Exercise 1

This exercise (or some points of this exercise) may be done during the Tutorium

Let $L = (L_t)_{0 \le t \le T_1}$ with $L_t := (L(T_1, T_2; t))_{0 \le t \le T_1}$ be the stochastic process representing the Libor rate payed between T_1 and T_2, with $T_1 < T_2$. Take $K > 0$ and consider a caplet fixed at T_1 and expiring at T_2 with strike rate K and notional N, i.e. the financial product paying off

$$N(T_2 - T_1)(L(T_1, T_2; T_1) - K)^+$$

at time T_2.

(a) Assume we are in a market where the Libor rate in the physical measure is driven by the dynamics

$$dL_t = \mu L_t dt + \sigma L_t dW_t, \quad 0 \le t \le T_1,$$

for some Brownian motion W and $\sigma > 0$. Write a JAVA method to calculate the value at time $t_0 = 0$ of the caplet for the parameters defining the model and for a given discount factor $P(T_2; 0)$.

(b) Write a test class in order to check that the result you have obtained is equal (up to a given tolerance) to the one you get with the method `blackModelCapletValue(double forward, double volatility, double optionMaturity, double optionStrike, double periodLength, double discountFactor)` of the class `net.finmath.functions.AnalyticFormulas`.

Exercise 2

This exercise (or some points of this exercise) may be done during the Tutorium

Consider now a Quanto Caplet on the Libor rate $L^f(T_1, T_2)$ of foreign currency, for $T_1 \le T_2$. Denote with f_t the forward FX rate at time t. The value of the quanto option in the domestic economy is

$$N \cdot C(T_2 - T_1)(L^f(T_1, T_2; T_1) - K)^+,$$

where $K > 0$ is the strike rate, N is the notional and C is a constant exchange rate converting the foreign currency in the domestic one. Assume that the processes $L^f = (L_t^f)_{0 \le t \le T_1}$ with $L_t^f = L^f(T_1, T_2; t)$ and $f = (f_t)_{0 \le t \le T_1}$ follow the lognormal dynamics

$$dL_t^f = \sigma_L L_t^f dW_t^{P(T_2)}, \quad 0 \le t \le T_1,$$

and

$$df_t = \sigma_f f_t dW_t^f, \quad 0 \le t \le T_1,$$

where $\sigma_f(\cdot)$ is a deterministic function of time and $W^{P(T_2)}$, W^f are $\mathbb{Q}^{P(T_2)}-$Brownian motions such that $d\langle W_t^{P(T_2)}, W_t^f \rangle = \rho dt$.

- Write a JAVA method (your program should be an easy extension of the one you wrote in Exercise 1) to compute the value of the quanto for the parameters that define the option.
- Compute the value of the option for $L_0^f = 0.05$ (initial value of the LIBOR), $\sigma_L = 0.3$, $\sigma_f = 0.2$, $\rho = 0.4$, $T_1 = 1$, $T_2 = 2$, $K = 0.044$, $P(T_2; 0) = 0.91$, $C = 0.9$, $N = 10000$. **Expected result:** 67.6973.

Exercise 3

Consider the quanto on the Libor rate of foreign currency $L^f(T_1, T_2)$, for $T_1 \le T_2$, assuming now that the process L^f defined in Exercise 2 has normal dynamics, i.e.,

$$dL_t^f = \sigma_L L_t^f dW_t^{P(T_2)}, \quad 0 \le t \le T_1,$$

where $\sigma_L(\cdot)$ is a deterministic functions of time.
Also suppose that $f = (f_t)_{0 \le t \le T_1}$ has dynamics given by

$$df_t = \sigma_f(t) f_t dW_t^f, \quad 0 \le t \le T_1,$$

where $\sigma_f(\cdot)$ is a deterministic function of time. Here $W^{P(T_2)}$ and W^f are again $\mathbb{Q}^{P(T_2)}-$Brownian motions such that $d\langle W_t^{P(T_2)}, W_t^f \rangle = \rho dt$. Derive a formula for the pricing of the Quanto Caplet under this setting.
Suppose now that f has normal dynamics as well, i.e.,

$$df_t = \sigma_f(t) dW_t^f, \quad 0 \le t \le T_1.$$

Can you derive a similar pricing formula as in the first part of the exercise? If not, what is the problem?