TINF Signali formule

Periodični signali

Snaga

$$\begin{aligned} P &= \lim_{k\to\infty}\left[\frac{1}{kT_0}\int_0^{kT_0}|x(t)|^2\,dt\right] \\ &= \frac{1}{T_0}\int_0^{T_0}|x(t)|^2\,dt \\ &= \sum_{k=-\infty}^{\infty}|c_k|^2 \end{aligned}$$

Za sinusni signal:

$$P = \frac{A^2}{2}$$

Za slijed pravokutnih impulsa:

$$P = A^2\frac{\tau}{T}$$

Fourierovi parovi

$$x(t) = \sin(\omega_0 t) \leftrightarrow -j\frac{A}{2}[\delta(f-f_0) - \delta(f+f_0)]$$

$$x(t) = \cos(\omega_0 t) \leftrightarrow \frac{A}{2}[\delta(f-f_0) + \delta(f+f_0)]$$

Neperiodični signali

$X(f) = |X(f)|e^{j\theta(f)}$, gdje je $|X(f)|$ amplitudni spektar, a $\theta(f)$ fazni spektar.

Parsevalov teorem

$$E = \int_{-\infty}^{\infty}|x(t)|^2\,dt = \int_{-\infty}^{\infty}|X(f)|^2\,df = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$$

Pravokutni impuls

$$P = 0 \quad \text{(beskonačnost)}, \quad E = A^2\tau$$

Slučajni signali

Pravila očekivanja (E)

- $E[c] = c, \quad c \in \mathbb{R}$
- $E[cX] = cE[X]$
- $E[X+Y] = E[X] + E[Y]$
- $E[XY] = E[X]E[Y]$

Stacionarnost

Uvjeti:

- $E[X(t)] = \mu_x$
- $\forall t_1, t_2, \quad R_x(t_1, t_2) = R_x(t_1 - t_2) = R_x(\tau)$

  Pri tome: R_x je parna funkcija, $|R_x(\tau)| \leq R_x(0) \geq 0$

Srednja snaga:

$$P = E[X^2(t)] = R_X(0) = \int_{-\infty}^{\infty}S_X(f)\,df$$

Ako $E[X] = 0$:

$$P = \text{var}(X) = \sigma_X^2$$

Bijeli šum

$W(t)$ je bijeli šum ako:

- $R_W(\tau) = C_1\delta(\tau)$
- $C_W(\tau) = C_2\delta(\tau)$

Svojstva:

- $\mu_W = 0$
- $R_W(\tau) = \sigma^2\delta(\tau) = N_0/2$
- $S_W(f) = \sigma^2\int_{-\infty}^{\infty}\delta(t)e^{-j2\pi ft}\,dt = \sigma^2 = N_0/2$

Gaussova razdioba:

$$f_x(x) = \frac{1}{\sigma_X\sqrt{2\pi}}e^{-(x-\mu_X)^2/(2\sigma_X^2)}$$

Prijenos

Izlazni signal:

$$y(t) = \int_{-\infty}^{\infty}x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty}h(\tau)x(t-\tau)\,d\tau$$

Prijenosna funkcija:

$$H(f) = \int_{-\infty}^{\infty}h(t)e^{-j2\pi ft}\,dt$$

Amplitudni odziv RC kruga:

$$20\log\frac{|H(f)|}{|H(0)|} = 20\log|H(f)|$$

Za idealni filtar:

$$|H(f)| = \begin{cases}1, & |f| \leq f_g \\ 0, & |f| > f_g\end{cases}$$

Impulsni odziv i prijenosna funkcija:

$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f)H(f)$$

Amplitudni odziv je parna funkcija, a fazni neparna:

$$|H(-f)| = |H(f)|$$

$$\theta(-f) = -\theta(f)$$

Ako je $X(t)$ stacionarni slučajni proces:

$$\mu_Y = \mu_X H(0)$$

$$S_Y(f) = S_X(f)|H(f)|^2$$

Ako je ulaz $x(t)$ sa spektrom $X(f) = |X(f)|e^{j\varphi(f)}$:

$$Y(f) = |Y(f)|e^{j\vartheta(f)}$$

$$|Y(f)| = |X(f)||H(f)|$$

$$\vartheta(f) = \varphi(f) + \theta(f)$$

Amplitudni odziv RC kruga:

$$|H(f)| = \left|\frac{U_{\text{izlaz}}(f)}{U_{\text{ulaz}}(f)}\right| = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

Uzorkovanje i kvantizacija

Frekvencija uzorkovanja u pomaknutom pojasu:

$$f_u = 2\frac{B+B_0}{M+1}, \quad M_m = \left\lfloor \frac{B_0}{B} + 1 \right\rfloor$$

Varijanca kvantizacijskog šuma (srednja snaga):

$$\mathrm{var}(Q) = \sigma_Q^2 = \frac{\Delta^2}{12} = \frac{1}{3} m_{\max}^2 2^{-2r}, \quad \Delta = \frac{2m_{\max}}{L}$$

Omjer srednje snage signala i snage kvantizacijskog šuma:

$$\frac{S}{N} = \frac{S}{\sigma_Q^2} = \left(\frac{3S}{m_{\max}^2}\right) 2^{2r}$$

U decibelima (samo za sinusni signal):

$$\left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6.02r$$

Brzina prijenosa:

$$R = f_u r \quad \left[\frac{\text{bit}}{s}\right]$$

Entropija u kontinuiranom kanalu

f su funkcije gustoće vjerojatnosti.

$$H(X) = E[-\log f_X(X)] = -\int_{-\infty}^{\infty} f_X(x) \log f_X(x)\, dx$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$$

$$H(X|Y) = E[-\log f_{X|Y}(X|Y)] = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \log\left(\frac{f(x,y)}{f_Y(y)}\right) dx\, dy$$

$$H(X,Y) = E[-\log f(X,Y)] = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \log f(x,y)\, dx\, dy$$

$$I(X;Y) = E[-\log f_{Y|X}(Y|X)] = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \log\left(\frac{f(x,y)}{f_X(x) f_Y(y)}\right) dx\, dy$$

Prijenos u prisutnosti aditivnog šuma:

$$f_{Y|X}(y|x) = f_Z(y-x)$$

$$I(X;Y) = H(Y) - H(Y|X) = H(Y) - H(Z)$$

Kapacitet:

$$C = \max I(X;Y) = \max\left[\frac{1}{2}\ln[2\pi e(\sigma_X^2+\sigma_Z^2)] - \frac{1}{2}\ln(2\pi e\sigma_Z^2)\right]$$

$$= \frac{1}{2}\ln\left(1+\frac{S}{N}\right) \quad \left[\frac{\text{bit}}{s}\right]$$

$$C = \frac{1}{2}\log_2\left(1+\frac{S}{N}\right) \quad [\text{bit/simbol}]$$

Maksimizacija entropije u kontinuiranom kanalu

- $x \in [a,b] \to f(x) = \frac{1}{b-a}, \quad H(X) = \ln(b-a)$ [nat/sym]
- $x \geq 0 \wedge E[X] = a > 0 \to f(x) = \frac{1}{a}e^{-x/a}, \quad H(X) = \ln(ae) = 1 + \ln a$
- $E[X] = 0 \wedge \exists \sigma_X \to f$ Gaussova, $\quad H(X) = \ln(\sigma_X\sqrt{2\pi e})$

Ostalo

Srednja kvadratna pogreška, u_{qi} kvantizacijske razine:

$$N_q^2 = \sum_{u_{qi}} \int_{u_{qi}-\Delta/2}^{u_{qi}+\Delta/2} (u-u_{qi})^2 f(u)\, du \quad [V^2]$$

Konverzije

U decibele (dB):

$$x \to 10\log_{10}(x)$$

Jedinice

$$c_k \leftrightarrow \left[\frac{V}{Hz}\right]$$

$$S_X(f) \leftrightarrow \left[\frac{W}{Hz}\right]$$