Exercise 1

Consider a LIBOR market model scenario for dates $T_0 = 0 < T_1 < \ldots < T_n$, and let the dynamics of the LIBOR rate $L(T_i, T_{i+1}; t)$ for $i = 0, \ldots n-1$ be driven by n correlated Brownian motions W^i, $i = 1, \ldots, n$, with exponentially decaying correlations

$$\rho_{i,j} = \exp(-a|T_i - T_j|), \quad a > 0, \quad 1 \le i, j \le n. \tag{1}$$

Factor reduction is performed in the `Finmath` library by the method `factorReduction(double[][] correlationMatrix, int numberOfFactors)` in the class `net.finmath.functions.linearAlgebra`. This method takes as arguments the original correlation matrix $R = (\rho_{i,j})_{0 \le i,j \le n}$ and the number of most relevant factors the user wants to take into account, and returns the matrix F^r defined at page 465 of the script. You can find a class

`com.andreamazzon.handout12.FactorReductionExponentialDecay`

which uses the latter method in order to test the impact of factor reduction, depending on the correlation decay parameter a in (1).

Read the class, and complete the methods when asked.

Also write a test class where you construct an object of type `FactorReductionExponentialDecay` with a semi-annual tenor discretization going up to 10 years, call the method

`getErrorFromFactorReduction(double corrDecay, int numberOfFactors)`

and print the value it returns, for a fixed value `numberOfFactors = 2` and letting `corrDecay` run from 0 to 1. Try to figure out what do you expect before looking at the results: the difference will increase or decrease when `corrDecay` increases?

Exercise 2

Change where needed the method

`com.andreamazzon.handout11.LIBORMarketModelConstructionWithDynamicsAndMeasureSpecification`,

in such a way that you can give it the number of factors you want to simulate.

Calling this modified method, repeat the tests you can find in

`com.andreamazzon.handout9.LMMDigitalCapletTest`

considering now only 4 factors. Compare the average error in the prices of the digital caplets with respect to the one we get without factor reduction. Do this for different values of the correlation decay parameter.

Exercise 3

Consider the tenor discretization $T_0 < T_1 < \cdots < T_n$ and the displaced LIBOR market model where the processes $L_i := L(T_i, T_{i+1})$, $i = 1, \ldots, n-1$ follow the dynamics

$$dL_i(t) = \mu_i(t)dt + (L_i(t) + d)\sigma_i^D(t)dW_i(t), \quad 0 \le t \le T_i,$$

where $d\langle W_i, W_j\rangle(t) = \rho_{i,j}(t)dt$, under the real-world measure $\mathbb{P}$. Also assume here that $d > 0$ and that $\sigma_i^D(\cdot)$ are deterministic functions.

- Derive an analytical approximation for the price of a swaption in this setting, in a similar way to what you have seen in the lecture for the log-nornal case, see pages 488-500 of the script.

  Hint: in order to solve the exercise, you first have to *guess* the dynamics of the par swap rate S. In particular, you can guess S to have displaced dynamics as well, with a displacement $d_S = d$. Try to see why having a look at Lemma 142 at page 107 of the script.

- Assume now that

  $$dL_i(t) = \mu_i(t)dt + (L_i(t) + d_i)\sigma_i^D(t)dW_i(t), \quad 0 \leq t \leq T_i, \quad 1 \leq i \leq n-1,$$

  with $d_i \neq d_j$ for at least one $i \neq j$. How would your analytical approximation change? Would you need one more approximation? Why?