Exercise 1

Given the tenor discretization $T_0 < T_1 < \cdots < T_n$, consider the caplets paying

$$\max\left(L(T_i, T_{i+1}; T_i) - K, 0\right)(T_{i+1} - T_i) \quad \text{in } T_{i+1},$$

$i = 1, \ldots, n-1$, for a given LIBOR market model represented by the processes $L_i := L(T_i, T_{i+1})$, $i = 1, \ldots, n-1$.
Derive the analytic formula of such caplets under the displaced lognormal model, that is, assuming that for any $i = 1, \ldots, n-1$, the process L_i has dynamics

$$dL_i(t) = \mu_i(t)dt + (L_i(t) + d_i)\sigma_i^D(t)dW_i(t), \quad t \geq 0,$$

under the real-world measure $\mathbb{P}$. Also assume here that $d_i > 0$ and that $\sigma_i^D(\cdot)$ are deterministic functions.

Exercise 2

Write a method similar to the method `createLIBORMarketModel` you can find in

`com.andreamazzon.handout7.LIBORMarketModelConstruction`,

where you construct an object of type

`net.finmath.montecarlo.interestrate.TermStructureMonteCarloSimulationModel`,

with the following enhancements:

- every time you call the method you can decide if you want normal or log-normal dynamics for the simulated LIBOR market model;
- every time you call the method you can decide if you want to construct the drift using the terminal or the spot measure.

Hints:

(a) You are free to decide how to communicate such a choice to the method (for example, by a `String` or a `boolean` value).

(b) Part of the exercise consists in finding the right class of the Finmath library to which you have to pass the information about the choice of the measure and the dynamics.

(c) You are also free to give a volatility structure of your choice, it can also be constant. In order to check if the values you give make sense, you can pass the object created in such a way to the `getValue` method of the product classes we have seen the last times. However: what do you observe if you give the same volatility structure both to the model with log-normal dynamics and with normal dynamics? If you see that some results do not make sense, you can think to rescale it when the dynamics are normal (or viceversa). You can do it manually or with the help of a class of the Finmath library.

Exercise 3

Test the method you have written in Exercise 2 by printing or plotting the value of a caplet for different strikes, first for log-normal dynamics of the LIBOR market model, and then for normal dynamics. Do it for initial values of the forward rates $L_i = 0.05$, $i = 1, \ldots, n-1$, and strike K going from 0.025 to 0.10. You can also try to see if things change depending on the measure you choose.
You can evaluate a caplet for a given underlying of type

`net.finmath.montecarlo.interestrate.TermStructureMonteCarloSimulationModel`

either writing a class extending

`net.finmath.montecarlo.interestrate.products.AbstractLIBORMonteCarloProduct,`

in a similar way as we did in the last weeks for the digital caplet and the exchange option, or using the appropriate class of the Finmath library.