Homework ※ 2 : Thermodynamics

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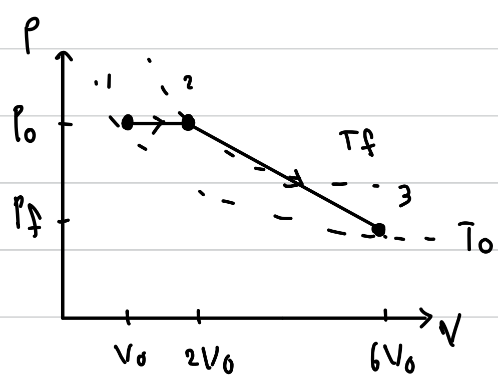
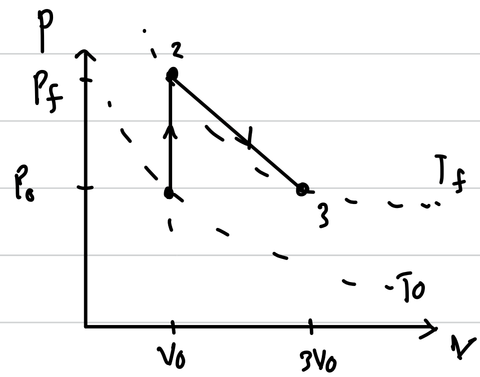
Problem # 1 : Gas Expansion



Process A



Process B

Gas is ideal with n moles and molar specific heat C_v in sealed container

[A] Determine the value of P_f and T_f in those process!

※ Process A

Process 1 to 2

$PV = nRT$

P = constant

$P = \frac{nRT}{V}$

$P_{o1} = P_{o2}$ (Isobarik)

$\frac{\cancel{nR} T_0}{V_0} = \frac{\cancel{nR} T_f}{2V_0}$

$T_f = 2T_0$

Process 2 to 3

The equation is

$PV = nRT$

nR = constant

$PV = T$

$\frac{P_2 V_2}{P_3 V_3} = \frac{T_2}{T_3}$

$\frac{P_0 \, 2\cancel{V_0}}{P_f \, 6\cancel{V_0}} = \frac{T_f}{T_0}$

We know that

$T_f = 2T_0$

$\frac{2P_0}{6P_f} = \frac{2\cancel{T_0}}{\cancel{T_0}}$

$P_0 = 6P_f$

$P_f = \frac{1}{6} P_0$

From the table, we know that $p_0 = \frac{nRT_0}{V_0} = \frac{nRT_f}{2V_0}$

$= \frac{nR2T_0}{2V} = \frac{nRT_0}{V_0}$

$P_f = \frac{1}{6} \frac{nRT_0}{V_0}$

$\boxed{\text{Thus, } P_f = \frac{nRT_0}{6V_0} \text{ and } T_f = 2T_0}$

※ Process B

Process 1 to 2 (isohoric)

$Pv = nRT$

$\frac{P}{T} = \frac{nR}{V}$ = constant

$\frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow \frac{P_0}{T_0} = \frac{P_f}{T_f}$

We know that $P_0 = \frac{nRT_0}{V_0}$

$\frac{nR\cancel{T_0}}{V_0\cancel{T_0}} = \frac{P_f}{T_f}$

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* Process 2 to 3 (isotermik)

$PV = nRT = \text{constant}$

$P_2 V_2 = P_3 V_3$

$P_f \cancel{V_f} = P_0 \, 3\cancel{V_0}$

$P_f = 3P_0$

Now subtitute $P_f = 3P_0$ into

$\frac{P_f}{T_f} = \frac{nR}{V_0}$

$\frac{3P_0}{T_f} = \frac{nR}{V_0}$

We know that $P_0 = \frac{nRT_0}{V_0}$

$\frac{3\cancel{nR}T_0}{T_f \cancel{V_0}} = \frac{\cancel{nR}}{\cancel{V_0}}$

$T_f = 3T_0$ //

Subtitute to $\frac{P_f}{T_f} = \frac{nR}{V_0}$

$\frac{P_f}{3T_0} = \frac{nR}{V_0}$

$P_f = nR\,3T_0 V_0$ //

$\boxed{\text{Thus, } T_f = 3T_0 \text{ and } P_f = nR\,3T_0V_0}$

B Determine the amount of heat Q added into the system during this proses.

We know that

$Q = \Delta E_{th} - W$

as $\Delta E_{th} = nC\Delta T$

and $W = -nR\Delta T$

* Process A

Q in process 1 to 2 (isobaric)

$Q_{12} = \Delta E_{th} - W$

$\Delta E_{th} = n C_v (T_f - T_0)$

$W = -nR(T_f - T_0)$

$Q_{12} = nC_v(T_f - T_0) + nR(T_f - T_0)$

$= n(T_f - T_0)(C_v + R)$

We know that

$T_f = 2T_0$

$Q_{12} = n(2T_0 - T_0)(C_v + R)$

$= nT_0(C_v + R)$

Q in process 2 to 3 (adiabatik)

$Q_{23} = \Delta E_{th} - W$

$\Delta E_{th} = nC_v\Delta T = nC_vT_0$

$W = -\frac{1}{\gamma - 1}(P_3V_3 - P_2V_2)$

we know that Laplace constant

$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V}$

$W = -\frac{1}{\frac{R}{C_V}}(P_3V_3 - P_2V_2)$

$W = -\frac{C_V}{R}[P_3V_3 - P_2V_2]$

$= -\frac{C_V}{R}[P_f 6V_0 - P_0 2V_0]$

$= -\frac{C_V V_0}{R}[6P_f - 2P_0]$

we know that $P_f = \frac{1}{6}P_0$

$W = -\frac{C_V V_0}{R}[P_0 - 2P_0]$

$W = \frac{C_V V_0 P_0}{R}$

$Q_{23} = \Delta E_{th} - W$

$= nC_VT_0 - \frac{C_V V_0 P_0}{R}$

$= C_V\left(nT_0 - \frac{V_0P_0}{R}\right)$

we know $P_0V_0 = nRT_0$

$= C_V\left(nT_0 - \frac{n\cancel{R}T_0}{\cancel{R}}\right) = 0$

$Q_{23} = 0$ // because adiabatic has no heat trasfer between system and environment.

So the total heat added into the system A is

$Q_{12} + Q_{23} = Q_{12} + 0$

$= nT_0(C_v + R)$ //

* Process B

Q in process 1 to 2 (isobaric)

$Q_{12} = \Delta E_{th} - W$

$\Delta E_{th} = nC_v\Delta T = nC_v(T_f - T_0)$

we know that $T_f = 3T_0$

$\Delta E_{th} = 2nC_vT_0$

$W = 0$ because isobaric has no change on system volume.

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$Q_{12} = \Delta E_{th} - 0$

$= 2nC_vT_0$

Q in process 2 to 3
(iso termic)

$Q_{23} = \Delta E_{th} - W$

Because it is isotermic process, the thermal energy is constant; thus $\Delta E_{th} = 0$

$Q_{23} = -W$

$W = \int_{V_0}^{V_f} p\,dV$; $p = \frac{nRT}{V}$

$$\int_{V_0}^{V_f} nRT \frac{dV}{V} = nRT\int_{V_0}^{V_f}\frac{dV}{V}$$

$V_f = 3V_0$

$$W = nRT\int_{V_0}^{3V_0}\frac{dV}{V} = nRT \ln\frac{3\cancel{V_0}}{\cancel{V_0}}$$

$W = nRT\ln 3$; $T_f = 3T_0$

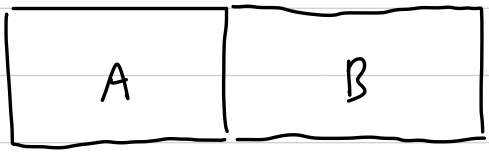
$W = 3nRT_0\ln 3$

So, the Q total in system B is

$Q_{12} + Q_{23} = 2nC_vT_0 + 3nRT_0\ln 3$

$= nT_0(2C_v + R\ln 3)$ //

Problem #2 : Thermal Equilibrium



M_a, C_a, T_a M_b, C_b, T_b

specific heat = C_a, C_b.

$\boxed{A}$ Determine the final temperature T_f of the two metal blocks!

Assume that $T_a > T_b$;

We know thermodynamic prinsip that Q released = Q accept, then the equation can be expressed as

$Q_{released} = Q_{accept}$

$Q_A = Q_B$

$M_aC_a(T_a - T_f) = M_bC_b(T_f - T_b)$

$M_aC_aT_a - M_aC_aT_f = M_bC_bT_f - M_bC_bT_b$

$M_aC_aT_a + M_bC_bT_b = M_aC_aT_f + M_bC_bT_f$

$M_aC_aT_a + M_bC_bT_b = T_f(M_aC_a + M_bC_b)$

$$T_f = \frac{M_aC_aT_a + M_bC_bT_b}{M_aC_a + M_bC_b}$$

So, the final termperature at equlibrium galory is equal to

$$T_f = \frac{M_aC_aT_a + M_bC_bT_b}{M_aC_a + M_bC_b}$$ //

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$\boxed{B}$ Determine the change in blocks' temperature $\Delta T_a = T_f - T_a$ and $\Delta T_b = T_f - T_b$

We know that $T_f = \dfrac{M_a C_a T_a + M_b C_b T_b}{M_a C_a + M_b C_b}$ //

$$\Delta T_a = T_f - T_a$$
$$= \frac{M_a \cancel{C_a T_a} + M_b C_b T_b - M_a \cancel{C_a} T_a - M_b C_b T_a}{M_a C_a + M_b C_b}$$
$$= \frac{M_b C_b (T_b - T_a)}{M_a C_a + M_b C_b} = \Delta T_a$$

$$\Delta T_b = T_f - T_b$$
$$= \frac{M_a C_a T_a + M_b C_b T_b}{M_a C_a + M_b C_b} - T_b$$
$$= \frac{M_a C_a T_a + M_b \cancel{C_b T_b} - M_a C_a T_b - M_b \cancel{C_b T_b}}{M_a C_a + M_b C_b}$$
$$= \frac{M_a C_a (T_a - T_b)}{M_a C_a + M_b C_b} = \Delta T_b$$

$\boxed{C}$ Assume $M_a = M_b$; $|\Delta T_a| > |\Delta T_b|$
Which metal block has higher specific heat?

We know that :

$$\Delta T_a = \frac{M_b C_b (T_b - T_a)}{M_a C_a + M_b C_b}$$

$$\Delta T_b = \frac{M_a C_a (T_a - T_b)}{M_a C_a + M_b C_b}$$

Now Subtitute to $Q_a = Q_b$

$$M_a C_a \Delta T_a = M_b C_b \Delta T_b$$

because $M_a = M_b$, then

$$C_a \Delta T_a = C_b \Delta T_b$$

$$\frac{C_a}{\Delta T_b} = \frac{C_b}{\Delta T_a}$$ we cannot compare in this way!

Now, because $|\Delta T_a| > |\Delta T_b|$ then

$$|\Delta T_a| = \frac{Q_a}{M_a C_a} > |\Delta T_b| = \frac{Q_b}{M_b C_b}$$

$$\frac{Q_a}{\cancel{M_a} C_a} > \frac{Q_b}{\cancel{M_b} C_b} \longrightarrow \frac{Q_a}{C_a} > \frac{Q_b}{C_b}$$

we know that $Q_a = Q_b$ at equillibrium temperature

$$\frac{\cancel{Q_a}}{C_a} > \frac{\cancel{Q_b}}{C_b}$$

$$\frac{1}{C_a} > \frac{1}{C_b}$$ it means,

$$C_b > C_a$$ //

Thus C_b has higher specific heat compare to C_a.

Problem $\cancel{\#}$ 3 : Maxwell - Boltzmann Distribution

Maxwell-Boltzmann Distribution :

$$N_v(v) = 4\pi N_0 \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$$

$\boxed{A}$ Prove $V_{rms} = \sqrt{\dfrac{3k_BT}{m}}$

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$$N_v(v) = 4\pi N_0 \left(\frac{m}{2\pi kBT}\right)^{3/2} v^2 e^{-mv^2/2kBT}$$

$$V_{rms} = \sqrt{\frac{3kBT}{m}}$$

We know that

$$N_v(v)\,dv = 4\pi N_0 \left(\frac{m}{2\pi kBT}\right)^{3/2} v^2 e^{-mv^2/2kBT}\,dv$$

The formula for Vrms in general is

$$V_{rms} = \left(\frac{\Sigma n_i v_i^2}{\Sigma n_i}\right)^{1/2}$$

Because it is a continous function, then we can use calculus for the zigma notation

$$V_{rms}^2 = \frac{\Sigma n_i v_i^2}{\Sigma n_i}$$

calculus notation:

$$V_{rms}^2 = \frac{\int_0^\infty v^2 N_v(v)\,dv}{\int_0^\infty N_v(v)\,dv}$$

We know that $N = \int_{V_1}^{V_2} N_v(v)\,dv$ so

$$= \int_0^\infty \frac{v^2 N_v(v)\,dv}{N_0}$$

$$= \frac{1}{\cancel{N_0}} \int_0^\infty v^2\, 4\pi \cancel{N_0} \left(\frac{m}{2\pi kBT}\right)^{\frac{3}{2}} v^2 e^{-mv^2/2kBT}$$

$$= 4\pi \left(\frac{m}{2\pi kBT}\right)^{\frac{3}{2}} \int_0^\infty v^4 e^{-mv^2/2kBT}\,dv$$

We know that $\int_0^\infty x^4 e^{-ax^2} = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$

where $a = \frac{m}{2kBT}$

Subtitute the equation into the integral equation

$$V_{rms}^2 = 4\pi \left(\frac{m}{2\pi kBT}\right)^{\frac{3}{2}} \int_0^\infty v^4 e^{-mv^2/2kBT}\,dv$$

$$= 4\pi \left(\frac{m}{2\pi kBT}\right)^{\frac{3}{2}} \left(\frac{3}{8}\sqrt{\frac{\pi}{\left(\frac{m}{2kBT}\right)^5}}\right)$$

$$= 4\pi \left(\frac{m}{2\pi kBT}\right)^{\frac{3}{2}} \frac{3}{8}\sqrt{\pi}\sqrt{\left(\frac{2kBT}{m}\right)^5}$$

$$= \cancel{4\pi}\,\frac{\cancel{m}}{\cancel{2\pi kBT}}\sqrt{\frac{\cancel{m}}{2\cancel{\pi}kBT}}\;\frac{3}{\cancel{8}\,2}\cancel{\sqrt{\pi}}\left(\frac{\cancel{2}kBT}{m}\right)^{2}\cancel{\sqrt{\frac{2kBT}{m}}}$$

$$V_{rms}^2 = \frac{3kBT}{m}$$

$$V_{rms} = \sqrt{\frac{3kBT}{m}}$$ proved

$\boxed{B}$ Prove $V_{avg} = \sqrt{\frac{8kBT}{\pi m}}$

The formula of Vavg in general is expressed as:

$$V_{avg} = \frac{\Sigma n_i v_i}{\Sigma n_i}$$ we can convert into integral notation

$$V_{avg} = \frac{\int_0^\infty v\,N_v(v)\,dv}{\int_0^\infty N_v(v)\,dv};$$

We know that $N = \int_{V_1}^{V_2} N_v(v)\,dv$; so

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$$V_{avg} = \frac{\int_0^\infty V\, N_v(V)\, dv}{N_0}$$

$$= \frac{1}{\cancel{N_0}} \int_0^\infty v\, 4\pi \cancel{N_0} \left(\frac{m}{2\pi KBT}\right)^{3/2} v^2 e^{-mv^2/2KBT} dV$$

$$= 4\pi \left(\frac{m}{2\pi KBT}\right)^{\frac{3}{2}} \int_0^\infty v^3 e^{-mv^2/2KBT} dv$$

We know from the question that:

$$\int_0^\infty x^3 e^{-ax^2} = \frac{1}{2a^2}$$

Where $a = \frac{m}{2KBT}$; so

$$V_{avg} = \cancel{4}^{2}\pi \left(\frac{m}{2\pi KBT}\right)^{\frac{3}{2}} \frac{1}{\cancel{2}\left(\frac{m}{2KBT}\right)^2}$$

$$= 2\pi \frac{m}{2\pi KBT} \sqrt{\frac{m}{2\pi KBT}} \left(\frac{2KBT}{m}\right)^2$$

$$= \frac{\cancel{m}}{\cancel{KBT}} \sqrt{\frac{m}{2\pi KBT}} \left(\frac{KBT}{m}\right)^2 4$$

$$V_{avg} = \sqrt{\frac{16 m\, KB^2 T^2}{2\pi KBT\, m^2}} = \sqrt{\frac{8KBT}{\pi m}}$$ proved //

$\boxed{c}$ Prove $V_{mp} = \sqrt{\frac{2KBT}{m}}$

V_{mp} is the maximum number of molecule at speed of V to V+dv.
So, the expression can be expressed as:

$$N_V(V)dv = 4\pi N_0 \left(\frac{m}{2\pi KBT}\right)^{3/2} V^2 e^{-mv^2/2KBT} dV$$

Now, the molecule that having the speed of V is equal to $\frac{NV(V)}{N}$; so

$$\frac{d\, NV(V)}{dV\, N_0} = 4\pi \left(\frac{m}{2\pi KBT}\right)^{\frac{3}{2}} V^2 e^{-mv^2/2KBT} dv$$

$$= 4\pi \left(\frac{m}{2\pi KBT}\right)^{\frac{3}{2}} \left(V^2 e^{-mv^2/2KBT} dv\right)$$

$$= 4\pi \left(\frac{m}{2\pi KBT}\right)^{\frac{3}{2}} \left(2Ve^{-mv^2/2KBT} + V^2 e^{-mv^2/2KBT} (-\cancel{2}mv/\cancel{2}KBT)\right)$$

$$= 4\pi \left(\frac{m}{2\pi KBT}\right)^{\frac{3}{2}} Ve^{-mv^2/2KBT} \left(2 - \frac{V^2 m}{KBT}\right)$$

The maximum distribution of speed had value of $\frac{d}{dv}\frac{Nv(V)}{N} = 0$ so

$$0 = \underbrace{4\pi \left(\frac{m}{2\pi KBT}\right)^{\frac{3}{2}} Ve^{-m^2/2KBT}}_{0} \left(2 - \frac{V^2 m}{KBT}\right)$$

$$2 - \frac{V^2 m}{KBT} = 0$$

$$\frac{V^2 m}{KBT} = 2 \longrightarrow V^2 = \frac{2KBT}{m}$$

$$V = \sqrt{\frac{2KBT}{m}}$$

Thus, the maximum / most posible distribution of speed:

$$V_{mp} = \sqrt{\frac{2KBT}{m}}$$ proved //