Qornain Aji
21/481707/TK/53170

3.13 The CDF of Random variable W is

$$F_W(w) = \begin{cases} 0 & w < -5 \\ (w+5)/8 & -5 \leq w < -3 \\ 1/4 & -3 \leq w < 3 \\ 1/4 + 3(w-3)/8 & 3 \leq w < 5 \\ 1 & w \geq 5 \end{cases}$$

$$\frac{1}{4} + 3(w-3)/8 = \frac{1}{2}$$

$$\frac{1}{2} + \frac{3w-9}{4} = 1$$

$$\frac{3w-9}{4} = +\frac{1}{2}$$

$$\frac{3w}{4} = +\frac{2}{4} + \frac{9}{4}$$

$$3w = 17$$

$$w = \frac{7}{3}$$

a) What is $P[W \leq 4]$?

$$P[W \leq 4] = 1/4 + 3((4)-3)/8$$
$$= 1/4 + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

b) What is $P[-2 < W \leq 2]$

$$P[-2 < W \leq 2] = P[W \leq 2] - P[W > -2]$$
$$= 1/4 - 1/4 = 0$$

c) What is $P[W > 0]$

$$P[W > 0] = 1 - P[W \leq 0]$$
$$= 1 - 1/4 = \frac{3}{4}$$

d) what is the value of a such that $P[W \leq a] = \frac{1}{2}$

Because of $\frac{1}{2}$ is bigger than the range of $-3 \leq W < 3$; thus, we can conclude that the only possible range is between $3 \leq w < 5$ with $F_W(\alpha) = 1/4 + 3\frac{(\alpha - 3)}{8}$

$$\frac{1}{2} = \frac{1}{4} + 3\frac{(\alpha - 3)}{8}$$

$$1 = \frac{1}{2} + \frac{3\alpha - 9}{4}$$

$$\frac{1}{2} = \frac{3\alpha}{4} - \frac{9}{4}$$

$$\frac{2}{4} = \frac{3\alpha}{4} - \frac{9}{4}$$

$$2 = 3a - 9$$

$$\frac{11}{3} = a$$

So, the value of α is $\frac{11}{3}$ //

3.2.3 Find the PDF of $f_U(u)$ of the random variable U in Problem 3.1.3

- $f_U(u < -5) = \frac{d(0)}{du} = 0$
- $f_U(-5 \leq u < -3) = \frac{d\left(\frac{u+5}{8}\right)}{du} = \frac{1}{8}$ //
- $f_U(-3 \leq u < 3) = \frac{d\left(\frac{1}{4}\right)}{du} = 0$
- $f_U(3 \leq u < 5) = \frac{d\left(\frac{1}{4} + 3\left(\frac{u-3}{8}\right)\right)}{du} = \frac{3}{8}$

• $f_U(u \geq 5) = \frac{d(1)}{du} = 0$

$$f_U(u) \begin{cases} 0 & u < -5 \\ 1/8 & -5 \leq u < -3 \\ 0 & -3 \leq u < 3 \\ 3/8 & 3 \leq u < 5 \\ 0 & u \geq 5 \end{cases}$$

$\boxed{3.3.7}$ a dan b saja

The CDF variable U

$$F_U(u) \begin{cases} 0 & u < -5 \\ (u+5)/8 & -5 \leq u < -3 \\ 1/4 & -3 \leq u < 3 \\ 1/4 + \frac{3(u-3)}{8} & 3 \leq u < 5 \\ 1 & u \geq 5 \end{cases}$$

a) What is $E[U]$

Find the PDF first ; because its the same function as number 3.1.3 ; the PDF function is as follows:

$$f_U(u) \begin{cases} 0 & u < -5 \\ 1/8 & -5 \leq u < -3 \\ 0 & -3 \leq u < 3 \\ 3/8 & 3 \leq u < 5 \\ 0 & u \geq 5 \end{cases}$$

General equation for $E[U]$ is

$$E[U] = \int_{-\infty}^{\infty} u \, f_U(u) \, du$$

So, the answer is:

$$E[U] = \cancel{\int_{-\infty}^{-5} u(0)\,du} + \int_{-5}^{-3} u\left(\tfrac{1}{8}\right) du + \cancel{\int_{-3}^{3} u(0)\,du} + \int_{3}^{5} u\left(3/8\right) du + \cancel{\int_{5}^{\infty} u(0)\,du}$$

$$= \int_{-5}^{-3} u\left(\tfrac{1}{8}\right) du + \int_{3}^{5} u\left(3/8\right) du$$

$$= \frac{1}{2}\cdot\frac{1}{8}\left(u^2\right)\Big]_{-5}^{-3} + \frac{1}{2}\cdot\frac{3}{8}\cdot u^2\Big]_{3}^{5}$$

$$= \frac{1}{16}\left(\underbrace{(9-25)}_{-16} + 3\underbrace{(25-9)}_{16}\right) = \frac{1}{16}(32) = 2$$

b) What is Var [U] ?

General equation:

$$Var[U] = \int_{-\infty}^{\infty} (u - E[u])^2 f_U(u)\, du = E[U^2] - E[U]^2$$

$\cdot)$ $E[U]^2 = 2^2 = 4$

$\cdot)$ $E[U^2] = \int_{-\infty}^{\infty} u^2 f_U(u)\, du$

$$= \int_{-5}^{-3} u^2 \frac{1}{8}\, du + \int_{3}^{5} u^2 \left(\frac{3}{8}\right) du$$

$$= \frac{1}{8}\frac{1}{3}\left(u^3 \right]_{-5}^{-3} + 3\, u^3 \Big]_3^5$$

$$= \frac{1}{24}\left(-27 + 125 + 3(125 - 27)\right)$$

$$= \frac{1}{24}\left(98 + 3(98)\right) = \frac{1}{\cancel{24}\,6} \cdot \cancel{4}\,(98) = \frac{49}{3}$$

$$Var[U] = \frac{49}{3} - 4 = \frac{49}{3} - \frac{12}{3} = \frac{37}{3}$$

$\boxed{4.1.3}$ The density function of continous random variable X, the total number of hours, in units of 100 hours, that a family runs a vaccum cleaner over a period one year, is given in Exercise 3.7 on page 92 as

$$f_X(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the average number of hours per year that families run their vacuum cleaner!

$$E[X] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

$$= \int_0^1 x \cdot x \cdot dx + \int_1^2 2x - x^2 \, dx$$

$$= \frac{1}{3} x^3 \Big]_0^1 + x^2 - \frac{1}{3} x^3 \Big]_1^2$$

$$= \frac{1}{3} + \left(4 - \frac{8}{3} - 1 + \frac{1}{3}\right) = 3 - \frac{6}{3} = 3 - 2 = 1$$

$E[X] = 1$ in units of 100 hours.

$\boxed{4.39}$ The Total number of hours, in units of 100 hours, that a family runs on vacuum cleaner over a period of one years is a random variable X having density function given in Excercise 4.13 on page 117. Find the variance of X

$$f_X(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{else where} \end{cases}$$

Find the variances: $Var[X] = E[X^2] - E[X]^2$

$E[X]^2 = 1^2 = 1$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$= \int_0^1 x^3\, dx + \int_1^2 2x^2 - x^3\, dx$$

$$= \frac{x^4}{4}\Big]_0^1 + \left(\frac{2}{3}x^3 - \frac{x^4}{4}\right]_1^2$$

$$= \frac{1}{4} + \left(\frac{16}{3} - \frac{16}{4} - \frac{2}{3} + \frac{1}{4}\right) = \frac{14}{3} - \frac{14}{4}$$

$$= 14\left(\frac{4-3}{12}\right) = \frac{14}{12} = \frac{7}{6}$$

$$Var[X] = \frac{7}{6} - 1 = \frac{1}{6} \,//$$

$\boxed{6.15}$ A Lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3,8 minutes. Assume the distribution of trip times to be normally distributed.

a) What is the probability that a trip will take at least 1/2 hour?

$Pr[X > 30 \text{ minutes}]$ with X = how long the trip takes.

$Pr[X > 30 \text{ minutes}]$ map into Z table

$$z = \frac{X - \mu}{\sigma} = \frac{30 - 24}{3,8} = \frac{30}{19} = 1,578$$

$$Pr[Z > 1,578] = 1 - Pr[x \leq 1,578]$$
$$= 1 - 0,9428$$
$$= 0,0571$$

b) If the office opens at 09.00 am and the lawyer leaves his house at 8:45 am, daily. what percentage of the time is he late for work

The passangers will late to go to the office if he is having a trip that longer than the 15 minute, so $Pr[X > 15]$

mapping to Z table : $$Z = \frac{15 - 24}{3,8} = -2,368$$

$$Pr[Z > -2,368] : 1 - Pr[Z \leq -2,368]$$
$$= 1 - 0,00914 = 0,99106 = 99.106\ \%$$

c) If he leaves the house at 8:35 am and coffe is served at the office from 8:50 am until 9.00 am., what is the prob-ability that he misses coffee

The passanger will likely misses the coffee if the trip is to fast or to late, which is less than at least 15 minutes or more than at least 25 minutes.

$Pr[X \leq 15] \cup Pr[X > 25]$; mapping in to Z

$Z_1 = \dfrac{15-24}{3.8} = -2{,}368$; $Z_2 = \dfrac{25-24}{3{,}8} = 0{,}263$

$Pr[Z_1 \leq -2{,}37] + Pr[Z_2 > 0{,}263]$

$0{,}00914 + (1 - Pr[Z_2 \leq 0{,}263])$

$0{,}00914 + (1 - 0{,}6026)$

$0{,}00914 + 0{,}3974$

$= 0{,}40654$ //

d) Find the leght of time above which we find the slowest 15 % of the trips

$Pr[Z > \alpha] = 0{,}15$ the only possible value for α is $1{,}04$

$1 - Pr[Z \leq \alpha] = 0{,}15$

$Pr[Z \leq \alpha] = 0{,}85$ if $Z = 1{,}04$ than $Z = \dfrac{X - \mu}{\sigma}$

$1{,}04 = \dfrac{X - 24}{3{,}8}$ for $-1{,}04 = \dfrac{X - 24}{3{,}8}$

$X = 27{,}952$ // $X = 20{,}048$

e) Find the probability that 2 of the next 3 trips will take at least 1/2 hour.

A combination between normal distribution and binomial distrubusion

$Pr[X > 30] = 0{,}0571$

The 2 of 3 trips were take at least 1/2 hour can be modeled as follows:

$\dfrac{T}{T} \quad \dfrac{T}{F} \quad \dfrac{F}{T}$

T = True
F = False

Which is $3C_2 = 3$

So; $b(k, n, p) = 3C_2 \cdot p^2 \cdot q^{3-2}$

$= 3(0{,}0571)^2 \cdot 0{,}9429 = 0{,}00922$ //

6.16 In the November 1990 issue of Chemical Engineering Progress, a study discussed the percent purity of oxygen from a certain supplier. Assume that the mean was 99,61 with a standard deviation of 0,08. Assume that the distribution of percent purity was approximately normal.

a) What precentage of the purity values would you expect between 99.5 and 99.7 ?

$$Z = \frac{99,5 - 99,61}{0,08} = -1,375$$

$$Z = \frac{99,7 - 99,61}{0,08} = 1,125$$

$$Pr[-1,375 < Z \leq 1,125] = Pr[Z \leq 1,125] - Pr[Z \leq -1,375]$$

$$0,8697 - 0,08455$$

$$= 0,78515$$

b) What purity value would you expect to exceed exactly 5 % of the population.

$$Pr[Z \leq \alpha] = 0,05$$

The most likely of α is -1.645 with $Pr[Z \leq -1,645] = 0,05$

$$-1,645 = \frac{X - 99,61}{0,08} \rightarrow X = 99,478$$

For the higher purity ; $Pr[Z \geqslant 1{,}645] = 0{,}05$

$$1{,}645 = \frac{X - 99{,}61}{0{,}08}$$

$$X = 99{,}7416$$