Exercise 1

Let $L = (L_t)_{0 \le t \le T_1}$ with $L_t := (L(T_1, T_2; t))_{0 \le t \le T_1}$ be the stochastic process representing the Libor rate payed between T_1 and T_2, with $T_1 < T_2$. Take $K > 0$ and consider a caplet fixed at T_1 and expiring at T_2 with strike rate K and notional N, i.e. the financial product paying off

$$N(T_2 - T_1)(L(T_1, T_2; T_1) - K)^+$$

at time T_2.

(a) Assume we are in a market where the Libor rate in the **physical** measure is driven by the dynamics

$$dL_t = \mu L_t dt + \sigma L_t dW_t, \quad 0 \le t \le T_1,$$

for some Brownian motion W and $\sigma > 0$. Write a JAVA method to calculate the value at time $t_0 = 0$ of the caplet for the parameters defining the model and for a given discount factor $P(T_2; 0)$. You can choose yourself the way you want to implement it: basing on some analytical formulas (but not the one that directly gives the value of a caplet) or on a Monte-Carlo method.

(b) Write a test class in order to check that the result you obtain for some parameters of your choice is equal (up to a tolerance that in your idea is suitable) to the one you get by calling the method `blackModelCapletValue(double forward, double volatility, double optionMaturity, double optionStrike, double periodLength, double discountFactor)` of the class `net.finmath.functions.AnalyticFormulas`.

Exercise 2

Let $V_{\text{swap}}(t)$ be the value at time t of a swap with tenor structure $0 = T_0 < T_1 < \cdots < T_n$ and swap rates $S_i = K$ for all $i = 1, \dots, n$. Consider a swaption with underlying V_{swap} and maturity T_1.

You have seen that pricing the swaption is equivalent to price a call option on the (par) swap rate S, where S_t is such that $V_{swap}(t) = 0$. In particular, if S is log-normally distributed, then the Black formula for swaptions holds:

$$V_{\text{swaption}}(t) = A_0 BS(S_0, 0, \sigma, T_1, K),$$

where σ is the volatility of the underlying and

$$A_t = \sum_{i=1}^{n-1} (T_{i+1} - T_i) P(T_{i+1}; t), \quad 0 \le t \le T_1,$$

is the so called swap annuity.

Use the formula above to write a Java method which returns the value of a swaption for given strike, tenure structure, zero coupon bond curve and log-volatility σ_S of the par swap rate, which is supposed to have log-normal dynamics. As usual, try to see if you can come up with some overloaded implementation depending on some particular features of the model.

Call the method to value a swaption with tenor structure $0 = T_0 < T_1 < \cdots < T_5$, $T_i = i$, $P(T_1; 0) = 0.98$, $P(T_2; 0) = 0.95$, $P(T_3; 0) = 0.92$, $P(T_4; 0) = 0.9$, $P(T_5; 0) = 0.87$, $\sigma_S = 0.3$, $K = 0.03$. The value you should get is 134.7204.