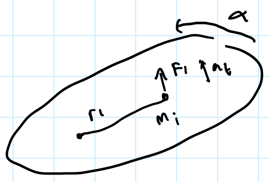
# Rotational Dynamics

Monday, 11 October 2021 10.50

$$\sum F = m.a$$




$$F_i = m_i \, a_t$$

$$a_t = \alpha \cdot r_i$$

$$F_i = m_i \, \alpha \, r_i$$

$$F_i r_i = m_i \, r_i^2 \, \alpha$$

$$\tau_i = (m_i r_i^2) \, \alpha$$

$$\sum_i \tau_i = \left( \sum_i m_i r_i^2 \right) \alpha \rightarrow \sum \tau = I \alpha$$


$$\Delta x_{cm} = V_{cm} \cdot T = 2\pi R$$

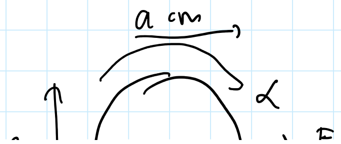
$T$ ↓ periode

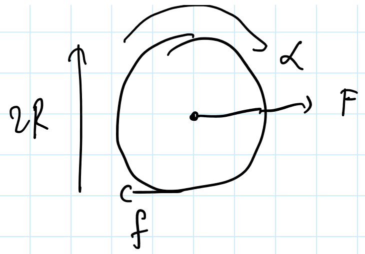
If no slip

$$V_{cm} = \left( \frac{2\pi}{T} \right) R$$

$$V_{cm} = \omega \cdot R$$

$$a_{cm} = \alpha R$$






Rotasi

$$\sum \tau = I\alpha$$

$$f \cdot R = I\alpha \quad \ldots (i)$$

F besar tidak menghasilkan Torsi

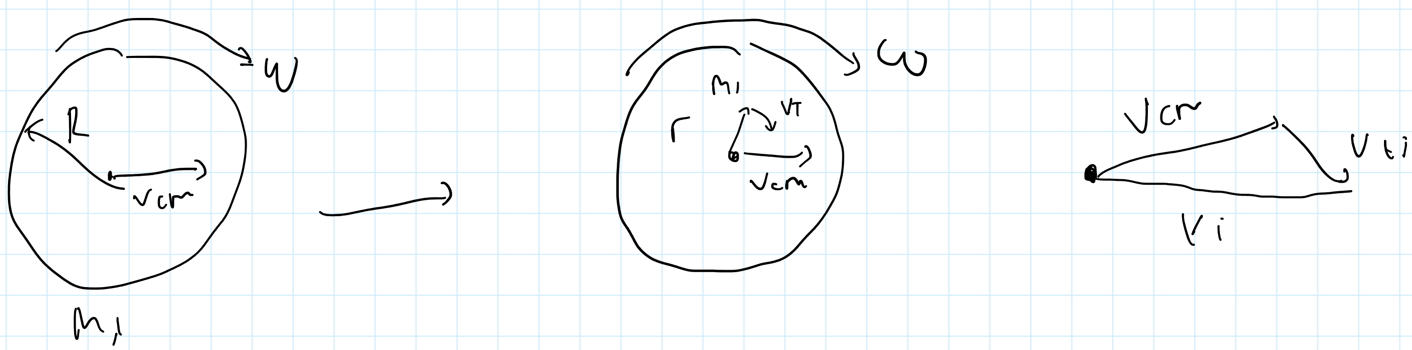
$$\sum F = M \cdot a_{cm}$$

$$F - f = M a_{cm} \quad \ldots (ii)$$

$$a_{cm} = \alpha R \quad \ldots (iii)$$

$$a_{cm} = \frac{F}{M + (I/R^2)}$$

(b) Kinetic energy of a Rolling Objek




Kinetic energy of a Rolling Object.

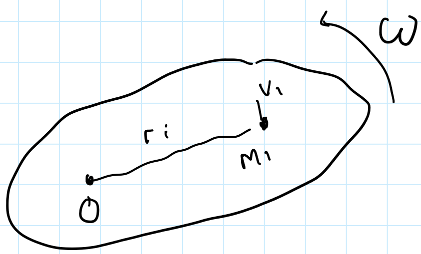
- The total kinetic energy of this object can be calculated as follow

$$K = \sum_i \frac{m_i v_i^2}{2} = \frac{v_{cm}^2}{2}\left(\sum_i m_i\right) + v_{cm}\omega\left(\sum_i m_i y_i\right) + \frac{\omega^2}{2}\left(\sum_i m_i r_i^2\right)$$
$$= \frac{M v_{cm}^2}{2} + M v_{cm}\omega y_{cm} + \frac{I\omega^2}{2}$$

- Because $y_{cm} = 0$ (why?), therefore the total kinetic energy is given by

$$K = \frac{M v_{cm}^2}{2} + \frac{I\omega^2}{2} = K_{translation} + K_{rotation} \qquad (17)$$

Angular Momentum




$P = m \cdot v$

$L = I \cdot \omega$

$L_i = m_i v_i r_i$

Angular Momentum and Torque

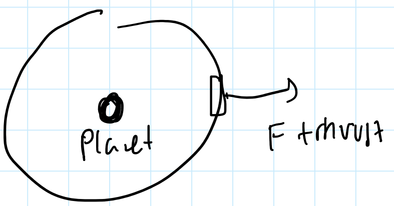
$$\vec{L} = \vec{r} \times \vec{p}$$

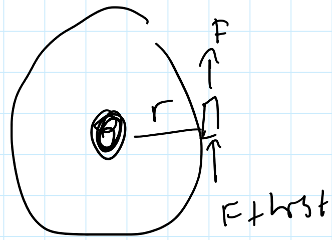
$$\Delta P = I = F \cdot \Delta t$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt}$$

$$dL = \vec{r} \times F = \boxed{d\vec{L} = \vec{\tau}}$$

$$\frac{dL}{dt} = \vec{r} \times F = \boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$




$$\Sigma \tau \neq 0$$

$$\Delta L \neq 0$$

$$\Sigma \tau = 0$$

$$\Delta L = 0$$