Exercise 1

Let $P(t, T)$ be the continuously compounded time-t price of a bond maturing at time T, and assume that it is a *deterministic* function of t and T: in other words

$$P(t, T) = e^{-\int_t^T r(u)du}$$

for some deterministic positive *short rate* function $r(t)$.

(a) Prove, via discussion of arbitrage possibilities, that for $t \leq T \leq S$ it has to hold

$$P(t, S) = P(t, T)P(T, S).$$

(b) Define the *continuously compounded forward rate prevailing at* t *of reset date* T *and maturing* S as the unique solution to the equation

$$e^{f(t,T,S)(S-T)} := \frac{P(t, T)}{P(t, S)},$$

and the instantaneous forward rate as

$$f(t, T) = \lim_{S \to T} f(t, T, S).$$

Prove that it holds

$$P(t, T) = \exp\left(-\int_t^T f(t, u)du\right)$$

and

$$r(t) = \lim_{T \to t} f(t, T).$$

(c) Conclude from (a) and (b) that

$$f(t, S) = r(S)$$

for all $t \leq S$.

(d) Establish in which of the points (a), (b), (c), the assumption of deterministic rates is necessary or can be relaxed to some class of stochastic short rates $r(t)$.

Exercise 2

A *swap* is an exchange payment of fixed rate for a floating rate. In particular, let

$$0 = T_0 < T_1 < T_2 < \cdots < T_n$$

denote a given tenor structure. A swap pays

$$N\left(L(T_i, T_{i+1}; T_i) - S_i\right) L(T_i, T_{i+1}; T_i)$$

in T_{i+1} for $i = 1, \ldots, n-1$, where $S_i \in \mathbb{R}$, $i = 1, \ldots, n-1$, denote the so called fixed swap rates, $L(T_i, T_{i+1}; T_i)$ denotes the forward rate from Definition 111 of the script, and N denotes the notional. The value of the swap is given by Theorem 139 in the script.

The par swap rate S at time t is the unique rate for which a swap with $S_i := S$ for every $i = 1, \ldots, n-1$ has value 0 at time t. Analytically compute the value of S at time t as a function of the times of the tenor structure and of the curve $P(T_i; t)$, $i = 1, \ldots, n$.

Exercise 3

Write a `SwapWithoutFinmath`, implementing the interface `com.andreamazzon.exercise4.Swap`

you find in the exercises repository.

In particular, you have to write methods that calculate the value of an interest rate Swap (for given fixed swap rates S_i, $i = 0, \dots, n-1$) as well as the par swap rate S, in correspondence to a tenure structure $0 = T_0 < T_1 < \dots < T_n = T$, at evaluation time $t = 0$.

Take into account the fact that the user can provide the forward rates $L(T_i, T_{i+1}; 0)$, $i = 0, \dots, n-1$ or the initial zero-bond curve $P(T_i; 0)$, $i = 0, \dots, n-1$, both as array of doubles.

Also try to give a different implementation to your methods in the case when the settlement dates $0 = T_0 < T_1 < \dots < T_n = T$ are evenly distributed, i.e., $\Delta_i := T_{i+1} - T_i = \delta > 0$ $\forall i = 1, \dots, n$: in this case, you can save time avoiding to compute Δ_i more than once.

Write then a JUnit test class which checks if the value for the Swap you get by choosing $S_i = S$ $\forall i$ is actually 0 (again, up to a suitable tolerance). For this purpose, take $T = 3$, $\Delta_i = \delta = 0.5$ (half year), and the following zero coupon bond curve:

Maturity	Price
6m	0.9986509108
1yr	0.9949129829
1.5yr	0.9897033769
2 yr	0.9835370208
2.5 yr	0.9765298116
3 yr	0.9689909565