Exercise 1

Let $L = (L_t)_{0 \le t \le T_1}$ with $L_t := (L(T_1, T_2; t))_{0 \le t \le T_1}$ be the stochastic process representing the Libor rate payed between T_1 and T_2, with $T_1 < T_2$. Take $K > 0$ and consider a caplet fixed at T_1 and expiring at T_2 with strike rate K and notional N, i.e. the financial product paying off

$$N \cdot (T_2 - T_1)(L(T_1, T_2; T_1) - K)^+$$

at time T_2.

(a) Assume we are in a market where the Libor rate in the **physical** measure is driven by the dynamics

$$dL_t = \mu L_t dt + \sigma L_t dW_t, \quad 0 \le t \le T_1,$$

for some Brownian motion W and $\sigma > 0$. Write a JAVA method to calculate the value at time $t_0 = 0$ of the caplet for the parameters defining the model and for a given discount factor $P(T_2; 0)$. You can choose yourself the way you want to implement it: basing on some analytical formulas (but not the one that directly gives the value of a caplet) or on a Monte-Carlo method.

(b) Write a test class in order to check that the result you obtain for some parameters of your choice is equal (up to a tolerance that in your idea is suitable) to the one you get by calling the method

```
blackModelCapletValue(double forward, double volatility, double optionMaturity,
        double optionStrike, double periodLength, double discountFactor)
```

of the class `net.finmath.functions.AnalyticFormulas`.

Exercise 2

Consider a Quanto Caplet on the Libor rate $L^f(T_1, T_2)$ of foreign currency, for $T_1 \le T_2$. Denote with f_t the forward FX rate at time t. The value of the quanto option in the domestic economy at maturity is

$$N \cdot C \cdot (T_2 - T_1)(L^f(T_1, T_2; T_1) - K)^+,$$

where $K > 0$ is the strike rate, N is the notional and C is a constant exchange rate converting the foreign currency in the domestic one. Assume that the processes $L^f = (L^f_t)_{0 \le t \le T_1}$ with $L^f_t = L^f(T_1, T_2; t)$ and $f = (f_t)_{0 \le t \le T_1}$ follow the lognormal dynamics

$$dL^f_t = \mu_L(t) L^f_t dt + \sigma_L L^f_t dW^{P(T_2)}_t, \quad 0 \le t \le T_1,$$

and

$$df_t = \sigma_f f_t dW^f_t, \quad 0 \le t \le T_1,$$

where $\mu_L(\cdot)$ is a deterministic function, $\sigma_L, \sigma_f > 0$ are constants and $W^{P(T_2)}$, W^f are $\mathbb{Q}^{P(T_2)}-$Brownian motions such that $d\langle W^{P(T_2)}_t, W^f_t \rangle = \rho dt$.

- Write a JAVA method to compute the value of the quanto for the parameters that define the option.
- Compute the value of the option for $L^f_0 = 0.05$ (initial value of the LIBOR), $\sigma_L = 0.3$, $\sigma_f = 0.2$, $\rho = 0.4$, $T_1 = 1$, $T_2 = 2$, $K = 0.05$, $P(T_2; 0) = 0.91$, $C = 0.9$, $N = 10000$. **Expected result:** $43,5456$.

Exercise 3

Consider again the quanto on the Libor rate of foreign currency $L^f(T_1, T_2)$, for $T_1 \leq T_2$, assuming now that the process L^f defined in Exercise 2 has normal dynamics, i.e.,

$$dL_t^f = \mu_L(t)dt + \sigma_L dW_t^{P(T_2)}, \quad 0 \leq t \leq T_1,$$

where $\sigma_L(\cdot)$ is a deterministic functions of time.
Also suppose that $f = (f_t)_{0 \leq t \leq T_1}$ has dynamics given by

$$df_t = \sigma_f(t) f_t dW_t^f, \quad 0 \leq t \leq T_1,$$

where $\sigma_f(\cdot)$ is a deterministic function of time. Here $W^{P(T_2)}$ and W^f are again $\mathbb{Q}^{P(T_2)}-$Brownian motions such that $d\langle W_t^{P(T_2)}, W_t^f \rangle = \rho dt$. Derive a formula for the pricing of the Quanto Caplet under this setting.
Suppose now that f has normal dynamics as well, i.e.,

$$df_t = \sigma_f(t) dW_t^f, \quad 0 \leq t \leq T_1.$$

Can you derive a similar pricing formula as in the first part of the exercise? If not, what is the problem?