Meteor science

Modelling & analysis of diurnal variation in meteor flux

C. Powell[1,2,3]

Temporal and spatial variations of peak hour and idealised sine function fit are considered in reflection of an extended model of the diurnal shift mechanism. This model is formed by extension of the currently understood mechanism, providing a mathematical argument focusing on orbital velocity. Hourly detection counts collected by forward-scatter radio detection are used as data to analyse the form of diurnal shift for each observer. The fit and mean peak hour of the diurnal shift and are considered across nearly 350 observers, analysing variation from 2000 to 2016, as well as variation between data from 9 latitude and 14 longitude categories spanning at most 10° each, to determine the agreement of data with the model. Modelling the orbital velocity of Earth as a primary factor behind diurnal variation is supported by the timezone corrected peak hours and correlation with longitude. The mechanism does not appear to vary with time, however the relative intensity of diurnal variation with respect to background detection counts is damped as a maximum in these hourly detection counts is observed. This provides a mathematical model of the diurnal shift mechanism, accompanied with support from a large dataset.

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1 Introduction

Diurnal variation, a notable increase and decrease in meteor flux over the time scale of a single day, has been observed and studied previously; observations are frequently reported, for example Kero et al. (2012) and Okamoto & Maegawa (2008). It is known that the variation relies on the rotation of Earth altering the component of orbital velocity contributing to a meteor's incident velocity. However, this has not been described in any mathematical detail. The motivation for this article is the formulation of a model describing the mechanism that causes diurnal variation, and subsequent comparison against data.

In order to compare the model to the data, a collection of hourly detection counts provided by forward-scatter radio detection is used to examine predictions made by the model. In conjunction with this, the temporal and spatial variation of sine function fit and peak hour is investigated.

Singer et al. (2004) have presented an analysis noting an annual variation of the diurnal shift, whilst Singer et al. (2005) observe an inverse proportionality between latitude and intensity. The data available for my own study covers a larger range of latitudes, perhaps supporting either of these studies. There are over 3.8 million available hourly counts, providing a large sample from which analysis of temporal variation can be made.

2 Modelling the diurnal variation mechanism

For a detailed overview of the 'standard' explanation for the mechanism behind diurnal variation, see Hines (1956). Despite being detailed, Hines (1956) is not a mathematical formulation of the explanation. I put forward a model with the aim of a more complete and mathematically rigorous description.

First, I assume that the number of meteors detected is proportional to the mean incident velocity of meteors to Earth. This is reasonable: the actual velocities are of course random. However, an overall larger mean incident velocity of a collection of meteors means that proportionally more will reach Earth, of those that are in a direction that could cause a collision with Earth's atmosphere. Of a group of meteors with random velocities (both random magnitude and direction), very few will be on a course tangential to Earth's atmosphere. However, this does not change as Earth rotates, so it is a negligible variable. I consider only those that are tangential with the atmosphere, and so only those that could cause a detection. We have, at a very basic level:

$$N \propto v_{\text{incident}} \tag{1}$$

The incident velocity is dependent on factors such as the Earth's rotational velocity, orbital velocity and the velocity of the sporadic meteors themselves. v_{meteor} is difficult to consider, since it (theoretically) has a random direction. We know that $v_{\text{orbit}} \approx 30\,000\,\text{ms}^{-1}$ and $v_{\text{rotation}} \approx 450\,\text{ms}^{-1}$.

Assuming that the path described by Earth's orbit around the Sun is a straight line through Earth when 'up close', let θ be the angle between a radius from Earth's centre to the observer's location, and the orbital path (Figure 1).

The set of meteors that can be detected appear, looking from above Earth's pole, to form a triangle. The average velocity of these meteors will be along the height of this triangle, perpendicular to the surface of Earth (v_{meteor}). This is clear simply from how meteor showers appear: all meteors appear to originate from the radiant. Since this perpendicular velocity v_{meteor} remains perpendicular despite Earth's rotation, the rotational velocity of Earth can be disregarded. Thus, the

[1]Exeter Mathematics School, Exeter, Devon, EX4 3PU, United Kingdom
[2]Norman Lockyer Observatory, Sidmouth, Devon, EX10 0NY, United Kingdom
[3]Email: cpowell@cwp.io

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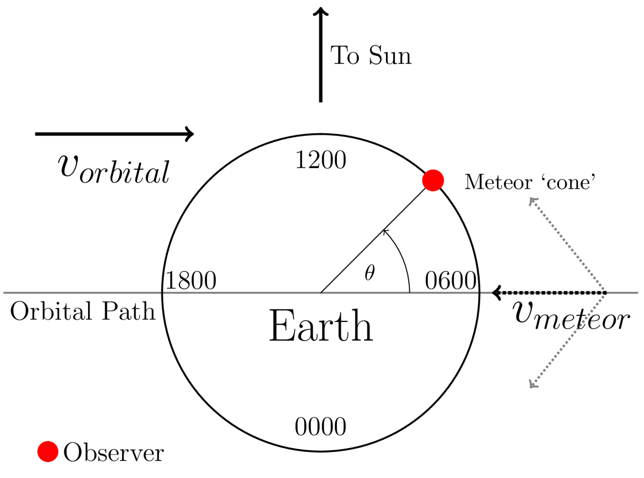

Figure 1 – Model diagram: Earth in orbit, showing an observer (red dot), the perceived meteor 'cone', and the incident meteor velocity v_{meteor}.

remaining velocity to consider is Earth's orbital velocity:

$$v_{\mathrm{incident}} \approx v_{\mathrm{orbit}} \cos\theta + v_{\mathrm{meteor}} \tag{2}$$

Clearly, this is at a maximum when $\theta = 0°$. For an observer at a longitude of 0°, this is 6:00. Since this is where the angle is defined from, it is clear that for any observer the peak hour will be 6:00[a] local time (assuming local time is based on longitude, not timezones). It should be noted that the peak hour can vary around this time, based on conditions for observing. Particularly important are changes in electron concentration in the ionosphere as the Sun rises, which increases reflection of radio signals, allowing for better reception of signals reflected by ionised meteor trails (Rana & Yadav, 2014). This could shift the peak hour. This effect may also contribute to a change in diurnal shift intensity with latitude, since the increased atmospheric ionisation caused by the Sun is dependent on latitude (Dabas, 2000).

3 Methodology

I use previous results to examine spatial and temporal variation of diurnal shift, including a measure of goodness-of-fit between a sine function and the diurnal shift curve.

3.1 Data source

The data used is a database of hourly detection counts from almost 350 observers across the globe, provided by the Radio Meteor Observation Bulletin (RMOB), accessible at `www.rmob.org`.

3.2 Sine function fit

The diurnal shift curve is generated by taking the mean of all data for a given hour after midnight for each observer. Time, for all observers, is recorded in UTC.

[a] All times given in 24-hour format

Table 1 – Sample sizes for location categories

Category	N° observers
Europe	220
Asia & Australia	12
North America	37

Using this curve I will compare the intensity of diurnal shift for three different location categories: Europe, Asia & Australia, and North America. The sample sizes for each category are shown in Table 1. Observers are sorted into these categories based on metadata provided with the RMOB data. In conjunction with this, I fit a sine curve, in order to test the model, indicating how well diurnal shift is described by a sinusoidal function.

3.3 Spatial analysis

QGIS (QGIS Development Team, 2009) is used to generate a map, with a dot representing each observer with a known location. These dots will be coloured based on the peak hour of diurnal shift, indicating how this changes with location. Further to this, I will analyse the variation of the peak hour of diurnal shift, as well as the error in fit for a sine function of the form $A\sin\left(\frac{2\pi}{24}t + \phi\right) + \mu$, indicating how well a sine curve fits the diurnal shift. This is calculated using the reduced covariance matrix X and the reduced chi-squared χ^2, where

$$\chi^2 = \frac{r^T I r}{N-3} \tag{3}$$

N is the number of data points, r is the matrix of residuals and I is the identity matrix. The numerator is the minimum value of the weighted objective function (the optimal parameters). Then the variance-covariance matrix of the parameters M^β is;

$$M^\beta = \chi^2 \left(X^T X\right)^{-1} \tag{4}$$

The parameter standard deviations is $\sigma_i = \sqrt{M^\beta_{ii}}$, and the sum of these three is taken. This analysis will be over two independent variables: latitude and longitude.

3.4 Temporal analysis

Finally I analyse how the peak hour and sinusoidal function fit vary over time, from 2000 to 2016. This will use the same calculated variables as the spatial analysis. These analyses will be completed using a Python program.

4 Results & Discussion

4.1 Sine function fit

Figure 2 shows the results for sine function fit. The optimal sine function and cubic are shown in red and green respectively. The sum of parameter standard deviations of the optimal sinusoidal function is 0.814, which indicates a good fit. This suggests that the sine function fits well. As a reference, a cubic is fitted. This shows how well a sine curve fits in comparison. This suggests that the cause of diurnal shift is based on a sine function of Earth's rotation (i.e. the hour of the day), which supports the presented model.

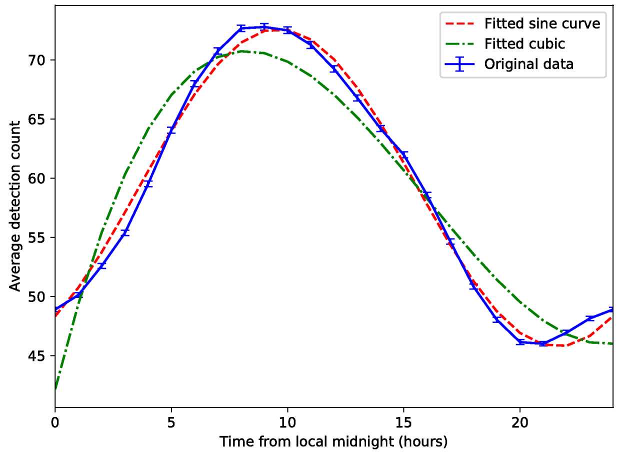

Figure 2 – Diurnal shift across all authors & sine function fit.

Figure 3 shows the same plot as Figure 2 for comparison, as well as diurnal shift curves for each respective location. "All" refers to the set of data from all observers. Surprisingly, each location category shows a clear sine curve. The peak of these sine curves appears to be correlated well with latitude. The average longitudes are, for Europe, Asia & Australia, and North America respectively: $\sim 15°$, $\sim 150°$ & $\sim -100°$. This supports the idea that the sine function can be used to describe the diurnal shift, since larger longitudes produce a greater hour of diurnal shift. The average latitudes are (respectively): $\sim 45°$, $\sim 35°$ & $\sim 15°$. The North American category has the largest intensity, followed by the European category and then Asia & Australia, though this is not the order of latitudes (seen in Figure 4). This appears to disagree with Singer et al. (2005), where it was suggested that the intensity (amplitude) of diurnal shift is dependent on latitude. It is difficult to comment on why the intensity of one category is different to another without knowledge of the exact detection setup, antenna type and observing conditions for a given observer.

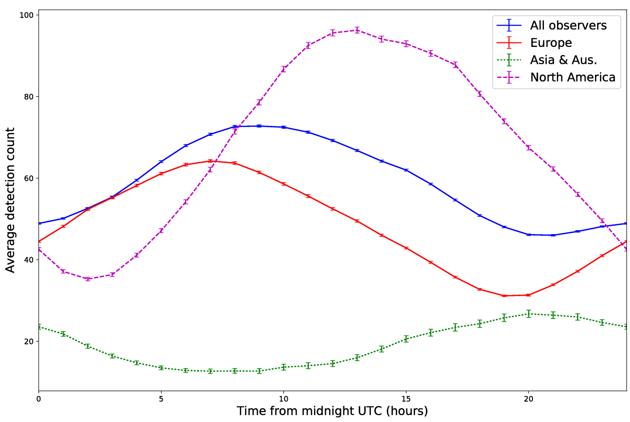

Figure 3 – Diurnal shifts for each individual location (overall shift of all observers included for reference).

Table 2 – Diurnal shift parameter results for location categories. Column 5 contains the sum of parameter standard deviations.

Category	A	ϕ	μ	$\Sigma\sigma$
All	13.4	−0.941	59.2	0.454
Europe	15.4	−0.302	48.4	0.498
Asia & Aus.	−7.19	−0.585	19.0	0.352
N. America	29.5	−2.05	68.1	1.16

The sum of parameter standard deviations for each location category are shown below. The positive numbers all indicate a positive correlation, as expected from the figures. Again, "all" refers to the diurnal shift curve when including data from all observers.

4.2 Spatial variation

Figure 4 contains a map showing the location of each observer in the data set. The dot for each circle is coloured (as in the legend) based on the peak hour of the diurnal shift. This gives a more visual demonstration of what is shown in Figures 3 and 7: the colours are clearly grouped based on location. Observers in Europe have diurnal shift peaks mostly around 6:00, North American observers have peaks around 15:00 and in Asia & Australia around 20:00. This supports implications from the previously stated figures. There are clearly anomalous results for some observers (note the blue dots in Europe), though there is often anomalous data, so this is not unexpected.

Latitude

There appears to be little correlation between latitude and the peak hour. Although, overall, the hour appears to decrease as the latitude varies from $-40°$ to $60°$, the error bars, indicating standard error, are substantial. The implication of this is there is little agreement within each category, suggesting no correlation at all. However, this is expected. There is no logical reason why the peak hour would be influenced by the latitude. Diurnal shift is modelled as being caused by Earth's rotation changing the average incident velocity of meteors. This does not change with latitude, hence no correlation is seen.

In Figure 6, little correlation is seen again. The standard deviations vary widely. There is no clear trend, suggesting that the latitude of an observer has no effect on how well collected data fits a sine function. Consequently, it would appear location does not influence diurnal shift.

Singer et al. (2005) analyse a variation of diurnal shift amplitude with latitude, over a relatively small range of latitudes. I find no correlation to support this finding, though the distribution of observer latitudes for my study is not uniform, making a definite conclusion difficult.

Longitude

Figure 7 shows agreement with Figures 3 and 4. The peak hour is lowest for a longitude of 0° and increases

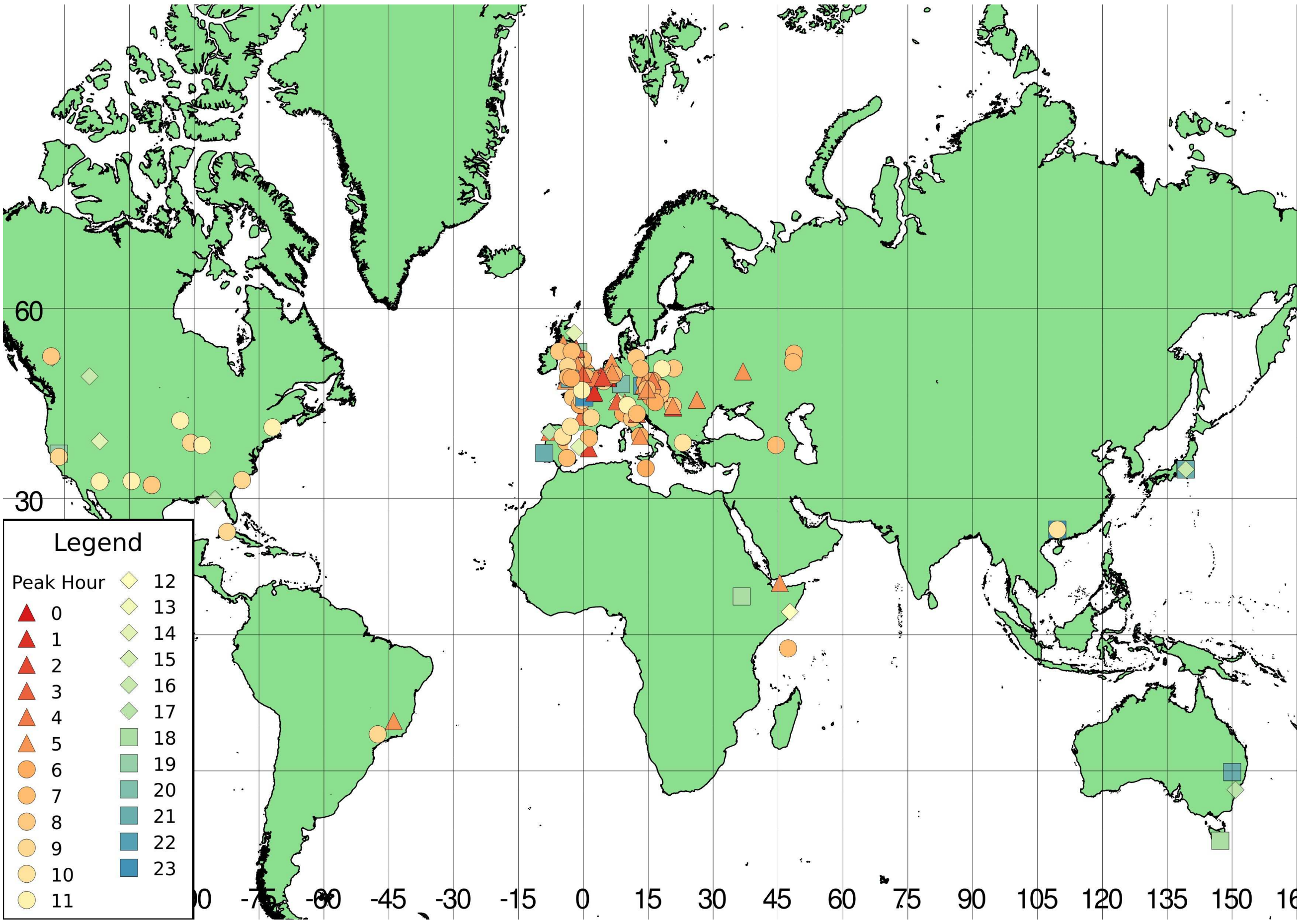

Figure 4 – Peak hour of diurnal shift for each observer.

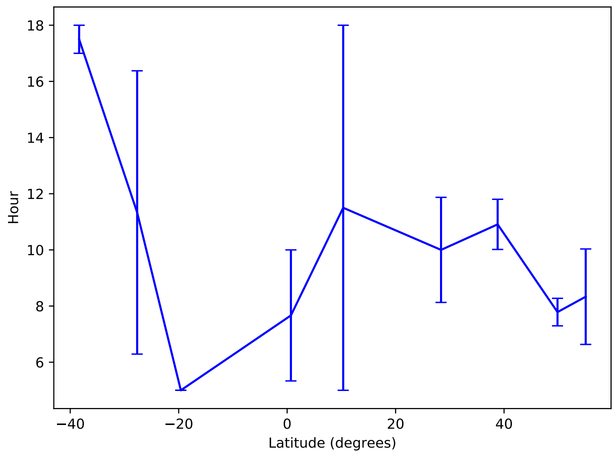

Figure 5 – Peak hour of diurnal shift against latitude.

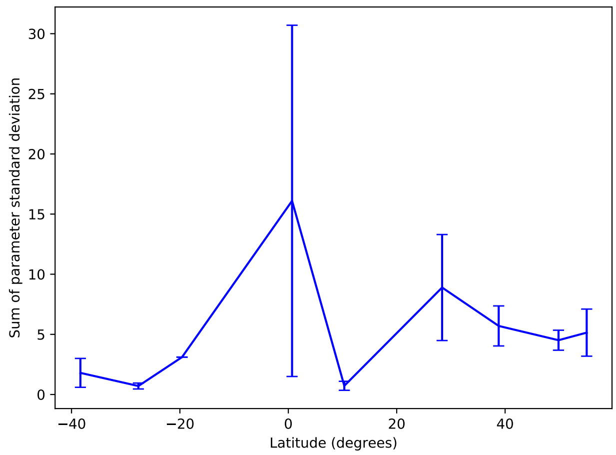

Figure 6 – Optimal sine function fit against latitude.

either side of this, as seen in the stated figures. The errors for some categories are reasonably large, though still fall in a range that fits the trend. There is, of course, minor variation though the trend is clear from the data: latitude and peak hour of diurnal shift are related.

In order to investigate this apparent trend further, I have corrected the peak hour of each location category such that $H_{\text{corrected}} = H_{\text{calculated}} + \frac{\phi}{15}$, provided the longitude ϕ is in degrees. This means that the hour will be the local time (not using timezones, only the time based on longitude). This is shown in Figure 8. Error bars, in this case, are such that 75% of the data for a given location category falls within the bars. The values appear to fluctuate around a peak hour of 6:00 (shown in green), supporting the model I have proposed. However, there is a large degree of variation.

In Figure 9 a histogram of the peak hour is shown. It is clear from this figure that the model is supported. Clearly most of the observers have a peak hour around

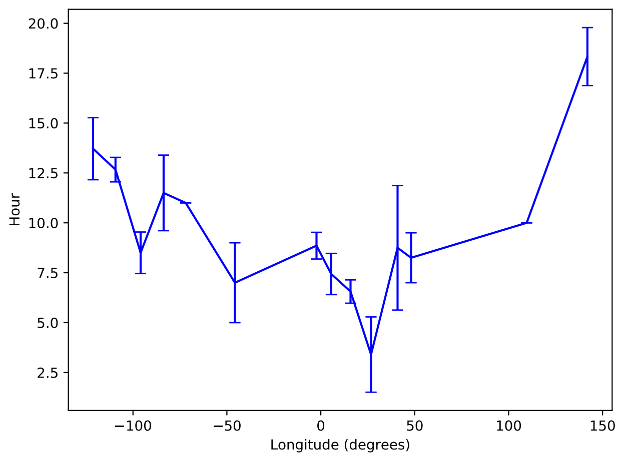

Figure 7 – Peak hour of diurnal shift against longitude.

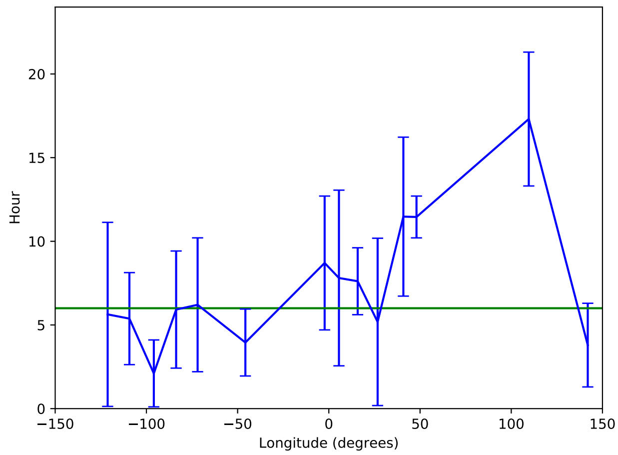

Figure 8 – Corrected peak hour of diurnal shift against longitude.

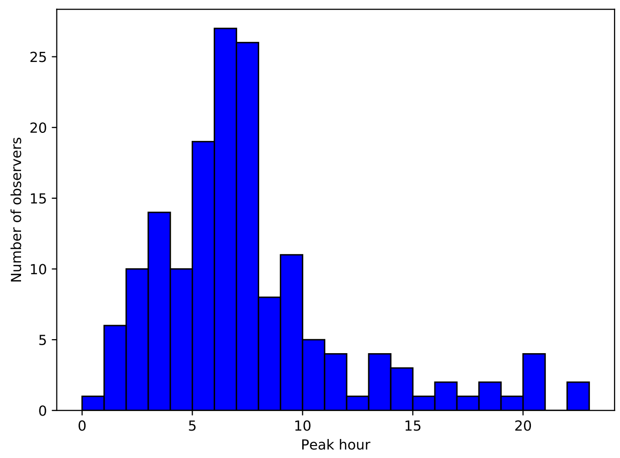

Figure 9 – Histogram of peak hours.

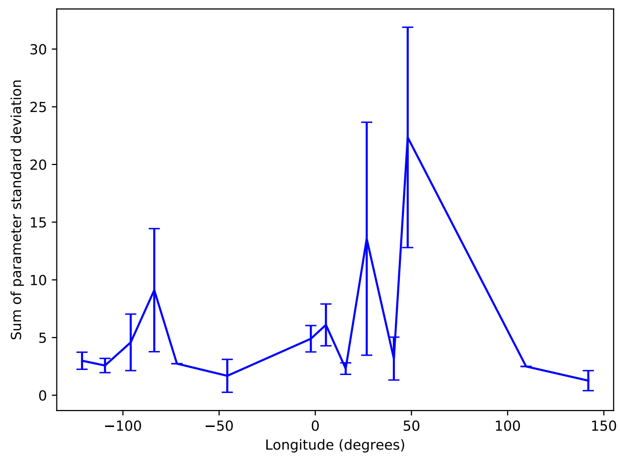

Figure 10 – Optimal sine function fit against longitude.

6, with some having a peak hour of 7. This is reasonable; as noted in section 2, the peak hour can vary slightly, resulting in some distribution either side of 6:00, so this is expected.

Most meteor detection setups are simply receiving stations. Typically a station, often a considerable distance away, emits a signal that reflects off the meteor's ionised trail and is received by the observer. This means that there is a difference between the longitude of the observer and the longitude of where the detection takes place. This is likely to have caused the observed distribution.

The sum of parameter standard deviations, indicating fit, varies a small amount with longitude for most categories. However, some have much greater variation, indicating less consistency in the data. There does not appear to be a clear trend. However, the poorest fits have large errors indicating that this is not a poor fit throughout observers in said categories. Consequently it would seem that the fit varies moderately across the globe.

4.3 Temporal variation

Figure 11 shows the same result as previously noted, namely a general diurnal behaviour. The peak hour for each location category varies around the hours expected, given that peak hour is correlated with longitude. There is a large amount of variation for the Asia & Australia category, so it is hard to make an analysis from this. It is clear that in more recent years, there is less variation. However, in general, no categories appear to increase or decrease over time, suggesting that the diurnal behaviour is constant, and not subject to a large degree of seasonal or annual change.

Generally, the fit, shown in Figure 12 is reasonably good. The fit may be considered a measure of the consistency of data obtained by each station. Thus the data indicates that there is less consistent data obtained between 2005 and 2011, since the fits are much poorer. This indicates either a weaker diurnal shift (an unlikely occurence) or a greater background detection rate. For periods outside this range, there is a low amount of variation. All categories have a similar fit and absent trend over time.

A question that may require a more detailed model of the mechanism behind diurnal shift is whether the phenomenon changes on a yearly time scale. Seasonal variation of diurnal shift may indicate an influence from sources such as the Sun, or the orientation of the Earth relative to its orbit. I do not make such an analysis, leaving scope for this in the future.

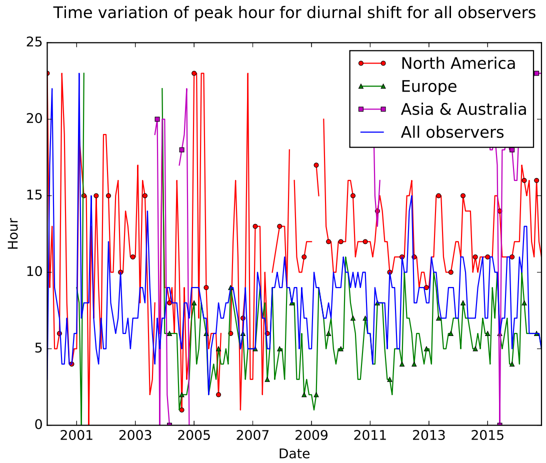

Figure 11 – Peak hour of diurnal shift change over time.

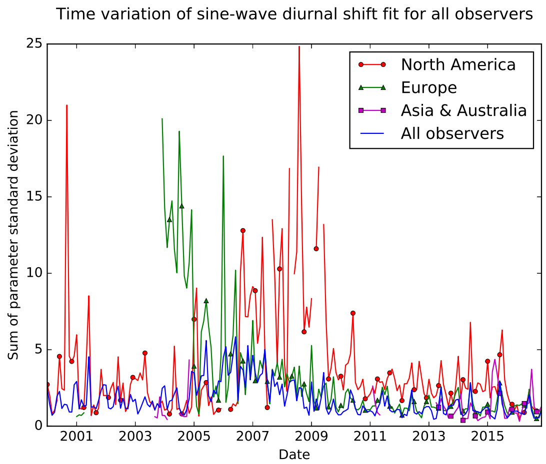

Figure 12 – Optimal sine function fit change over time.

5 Conclusion

1. There is a clear correlation between longitude and peak hour of diurnal shift, as suggested by the proposed model. This is apparent from several results, including a histogram of peak hours.

2. The good agreement indicates that the proposed model is a valid explanation of the mechanism and is supported by the data. Thus it can be concluded that the influence of orbital velocity on incident meteor velocity is the primary mechanism behind diurnal shift.

3. There is little correlation between location and a sine function fit, suggesting that the intensity of diurnal shift (in the sense of relative intensity compared to background detection rates) is roughly uniform across the globe.

4. There is no clear link to be made between the intensity of diurnal variation and latitude.

5. There are differences in amplitude between different location categories, however I find no explanation for this.

6. During the period of increased detection rates, the relative intensity of diurnal variation decreases.

6 Acknowledgements

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