TINF Signali bilješke

Dirac delta izvodi

$e \longrightarrow sinc$

$$\int_{T}^{-T} e^{i(\omega-\omega')t} dt = \frac{e^{i(\omega-\omega')t} - e^{-i(\omega-\omega')t}}{i(\omega-\omega')} = \frac{2T sin((\omega-\omega')T)}{(\omega-\omega')T}$$

Sada za $T \longrightarrow \infty$ se ponaša kao:

- $\omega \neq \omega'$: oscilira zauvijek, kada se koristi unutar integrala (isto kao δ) ta površina nestaje.
- $\omega = \omega'$: sve osim $2T$ postaje 1, tkd. $2T \longrightarrow \infty$, no opet, unutar nekog drugog integrala površina $\longrightarrow 1$

Pa je time:

$$\int_{T}^{-T} e^{i\omega t} * e^{-i\omega' t} dt = 2\pi\delta(\omega-\omega')$$

Dirac comb funkcija

Dirac comb funkcija ovdje označena s S:

$$S_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

Fourier koeficijenti periodične S funkcije, pretpostavka da je linearnost očuvana:

$$c_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} S_{T_0}(t) e^{-jk\omega_0 t} dt =$$
$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (\sum_{n=-\infty}^{\infty} \delta(t-nT_0)) e^{-jk\omega_0 t} dt =$$
$$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} (\int_{-T_0/2}^{T_0/2} e^{-jk\omega_0 t} \delta(t-nT_0)) dt = ...$$

Sad $\delta(t - nT_0) = 0, \forall n > 0$:

$$... = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} e^{-jk\omega_0 t} \delta(t) dt =$$
$$\frac{1}{T_0} e^{-jk\omega_0 * 0} = \frac{1}{T_0}$$

Sad fourierov par:

$$S(t) \longleftrightarrow f_0 \sum_{k=-\infty}^{\infty} \delta(f - kf_0) = f_0 * S_{f_0}(f)$$

Tj.:

$$x(t) * S_{T_0}(t) \longleftrightarrow X(f) * f_0 * S_{f_0}(f)$$