Exercise 1

Consider two dates $0 < T_1 < T_2$, and suppose you know the zero coupon bonds values $P(T_1; 0)$, $P(T_2; 0)$. Also suppose that the process $L = (L_t)_{t \geq 0}$ defined by

$$L_t := L(T_1, T_2; t) = \frac{1}{T_2 - T_1} \frac{P(T_1; t) - P(T_2; t)}{P(T_2; t)}, \quad t \geq 0$$

has log-normal dynamics, and that you know the Libor volatility σ_L.

- Write an `abstract` class that implements the valuation of the price of a European option with a general payoff $V(T_1) = f(L_{T_1})$, both when it is payed in T_2 and in arrears, for the parameters listed above. Try to see which methods can be implemented here and which ones are instead specific of the derived classes.
- Extend your `abstract` class to three derived classes taking care of such a valuation for a caplet, for a digital caplet and for a floater, respectively. These derived classes can eventually have other option specific fields (for example, the strike for a caplet).
- Check if, for the same parameters, the valuations of the caplet and of the floater both for payment in $P(T_2; 0)$ and in arrears corresponds to the ones you derive with the methods of Handout 5.

Hint: Look at the solution of the theoretical part of Exercise 2 of Handout 5.

Exercise 2

Do the following:

(a) Write a class whose main goal is to get the solution of a linear system

$$Ax = y,$$

for general matrix A in the space of matrices with n rows and n columns, for $n \in \mathbb{N}$, and general vector y of length n. You can of course proceed as you like to achieve this.

Also take care of the possible errors one would get when specifying these parameters in the wrong way (for example, A is not a square matrix or it has determinant zero, y has not the same length as the dimension of A, and so on).

(b) In another class, write a method that computes the values for different strikes and fixed maturity, volatility, forward and discount factor ($F(T; 0)$ and $M(T; 0)$ at pages 347-348 of the script) of some possibly collateralized calls via the *generalized* Black-Scholes formula, see page 345 of the script, does the same for put options and returns the vector of the differences.

(c) Also write a method that, taking as an argument a vector representing the difference between such call and put options, and knowing the values of the corresponding strikes, outputs the values of forward and discount factor via a linear regression performed at point (a) of this exercise.

(d) In a test class check if, providing to this last method the differences computed at point (b) of this exercise for some parameters of your choice, you get back the forward and discount factor you have chosen.