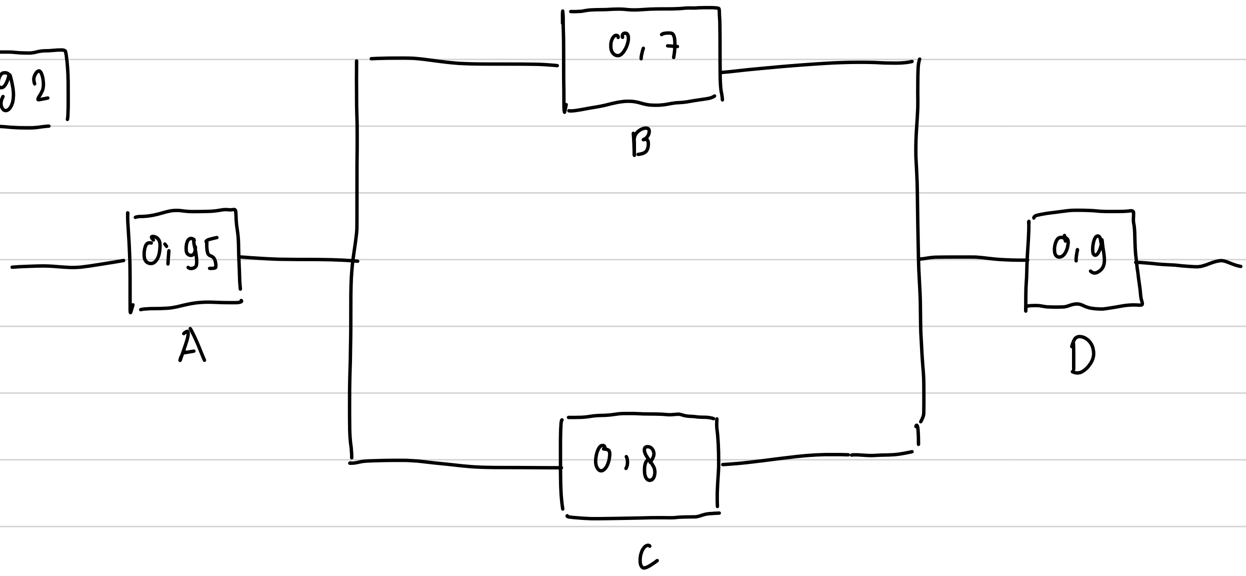
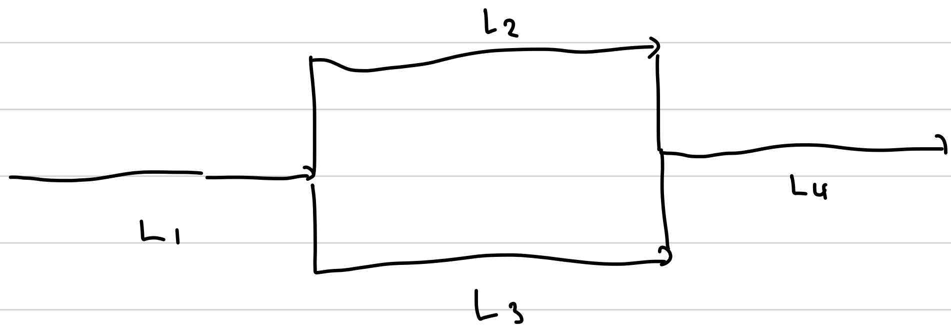
Qornain Aji
21/481767/TK/53170

2.92



Suppose the diagram of an electrical system is as given in Figure 2.10. What is the probability that the system works? Assume the components fail independently.

If we assume that there is in total 4 lane in the system that are $L_1, L_2, L_3, \& L_4$. We can draw the system as.



In order to make the system working, L_1 and L_4 must not fail. While L_2 and L_3 only need one of the at least working to make the system working properly. So we can conclude that

$(L_1 \cap (L_2 \cup L_3) \cap L_4)$ = The ways system ar working.

So the probability of system working properly are

$Pr(L_1 \cap (L_2 \cup L_3) \cap L_4)$.

First calculate the $Pr(L_2 \cup L_3) = Pr(L_2) + Pr(L_3) - Pr(L_2 \cap L_3)$

We know that :

$Pr(L_2) = Pr(B) = 0,7$

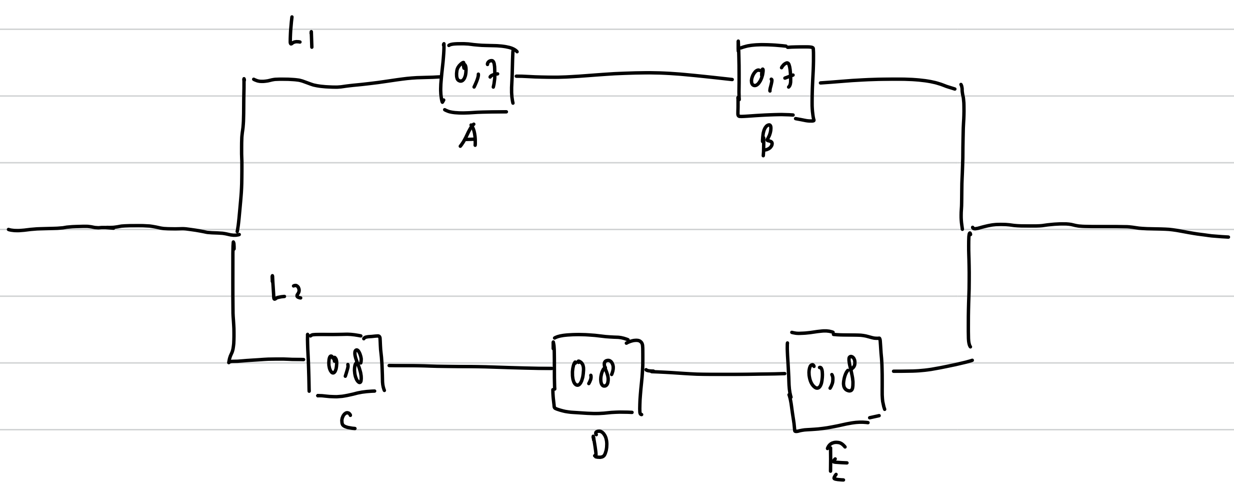
$Pr(L_3) = Pr(C) = 0,8$

$Pr(L_2 \cap L_3) = Pr(A \cap B) = Pr(A) . Pr(B) = 0,7 \times 0,8 = 0,56$

$Pr(L_2 \cup L_3) = 0,7 + 0,8 - 0,56 = 0,94$

Then, $Pr(L_1 \cap (L_2 \cup L_3) \cap L_4) = 0,95 \times 0,94 \times 0,9 = 0,8037$

2.93



A circuit system is given in Figure 2.11. Assume the components fail independently.

a) What is the probability that the entire system works?
b) Given that the system works, what is the probability that the component A is not working?

a) Same as the 2.92, we can assume that the system have 2 lane which is L_1 and L_2. In order to work properly, the system need at least one of the lane to work and not failing. So, we can conclude that :

The ways that the system working = $L_1 \cup L_2$
So, we can express the probability for the system to work properly to be:

$$Pr(L_1 \cup L_2) = Pr(L_1) + Pr(L_2) + Pr(L_1 \cap L_2)$$

We know that :
- $Pr(L_1) = Pr(A \cap B) = Pr(A) \times Pr(B) = 0{,}7 \times 0{,}7 = 0{,}49$ //
- $Pr(L_2) = Pr(C \cap D \cap E) = Pr(C) \times Pr(D) \times Pr(E) = (0{,}8)^3 = 0{,}512$
- $Pr(L_1 \cap L_2) = Pr(L_1) \times Pr(L_2) = 0{,}49 \times 0{,}512 = 0{,}25088$

So, $Pr(L_1 \cup L_2) = Pr(L_1) + Pr(L_2) + Pr(L_1 \cap L_2)$
$= 0{,}49 + 0{,}512 - 0{,}25088$
$= 0{,}75112$ //

b) Probability that A is not working (A') given that the system is working = $P(A' \mid \text{works})$

The formula is

$$Pr(A' \mid \text{Works}) = \frac{Pr(A' \cap \text{works})}{Pr(\text{works})}$$

The way A' but the whole sistem still working is,

$$(A' \cap B \cap C \cap D \cap E) \quad \text{or} \quad (A' \cap B' \cap C \cap D \cap E)$$

So $Pr(A' \cap B \cap C \cap D \cap E) + Pr(A' \cap B' \cap C \cap D \cap E) =$
$\big(Pr(A') \cdot Pr(B) \cdot Pr(C) \cdot Pr(D) \cdot Pr(E)\big) + \big(Pr(A') \cdot Pr(B') \cdot Pr(C) \cdot Pr(D) \cdot Pr(E)\big)$

$$Pr(A') = 1 - Pr(A) = 1 - 0{,}7 = 0{,}3$$
$$Pr(B') = 1 - Pr(B) = 1 - 0{,}7 = 0{,}3$$

$$Pr(A' \cap B \cap C \cap D \cap E) + Pr(A' \cap B' \cap C \cap D \cap E)$$
$$= (0{,}3)(0{,}7)(0{,}8)^3 + (0{,}3)(0{,}3)(0{,}8)^3$$
$$= 0{,}10752 + 0{,}04608 = 0{,}1536 = Pr(A' \cap \text{Works})$$

So, $$Pr(A' \mid \text{Works}) = \frac{Pr(A' \cap \text{Works})}{Pr(\text{Works})} = \frac{0{,}01536}{0{,}75112}$$

$$Pr(A' \mid \text{Works}) = 0{,}204495$$

Hence, the probability A is not working given that the system working is
$0{,}204495 \approx \underline{\underline{0{,}2045}}$

$\boxed{2.95}$ In a certain region of the country it is known from past experience that the probablity of selecting an adult over 40 years of age with cancer is 0,05. If the probability of a doctor correctly diagnosing a person with cancer as having a disease is 0,78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0,06; what is the probability that an adult over 40 years of age is diagnosed as having cancer

First, mention all the data:

- Pr (Adult over 40 years old with cancer) = $Pr(A) = 0{,}05$
- Pr (Docter correctly diagnozed a Person) = $Pr(DC \mid A) = 0{,}78$
- Pr (Doctor incorecty diagnozed a Person) = $Pr(DC \mid A')$ $0{,}06$

Question => $Pr(D_c)$

Bayes law => $Pr(D_c) = Pr(DC \cap A) + Pr(DC \cap A')$

$$= \quad Pr(D_c) = Pr(DC \mid A)\, Pr(A) + Pr(D_c \mid A')\, Pr(A')$$

$$= 0{,}78 \times 0{,}05 + 0{,}06 \times 0{,}95$$

$$Pr(D_c) = 0{,}096 \,//$$

$\boxed{2.97}$ Refering 2.95, what is the probability that a person diagnosed as having cancer actually has the disease?

The quetion is what is the probability that aperson have cancer given that have been diagnosed cancer?

Bayes probality : $Pr(A \mid Dc) = \frac{Pr(A \cap Dc)}{Pr(Dc)} = \frac{Pr(Dc \cap A)}{Pr(Dc)}$

$$= \frac{Pr(Dc \mid A)\, Pr(A)}{Pr(Dc)} = \frac{0{,}78 \cdot 0{,}05}{0{,}096}$$

$$= \frac{0{,}039}{0{,}096} = 0{,}406 \;//$$

So the probability that aperson have cancer given that have been diagnored cancer is 0,406 //