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21/481767/TK/53170

PR Tutor TVM 6

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ -1 & -3 & -5 \end{bmatrix}$$

* Basis column space

$A \xrightarrow{\text{rref}} R$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ -1 & -3 & -5 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & \underline{2} & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{rank} = 2$$

- Basis column space : $C(A)$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \& \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

This because the rank indicate the possition of the basis

- Basis row space $C(A^T)$:

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}^T \& \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}^T$$

- Basis Null space $N(A)$:

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

free variable

x_3	x_1 x_2
1	$-\frac{1}{2}$ $-\frac{3}{2}$
0	0 0

S_1

$$N(A) = \begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ 1 \end{bmatrix}$$ this is the basis for $N(A)$

- Basis for left null space $N(A^T)$

$$A^T = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & -3 \\ 2 & 3 & -5 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 2 & 3 & -5 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 0 & 3 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix} = R^T$$

$$R^T x = 0 \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

free variable

x_3	x_1 x_2
1	1 1
0	0 0

$\rightsquigarrow S_2$

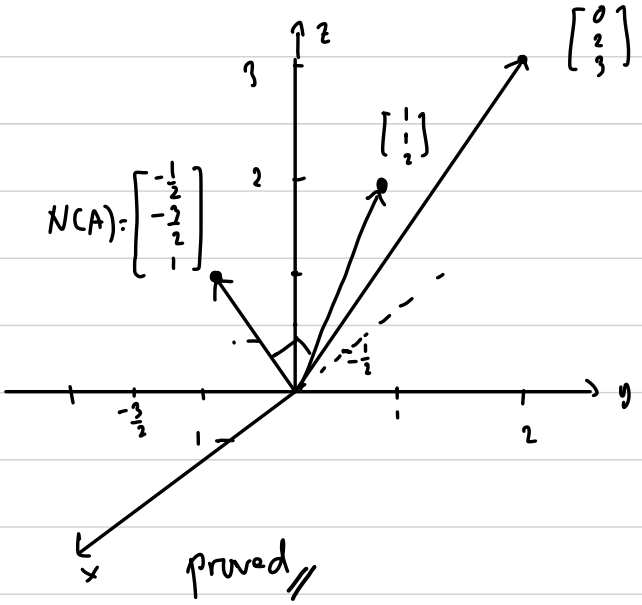
S_0, $N(A^T) = [1, 1, 1)$ is the basis of $N(A^T)$

Proof of $N(A) \perp C(A^T)$ and $N(A^T) \perp C(A)$

next page

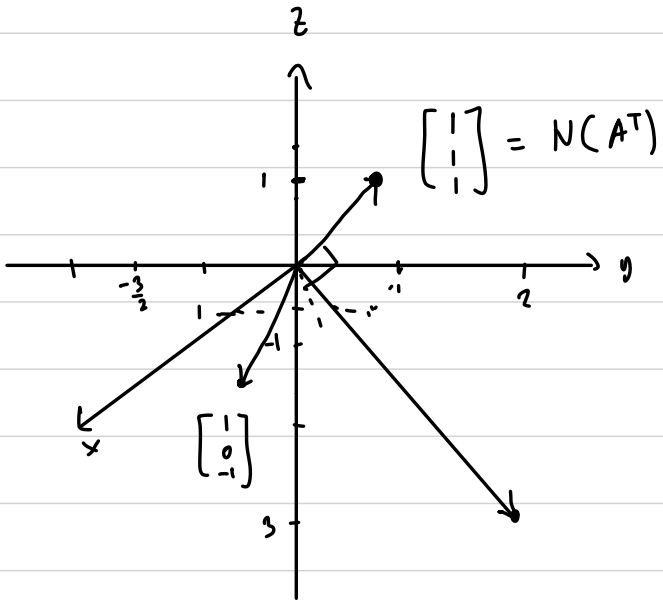
With picture !

$N(A) \perp C(A^T)$



proved //

$N(A^T) \perp C(A)$



proved //