Exercise 1

Consider the Cap defined at page 145 of the script, involving three dates $T_1 < T_2 < T_3$ and the associated Libors $L(T_1, T_2; T_1)$ and $L(T_2, T_3; T_2)$. Write a method to get the value of such a contract, with strikes $K_1, K_2 > 0$ and unique notional $N > 0$, in the case when the dynamics of the two Libors $L_1 = (L_1(t))_{t\in[0,T_1]}$ with $L_1(t) := L(T_1, T_2; t)$ and $L_2 = (L_2(t))_{t\in[0,T_2]}$ with $L_2(t) := L(T_2, T_3; t)$ are described by

$$\begin{aligned} dL^1_t &= \sigma_1 S^1_t dW^1_t, \quad 0 \le t \le T_1, \\ dL^2_t &= \sigma_2 S^2_t dW^2_t, \quad 0 \le t \le T_2, \end{aligned}$$

with constant volatilities $\sigma_1, \sigma_2 > 0$ and correlated Brownian motions $\langle W^1, W^2\rangle_t = \rho t$, $\rho \in [-1, 1]$.
Test your implementation for $T_1 = 1$, $T_2 = 1.5$, $T_3 = 2$, $P(T_2; 0) = 0.91$, $P(T_3; 0) = 0.82$, $L(T_1, T_2; 0) = 0.05$, $L(T_2, T_3; 0) = 0.04$, $\sigma_1 = 0.3$, $\sigma_2 = 0.25$, $\rho = 0.2$, $K_1 = 0.05$, $K_2 = 0.04$, notional 1000.
Expected result: close to 4.7.
Hint: you can see the contract as the sum of the payoffs of two call options, one with strike K_1, underlying L_1 and maturity T_1 (but discounted at T_2 by $P(T_2; 0)$) and one with strike K_2, underlying L_2 and maturity T_2 (but discounted at T_3 by $P(T_3; 0)$). Also consider that you have to multiply the first payoff by $(T_2 - T_1)$ (times the notional) and the second one by $(T_3 - T_2)$ (also times the notional).
The complication is the possible correlation between the Brownian motions. In order to account for that, one possibility would be to construct an object of a to suitable class implementing

`net.finmath.montecarlo.MonteCarloSimulationModel`

and give such an object to the `getValue` method of a class representing the sum of the calls and extending

`net.finmath.montecarlo.assetderivativevaluation.products.AbstractAssetMonteCarloProduct`,

Exercise 2

Consider two dates $0 < T_1 < T_2$. Do the following experiments about convexity adjustments:

- Consider first the natural floater paying

$$N(T_2 - T_1)L(T_1, T_2; T_1) \tag{1}$$

  in T_2, and then the LIBOR in arrears paying the value (1) in T_1. Write a `Junit` test class where you set $T_1 = 1$, $T_2 = 2$, let the LIBOR follow log-normal dynamics with volatility $\sigma = 0.25$, let the notional be $N = 10000$ and the prices of the zero coupon bonds $P(T_1; 0) = 0.95$, $P(T_2; 0) = 0.9$. For such parameters, do the following:

  (a) compute the analytic value of a natural floater;
  (b) simulate the process $L = (L_t)_{t\in[0,T_1]}$ with $L_t := L(T_1, T_2; t)$ under $Q^{P(T_2)}$, and test if the resulting Monte-Carlo value of a natural floater is equal (up to a given tolerance) to the value computed in (a);
  (c) find the analytic formula for the value of the floater in arrears and compute the value in the present case;
  (d) compute the value of the floater in arrears by simulating the process L under $Q^{P(T_2)}$ and compare this value with the one found in (c).

- A caplet is said to be paid *in arrears* if the payment of the option on the observed LIBOR rate $L(T_1, T_2; T_1)$ is made at T_1 instead of T_2. Find the formula for the price of a caplet in arrears by a suitable convexity adjustment, and use it to write a method that prices this product.

  Compute then the difference between the valuation of the caplet in Exercise 1 of Handout 4 and the current one, setting $T_1 = 1$, $T_2 = 2$, $L(T_1, T_2; 0) = 0.05$, LIBOR volatility $\sigma = 0.3$, strike $K = 0.044$, discount factor $P(T_2; 0) = 0.91$, notional $N = 10000$: this is the market price of the convexity adjustment. **Expected result for the difference:** 5.7819.