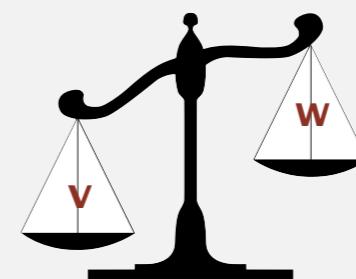
`v.compareTo(w) <= 0`  
means `v` is less than or equal to `w`



**less than**  
**(return negative integer)**



**equal to**  
**(return 0)**



**greater than**  
**(return positive integer)**

Built-in comparable types. `Integer`, `Double`, `String`, `Date`, `File`, ...

User-defined comparable types. Implement the Comparable interface.

# Implementing the Comparable interface

Date data type. Simplified version of java.util.Date.

```
public class Date implements Comparable<Date>
{
    private final int month, day, year;

    public Date(int m, int d, int y)
    {
        month = m;
        day   = d;
        year  = y;
    }

    public int compareTo(Date that)
    {
        if (this.year < that.year) return -1;
        if (this.year > that.year) return +1;
        if (this.month < that.month) return -1;
        if (this.month > that.month) return +1;
        if (this.day   < that.day)  return -1;
        if (this.day   > that.day)  return +1;
        return 0;
    }
}
```

can compare Date objects  
only to other Date objects

# Algorithms

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## 2.1 ELEMENTARY SORTS

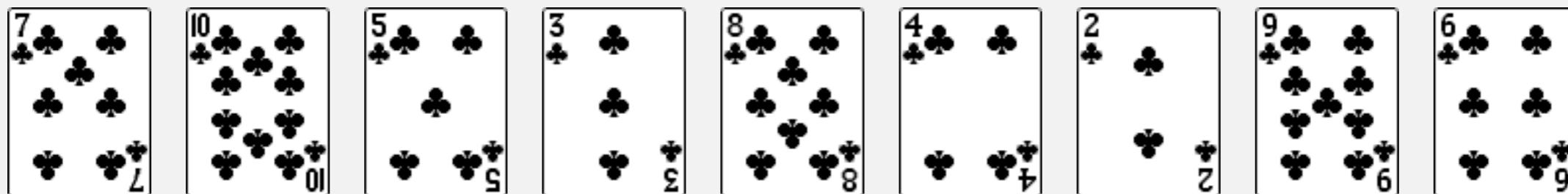
---

- ▶ *rules of the game*
- ▶ ***selection sort***
- ▶ *insertion sort*
- ▶ *shuffling*
- ▶ *comparators*
- ▶ *stability*

# Selection sort demo

---

- In iteration  $i$ , find index  $\text{min}$  of smallest remaining entry.
- Swap  $a[i]$  and  $a[\text{min}]$ .



initial



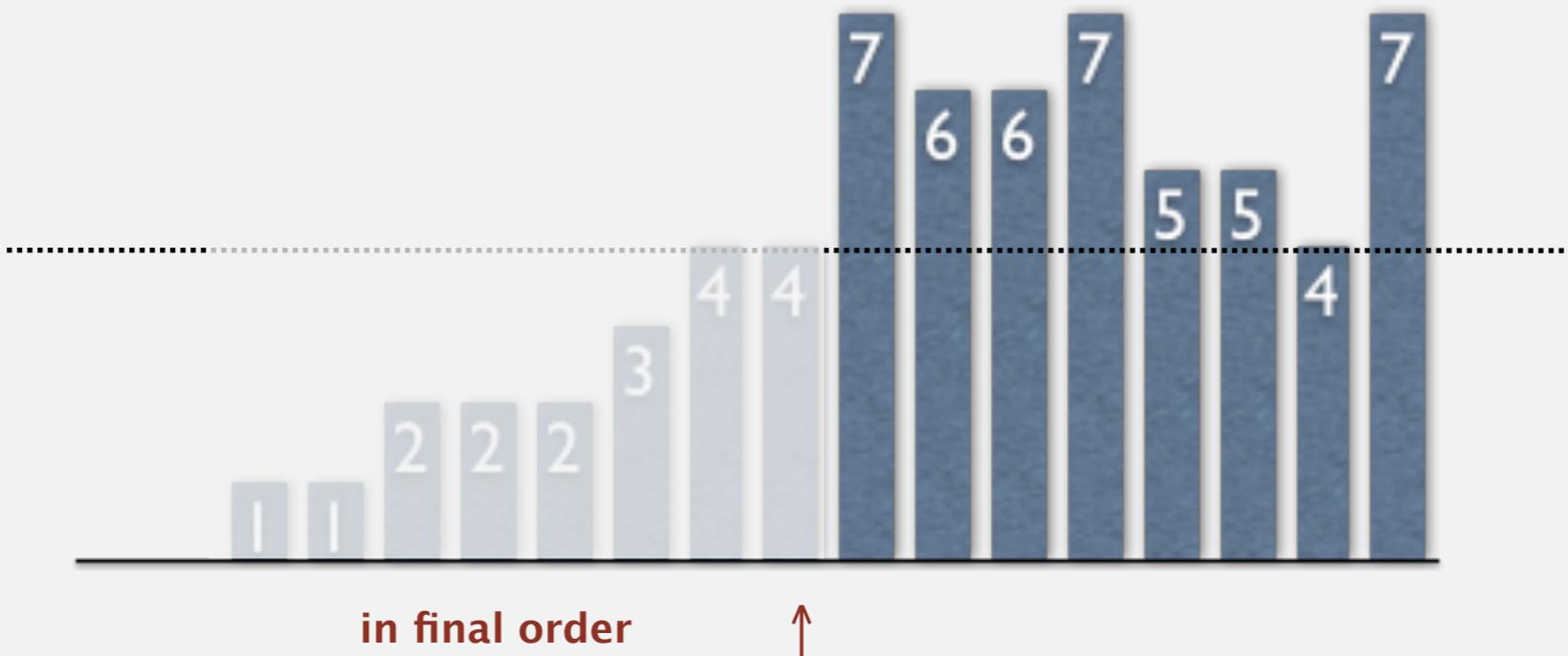
# Selection sort

---

Algorithm.  $\uparrow$  scans from left to right.

## Invariants.

- Entries to the left of  $\uparrow$  (including  $\uparrow$ ) fixed and in ascending order.
- No entry to right of  $\uparrow$  is smaller than any entry to the left of  $\uparrow$ .



# Selection sort inner loop

To maintain algorithm invariants:

- Move the pointer to the right.

```
i++;
```



- Identify index of minimum entry on right.

```
int min = i;
for (int j = i+1; j < n; j++)
    if (less(a[j], a[min]))
        min = j;
```



- Exchange into position.

```
exch(a, i, min);
```



## Two useful sorting abstractions

---

Helper functions. Refer to data only through **compares** and **exchanges**.

Less. Is item  $v$  less than  $w$ ?

```
private static boolean less(Comparable v, Comparable w)
{   return v.compareTo(w) < 0; }
```

Exchange. Swap item in array  $a[]$  at index  $i$  with the one at index  $j$ .

```
private static void exch(Object[] a, int i, int j)
{
    Object swap = a[i];
    a[i] = a[j];
    a[j] = swap;
}
```

↑  
polymorphism: you can treat any  
object as an object of type Object

## Selection sort: Java implementation

---

```
public class Selection
{
    public static void sort(Comparable[] a)
    {
        int n = a.length;
        for (int i = 0; i < n; i++)
        {
            int min = i;
            for (int j = i+1; j < n; j++)
                if (less(a[j], a[min]))
                    min = j;
            exch(a, i, min);
        }
    }

    private static boolean less(Comparable v, Comparable w)
    { /* see previous slide */ }

    private static void exch(Object[] a, int i, int j)
    { /* see previous slide */ }
}
```

<https://algs4.cs.princeton.edu/21elementary/Selection.java.html>

# Generic methods

---

Oops. The compiler complains.

```
% javac-algs4 Selection.java
Selection.java:83: warning: [unchecked] unchecked call to
compareTo(T) as a member of the raw type java.lang.Comparable
    return (v.compareTo(w) < 0);
                           ^
1 warning
```

Q. How to appease the compiler?

# Generic methods

Pedantic (type-safe) version. Compiles without any warnings.

generic type variable (for a static method)  
(type inferred from argument; must be Comparable)

```
public class SelectionPedantic
{
    public static <Key extends Comparable<Key>> void sort(Key[] a)
    { /* as before */ }

    private static <Key extends Comparable<Key>> boolean less(Key v, Key w)
    { /* as before */ }

    private static void exch(Object[] a, int i, int j)
    { /* as before */ }
}
```

<https://algs4.cs.princeton.edu/21elementary/SelectionPedantic.java.html>

Remark. Use type-safe version in system code (but not in lecture).

and Assignment 3

# Selection sort: animations

## 20 random items



<http://www.sorting-algorithms.com/selection-sort>



How many compares does selection sort make to sort an array of  $n$  distinct items in reverse order?

- A.  $\sim n$
- B.  $\sim 1/4 n^2$
- C.  $\sim 1/2 n^2$
- D.  $\sim n^2$

## Selection sort: mathematical analysis

**Proposition.** Selection sort makes  $(n - 1) + (n - 2) + \dots + 1 + 0 \sim n^2/2$  compares and  $n$  exchanges to sort **any** array of  $n$  items.

		a[]										
i	min	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
0	6	S	O	R	T	E	X	A	M	P	L	E
1	4	A	O	R	T	E	X	S	M	P	L	E
2	10	A	E	R	T	O	X	S	M	P	L	E
3	9	A	E	E	T	O	X	S	M	P	L	R
4	7	A	E	E	L	O	X	S	M	P	T	R
5	7	A	E	E	L	M	X	S	O	P	T	R
6	8	A	E	E	L	M	O	S	X	P	T	R
7	10	A	E	E	L	M	O	P	X	S	T	R
8	8	A	E	E	L	M	O	P	R	S	T	X
9	9	A	E	E	L	M	O	P	R	S	T	X
10	10	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

entries in black are examined to find the minimum

entries in red are  $a[min]$

entries in gray are in final position

**Running time insensitive to input.** Quadratic time, even if input is sorted.

**Data movement is minimal.** Linear number of exchanges—exactly  $n$ .

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- ▶ *shuffling*
- ▶ *comparators*
- ▶ *stability*

## Insertion sort demo

---

- In iteration  $i$ , swap  $a[i]$  with each larger entry to its left.



<https://www.youtube.com/watch?v=ROalU379I3U>

# Insertion sort

---

Algorithm.  $\uparrow$  scans from left to right.

## Invariants.

- Entries to the left of  $\uparrow$  (including  $\uparrow$ ) are in ascending order.
- Entries to the right of  $\uparrow$  have not yet been seen.



## Insertion sort: inner loop

To maintain algorithm invariants:

- Move the pointer to the right.

```
i++;
```



- Moving from right to left, exchange  $a[i]$  with each larger entry to its left.

```
for (int j = i; j > 0; j--)  
    if (less(a[j], a[j-1]))  
        exch(a, j, j-1);  
    else break;
```



# Insertion sort: Java implementation

---

```
public class Insertion
{
    public static void sort(Comparable[] a)
    {
        int n = a.length;
        for (int i = 0; i < n; i++)
            for (int j = i; j > 0; j--)
                if (less(a[j], a[j-1]))
                    exch(a, j, j-1);
                else break;
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }

    private static void exch(Object[] a, int i, int j)
    { /* as before */ }
}
```

<https://algs4.cs.princeton.edu/21elementary/Insertion.java.html>



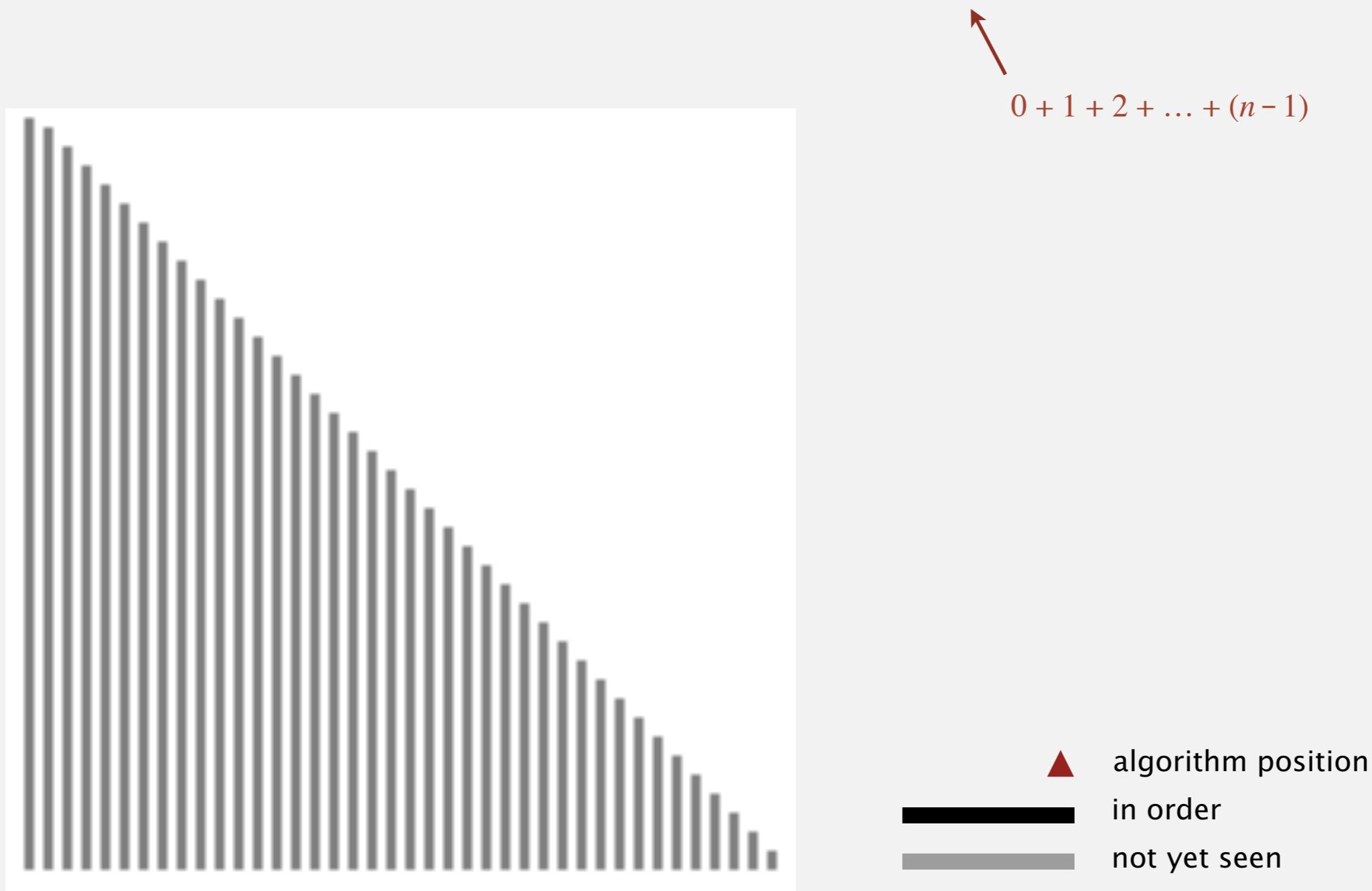
How many compares does insertion sort make to sort an array of  $n$  distinct keys in reverse order?

- A.  $\sim n$
- B.  $\sim 1/4 n^2$
- C.  $\sim 1/2 n^2$
- D.  $\sim n^2$

## Insertion sort: analysis

**Worst case.** Insertion sort makes  $\sim \frac{1}{2} n^2$  compares and  $\sim \frac{1}{2} n^2$  exchanges to sort an array of  $n$  distinct keys in reverse order.

**Pf.** Exactly  $i$  compares and exchanges in iteration  $i$ .





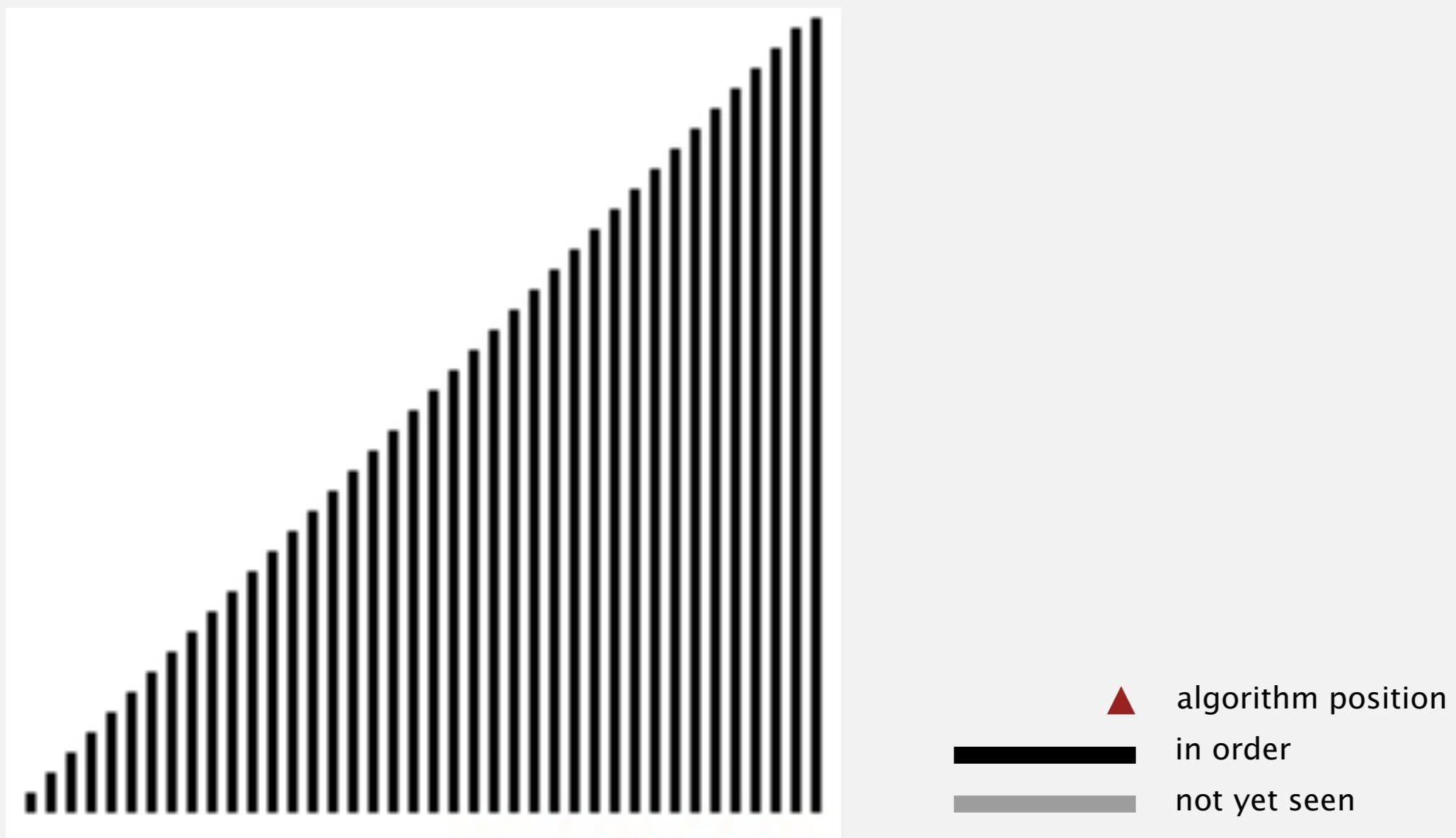
**Which is faster in practice to sort an array of  $n$  items,  
selection sort or insertion sort?**

- A. Selection sort.
- B. Insertion sort.
- C. No significant difference.
- D. It depends.

## Insertion sort: analysis

---

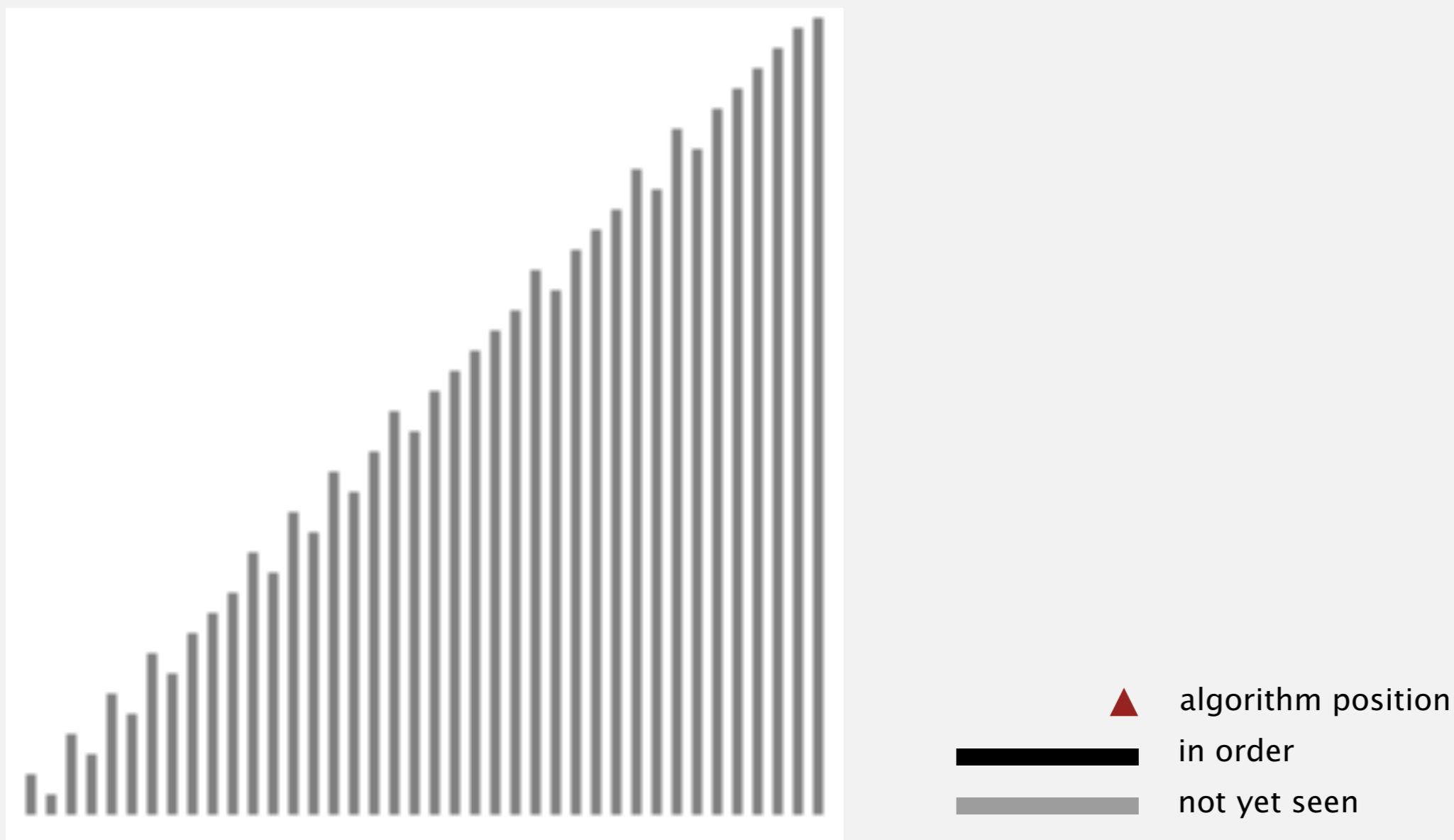
**Best case.** Insertion sort makes  $n-1$  compares and 0 exchanges to sort an array of  $n$  distinct keys in ascending order.



## Insertion sort: analysis

**Good case.** Insertion sort is linear time for “partially sorted” arrays.

Q. What do we mean by partially sorted?



## Insertion sort: partially sorted arrays

---

Def. An **inversion** is a pair of keys that are out of order.

A	E	E	L	M	O	T	R	X	P	S	
						1	2	3	4	5	6
<hr/>											
						T-R	T-P	T-S	R-P	X-P	X-S
(6 inversions)											

Def. A family of arrays is **partially sorted** if the number of inversions is  $\leq c n$ .

- Ex 1. A sorted array.
- Ex 2. A subarray of length 10 appended to a sorted subarray of length  $n$ .

$$\leq 10n + 45 \text{ inversions}$$

Proposition. Insertion sort runs in linear time on partially sorted arrays.

Pf.

- Number of exchanges in insertion sort = number of inversions.
- Number of compares  $\leq$  number of exchanges +  $(n - 1)$ .

each compare in iteration  $i$  triggers one exchange  
(except possibly last one in iteration)

exchange decreases number of inversions by 1

## Insertion sort: practical improvements

---

**Half exchanges.** Shift items over (instead of exchanging).

- Eliminates unnecessary data movement.
- No longer uses only `less()` and `exch()` to access data.

A C H H I M N N P Q X Y K B I N A R Y

**Binary insertion sort.** Use binary search to find insertion point.

- Number of compares  $\sim n \log_2 n$ .
- But still a quadratic number of array accesses.

A C H H I M N N P Q X Y K B I N A R Y



binary search for first key > K

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## 2.1 ELEMENTARY SORTS

---

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- ▶ *stability*

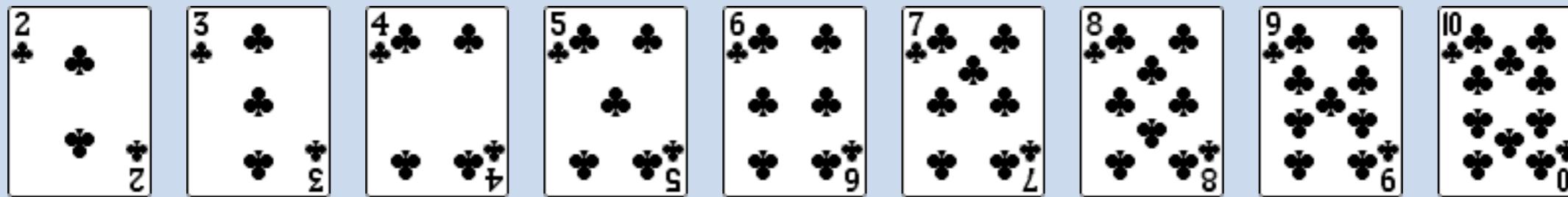
# INTERVIEW QUESTION: SHUFFLE AN ARRAY



Goal. Rearrange array so that result is a uniformly random permutation.



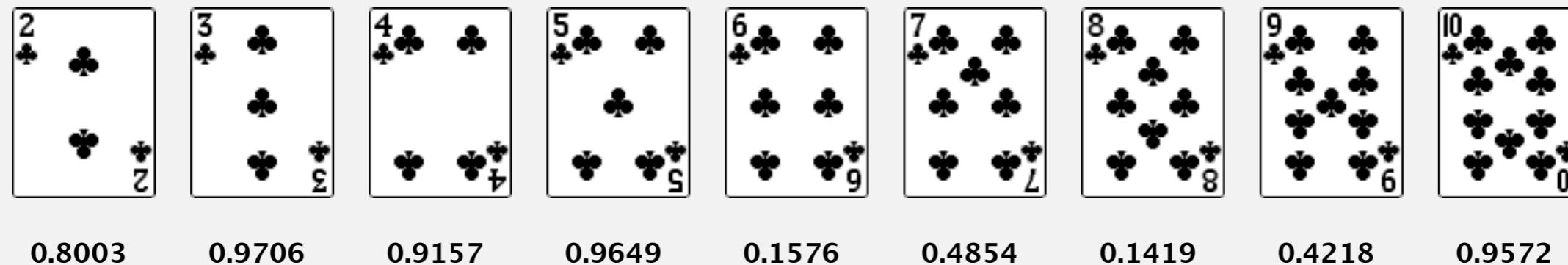
all  $n!$  permutations  
equally likely



# Shuffling by sorting

---

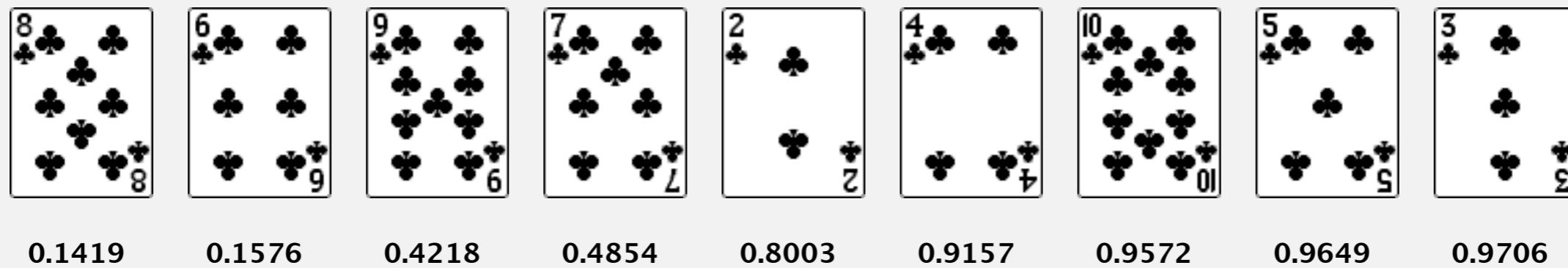
- Generate a random real number for each array entry.
- Sort the array.



## Shuffling by sorting

---

- Generate a random real number for each array entry.
- Sort the array.



Proposition. Shuffle sort produces a uniformly random permutation.

Application. Shuffle columns in a spreadsheet.

assuming real numbers are  
uniformly random (and no ties)

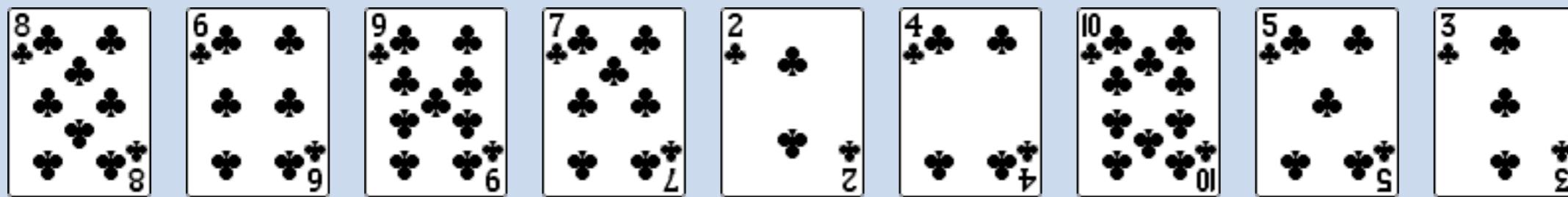
# INTERVIEW QUESTION: SHUFFLE AN ARRAY



Goal. Rearrange array so that result is a uniformly random permutation.



all  $n!$  permutations  
equally likely



Shuffling by sorting.

- Quadratic time (with insertion sort or selection sort).
- Linearithmic time (with mergesort).

Challenge. Design a linear-time algorithm (without sorting).

# SHUFFLING HALL OF SHAME (MICROSOFT)



Microsoft antitrust probe by EU. Microsoft agreed to provide a randomized ballot screen for users to select browser.

## Select your web browser(s)



**Google  
chrome**

A fast new browser from Google. Try it now!



**Safari**

Safari for Windows from Apple, the world's most innovative browser.



**mozilla  
Firefox**

Your online security is Firefox's top priority. Firefox is free, and made to help you get the most out of the



**Opera™  
browser**

The fastest browser on Earth. Secure, powerful and easy to use, with excellent privacy protection.



**Windows®  
Internet  
Explorer®8**

Designed to help you take control of your privacy and browse with confidence. Free from Microsoft.



appeared last 50% of the time

# SHUFFLING HALL OF SHAME (PLANETPOKER.COM)



Texas hold'em poker. Software must shuffle electronic cards.



## How We Learned to Cheat at Online Poker: A Study in Software Security

<https://www.developer.com/tech/article.php/616221/How-We-Learned-to-Cheat-at-Online-Poker-A-Study-in-Software-Security.htm>

# SHUFFLING HALL OF SHAME (PRINCETON)



Ivy league school room draw. Students assigned random room draw times.

## U. claims error in room draw process, provides compensation to affected students

By [Rebecca Han](#) | Apr 26, 2019

Print



Photo Credit: Jon Ort / The Daily Princetonian

# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

<https://algs4.cs.princeton.edu>

## 2.1 ELEMENTARY SORTS

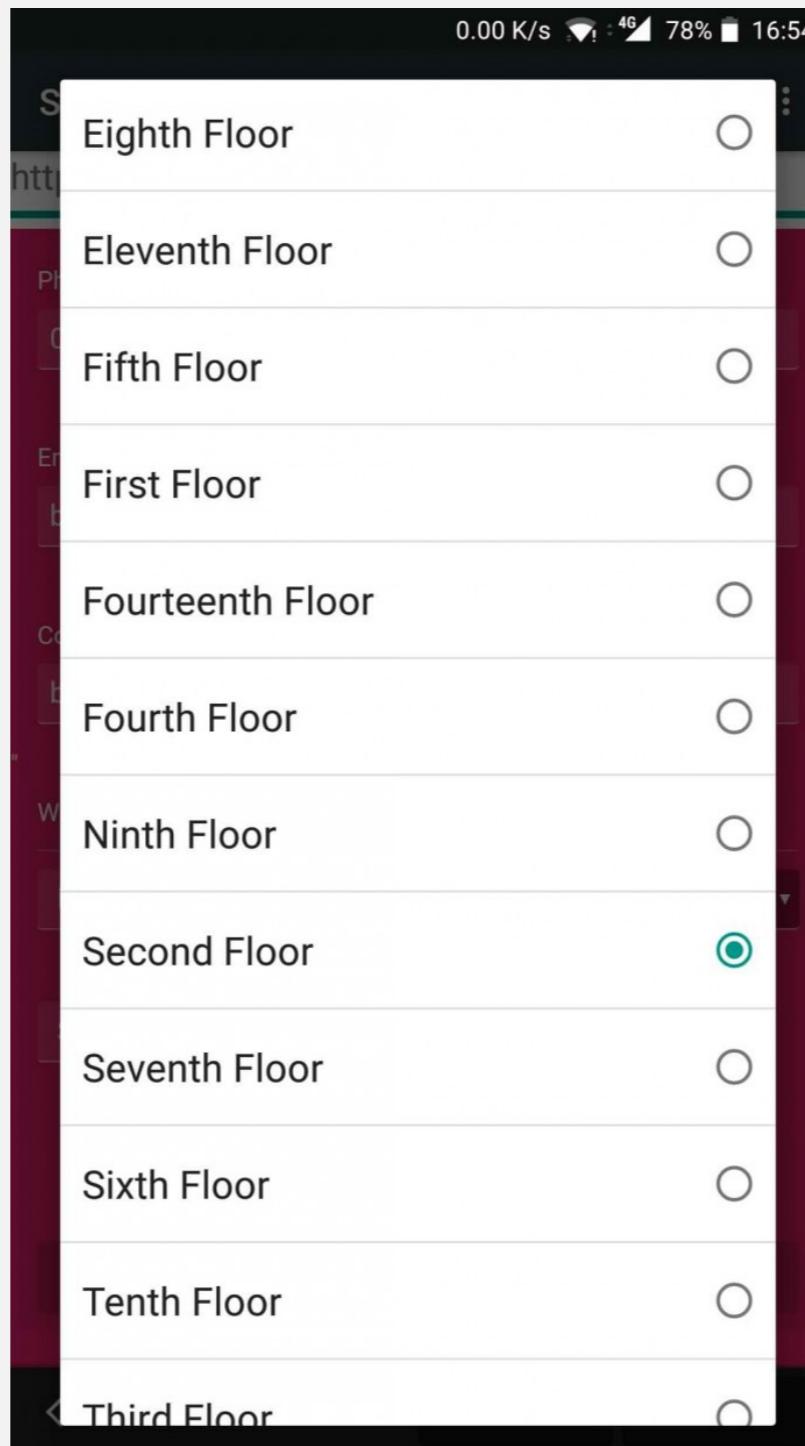
---

- ▶ *rules of the game*
- ▶ *selection sort*
- ▶ *insertion sort*
- ▶ *shuffling*
- ▶ **comparators**
- ▶ *stability*

# Different orderings

---

Q. When might we need to define different sort orderings?

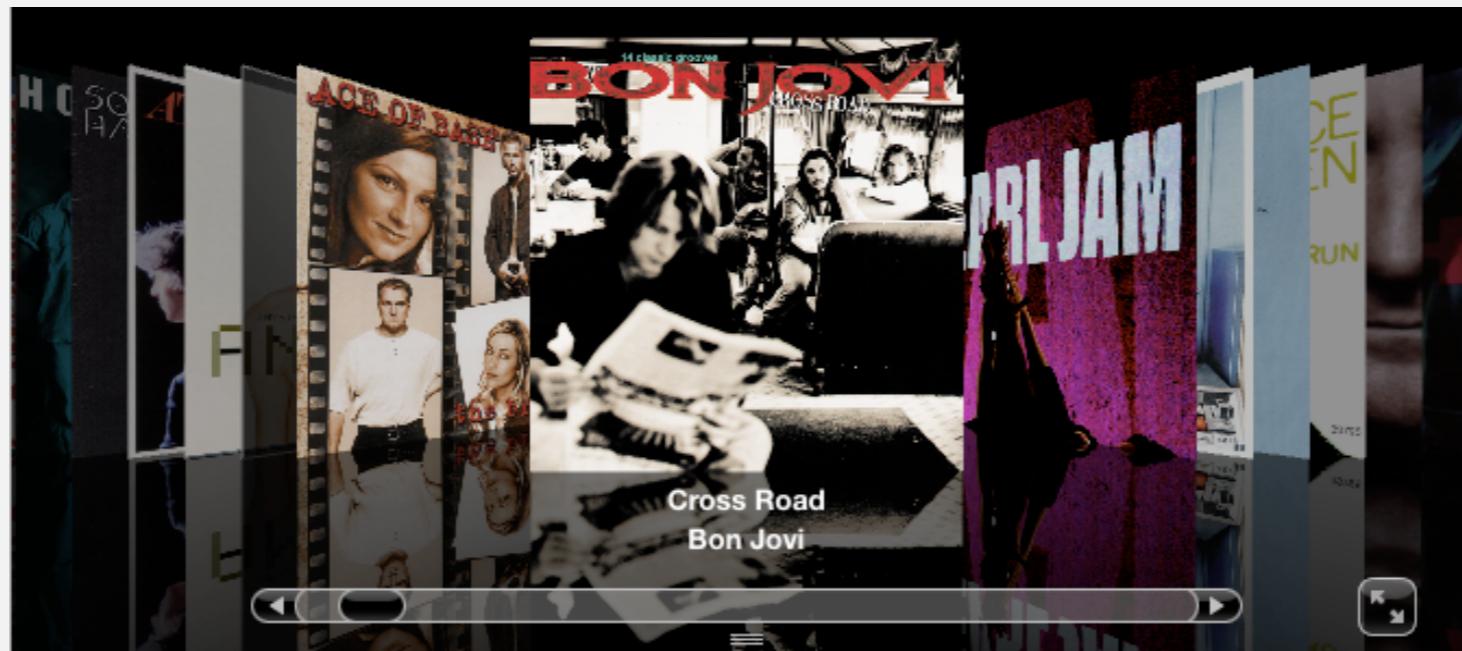


# Sort music library by artist

	Name	Artist	Time	Album
12	<input checked="" type="checkbox"/> Let It Be	The Beatles	4:03	Let It Be
13	<input checked="" type="checkbox"/> Take My Breath Away	BERLIN	4:13	Top Gun - Soundtrack
14	<input checked="" type="checkbox"/> Circle Of Friends	Better Than Ezra	3:27	Empire Records
15	<input checked="" type="checkbox"/> Dancing With Myself	Billy Idol	4:43	Don't Stop
16	<input checked="" type="checkbox"/> Rebel Yell	Billy Idol	4:49	Rebel Yell
17	<input checked="" type="checkbox"/> Piano Man	Billy Joel	5:36	Greatest Hits Vol. 1
18	<input checked="" type="checkbox"/> Pressure	Billy Joel	3:16	Greatest Hits, Vol. II (1978 - 1985) (Disc 2)
19	<input checked="" type="checkbox"/> The Longest Time	Billy Joel	3:36	Greatest Hits, Vol. II (1978 - 1985) (Disc 2)
20	<input checked="" type="checkbox"/> Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
21	<input checked="" type="checkbox"/> Sunday Girl	Blondie	3:15	Atomic: The Very Best Of Blondie
22	<input checked="" type="checkbox"/> Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
23	<input checked="" type="checkbox"/> Dreaming	Blondie	3:06	Atomic: The Very Best Of Blondie
24	<input checked="" type="checkbox"/> Hurricane	Bob Dylan	8:32	Desire
25	<input checked="" type="checkbox"/> The Times They Are A-Changin'	Bob Dylan	3:17	Greatest Hits
26	<input checked="" type="checkbox"/> Livin' On A Prayer	Bon Jovi	4:11	Cross Road
27	<input checked="" type="checkbox"/> Beds Of Roses	Bon Jovi	6:35	Cross Road
28	<input checked="" type="checkbox"/> Runaway	Bon Jovi	3:53	Cross Road
29	<input checked="" type="checkbox"/> Rasputin (Extended Mix)	Boney M	5:50	Greatest Hits
30	<input checked="" type="checkbox"/> Have You Ever Seen The Rain	Bonnie Tyler	4:10	Faster Than The Speed Of Night
31	<input checked="" type="checkbox"/> Total Eclipse Of The Heart	Bonnie Tyler	7:02	Faster Than The Speed Of Night
32	<input checked="" type="checkbox"/> Straight From The Heart	Bonnie Tyler	3:41	Faster Than The Speed Of Night
33	<input checked="" type="checkbox"/> Holding Out For A Hero	Bonny Tyler	5:49	Meat Loaf And Friends
34	<input checked="" type="checkbox"/> Dancing In The Dark	Bruce Springsteen	4:05	Born In The U.S.A.
35	<input checked="" type="checkbox"/> Thunder Road	Bruce Springsteen	4:51	Born To Run
36	<input checked="" type="checkbox"/> Born To Run	Bruce Springsteen	4:30	Born To Run
37	<input checked="" type="checkbox"/> Jungleland	Bruce Springsteen	9:34	Born To Run
38	<input checked="" type="checkbox"/> Turn! Turn! Turn! (To Everything)	The Byrds	3:57	Forrest Gump The Soundtrack (Disc 2)

# Sort music library by song name

---



	Name	Artist	Time	Album
1	<input checked="" type="checkbox"/> Alive	Pearl Jam	5:41	Ten
2	<input checked="" type="checkbox"/> All Over The World	Pixies	5:27	Bossanova
3	<input checked="" type="checkbox"/> All Through The Night	Cyndi Lauper	4:30	She's So Unusual
4	<input checked="" type="checkbox"/> Allison Road	Gin Blossoms	3:19	New Miserable Experience
5	<input checked="" type="checkbox"/> Ama, Ama, Ama Y Ensancha El ...	Extremoduro	2:34	Deltoya (1992)
6	<input checked="" type="checkbox"/> And We Danced	Hooters	3:50	Nervous Night
7	<input checked="" type="checkbox"/> As I Lay Me Down	Sophie B. Hawkins	4:09	Whaler
8	<input checked="" type="checkbox"/> Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
9	<input checked="" type="checkbox"/> Automatic Lover	Jay-Jay Johanson	4:19	Antenna
10	<input checked="" type="checkbox"/> Baba O'Riley	The Who	5:01	Who's Better, Who's Best
11	<input checked="" type="checkbox"/> Beautiful Life	Ace Of Base	3:40	The Bridge
12	<input checked="" type="checkbox"/> Beds Of Roses	Bon Jovi	6:35	Cross Road
13	<input checked="" type="checkbox"/> Black	Pearl Jam	5:44	Ten
14	<input checked="" type="checkbox"/> Bleed American	Jimmy Eat World	3:04	Bleed American
15	<input checked="" type="checkbox"/> Borderline	Madonna	4:00	The Immaculate Collection
16	<input checked="" type="checkbox"/> Born To Run	Bruce Springsteen	4:30	Born To Run
17	<input checked="" type="checkbox"/> Both Sides Of The Story	Phil Collins	6:43	Both Sides
18	<input checked="" type="checkbox"/> Bouncing Around The Room	Phish	4:09	A Live One (Disc 1)
19	<input checked="" type="checkbox"/> Boys Don't Cry	The Cure	2:35	Staring At The Sea: The Singles 1979–1985
20	<input checked="" type="checkbox"/> Brat	Green Day	1:43	Insomniac
21	<input checked="" type="checkbox"/> Breakdown	Deerheart	3:40	Deerheart
22	<input checked="" type="checkbox"/> Bring Me To Life (Kevin Roen Mix)	Evanescence Vs. Pa...	9:48	
23	<input checked="" type="checkbox"/> Californication	Red Hot Chili Pepp...	1:40	
24	<input checked="" type="checkbox"/> Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
25	<input checked="" type="checkbox"/> Can't Get You Out Of My Head	Kylie Minogue	3:50	Fever
26	<input checked="" type="checkbox"/> Celebration	Kool & The Gang	3:45	Time Life Music Sounds Of The Seventies – C
27	<input checked="" type="checkbox"/> Chaiwa Chaiwa	Salbhawinder Singh	5:11	Bombay Dreams

# Comparable interface: review

Comparable interface: sort using a type's **natural order**.

```
public class Date implements Comparable<Date>
{
    private final int month, day, year;

    public Date(int m, int d, int y)
    {
        month = m;
        day   = d;
        year  = y;
    }

    ...

    public int compareTo(Date that)
    {
        if (this.year < that.year) return -1;
        if (this.year > that.year) return +1;
        if (this.month < that.month) return -1;
        if (this.month > that.month) return +1;
        if (this.day   < that.day)  return -1;
        if (this.day   > that.day)  return +1;
        return 0;
    }
}
```



natural order

# Comparator interface

Comparator interface: sort using an alternate order.

```
public interface Comparator<Item>
{
    public int compare(Item v, Item w);
}
```

Required property. Must be a total order.

string order	example
natural order	Now is the time
case insensitive	is Now the time
Spanish language	café cafetero cuarto churro nube ñoño
British phone book	McKinley Mackintosh

pre-1994 order for  
digraphs ch and ll and rr



# Comparator interface: system sort

To use with Java system sort:

- Create Comparator object.
- Pass as second argument to `Arrays.sort()`.

```
String[] a;
...
Arrays.sort(a);           uses natural order
...
Arrays.sort(a, String.CASE_INSENSITIVE_ORDER);    uses alternate order defined by
...
Arrays.sort(a, Collator.getInstance(new Locale("es")));
...
Arrays.sort(a, new BritishPhoneBookOrder());
...
```

**Bottom line.** Decouples the definition of the data type from the definition of what it means to compare two objects of that type.

# Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the Comparator interface.
- Implement the compare() method.
- Provide client access to Comparator.

```
import java.util.Comparator;

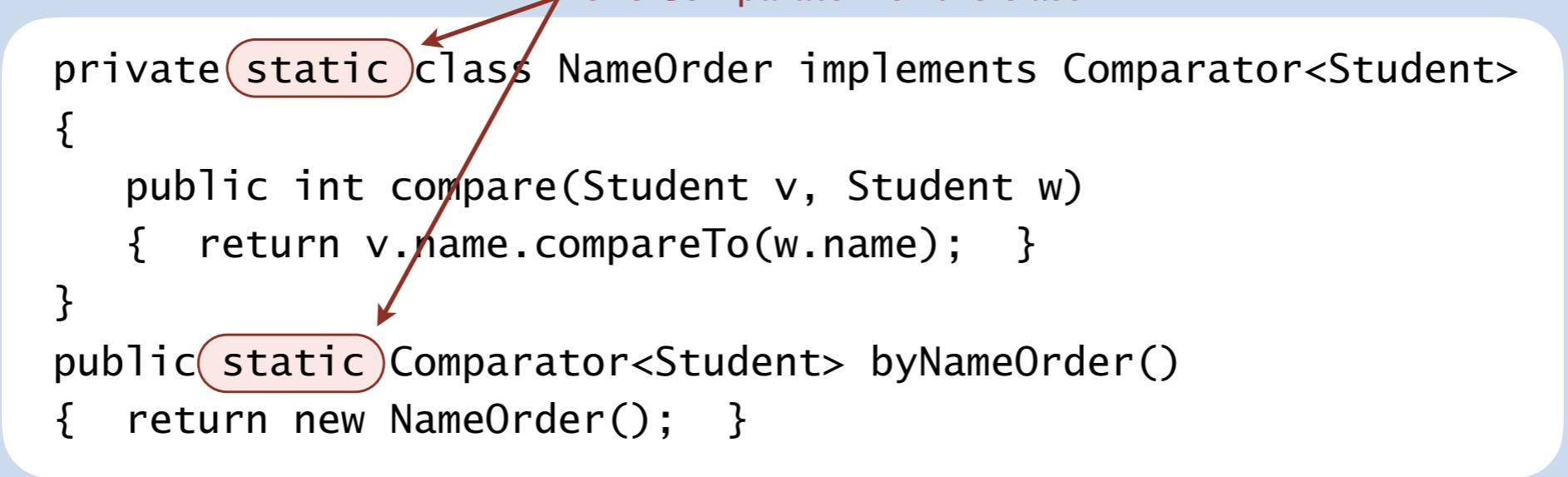
public class Student
{
    private final String name;
    private final int section;
    ...

    private static class NameOrder implements Comparator<Student>
    {
        public int compare(Student v, Student w)
        {   return v.name.compareTo(w.name);   }
    }

    public static Comparator<Student> byNameOrder()
    {   return new NameOrder();   }

}
```

one Comparator for the class



<https://algs4.cs.princeton.edu/12oop/Student.java.html>

# Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the Comparator interface.
- Implement the compare() method.
- Provide client access to Comparator.

```
import java.util.Comparator;

public class Student
{
    private final String name;
    private final int section;
    ...

    private static class SectionOrder implements Comparator<Student>
    {
        public int compare(Student v, Student w)
        {   return Integer.compare(v.section, w.section); }
    }

    public static Comparator<Student> bySectionOrder()
    {   return new SectionOrder(); }

}
```

useful library  
method

# Comparator interface: implementing

---

To implement a comparator:

- Define a (nested) class that implements the Comparator interface.
- Implement the compare() method.
- Provide client access to Comparator.

`Insertion.sort(a, Student.byNameOrder());`

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

`Insertion.sort(a, Student.bySectionOrder());`

Furia	1	A	(766) 093-9873	101 Brown
Rohde	2	A	(232) 343-5555	343 Forbes
Andrews	3	A	(664) 480-0023	097 Little
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# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

<https://algs4.cs.princeton.edu>

## 2.1 ELEMENTARY SORTS

---

- ▶ *rules of the game*
- ▶ *selection sort*
- ▶ *insertion sort*
- ▶ *shuffling*
- ▶ *comparators*
- ▶ ***stability***

skipped in lecture  
(see precept)

# Stability

---

A typical application. First, sort by name; then sort by section.

`Selection.sort(a, Student.byNameOrder());`

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

`Selection.sort(a, Student.bySectionOrder());`

Furia	1	A	(766) 093-9873	101 Brown
Rohde	2	A	(232) 343-5555	343 Forbes
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Andrews	3	A	(664) 480-0023	097 Little
Kanaga	3	B	(898) 122-9643	22 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Battle	4	C	(874) 088-1212	121 Whitman

@#%&@! Students in section 3 no longer sorted by name.

A **stable** sort preserves the relative order of items with equal keys.





Which sorting algorithm(s) are stable?

- A. Selection sort.
- B. Insertion sort.
- C. Both A and B.
- D. Neither A nor B.

## Stability: insertion sort

Proposition. Insertion sort is **stable**.

```
public class Insertion
{
    public static void sort(Comparable[] a)
    {
        int n = a.length;
        for (int i = 0; i < n; i++)
            for (int j = i; j > 0 && less(a[j], a[j-1]); j--)
                exch(a, j, j-1);
    }
}
```

i	j	0	1	2	3	4
0	0	B <sub>1</sub>	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	B <sub>2</sub>
1	0	A <sub>1</sub>	B <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	B <sub>2</sub>
2	1	A <sub>1</sub>	A <sub>2</sub>	B <sub>1</sub>	A <sub>3</sub>	B <sub>2</sub>
3	2	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	B <sub>1</sub>	B <sub>2</sub>
4	4	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	B <sub>1</sub>	B <sub>2</sub>
		A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	B <sub>1</sub>	B <sub>2</sub>

Pf. Equal items never move past each other.

## Stability: selection sort

Proposition. Selection sort is not stable.

```
public class Selection
{
    public static void sort(Comparable[] a)
    {
        int n = a.length;
        for (int i = 0; i < n; i++)
        {
            int min = i;
            for (int j = i+1; j < n; j++)
                if (less(a[j], a[min]))
                    min = j;
            exch(a, i, min);
        }
    }
}
```

i	min	0	1	2
0	2	B <sub>1</sub>	B <sub>2</sub>	A
1	1	A	B <sub>2</sub>	B <sub>1</sub>
2	2	A	B <sub>2</sub>	B <sub>1</sub>
		A	B <sub>2</sub>	B <sub>1</sub>

Pf by counterexample. Long-distance exchange can move an equal item past another one.