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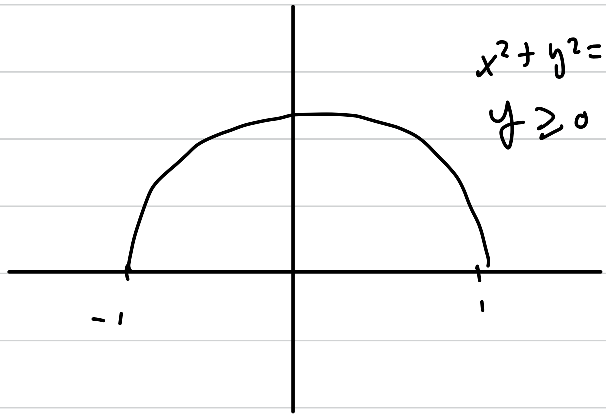
21/481767/TK/53170

Latihan Soal 5

1) $\int_c (2 + x^2 y)$



$x^2 + y^2 = 1$
$y \geq 0$

$ds = \sqrt{(-\sin t)^2 + (-\cos t)^2}\, dt$
$= \sqrt{\sin^2 t + \cos^2 t}\, dt = \sqrt{1}\, dt$
$= 1\, dt$

$x^2 + y^2 = r^2$
$r = 1$
$x = r \cos t = 1 \quad \vee \quad r \cos t = -1$
$\cos t = 1$
$t = 0 \quad \vee \quad t = \pi$
$0 \leq t \leq \pi$

$2 + x^2 y = 2 + \cos^2 t \sin t$

$\int_c (2 + \cos^2 t \sin t)\, 1\, dt = \int_0^{\pi} (2 + \cos^2 t \sin t)\, dt$

$= \left(2t - \frac{\cos^3 t}{3}\right)_0^{\pi} = 2\pi + \frac{1}{3} - \left(0 - \frac{1}{3}\right) = 2\pi + \frac{2}{3}$

2) $\int_C F \cdot dr \qquad 0 \le t \le 1$

$F(x,y,z) = z i + xy j - y^2 k$

$C\ r t = t^2 i + t j + \sqrt{t}\, k \qquad 0 \le t \le 1$

$F(r t) = \sqrt{t}\, i + t^3 j - t^2 k$

$r' t = 2t\, i + 1 j + \frac{1}{2\sqrt{t}}$

$\int_0^1 F \cdot dr$

$\int_0^1 (\sqrt{t}, t^3, -t^2)(2t, 1, \frac{1}{2\sqrt{t}})\, dt$

$\int_0^1 2t\sqrt{t} + t^3 + \frac{-t^{\cancel{2}}\sqrt{t}}{2\sqrt{\cancel{t}}}\, dt$

$= \int_0^1 \frac{3}{2} t\sqrt{t} + t^3\, dt$

$= \left(\frac{3}{2}\left(\frac{2}{5}(t)^{\frac{5}{2}}\right) + \frac{1}{4} t^3\right)\Big|_0^1$

$= \frac{3}{2}\frac{2}{5} + \frac{1}{4} = \frac{3}{5} + \frac{1}{4} = \frac{12+5}{20} = \frac{17}{20}$

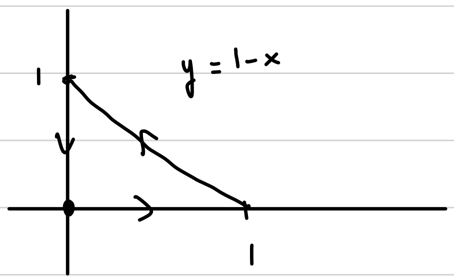
Theorem Green

1. Evaluate $\int x^4\, dx\ C + xy\, dy$

line segment $(0,0) \rightarrow (1,0)$

$(1,0) \rightarrow (0,1)$

$(0,1) \rightarrow (0,0)$



$= \iint_D \left(\frac{xy}{dx} - \frac{x^4}{dy}\right) dx\, dy$

$= \iint_D y\, dx\, dy = \int_0^{1-x}\int_0^1 y\, dx\, dy$

$= \int_0^1 \left(\frac{y^2}{2}\right)_0^{1-x} dx = \int \frac{(1-x^2)}{2}\, dx$

$\rightarrow -\left[\frac{(-x)^3}{6}\right]_0^1$

$-\left(\frac{0-1}{6}\right)$

$= \frac{1}{6}$

4) F(x,y) Fulida

$F(x,y) = (-3y, 3x)$

$\quad\quad\quad\quad\quad\quad M \quad N$

$$\oint_C = \vec{F}\, N\, ds$$

$M_x = 0$

$N_y = 0$

$$\int_0^{2\pi}\int_0^1 (0+0)\, r\,dr\,d\theta = 0 \text{ // Fluks}$$

Sirkulasi

$$\int_0^{2\pi}\int_0^1 3+3 \quad r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 6r\,dr\,d\theta$$

$$= \int_0^{2\pi} \left| 3r^2 \right|_0^1 d\theta$$

$$= \int_0^{2\pi} 3\,d\theta$$

$$= \left| 3\theta \right|_0^{2\pi}$$

$$= 6\pi \text{ //}$$