Exercise 1

In Exercise 2 of Handout 4 we have seen the implementation of the valuation of a Swaption for tenor structure $0 = T_0 < T_1 < \cdots < T_n$, zero coupon curve $P(T_1; 0), \ldots, P(T_n; 0)$, strike K and maturity T_1 under the Black model for the par swap rate, i.e., supposing that the par swap rate $S = (S_t)_{t \geq 0}$ has log-normal dynamics. That is,

$$dS_t = \mu S_t dt + \sigma S_t dW_t^P, \quad 0 \leq t \leq T_1$$

under the physical measure P.

Write now a method computing the value of such a swaption under the Bachelier model for the par swap rate, i.e., supposing that S has normal dynamics. That is,

$$dS_t = \mu dt + \sigma dW_t^P, \quad 0 \leq t \leq T_1$$

under the physical measure P.

- Call the method to compute the price of a swaption with tenor structure $0 = T_0 < T_1 < \cdots < T_5$, $T_i = i$, $P(T_1; 0) = 0.98$, $P(T_2; 0) = 0.95$, $P(T_3; 0) = 0.92$, $P(T_4; 0) = 0.9$, $P(T_5; 0) = 0.87$, $\sigma = 0.3$, $K = S_0$. **Expected result with seed equal to** 1897**:** 4356.45.
- Let then $P(T_2; 0)$ vary from 0.93 to 0.97 and see how the price changes. What do you observe with respect to what we have seen for the Swaption under the Black model, in the last exercise session? Try to investigate this behaviour as we did for the Black model (see the test class `SwaptionTest`, lines 44-88).
- Try to see if you get a different behaviour changing some parameters.

Exercise 2

Consider the Cap defined at page 126 of the script, involving three dates $T_1 < T_2 < T_3$ and the associated Libors $L(T_1, T_2; T_1)$ and $L(T_2, T_3; T_2)$. Write a method to get the value of such a contract, with strikes $K_1, K_2 > 0$ and unique notional $N > 0$, in the case when the dynamics of the two Libors $L_1 = (L_1(t))_{t \in [0, T_1]}$ with $L_1(t) := L(T_1, T_2; t)$ and $L_2 = (L_2(t))_{t \in [0, T_2]}$ with $L_2(t) := L(T_2, T_3; t)$ are described by

$$dL_t^1 = \mu_1 L_t^1 dt + \sigma_1 S_t^1 dW_t^1, \quad 0 \leq t \leq T_1, \tag{1}$$
$$dL_t^2 = \mu_2 L_t^2 dt + \sigma_2 S_t^2 dW_t^2, \quad 0 \leq t \leq T_2, \tag{2}$$

with constant volatilities $\sigma_1, \sigma_2 > 0$ and correlated Brownian motions $\langle W^1, W^2 \rangle_t = \rho t$, $\rho \in [-1, 1]$.

Test your implementation for $T_1 = 0.5$, $T_2 = 1$, $T_3 = 1.5$, $P(T_2; 0) = 0.91$, $P(T_3; 0) = 0.82$, $L(T_1, T_2; 0) = 0.05$, $L(T_2, T_3; 0) = 0.04$, $\sigma_1 = 0.3$, $\sigma_2 = 0.25$, $\rho = 0.2$, $K_1 = 0.05$, $K_2 = 0.04$, notional 1000. Expected result: close to 4.74.

Hint: you can see the contract as the sum of the payoffs of two call options, one with strike K_1, underlying L_1 and maturity T_1 (but discounted at T_2 by $P(T_2; 0)$) and one with strike K_2, underlying L_2 and maturity T_2 (but discounted at T_3 by $P(T_3; 0)$). Also consider that you have to multiply the first payoff by $(T_2 - T_1)$ (times the notional) and the second one by $(T_3 - T_2)$ (also times the notional).

The complication is the possible correlation between the Brownian motions. In order to account for that, one possibility would be to construct an object of type `MonteCarloMultiAssetBlackScholesModel`, a class that you find in

`net.finmath.montecarlo.assetderivativevaluation.MonteCarloMultiAssetBlackScholesModel`.

You can then give such an object to the `getValue` method of a class extending `net.finmath.montecarlo.assetderivativevaluation.products.AbstractAssetMonteCarloProduct`,

and representing the sum of the calls.