

# An Examination of Sport Climbing Competition Format and Scoring System

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## Abstract

Sport climbing, which made its Olympic debut at the 2020 Summer Games, generally consists of three separate disciplines: speed climbing, bouldering, and lead climbing. However, the International Olympic Committee (IOC) only allowed one set of medals per gender for sport climbing at Tokyo 2020. As a result, the governing body of sport climbing, rather than choosing only one of the three disciplines to include in the Olympics, decided to create a competition combining all three disciplines. In order to determine a winner, a combined scoring system was created using the product of the ranks across the three disciplines to determine an overall score for each climber. In this work, the sport climbing scoring system is evaluated through simulations to investigate its general features such as the advancement probabilities given certain placements and the sensitivity of the scoring system to small changes. In addition, analyses of historical climbing contest results are presented and real examples of violations of the independence of irrelevant alternatives are illustrated. Finally, this work finds evidence that the current competition format is putting speed climbers at a disadvantage.

*Keywords:* sport climbing, rankings, 2020 Summer Olympics

# 1 Introduction

In 2016, the International Olympic Committee (IOC) announced the addition of five new sports to the 2020 Summer Olympics in Tokyo, Japan, which would then reschedule for 2021 due to the impact of the COVID-19 global pandemic. The five new events added to Tokyo 2020’s competition program were baseball/softball, karate, skateboard, surfing, sports climbing (IOC 2016). One of the new sports, sport climbing, is particularly interesting because of its scoring system which uses the product of ranks across three disciplines to determine the medalists, with the lowest rank product declared the winner.

The three disciplines that comprise sport climbing at Tokyo 2020 are: speed climbing, bouldering, and lead climbing. Speed climbing takes place on a standardized course and competitors try to reach the top of the course as fast as possible. For Tokyo 2020, speed climbing is being contested in a head-to-head format with ranks determined by how far a competitor advances in the bracket. In bouldering, contestants have a fixed amount of time to complete as many courses as they can. Winners are determined based on who completes the most courses and ties are broken based on who had the fewest attempts. Ties are further broken by the competitor achieved the most “zone holds”, which are holds approximately halfway through each course. Finally, in lead climbing, an athlete gets one point for each hold that they reach, so whoever reaches the highest point on the wall is the winner. Each lead climber only gets one attempt and when they fall their attempt is over.

At the 2020 Summer Olympics, both sport climbing competitions for male and female at the begin with 20 climbers who has previously qualified for the Olympics from qualifying events held in 2019 and 2020. All 20 athletes compete in each of the three disciplines in the qualification round, and their performances in each concentration are ranked from 1 to 20. A competitor’s combined score is computed as the product of their ranks in the three events; specifically,

$$Score_i = R_i^S \times R_i^B \times R_i^L, \quad (1)$$

where  $R_i^S$ ,  $R_i^B$ , and  $R_i^L$  are the ranks of the  $i$ -th competitor in speed climbing, bouldering, and lead climbing, respectively.

The 8 climbers with the lowest score (in terms of product of ranks across the three disciplines) in the qualification round advance to the finals where they once again compete

in all three disciplines. The final score for each person in the final stage is determined by multiplying their ranks in each discipline, similar to the qualification round, with the main difference being that there are only 8 competitors in the final as opposed to 20 in the qualification round. The climbers with the lowest, second lowest, and third lowest product of ranks in the final wins the gold, silver, and bronze medal, respectively. This type of scoring system heavily rewards high finishes and relatively ignores poor finishes. For instance, if climber A finished 1st, 20th, and 20th and climber B finished 10th, 10th, and 10th, climber B would have a score of 1000 whereas climber A would have a much better score of 400, despite finishing last in 2 out of 3 of the events.

To the best of our knowledge, we know of no sporting event, team or individual, that uses the product of ranks to determine an overall rankings. There are examples of team sports that use the sum of ranks to determine the winning team such as cross country, where the team with the lowest sum of ranks of the top five runners is determined to be the winner (Hammond 2007, Boudreau et al. 2018). In addition, some individual sports such as the decathlon and heptathlon rely on a sum of scores from the ten or seven events, however, these scores are not determined based on the ranks of the competitors. That is, a decathlete’s score is entirely based upon their times, distances, and heights, and their overall score will be exactly the same if the times, distances, and heights remain the same regardless of the performance of other individuals (Westera 2006). Furthermore, there are other individual competitions consisted of several events combined that do base their scoring on ranks, such as crossfit competitions. In each event, points are earned based on the competitor’s rank in the event based on a scoring table, and a competitors final score is based on the sum of their scores across all the events (CrossFit 2021).

The decision to combine the three climbing events and only award one set of medals for both men’s and women’s events in the Olympics has received a large amount of criticism from climbing athletes all over the world. In a series of interviews conducted by Climbing Magazine in 2016, a number of climbers shared their thoughts and concerns about the new Olympics climbing format. Climber Lynn Hill compared the idea of combining speed climbing, bouldering, and lead climbing to “asking a middle distance runner to compete in the sprint”. She then added “Speed climbing is a sport within our sport”. Other climbers

also hold the same opinion as Hill regarding speed climbing, using words and phrases like “bogus”, “a bummer”, “less than ideal”, “not in support”, and “cheesy and unfair” to describe the new combined competition format. Courtney Woods stated “Speed climbers will have the biggest disadvantage because their realm isn’t based on difficult movements”. Mike Doyle believed “Honestly, the people that will suffer the most are the ones that focus only on speed climbing. Those skills/abilities don’t transfer as well to the other disciplines”. Most climbers also expressed their hope for a change in the format in future tournaments, with some calling for each discipline to have its own set of medals. See Blanchard (2016) for more information on these interviews.

In this paper, we attempt to use simple statistical analysis to address the limitations of sport climbing’s combined competition format and ranking system. The manuscript is outlined as follows. We first begin with some descriptions of the data and methods in Section 2. Our analyses and results are then presented in Sections 3 and 4. Finally, in Section 5, we summarize up our main findings and provide a discussion to close out the paper.

## 2 Data and Methods

Our first sets of data come from a simulation study that we conducted, with the purpose of examining the rankings and scoring for climbers in both qualification and final rounds. For each round, we performed 10000 simulations, and this was accomplished by randomly assigning the ranks of each event to every participant, with the assumption that the ranks are uniformly distributed. After the completion of the simulations, we calculated the total scores for every simulated round, as well as the final standings for the climbing athletes. The simulation results allow to answer questions about various topics, including the distributions of scores for qualifying and final rounds, and the probabilities of advancing to the finals or winning a medal, given certain conditions. First of all, Table 1 and Figure 1 are numerical and visual summaries of climbing total score obtained from our simulated data for the qualification and final rounds.

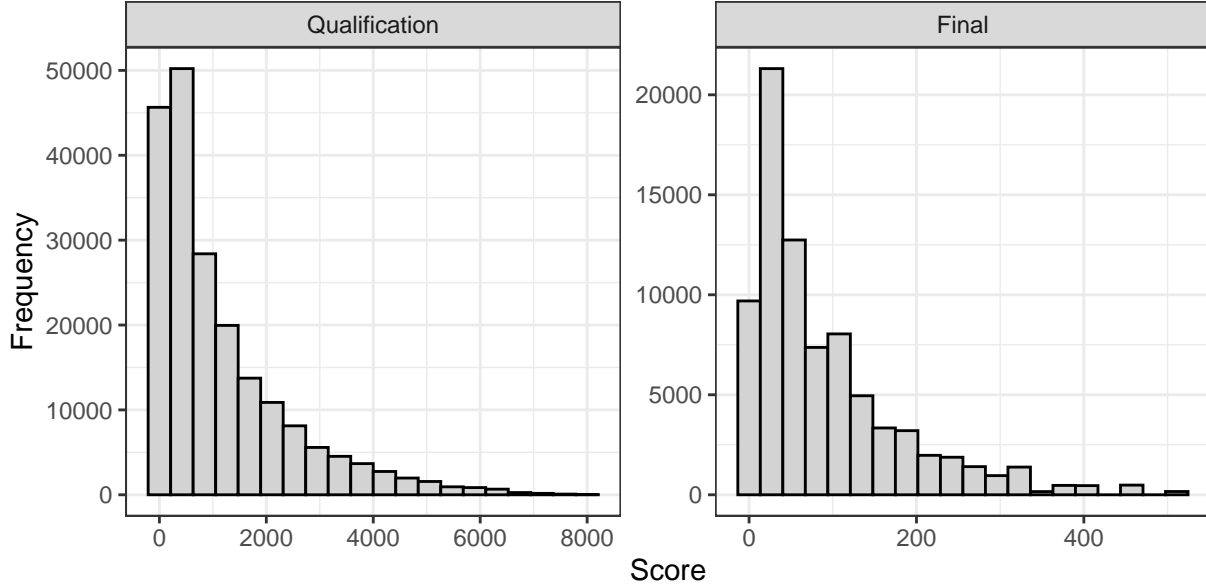


Figure 1: Histogram of simulated scores for qualification and final rounds

Table 1: Descriptive statistics of simulated scores for qualification and final rounds

round	min	Q1	median	Q3	max	mean	sd	n
Qualification	1	240	684	1638	8000	1158.84	1273.48	200000
Final	1	24	60	126	512	91.20	91.11	80000

Additionally, we rely historical results from major climbing competitions in recent years. We collected data on climbing contests that took place between 2018 and 2020, where the combined format was used to determine the scores and ranks of climbers. The events include the 2020 Continental Championships of Europe, Africa, Oceania, Pan-America; 2019 and 2018 World Championships; 2018 Asian Games; and 2018 Youth Olympics. Data were obtained from various sources, including the event websites, Wikipedia, and the International Federation of Sport Climbing (IFSC). The main attributes of our datasets are the name and nationality of the climbers; bib number (for some competitions); the finishing place of climbers in speed climbing, bouldering, and lead climbing; the total score (which equals the product of event ranks); and the final rank. We utilize this data to compute the correlations between the event ranks and final table position, as well as to look at how

often the final orderings change if one athlete is dropped and the ranks for each discipline as well as the total score are re-calculated.

Since we are interested in applying the “leave-one-climber-out” type of analysis to sport climbing, there is a connection between this situation and the idea of independence of irrelevant alternatives (IIA). The IIA criterion is a property of a voting system which states that after a winner is determined, if one of the losing candidates drops out and the votes are recounted, there should not be a change in the winner. First mentioned by Arrow (1951), the IIA condition is also known as Luce’s choice axiom (Luce 1959) in probability theory, and it has had a number of applications in the fields of decision theory, economics, and psychology over the years.

We notice a link between the concept of IIA and the topic of ranking system in sports. As an illustration, suppose we have 3 players A, B, and C participating in a competition. If A finishes in the first place and C is later disqualified and removed, A should still win. If the original winner (A) loses the modified competition (with C removed), then the Independence of Irrelevant Alternatives has been violated. For our particular case, this sort of analysis can be helpful in examining the results of a climbing competition, specifically how a disqualification can affect the standings of medalists in the final round.

Paragraph on rank swapping

Paragraph on PCA

## 3 Simulations

### 3.1 Uniform Ranks

In this section, we discuss the results of our simulations described in Section 2. We first look at the chances of advancing to the final round for a qualifier, given that 1) they finish first in any of the three disciplines, and 2) they win the first event (which happens to be speed climbing in reality). Our simulation study shows that a climber is almost guaranteed to make the final round if they win at least one of the three climbing concentrations or if they win the opening event, as the probabilities of advancing for these two situations are 0.9948 and 0.9951, respectively (see Table 2). Moreover, after winning the first event, a

climber is also more likely to finish first overall (see Table 2) than any other positions in the overall qualification rankings. Therefore, this product of ranks scoring format highly rewards climbers who can get off to a strong start, which is a major strategy climbing athletes should take note of.

In addition, the average score for qualification positions 1 to 8 are displayed in Table 3. We observe that on average, the minimum score that one should aim for in order to move on to the final round is about 434 (for 8th rank).

Table 2: This table shows every possible qualification rank obtained from simulations for a climber given that a they win at least one discipline or finish first in the first discipline. For each rank, information on the number of times that climbers finish at the given rank, the probability of finishing at exactly the given rank, and the probability of finishing at or below the given rank are included.

Win Any				Win First			
Rank	Count	Probability	Cumulative	Rank	Count	Probability	Cumulative
1	7890	0.2762	0.2762	1	2999	0.2999	0.2999
2	5943	0.2080	0.4842	2	1963	0.1963	0.4962
3	4644	0.1626	0.6467	3	1559	0.1559	0.6521
4	3651	0.1278	0.7745	4	1269	0.1269	0.7790
5	2766	0.0968	0.8714	5	957	0.0957	0.8747
6	1960	0.0686	0.9400	6	686	0.0686	0.9433
7	1118	0.0391	0.9791	7	363	0.0363	0.9796
8	448	0.0157	0.9948	8	155	0.0155	0.9951
9	127	0.0044	0.9992	9	42	0.0042	0.9993
10	19	0.0007	0.9999	10	6	0.0006	0.9999
11	3	0.0001	1.0000	11	1	0.0001	1.0000

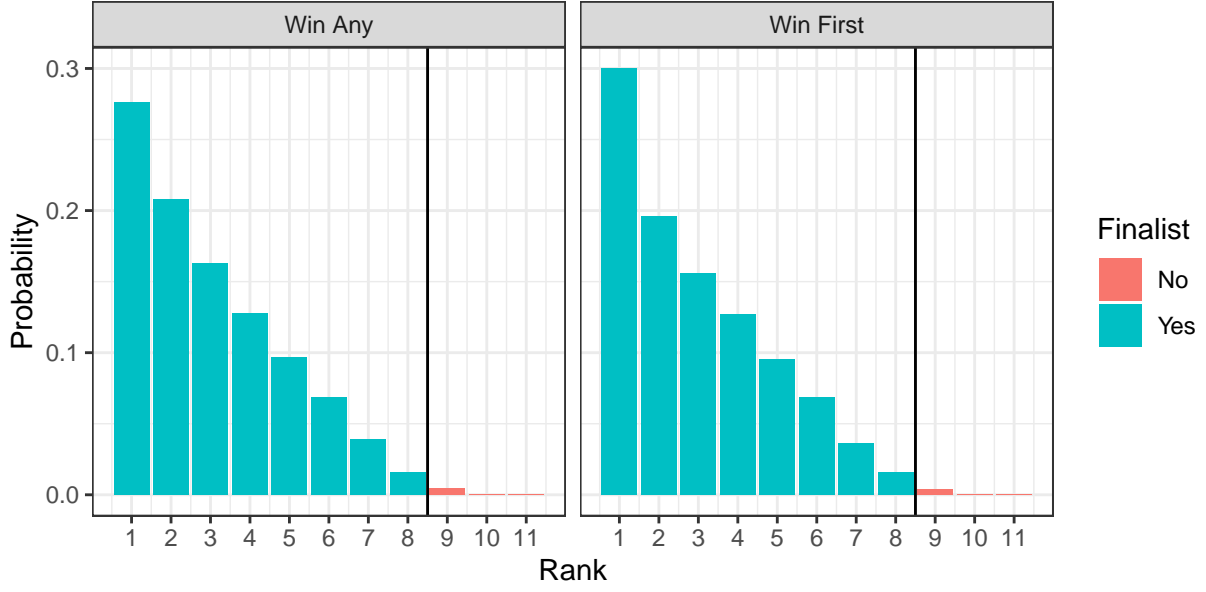


Figure 2: Distribution plots of qualification ranks, given 1) winning any event and 2) winning first event

Table 3: The average scores for the top 10 qualification ranks according to our simulations. A climber will secure a finalist spot if they finish in the top 8.

Rank	Average score
1	36.0187
2	73.6111
3	115.3954
4	162.2263
5	216.0041
6	278.1649
7	350.3272
8	434.5932
9	532.1383
10	642.3298



Regarding the finals, Table 4 indicates that a climber is very likely to finish in the top 3 and hence earn a medal if they win the first event (83.03% chance) or any event (85.01% chance). Furthermore, according to our final simulations, in order to obtain a climbing medal, the average scores (rounded down) that put an athlete in position to receive gold, silver, and bronze medals are 9, 20, and 33, respectively (see Table 5).

Table 4: This table shows every possible final rank obtained from simulations for a climber given that a they win at least one discipline or finish first in the first discipline. For each rank, information on the number of times that climbers finish at the given rank, the probability of finishing at exactly the given rank, and the probability of finishing at or below the given rank are included.

Win Any				Win First			
Rank	Count	Probability	Cumulative	Rank	Count	Probability	Cumulative
1	9277	0.3512	0.3512	1	4295	0.4295	0.4295
2	7409	0.2805	0.6317	2	2505	0.2505	0.6800
3	5257	0.1990	0.8308	3	1702	0.1702	0.8502
4	3078	0.1165	0.9473	4	1020	0.1020	0.9522
5	1187	0.0449	0.9922	5	393	0.0393	0.9915
6	198	0.0075	0.9997	6	83	0.0083	0.9998
7	7	0.0003	1.0000	7	2	0.0002	1.0000

### 3.2 Leave-one-climber-out

Another interesting question that we are interested in investigating is “What would happen to the rankings if a single climber is removed?”. This is related to the concept of independence of irrelevant alternatives (IIA), as mentioned in Section 2.

We would expect that if a climber with a high combined ranking is removed from the list of competitors, the placement of lower-ranked climbers will shift. In the most trivial case, if we were to remove the 3rd place climber, we would see 4th place climber shift to 3rd,

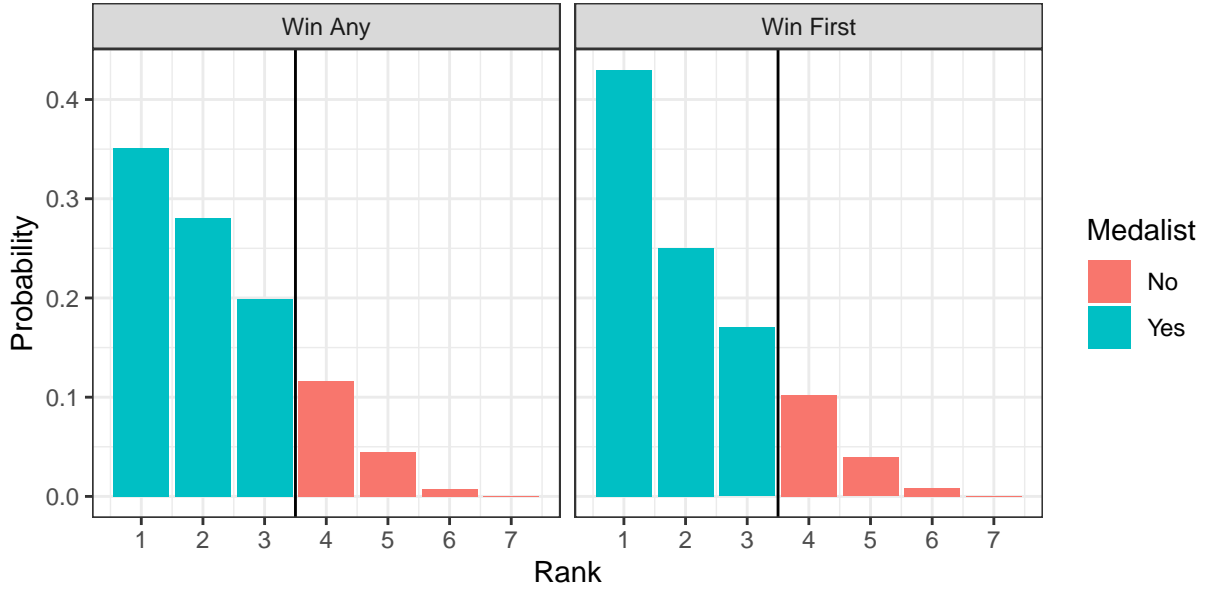


Figure 3: Distribution plots of final ranks, given 1) winning any event and 2) winning first event

Table 5: The average scores for all final ranks according to our simulations. Ranks 1, 2, and 3 are table positions that guarantee medalist status for climbers.

Rank	Average score
1	9.6687
2	20.1844
3	33.3658
4	50.5734
5	75.2499
6	110.7070
7	164.8258
8	265.0198

5th place climber shift to 4th, etc. It is maybe less trivial that when a climber is removed from the list, the placement of their higher-ranking competitors may also shift. In other words, there is a non-zero probability of seeing a change in the placement - particularly of medalists - regardless of the fact that no changes occurred in the performance of the remaining climbers. Real examples are given in Section 4. In this section, we conduct simulations to analyze the probability of seeing a reordering of medalists when one non-medalist is removed from the list of finalists.

### 3.2.1 Probability of Reordering in Higher Ranks

We first look at the rate at which the ranks above that of which is dropped are reordered.

(Need a more detailed description)

Dropped Rank	Simulations	Avg. Probability	Median Probability	St. Deviation
4	100	0.25869	0.2590	0.0145433
5	100	0.34032	0.3405	0.0151263
6	100	0.38012	0.3820	0.0152430
7	100	0.40636	0.4060	0.0149582
8	100	0.39439	0.3930	0.0151023

### 3.2.2 Probability of Change in Finalists

A potential event of interest is the replacement of one finalist with another, taking the chance away from the original finalist to win a medal in the finals round. Here we compute the probability that a finalist will be replaced with a non-finalist after removing another non-finalist from the rankings. Below is a table summarizing the results.

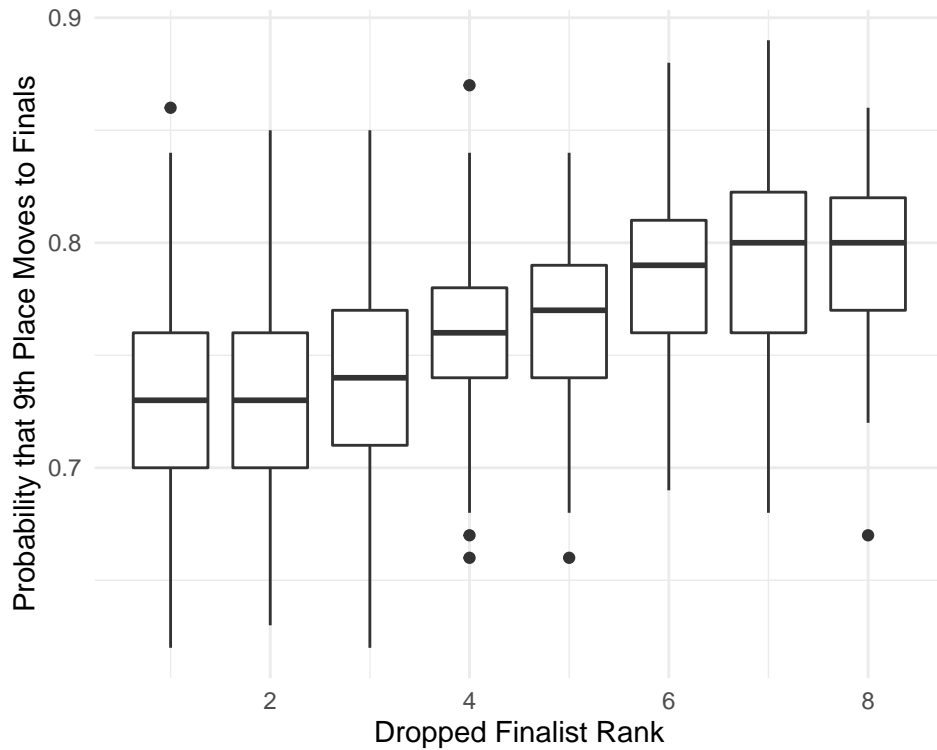
Dropped Rank	Simulations	Avg. Probability	Median Probability	St. Deviation
9	100	0.0505	0.050	0.0217597
10	100	0.1538	0.150	0.0318386
11	100	0.1603	0.160	0.0343616
12	100	0.1416	0.140	0.0353259
13	100	0.1365	0.140	0.0336162
14	100	0.1217	0.120	0.0344965
15	100	0.1213	0.120	0.0308353
16	100	0.1159	0.120	0.0306197
17	100	0.1086	0.105	0.0314312
18	100	0.1026	0.100	0.0329867
19	100	0.0886	0.090	0.0291641
20	100	0.0700	0.070	0.0258590

as can be seen, the probability is relatively low when the 9th place competitor is removed. My hypothesis is that there is enough separation between the original finalists and the 10th+ ranking competitors in the individual events that there isn't a high chance that the total rankings of the finalists can be overtaken by 10+th place competitors. Note: it might be worth computing the probability of getting a low ranking in any of the individual events, given that a competitor is a finalist (ie:  $P(r_1 > 8 \text{ or } r_2 > 8 \text{ or } r_3 > 8 \mid R_T \leq 8)$ ).

Olympic Finalist Bassa Mawem pulled out of the sport climbing finals due to an injury in his left arm. Reuters reported that Megos, originally 9th in the total qualification rankings would not be replacing him. We suspect that this might be part of the unclear format of the Olympic Sport Climbing format. However, according to the climber's rankings, Megos technically should not have been next in line to fill in the missing finalist spot. Upon removing B. Mawem from the list and reranking the remaining competitors, we find that Chon would have moved into 8th place with a 640 combined score and Megos would have remained in 9th with 648. B. Mawem originally ranked first in the speed climbing, and Megos did comparatively poorly with 19th place. Both performed better than Mawem in bouldering and lead, leaving those rankings unchanged.

Prior, we considered a change in finalists if a non-finalist was removed. It is clear now though that the more interesting scenario is would would replace an original finalist who is removed from the rankings. Should it be the original 9th place competitor, or does it make more sense with such a scoring system to rerank? Surely Chon should be rewarded for his superior performance in speed compared to Megos with the chance to win a medal?

The next set of simulations will evaluate the probability that a removed finalist is replaced with someone other than the original 9th place competitor.



### 3.2.3 Probability that Medalists are Reordered or Changed

The second scenario of interest is if a medalist is displaced by another medalist, or even a non-medalist. Below are the results summarizing the probability of this scenario occurring under uniform random ranking conditions.

Dropped Rank	Simulations	Avg. Probability	Median Probability	St. Deviation
4	100	0.25872	0.2580	0.0137106
5	100	0.31988	0.3190	0.0161416
6	100	0.28741	0.2870	0.0127969
7	100	0.25631	0.2545	0.0145301
8	100	0.20631	0.2050	0.0120283

## 4 Data Analysis

### 4.1 Correlation Analysis

#### 4.1.1 Rank Correlation

For our analysis on the relationship between the rankings of the events and the final result, we used data from the 2018 Youth Olympics Women’s Qualification. Figure 2 is a scatterplot and correlation matrix between the ranks of the individual events and the final standings, with Kendall’s Tau (Kendall 1938) as our measure of ordinal association between the quantities. It is evidently clear that there exists a strong and positive correlation between the ranks of bouldering and lead climbing, and as a results, the standings of these two events are highly correlated with the final rankings. On the other hand, the correlation with the final rank is not as strong for speed climbing as the other two events. Thus, speed climbers are facing a huge disadvantage in this scoring system, compared to those that are specialized in the other two concentrations. This validates the concerns of climbers in the interview mentioned in Section 1. We also have evidence for these correlations from the qualification rounds of the 2018 Asian Games (Table 4), 2019 World Championship (Table 5), 2020 European Championship (Table 6), and 2020 Pan-American Championships (Table 7).

#### 4.1.2 Principal Component Analysis

Olympic results

Women: lead and bouldering belong to the same component, the other is speed.

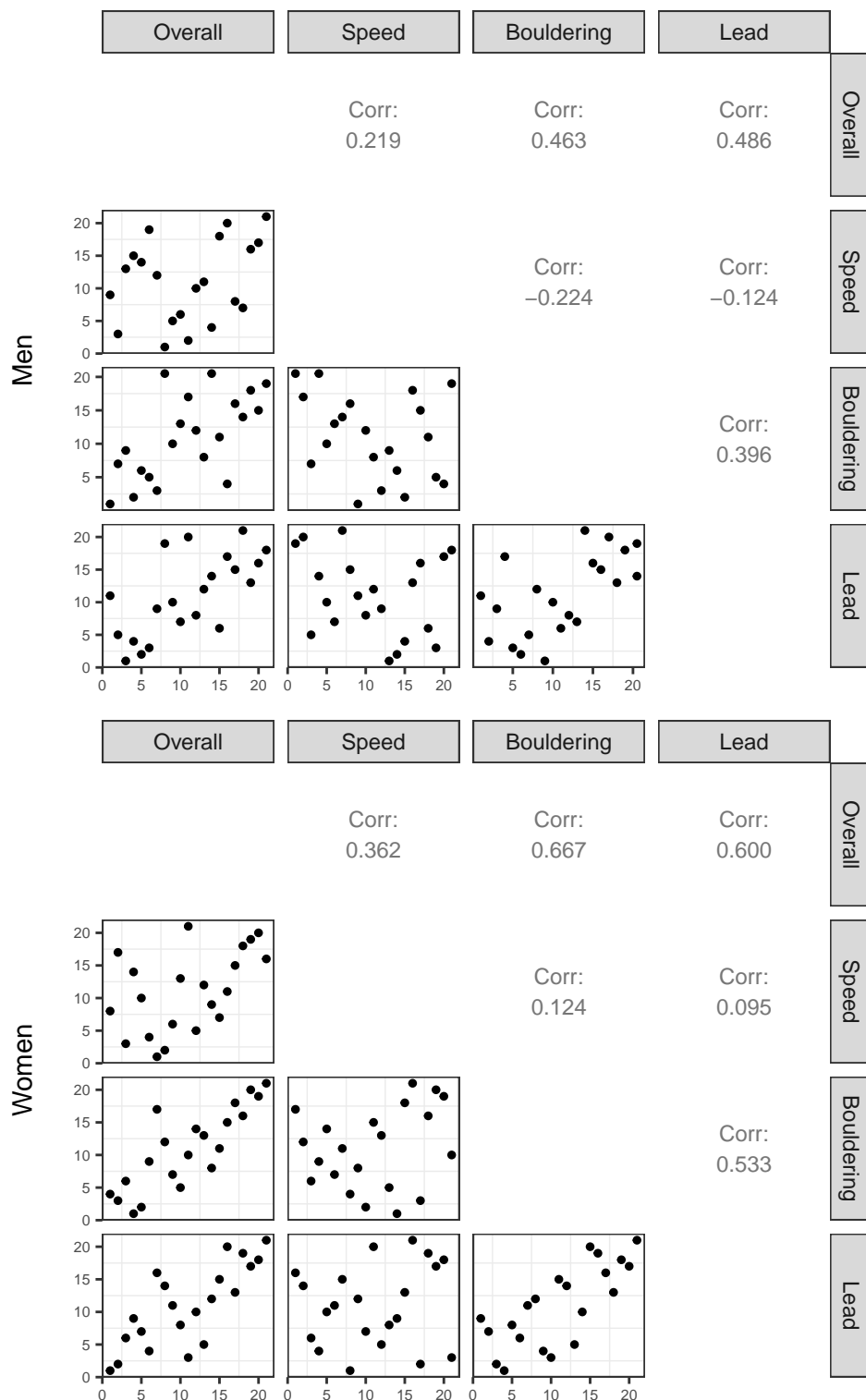


Figure 4: Kendall's rank correlations - 2018 World Championship, Men's and Women's Qualifications



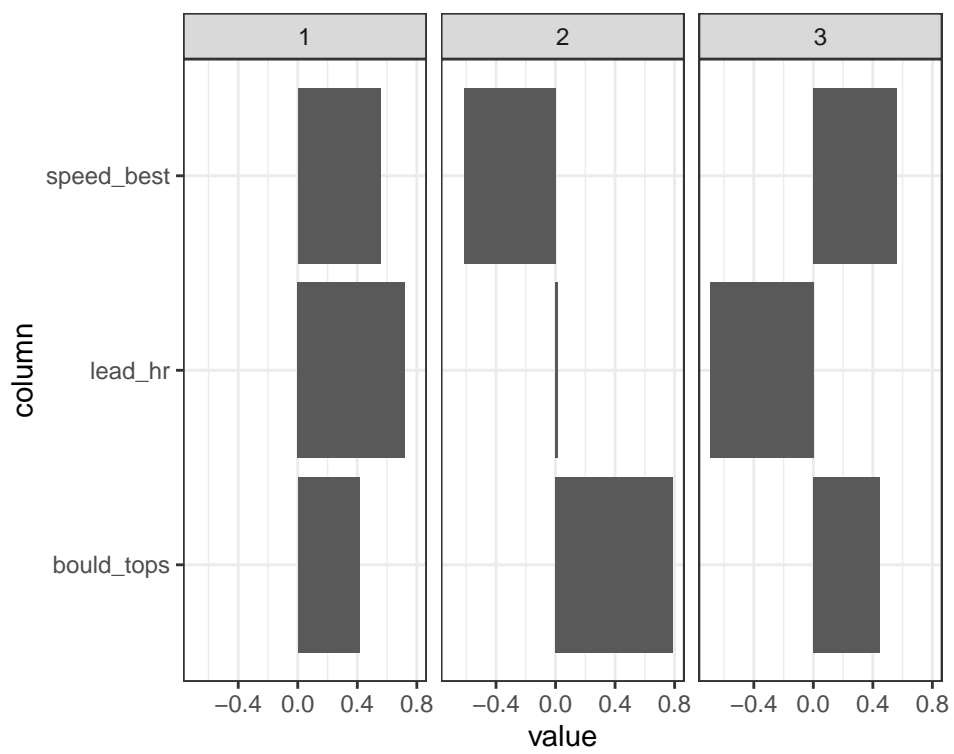
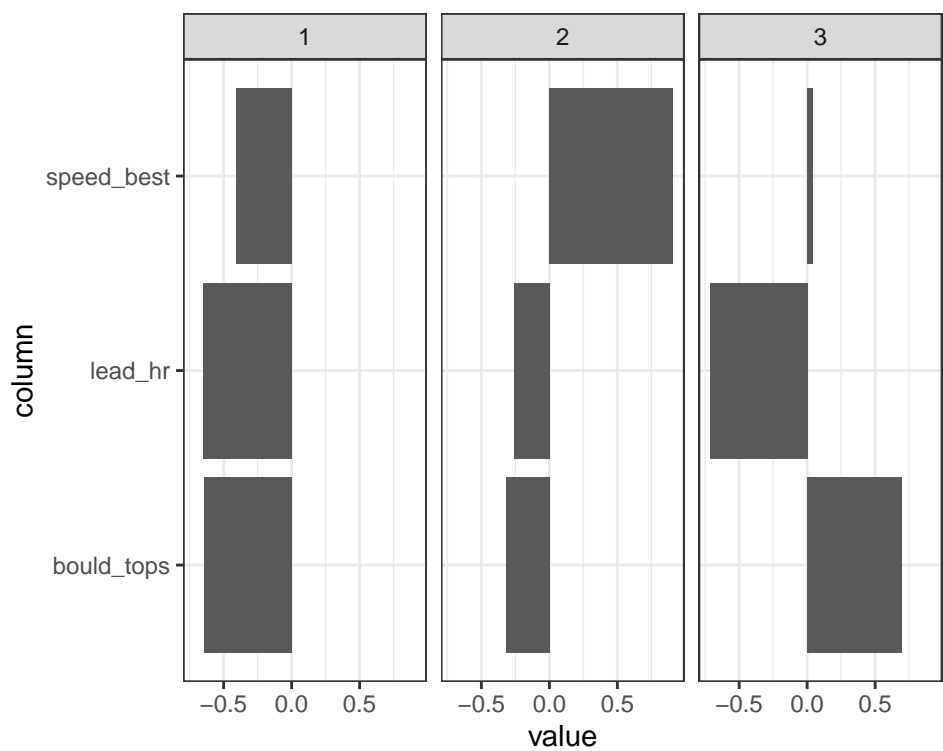
Figure 5: Women's Qualification PCA

“overall ability”

“power”

How each variable contributes to each principal component





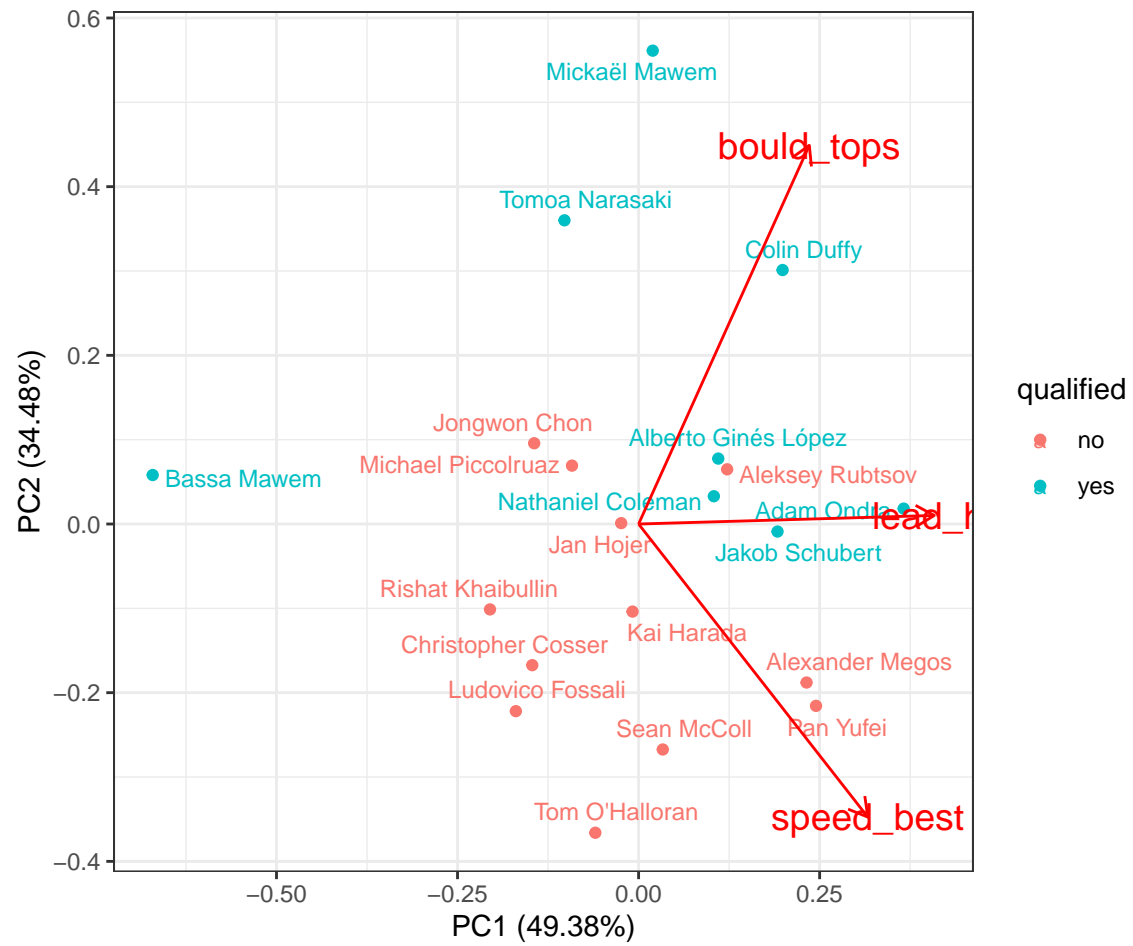


Figure 6: Men's Qualification PCA

## 4.2 Sensitivity Analysis

### 4.2.1 Leave-one-climber-out Analysis

Next, we perform analysis on the situations where a climber is dropped from the original standings. We once again make use of data from the 2018 Youth Olympics for this analysis, but this time we examine the final round of both men’s and women’s competitions.

Figure 3 shows the modified versions of the rankings after each ranked climber is excluded for both gender events. We have clear evidence from this plot that removing a single climber changes the rankings drastically, especially order of medalists. One particular interesting case is where an athlete’s position change when someone who originally finished behind them drops out. This situation is illustrated by panel 5 of the women’s competition, where the fifth-place climber, Krasovskaia, was excluded; and Meul, whose actual final rank was fourth, moved up to the second spot and claim the silver medal. Furthermore, this implies that the ranking system of Olympics sport climbing violates the IIA criterion.

Likewise, dropping a higher-ranked athlete could also result in a major shake-up to the new orderings of players. This type of ranking change is demonstrated by panel 1 and 2 of the men’s facet. After dropping the first ranked male contestant, Dohi; Schenk, who previously finished fourth and outside of the medalist group now has the lowest score and become the gold medal winner. A similar situation occurs in the case where the second best competitor (Tanaka) was removed, which results in a jump from fourth place to second place for Pan.

### 4.2.2 Rank Swapping Analysis

## 5 Conclusion and Discussion

Overall

Speed climbers

There is a great dependence on irrelevant party.

Future work

Recommendations: climbers were right, need a change in format

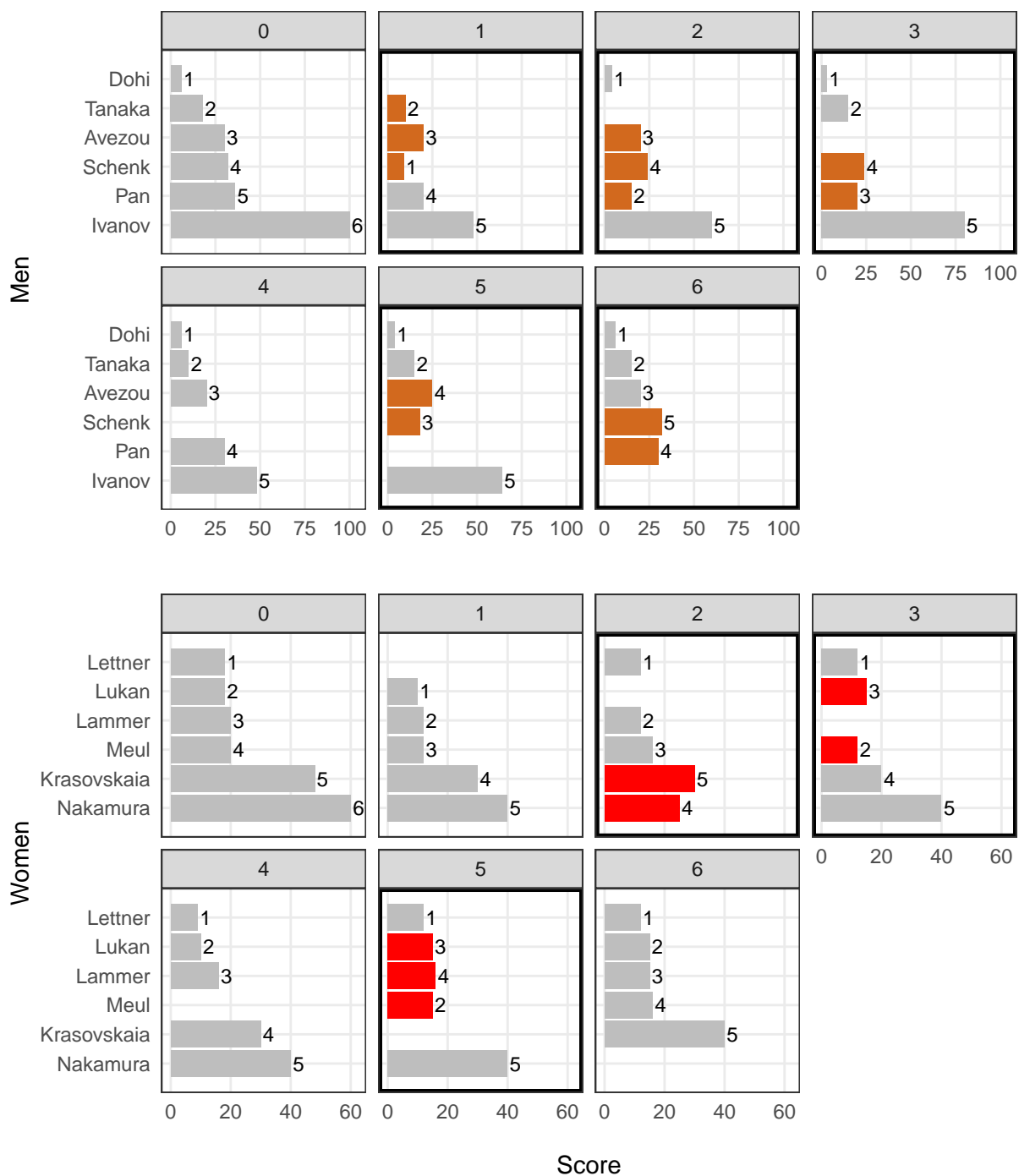


Figure 7: This figure illustrates the changes to the final rankings of the 2018 Youth Olympics Men's and Women's Finals when we leave out one climber. For each gender competition, each panel represents the rank of the drop-out athlete, with 0 being the original final results. Each case with a change in rank orderings is highlighted by a black panel border, and any player with a rank change is represented by a red-filled bar

# Supplementary Material

All of the materials related to this manuscript are publicly available on GitHub at <https://github.com/qntkhvn/climbing>.

Table 6: This table shows Kendall’s rank correlation coefficients between the overall and individual discipline ranks for men’s and women’s sport climbing qualifications at the 2018 Asian Games

	Men				Women			
	Overall	Speed	Bouldering	Lead	Overall	Speed	Bouldering	Lead
Overall	1.0000	0.5731	0.6760	0.6126	1.0000	0.5474	0.6878	0.7053
Speed	0.5731	1.0000	0.3022	0.2174	0.5474	1.0000	0.2540	0.2947
Bouldering	0.6760	0.3022	1.0000	0.6203	0.6878	0.2540	1.0000	0.6878
Lead	0.6126	0.2174	0.6203	1.0000	0.7053	0.2947	0.6878	1.0000

Table 7: This table shows Kendall’s rank correlation coefficients between the overall and individual discipline ranks for men’s and women’s sport climbing qualifications at the 2019 World Championships

	Men				Women			
	Overall	Speed	Bouldering	Lead	Overall	Speed	Bouldering	Lead
Overall	1.0000	0.2632	0.3684	0.3474	1.0000	0.2316	0.5013	0.2526
Speed	0.2632	1.0000	-0.0947	-0.3263	0.2316	1.0000	0.0686	-0.4526
Bouldering	0.3684	-0.0947	1.0000	0.2000	0.5013	0.0686	1.0000	0.2902
Lead	0.3474	-0.3263	0.2000	1.0000	0.2526	-0.4526	0.2902	1.0000

Table 8: This table shows Kendall’s rank correlation coefficients between the overall and individual discipline ranks for men’s and women’s sport climbing qualifications at the 2020 European Championship

	Men				Women			
	Overall	Speed	Bouldering	Lead	Overall	Speed	Bouldering	Lead
Overall	1.0000	0.0947	0.3810	0.2632	1.0000	0.1847	0.5358	0.5368
Speed	0.0947	1.0000	-0.1587	-0.4316	0.1847	1.0000	-0.1915	-0.0897
Bouldering	0.3810	-0.1587	1.0000	0.2751	0.5358	-0.1915	1.0000	0.5040
Lead	0.2632	-0.4316	0.2751	1.0000	0.5368	-0.0897	0.5040	1.0000

Table 9: This table shows Kendall’s rank correlation coefficients between the overall and individual discipline ranks for men’s and women’s sport climbing qualifications at the 2020 Pan American Championship

	Men				Women			
	Overall	Speed	Bouldering	Lead	Overall	Speed	Bouldering	Lead
Overall	1.0000	0.5906	0.5556	0.6491	1.0000	0.5789	0.6140	0.5906
Speed	0.5906	1.0000	0.2164	0.4035	0.5789	1.0000	0.2632	0.3099
Bouldering	0.5556	0.2164	1.0000	0.2749	0.6140	0.2632	1.0000	0.3918
Lead	0.6491	0.4035	0.2749	1.0000	0.5906	0.3099	0.3918	1.0000

gold medal

order of medalists

3 medalists

any rankings

Structure:

sensitivity to:

results

tournament format

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