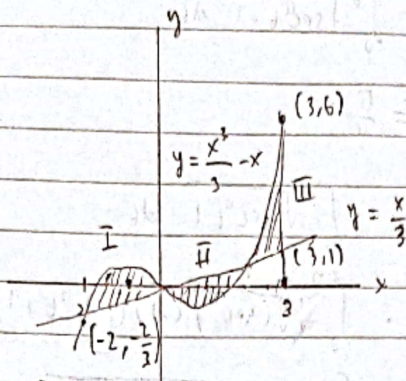


1. Cari lah luas dari wilayah berikut ini.



$$\begin{aligned} \text{Area I} &= \int_{-2}^0 \left(\frac{x^3}{3} - x \right) - \frac{x}{3} dx \\ &= \int_{-2}^0 \frac{x^3 - 4x}{3} dx = \frac{1}{3} \int_{-2}^0 (x^3 - 4x) dx \\ &= \frac{1}{3} \left[\frac{1}{4} x^4 - 2x^2 \right]_{-2}^0 \\ &= \frac{1}{3} (0 - (4 - 8)) = \frac{1}{3} \cdot 4 = \frac{4}{3} \end{aligned}$$

Area II = Mencari rentang

$$\frac{x^3}{3} - x = x$$

$$\frac{x^3}{3} - \frac{4}{3}x = 0$$

$$x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$$x = 0, x = -2, x = 2$$

$$\begin{aligned} \therefore \text{Luas} &= \int_0^2 \left(\frac{x^3}{3} - x \right) - \frac{x}{3} dx \\ &= \int_0^2 \frac{4x - x^3}{3} dx = \frac{1}{3} \int_0^2 (4x - x^3) dx \\ &= \frac{1}{3} \left(2x^2 - \frac{1}{4}x^4 \right) \Big|_0^2 \\ &= \frac{1}{3} (8 - 4 - 0) = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \text{Area III} &= \int_2^3 \left(\frac{x^3}{3} - x \right) - \frac{x}{3} dx \\ &= \frac{1}{3} \int_2^3 (x^3 - 4x) dx = \frac{1}{3} \left[\frac{1}{4}x^4 - 2x^2 \right]_2^3 \\ &= \frac{1}{3} \left(\frac{81}{4} - 18 - (8 - 4) \right) = \frac{1}{3} \left(\frac{81}{4} - 22 \right) = \frac{1}{3} \left(\frac{81 - 88}{4} \right) = \frac{1}{3} \left(-\frac{7}{4} \right) = -\frac{7}{12} \end{aligned}$$

$$\frac{1}{3} (81 - 18 - (8 - 4)) = \frac{1}{3} (63 - 4)$$

$$\frac{1}{3} \cdot 49 = \frac{49}{3}$$

$$\text{Area total} = \frac{49}{3} + \frac{4}{3} + \frac{4}{3} = \frac{57}{3} = 19$$

2. Tentukan integral berikut.

$$\int \frac{(2r-1) \cos \sqrt{3(2r-1)^2 + 6}}{\sqrt{3(2r-1)^2 + 6}} dr$$

$$u = 3(2r-1)^2 + 6$$

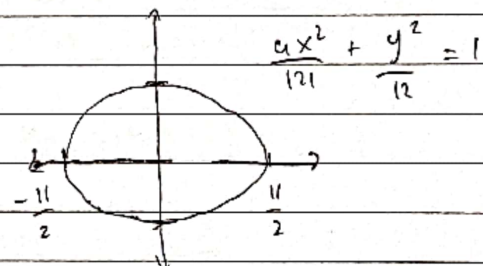
$$du = 12(2r-1) dr$$

$$\int \frac{(2r-1) \cos \sqrt{u}}{\sqrt{u}} \frac{du}{12(2r-1)}$$

$$\int \frac{\cos \sqrt{u}}{\sqrt{u}} du = 2 \sin \sqrt{u}$$

$$2 \sin \sqrt{u} = 2 \sin \sqrt{3(2r-1)^2 + 6} + C$$

3. Carilah volum benda yang dibentuk dengan



$$\text{Volume} = \frac{64}{7}$$

4) Tentukan Panjang kurva

$$x = \int_0^y \sqrt{\sec^4 t - 1} dt$$

$$-\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$$

$$x = \int_0^y \sqrt{\sec^4 t - 1} dt$$

$$= \int_0^y \sqrt{(\sec^2 t - 1)(\sec^2 t + 1)} dt$$

$$= \int_0^y \sqrt{\tan^2 t (\sec^2 t + 1)} dt$$

$$= \int_0^y \sqrt{\tan^2 t \sec^2 t + \sec^2 t} dt$$

$$= \int_0^y (\tan^2 t \sec^2 t + \sec^2 t)^{\frac{1}{2}} dt$$

$$= \int_0^y \left(\frac{\sin^2 t}{\cos^4 t} + \frac{1}{\cos^2 t} \right)^{\frac{1}{2}} dt$$

$$= \int_0^y \left(\frac{\sin^2 t + \cos^2 t}{\cos^4 t} \right)^{\frac{1}{2}} dt$$

$$= \int_0^y \left(\frac{1}{\cos^4 t} \right)^{\frac{1}{2}} dt = \int_0^y (\sec^2 t) dt$$

$$= \int_0^y \sec^2 t dt = \int_0^y \tan t dt$$

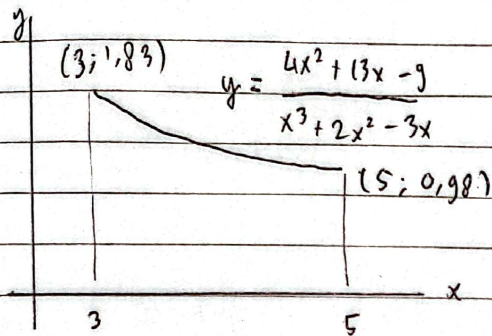
$$x = \tan y$$

$$\text{Panjang kurva} : \int_a^b \sqrt{1 + \left(\frac{dx}{dy} \right)^2} dy$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \left(\frac{d(\tan y)}{dy} \right)^2} dy$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \sec^4 y} dy$$

5 Centroid



Centroid Sumbu x :

$$\begin{aligned}\bar{x} &= \frac{\int_3^5 x f(x) dx}{\int_3^5 f(x) dx} = \frac{\int_3^5 \frac{4x^2 + 13x - 9}{x^3 + 2x^2 - 3x} dx}{\int_3^5 \frac{4x^2 + 13x - 9}{x^3 + 2x^2 - 3x} dx} \\ &= \frac{\int_3^5 (4x^2 + 13x - 9) dx}{\int_3^5 (x^3 + 2x^2 - 3x) dx} \\ &= \frac{\left[\frac{4}{3}x^3 + \frac{13}{2}x^2 - 9x \right]_3^5}{\left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2 \right]_3^5} \\ &= \frac{216.6}{177.3} \\ &= 1.220\end{aligned}$$

6 Trapezoid Rule $n=6$

$$\int_0^{\pi} 2 \sin^2 x dx$$

$$\frac{\pi - 0}{6} = \frac{\pi}{6} = 0.523$$

$$n_1 = 0.523 = \Delta x$$

$$n_2 = 1.046 = \Delta x$$

$$n_3 = 1.57$$

$$n_4 = 2.093$$

$$n_5 = 2.616$$

$$n_6 = 3.14$$

$$L_{\text{true}} = \frac{\Delta x}{2} (y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + y_6)$$

$$= \frac{0.523}{2} (0.498 + 2(1.497) + 2(2) +$$

$$2(1.502) + 2(0.503) + (5.07 \times 10^{-6}))$$

$$= 3.00777$$

$$\text{Error} = \frac{M(b-a)^3}{12n^2}$$

$$f'(x) = 2 \sin 2x$$

$$f''(x) = 4 \cos 2x$$

$$M = 4 \cos 2(\pi) = 4$$

$$\text{Error} = \frac{4(\pi)^3}{12(36)} = 0.287095$$

$$7] \text{ Simpson Rule} = \frac{\Delta x}{3} (y_0 + 4y_1 + 2y_2 + \dots + 2y_{n-2} + 4y_{n-1} + y_n)$$

$$n = 8$$

$$\Delta x = 15$$

$$= \frac{15}{3} (0 + 4(76) + 2(54) + 4(51) + 2(49,5) + 4(54) + 2(64,4) + 4(67,5) + 42)$$

$$= 6059$$

8] Ucah menarik benda berat 100 N
tali memiliki berat 0,4 N/m dengan
panjang = 20 m

$$\therefore W_{\text{benda}} = F_{\text{benda}} \cdot h$$

$$= (100) (20) = 2000 \text{ N}\cdot\text{m}$$

$$\therefore W_{\text{tali}} = \int_0^{20} 0,4(20-x) dx$$

$$= \int_0^{20} (8 - 0,4x) dx$$

$$= \left[8x - 0,2x^2 \right]_0^{20}$$

$$= 160 - 80 = 80 \text{ N}\cdot\text{m}$$

$$W_{\text{total}} = W_{\text{benda}} + W_{\text{tali}}$$

$$= 2000 + 80 = 2080 \text{ N}\cdot\text{m}$$