# Remark to exercise 1

Suppose without loss of generality that $\{x_1, \dots, x_n\}$ is already sorted and that $x_n \neq 1$, i.e.,

$$x_1 < x_2 < \cdots < x_n < 1.$$

It can be seen that in

$$D(\{x_1, \dots, x_n\}) = \max_{a \in \{0, x_1, \dots, x_n\}} \max_{b \in \{x_1, \dots, x_n, 1\}, b > a} \max \left( b - a - \frac{|x_i \in (a,b)|}{n}, \frac{|x_i \in [a,b]|}{n} - (b-a) \right), \quad (1)$$

we don't need to consider $a = x_n$, $b = 1$. Indeed,

$$\begin{aligned} &\max \left( 1 - x_n - \frac{|x_i \in (x_n, 1)|}{n}, \frac{|x_i \in [x_n, 1]|}{n} - (1 - x_n) \right) \\ &= \max \left( 1 - x_n, \frac{1}{n} - (1 - x_n) \right) \\ &\leq \max \left( 1 - (x_n - x_1), \frac{1}{n} \right). \end{aligned} \quad (2)$$

But the first value is $\frac{|x_i \in [x_1, x_n]|}{n} - (x_n - x_1)$, so the one we obtain when we consider the closed interval $[x_1, x_n]$, and the second one is smaller or equal than the maximum distance between two consecutive points in $\{x_1, \dots, x_n\}$, that we get when we consider all the possible open interval between two consecutive points in the set.

So, this value will not be an absolute maximum.