Exercise 1

Consider a Quanto Caplet on the Libor rate $L^f(T_1, T_2)$ of foreign currency, for $T_1 \le T_2$. Denote with f_t the forward FX rate at time t. The value of the quanto option in the domestic economy at maturity is

$$N \cdot C(T_2 - T_1)(L^f(T_1, T_2; T_1) - K)^+,$$

where $K > 0$ is the strike rate, N is the notional and C is a constant exchange rate converting the foreign currency in the domestic one. Assume that the processes $L^f = (L^f_t)_{0 \le t \le T_1}$ with $L^f_t = L^f(T_1, T_2; t)$ and $f = (f_t)_{0 \le t \le T_1}$ follow the lognormal dynamics

$$dL^f_t = \mu_L(t) L^f_t dt + \sigma_L L^f_t dW^{P(T_2)}_t, \quad 0 \le t \le T_1,$$

and

$$df_t = \sigma_f f_t dW^f_t, \quad 0 \le t \le T_1,$$

where $\mu_L(\cdot)$ is a deterministic function, $\sigma_L, \sigma_f > 0$ are constants and $W^{P(T_2)}$, W^f are $\mathbb{Q}^{P(T_2)}-$Brownian motions such that $d\langle W^{P(T_2)}_t, W^f_t \rangle = \rho dt$.

- Write a JAVA method (your program should be an extension of the one you wrote for Exercise 1 Handout 4) to compute the value of the quanto for the parameters that define the option.
- Compute the value of the option for $L^f_0 = 0.05$ (initial value of the LIBOR), $\sigma_L = 0.3$, $\sigma_f = 0.2$, $\rho = 0.4$, $T_1 = 1$, $T_2 = 2$, $K = 0.044$, $P(T_2; 0) = 0.91$, $C = 0.9$, $N = 10000$. **Expected result:** 67.6973.

Exercise 2

Consider again the quanto on the Libor rate of foreign currency $L^f(T_1, T_2)$, for $T_1 \le T_2$, assuming now that the process L^f defined in Exercise 2 has normal dynamics, i.e.,

$$dL^f_t = \mu_L(t) dt + \sigma_L dW^{P(T_2)}_t, \quad 0 \le t \le T_1,$$

where $\sigma_L(\cdot)$ is a deterministic functions of time.
Also suppose that $f = (f_t)_{0 \le t \le T_1}$ has dynamics given by

$$df_t = \sigma_f(t) f_t dW^f_t, \quad 0 \le t \le T_1,$$

where $\sigma_f(\cdot)$ is a deterministic function of time. Here $W^{P(T_2)}$ and W^f are again $\mathbb{Q}^{P(T_2)}-$Brownian motions such that $d\langle W^{P(T_2)}_t, W^f_t \rangle = \rho dt$. Derive a formula for the pricing of the Quanto Caplet under this setting. Suppose now that f has normal dynamics as well, i.e.,

$$df_t = \sigma_f(t) dW^f_t, \quad 0 \le t \le T_1.$$

Can you derive a similar pricing formula as in the first part of the exercise? If not, what is the problem?