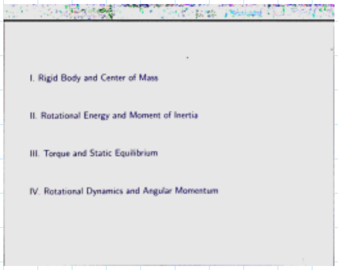
Rotational of Rigid Body

Monday, 04 October 2021 10.03



I. Rigid Body and Center of Mass

II. Rotational Energy and Moment of Inertia

III. Torque and Static Equilibrium

IV. Rotational Dynamics and Angular Momentum

Point of study.

I. Rigid Body and Center of Mass

* Particle model : An object is represented as mass in single point in space

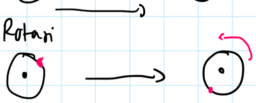
* Rigid Body ; have shape and sizes
  ex: Wheel, Rod, Ball, tires, etc.

Bentuk takkan berubah jika diberi gaya

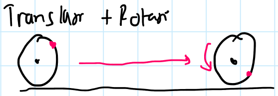
Gerak Translasi, Rotasi, Translasi + Rotasi

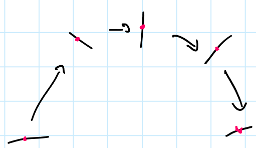
* Translasi

* Rotasi



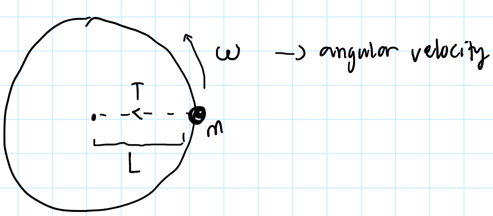
* Translasi + Rotasi



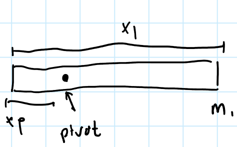


Benda yg tdk di batasi oleh sesuatu lalu diputar, maka akan terdapat titik pusat massa (Center of mass)

How to calculate?



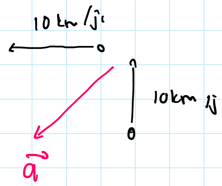
$$\boxed{a_c = \omega^2 L}$$

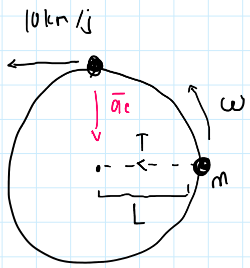


$$\frac{\Delta \vec{v}}{\Delta t} = \vec{a}$$

10 km/j → 20 km/jm dipercepat

10 km/j → 10 km/j



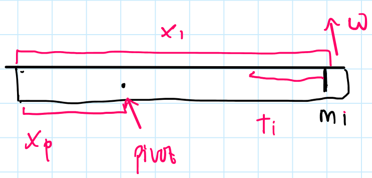


acceleration centripetal

* Gaya Centripetal

$$\boxed{T = m \cdot \vec{a}_c = m \omega^2 L}$$

* Menghitung Center of mass



$$L = (x_i - x_p)$$

* $\vec{a_i} = \omega^2 (\vec{x_p} - \vec{x_i})$

* $\vec{T_i} = m_i \omega^2 (\vec{x_p} - \vec{x_i}) = m_i \cdot \vec{a_i}$

* $\sum \vec{T_i} = \sum m_i \omega^2 (\vec{x_p} - x_i) = \omega^2 \sum_i m_i (\vec{x_p} - \vec{x_i})$

Jika titik pivot = Center of mass

$$\boxed{\sum \vec{T_i} = 0}$$

$$\sum m_i (x_{cm} - x_i) = 0$$

$$\sum m_i x_{cm} - \sum m_i x_i = 0$$

$$\boxed{x_p = x_{cm}}$$

$$x_{cm} \sum m_i - \sum m_i x_i = 0$$

$$\boxed{x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + m_3}}$$ → Benda Diskrit

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

$$z_{cm} = \frac{\sum m_i z_i}{\sum m_i}$$

Ex:

(3,7) ○ m_3

m_1 ○ (0,0)

○ m_2 (6,0)

$m_1 = 1$ kg

$m_2 = 2$ kg

$m_3 = 3$ kg

$x_{cm} = \; = 3,5$

$y_{cm} = 1,5$

$2x - 5y = 0$

$2x = 5y$

$\frac{2}{5} = \frac{y}{x}$

$y = \frac{2}{5}x$

$10 = x + \frac{2}{5}x$

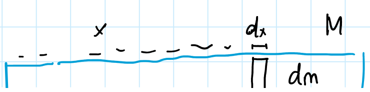
$10 = \frac{7x}{5}$

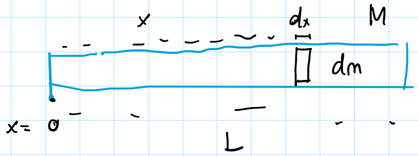
$50 = 7x$

$\frac{50}{7} = x$

kalo digerakin pindahnya, CM nya tetap tp V tangensial yg berubah.

Center of Mass Benda continue





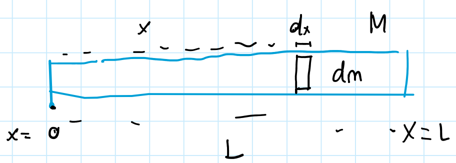
$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$ → bend diskrit

$$\boxed{x_{cm} = \frac{\int \vec{x} \cdot dm}{\int dm}}$$ → benda continue

$$y_{cm} = \frac{\int \vec{y}\ dm}{\int dm}$$

$$z_{cm} = \frac{\int \vec{z}\ dm}{\int dm}$$

$dm = ?$



$L \rightarrow M$

$$dm \rightarrow \frac{dx}{L} \cdot M$$

$$x_{cm} = \frac{\int x\ dm}{\int dm} = \frac{\int x \left(\frac{M}{L}\right) dx}{\int \frac{M}{L}\ dx}$$

$$= \frac{\frac{M}{L} \int x\, dx}{\frac{M}{L} \int dx}$$

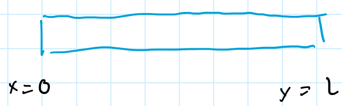
$0 \rightarrow L$

$0 \to L$

$$x_{cm} = \frac{\frac{M}{L}\int_0^L x\,dx}{\frac{M}{L}\int_0^L dx} = \frac{\int_0^L x\,dx}{\int_0^L dx}$$

$$= \frac{\frac{1}{2}x^2\Big]_0^L}{x\Big]_0^L} = \frac{\frac{1}{2}L^2}{L} = \frac{L}{2}$$

* Another case — kalo tidak uniform

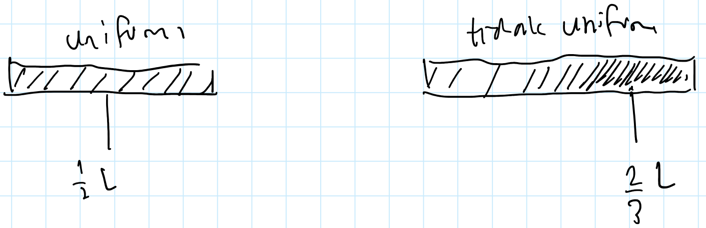


$$\frac{dm}{dx} = \lambda = \alpha x$$

$$dm = \alpha x \, dx$$

$$x_{cm} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x\,\alpha x\,dx}{\int_0^L \alpha x\,dx}$$

$$= \frac{\alpha\int_0^L x^2\,dx}{\alpha\int_0^L x\,dx} = \frac{\frac{1}{3}x^3\Big]_0^L}{\frac{1}{2}x^2\Big]_0^L} = \frac{2L}{3}$$



Kecepatan Pusat Massa

$m_1 \; o^{x_1} \longrightarrow v_1$

$m_2 \; o^{x_2} \longrightarrow v_2$

$m_3 \; o^{x_3} \longrightarrow v_3$

- $$X_{cm} = \frac{m x_1 + m_2 + x_3 m_3}{m_1 + m_2 + m_3}$$
- $$X_{cm}' = \frac{m_1(x_1 + v_1\Delta t) + m_2(x_2 + v_2\Delta t) + m_3(x_3 + v_3\Delta t)}{m_1 + m_2 + m_3}$$

$$X_{cm}' - X_{cm} = \underline{m_1 v_1 \Delta t + m_2 v_2 \Delta t + m_3 v_3 \Delta t}$$

$m_1 + m_2 + m_3$

$$\frac{x'_{cm} - x_{cm}}{\Delta t} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3} = v$$

$$v = \frac{\sum m_i v_i}{\sum m_i}$$

$$a_{cm} = \frac{V_{cm}' - V_{cm}}{\Delta t} = \frac{\sum m_i a_i}{\sum m_i}$$

$m_1 \longrightarrow F_1$ $\qquad a_1 = \frac{F_1}{m_1}$

$m_2 \longrightarrow F_2$ $\qquad a_2 = \frac{F_2}{m_2}$

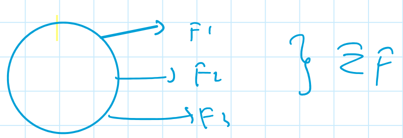
$m_3 \longrightarrow F_3$ $\qquad a_3 = \frac{F_3}{m_3}$

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2 + m_3 a_3}{m_1 + m_2 + m_3}$$

$$= \frac{m_1\left(\frac{F_1}{m_1}\right) + m_2\left(\frac{F_2}{m_2}\right) + m_3\left(\frac{F_3}{m_3}\right)}{m_1 + m_2 + m_3}$$

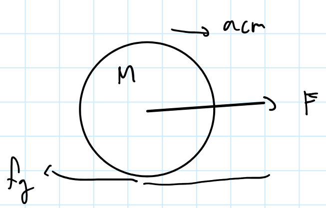
$$A_{cm} = \frac{F_1 + F_2 + F_3}{m_1 + m_2 + m_3}$$

$$a_{cm} = \frac{\sum F}{\sum m}$$ percepatan pusat massa



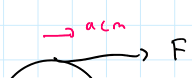
$$\boxed{\sum F = \left(\sum m\right) a_{cm}}$$

Ex :

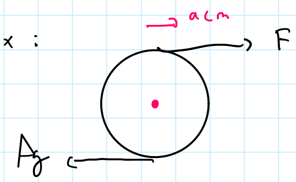


$$\boxed{a_{cm} = \frac{F - f}{M}}$$

Ex :



Ex :



$$\boxed{a_{cm} = \frac{F - f}{M}}$$

* Menghitung kecepatan pusat massa

m_1 o $\longrightarrow a_1$

m_2 o $\longrightarrow a_2$

$$u_1 = V_1 + a_1 \Delta t$$

$$u_2 = V_2 + a_2 \Delta t$$

$$V_{cm} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$$

$$V_{cm}' = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$\Delta V_{cm} = \frac{(m_1 u_1 + m_2 u_2) - (m_1 V_1 + m_2 V_2)}{m_1 + m_2}$$

m_1 o $\longrightarrow u_1$ $\quad$ m_2 o $\longrightarrow u_2$ $\Bigg\}$ m_1 o $\longrightarrow V_1$ $\quad$ m_2 o $\longrightarrow V_2$

$$\Delta V_{cm} = \frac{P_{akhir} - P_{awal}}{m_1 + m_2}$$

$$a_{cm} \cdot \Delta t = \frac{P_{akhir} - P_{awal}}{m_1 + m_2}$$

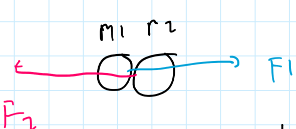
$$\frac{\Sigma F}{m_1 + m_2} \Delta t = \frac{P_{akhir} - P_{awal}}{m_1 + m_2} \longrightarrow \Sigma F \Delta t = P_{akhir} - P_{awal}$$

Yang perlu ditekankan adalah ini berlaku jika

$$\Sigma F = 0 \longrightarrow P_{akhir} = P_{awal}$$

Hkm kekekalan Momentum

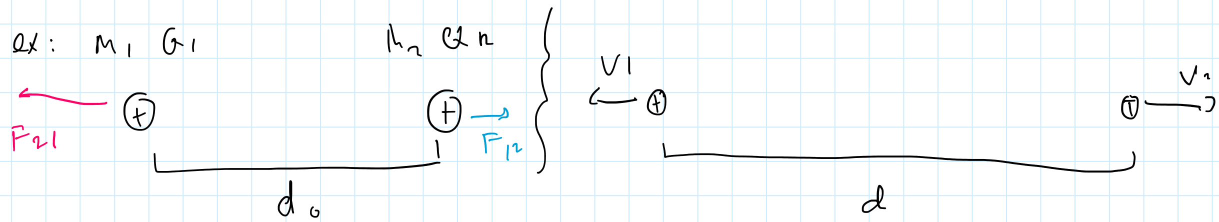
m_1 o $\longrightarrow v_1$ $\quad$ $v_2 \longleftarrow$ o m_2 $\Bigg\}$



$$|F_1| = |F_2|$$

arah berkebalikan

$\Sigma F = 0$



$|F_{21}| = |F_{12}|$

$\Sigma F = 0$

total momentum $= 0$