Entropy and information

11.1 Fair coin:

H(1/2, 1/2) = $\left(-\frac{1}{2}\log\frac{1}{2}\right)\times 2 = 1$

Fair die: H(p) = $\left(-\frac{1}{6}\log\frac{1}{6}\right)\times 6 = \log 6$.

The entropy decreases if the coin or die is unfair.

11.2

From assumption $I(pq) = I(p) + I(q)$.

$\partial I(pq)\frac{}{\partial p = \frac{\partial I(p)}{\partial p} + 0 = \frac{\partial I(p)}{\partial p}\ \frac{\partial I(pq)}{\partial q} = 0 + \frac{\partial I(q)}{\partial q} = \frac{\partial I(q)}{\partial q}}$

$\partial I(pq)\frac{}{\partial p = \frac{\partial I(pq)}{\partial (pq)}\ \frac{\partial (pq)}{\partial p} = q\frac{\partial I(pq)}{\partial (pq)} \Rightarrow \frac{\partial I(pq)}{\partial (pq)} = \frac{1}{q}\ \frac{\partial I(p)}{\partial p}\ \frac{\partial I(pq)}{\partial q} = \frac{\partial I(pq)}{\partial (pq)}\ \frac{\partial (pq)}{\partial q} = p\frac{\partial I(pq)}{\partial (pq)} \Rightarrow \frac{\partial I(pq)}{\partial (pq)} = \frac{1}{p}\ \frac{\partial I(q)}{\partial q}}$

Thus

$1\frac{}{q\frac{\partial I(p)}{\partial p} = \frac{1}{p}\ \frac{\partial I(q)}{\partial q}\quad p\frac{dI(p)}{dp} = q\frac{dI(q)}{dq}\qquad for all p, q \in [0,1].}$

Then $p(dI(p)/dp)$ is constant.

If $p(dI(p)/dp) = k$, $k \in R$. Then $I(p) = k\ln p = k'\log p$ where $k' = k/\log e$.

11.3

$H_{bin}(p) = -p\log p - (1-p)\log(1-p)$.

d $\mathrm{H}_{bin}(p)\frac{}{dp = \frac{1}{\ln 2}(-\log p - 1 + \log(1-p) + 1) = \frac{1}{\ln 2}\ln\frac{1-p}{p} = 0 \Rightarrow \frac{1-p}{p} = 1 \Rightarrow p = 1/2.}$

11.4

11.5