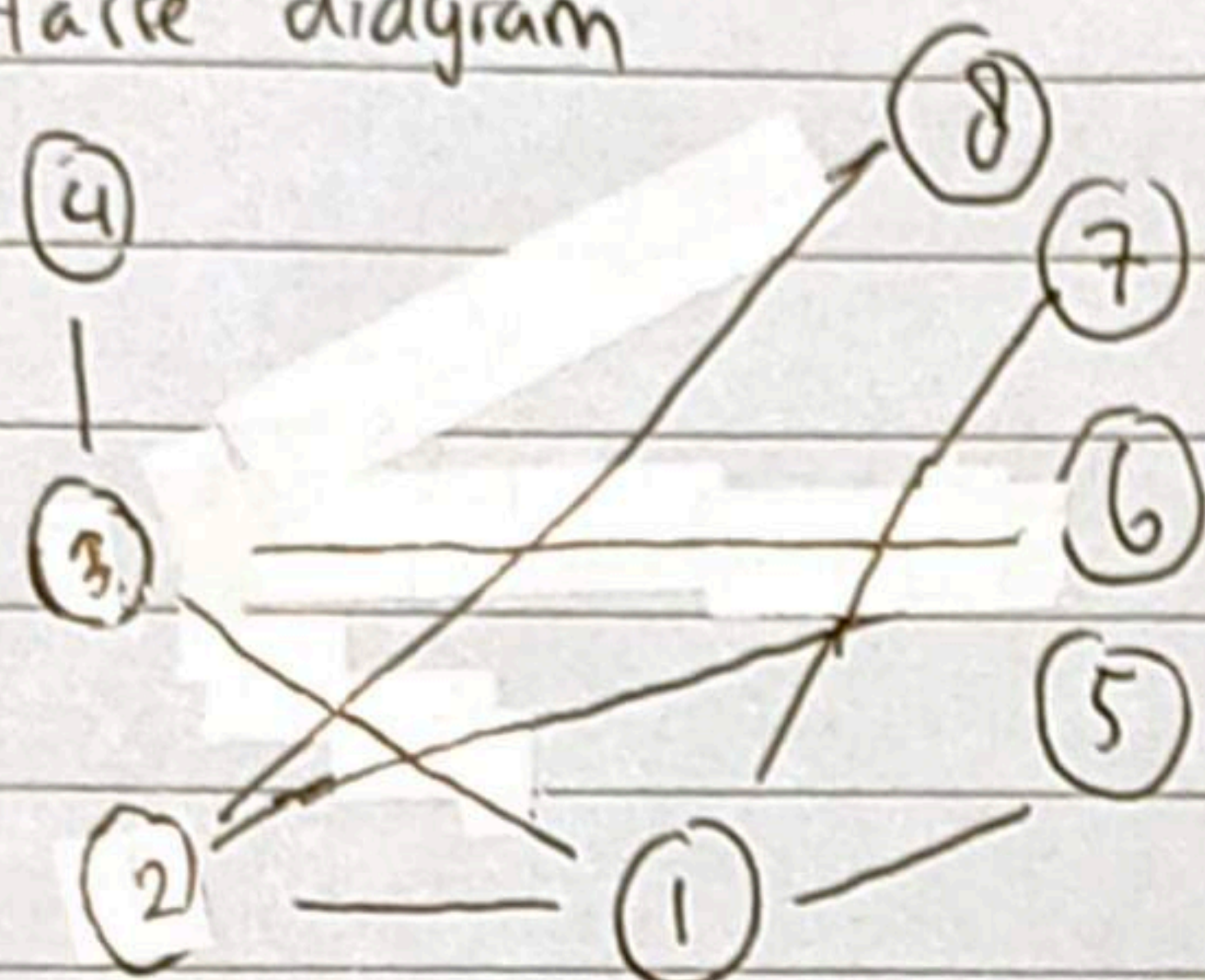


Type A

$$R = \{(a, b) \mid a \text{ divides } b\}$$

[A1] a)  $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8)\}$

b) Hasse diagram



c) An equivalence relation sets need to be reflective, symetry, and transitif

∴ This sets is reflective because  $(\forall x \in A) x R x$ . In other words, every  $x$  element in the sets have loop.

∴ This sets is not symmetrical, because  $(\forall x, y \in A) x R y \rightarrow y R x$ . For proof, there is  $(1, 2)$  but there is no  $(2, 1)$ . Thus the sets is not symetry and the sets is not equivalence relations.

c) Determine if these sets is equivalence relation!

∴ This sets is reflective because  $(\forall x \in A) x R x$ . In other words, every  $x$  element in the sets have loop.

∴ This sets is not symmetrical because  $(\forall x, y \in A) x R y \rightarrow y R x$ . For proof, there is  $(5, 10)$  but there is no  $(10, 5)$ . Thus the sets is not symetry and the sets is not equivalence relation.

Type B

[B1] Proof that Least Common Multiple of  $j, k$  is  $\frac{jk}{i} \rightarrow \text{lcm}(j, k) = \frac{jk}{i}$

∴ We know that if  $ab$  is an integer, then:

$$ab = \text{gcd}(a, b) \cdot \text{lcm}(a, b)$$

Now, we know that  $\text{gcd}(j, k) = i$  while

$$jk = \text{gcd}(j, k) \cdot \text{lcm}(j, k)$$

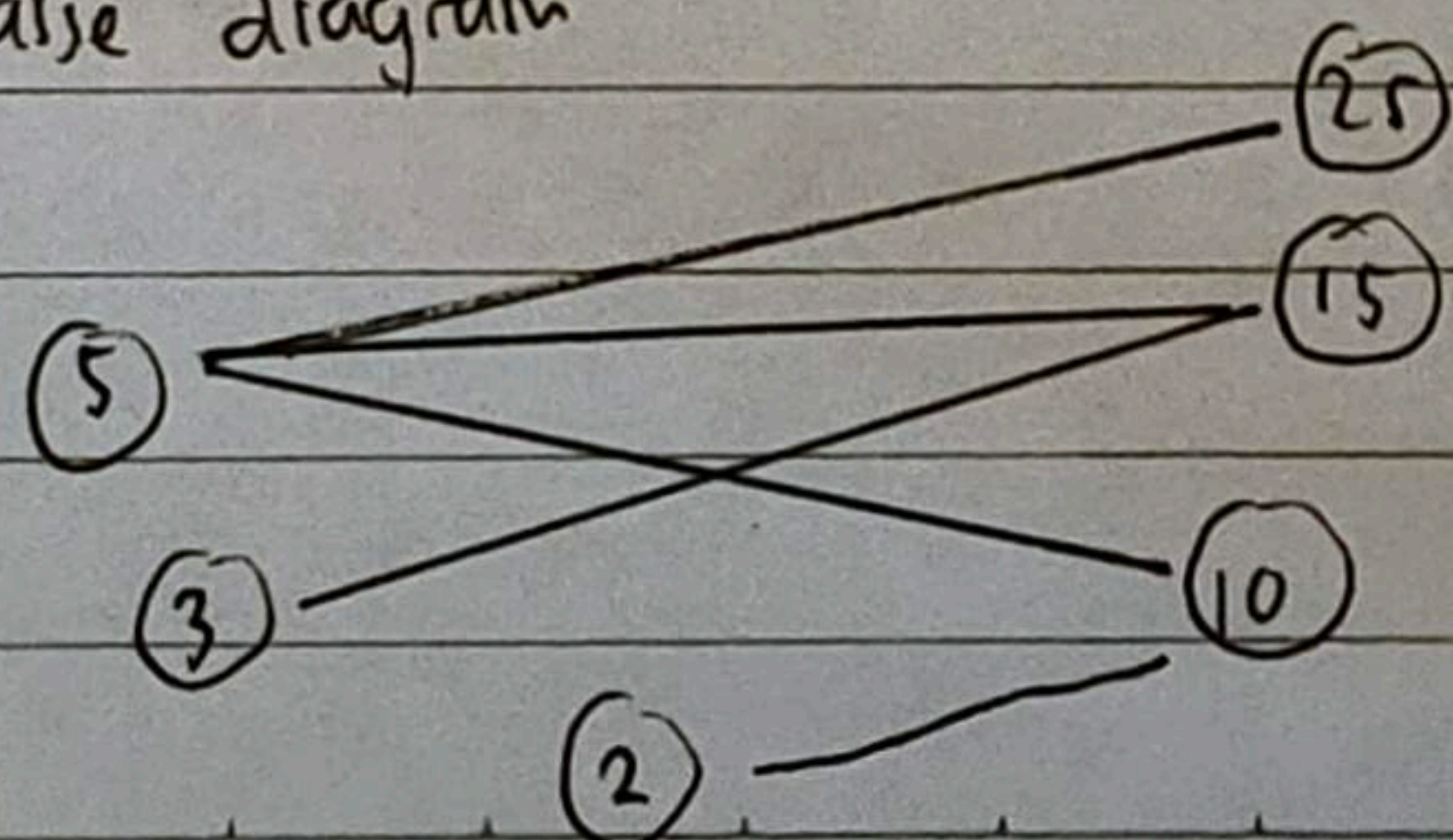
Thare proof that

$$\text{lcm}(j, k) = \frac{jk}{\text{gcd}(j, k)} = \frac{jk}{i}$$

Thus,  $\text{lcm}(j, k) = \frac{jk}{i}$  is true  $\square$

[A2] a)  $R = \{(2,2), (2,10), (3,3), (3,15), (5,5), (5,10), (5,15), (5,25), (10,10), (11,11), (15,15), (25,25)\}$

b) Hasse diagram



[B2] Proof that  $r^{q-1} \equiv 1 \pmod{q}$   
 $\text{gcd}(q, r) = 1$

$$ir \equiv jr \pmod{q} \quad 1 \leq i, j \leq (q-1)$$

Thus the proof of  $r^{q-1} \equiv 1 \pmod{q}$  is true  $\square$



Type C

ID Number : 21/481767 / TK / 53170

xy xi xi

[C<sub>0</sub>] Set  $A = (8-7) \bmod 3 + 1$

Set  $B = (0+1+3) \bmod 4 + 1$

Set  $A = 1 \bmod 3 + 1 = 2$

Set  $B = 4 \bmod 4 + 1 = 1$

[C<sub>1</sub>] Recursive algorithm to compute  $\sum_{i=0}^n \frac{A}{B^i} !$

$$\sum_{i=0}^n \frac{A}{B^i} !$$

Initial value :  $f(0) = \frac{2}{1^0} ! = 2 ! = 2$

$f(1) = \frac{2}{1^1} ! = \frac{2}{1} ! = 2$

The recursive algorithm is

$f(i+1) = f(i) \cdot 1$

Because all iteration will resulting to divide A with 1, then the recursive algorithm should be true.

[C<sub>2</sub>] We are going to prove inductively, if  $i \in \mathbb{Z}$ , then  $k \cap k+1 \in \mathbb{Z}$ .

Set proposition:  $P(n) = f(n+1) = f(n) \cdot 1$

Basis step :  $P(0) : f(0+1) = f(0) \cdot 1$

when  $f(0) = 2$  then  $f(0+1) = 2$

Inductive Steps:  $P(k) \rightarrow P(k+1)$

Inductive Hipotesis :

$P(k) : f(k+1) = f(k)$

Show :  $P(k+1) : f(k+2) = f(k+1)$

$f(k+2) = f(k+1) = f(k)$

For  $P(k+2)$ , the proposition is always true

Thus  $P(n)$  is true

[C<sub>3</sub>] How many ways to place A student from different department tutor assistant in  $A+B-1$  parallel classes at DETI

$A = 2 = n$

$A+B-1 = 2$

The way to put it is using DOUB!

$\sum_{j=1}^k S(n,j)$  while

$S(n,j) = \frac{1}{j!} \sum_{i=0}^{j-1} (-1)^i \binom{j}{i} (j-i)^n$

$S(2,1) = \frac{1}{1!} \sum_{i=0}^0 (-1)^i \binom{2}{i} (2-i)^2$

$= 1$

$S(2,2) = \frac{1}{2!} \sum_{i=0}^1 (-1)^i \binom{2}{i} (2-i)^2$

$= 2 + (-1) = 2$

$\sum_{j=1}^k S(n,j) = 1 + 2 = 3$  ways.

[C<sub>4</sub>] Construct language :  $G = (V, T, S, P)$

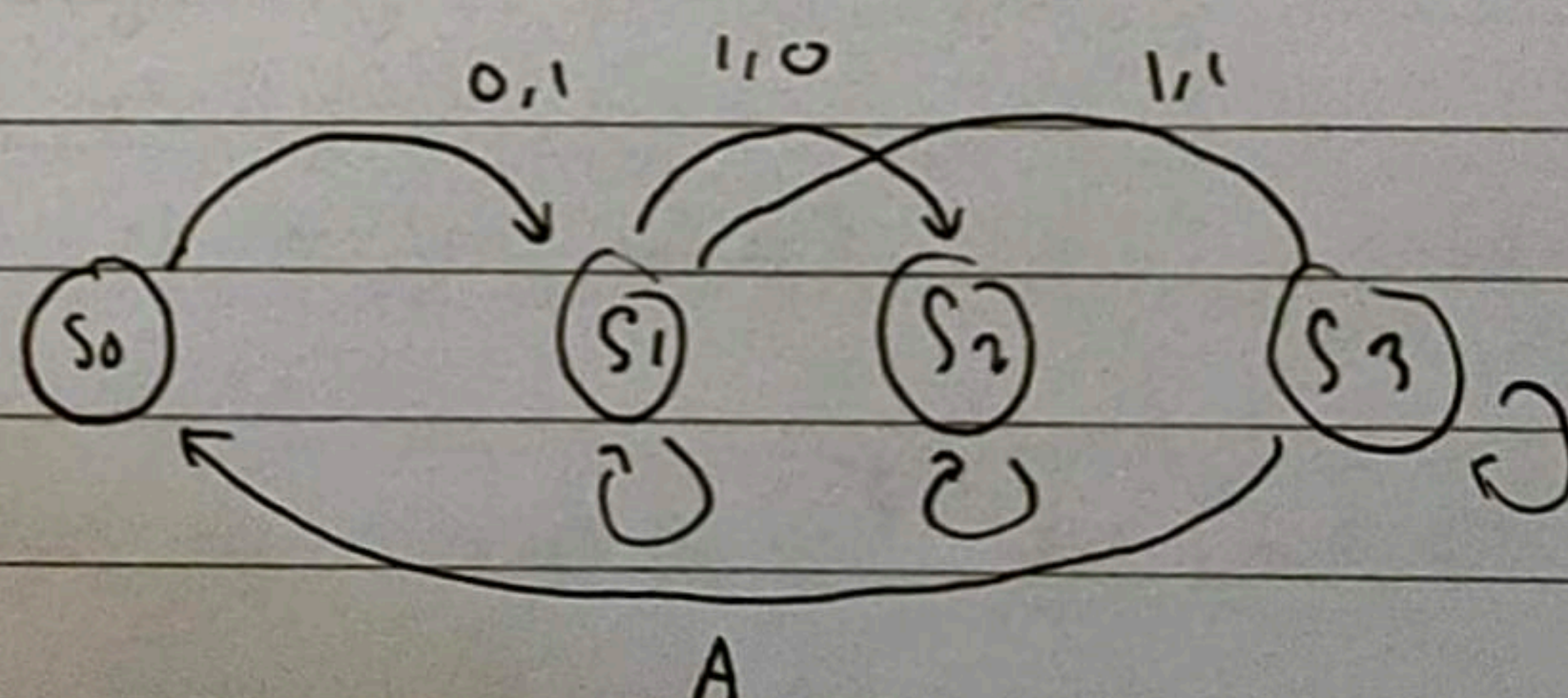
$V = \{ A \leq x < 10 \cup B \leq x < 10 \}$

$T = \{ A, x, B \}$ ,  $A=2$  dan  $B=2$

$P = \{ S \rightarrow AB, S \rightarrow Ax, x \rightarrow AB, x \rightarrow Ax, x \rightarrow xB, x \rightarrow Bx \}$

[C<sub>5</sub>]  $I = \{ mango, durian, apple, duku \}$

FSM





### Type D

- [1] Find route with shortest time from Boston to Los Angeles!

Index	L	V-L	-	D <sub>c</sub>	D <sub>N</sub>	D <sub>Da</sub>	D <sub>De</sub>	D <sub>S</sub>	D <sub>L</sub>
-	AF	{source}		3	2	∞	∞	∞	∞
N	{N}	{-N}		3		8	∞	∞	∞
C <sub>NY</sub>	{N, C}	{source, -N, C}			10	8	9	8	10
D <sub>NY</sub>	{N, C, D <sub>NY</sub> }	{source, -N, C, D <sub>NY</sub> }					9	8	10
D <sub>S</sub>	{S, N, C, D <sub>NY</sub> }	{source, -N, C, D <sub>NY</sub> }					9		16
D <sub>c</sub>	{S, N, C, D <sub>NY</sub> , D <sub>c</sub> }	{source, -N, C, D <sub>NY</sub> , D <sub>c</sub> }							10

The shortest route has total time of 10 second

- [2] Minimum Spanning Tree

B → NY → P<sub>1</sub> → A → NO → ME  
CH → CI → I, A → MI

$$3 + 5 + 7 + 6 + 4 + 6 + 7 + 6$$

$$= 44$$

