Exercise 1

Consider two dates $0 < T_1 < T_2$. Do the following experiments about convexity adjustment:

(a) Consider first the natural floater paying

$$N(T_2 - T_1)L(T_1, T_2; T_1) \tag{1}$$

in T_2, and then the LIBOR in arrears paying the value (1) in T_1. Write a `Junit` test class where you set $T_1 = 1$, $T_2 = 2$, let the LIBOR follow log-normal dynamics with volatility $\sigma = 0.25$, let the notional be $N = 10000$ and the prices of the zero coupon bonds $P(T_1; 0) = 0.95$, $P(T_2; 0) = 0.9$. For such parameters, do the following:

- compute the analytic value of a natural floater;
- simulate the process $L = (L_t)_{t\in[0,T_1]}$ with $L_t := L(T_1, T_2; t)$ under $Q^{P(T_2)}$, and test if the resulting Monte-Carlo value of a natural floater is equal (up to a given tolerance) to the value computed in (a);
- find the analytic formula for the value of the floater in arrears and compute the value in the present case;
- compute the value of the floater in arrears by simulating the process L under $Q^{P(T_2)}$ and compare this value with the one found in (c).

(b) A caplet is said to be paid *in arrears* if the payment of the option on the observed LIBOR rate $L(T_1, T_2; T_1)$ is made at T_1 instead of T_2. Find the formula for the price of a caplet in arrears by a suitable convexity adjustment, and use it to write a method that prices this product.

Compute then the difference between the valuation of the caplet in Exercise 1 of Handout 6 and the current one, setting $L(T_1, T_2; 0) = 0.05$, LIBOR volatility $\sigma = 0.3$, strike $K = 0.044$, discount factor $P(T_2; 0) = 0.91$, notional $N = 10000$: this is the market price of the convexity adjustment. **Expected result for the difference:** 5.7819.