

TINF Signali formule

Slučajni signali

Srednja vrijednost

$$\mu_x(t) = E[X(t)] = \int_{-\infty}^{\infty} x f_X(x, t) dx$$

Periodični signali

Fourierov razvoj:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \text{ gdje } c_k = \frac{1}{T} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt$$

Snaga

$$\begin{aligned} P &= \lim_{k \rightarrow \infty} \left[\frac{1}{kT_0} \int_0^{kT_0} |x(t)|^2 dt \right] \\ &= \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2 \end{aligned}$$

$$\text{Sinusni signal: } P = \frac{A^2}{2}$$

$$\text{Slijed pravokutnih impulsa: } P = A^2 \frac{T}{T}$$

Fourierovi parovi

$$x(t) = \cos(\omega_0 t) \leftrightarrow \frac{A}{2} [\delta(f - f_0) + \delta(f + f_0)]$$

$$x(t) = \sin(\omega_0 t) \leftrightarrow -j \frac{A}{2} [\delta(f - f_0) - \delta(f + f_0)]$$

Modulacijsko pravilo:

$$x(t) \cos(2\pi f_0 t) \leftrightarrow \frac{1}{2} [X(f - f_0) + X(f + f_0)]$$

Neperiodični signali

Energija i snaga

$$\text{Energija } \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt, \quad P = \frac{E}{2T}$$

Spektar

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt, \quad x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

$$X(f) = |X(f)| e^{j\theta(f)}$$

, gdje je

$$|X(f)|$$

amplitudni spektar, a

$$\theta(f)$$

fazni spektar.

Parsevalov teorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

Pravokutni impuls

$$P = 0 \quad (\text{beskonačnost}), \quad E = A^2 \tau$$

Autokorelacijska funkcija

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)]$$

Autokovarijanca

$$C_X(t_1, t_2) = E \{ [X(t_1) - \mu_x(t_1)] [X(t_2) - \mu_x(t_2)] \}$$

Pravila očekivanja (E)

$$E[c] = c, \quad c \in \mathbb{R}, \quad E[cX] = cE[X]$$

$$E[X + Y] = E[X] + E[Y], \quad E[XY] = E[X]E[Y]$$

Stacionarnost

Uvjeti:

- $E[X(t)] = \mu_x$

- $\forall t_1, t_2, \quad R_x(t_1, t_2) = R_x(t_1 - t_2) = R_x(\tau)$

Pri tome: R_x je parna funkcija, $|R_x(\tau)| \leq R_x(0) \geq 0$

Srednja snaga:

$$P = E[X^2(t)] = R_x(0) = \int_{-\infty}^{\infty} S_X(f) df$$

$$E[X] = 0 \longrightarrow P = \text{var}(X) = \sigma_X^2$$

Spektralna gustoća snage

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau} d\tau \quad \left[\frac{W}{Hz} \right]$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{j2\pi f \tau} df$$

Bijeli šum

$W(t)$ je bijeli šum ako:

$$R_W(\tau) = C_1 \delta(\tau) \quad \wedge \quad C_W(\tau) = C_2 \delta(\tau)$$

Svojstva:

$$\mu_W = 0, \quad R_W(\tau) = \sigma^2 \delta(\tau) = N_0/2$$

$$S_W(f) = \sigma^2 \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt = \sigma^2 = N_0/2$$

Gaussova razdioba:

$$f_x(x) = \frac{1}{\sigma_X \sqrt{2\pi}} e^{-(x - \mu_X)^2 / (2\sigma_X^2)}$$

Prijenos

Izlazni signal:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau) d\tau$$

Prijenosna funkcija:

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft} dt$$

Amplitudni odziv RC kruga:

$$20 \log \frac{|H(f)|}{|H(0)|} = 20 \log |H(f)|$$

Za idealni filter:

$$|H(f)| = \begin{cases} 1, & |f| \leq f_g \\ 0, & |f| > f_g \end{cases}$$

Impulsni odziv i prijenosna funkcija:

$$y(t) = x(t) * h(t), \quad Y(f) = X(f)H(f)$$

Amplitudni odziv je parna funkcija, a fazni neparna:

$$|H(-f)| = |H(f)|, \quad \theta(-f) = -\theta(f)$$

Ako je $X(t)$ stacionarni slučajni proces:

$$\mu_Y = \mu_X H(0), \quad S_Y(f) = S_X(f)|H(f)|^2$$

Ako je ulaz $x(t)$ sa spektrom $X(f) = |X(f)|e^{j\varphi(f)}$:

$$Y(f) = |Y(f)|e^{j\vartheta(f)}, \quad |Y(f)| = |X(f)||H(f)|$$

$$\vartheta(f) = \varphi(f) + \theta(f)$$

Amplitudni odziv RC kruga:

$$|H(f)| = \left| \frac{U_{\text{izlaz}}(f)}{U_{\text{ulaz}}(f)} \right| = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}$$

Uzorkovanje i kvantizacija

Frekvencija uzorkovanja u pomaknutom pojasu:

$$f_u = 2 \frac{B + B_0}{M + 1}, \quad M_m = \left\lfloor \frac{B_0}{B} + 1 \right\rfloor$$

Varianca kvantizacijskog šuma (srednja snaga):

$$\text{var}(Q) = \sigma_Q^2 = \frac{\Delta^2}{12} = \frac{1}{3} m_{\max}^2 2^{-2r}, \quad \Delta = \frac{2m_{\max}}{L}$$

Omjer srednje snage signala i snage kvantizacijskog šuma:

$$\frac{S}{N} = \frac{S}{\sigma_Q^2} = \left(\frac{3S}{m_{\max}^2} \right) 2^{2r}$$

U decibelima (samo za sinusni signal):

$$\left(\frac{S}{N_q} \right)_{dB} = 1.76 + 6.02r$$

Brzina prijenosa:

$$R = f_u r \quad \left[\frac{\text{bit}}{\text{s}} \right]$$

Entropija u kontinuiranom kanalu

f su funkcije gustoće vjerojatnosti.

$$H(X) = E[-\log f_X(X)] = - \int_{-\infty}^{\infty} f_X(x) \log f_X(x) dx$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy, \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$H(X|Y) = E[-\log f_{X|Y}(X|Y)]$$

$$= - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \log \left(\frac{f(x, y)}{f_X(x)f_Y(y)} \right) dx dy$$

$$H(X, Y) = E[-\log f(X, Y)]$$

$$= - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \log f(x, y) dx dy$$

$$I(X; Y) = E[-\log f_{Y|X}(Y|X)]$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \log \left(\frac{f(x, y)}{f_X(x)f_Y(y)} \right) dx dy$$

Prijenos u prisutnosti aditivnog šuma:

$$f_x(y|x) = f_x(z + x|x) = \phi(z)$$

$$I(X; Y) = H(Y) - H(Y|X) = H(Y) - H(Z)$$

Kapacitet:

$$\begin{aligned} C &= \max I(X; Y) = \max \left[\frac{1}{2} \ln[2\pi e(\sigma_X^2 + \sigma_Z^2)] - \frac{1}{2} \ln(2\pi e \sigma_Z^2) \right] \\ &= \frac{1}{2} \ln \left(1 + \frac{S}{N} \right) \quad \left[\frac{\text{nat}}{s} \right] \\ C &= \frac{1}{2} \log_2 \left(1 + \frac{S}{N} \right) \quad [\text{bit/simbol}] \end{aligned}$$

Maksimizacija entropije u kontinuiranom kanalu

- $x \in [a, b] \rightarrow f(x) = \frac{1}{b-a}$, $H(X) = \ln(b-a)$ [nat/sym]
- $x \geq 0 \wedge E[X] = a > 0 \rightarrow f(x) = \frac{1}{a} e^{-x/a}$, $H(X) = \ln(ae) = 1 + \ln a$
- $E[X] = 0 \wedge \exists \sigma_X \rightarrow f$ Gaussova, $H(X) = \ln(\sigma_X \sqrt{2\pi e})$

Inf. kapacitet AWGN kanala

Za kanal s $f_u = 2B$...

$$n = 2B \longrightarrow B \log_2 \left(1 + \frac{S}{N} \right) \quad [\text{bit/s}], \quad C = 2BD$$

E_b , srednja energija po svakom bitu...

$$\text{uz... } E_b = S/R_b, \quad S = E_b C, \quad \frac{C}{B} = \log_2 \left(1 + \frac{E_b}{N_0} \frac{C}{B} \right)$$

$$\frac{E_b}{N_0} = \frac{2^{C/B} - 1}{C/B}, \quad \lim_{B \rightarrow \infty} \left(\frac{E_b}{N_0} \right) = \log(2), \quad \lim_{B \rightarrow \infty} C = \frac{S}{N_0} \log_2 e$$

Konverzije

Pojačanje. U decibele (dB): $x \rightarrow 10 \log_{10}(x)$

Jedinice

$$c_k \leftrightarrow \left[\frac{V}{Hz} \right], \quad S_X(f) \leftrightarrow \left[\frac{W}{Hz} \right]$$

Ostalo

Neka svojstva operatora Fourierove transformacije:

$$\text{Linearnost} \quad \mathcal{F}\{ax + by\} = a\mathcal{F}\{x\} + b\mathcal{F}\{y\}$$

Riemann-Lebesgue lema na realnom / kompleksnom skupu.
 x je L^1 ako:

$$\int_{\mathbb{R}^n} |x(t)| dt < \infty$$

Za fourierov transformat $X(f)$ tada vrijedi:

$$|X(f)| \rightarrow 0 \text{ kada } |f| \rightarrow 0$$

Srednja kvadratna pogreška, u_{qi} kvantizacijske razine:

$$N_q^2 = \sum_{u_{qi}} \int_{u_{qi}-\Delta/2}^{u_{qi}+\Delta/2} (u - u_{qi})^2 f(u) du \quad [V^2]$$

Hamming bound generalno:

$$M \leq \frac{|F_q|^n}{\int_{i=0}^t \binom{n}{i} (|F_q| - 1)^i}$$

Sistematsko kodiranje / dekodiranje:

$$\text{Kodiranje } c(x) = d(x) * x^r + [d(x) * x^r \pmod{g(x)}]$$

$$\text{Prijenos } y(x) = c(x) + e(x)$$

$$\text{Dekodiranje } S[e(x)] = y(x) * x^r \pmod{g(x)}$$

$$\rightarrow y'(x) = y(x) - e(x)$$

$$\rightarrow d(x) = \frac{y'(x)}{x^r}$$

Nesistematsko:

$$\text{Kodiranje } c(x) = d(x) * g(x)$$

$$\text{Prijenos } y(x) = c(x) + e(x)$$

$$\text{Dekodiranje } \#e(x) \rightarrow \frac{y(x)}{g(x)}, \text{ otherwise } \frac{y(x) - e(x)}{g(x)}$$

Entropija slučajnog vektora

$$\begin{aligned} H(\mathbf{X}) &= E[-\log \{X_1, \dots, X_n\}] \\ &= - \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} f_{\mathbf{X}}(x_1, \dots, x_n) \log [f_{\mathbf{X}}(x_1, \dots, x_n)] dx_1 \dots dx_n \end{aligned}$$

Inf. kapacitet AWGN kanala

Pri uzorkovanju:

$$\mathbf{X} = [X_1, X_2, \dots, X_n]$$

$$\mathbf{Y} = \mathbf{X} + \mathbf{Z}$$

$$E[X_k] = 0, \quad E[X_k^2] = \sigma_{xk^2}$$

$$\phi(\mathbf{z}) = \prod_{k=1}^n \left[\frac{1}{\sigma_{z_k} \sqrt{2\pi}} e^{-z_k^2/2\sigma_{z_k}^2} \right]$$

$$H(\mathbf{Y}|\mathbf{X}) = H(\mathbf{Z}) = - \int_{-\infty}^{\infty} \phi(\mathbf{z}) \log [\phi(\mathbf{z})] = \sum_{k=1}^n \log(\sigma_{z_k} \sqrt{2\pi e})$$

$$I(\mathbf{X}; \mathbf{Y}) = H(\mathbf{Y}) - \sum_{k=1}^n \log(\sigma_{z_k} \sqrt{2\pi e})$$

Ako su sve varijance jednake...

$$\begin{aligned} I_{\max}(\mathbf{X}; \mathbf{Y}) &= \frac{n}{2} \log \left(1 + \frac{\sigma_x^2}{\sigma_z^2} \right) \quad [\text{bit/simbol}] \\ &= \frac{n}{2} \log \left(1 + \frac{S}{N} \right) \end{aligned}$$

Vjerojatnost

$$\text{Znano } B \text{ tražimo } A : P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Slike

