Homework 5

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o) Solve the following non-homogeneous diff eq initial value problem!

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^x$$

$y(0) = 0$

$\frac{dy}{dx}(0) = 3$

Find y_c,

Characteristic equation :

$r^2 + 3r + 2 = 0$

$(r+1)(r+2) = 0$

$r = -1$; $r = -2$

$y_c = C_1 e^{-1x} + C_2 e^{-2x}$

$y_p = A e^x$

$y_p' = A e^x$

$y_p'' = A e^x$; Check the y_p

$Ae^x + 3Ae^x + 2Ae^x = e^x$

$6Ae^x = e^x$

$A = \frac{1}{6}$

$y_p = \frac{1}{6} e^x$

Now we get :

$y = y_c + y_p$

$= C_1 e^{-x} + C_2 e^{-2x} + \frac{1}{6} e^x$

Using the initial value :

$y(0) = C_1 e^{(0)} + C_2 e^{(0)} + \frac{1}{6} e^{(0)} = 0$

$= C_1 + C_2 + \frac{1}{6} = 0$

$y' = -C_1 e^{-x} - 2C_2 e^{-2x} + \frac{1}{6} e^x$

$y'(0) = -C_1 - 2C_2 + \frac{1}{6} = 3$

Elimination :

$$\begin{array}{r} C_1 + C_2 + \frac{1}{6} = 0 \\ -C_1 - 2C_2 + \frac{1}{6} = 3 \\ \hline -C_2 + \frac{1}{3} = 3 \end{array} +$$

$-C_2 = \frac{8}{3}$

$C_2 = -\frac{8}{3}$

$$c_1 + \left(-\frac{8}{3}\right) + \frac{1}{6} = 0$$

$$c_1 = \frac{16}{6} - \frac{1}{6} = \frac{15}{6} = \frac{5}{2}$$

Hence, the particular solution is:

$$y = \frac{1}{6} e^{x} + \frac{5}{2} e^{-x} - \frac{8}{3} e^{-2x} \text{ ,,}$$

$$y'' + 2y' + 2y = \sin 3x$$

$$y(0) = 2$$

$$y'(0) = 0$$

Find y_c using characteristic equation

$$r + 2r + 2 = 0$$

$$r_{12} = \frac{-2 \pm \sqrt{4 - 8}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm j\sqrt{4}}{2}$$

$$= \frac{-2 \pm j2}{2} = -1 \pm j$$

$r_1 = -1 + j$; $r_2 = -1 - j$

with $r = f \pm j g$

$$y_c = e^{-1x} \left(c_1 \cos(-1x) + c_2 \sin(1x) \right)$$
$$= e^{-x} \left(c_1 \cos(x) + c_2 \sin(x) \right)$$

Now; find the y_p;

$$y_p = C_3 \cos 3x + C_4 \sin 3x$$

Test the equation

$$y_p' = -3C_3 \sin 3x + 3C_4 \cos 3x$$
$$y_p'' = -9C_3 \cos 3x - 9C_4 \sin 3x$$

Subtitute into: $y'' + 2y' + 2y = \sin 3x$

$$-9C_3 \cos 3x - 9C_4 \sin 3x + 2(-3C_3 \sin 3x + 3C_4 \cos 3x) + 2C_3 \cos 3x$$
$$+ 2C_4 \sin 3x = \sin 3x$$

$$-7C_3 \cos 3x - 7C_4 \sin 3x - 6C_3 \sin 3x + 6C_4 \cos 3x = \sin 3x$$
$$-7C_3 \cos 3x + 6C_4 \cos 3x - 7C_4 \sin 3x - 6C_3 \sin 3x = \sin 3x$$
$$(-7C_3 + 6C_4) \cos 3x + (-7C_4 - 6C_3) \sin 3x = \sin 3x$$

$$-7C_3 + 6C_4 = 0 \quad \text{...(i)} \quad \times 7 \qquad -49C_3 + 42C_4 = 0$$
$$-7C_4 - 6C_3 = 1 \quad \text{...(ii)} \quad \times 6 \qquad -42C_4 - 36C_3 = 6 \; +$$

$$-85C_3 = 6$$
$$C_3 = \frac{6}{-85}$$

$$-7\left(\frac{-6}{85}\right) + 6C_4 = 0 \longrightarrow \frac{42}{85} + 6C_4 = 0 \longrightarrow 42 + 510C_4 = 0$$
$$510C_4 = -42 \longrightarrow C_4 = \frac{-7}{85}$$

$$y = y_c + y_p$$

$$= e^{-x}(C_1 \cos x + C_2 \sin x) - \frac{6}{85}\cos 3x - \frac{7}{85}\sin 3x$$

- Using the initial value:

$$\begin{array}{r} {}^{1}\,85 \\ 85 \\ \hline 170 \end{array}$$

$$y(0) = 1(C_1 + 0) - \frac{6}{85} + 0 = 2 \quad \text{---} \; (i)$$

$$= C_1 - \frac{6}{85} = 2 \longrightarrow \quad C_1 = \frac{170+6}{85} = \frac{176}{85}$$

$$y' = -e^{-x}(C_1 \cos x + C_2 \sin x) + (-C_1 \sin x + C_2 \cos x)e^{-x} + \frac{18}{85}\sin 3x - \frac{21}{85}\cos 3x$$

$$= e^{-x}(-C_1 \cos x - C_2 \sin x - C_1 \sin x + C_2 \cos x) + \frac{18}{85}\sin 3x - \frac{21}{85}\cos 3x$$

$$y'(0) = -C_1 + C_2 - \frac{21}{85} = 0 \quad \text{-----} \; (ii)$$

$$= \left(-\frac{176}{85}\right) + C_2 - \frac{21}{85} = 0$$

$$C_2 = \frac{197}{85}$$

For that; we have the complete equation at:

$$y = e^{-x}\left(\frac{176}{85}\cos x + \frac{197}{85}\sin x\right) - \frac{6}{85}\cos 3x - \frac{7}{85}\sin 3x \quad //$$