

## TINF Signali formule

$$E = I(Ri^2(t)) = I\left(\frac{u^2(t)}{R}\right), \quad P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T_0/2}^{T_0/2} Ri^2(t) dt$$

### Periodični signali

Fourierov razvoj:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \text{ gdje } c_k = \frac{1}{T} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt$$

### Snaga

$$P = \lim_{k \rightarrow \infty} \left[ \frac{1}{kT_0} \int_0^{kT_0} |x(t)|^2 dt \right] \\ = \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2$$

$$\text{Sinusni signal: } P = \frac{A^2}{2}$$

$$\text{Slijed pravokutnih impulsa: } P = A^2 \frac{\tau}{T}$$

### Fourierovi parovi

$$x(t) = \cos(\omega_0 t) \leftrightarrow \frac{A}{2} [\delta(f - f_0) + \delta(f + f_0)]$$

$$x(t) = \sin(\omega_0 t) \leftrightarrow -j \frac{A}{2} [\delta(f - f_0) - \delta(f + f_0)]$$

Modulacijsko pravilo:

$$x(t) \cos(2\pi f_0 t) \leftrightarrow \frac{1}{2} [X(f - f_0) + X(f + f_0)]$$

### Neperiodični signali

#### Energija i snaga

$$\text{Energija } \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt, \quad P = \frac{E}{2T}$$

#### Spektar

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt, \quad x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

$$X(f) = |X(f)| e^{j\theta(f)}$$

, gdje je

$$|X(f)|$$

amplitudni spektar, a

$$\theta(f)$$

fazni spektar.

#### Parsevalov teorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

#### Pravokutni impuls

$$P = 0 \quad (\text{beskonačnost}), \quad E = A^2 \tau$$

### Slučajni signali

Srednja vrijednost

$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x f_X(x, t) dx$$

Autokorelacijska funkcija

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)]$$

Autokovarijanca

$$C_X(t_1, t_2) = E\{[X(t_1) - \mu_x(t_1)][X(t_2) - \mu_x(t_2)]\}$$

### Pravila očekivanja (E)

$$E[c] = c, \quad c \in \mathbb{R}, \quad E[cX] = cE[X]$$

$$E[X + Y] = E[X] + E[Y], \quad E[XY] = E[X]E[Y]$$

### Stacionarnost

Uvjeti:

$$\bullet E[X(t)] = \mu_x$$

$$\bullet \forall t_1, t_2, \quad R_X(t_1, t_2) = R_X(t_1 - t_2) = R_X(\tau)$$

$$\text{Pri tome: } R_X \text{ je parna funkcija, } |R_X(\tau)| \leq R_X(0) \geq 0$$

Srednja snaga:

$$P = E[X^2(t)] = R_X(0) = \int_{-\infty}^{\infty} S_X(f) df$$

$$E[X] = 0 \longrightarrow P = \text{var}(X) = \sigma_X^2$$

### Spektralna gustoća snage

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau} d\tau \quad \left[ \frac{W}{Hz} \right]$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{j2\pi f \tau} df$$

### Bijeli šum

$W(t)$  je bijeli šum ako:

$$R_W(\tau) = C_1 \delta(\tau) \quad \wedge \quad C_W(\tau) = C_2 \delta(\tau)$$

Svojstva:

$$\mu_W = 0, \quad R_W(\tau) = \sigma^2 \delta(\tau) = N_0/2$$

$$S_W(f) = \sigma^2 \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi f t} dt = \sigma^2 = N_0/2$$

Gaussova razdioba:

$$f_x(x) = \frac{1}{\sigma_X \sqrt{2\pi}} e^{-(x-\mu_X)^2/(2\sigma_X^2)}$$

## Prijenos

Izlazni signal:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau) d\tau$$

Prijenosna funkcija:

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft} dt$$

Amplitudni odziv RC kruga:

$$20 \log \frac{|H(f)|}{|H(0)|} = 20 \log |H(f)|$$

Za idealni filter:

$$|H(f)| = \begin{cases} 1, & |f| \leq f_g \\ 0, & |f| > f_g \end{cases}$$

Impulsni odziv i prijenosna funkcija:

$$y(t) = x(t) * h(t), \quad Y(f) = X(f)H(f)$$

Amplitudni odziv je parna funkcija, a fazni neparna:

$$|H(-f)| = |H(f)|, \quad \theta(-f) = -\theta(f)$$

Ako je  $X(t)$  stacionarni slučajni proces:

$$\mu_Y = \mu_X H(0), \quad S_Y(f) = S_X(f)|H(f)|^2$$

Ako je ulaz  $x(t)$  sa spektrom  $X(f) = |X(f)|e^{j\varphi(f)}$ :

$$Y(f) = |Y(f)|e^{j\vartheta(f)}, \quad |Y(f)| = |X(f)||H(f)|$$

$$\vartheta(f) = \varphi(f) + \theta(f)$$

Amplitudni odziv RC kruga:

$$|H(f)| = \left| \frac{U_{\text{izlaz}}(f)}{U_{\text{ulaz}}(f)} \right| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

## Uzorkovanje i kvantizacija

Frekvencija uzorkovanja u pomaknutom pojasu:

$$f_u = 2 \frac{B + B_0}{M + 1}, \quad M_m = \left\lfloor \frac{B_0}{B} + 1 \right\rfloor$$

Varijanca kvantizacijskog šuma (srednja snaga):

$$\text{var}(Q) = \sigma_Q^2 = \frac{\Delta^2}{12} = \frac{1}{3} m_{\max}^2 2^{-2r}, \quad \Delta = \frac{2m_{\max}}{L}$$

Omjer srednje snage signala i snage kvantizacijskog šuma:

$$\frac{S}{N} = \frac{S}{\sigma_Q^2} = \left( \frac{3S}{m_{\max}^2} \right) 2^{2r}$$

U decibelima (samo za sinusni signal):

$$\left( \frac{S}{N_q} \right)_{dB} = 1.76 + 6.02r$$

Brzina prijenosa:

$$R = f_u r \quad \left[ \frac{\text{bit}}{s} \right]$$

## Entropija u kontinuiranom kanalu

$f$  su funkcije gustoće vjerojatnosti.

$$H(X) = E[-\log f_X(X)] = - \int_{-\infty}^{\infty} f_X(x) \log f_X(x) dx$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy, \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$H(X|Y) = E[-\log f_{X|Y}(X|Y)] \\ = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \log \left( \frac{f(x, y)}{f_Y(y)} \right) dx dy$$

$$H(X, Y) = E[-\log f(X, Y)] \\ = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \log f(x, y) dx dy$$

$$I(X; Y) = E[-\log f_{Y|X}(Y|X)] \\ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \log \left( \frac{f(x, y)}{f_X(x)f_Y(y)} \right) dx dy$$

Prijenos u prisutnosti aditivnog šuma:

$$f_x(y|x) = f_x(z+x|x) = \phi(z)$$

$$I(X; Y) = H(Y) - H(Y|X) = H(Y) - H(Z)$$

Kapacitet:

$$C = \max I(X; Y) = \max \left[ \frac{1}{2} \ln[2\pi e(\sigma_X^2 + \sigma_Z^2)] - \frac{1}{2} \ln(2\pi e\sigma_Z^2) \right]$$

$$= \frac{1}{2} \ln \left( 1 + \frac{S}{N} \right) \quad \left[ \frac{\text{nat}}{s} \right]$$

$$C = \frac{1}{2} \log_2 \left( 1 + \frac{S}{N} \right) \quad [\text{bit/simbol}]$$

## Maksimizacija entropije u kontinuiranom kanalu

- $x \in [a, b] \rightarrow f(x) = \frac{1}{b-a}, \quad H(X) = \ln(b-a) \quad [\text{nat/sym}]$
- $x \geq 0 \wedge E[X] = a > 0 \rightarrow f(x) = \frac{1}{a} e^{-x/a}, \quad H(X) = \ln(ae) = 1 + \ln a$
- $E[X] = 0 \wedge \exists \sigma_X \rightarrow f \text{ Gaussova}, \quad H(X) = \ln(\sigma_X \sqrt{2\pi e})$

## Inf. kapacitet AWGN kanala

Za kanal s  $f_u = 2B...$

$$n = 2B \rightarrow B \log_2 \left( 1 + \frac{S}{N} \right) \quad [\text{bit/s}], \quad C = 2BD$$

$E_b$ , srednja energija po svakom bitu...

$$\text{uz... } E_b = S/R_b, \quad S = E_b C, \quad \frac{C}{B} = \log_2 \left( 1 + \frac{E_b}{N_0} \frac{C}{B} \right)$$

$$\frac{E_b}{N_0} = \frac{2^{C/B} - 1}{C/B}, \quad \lim_{B \rightarrow \infty} \left( \frac{E_b}{N_0} \right) = \log(2), \quad \lim_{B \rightarrow \infty} C = \frac{S}{N_0} \log_2 e$$

## Konverzije

Pojačanje. U decibele (dB):  $x \rightarrow 10 \log_{10}(x)$

## Jedinice

$$c_k \leftrightarrow \left[ \frac{V}{Hz} \right], \quad S_X(f) \leftrightarrow \left[ \frac{W}{Hz} \right]$$

## Ostalo

Neka svojstva operatora Fourierove transformacije:

$$\text{Linearnost} \quad \mathcal{F}\{ax + by\} = a\mathcal{F}\{x\} + b\mathcal{F}\{y\}$$

Riemann-Lebesgue lema na realnom / kompleksnom skupu.  
 $x$  je  $L^1$  ako:

$$\int_{\mathbb{R}^n} |x(t)| dt < \infty$$

Za fourierov transformat  $X(f)$  tada vrijedi:

$$|X(f)| \rightarrow 0 \text{ kada } |f| \rightarrow 0$$

Srednja kvadratna pogreška,  $u_{qi}$  kvantizacijske razine:

$$N_q^2 = \sum_{u_{qi}} \int_{u_{qi}-\Delta/2}^{u_{qi}+\Delta/2} (u - u_{qi})^2 f(u) du \quad [V^2]$$

Hamming bound generalno:

$$M \leq \frac{|F_q|^n}{\sum_{i=0}^t \binom{n}{i} (|F_q| - 1)^i}$$

Sistematsko kodiranje / dekodiranje:

$$\text{Kodiranje } c(x) = d(x) * x^r + [d(x) * x^r \mod g(x)]$$

$$\text{Prijenos } y(x) = c(x) + e(x)$$

$$\text{Dekodiranje } S[e(x)] = y(x) * x^r \pmod{g(x)}$$

$$\rightarrow y'(x) = y(x) - e(x)$$

$$\rightarrow d(x) = \frac{y'(x)}{x^r}$$

Nesistematsko:

$$\text{Kodiranje } c(x) = d(x) * g(x)$$

$$\text{Prijenos } y(x) = c(x) + e(x)$$

$$\text{Dekodiranje } \hat{e}(x) \rightarrow \frac{y(x)}{g(x)}, \text{ otherwise } \frac{y(x) - e(x)}{g(x)}$$

## Entropija slučajnog vektora

$$H(\mathbf{X}) = E[-\log\{X_1, \dots, X_n\}]$$

$$= - \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} f_{\mathbf{X}}(x_1, \dots, x_n) \log[f_{\mathbf{X}}(x_1, \dots, x_n)] dx_1 \dots dx_n$$

## Inf. kapacitet AWGN kanala

Pri uzorkovanju:

$$\mathbf{X} = [X_1, X_2, \dots, X_n]$$

$$\mathbf{Y} = \mathbf{X} + \mathbf{Z}$$

$$E[X_k] = 0, \quad E[X_k^2] = \sigma_{xk}^2$$

$$\phi(\mathbf{z}) = \prod_{k=1}^n \left[ \frac{1}{\sigma_{zk} \sqrt{2\pi}} e^{-z_k^2 / 2\sigma_{zk}^2} \right]$$

$$H(\mathbf{Y}|\mathbf{X}) = H(\mathbf{Z}) = - \int_{-\infty}^{\infty} \phi(\mathbf{z}) \log[\phi(\mathbf{z})] = \sum_{k=1}^n \log(\sigma_{zk} \sqrt{2\pi e})$$

$$I(\mathbf{X}; \mathbf{Y}) = H(\mathbf{Y}) - \sum_{k=1}^n \log(\sigma_{zk} \sqrt{2\pi e})$$

Ako su sve varijance jednake...

$$\begin{aligned} I_{\max}(\mathbf{X}; \mathbf{Y}) &= \frac{n}{2} \log \left( 1 + \frac{\sigma_x^2}{\sigma_z^2} \right) \quad [bit/simbol] \\ &= \frac{n}{2} \log \left( 1 + \frac{S}{N} \right) \end{aligned}$$

## Vjerojatnost

$$\text{Znano } B \text{ tražimo } A : P(A|B) = \frac{P(A \cap B)}{P(B)}$$

## Slike

