Exercise 2

In the first part of the lecture we have seen analytic formulas for the valuation of some European Options on interest rates. Some of such formulas are given under the Black Model. We now consider instead the Libor Market Model, i.e., we assume log-normal dynamics for n Libors $L_i(\cdot) := L(T_i, T_{i+1}; \cdot)$. Find at least one formula derived under the Black model that still holds in the Libor Market Model setting and at least one that does not hold (because its assumption cannot be valid in general) in the Libor Market Model setting. Motivate your answer.

Solution

A caplet on the Libor $L(T_i, T_{i+1})$ with strike $K > 0$ pays

$$V(T_i) = N \cdot (T_{i+1} - T_i)\,(L(T_i, T_{i+1}; T_i) - K)^+$$

at T_{i+1}. The Black model for the valuation of a caplet simply assumes log-normal dynamics for the underlying forward rate, with deterministic coefficients. These requirements are satisfied in the Libor Market Model, so the formula still holds.

In contrast, the Black model for a swaption postulates log-normal dynamics with deterministic volatility function for the swap rate. As it can be seen for example at page 674 of the script, this assumption does not hold in the Libor Market Model (indeed we have to make an approximation). So the Black formula does not hold in this setting.