TINF Signali formule

$$E = I(Ri^2(t)) = I(\frac{u^2(t)}{R}), \quad P = \lim_{T\to\infty} \frac{1}{T}\int_{-T_0/2}^{T_0/2} Ri^2(t)dt$$

Periodični signali

Fourierov razvoj:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \text{ gdje } c_k = \frac{1}{T}\int_{-T_0/2}^{T_0/2} x(t)e^{-jk\omega_0 t}dt$$

Snaga

$$\begin{aligned} P &= \lim_{k\to\infty}\left[\frac{1}{kT_0}\int_0^{kT_0} |x(t)|^2\, dt\right] \\ &= \frac{1}{T_0}\int_0^{T_0} |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |c_k|^2 \end{aligned}$$

$$\text{Sinusni signal:} \quad P = \frac{A^2}{2}$$

$$\text{Slijed pravokutnih impulsa:} \quad P = A^2\frac{\tau}{T}$$

Fourierovi parovi

$$x(t) = \cos(\omega_0 t) \leftrightarrow \frac{A}{2}[\delta(f-f_0)+\delta(f+f_0)]$$

$$x(t) = \sin(\omega_0 t) \leftrightarrow -j\frac{A}{2}[\delta(f-f_0)-\delta(f+f_0)]$$

Modulacijsko pravilo:

$$x(t)cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}\left[X(f-f_0)+X(f+f_0)\right]$$

Neperiodični signali

Energija i snaga

$$\text{Energija} \quad \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2 dt = \int_{-\infty}^{\infty}|x(t)|^2 dt, \quad P = \frac{E}{2T}$$

Spektar

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt, \quad x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

$$X(f) = |X(f)|e^{j\theta(f)}$$

, gdje je

$$|X(f)|$$

amplitudni spektar, a

$$\theta(f)$$

fazni spektar.

Parsevalov teorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

Pravokutni impuls

$$P = 0 \quad \text{(beskonačnost)}, \quad E = A^2\tau$$

Slučajni signali

Srednja vrijednost

$$\mu_X(t) = E\left[X(t)\right] = \int_{-\infty}^{\infty} x f_X(x,t)dx$$

Autokorelacijska funkcija

$$R_X(t_1,t_2) = E\left[X(t_1)X(t_2)\right]$$

Autokovarijanca

$$C_X(t_1,t_2) = E\left\{\left[X(t_1)-\mu_x(t_1)\right]\left[X(t_2)-\mu_x(t_2)\right]\right\}$$

Pravila očekivanja (E)

$$E[c] = c, \quad c\in\mathbb{R}, \quad E[cX] = cE[X]$$

$$E[X+Y] = E[X]+E[Y], \quad E[XY] = E[X]E[Y]$$

Stacionarnost

Uvjeti:

- $E[X(t)] = \mu_x$
- $\forall t_1, t_2, \quad R_x(t_1,t_2) = R_x(t_1-t_2) = R_x(\tau)$

  Pri tome: R_x je parna funkcija, $|R_x(\tau)| \leq R_x(0) \geq 0$

Srednja snaga:

$$P = E[X^2(t)] = R_X(0) = \int_{-\infty}^{\infty} S_X(f)\, df$$

$$E[X] = 0 \longrightarrow P = \text{var}(X) = \sigma_X^2$$

Spektralna gustoća snage

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)e^{-j2\pi f\tau}d\tau \quad \left[\frac{W}{Hz}\right]$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f)e^{j2\pi f\tau}df$$

Bijeli šum

$W(t)$ je bijeli šum ako:

$$R_W(\tau) = C_1\delta(\tau) \quad \wedge \quad C_W(\tau) = C_2\delta(\tau)$$

Svojstva:

$$\mu_W = 0, \quad R_W(\tau) = \sigma^2\delta(\tau) = N_0/2$$

$$S_W(f) = \sigma^2\int_{-\infty}^{\infty}\delta(t)e^{-j2\pi ft}\, dt = \sigma^2 = N_0/2$$

Gaussova razdioba:

$$f_x(x) = \frac{1}{\sigma_X\sqrt{2\pi}}e^{-(x-\mu_X)^2/(2\sigma_X^2)}$$

Prijenos

Izlazni signal:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

Prijenosna funkcija:

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt$$

Amplitudni odziv RC kruga:

$$20\log\frac{|H(f)|}{|H(0)|} = 20\log|H(f)|$$

Za idealni filtar:

$$|H(f)| = \begin{cases} 1, & |f| \le f_g \\ 0, & |f| > f_g \end{cases}$$

Impulsni odziv i prijenosna funkcija:

$$y(t) = x(t) * h(t), \quad Y(f) = X(f)H(f)$$

Amplitudni odziv je parna funkcija, a fazni neparna:

$$|H(-f)| = |H(f)|, \quad \theta(-f) = -\theta(f)$$

Ako je $X(t)$ stacionarni slučajni proces:

$$\mu_Y = \mu_X H(0), \quad S_Y(f) = S_X(f)|H(f)|^2$$

Ako je ulaz $x(t)$ sa spektrom $X(f) = |X(f)|e^{j\varphi(f)}$:

$$Y(f) = |Y(f)|e^{j\vartheta(f)}, \quad |Y(f)| = |X(f)||H(f)|$$

$$\vartheta(f) = \varphi(f) + \theta(f)$$

Amplitudni odziv RC kruga:

$$|H(f)| = \left|\frac{U_{\text{izlaz}}(f)}{U_{\text{ulaz}}(f)}\right| = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

Uzorkovanje i kvantizacija

Frekvencija uzorkovanja u pomaknutom pojasu:

$$f_u = 2\frac{B+B_0}{M+1}, \quad M_m = \left\lfloor \frac{B_0}{B} + 1 \right\rfloor$$

Varijanca kvantizacijskog šuma (srednja snaga):

$$\text{var}(Q) = \sigma_Q^2 = \frac{\Delta^2}{12} = \frac{1}{3}m_{\max}^2 2^{-2r}, \quad \Delta = \frac{2m_{\max}}{L}$$

Omjer srednje snage signala i snage kvantizacijskog šuma:

$$\frac{S}{N} = \frac{S}{\sigma_Q^2} = \left(\frac{3S}{m_{\max}^2}\right)2^{2r}$$

U decibelima (samo za sinusni signal):

$$\left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6.02r$$

Brzina prijenosa:

$$R = f_u r \quad \left[\frac{\text{bit}}{s}\right]$$

Entropija u kontinuiranom kanalu

f su funkcije gustoće vjerojatnosti.

$$H(X) = E[-\log f_X(X)] = -\int_{-\infty}^{\infty} f_X(x)\log f_X(x)\,dx$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy, \quad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$\begin{aligned} H(X|Y) &= E[-\log f_{X|Y}(X|Y)] \\ &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\log\left(\frac{f(x,y)}{f_Y(y)}\right)dx\,dy \end{aligned}$$

$$\begin{aligned} H(X,Y) &= E[-\log f(X,Y)] \\ &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\log f(x,y)\,dx\,dy \end{aligned}$$

$$\begin{aligned} I(X;Y) &= E[-\log f_{Y|X}(Y|X)] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\log\left(\frac{f(x,y)}{f_X(x)f_Y(y)}\right)dx\,dy \end{aligned}$$

Prijenos u prisutnosti aditivnog šuma:

$$f_x(y|x) = f_x(z+x|x) = \phi(z)$$

$$I(X;Y) = H(Y) - H(Y|X) = H(Y) - H(Z)$$

Kapacitet:

$$C = \max I(X;Y) = \max\left[\frac{1}{2}\ln[2\pi e(\sigma_X^2+\sigma_Z^2)] - \frac{1}{2}\ln(2\pi e\sigma_Z^2)\right]$$

$$= \frac{1}{2}\ln\left(1+\frac{S}{N}\right) \quad \left[\frac{\text{nat}}{s}\right]$$

$$C = \frac{1}{2}\log_2\left(1+\frac{S}{N}\right) \quad [\text{bit/simbol}]$$

Maksimizacija entropije u kontinuiranom kanalu

- $x \in [a,b] \to f(x) = \frac{1}{b-a}, \quad H(X) = \ln(b-a)$ [nat/sym]
- $x \ge 0 \wedge E[X] = a > 0 \to f(x) = \frac{1}{a}e^{-x/a}, \quad H(X) = \ln(ae) = 1 + \ln a$
- $E[X] = 0 \wedge \exists\sigma_X \to f$ Gaussova, $H(X) = \ln(\sigma_X\sqrt{2\pi e})$

Inf. kapacitet AWGN kanala

Za kanal s $f_u = 2B$...

$$n = 2B \longrightarrow B\log_2\left(1+\frac{S}{N}\right) \quad [bit/s], \quad C = 2BD$$

E_b, srednja energija po svakom bitu...

$$\text{uz... } E_b = S/R_b, \quad S = E_b C, \quad \frac{C}{B} = \log_2\left(1+\frac{E_b}{N_0}\frac{C}{B}\right)$$

$$\frac{E_b}{N_0} = \frac{2^{C/B}-1}{C/B}, \quad \lim_{B\to\infty}\left(\frac{E_b}{N_0}\right) = \log(2), \quad \lim_{B\to\infty} C = \frac{S}{N_0}\log_2 e$$

Konverzije

Pojačanje. U decibele (dB): $x \to 10\log_{10}(x)$

Jedinice

$$c_k \leftrightarrow \left[\frac{V}{Hz}\right], \quad S_X(f) \leftrightarrow \left[\frac{W}{Hz}\right]$$

Ostalo

Neka svojstva operatora Fourierove transformacije:

$$\text{Linearnost} \qquad \mathcal{F}\{ax + by\} = a\mathcal{F}\{x\} + b\mathcal{F}\{y\}$$

Riemann-Lebesgue lema na realnom / kompleksnom skupu.
x je L^1 ako:

$$\int_{\mathbb{R}^n} |x(t)| dt < \infty$$

Za fourierov transformat $X(f)$ tada vrijedi:

$$|X(f)| \to 0 \text{ kada } |f| \to 0$$

Srednja kvadratna pogreška, u_{qi} kvantizacijske razine:

$$N_q^2 = \sum_{u_{qi}} \int_{u_{qi}-\Delta/2}^{u_{qi}+\Delta/2} (u - u_{qi})^2 f(u)\, du \quad [V^2]$$

Entropija slučajnog vektora

$$\begin{aligned} H(\mathbf{X}) &= E\left[-\log\{X_1, ..., X_n\}\right] \\ &= -\int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} f_{\mathbf{X}}(x_1, ..., x_n) \log\left[f_{\mathbf{X}}(x_1, ..., x_n)\right] dx_1 ... dx_n \end{aligned}$$

Inf. kapacitet AWGN kanala

Pri uzorkovanju:

$$\mathbf{X} = [X_1, X_2, ..., X_n]$$

$$\mathbf{Y} = \mathbf{X} + \mathbf{Z}$$

$$E[X_k] = 0, \quad E[X_k^2] = \sigma_{xk^2}$$

$$\phi(\mathbf{z}) = \prod_{k=1}^{n} \left[\frac{1}{\sigma_{z_k}\sqrt{2 * \pi}} e^{-z_k^2/2\sigma_{z_k}^2}\right]$$

$$H(\mathbf{Y}|\mathbf{X}) = H(\mathbf{Z}) = -\int_{-\infty}^{\infty} \phi(\mathbf{z}) \log\left[\phi(\mathbf{z})\right] = \sum_{k=1}^{n} \log(\sigma_{z_k}\sqrt{2\pi e})$$

$$I(\mathbf{X};\mathbf{Y}) = H(\mathbf{Y}) - \sum_{k=1}^{n} \log(\sigma_{z_k}\sqrt{2\pi e})$$

Ako su sve varijance jednake...

$$\begin{aligned} I_{\max}(\mathbf{X};\mathbf{Y}) &= \frac{n}{2}\log\left(1 + \frac{\sigma_x^2}{\sigma_z^2}\right) \qquad [bit/simbol] \\ &= \frac{n}{2}\log\left(1 + \frac{S}{N}\right) \end{aligned}$$

Slike