isis Pr 4

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Permasalahan 1

∴ Diketahui isyarat kontinu x(t) yang diberikan oleh persamaan berikut ini:

$$x(t) = \frac{6\sin(Bt)}{\pi t} + \frac{2\sin(Bt)}{\pi t} e^{j2Bt} + \frac{3\sin(Bt)}{\pi t} e^{-j2Bt}$$

∴ Isyarat tersebut akan dilewatkan melalui suatu frekuensi shaping filter yang memiliki tanggapan impuls:

$$h(t) = \frac{\sin(Bt)}{\pi t} e^{jBt} + \frac{\sin(Bt)}{4\pi t} e^{-jBt} + \frac{\sin(Bt/2)}{2\pi t} e^{\frac{1}{2}jBt}$$

a) Tentukan spektrum dari x(t) dan sediakan plot spektrum tersebut.

$$x(t) \rightarrow X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Dengan melihat tabel transformasi fourier:

$$x(t) = \frac{\sin(Bt)}{\pi t} \xrightarrow{F} X(j\omega) = \int_{-\infty}^{\infty} \frac{\sin(Bt)}{\pi t} e^{-j\omega t} dt = \begin{cases} 1, & |\omega| < B \\ 0, & |\omega| > B \end{cases}$$

Sehingga:

$$x(t) = \frac{\sin(Bt)}{\pi t} e^{j\omega_0 t} \xrightarrow{F} X(j\omega) = \int_{-\infty}^{\infty} \frac{\sin(Bt)}{\pi t} e^{-j(\omega-\omega_0)t} dt$$

Lalu, gunakan persamaan tersebut;

$$\therefore x_1(t) = \frac{6\sin(Bt)}{\pi t} \xrightarrow{\mathcal{F}} x(j\omega) = \int_{-\infty}^{\infty} \frac{6\sin(Bt)}{\pi t} e^{-j\omega t}\,dt = \begin{cases} 6, & |\omega| < B \\ 0, & |\omega| > B \end{cases}$$

$$\boxed{\begin{cases} 6, & -B < \omega < B \\ 0, & \text{elsewhere.} \end{cases}}$$

$$\therefore x_2(t) = \frac{2\sin(Bt)}{\pi t} e^{j2Bt} \xrightarrow{\mathcal{F}} x(j\omega) = \int_{-\infty}^{\infty} \frac{2\sin(Bt)}{\pi t} e^{-j(\omega-2Bt)}\,dt$$

$$x_2(t) = \begin{cases} 2, & |\omega - 2B| < B \\ 0, & |\omega - 2B| > B \end{cases} \Rightarrow \boxed{\begin{cases} 2, & B < \omega < 3B \\ 0, & 3B < \omega < B \sim \text{elsewhere} \end{cases}}$$

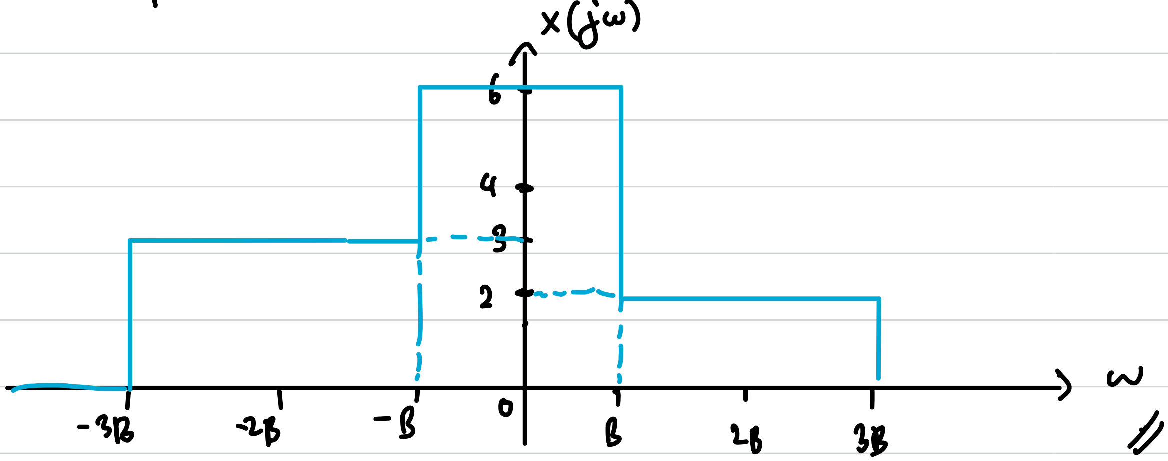
$$\therefore X_3(t) = \frac{3\sin(Bt)}{\pi t} e^{-j2Bt} \xrightarrow{\mathcal{F}} x(j\omega) = \int_{-\infty}^{\infty} \frac{3\sin(Bt)}{\pi t} e^{-j(\omega+2B)t}\,dt$$

$$X_3(t) = \begin{cases} 3, & |\omega + 2B| < B \\ 0, & |\omega + 2B| > B \end{cases} = \boxed{\begin{cases} 3, & -3B < \omega < -B \\ 0, & \text{elsewhere} \end{cases}}$$

Hasil diatas kita bisa tentukan hasil akhir dari transformasi $x(t)$ berupa:

$$X(j\omega) = \begin{cases} 3, & -3B < \omega < -B \\ 6, & -B < \omega < B \\ 2, & B < \omega < 3B \\ 0, & \text{elsewhere} \end{cases}$$

Plot dari spektrum x(t) atau X(jw)



b) Tentukan tanggapan frekuensi dari frequency shaping filter di atas serta sediakan plot bagi tanggapan frekuensi tersebut :

$\therefore h_1(t) = \frac{\sin(Bt)}{\pi t} e^{jBt} \xrightarrow{F} h_1(j\omega) = \int_{-\infty}^{\infty} \frac{\sin(Bt)}{\pi t} e^{-j(\omega - B)t} dt$

$= \begin{cases} 1, & |\omega - B| < B \\ 0, & |\omega - B| > B \end{cases} = \boxed{\begin{cases} 1, & 0 < \omega < 2B \\ 0, & \text{elsewhere} \end{cases}}$

$\therefore h_2(t) = \frac{\sin(Bt)}{4\pi t} e^{-jBt} \xrightarrow{F} h_2(j\omega) = \int_{-\infty}^{\infty} \frac{\sin(Bt)}{4\pi t} e^{-j(\omega + B)} dt$

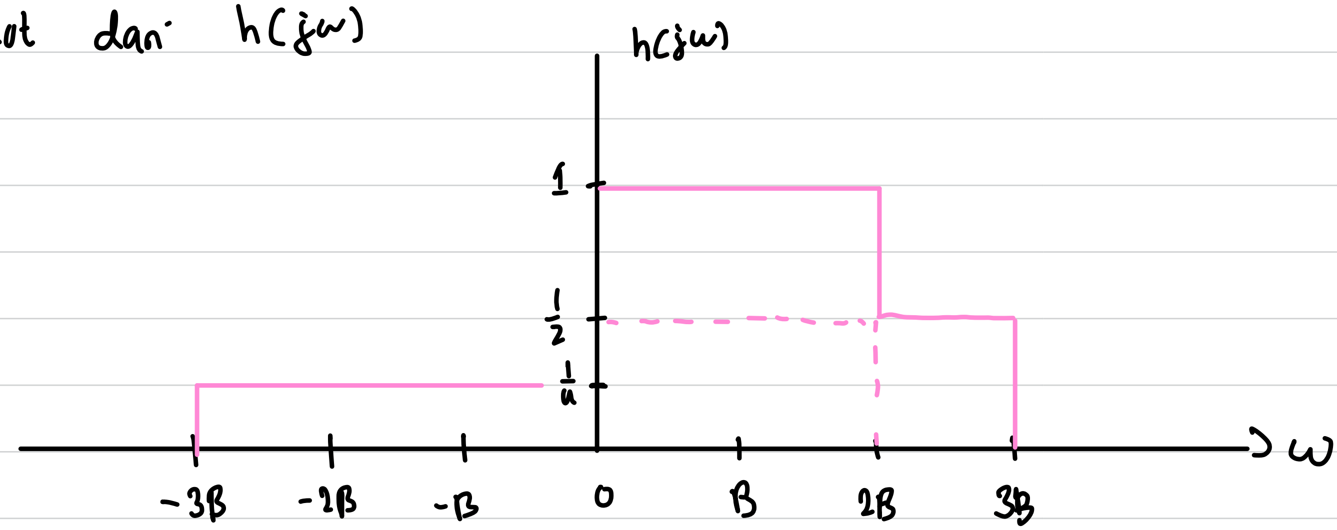
$= \boxed{\begin{cases} 1/4, & |\omega + B| < B \\ 0, & \text{else where} \end{cases}}$

$\therefore h_3(t) = \frac{\sin(Bt/2)}{2\pi t} e^{1/2\, jBt} \xrightarrow{F} h_3(j\omega) = \int_{-\infty}^{\infty} \frac{\sin(Bt/2)}{2\pi t} e^{-j(\omega - \frac{B}{2})t} dt$

$= \begin{cases} 1/2, & |\omega - B/2| < B/2 \\ 0, & \text{else where} \end{cases} = \begin{cases} 1/2, & 2B < \omega < 3B \\ 0, & \text{elsewhere} \end{cases}$

Spektrum $h(j\omega) = \begin{cases} 1/4, & -2B < \omega < 0 \\ 1, & 0 < \omega < 2B \\ 1/2, & 2B < \omega < 3B \\ 0, & \text{elsewhere.} \end{cases}$

plot dari $h(j\omega)$



c.) Jika keluaran dari frekuensi shaping filter di atas adalah suatu isyarat $y(t)$ maka tentukanlah $y(t)$ maupun representasi $y(t)$ di kawasan frekuensi atau spektrum dari $y(t)$. Sediakan plot bagi spektrum $y(t)$. Bila perlu gunakan Tabel Transformasi Fourier.

∴ Cari $y(t) = x(t) * h(t)$

$y(j\omega) = X(j\omega) \cdot h(j\omega)$

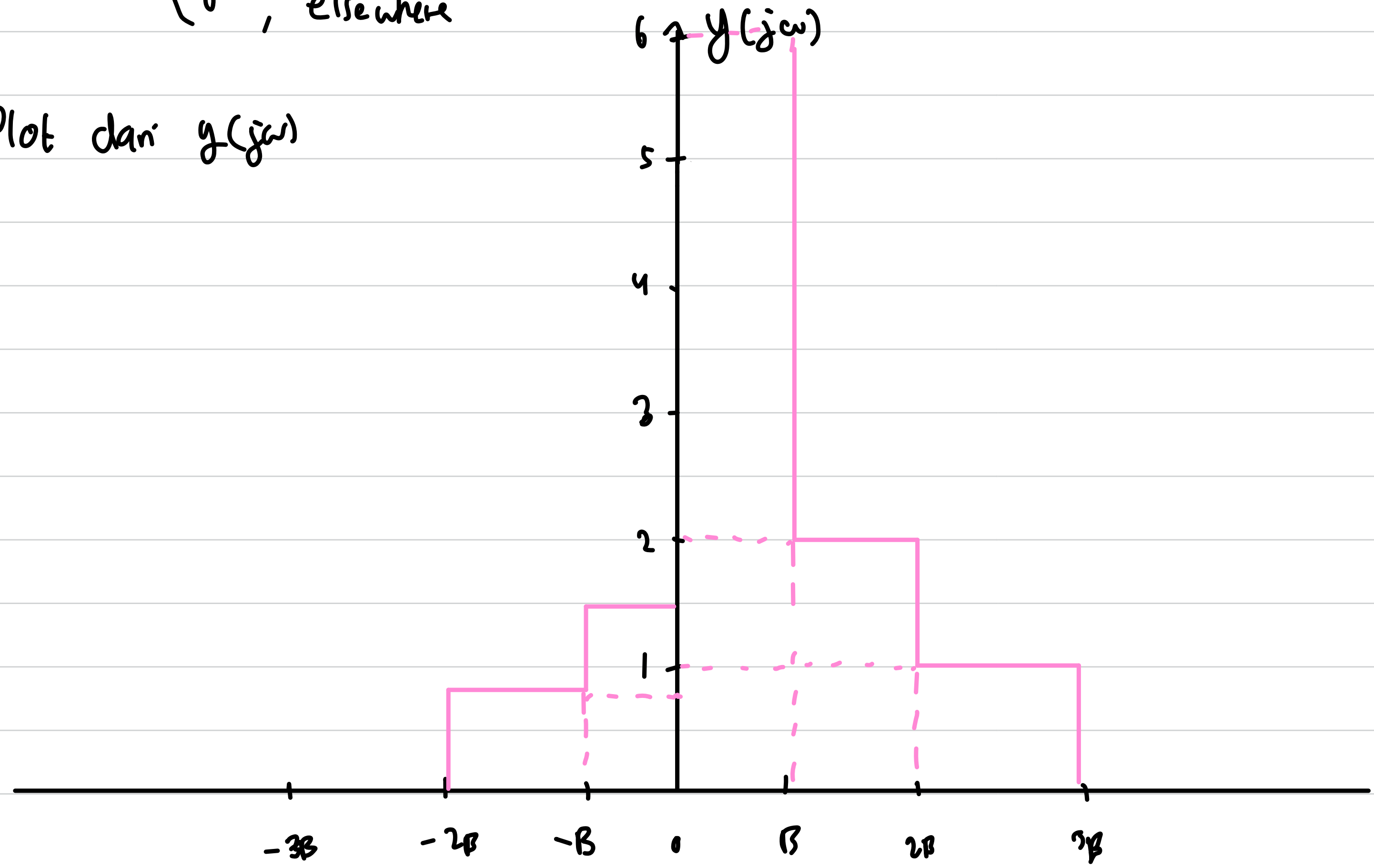
Atau dapat direpresentasikan sebagai berikut.

$$y(j\omega) = \left[\begin{cases} 3, & -3B < \omega < -B \\ 6, & -B < \omega < B \\ 2, & B < \omega < 3B \\ 0, & \text{elsewhere} \end{cases} \right] \cdot \left[\begin{cases} 1/4, & -2B < \omega < 0 \\ 1, & 0 < \omega < 2B \\ 1/2, & 2B < \omega < 3B \\ 0, & \text{elsewhere.} \end{cases} \right]$$

$$y(j\omega) = \left[\begin{cases} 3, & -3B < \omega < -2B \\ 3, & -2B < \omega < -B \\ 6, & -B < \omega < 0 \\ 6, & 0 < \omega < B \\ 2, & B < \omega < 2B \\ 2, & 2B < \omega < 3B \\ 0, & \text{elsewhere} \end{cases}\right] \cdot \left[\begin{cases} 0, & -3B < \omega < -2B \\ 1/4, & -2B < \omega < -B \\ 1/4, & -B < \omega < 0 \\ 1, & 0 < \omega < B \\ 1, & B < \omega < 2B \\ 1/2, & 2B < \omega < 3B \\ 0, & \text{elsewhere} \end{cases}\right]$$

$$y(j\omega) = \begin{cases} 0, & -3B < \omega < -2B \\ 3/4, & -2B < \omega < -B \\ 3/2, & -B < \omega < 0 \\ 6, & 0 < \omega < B \\ 2, & B < \omega < 2B \\ 1, & 2B < \omega < 3B \\ 0, & \text{elsewhere} \end{cases}$$

Plot dari $y(j\omega)$



Permasalahan 2

∴ Diketahui suatu sistem LTI dengan relasi antara isyarat masukan $x(t)$ dan isyarat keluaran $y(t)$ diberikan oleh persamaan differensial berikut ini :

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} - 24\frac{dy(t)}{dt} + 28\,y(t) = 12x(t) + 2\frac{dx(t)}{dt} + 5\frac{d^2x(t)}{dt^2}$$

∴ Tentukan tanggapan frekuensi dari sistem LTI diatas.

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} - 24\frac{dy(t)}{dt} + 28y(t) = 12x(t) + 2\frac{dx(t)}{dt} + 5\frac{d^2x(t)}{dt^2}$$

$$(j\omega)^3 y(j\omega) + 3(j\omega)^2 y(j\omega) - 24(j\omega)y(j\omega) + 28y(j\omega) = 12x(j\omega) + 2(j\omega)x(j\omega) + 5(j\omega)^2 x(j\omega)$$

$$y(j\omega)\left[(j\omega)^3 + 3(j\omega)^2 - 24(j\omega) + 28\right] = x(j\omega)\left[12 + 2(j\omega) + 5(j\omega)^2\right]$$

$$h(j\omega) = \frac{y(j\omega)}{x(j\omega)} = \frac{\left[12 + 2(j\omega) + 5(j\omega)^2\right]}{\left[(j\omega)^3 + 3(j\omega)^2 - 24(j\omega) + 28\right]} \quad //$$