Practice SVD 1

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Construct the SVD of Each matrix below (by hand)!

$$\begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$$

1) $A^TA = \begin{bmatrix} 2 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$

2) Eigen values and eigen vectors

$$\begin{vmatrix} 8-\lambda & 2 \\ 2 & 5-\lambda \end{vmatrix} = 0 \rightarrow (8-\lambda)(5-\lambda) - 4 = 0$$

$$40 - 8\lambda - 5\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 13\lambda + 36 = 0$$

$$(\lambda - 9)(\lambda - 4) = 0$$

$\lambda_1 = 9$

$\lambda_2 = 4$ } eigen value

$\lambda_1 = 9$

$$\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} X = 0 \qquad R_2 = R_2 + 2R_1 = -4 + 2(2)$$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} X = 0$$

x_2	x_1
0	0
1	2

$$x_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\lambda_2 = 4$

$$\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} X = 0 \qquad R_2 = R_2 - \frac{1}{2}R_1$$

$$\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} X = 0$$

x_2	x_1
0	0
1	$-\frac{1}{2}$

$$x_2 = \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -\frac{1}{2} \\ 1 & 1 \end{bmatrix} \quad \text{normalize} \rightsquigarrow$$

$x_1 \quad x_2$

$$|x_1| = \sqrt{4+1} = \sqrt{5}$$

$$|x_2| = \sqrt{\frac{1}{4}+1} = \sqrt{\frac{5}{4}} = \frac{1}{2}\sqrt{5}$$

$$V = \begin{bmatrix} \frac{2}{5}\sqrt{5} & \frac{-\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{bmatrix} \qquad V^T = \begin{bmatrix} \frac{2}{5}\sqrt{5} & \frac{\sqrt{5}}{5} \\ \frac{-\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{bmatrix}$$

$$\Lambda = \Sigma^2 = \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix}; \quad \Sigma = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}; \quad \Sigma^T =$$

$$A = U \Sigma V^T$$

$$U = A V \Sigma^{-1}$$

$$U = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} \frac{2}{5}\sqrt{5} & \frac{-\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$= \frac{1}{\sqrt{5}} \begin{bmatrix} 3 & -4 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

Hences: $A = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

$U \qquad \Sigma \qquad V^T$

•) $\begin{bmatrix} 7 & 1 \\ 0 & 0 \\ 5 & 5 \end{bmatrix}$

$$A^T A = \begin{bmatrix} 7 & 1 \\ 0 & 0 \\ 5 & 5 \end{bmatrix} \cdot \begin{bmatrix} 7 & 0 & 5 \\ 1 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 50 & 0 & 40 \\ 0 & 0 & 0 \\ 40 & 0 & 50 \end{bmatrix}$$

•) Eigenvalue and eigen vector :

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 50-\lambda & 0 & 40 \\ 0 & -\lambda & 0 \\ 40 & 0 & 50-\lambda \end{vmatrix} = -\lambda^3 + 100\lambda^2 - 900\lambda = -\lambda(\lambda^2 - 100\lambda + 900) = 0$$

$$= -\lambda(\lambda-10)\cdot(\lambda-90) = 0$$

$$\lambda = 0$$

$$\lambda_2 = 10$$

$$\lambda_3 = 90$$

$\lambda_1 = 0$

$$\begin{bmatrix} 50 & 0 & 40 \\ 0 & 0 & 0 \\ 40 & 0 & 50 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & \frac{4}{5} \\ 0 & 0 & 0 \\ 40 & 0 & 50 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & \frac{4}{5} \\ 0 & 0 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & \frac{4}{5} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = 0 \qquad X_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda_2 = 10$

$$\begin{bmatrix} 40 & 0 & 40 \\ 0 & -10 & 0 \\ 40 & 0 & 40 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -10 & 0 \\ 40 & 0 & 40 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -10 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda_3 = 90$

$$\begin{bmatrix} -40 & 0 & 40 \\ 0 & -90 & 0 \\ 40 & 0 & -40 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & -90 & 0 \\ 40 & 0 & -40 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & -90 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad X_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$V = \frac{X}{|X|} \qquad |X_1| = 1$$

$$|X_2| =$$

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