

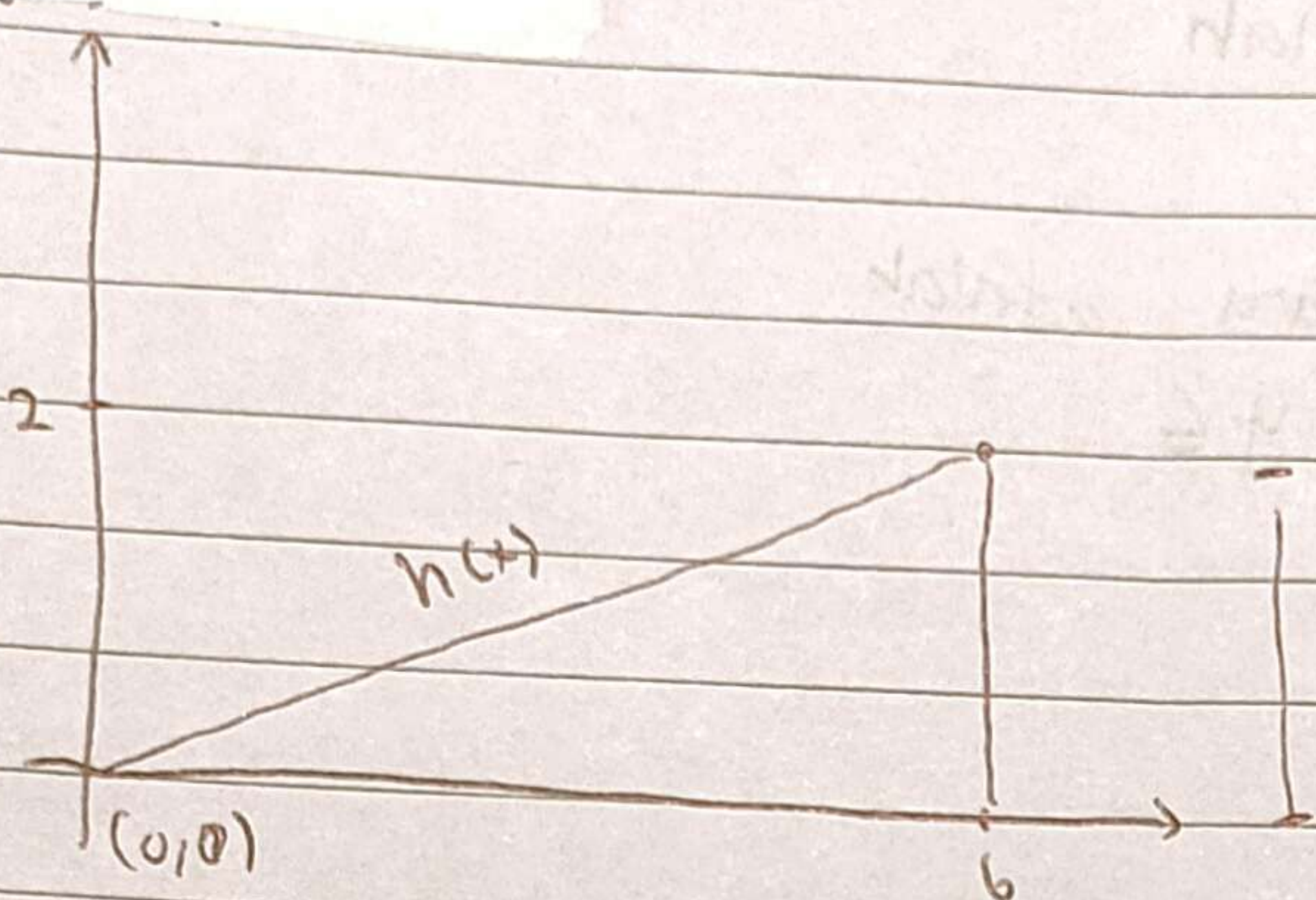
Quiz 1

1. Volume

$$f(x, y) = 3 + 2 \cos x^2$$

segitiga $(0,0)$, $(6,0)$ dan $(6,2)$

Batas = $0 \leq x \leq 6$; $0 \leq y \leq h(x)$



mencari $h(x) \rightarrow y - y_1 = m(x - x_1)$

$$m = \frac{2}{6} = \frac{1}{3}$$

$$y - 0 = \frac{1}{3}(x - 0)$$

$$y = \frac{1}{3}x$$

$$\text{Volume} = \int_0^6 \int_0^{\frac{1}{3}x} (3 + 2 \cos x^2) dy dx$$

$$= \int_0^6 \left[3y + 2y \cos x^2 \right]_0^{\frac{1}{3}x} dx$$

$$= \int_0^6 \left(x + \frac{2}{3}x \cos x^2 \right) dx$$

$$= \left[\frac{1}{2}x^2 + \frac{2}{3} \left(\frac{1}{2}x^2 \cdot \frac{1}{2x} \sin x^2 \right) \right]_0^6$$

$$= 18 + \frac{2}{12} (6 \sin 36)$$

$$= 18 + \sin 36$$

$$= 18 + (-0,99)$$

$$= 17,01 //$$

2) Volume S

$$z^2 \leq y \leq 2 - 2x^2 - z^2, x \geq 0$$

$$f(x, y, z) =$$

$$f(x, z) = 2 - 2x^2 - z^2 - z^2$$

$$(f(x, y, z) - f(x, y, 0)) \cdot n_z = (f(x, y, z) - f(x, y, 0)) \cdot n_z = (2 - 2x^2 - z^2 - z^2 - 2) \cdot (-1) = 2z^2$$

Setelah melihat Geogebra batasnya menjadi

$$0 \leq x \leq 1, -1 \leq z \leq 1$$

$$\int_0^1 \int_{-1}^1 (2 - 2x^2 - 2z^2) dz dx = \int_0^1 \int_{-1}^1 (2 - 2x^2 - 2z^2) dz dx$$

$$= \int_0^1 \left(2z - 2xz^2 - \frac{2}{3} z^3 \right) \Big|_{-1}^1 dx$$

$$= \int_0^1 \left(\frac{8}{3} - 4x^2 \right) dx$$

$$= \left[\frac{8}{3} x - \frac{4}{3} x^3 \right]_0^1 = \frac{8}{3} - \frac{4}{3} = \frac{4}{3}$$

3) Tentukan $\int_C f(x,y) ds$; $f(x,y) = x(y+1) + y$

$$C = x^2 + y^2 = 4$$

$$C = \begin{cases} x = t \\ y = \sqrt{4-t^2} \end{cases}$$

$$ds = \sqrt{1^2 + \left(\frac{-t}{\sqrt{4-t^2}}\right)^2} dt$$

$$= \sqrt{1 + \frac{t^2}{4-t^2}} dt$$

$$-\sqrt{3} \leq x \leq 0 \rightarrow -\sqrt{3} \leq t \leq 0$$

$$\int_{-\sqrt{3}}^0 (x(y+1) + y) \frac{2}{\sqrt{4-t^2}} dt = \int_{-\sqrt{3}}^0 (t(\sqrt{4-t^2} + 1) + \sqrt{4-t^2}) \frac{2}{\sqrt{4-t^2}} dt$$

$$= \int_{-\sqrt{3}}^0 (t(\sqrt{4-t^2} + 1) + \sqrt{4-t^2}) \frac{2}{\sqrt{4-t^2}} dt = \int_{-\sqrt{3}}^0 (t\sqrt{4-t^2} + t + \sqrt{4-t^2}) \frac{2}{\sqrt{4-t^2}} dt$$

$$= \int_{-\sqrt{3}}^0 2 \left(t\sqrt{4-t^2} + t + \sqrt{4-t^2} \right) dt = \int_{-\sqrt{3}}^0 2 \left(t + \frac{t}{\sqrt{4-t^2}} + 1 \right) dt$$

$$= 2 \left(\frac{1}{2} t^2 + (-\sqrt{4-t^2}) + t \right) \Big|_{-\sqrt{3}}^0 = 2 \left(\frac{1}{2} + (-\sqrt{4-1}) + (-\sqrt{3}) \right)$$

$$= 2 \left(\frac{1}{2} - 1 - \sqrt{3} \right) = 1 - 2\sqrt{3}$$