Exercise 1

Consider two dates $0 < T_1 < T_2$, and suppose you know the zero coupon bonds values $P(T_1; 0)$, $P(T_2; 0)$. Also suppose that the process $L = (L_t)_{t\geq 0}$ defined by

$$L_t := L(T_1, T_2; t) = \frac{1}{T_2 - T_1} \frac{P(T_1; t) - P(T_2; t)}{P(T_2; t)}, \quad t \geq 0$$

has log-normal dynamics, and that you know the Libor volatility σ_L.

- Write an `abstract` class that implements the valuation of the price of a European option with a general payoff $V(T_1) = f(L_{T_1})$, both when it is payed in T_2 and in arrears, for the parameters listed above. Try to see which methods can be implemented here and which ones are instead specific of the derived classes.
- Extend your `abstract` class to three derived classes taking care of such a valuation for a caplet, for a digital caplet and for a floater, respectively. These derived classes can eventually have other option specific fields (for example, the strike for a caplet).
- Check if, for the same parameters, the valuations of the caplet and of the floater both for payment in $P(T_2; 0)$ and in arrears corresponds to the ones you derive with the methods of Handout 7.

Hint: Look at the solution of the theoretical part of Exercise 1 of Handout 7.

Exercise 2

Given the tenor discretization $T_0 < T_1 < \cdots < T_n$, consider the caplets paying

$$\max\left(L(T_i, T_{i+1}; T_i) - K, 0\right)(T_{i+1} - T_i) \quad \text{in } T_{i+1},$$

$i = 1, \ldots, n-1$, for a given LIBOR market model represented by the processes $L_i := L(T_i, T_{i+1})$, $i = 1, \ldots, n-1$.

Derive the analytic formula of such caplets under the displaced lognormal model, that is, assuming that for any $i = 1, \ldots, n-1$, the process L_i follows the dynamics

$$dL_i(t) = \mu_i(t)dt + (L_i(t) + d_i)\sigma_i^D(t)dW_i(t), \quad t \geq 0,$$

under the real-world measure $\mathbb{P}$. Also assume here that $d_i > 0$ and that $\sigma_i^D(\cdot)$ are deterministic functions.