Exercise 1

This exercise may be done during the Tutorium

Let $V_{\text{swap}}(t)$ be the value at time t of a swap with tenor structure $0 = T_0 < T_1 < \cdots < T_n$ and swap rates $S_i = K$ for all $i = 1, \ldots, n$. Consider a swaption with underlying V_{swap} and maturity T_1.

You have seen that pricing the swaption is equivalent to price a call option on the (par) swap rate S, where S_t is such that $V_{swap}(t) = 0$. In particular, if S is log-normally distributed, then the Black formula for swaptions holds:

$$V_{\text{swaption}}(t) = A_0 BS(S_0, 0, \sigma, T_1, K),$$

where σ is the volatility of the underlying and

$$A_t = \sum_{i=1}^{n-1} (T_{i+1} - T_i) P(T_{i+1}; t), \quad 0 \le t \le T_1,$$

is the so called swap annuity.

Use the formula above to write a Java method which returns the value of a swaption for given strike, tenure structure, zero coupon bond curve and log-volatility σ_S of the par swap rate, which is supposed to have log-normal dynamics. Call the method to value a swaption with tenor structure $0 = T_0 < T_1 < \cdots < T_5$, $T_i = i$, $P(T_1; 0) = 0.98$, $P(T_2; 0) = 0.95$, $P(T_3; 0) = 0.92$, $P(T_4; 0) = 0.9$, $P(T_5; 0) = 0.87$, $\sigma_S = 0.3$, $K = 0.03$.

Exercise 2

Consider two dates $0 < T_1 < T_2$. Do the following experiments about convexity adjustment:

(a) **This part of the exercise may be done during the Tutorium**

Consider first the natural floater paying

$$N(T_2 - T_1) L(T_1, T_2; T_1) \tag{1}$$

in T_2, and then the LIBOR in arrears paying the value (1) in T_1. Write a `Junit` test class where you set $T_1 = 1$, $T_2 = 2$, let the LIBOR follow log-normal dynamics with volatility $\sigma = 0.25$, let the notional be $N = 10000$ and the prices of the zero coupon bonds $P(T_1; 0) = 0.95$, $P(T_2; 0) = 0.9$. For such parameters, do the following:

- compute the analytic value of a natural floater;
- simulate the process $L = (L_t)_{t \in [0, T_1]}$ with $L_t := L(T_1, T_2; t)$ under $Q^{P(T_2)}$, and test if the resulting Monte-Carlo value of a natural floater is equal (up to a given tolerance) to the value computed in (a);
- find the analytic formula for the value of the floater in arrears and compute the value in the present case;
- compute the value of the floater in arrears by simulating the process L under $Q^{P(T_2)}$ and compare this value with the one found in (c).

(b) A caplet is said to be paid *in arrears* if the payment of the option on the observed LIBOR rate $L(T_1, T_2; T_1)$ is made at T_1 instead of T_2. Find the formula for the price of a caplet in arrears by a suitable convexity adjustment, and use it to write a method that prices this product.

Compute then the difference between the valuation of the caplet in Exercise 1 of Handout 6 and the current one, setting $L(T_1, T_2; 0) = 0.05$, LIBOR volatility $\sigma = 0.3$, strike $K = 0.044$, discount factor $P(T_2; 0) = 0.91$, notional $N = 10000$: this is the market price of the convexity adjustment. **Expected result for the difference:** 5.7819.