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21/481767/TK/53170

PVA Tugas 5

4.1.1 Random variables X and Y have the joint CDF

$$F_{X,Y}(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}) & x \geq 0 \cap y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a) What is $P[X \leq 2, Y \leq 3]$?

$$P[X \leq 2 ; Y \leq 3] = (1-e^{-2})(1-e^{-3})$$
$$= 1 - e^{-3} - e^{-2} + e^{-5}$$

b) What is the marginal CDF, $F_X(x)$?

at $y = \infty \rightarrow$ $F_X(x) = \begin{cases} (1-e^{-x})(1) & x \geq 0, \\ 0 & \text{otherwise} \end{cases}$

c) What is the marginal CDF, $F_Y(y)$?

at $x = \infty \rightarrow$ $F_Y(y) = \begin{cases} (1)(1-e^{-y}) & y \geq 0, \\ 0 & \text{otherwise} \end{cases}$

4.1.2 Express the following extreme values of $F_{X,Y}(x,y)$ in terms of the marginal cumulative distribution function $F_X(x)$ and $F_Y(y)$.

a) $F_{X,Y}(x,-\infty) = 0$

b) $F_{X,Y}(x,\infty) = F_X(x)$

c) $F_{X,Y}(-\infty,\infty) = 0$

d) $F_{X,Y}(-\infty,y) = 0$

e) $F_{X,Y}(\infty,y) = F_Y(y)$

4.2.1 Random variables X and Y have the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} cxy & x = 1, 2, 4 \quad y = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

a) What is the value of the constant c ?

The possible answer : (1;1) , (1;3) , (2;1) , (2;3) , (4;1) , (4;3)

We know that sum of all possible PMF = 1 , So

$$\sum_x \sum_y P_{X,Y}(x,y) = c + 3c + 2c + 6c + 4c + 12c = 1$$

$$28C = 1$$

$$C = \frac{1}{28}$$

b) What is $P[Y < X]$?

Possible answer : (2;1) , (4;1) , (4;3)

$$\sum_x \sum_y P_{X,Y}(x,y) = \frac{1}{28}(2) + \frac{1}{28}(4) + \frac{1}{28}(12)$$

$$= \frac{18}{28} = \frac{9}{14}$$

c) What is $P[Y > X]$?

Possible answer : (1;3) , (2;3)

$$\sum_x \sum_y P_{X,Y}(x,y) = \frac{1}{28}(3) + \frac{1}{28}(6) = \frac{9}{28}$$

d) What is $P[Y = X]$?

Possible answer : (1;1)

$$\sum_x \sum_y P_{X,Y}(x,y) = \frac{1}{28}$$

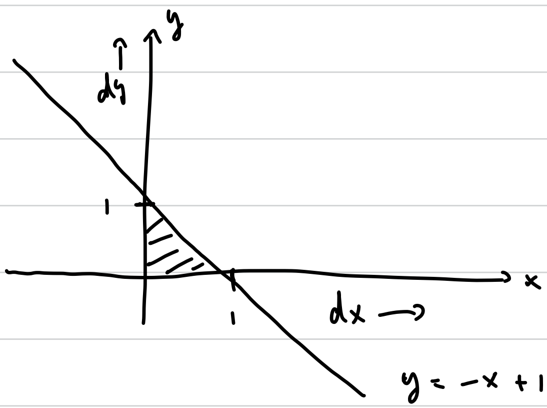
e) What is $P[Y = 3]$? → Possible answer : (1 ; 3), (2 ; 3) , (4 ; 3)

$$\sum_x \sum_y P_{X,Y}(x,y) = \frac{3}{28} + \frac{6}{28} + \frac{12}{28} = \frac{21}{28} = \frac{3}{4}$$

4.4.1 Random variables X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} c & x+y \le 1,\ x \ge 0,\ y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

a) What is the value of the constant c?



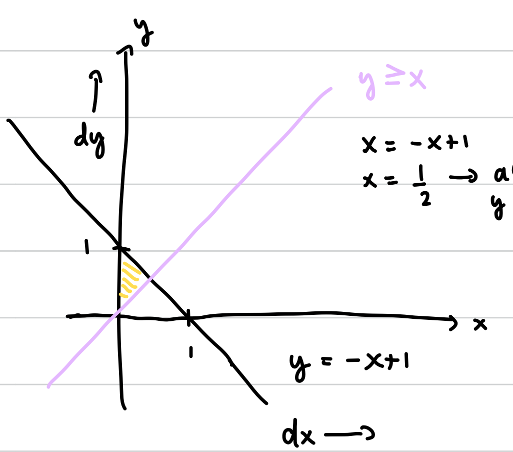
$$F_{X,Y}(x,y) = \int_0^1 \int_0^{-x+1} c \; dy\, dx = 1$$

$$= \int_0^1 cy \Big]_0^{-x+1} dx = \int_0^1 -cx + c \; dx$$

$$= -\frac{1}{2}cx^2 + cx \Big]_0^1 = -\frac{1}{2}c + c = \frac{1}{2}c = 1$$

$c = 2$

b) What is $P[X \le Y]$? There is a line of $y \ge x$ in the graph.



$x = -x+1$

$x = \frac{1}{2}$ → as the y boundry

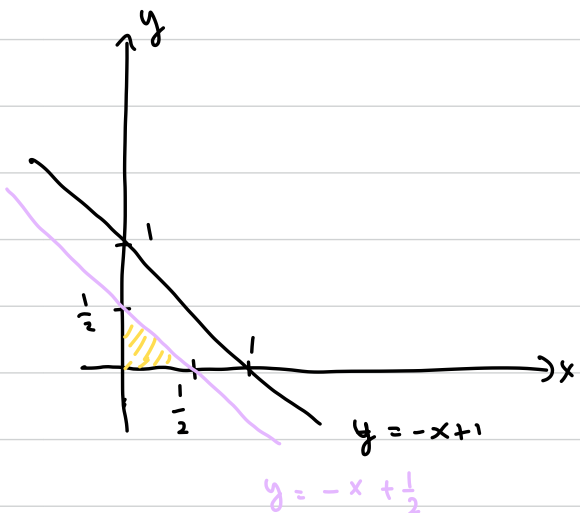
$$P[X \le Y] = \int_0^{\frac{1}{2}} \int_x^{-x+1} c \; dy\, dx = \int_0^{\frac{1}{2}} cy \Big]_x^{-x+1} dx$$

$$= \int_0^{\frac{1}{2}} (-cx + c - cx)\, dx = \int_0^{\frac{1}{2}} -2cx + c \; dx$$

$$= -cx^2 + cx \Big]_0^{\frac{1}{2}} = -c\frac{1}{4} + c\frac{1}{2} = \frac{1}{4}c$$

$= \frac{1}{2}$ // or just simply half of all probabilty.

What is $\Pr[X+Y \le 1/2]$?



$$\Pr[X+Y \le 1/2] = \int_0^{\frac{1}{2}} \int_0^{-x+\frac{1}{2}} 2 \, dy \, dx$$

$$= \int_0^{\frac{1}{2}} 2y \Big]_0^{-x+\frac{1}{2}} dx$$

$$= \int_0^{\frac{1}{2}} -2x + 1 \, dx$$

$$= -x^2 + x \Big]_0^{\frac{1}{2}} = -\frac{1}{4} + \frac{1}{2}$$

$$= \frac{1}{4} \, {}_{''}$$