Qornain Aji
21/481767/TK/53170

Tugas I

1. Tentukan semua $z \in \mathbb{C}$ sehingga $\cos z = -i$

$\cos z = \underbrace{\cos x \cosh y}_{0} - i \sin x \sinh y = -i$

$\Leftrightarrow$

$\cos x \cosh y = 0$

- $\cosh y > 0$; $\cos x = 0 \Leftrightarrow \cos x = \cos\left(\frac{1}{2}+n\right)\pi \Leftrightarrow x = \left(\frac{1}{2}+n\right)\pi$; $n \in R$
- $\sin x = \pm 1$

* Kasus : $\sin x = -1$; $x = \left(\frac{1}{2}+2n+1\right)\pi$

$\sinh y = -1$

$\Leftrightarrow \frac{e^y - e^{-y}}{2} = -1 \Leftrightarrow e^y - e^{-y} = -2 \Leftrightarrow e^{2y} - 1 + 2e^y = 0$

$\boxed{e^y = r}$

$\Leftrightarrow r^2 + 2r - 1 = 0$

$\Leftrightarrow r_{1,2} = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$\Leftrightarrow e^y = -1 \pm \sqrt{2}$

$y = \ln|-1 \pm \sqrt{2}| = \ln(-1+\sqrt{2})$

Penyelesaian kasus

$z = \left(\frac{1}{2}+2n+1\right)\pi + i\ln(-1+\sqrt{2})$

* Kasus 2 : $\sin x = +1$; $x = \left(\frac{1}{2}+2n\right)\pi$

$\sinh y = 1$

$\Leftrightarrow \frac{e^y - e^{-y}}{2} = 1 \Leftrightarrow e^y - e^{-y} = 2 \Leftrightarrow e^{2y} - 1 - 2e^y = 0$

$\Leftrightarrow r^2 - 2r - 1 = 0$ $\boxed{e^y = r}$

$r_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

$\Leftrightarrow e^y = 1 \pm \sqrt{2}$

$\Leftrightarrow y = \ln|1 \pm \sqrt{2}|$

$\Leftrightarrow y = \ln(1+\sqrt{2})$

Penyelesaian kasus

$$z = \left(\tfrac{1}{2} + 2n\right)\pi + i \ln\left(1+\sqrt{2}\right)$$

$\boxed{2}$ Tentukan semua $z \in \mathbb{C}$ sehingga $\tanh z = 0$

$$\tanh z = \frac{\sinh z}{\cosh z} = \frac{\sinh x \cos y + i \cosh x \sin y}{\cosh x \cos y + i \sinh x \sin y} = 0$$

$$\Leftrightarrow \frac{\sinh x \cos y + i \cosh x \sin y}{\cosh x \cos y + i \sinh x \sin y} \cdot \frac{(\cosh x \cos y - i \sinh x \sin y)}{(\cosh x \cos y - i \sinh x \sin y)} = 0$$

$$\Leftrightarrow \frac{\cos^2 y \sinh x \cosh x - i \sinh^2 x \sin y \cos y + i \cosh^2 x \sin y \cos y + \sin^2 y \sinh x \cosh x}{\cosh^2 x \cos^2 y + \sinh^2 x \sin^2 y} = 0$$

$$\Leftrightarrow \frac{\sinh x \cosh x (1) + i \sin y \cos y (1)}{\cosh^2 x \cos^2 y + \sinh^2 x \sin^2 y} = \frac{\sinh x \cosh x}{\cosh^2 x \cos^2 y + \sinh^2 x \sin^2 y} + i \frac{\sin y \cos y}{\cosh^2 x \cos^2 y + \sinh^2 x \sin^2 y}$$

•) $\sinh x \cosh x = 0$

$\cosh x > 0$; $\sinh x = 0$

$$\Leftrightarrow \sinh x = \frac{e^x - e^{-x}}{2} = 0 \Leftrightarrow e^{2x} = 1 \Leftrightarrow e^x = \pm 1 \Leftrightarrow x = \ln 1 = 0$$

$$\cosh(0) = \frac{e^0 + e^{-0}}{2} = \frac{2}{2} = 1$$

Diketahui : $\sinh(0) = 0$; $\cosh(0) = 1$; $x = 0$

Substitusi ke Imajiner:

$$\text{•) } \frac{i \sin y \cos y}{\cosh^2 x \cos^2 y + \sinh^2 x \sin^2 y} \Leftrightarrow \frac{i \sin y \cos y}{(1)\cos^2 y + (0)\sin^2 y} \Leftrightarrow i \frac{\sin y}{\cos y} = 0$$

$\Leftrightarrow \tan y = 0$

$\Leftrightarrow \tan^{-1} 0 = y \Leftrightarrow y = k\pi$

Penyelesaian:

$z = 0 + i k\pi$

$\boxed{3}$ Tentukan semua $z \in \mathbb{C}$ sehingga $\log z = 2 + i\,3\pi$

$\log z = \ln r + i(\theta + 2k\pi) = 2 + i\,3\pi$

* $\Leftrightarrow \ln r = 2$

$\Leftrightarrow r = e^2$

* $i(\theta + 2k\pi) = i\,3\pi$

$\Leftrightarrow \theta + 2k\pi = 3\pi$

$\theta = 3\pi - 2k\pi$

Sehingga,

$\log z = 2 + i(3\pi)$

$z = e^{2 + i(3\pi)}$

$=$

$\boxed{4}$ Tentukan

a) $\text{Log}(-e^2 i)$

$\Leftrightarrow \text{Log}(-e^2 i) = \log(r e^{i\theta})$

$r = e^2$; sehingga $e^{i\theta} = -i$

$\theta = -\frac{\pi}{2}$

$\Leftrightarrow \text{Log}(-e^2 i) = \ln r + i\theta$

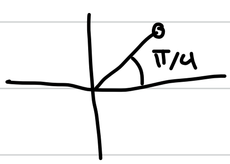
$= 2 - i\frac{\pi}{2}$,

b) $\log(1+i) = \ln|r| + i\theta$

∴ $1 + i = r e^{i\theta}$

$r = \sqrt{1+1} = \sqrt{2}$

$\theta = \frac{\pi}{4}$



$\log(1+i) = \ln\sqrt{2} + i\frac{\pi}{4} + 2k\pi$

c) $\log\left(i^{\frac{1}{2}}\right)$

$$i^{\frac{1}{2}} = r^{\frac{1}{2}} \operatorname{cis} \frac{\theta}{2} = r^{\frac{1}{2}} \operatorname{cis} \frac{\theta}{2}$$

$r = 1$

$\theta = \frac{\pi}{2}$

$$\log\left(i^{\frac{1}{2}}\right) = \ln|r| + i\,\frac{\theta + 2k\pi}{2}$$

$$\Leftrightarrow \ln 1 + i\frac{\pi}{4} + k\pi \Leftrightarrow i\frac{\pi}{4} + k\pi$$

∘) $k = 0$; maka :

$$\log\left(i^{\frac{1}{2}}\right) = i\frac{\pi}{4} + 2k\pi$$

∘) $k = 1$; maka

$$\log\left(i^{\frac{1}{2}}\right) = i\frac{5\pi}{4} + 2k\pi = -i\frac{3}{4}\pi + 2k\pi \ //$$

d) $(1-i)^{1-i}$

$$\Leftrightarrow e^{(1-i)\log(1-i)}$$

∘) $1 - i = r e^{i\theta}$

∘) $r = \sqrt{2}$

∘) $\theta = -\frac{\pi}{4}$

$$1 - i = \sqrt{2}\, e^{-i\frac{\pi}{4}}$$

Sehingga :

$$e^{(1-i)\log(1-i)} = e^{(1-i)\ln\sqrt{2} - i\frac{\pi}{4} + 2k\pi} = e^{(\ln\sqrt{2} - i\left(\frac{1}{4} + 2k\right)\pi - i\ln\sqrt{2} - \left(\frac{1}{4} + 2k\right)\pi)}$$

$$\Leftrightarrow e^{(\ln\sqrt{2} - \left(\frac{1}{4} + 2k\right)\pi) - i(\ln\sqrt{2} + \left(\frac{1}{4} + 2k\right)\pi)}$$

untuk $k \in \mathbb{Z}$

4 e) $(-1)^{\frac{1}{\pi}}$

$\Leftrightarrow \left(e^{i\cancel{\pi}}\right)^{\frac{1}{\cancel{\pi}}}$

$\Leftrightarrow e^{i}$ //

$\boxed{5}$ $\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$

coba kita buat $\frac{i}{2} \log \frac{i+z}{i-z} = \theta$

$\tan \theta = ?$

$$\Leftrightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{e^{i\theta} - e^{-i\theta}}{\cancel{2}i}}{\frac{e^{i\theta} + e^{-i\theta}}{\cancel{2}}}$$

$$\Leftrightarrow \tan\theta = \frac{e^{i\theta} - e^{-i\theta}}{i(e^{i\theta} + e^{-i\theta})} = z$$

$$\Leftrightarrow e^{i\theta} - e^{-i\theta} = zi(e^{i\theta} + e^{-i\theta})$$

$$\Leftrightarrow e^{i\theta} - e^{-i\theta} = ize^{i\theta} + ize^{-i\theta}$$

$$\Leftrightarrow e^{i\theta} - ize^{i\theta} = ize^{-i\theta} + e^{-i\theta}$$

$$\Leftrightarrow e^{i\theta}(1 - iz) = e^{-i\theta}(1 + iz)$$

$$\Leftrightarrow \frac{e^{i\theta}}{e^{-i\theta}} = \frac{(1+iz)}{(1-iz)}$$

$$\Leftrightarrow e^{2i\theta} = \frac{(1+iz)}{(1-iz)}$$

$$\Leftrightarrow 2i\theta = \log \frac{(1+iz)}{(1-iz)}$$

$$\Leftrightarrow \theta = \frac{1}{2i} \log \frac{(1+iz)}{(1-iz)}$$ ▨ terbukti

Sehingga:

$$\tan^{-1} z = \theta \Leftrightarrow \frac{1}{2i} \log \frac{(1+iz)}{(1-iz)},$$