Exercise 1

In Exercise 1 of Handout 7 we have seen the implementation of the valuation of a Swaption for tenor structure $0 = T_0 < T_1 < \cdots < T_n$, zero coupon curve $P(T_1; 0), \dots, P(T_n; 0)$, strike K and maturity T_1 under the Black model for the par swap rate, i.e., supposing that the par swap rate $S = (S_t)_{t \geq 0}$ has log-normal dynamics. That is,

$$dS_t = \mu S_t dt + \sigma S_t dW_t^P, \quad 0 \leq t \leq T_1$$

under the physical measure P.
Write now a method computing the value of such a swaption under the Bachelier model for the par swap rate, i.e., supposing that S has normal dynamics. That is,

$$dS_t = \mu dt + \sigma dW_t^P, \quad 0 \leq t \leq T_1$$

under the physical measure P.

- Call the method to compute the price of a swaption with tenor structure $0 = T_0 < T_1 < \cdots < T_5$, $T_i = i$, $P(T_1; 0) = 0.98$, $P(T_2; 0) = 0.95$, $P(T_3; 0) = 0.92$, $P(T_4; 0) = 0.9$, $P(T_5; 0) = 0.87$, $\sigma = 0.3$, $K = S_0$. **Expected result:** 4356.45.
- Let then $P(T_2; 0)$ vary from 0.93 to 0.97 and see how the price changes. What do you observe with respect to what we have seen for the Swaption under the Black model, in the last exercise session? Try to investigate this behaviour as we did for the Black model (see the test class `InterestRateProductsTest`, lines 59-88).
- Try to see if you get a different behaviour changing some parameters.

Exercise 2

This exercise may be done during the Tutorium

Consider two dates $0 < T_1 < T_2$, and suppose you know the zero coupon bonds values $P(T_1; 0)$, $P(T_2; 0)$. Also suppose that the process $L = (L_t)_{t \geq 0}$ defined by

$$L_t := L(T_1, T_2; t) = \frac{1}{T_2 - T_1} \frac{P(T_1; t) - P(T_2; t)}{P(T_2; t)}, \quad t \geq 0$$

has log-normal dynamics, and that you know the Libor volatility σ_L.

- Write an `abstract` class that implements the valuation of the price of a European option with a general payoff $V(T_1) = f(L_{T_1})$, both when it is payed in T_2 and in arrears, for the parameters listed above. Try to see which methods can be implemented here and which ones are instead specific of the derived classes.
- Extend your `abstract` class to three derived classes taking care of such a valuation for a caplet, for a digital caplet and for a floater, respectively. These derived classes can eventually have other option specific fields (for example, the strike for a caplet).
- Check if, for the same parameters, the valuations of the caplet and of the floater both for payment in $P(T_2; 0)$ and in arrears corresponds to the ones you derive with the methods of Handout 7.

Hint: Look at the solution of the theoretical part of Exercise 2 of Handout 7.