Tugas 1

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Bagian 1.1

9 Express the area and perimeter of an equilateral triangle as a function of the triangle's side length x.

- Fungsi keliling segitiga dinyatakan sebagai $f(x)$ sehingga

$$f(x) = x + x + x = 3x$$

- Fungsi luas segitiga dinyatakan sebagai $g(x)$ sehingga

$$g(x) = \text{luas segitiga sama sisi}$$

Rumus umum segitig adalah

$$L_\Delta = \frac{1}{2} x \cdot b \quad ; \quad \begin{array}{l} x = \text{sisi horizontal} \\ b = \text{sisi vertikal} \end{array}$$

Mencari sisi b dapat dilakukan dengan rumus pitagoras dengan rumus umum

$$c = \sqrt{a^2 + b^2} \quad ; \quad \begin{array}{l} c = x \\ a = \frac{1}{2} x \end{array}$$

Panjang sisi b adalah sebagai berikut.

$$b = \sqrt{c^2 - a^2} = \sqrt{x^2 - \frac{1}{4}x^2}$$
$$= \frac{x}{2}\sqrt{3}$$

Sehingga rumus fungsi dari luas segitiga sama sisi adalah :

$$g(x) = \frac{1}{2} x \cdot \frac{x}{2}\sqrt{3} = \frac{1}{4}x^2\sqrt{3}$$

11 Express the length of a cube as a function of the cube's diagonal lenght d. Then express the surface area and volume of the cube as a function of the diagonal length.

- Fungsi panjang sisi dari kubus dinyatakan sebagai

$f(d) =$ panjang sisi

kubus dinyatakan sebagai

$f(d)$ = panjang sisi

Hubungan antara panjang sisi kubus dengan panjang diagonal kubus dinyatakan dalam rumus umum sebagai berikut.

d = panjang sisi $\cdot \sqrt{3}$

panjang sisi $= \dfrac{d}{\sqrt{3}}$

sehingga fungsi panjang sisi dari kubus adalah

$$f(d) = \frac{d}{\sqrt{3}}$$

- Fungsi luas permukaan dari kubus dinyatakan sebagai $\longrightarrow$ Rumus umum dari luas permukaan kubus adalah sebagai berikut.

$g(d)$ = luas permukaan

$$\begin{aligned} L_p &= 6 \cdot \text{Luas Persegi} \\ &= 6 \cdot \text{panjang sisi}^2 \\ &= 6 \cdot \left(\frac{d}{\sqrt{3}}\right)^2 \end{aligned}$$

Sehingga fungsi panjang sisi dari kubus adalah

$$g(d) = 6\left(\frac{d}{\sqrt{3}}\right)^2 = \cancel{6} \cdot \frac{d^2}{\cancel{3}} = 2 \cdot d^2$$

- Fungsi volume dari kubus dinyatakan sebagai,

$h(d)$ = volume kubus

$h(d)$ = panjang sisi^3 $= \left(\dfrac{d}{\sqrt{3}}\right)^3$

Sehingga fungsi panjang sisi dari kubus adalah

$$h(d) = \frac{d^3}{3\sqrt{3}}$$

59 The variable s is proportional to t, and $s = 25$ when $t = 75$. Determine t when $s = 60$

t is proportional to s can be express as

$\dfrac{t}{s} = \dfrac{t'}{s'}$ When $t = t' = 75$; the $s = s'$ is equal to 25.

So, the result of t when s is equal to 60 is

$\dfrac{t}{s} = \dfrac{t'}{s'}$ $t = 100$

So, when s is equal to 60,

$$\frac{t}{s} = \frac{t'}{s'}$$

$$\frac{t}{60} = \frac{\cancel{75}^{3}}{\cancel{25}_{1}}$$

$t = 180$

So, when s is equal to 60, t is equal to 180

62 Boyle's Law says that the volume *V* of a gas at constant temperature increase whenever the pressure *P* decreases, so that *V* and *P* are Inversely proportional. If *P* = 147lbs/in^2 when *V* = 1000 in^3, then What is *V* when *P* = 23.4 lbs/in^2?

*) Boyle's Law in general is express as $P_1 V_1 = P_2 V_2$
It is said to be inversely proportional because when P is increase, V is decreasing as shown below

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

If $P = 147\ lbs/in^2$ when $V = 1000\ in^3$. What is V when $P = 23.4\ lbs/in^2$?

$$\frac{23.4}{147} = \frac{1000}{V}$$

$$V = 15{,}9\ in^3$$

So V is equal to $15.9\ in^3$

64 The accompanying figure shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.
a. Express the y-coordinate of P in terms of x.
(You might start by writing an equation for the line AB.)
b. Express the area of the rectangle in terms of x.

a. To find the y-coordinate in terms of x we need to determined what variable we will use for the y-cordirate.

y = y-coordinat

We know that x has a tangent point to the AB line. From that, we can determine the value of y by expressing the AB slope.

$$m\,AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-0}{0-1} = \boxed{-1}$$

Because it is a special form of isosceles triangle, we can conclude that the y_2 variable is also have a value of 1. So now, we can complete the equation from AB slope.

we can now find the second equation using the slope function in the tangent point.

$$m\,AB = -1 = \frac{\Delta y}{\Delta x}$$

$$-1 = \frac{y-0}{x-1}$$

$$(x-1) - 1 = y$$

$$1 - x = y$$

So the function is $y = 1 - x$ //

b. The area of the rectangle in term of x can be written as

A = the area of the square

= leght · width

= $2x \cdot y$

= $2x(1-x)$

$A = 2x - 2x^2$

72 A power plant sits next to a river 900 f wide. the cost for laying a new cable from plant to new location cost 180 dollar per foot across the river and 120 dollar per foot on land.

a. To know the $C(x)$ function, we must calculate the cost function by crossing the river from Power Plan to Q and added with the function by walking down the land.

* The cost function from crossing down the river can be express as

$$f(x) = \frac{\text{cost}}{\text{foot}} \cdot \text{distance}$$

$$= \frac{180\ \$}{\text{foot}} \cdot \text{distance to Q}$$

The distance to Q can be find using pythagoras formula.

$$\text{Distance to } Q = \sqrt{(\text{Power Plant})^2 + (PQ)^2} \text{ foot}$$
$$= \sqrt{(800)^2 + x^2} \text{ foot}$$
$$= \sqrt{640000 + x^2} \text{ foot}$$

Now we can complete the $f(x)$ function.

$$f(x) = \frac{180\ \$}{\cancel{\text{foot}}} \cdot \sqrt{64 \cdot 10^4 + x^2}\ \cancel{\text{foot}}$$
$$= \left(180\sqrt{64 \cdot 10^4 + x^2}\right)$$

*) The cost function from crossing down the land can be express as

$$g(x) = \frac{\text{cost}}{\text{foot}} \cdot \text{distance to the city}$$

The distance to the city can be find by

$$\text{Distance to city} = (P \text{ to city}) - PQ \text{ foot}$$
$$= 2 \text{ mile} - x \text{ foot}$$
$$= (10560 - x) \text{ foot}$$

Note-

1 mile = 5280 ft

Now we can complete the $g(x)$ function.

$$g(x) = \frac{100\ \$}{\cancel{\text{foot}}} \cdot (10560 - x)\ \cancel{\text{foot}}$$
$$= (1056000 - 100x)$$

So to complete the $C(x)$ function, we need to add the $f(x)$ and $g(x)$ function.

$$C(x) = f(x) + g(x)$$
$$= \left(180\sqrt{64 \cdot 10^4 + x^2}\right) + (1056000 - 100x)$$

b. $C(x) = \left(180\sqrt{(800)^2 + x^2}\right) + (1056000 - 100x)$

x	$C(x)$ (\$)
2400	1.271.367,98
2200	1.257.369,19
2000	1.243.731,86
1800	1.230.558,88
1600	1.217.993,78

From the table, we can conclude that $x < 2000$ ft has cheaper cost for laying a new cable.

Bagian 1.3

12 $\sin x = -\frac{1}{2}$; $\pi \leq x \leq \frac{3\pi}{2}$

To find "x" in the interval, we can use "arc" function. But in my case, I will use another approach.

$\sin x = -\frac{1}{2}$; $\sin \frac{7}{12}\pi = -\frac{1}{2}$

$\sin \frac{\pi}{6} = \frac{1}{2}$

$\pi \leq x \leq \frac{3}{2}\pi$

$\sin \frac{7}{12}\pi = -\frac{1}{2}$

a) $\cos \frac{7}{12}\pi = -\frac{1}{2}\sqrt{3}$

b) $\tan \frac{7}{12}\pi = \frac{1}{3}\sqrt{3}$

19 $\cos\left(x - \frac{\pi}{2}\right)$

0	$\pi/2$	π	$\frac{3}{2}\pi$	2π		
0	1	0	-1	0		

domain : $-\infty < x < \infty$
Range : $-1 \leq x \leq 1$
periode : 2π

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a. $y = \cos x$
$y = \sec x$ $\Big\}$ $-\frac{3}{2}\pi \leq x \leq \frac{3}{2}\pi$

Tabel of $\cos x$

X	$-3/2\pi$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3/2\pi$
y	0	-1	0	1	0	-1	

Tabel of $\sec x = \frac{1}{\cos x}$

X	$-3/2\pi$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3/2\pi$
y	$\frac{1}{0} = \pm\infty$	$\frac{1}{-1} = -1$	$\pm\infty$	1	$\pm\infty$	-1	$\pm\infty$

x	$-3/2\pi$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3/2\pi$
y	$\frac{1}{0} = \pm\infty$	$\frac{1}{-1} = -1$	$\pm\infty$	1	$\pm\infty$	-1	$\pm\infty$

Graph :

The behavior of cos x in relation to sec x is that they do have tangent point on the vertex.

b. $y = \sin x$
$y = \operatorname{cosec} x$ $\Big\}$ $-\pi \leq x \leq 2\pi$

Table of $y = \sin x$

x	$-\pi$	$-\frac{\pi}{2}$	0	$\pi/2$	π	$\frac{3}{2}\pi$	2π
y	0	-1	0	1	0	-1	0

Table of $y = \operatorname{cosec} x = \frac{1}{\sin x}$

x	$-\pi$	$-\frac{\pi}{2}$	0	$\pi/2$	π	$\frac{3}{2}\pi$	2π
y	∞	-1	∞	1	∞	-1	∞

Both functions $y = \sin x$ and $y = \csc x$ have tangent point in the same vertex or maximum value of y.

38 If the $B = 2\pi$ is added to the addition formula, the formula will be proven by using $\cos(x+B)$ formula.

$$\begin{aligned}\cos(x+B) &= \cos x \cos B - \sin x \sin B \\ &= \cos x \cos 2\pi - \sin x \sin 2\pi \\ &= \cos x\,(1) - \sin x\,(0) \\ &= \cos x\end{aligned}$$

So, the result is true for $B = 2\pi$ because $\cos(x+B) = \cos x$. We can agree that it is true.

40 $\sin(2\pi - x)$ into $\cos x$ and $\sin x$

$$\begin{aligned}\sin(2\pi - x) &= \sin 2\pi \cos x - \cos 2\pi \cdot \sin x \\ &= (0)\cos x - (1)\sin x \\ &= -\sin x\end{aligned}$$