$$\Phi_{S_T}(S_T) = \frac{1}{\sigma\sqrt{T}}\,\Phi_{Std}\Bigg(\overbrace{\underbrace{\frac{1}{\sigma\sqrt{T}}\Big(\log\Big(\frac{S_T}{S_0}\Big)}\,-rT+\frac{1}{2}\sigma^2 T\Big)}^{:=\,(\nabla)}\Bigg)\frac{1}{S_T}$$

THE DERIVATIVE OF THIS WITH RESPECT TO S_0 IS $\underbrace{-\frac{1}{S_0}\cdot\frac{1}{\sigma\sqrt{T}}}$

WE HAVE

$$\Phi'_{Std}(x) = -x\,\Phi_{Std}(x)$$

SO,

$$\frac{d\Phi_{S_T}(S_T)}{dS_0}\,\frac{1}{\Phi_{S_T}(S_T)} = \frac{\left(-\frac{1}{S_0}\frac{1}{\sigma\sqrt{T}}\right)\cdot(-\nabla)\cdot\Phi_{S_T}(S_T)}{\Phi_{S_T}(S_T)}$$

$$= \frac{1}{\sigma^2 T}\,\frac{1}{S_0}\left(\log\Big(\frac{S_T}{S_0}\Big)-rT+\frac{1}{2}\sigma^2 T\right)$$