12.1

12.2

12.3
Equality of (12.14) will happen when ρ_x has orthogonal support. It is obvious that n qubits have at most n orthogonal ρ_xs, and from (12.6),

$$H(X:Y) \leq \chi(\rho) \leq H(X) \leq n \tag{1}$$

So, n qubits can be used to at most n bits of classical information.
12.6

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12.31
Eve makes her qubits entangled with $|\beta_{00}\rangle$, and gets ρ^E.

$$|ABE\rangle = U\,|\beta_{00}^{\otimes n}\rangle\,|0\rangle_E \tag{2}$$
$$\rho^E = tr_{AB}(|ABE\rangle\,\langle ABE|) \tag{3}$$

Note that Eve's mutual information with Alice and Bob measurements does not depend on whether Eve measures ρ^E before Alice and Bob's measurement or after. So we can assume that Eve measures ρ^E after Alice and Bob's measurement. Alice and Bob measure their Bell state, getting binary string $\vec{k}$ as an outcome. Let ρ_k^E and p_k are the corresponding Eve's states and probabilities. Note,

$$\rho_E = \sum_k p_k \rho_k^E. \tag{4}$$

Let K is a variable of $\vec{k}$ and e is an outcom of a measurement of ρ^E, and E is its variable. From Holevo bound,

$$H(K:E) \leq S(\rho^E) - \sum_k p_k \rho_k^E \leq S(\rho^E) = S(\rho). \tag{5}$$