Matdis B - Assignment 5

Hal 362.

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3. $P(n)$: postage $n \geq 8$ cent
can be formed using just 3 cent & 5 cent
Use strong induction to proof that $P(n)$ is true for all integers $n \geq 8$

a) $P(8)$, $P(9)$, $P(10)$ is proved to be true and completing the basis step of a proof by strong induction that $P(n)$ is true for all integers $n \geq 8$.

Basis Step :

$P(8)$: $8 = 3 + 5$ cent ✓
$P(9)$: $9 = 3 \cdot 3$ cent ✓
$P(10)$: $10 = 2 \cdot 5$ cent ✓

b) Strong Inductive Hypothesis

$P(j)$ is true $\wedge$ $8 \leq j \leq k$
$\Longrightarrow P(k+1)$

c) We know that $P(8)$, $P(9)$, $P(10)$ is true. We also know that $8 \leq k$, so it can be rewritten as if $k \geq 11$;
$P(k-3)$ is also $\equiv P(8) \longrightarrow P(k+1)$

$P(k+1)$: $P(k-3) + 3 \cdot 3$ cent $- 5$ cent $=$

From the result, we can ensure that $P(k+1)$ can be done and thus we can say that

$P(n)$ is true for integers $n \geq 8$ ▨

d) If $k \geq 10$, then the statement of $P(8)$ can be rewritten as $P(k-2)$ is true

$P(k-2) \longrightarrow P(k+1)$ is true

$P(k+1)$: $P(k-2) + 3$ cent ✓

▨

Because of $k \geq 10$; so $k+1 \geq 11$ cent is consider to be true.
Thus, $P(n)$ is true for $n \geq 8$ integer

e) These steps is called strong induction, shows that $P(n)$ is

e) These steps is called strong induction, shows that $P(n)$ is is true when $n \geq 8$ because if $P(8)$ is able to formed by 3 & 5 cents, then if $k \geq 10$; then $(K-2)^{th}$ postage is equivalen to 8 cent postage. The k cent postage cant be earned by adding 3 cent stamp. The result would be 10 cent equivalen of stamp. If k is accessable, than $(k+1)$ is also accessable. Thus, the $P(n)$ postage can be formed from 3 & 5 cent for $n \geq 8$ ▨

$\boxed{4}$ $P(n)$ be the n cents postage can be formed by 4 & 7 cents stamps if $n \geq 18$. We'll use strong induction to proof that $P(n)$ is true for $n \geq 8$

a) Show the statement $P(18)$, $P(18)$, $P(20)$ and $P(21)$ is true

Basis Step:

$P(18)$: $2 \cdot 7 + 4$ cent ✓

$P(19)$: $3 \cdot 4 + 7$ cent ✓

$P(20)$: $5 \cdot 4$ cent ✓

$P(21)$: $3 \cdot 7$ cent ✓

b) The inductive hypothesis of a proof by strong induction that $P(n)$ is true for all integers $n \geq 18$ is

$$P(j) \text{ is true} \wedge 18 \leq j \leq k$$
$$\Rightarrow P(k+1)$$

c) We know that $P(18), P(19), P(20), \& P(21)$ is true, We also know that $18 \leq k$, so it can be rewritten as if $k \geq 22$;

$P(k-4)$ is also $\equiv P(18) \longrightarrow P(k+1)$

$P(k+1)$: $P(k-4) + 3 \cdot 4$ cent $- 7$ cent

: $18 + 5$ cent $= 23$ cent

From the result, we can ensure that $P(k+1)$ can be done and thus we can say that

$P(n)$ is true for integers $n \geq 18$ ▨

d) If $k \geq 21$, then the statement of $P(18)$ can be rewritten

d) If $k \geq 21$, then the statement of $P(18)$ can be rewritten as $P(k-3)$ is true

$P(k-3) \longrightarrow P(k+1)$

$P(k+1) : P(k-3) + 4$ ✓

Because of $k \geq 21$; so $k+1 \geq 22$ cent is consider to be true.
Thus, $P(n)$ is true for $n \geq 18$ integer

e) These steps is called strong induction, shows that $P(n)$ is is true when $n \geq 18$ because if $P(18)$ is able to formed by 4 & 7 cents, then if $k \geq 21$; then $(K-3)^{th}$ postage is equivalen to 18 cent postage. The k cent postage cant be earned by adding 4 cent stamp. The result would be 10 cent equivalen of stamp. If k is accessable, than $(k+1)$ is also accessable.
Thus, the $P(n)$ postage can be formed from 3 & 5 cent for $n \geq 8$

Hal 378 no 2.6

$\boxed{2}$ Find $f(2)$, $f(3)$, $f(4)$, & $f(5)$.
$f(0) = 3$

a) $f(n+1) = -2f(n)$

* $f(1) = -2f(0) = -2(3) = -6$
* $f(2) = -2f(1) = -2(-6) = 12$
* $f(3) = -2f(2) = -2(12) = -24$
* $f(4) = -2f(3) = -2(-24) = 48$
* $f(5) = -2f(4) = -2(48) = -96$

b) $f(n+1) = 3f(n) + 7$

* $f(1) = 3f(0) + 7 = 3 \cdot 3 + 7 = 9+7 = 16$
* $f(2) = 3f(1) + 7 = 3(16) + 7 = 48+7 = 55$
* $f(3) = 3f(2) + 7 = 3(55)+7 = 165+7 = 172$
* $f(4) = 3f(3) + 7 = 3(172)+7 = 516+7 = 523$
* $f(5) = 3f(4) + 7 = 3(523)+7 = 1569+7 = 1576$

c) $f(n+1) = f(n)^2 - 2f(n) - 2$

* $f(1) = f(0)^2 - 2f(0) - 2 = 9 - 6 - 2 = 1$

c) $f(n+1) = f(n)^2 - 2f(n) - 2$

※ $f(1) = f(0)^2 - 2f(0) - 2 = 9 - 6 - 2 = 1$

※ $f(2) = f(1)^2 - 2f(1) - 2 = 1 - 2 - 2 = -3$

※ $f(3) = f(2)^2 - 2f(2) - 2 = 9 + 6 - 2 = 14$

※ $f(4) = f(3)^2 - 2f(3) - 2 = 196 - 2(14) - 2 = 196 - 28 - 2 = 166$

※ $f(5) = f(4)^2 - 2f(4) - 2 = (166)^2 - 2(166) - 2 = 27556 - 332 - 2 = 27222$

d) $f(n+1) = 3^{f(n)/3}$

※ $f(1) = 3^{f(0)/3} = 3^{3/3} = 3$

※ $f(2) = 3^{f(1)/3} = 3^{3/3} = 3$

※ $f(3) = 3^{f(2)/3} = 3^{3/3} = 3$

※ $f(4) = 3^{f(4)/3} = 3^{3/3} = 3$

※ $f(5) = 3^{f(4)/3} = 3^{3/3} = 3$

$\boxed{6}$ a) $f(0) = 1$

$f(n) = -f(n-1)$; for $n \geq 1$

※ Basis step:

$f(1) = -f(1-1) = -1$ ✓

$f(2) = -f(2-1) = -f(1) = 1$ ✓) $\times(-1)$

$f(3) = -f(3-1) = -f(2) = -1$ ✓) $\times(-1)$

※ Recursion formula

$f(n+1) = -f(n)$ for $n \geq 0$

$f(1) = -f(0) = -1$ ✓

$f(2) = -f(1) = -(-1) = 1$ ✓

$f(3) = -f(2) = -1$ ✓

※ Inductive Steps:

For $0 \geq j \geq k$ is true ; proof $(k+1)$ also true

$k \geq 3$

Proof : $f(k+1) = -f(k) = -f(3) = -(-1) = 1$ ✓

Thus, $(k+1)$ is true for all nonnegative integer. ▨

b) $f(0) = 1$; $f(1) = 0$; $f(2) = 2$; $f(n) = 2f(n-3)$; for $n \geq 3$

※ Basis Steps :

- $f(3) = 2f(0) = 2$
- $f(4) = 2f(1) = 0$
- $f(5) = 2f(2) = 4$
- $f(6) = 2f(3) = 4$
- $f(7) = 2f(4) = 0$
- $f(8) = 2f(5) = 8$
- $f(9) = 2f(6) = 8$
- $f(10) = 2f(7) = 0$
- $f(11) = 2f(8) = 16$
- $f(12) = 2f(9) = 16$

※ Recursive Steps ;

For $i > 1$; n is nonnegative integer; $U_0 = 0$; $U_1 = 1$

$U_i = U_0 + 3i$;

then, $\left.\begin{array}{l} f(n) = f(U_i) \\ f(n-1) = f(U_i - 1) \end{array}\right\} = 2^i$

$f(n+1) = f(U_{i+1}) = 0$

※ Inductive Steps

When i is true for $1 < i \leq k$; then proves that $k+1$ is also true if $k \geq 5$; U_i is multiple of 3 from n integer.

Strong induction :

- $U_{k+1} = U_0 + (k+1)3 = 3k+3$

then ; $\left.\begin{array}{l} f(3k+3) \\ f(3k+2) \end{array}\right\} = 2^{k+1}$

$f(\quad = 0$

if $k = 5$; then

$f(18) = 2^6$ ✓

$f(17) = 2^6$ ✓

$f(19) = 0$ ✓ ∎

$k+1$ is true for all nonnegative integer ;

c) $f(0) = 0$; $f(1) = 1$; $f(n) = 2f(n+1)$ for $n \geq 2$

※ Basis Steps:

- $f(2) = 2f(2+1) = 2f(3)$ ✗
- $f(3) = 2f(3+1) = 2f(4)$ ✗

For this recursion function, it is not true for all non negatif integer. It is not Valid

d) $f(0) = 0$; $f(1) = 1$; $f(n) = 2f(n-1)$ for $n \geq 1$

※ Basis Steps :

- $f(1) = 1 = 2(f(1-1)) = 2(f(0)) = 2.(0) = 0$ ✗

For this recursion function, it is not true for all nonnegatif integer. It is not valid.

e) $f(0) = 2$; $f(n) = f(n-1)$ if n is odd and $n \geq 1$

$f(n) = 2f(n-2)$ if $n \geq 2$

※ Basis Steps :

1) $f(n) = f(n-1)$ if n is odd and $n \geq 1$ and

$f(n) = 2f(n-2)$ if $n \geq 2$

- $f(1) = f(1-1) = f(0) = 2$ ✓
- $f(2) = 2f(2-2) = 2f(0) = 4$ ✓
- $f(3) = f(3-1) = f(2) = 4$ ✓
- $f(4) = 2f(4-2) = 2f(2) = 8$ ✓
- $f(5) = f(5-1) = f(4) = 8$ ✓
- $f(6) = 2f(4) = 16$

※ Recursion Formula

- For n is even number and $n \geq 0$; $f(0) = 2$; $f(1) = 2$

  $f(n+2) = f(n+1) + f(n)$

- For n is odd number and $n \geq 1$; $f(0) = 2$; $f(1) = 2$

  $f(n+2) = f(n) + f(n-1)$

※ Inductive Step

IH : If j is odd number with range $1 \leq j \leq k$

IH : If j is odd number with range $1 \leq j \leq k$
If j is even number with range $0 \leq j \leq k$

For k is true ; than show $(k+1)$ also true.

Show : Case : 1) If $k+1$ is odd number ; then :

$$\underset{\text{odd}}{f(k+1)} = \underset{\text{odd}}{f(k-1)} + \underset{\text{even}}{f(k-2)}$$

$$\underset{\text{odd}}{f(n+2)} = \underset{\text{odd}}{f(n)} + \underset{\text{even}}{f(n-1)} \quad \checkmark$$

$K+1$ is true for odd number

2) If $k+1$ is even number ; then :

$$\underset{\text{even}}{f(k+1)} = \underset{\text{odd}}{f(k)} + \underset{\text{even}}{f(k-1)}$$

If k is odd and $k-1$ is even ; the sum of $f(k)$ and $f(k-1)$ will resulting to $f(k+1)$ to be true be cause if $k \geq 5$; then $f(6) = f(5) + f(4) = 16$ ✓

$f(k+1)$ is true for $\forall n \in \mathbb{Z}$, n is even number and odd number