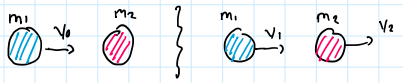
Homework # 1 Qornain Aji
Friday, 08 October 2021 14.38 21/481767/TK/53170

Classic Mechanics :
Conservation of Linear Momentum

Problem # 1 : Collision between Several Balls

A. For case A, to determine the value of v_1 and v_2, we need to find what are the variable that are involve in the case A.

We know :



∴ m_1 as the mass of blue ball
∴ m_2 as the mass of red ball
∴ V_0 as the initial velocity of blue ball
∴ V_1 as the final velocity of blue ball
∴ V_2 as the final velocity of blue ball.

∴ e as the modulo elastic between two balls is:
$e = 1$; which mean the two balls is perfectly elastic.

The equation of e (elastic modulus) can be express as

$$e = \frac{-(V_{2f} - V_{1f})}{V_{2i} - V_{1i}}$$

The variable V_{2f}, V_{1f}, V_{2i}, and V_{1i} can be replaced in consecutively as V_2, V_1, 0, and V_0. So we can obtain a linear equation of

$$e = \frac{-(V_2 - V_1)}{0 - V_0} = 1$$

$$V_2 - V_1 = V_0 \quad \ldots (i)$$

After we find the first equation, we can find another equation by using the law of conservation of momentum. The equation can be expressed as

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

As we know, $V_{1i} = V_0$; $V_{2i} = 0$; $V_{1f} = V_1$; and $V_{2f} = V_2$
We will replaced the following variable as

$$m_1 V_0 + m_2 (0) = m_1 V_1 + m_2 V_2$$

Know we'll solve the expression

$$m_1 V_0 + 0 = m_1 V_1 + m_2 V_2$$
$$m_1 V_0 = m_1 V_1 + m_2 V_2 \quad \ldots (ii)$$

∴ Know, with the help of equation (i); we can know find the value of V_1 and V_2. To find the value of V_1 we can do some substitution on the variable as

$$V_2 - V_1 = V_0$$

$$V_2 = V_0 + V_1$$

Replace V_2 in the equation (ii) with V_2 in the (i) equation.

$$m_1 V_0 = m_1 V_1 + m_2 V_2$$
$$m_1 V_0 = m_1 V_1 + m_2 (V_0 + V_1)$$
$$m_1 V_0 = m_1 V_1 + m_2 V_0 + m_2 V_1$$
$$m_1 V_0 - m_2 V_0 = m_1 V_1 + m_2 V_1$$
$$V_0 (m_1 - m_2) = V_1 (m_1 + m_2)$$
$$V_1 = \frac{V_0 (m_1 - m_2)}{m_1 + m_2}$$

The value of V_1 is $\frac{V_0 (m_1 - m_2)}{m_1 + m_2}$.

∴ Now, we will find the value of V_2. Simply subtitute the V_1 variable with the (i) equation

$$V_2 - V_1 = V_0$$
$$-V_1 = V_0 - V_2$$
$$V_1 = V_2 - V_0$$

Subtitute the equation to the (ii) equation

$$m_1 V_0 = m_1 V_1 + m_2 V_2$$
$$m_1 V_0 = m_1 (V_2 - V_0) + m_2 V_2$$
$$m_1 V_0 = m_1 V_2 - m_1 V_0 + m_2 V_2$$
$$2 m_1 V_0 = m_1 V_2 + m_2 V_2$$
$$2 m_1 V_0 = V_2 (m_1 + m_2)$$
$$V_2 = \frac{2 m_1 V_0}{m_1 + m_2}$$

The value of V_2 is $\frac{2 m_1 V_0}{m_1 + m_2}$

B. Same as case A, we want to find the value of V_1 and V_2. But, the collision between the three balls form a θ angle on ball 2nd and ball 3rd as illustrated below.

There are two axis that the direction of the balls go, X-axis and y axis

To find the value of V_1, first we need to know the law

To find the value of V_1, first we need to know the law of conservation momentum. The expression from the X-axis is

$$m_1 V_{1ix} + m_2 V_{2ix} + m_3 V_{3ix} = m_1 V_{1fx} + m_2 V_{2fx} + m_3 V_{3fx} \quad \ldots (i)$$

The expression from the Y-axis can be written as:

$$m_1 V_{1iy} + m_2 V_{2iy} + m_3 V_{3iy} = m_1 V_{1fy} + m_2 V_{2fy} + m_3 V_{3fy} \quad (ii)$$

∴ We will find the value of V_1. First we will calculate using the X-axis.

As we know :

$V_{1ix} = V_0$
$m_2 = m_3 = m_2$
$V_{1fx} = V_1$
$V_{2fx} = V_{3fx} = V_2 \cos\theta$

$V_{2ix} = V_{3ix} = 0$

The equation will be :

$$m_1 V_{1ix} + m_2 V_{2ix} + m_3 V_{3ix} = m_1 V_{1fx} + m_2 V_{2fx} + m_3 V_{3fx} \quad \ldots (i)$$
$$m_1 V_0 + 2 m_2 (0) = m_1 V_1 + 2 m_2 V_2 \cos\theta$$
$$m_1 V_0 = m_1 V_1 + 2 m_2 V_2 \cos\theta$$

The e can be express as

$$e = 1 = \frac{-(V_2 \cos\theta - V_1)}{0 - V_0}$$

$$V_0 = V_2 \cos\theta - V_1$$
$$V_2 \cos\theta = V_0 + V_1 \quad \ldots (iii)$$

Subtitute the $V_2 \cos\theta$ into (i) equation

$$m_1 V_0 = m_1 V_1 + 2m_2 V_2 \cos\theta$$
$$m_1 V_0 = m_1 V_1 + 2m_2 (V_0 + V_1)$$
$$m_1 V_0 = m_1 V_1 + 2m_2 V_0 + 2m_2 V_1$$
$$m_1 V_0 - 2m_2 V_0 = m_1 V_1 + 2m_2 V_1$$
$$V_0 (m_1 - 2m_2) = V_1 (m_1 + 2m_2)$$

$$V_1 = \frac{V_0 (m_1 - 2m_2)}{m_1 + 2m_2}$$

We get the value of V_1 is equal to $\frac{V_0 (m_1 - 2m_2)}{m_1 + 2m_2}$

∴ Now, we will find the value of the V_2. Apparently, after a quick calculation, we cannot calculate the value of V_2 by using the y-axis. So, we will continue using the X-axis as our resource. From the V_1 calculation, before we get an equation express as

$$m_1 V_0 = m_1 V_1 + 2m_2 V_2 \cos\theta$$

Now, we will substitute V_1 from the equation (iii).

$$V_2 \cos\theta = V_0 + V_1$$
$$V_1 = V_2 \cos\theta - V_0$$

Subtitute the V_1

$$m_1 V_0 = m_1 (V_2 \cos\theta - V_0) + 2m_2 V_2 \cos\theta$$
$$m_1 V_0 = m_1 V_2 \cos\theta - m_1 V_0 + 2m_2 V_2 \cos\theta$$
$$2m_1 V_0 = m_1 V_2 \cos\theta + 2m_2 V_2 \cos\theta$$
$$2m_1 V_0 = V_2 \cos\theta (m_1 + 2m_2)$$

$$V_2 \cos\theta = \frac{2m_1 V_0}{(m_1 + 2m_2)}$$
$$V_2 = \frac{2m_1 V_0}{(m_1 + 2m_2)\cos\theta}$$

$$V_2 = \frac{2m_1 V_0 \sec\theta}{m_1 + 2m_2}$$

So, the value of V_2 is equal to $\frac{2m_1 V_0 \sec\theta}{m_1 + 2m_2}$

c. We have a data that says

$$m_1 = \eta m_2 \ldots\ldots$$

For case A, the value of η to make m_1 stationary after the collision, we can calculate as

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

As we know :

$$V_{1i} = V_0$$
$$V_{2i} = 0$$
$$V_{1f} = V_1$$
$$V_{2f} = V_2$$

The goal is to make $V_1 = 0$ so that the m_1 can be stationary.

$$m_1 V_0 + 0 = m_1 V_1 + m_2 V_2$$
$$m_1 V_0 = m_1 V_1 + m_2 V_2$$

$$m_1 V_1 = m_1 V_0 - m_2 V_2$$
$$V_1 = \frac{m_1 V_0 - m_2 V_2}{m_1} = 0$$

Subtitute $m_1 = \eta m_2$ into the equation above.

$$\frac{m_1 V_0 - m_2 V_2}{m_1} = 0$$

$$\frac{\eta m_2 V_0 - m_2 V_2}{\eta m_2} = 0$$

$$\eta m_2 V_0 - m_2 V_2 = 0$$
$$\eta \cancel{m_2} V_0 = \cancel{m_2} V_2$$
$$\eta V_0 = V_2$$

$$V_2 - V_1 = V_1' - V_2'$$
$$-V_0 =$$

$$\eta = \frac{V_2}{V_0}$$

We get the value of η is equal to $\frac{V_2}{V_0}$ =

D. For case B, there is η and θ that make m_1 is stationary after the collision. We will use the Law of conservation momentum for this.

$$m_1 V_0 = m_1 V_1 + 2m_2 V_2 \cos\theta$$
$$m_1 V_0 - 2m_2 V_2 \cos\theta = m_1 V_1$$
$$V_1 = \frac{m_1 V_0 - 2m_2 V_2 \cos\theta}{m_1} = 0$$
$$V_1 = m_1 V_0 - 2m_2 V_2 \cos\theta = 0$$

The goal is to make m_1 has the value of $V_1 = 0$
As we know, $m_1 = \eta m_2$. Subtitute!

$$m_1 V_0 - 2m_2 V_2 \cos\theta = 0$$
$$\eta m_2 V_0 - 2m_2 V_2 \cos\theta = 0$$

$$\cancel{m_2}(\eta V_0 - 2V_2 \cos\theta) = 0$$

$$\eta V_0 - 2V_2 \cos\theta = 0$$

$$\eta V_0 = 2V_2 \cos\theta$$

$$\eta = \frac{2V_2 \cos\theta}{V_0}$$

E. For case B, there is η that greater than η_{max}. If $\eta > \eta_{max}$, it is impossible to make m_1 is stationary after the collision.
Now, let us make an asumption. if we know the value of V_2, $\cos\theta$,

$\eta > \eta_{max}$, it is impossible to make m_1 is stationary after the collision.

Now, let us make an asumption. if we know the value of V_2, $\cos\theta$, and V_0. Let say that

$$V_2 = 1$$
$$\cos\theta = 0.5$$
$$V_0 = 2$$

Than, η_{max} will be ;

$$\eta_{max} = \frac{2(1) \cdot \frac{1}{2}}{2}$$
$$\eta_{max} = \frac{1}{2}$$

If $m_1 = 1$. Then, $m_1 = \eta_{max}(m_2)$ will be $1 = \frac{1}{2} m_2$ or $m_2 = 2$. Put the value into the equation below.

$$m_1 V_0 = m_1 V_1 + 2 m_2 V_2 \cos\theta$$
$$1(2) = 1(V_1) + 2(\cancel{2})(1)\left(\frac{1}{\cancel{2}}\right)$$
$$2 = V_1 + 2$$
$$V_1 = 0$$

Now if we use $\eta > \eta_{max}$ while the value of η is 1, we will get

$$m_1 = 1 \quad ; \quad m_2 = 1$$

Subtitute!

$$m_1 V_0 = m_1 V_1 + 2 m_2 V_2 \cos\theta$$
$$1(2) = 1(V_1) + 2(1)(1)\left(\frac{1}{2}\right)$$
$$2 = V_1 + 1$$
$$V_1 = 1$$

This prove that if $\eta > \eta_{max}$; the m_1 is not stationary.

Problem # 2 : Inelastic Head on Collision

The 2 masses m_1 and m_2 with intial move with velocity v_1 and v_2 have final velocity of u_1 and u_2. We know that the equation for coefficient of restitution from these variables are

$$e = -\left(\frac{u_1 - u_2}{V_1 - V_2}\right) \quad \ldots (i)$$

A. To know the value of u_1 and u_2, first we need to use the law of conservation momentum to find the first equation.

$$m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 u_2 \quad \ldots (ii)$$

∴ To find the value of u_2, we can subtitute the value of u_1 from equation (i) into equation (ii).

$$e = -\left(\frac{u_1 - u_2}{V_1 - V_2}\right)$$

$$eV_1 - eV_2 = u_2 - u_1$$

$$eV_1 - eV_2 - u_2 = -u_1$$

$$u_2 + eV_2 - eV_1 = u_1$$

Now subtitute !

$$m_1V_1 + m_2V_2 = m_1u_1 + m_2u_2$$
$$m_1V_1 + m_2V_2 = m_1u_2 + m_1eV_2 - m_1eV_1 + m_2u_2$$
$$m_1V_1 + m_1eV_1 + m_2V_2 - m_1eV_2 = m_1u_2 + m_2u_2$$
$$V_1m_1(1+e) + V_2(m_2 - m_1e) = u_2(m_1 + m_2)$$

$$u_2 = \frac{m_1V_1(1+e) + V_2(m_2 - m_1e)}{m_1 + m_2}$$

The value of u_2 is $= \dfrac{m_1V_1(1+e) + V_2(m_2 - m_1e)}{m_1 + m_2}$

∴ To find the value of u_1, we can subtitute the value of u_2 from equation (i) into equation (ii).

$$eV_1 - eV_2 = u_2 - u_1$$
$$eV_1 - eV_2 + u_1 = u_2$$

Now subtitute!

$$m_1V_1 + m_2V_2 = m_1u_1 + m_2u_2$$
$$m_1V_1 + m_2V_2 = m_1u_1 + m_2eV_1 - m_2eV_2 + m_2u_1$$
$$m_1V_1 - m_2eV_1 = m_1u_1 + m_2u_1 - m_2eV_2 - m_2V_2$$
$$V_1(m_1 - m_2e) = u_1(m_1 + m_2) - m_2V_2(e+1)$$
$$V_1(m_1 - m_2e) + m_2V_2(e+1) = u_1(m_1 + m_2)$$

$$u_1 = \frac{V_1(m_1 - m_2e) + m_2V_2(e+1)}{m_1 + m_2}$$

The value of u_1 is $= \dfrac{V_1(m_1 - m_2e) + m_2V_2(e+1)}{m_1 + m_2}$

B. The velocity of the center of mass before collission (V_{cm}) can be express by :

$$V_{cm} = \frac{\sum m_i v_i}{\sum m_i} = \frac{m_1V_1 + m_2V_2}{m_1 + m_2}$$

The value of V_{cm} is $= \dfrac{m_1V_1 + m_2V_2}{m_1 + m_2}$

The velocity of the center of mass after collision (u_{cm}) can be express by:

$$u_{cm} = \frac{\sum m_i u_i}{\sum m_i} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

The value of u_{cm} is $= \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

c. Prove that the final velocity u_1 and u_2 can be written as follows:

$$u_1 = V_{cm} - \alpha_1 (V_1 - V_2) \; ; \quad u_2 = V_{cm} - \alpha_2 (V_2 - V_1)$$

∴ First, we will calculate the u_1 expression

We know that $e = -\left(\frac{u_1 - u_2}{V_1 - V_2}\right)$. From that, we can get an equation of

$$V_1 e - V_2 e + u_1 = u_2 \text{ ..... (i)}$$

We need another equation taken from the Law of Conservation Momentum.

$$m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 u_2 \text{ ..... (ii)}$$

Now subtitute equation (i) into equation (ii) in order to obtain the u_1 result.

$$m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 (V_1 e - V_2 e + u_1)$$
$$m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 V_1 e - m_2 V_2 e + m_2 u_1$$
$$m_1 V_1 + m_2 V_2 = u_1 (m_1 + m_2) + m_2 e (V_1 - V_2)$$

We know that: $V_{cm} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$

$$m_1 V_1 + m_2 V_2 = V_{cm} (m + m_2) \text{ ...(iii)}$$ subtitute this equation

$$V_{cm} (m_1 + m_2) = u_1 (m_1 + m_2) + m_2 e (V_1 - V_2)$$

$$V_{cm} = u_1 + \frac{m_2 e (V_1 - V_2)}{m_1 + m_2}$$

$$u_1 = V_{cm} - \frac{m_2 e}{m_1 + m_2} (V_1 - V_2)$$ is the same as

$$u_1 = V_{cm} - \alpha_1 (V_1 - V_2).$$

Those expression is proved! Now, the value of α_1 is $= \frac{m_2 e}{m_1 + m_2}$ //

∴ To prove the u_2, same as we prove u_1, we will use equation (i) to substitute into equation (ii).

$$u_1 = u_2 - v_1 e + v_2 e$$

Now subtitute into equation (ii)

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$
$$m_1 v_1 + m_2 v_2 = m_1 u_2 - m_1 v_1 e + m_1 v_2 e + m_2 u_2$$
$$m_1 v_1 + m_2 v_2 = u_2 (m_1 + m_2) + m_1 e (v_2 - v_1)$$

We have equation (iii), subtitute it into the current equation!

$$m_1 v_1 + m_2 v_2 = u_2 (m_1 + m_2) + m_1 e (v_2 - v_1)$$
$$v_{cm} (m_1 + m_2) = u_2 (m_1 + m_2) + m_1 e (v_2 - v_1)$$
$$v_{cm} = u_2 + \frac{m_1 e}{m_1 + m_2} (v_2 - v_1)$$

$$u_2 = v_{cm} - \frac{m_1 e}{m_1 + m_2} (v_2 - v_1)$$, is the same as

$$u_2 = v_{cm} - \alpha_2 (v_2 - v_1)$$

So, now we know the value of α_2 is $\frac{m_1 e}{m_1 + m_2}$ and is proved to be true.

D. Prove that u_1 and u_2 can be written as follows.

$$u_1 = v_1 - \beta_1 (v_1 - v_2) \quad ; \quad u_2 = v_2 - \beta_2 (v_2 - v_1).$$

Then, determine the value of β_1 and β_2!

∴ First we will the $u_1 = v_1 - \beta_1 (v_1 - v_2)$.

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$
$$m_1 u_1 = m_1 v_1 + m_2 v_2 - m_2 u_2$$
$$u_1 = v_1 + \frac{m_2}{m_1} v_2 - \frac{m_2}{m_1} u_2$$

Subtitute this equation

$$v_1 e - v_2 e + u_1 = u_2 \quad \text{.....} \quad (i)$$

$$u_1 = v_1 + \frac{m_2 v_2 - m_2 (v_1 e - v_2 e + u_1)}{m_1}$$

$$u_1 = v_1 + \frac{m_2 v_2 - m_2 v_1 e + m_2 v_1 e - m_2 u_1}{m_1}$$

$$u_1 = v_1 + \frac{m_2 v_2 - m_2 e (v_1 - v_2) - m_2 u_1}{m_1}$$

$$u_1 = v_1 - \left(\frac{-m_2 v_2 + m_2 e (v_1 - v_2) + m_2 u_1}{m_1} \right)$$

$$u_1 = v_1 - \left(\frac{-m_2 v_2 + m_2 u_1}{m_1 (v_1 - v_2)} + \frac{m_2 e}{m_1} \right) (v_1 - v_2)$$

Now the equation is in the same form of

$$u_1 = v_1 - \beta_1 (v_1 - v_2)$$

The value of β_1 is $= \left(- \frac{m_2 v_2 + m_2 u_1}{m_1 (v_1 - v_2)} + \frac{m_2 e}{m_1} \right)$

∴ To prove the $u_2 = v_2 - \beta_2 (v_2 - v_1)$, we're gonna use the same process as before.

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$m_2 u_2 = m_1 v_1 + m_2 v_2 - m_1 u_1$$

$$u_2 = \frac{m_1}{m_2} v_1 + v_2 - \frac{m_1}{m_2} u_1$$

Subtitute the equation (i)

$$u_1 = u_2 - v_1 e + v_2 e \quad \ldots\ldots i$$

into the current equation!

$$u_2 = v_2 + \frac{m_1 v_1}{m_2} - \frac{m_1}{m_2} (u_2 - v_1 e + v_2 e)$$

$$u_2 = v_2 + \frac{m_1 v_1 - m_1 u_2 - m_1 v_1 e + m_1 v_2 e}{m_2}$$

$$u_2 = v_2 + \frac{m_1 v_1 - m_1 u_2 + m_1 e (v_2 - v_1)}{m_2}$$

$$u_2 = v_2 + \left(\frac{m_1 e}{m_2} + \frac{m_1 v_1 - m_1 u_2}{m_2 (v_2 - v_1)} \right) (v_2 - v_1)$$

$$u_2 = v_2 - \left(\frac{-m_1 e}{m_2} + \left(\frac{-m_1 v_1 + m_2 u_2}{m_2 (v_2 - v_1)} \right) \right) (v_2 - v_1)$$

Now the equation is in the same form of

$$u_2 = v_2 - \beta_2 (v_2 - v_1)$$

The value of β_2 is $= \left(\frac{-m_1 e}{m_2} + \left(\frac{-m_1 v_1 + m_2 u_2}{m_2 (v_2 - v_1)} \right) \right)$

E. The lost in kinetic energy (ΔE_k) during collision can be written as

$$\Delta E_k = \eta \left(\frac{1}{2} \right) \left(\frac{m_1 m_2}{m_1 + m_2} \right) (v_1 - v_2)^2$$

We also know that $\Delta E_k = \frac{1}{2} m (v_f^2 - v_i^2)$

So, we cant calculate the equation as

$$\frac{1}{2} m_2 \left(u_2^2 - v_2^2 \right) + \frac{1}{2} m_1 \left(u_1^2 - v_1^2 \right) = \Delta E_K$$

$$\frac{1}{2} m_2 \left((v_2 - \beta_2 (v_2 - v_1))^2 - v_2^2 \right) + \frac{1}{2} m_1 \left((v_1 - \beta_1 (v_1 - v_2))^2 - v_1^2 \right) = \Delta E_k$$

$$\frac{1}{2} m_2 \left(v_2^2 - 2 v_2 \beta_2 (v_2 - v_1) + \beta_2^2 (v_2 - v_1)^2 - v_2^2 \right) + \frac{1}{2} m_1 \left(v_1^2 - 2 v_1 \beta_1 (v_1 - v_2) + \beta_1^2 (v_1 - v_2)^2 - v_1^2 \right) = E_k$$

$$\frac{1}{2} m_2 \left(-2 v_2 \beta_2 (v_2 - v_1) + \beta_2^2 (v_2 - v_1)^2 \right) + \frac{1}{2} m_1 \left(-2 v_1 \beta_1 (v_1 - v_2) + \beta_1^2 (v_1 - v_2)^2 \right) = \Delta E_k$$

$$\frac{1}{2} m_2 \beta_2 (V_2 - V_1)\left(-2V_2 + \beta_2 (V_2 - V_1)\right) + \frac{1}{2} m_1 \beta_1 (V_1 - V_2)\left(-2V_1 + \beta_1 (V_1 - V_2)\right) = \Delta E_k$$

We know that the value of β_1 is

$$u_1 = \frac{V_1(m_1 - m_2 e) + m_2 V_2 (e+1)}{m_1 + m_2} = V_1 - \beta_1 (V_1 - V_2)$$

$$\frac{V_1 m_1 - V_1 m_2 e + m_2 V_2 e + m_2 V_2}{m_1 + m_2} - V_1 m_1 - V_1 m_2 = -\beta_1 (V_1 - V_2)$$

$$\frac{m_2 V_2 (e+1) - m_2 V_1 (e+1)}{m_1 + m_2} = -\beta_1 (V_1 - V_2)$$

$$\frac{m_2 (e+1)(\cancel{V_2 - V_1})}{m_1 + m_2} = \beta_1 (\cancel{V_2 - V_1})$$

$$\beta_1 = \frac{m_2 (e+1)}{m_1 + m_2}$$; and β_2 is

$$\beta_2 = \frac{m_1 (1+e)}{m_1 + m_2}$$ Subtitute the value !

$$\cancel{\frac{1}{2}} m_2 \left(\frac{m_1 (1+e)}{\cancel{m_1 + m_2}}\right)(\cancel{V_2 - V_1})\left(-2V_2 + \left(\frac{m_1 (1+e)}{m_1 + m_2}\right)(V_2 - V_1)\right) + \cancel{\frac{1}{2}} m_1 \left(\frac{m_2 (e+1)}{\cancel{m_1 + m_2}}\right)(\cancel{V_1 - V_2})\left(-2V_1 + \left(\frac{m_2 (e+1)}{m_1 + m_2}\right)(V_1 - V_2)\right) = \cancel{\frac{1}{2}} \eta \left(\frac{m_1 m_2}{\cancel{m_1 + m_2}}\right)(V_1 - V_2)^{\cancel{2}}$$

$$-\cancel{m_2}(\cancel{m_1}(1+e)\left(-2V_2 + \left(\frac{m_1 (1+e)}{m_1 + m_2}\right)(V_2 - V_1)\right) + \cancel{m_1}(\cancel{m_2}(e+1))\left(-2V_1 + \frac{(m_2 (e+1)}{m_1 + m_2}(V_1 - V_2)\right) = \eta (\cancel{m_1 m_2})(V_1 - V_2)$$

$$(1+e)\left(2V_2 + \beta_2 (V_1 - V_2) + (e+1)\left(-2V_1 + \beta_1 (V_1 - V_2)\right)\right) = \eta (V_1 - V_2)$$

$$(1+e)\left(2V_2 + \beta_2 (V_1 - V_2) - 2V_1 + \beta_1 (V_1 - V_2)\right) = \eta (V_1 - V_2)$$

$$(1+e)\left(-2(V_1 - V_2) + (V_1 - V_2)(\beta_2 + \beta_1)\right) = \eta (V_1 - V_2)$$

$$(1+e)\left(-2(\cancel{V_1 - V_2}) + (\cancel{V_1 - V_2})\left(\frac{m_1 (1+e) + m_2 (e+1)}{m_1 + m_2}\right)\right) = \eta (\cancel{V_1 - V_2})$$

$$(1+e)\left(-2 + (1+e)\left(\frac{\cancel{m_1 + m_2}}{\cancel{m_1 + m_2}}\right)\right) = \eta$$

$$(1+e)(1-e) = \eta$$

$$1 - e^2 = \eta$$

The equation is proved to be true and the value of η is $1 - e^2$

F. How about the kinetic energy that lost during collision when ($e = 1$) and when ($e = 0$)?

We know that the kinetic energy lost is equal to

$$\Delta E_k = \eta \left(\frac{1}{2}\right)\left(\frac{m_1 m_2}{m_1 + m_2}\right)(V_1 - V_2)^2 \; ; \; \eta = 1 - e^2$$

∴ When $e = 1$

$$\eta = 1 - (1)^2 = 0$$

So the total energy lost is

$$\Delta E_k = 0 \left(\frac{1}{2}\right)\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_1 - v_2)^2 = 0$$

When $e = 1$; there is no kinetic energy lost.

∴ When $e = 0$

$$\eta = 1 - 0 = 1$$

So, the total kinetic energy lost is

$$\Delta E_k = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_1 - v_2)^2$$

When $e = 0$; the total kinetic energy lost is equal to

$$\Delta E_k = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_1 - v_2)^2$$

An elastic collision will rebounce all the kinetic energy and causing no energy loses. But, for an inelastic collision, all the energy is absorbed and converted into calor energy.