Tugas 2 KVT

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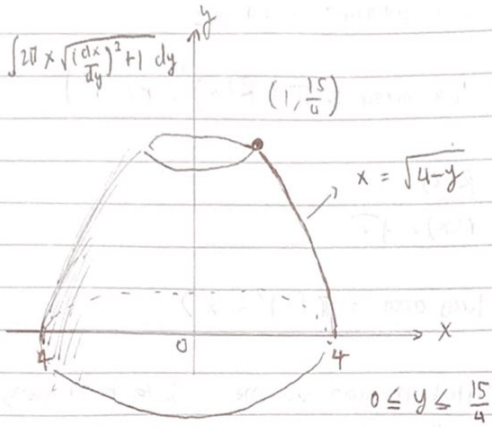
21/481767/TK/53170

1 Tentukan luas permukaan berikut:

$L = \int 2\pi \times \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$



$0 \le y \le \frac{15}{4}$

$$L_{\text{area}} = \int_0^{\frac{15}{4}} 2\pi (4-y)^{\frac{1}{2}} \sqrt{\left[\frac{1}{2}\left(\frac{1}{\sqrt{4-y}}\right)\right]^2 + 1}\, dy$$

$$= 2\pi \int_0^{\frac{15}{4}} (4-y)^{\frac{1}{2}} \sqrt{\frac{-1}{4(4-y)} + 1}\, dy$$

$$= 2\pi \int_0^{\frac{15}{4}} \left((4-y)\left(\frac{-1+4(4-y)}{4(4-y)}\right)\right)^{\frac{1}{2}} dy$$

$$= 2\pi \int_0^{\frac{15}{4}} \left(\frac{-1+4(4-y)}{4}\right)^{\frac{1}{2}} dy$$

$$= 2\pi \cdot \frac{1}{2} \int_0^{\frac{15}{4}} \left(-1+4(4-y)\right)^{\frac{1}{2}} dy$$

$$= \pi \int_0^{\frac{15}{4}} (15-4y)^{\frac{1}{2}} dy$$

$$= \pi \left[\frac{-(15-4y)^{\frac{3}{2}}}{6}\right]_0^{\frac{15}{4}}$$

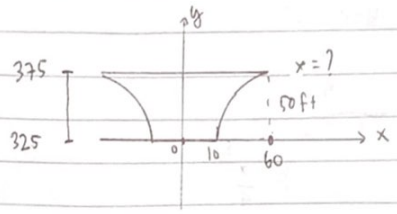
$$= \pi \left[\frac{-(15-4y)\sqrt{15-4y}}{6}\right]_0^{\frac{15}{4}}$$

$$= \pi\left(\frac{-(0)(0)}{6}\right) - \pi\left(\frac{-(15)\sqrt{15}}{6}\right)$$

$$= \frac{15\pi\sqrt{15}}{6} = \frac{5\pi\sqrt{15}}{2}$$

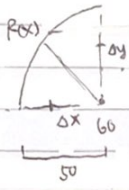
∴ Nilai dari luas permukaan bangun ruang kresbut adalah $\frac{5\pi\sqrt{15}}{2}$

2 Tentukan usaha yang diperlukan untuk memompa air dari pipa berikut



Diketahui : berat jenis dari air adalah 62,4 lb/ft³

* Mencari jari-jari bagian dalam berdungsin.



$\therefore R(y) = 60 - \Delta x$

$\therefore \Delta x = \sqrt{(\text{jari-jari luar})^2 - (\Delta y)^2}$

$= \sqrt{(50)^2 - (y-325)^2}$

$R(y) = 60 - \sqrt{50^2 - (y-325)^2}$

$\Delta \text{Volume} = \pi R(y)^2\, dy$

$\Delta F(y) = 62{,}4 \cdot \Delta V = 62{,}4 \cdot \pi R(y)^2 \cdot dy$

$\Delta W = 62{,}4 \cdot \pi R(y)^2 (375-y)\, dy$

∴ sehingga total usaha yang dibutuhkan untuk memompa air keluar adalah :

$$W = \int_{325}^{375} 62{,}4 \cdot \pi \cdot \left(60 - \sqrt{50^2 - (y-325)^2}\right)^2 (375-y)\, dy$$

$$W = 62{,}4\,\pi \int_{325}^{375} \left(60 - \sqrt{50^2 - (y-325)^2}\right)^2 (375-y)\, dy$$

$$W = 62{,}4\,\pi \left(\frac{350000}{3} - 31250\right)$$

$$\approx 62{,}4\,\pi\,(18491) \approx 3{,}62489 \cdot 10^6$$

Jadi total usaha yang dibutuhkan untuk memompa air adalah $3{,}62489 \times 10^6$ joule