Persdif

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1) Find the position function of time $y(t)$ if initial height is y_0. We know that

$$v = \left(v_0 + \frac{mg}{k}\right)e^{-\frac{k}{m}t} - \frac{mg}{k} = \frac{dy}{dt}$$

$$\int dy = \int \left(v_0 + \frac{mg}{k}\right)e^{-\frac{kt}{m}} - \frac{mg}{k}\, dt$$

$$y = -\frac{m}{k}\left(v_0 + \frac{mg}{k}\right)e^{-\frac{k}{m}t} - \frac{mg}{k}t + C$$

$$y = \left(\left(v_0 + \frac{mg}{k}\right)e^{-\frac{k}{m}t} + gt\right)\left(-\frac{m}{k}\right) + C$$

if $y = y_0$ than

$$y_0 = \left(\left(v_0 + \frac{mg}{k}\right)e^{-\frac{k}{m}t} + gt\right)\left(-\frac{m}{k}\right) + C$$

$$y_0 - \left(\left(v_0 + \frac{mg}{k}\right)e^{-\frac{kt}{m}} + gt\right)\left(-\frac{m}{k}\right) = C$$ so the complete solution is

$$y = \left(\left(v_0 + \frac{mg}{k}\right)e^{-\frac{k}{m}t} + gt\right)\left(-\frac{m}{k}\right) + y_0 - \left(\left(v_0 + \frac{mg}{k}\right)e^{-\frac{kt}{m}} + gt\right)\left(-\frac{m}{k}\right)$$

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2) Solve the following differential equation

a) $\frac{dy}{dx} = -\frac{3x^2y + 2}{x^3 + y}$

$(x^3 + y)dy + (3x^2y + 2)dx = 0$

$\downarrow \qquad\qquad \downarrow$

$N(x,y) \qquad M(x,y)$

$\frac{\delta N(x,y)}{\delta x} = \frac{\delta M(x,y)}{\delta y}$

$\frac{\delta(x^3+y)}{\delta x} = \frac{\delta(3x^2y+2)}{\delta y}$

$3x^2 = 3x^2$

if the parsial derivateve resulting the same value, it shows that the equation has exact solution and the equation is continue.

$$M(x,y) = \frac{\delta F(x,y)}{\delta x}$$

$$\hookrightarrow F(x,y) = \int M(x,y)\, dx$$

$$= \int 3x^2 y + 2 \, dx = x^3 y + 2x + g(y)$$

Compared

$$N(x,y) = x^3 + y = \frac{\delta (F(x,y))}{\delta y} = \frac{\delta (x^3 y + 2x + g(y))}{\delta y}$$

$$x^3 + y = x^3 + g'(y)$$

$$y = g'(y)$$

$$\frac{1}{2}y^2 + C = g(y)$$

$$F(x,y) = x^3 y + 2x + \frac{1}{2}y^2 + C = 0$$ //

a) $x\frac{dy}{dx} + 3y = 4x^2 - 3x$

$$x\frac{dy}{dx} = 4x^2 - 3x - 3y$$

$$\frac{dy}{dx} = 4x - 3 - \frac{3y}{x}$$

we can do:

$$\frac{y}{x} = v$$

$$y = vx$$

$$\frac{dy}{dx} = \frac{dv}{dx}x + \cancel{\frac{dx}{dx}}v$$

$$\frac{dy}{dx} = \frac{dv}{dx}x + v$$

Subtitute to the main equation

$$\frac{dv}{dx}x + v = 4x - 3 - 3\frac{y}{x} \rightarrow \frac{y}{x} = v$$

$$\frac{dv}{dx}x + v = 4x - 3 - 3v$$

no solution for substitution method.

Now with Linear first order equation

$$x\frac{dy}{dx} + 3y = 4x^2 - 3x$$

find $\rho = e^{\int P(x)\,dx}$

$$P(x) = \frac{3}{x}$$

$$\frac{dy}{dx} + \frac{3y}{x} = 4x - 3$$

$$\rho = e^{\int \frac{3}{x}dx}$$

$$\rho = e^{3\ln|x|}$$

$$\rho\frac{dy}{dx} + \rho\frac{3y}{x} = 4x - 3$$

$$e^{3\ln x}\frac{dy}{dx} + e^{3\ln x}\frac{3y}{x} = (4x - 3)e^{3\ln x}$$

$$\frac{d}{dx}(e^{3\ln x}y) = (4x - 3)^{3\ln x}$$

$$\int d\left(e^{3\ln x} y\right) = \int (4x-3)\, e^{3\ln x}\, dx$$

$$e^{3\ln x} y = \int 4x e^{3\ln x} - 3 e^{3\ln x}\, dx$$

$$= \int 4x e^{e\log x^3} - 3 e^{e\log x^3}\, dx$$

$$= \int 4x\, x^3 - 3x^3\, dx$$

$$x^3 y = \frac{4x^5}{5} - \frac{3}{4}x^3 + C$$

$$y = \frac{4}{5}x^2 - \frac{3}{4} + D \quad //$$