Latihan Soal

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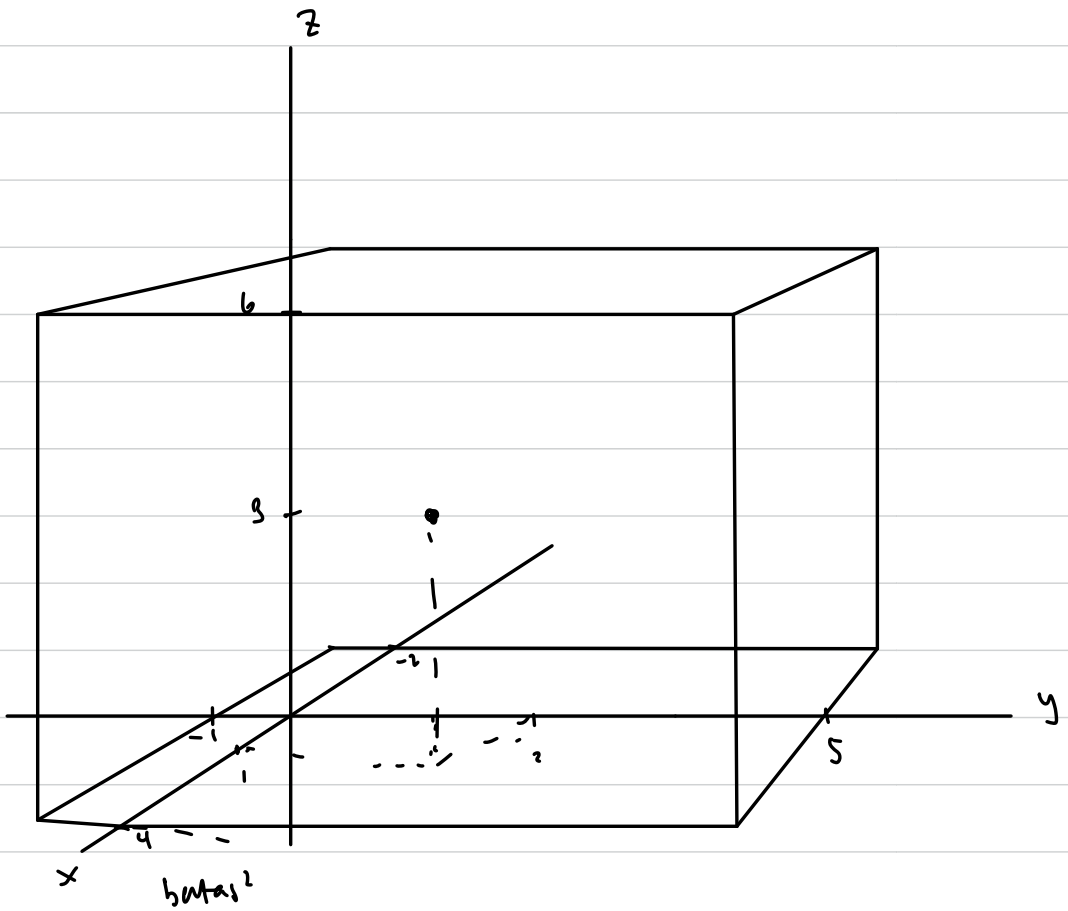
21/481767/TK/53170

1) Hitung integral lipat 3
$f(x,y,z) = 5xyz$ dengan batas kubus berpusat di $(1,2,3)$ dan panjang sisi 6



batas²

$-2 \leq x \leq 4$

$-1 \leq y \leq 5$

$0 \leq z \leq 6$

jawab

$$\int_{-1}^{5}\int_{-2}^{4}\int_{0}^{6} 5xyz \; dz\, dx\, dy$$

$$= \int_{-1}^{5}\int_{-2}^{4} \left(\frac{5xyz^2}{2}\right)_0^6 dx\, dy$$

$$= \int_{-1}^{5}\int_{-2}^{4} (90xy)\, dx\, dy$$

$$= \int_{-1}^{5} \left(45x^2y\right)_{-2}^{4} dy$$

$$\rightarrow \int_{-1}^{5} (720y - 180y)\, dy = \int_{-1}^{5} 540y\, dy$$

$$= \left(270y^2\right)_{-1}^{5} = 6750 - 270$$

$$= 6480 \;//$$

2) Hasil $\iiint_V 2x^2 + 2y^2 \, dz \, dy \, dx$

$V: z = x^2 + y^2$

$z = 4$

$z = r^2$

$r = \sqrt{z}$

$0 \le z \le 4$

$0 \le \theta \le 2\pi$

$0 \le r \le \sqrt{z}$

Jawab

$$\iiint 2(r^2)\, r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{z}} \int_0^{4} 2(r^2)\, r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^{4} \int_0^{\sqrt{z}} 2r^3 \, dr \, dz \, d\theta$$

$$= \int_0^{2\pi} \int_0^{4} \frac{1}{2} r^4 \Big]_0^{\sqrt{z}} \, dz \, d\theta$$

$$= \int_0^{2\pi} \int_0^{4} \frac{1}{2} z^2 \, dz \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{6} z^3 \Big]_0^{4} \, d\theta$$

$$= \int_0^{2\pi} \frac{64}{6} \, d\theta$$

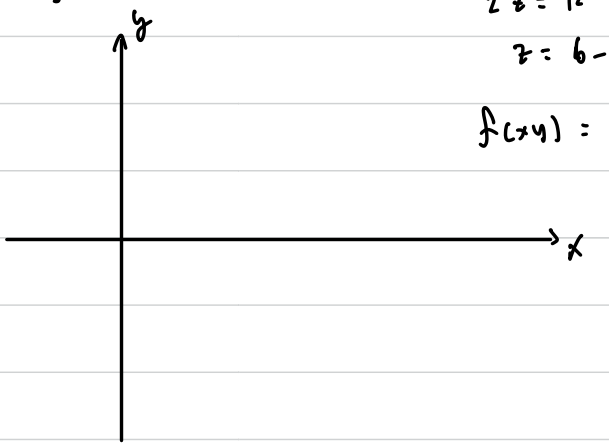
$$= \frac{64(2\pi)}{6} = \frac{64}{3}\pi$$

$\iint_S (3xyz)\, dS$; $S: 3x + 4y + 2z = 12$

$2z = 12 - 3x - 4y$

$z = 6 - \frac{3}{2}x - 2y$

$f(x,y) = 6 - \frac{3}{2}x - 2y$



$f_x = -\frac{3}{2}$; $f_y = -2$

$3xy\left(6 - \frac{3}{2}x - 2y\right) = 18xy - \frac{9}{2}x^2y - 6y^2x$

batas

1) $0 = 6 - 0 - 2y$

$3 = y \quad (0, 3)$ $\qquad \frac{0-3}{4-0} = -\frac{3}{4}$

2) $0 = 6 - \frac{3}{2}x - 0$

$12 = 3x$ $\qquad y - 3 = \frac{-3}{4}x$

$4 = x \quad (4, 0)$ $\qquad y = -\frac{3}{4}x + 3$

$$dS = \sqrt{f_x^2 + f_y^2 + 1}\, dA = \sqrt{\frac{9}{4} + 4 + 1}\, dA$$

$$= \frac{1}{2}\sqrt{29}\, dA$$

$$\int_0^{4} \int_0^{-\frac{3}{4}x+3} 18xy - \frac{9}{2}x^2y - 6y^2x \cdot \frac{1}{2}\sqrt{29} \, dy \, dx$$

$$= \frac{1}{2}\sqrt{29} \int_0^{4} 9xy^2 - \frac{9}{4}x^2y^2 - 2y^3x \Big]_0^{-\frac{3}{4}x+3}$$

Maaf tak ngga sempat garap

$= \frac{12}{4}$