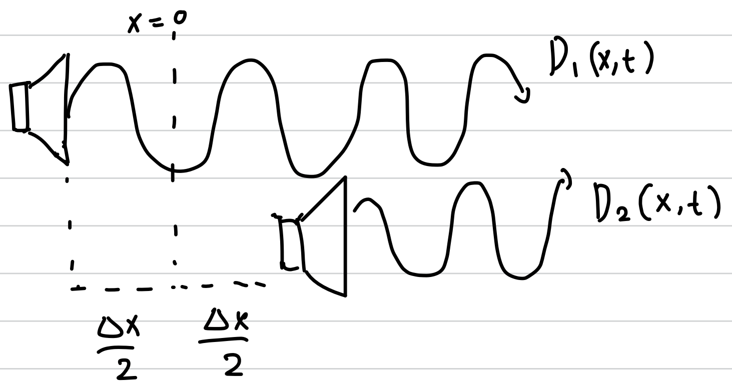
Qornain Aji
21/481767/TK/53170

Homework #1 : Wave & Acoustics

1. Problem ※ 1 : Interference of 2 Sound Waves.



$D_1(x,t) = A_1 \sin(k(x-\Delta x/2) - \omega t + \phi_1)$
$D_2(x,t) = A_2 \sin(k(x+\Delta x/2) - \omega t + \phi_2)$

A) We know :
$A_1 = A_0 - \Delta A/2$
$A_2 = A_0 + \Delta A/2$
$\phi_1 = \phi_2 = 0$

Determine : Δx_{min} and Δx_{max}

Answer :

$D_1(x,t) = A_1 \sin(k(x-\Delta x/2) - \omega t) = (A_0 - \Delta A/2)\sin(k(x-\Delta x/2)-\omega t)$
$D_2(x,t) = A_2 \sin(k(x+\Delta x/2)-\omega t) = (A_0 + \Delta A/2)\sin(k(x+\Delta x/2)-\omega t)$

$D_1(x,t) + D_2(x,t) = (A_0 - \Delta A/2)\sin(k(x-\Delta x/2)-\omega t) + (A_0 + \Delta A/2)\sin(k(x+\Delta x/2)-\omega t)$

$= A_0 \sin(k(x-\Delta x/2)-\omega t) - \Delta A/2 \sin(k(x-\Delta x/2)-\omega t) + A_0 \sin(k(x+\Delta x/2)-\omega t)$
$+ \Delta A/2 \sin(k(x+\Delta x/2)-\omega t)$

$= A_0(\sin(k(x-\Delta x/2)-\omega t) + \sin(k(x+\Delta x/2)-\omega t)) - \Delta A/2(\sin(k(x-\Delta x/2)-\omega t) - \sin(k(x+\Delta x/2)$
$-\omega t))$

$= A_0(\sin(\underbrace{kx-\omega t}_{x} - \underbrace{k\Delta x/2}_{y}) + \sin(kx-\omega t + k\Delta x/2)) - \Delta A/2(\sin(kx-\omega t - k\Delta x/2) - \sin(kx-\omega t$
$+ k\Delta x/2))$

$= A_0(\sin x\cos y - \cos x\sin y + \sin x\cos y + \cos x \sin y) - \Delta A/2(\sin x\cos y - \cos x\sin y - \sin x\cos y - \cos x$
$\sin y)$

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$= A_0(2\sin x\cos y) - \Delta A/2(-2\cos x\sin y) = A_0(2\sin x\cos y) + \Delta A/2(2\cos x\sin y)$
$= A_0(2\sin(kx-\omega t)\cos(k\Delta x/2)) + \Delta A/2(2\cos(kx-\omega t)\sin(k\Delta x/2))$
$= 2A_0\sin(kx-\omega t)\cos(k\Delta x/2) + \Delta A\cos(kx-\omega t)\sin(k\Delta x/2)$

• $A_0\cos(k\Delta x/2) = A\cos\alpha = A(x)$ | $\Delta A\sin(k\Delta x/2) = A\sin\alpha = A(x)$

$= A\sin(kx-\omega t)\cos\alpha + A\cos(kx-\omega t)\sin\alpha$

Use different approach using $I = CA^2$

• $A^2 = A^2\cos^2\alpha + A^2\sin^2\alpha = 4A_0^2\cos^2\left(\frac{k\Delta x}{2}\right) + \Delta A^2\sin^2\left(\frac{k\Delta x}{2}\right)$

$\hookrightarrow I = c\left(4A_0^2\cos^2\left(\frac{k\Delta x}{2}\right) + \Delta A^2\sin^2\left(\frac{k\Delta x}{2}\right)\right)$

$= c\left(4A_0^2\underbrace{\cos^2\left(\frac{k\Delta x}{2}\right) + \Delta A^2\left(1-\cos^2\left(\frac{k\Delta x}{2}\right)\right)}_{\text{The only matter}}\right)$

$I_{max} \rightarrow \cos\left(\frac{k\Delta x}{2}\right) = 1$

$\cos\left(\frac{k\Delta x}{2}\right) = \cos m\pi \quad ; m = 0,1,2 \ldots$

$\frac{k\Delta x}{2} = m\pi$

$\Delta x = \frac{2m\pi}{k} \rightarrow \frac{2m\pi}{\frac{2\pi}{\lambda}} = \underline{\underline{m\lambda}} = \Delta x_{max}$

$I_{min} \rightarrow \cos\left(\frac{k\Delta x}{2}\right) = 0 = \cos m + \frac{\pi}{2}$

$\frac{k\Delta x}{2} = m + \frac{\pi}{2} \quad ; m = 0,1,2 \ldots$

$\Delta x = \frac{2\left(m+\frac{\pi}{2}\right)}{\frac{2\pi}{\lambda}} = \left(\frac{m}{\pi} + \frac{1}{2}\right)\lambda = \Delta x_{min}$

B) Now the wave have phase shift
$\phi_1 = -\phi_0/2$ and $\phi_0/2$
$A_1 = A_2 = A_0$

$D_1(x,t) = A_0\sin(k(x-\Delta x/2) - \omega t - \phi_0/2)$
$D_2(x,t) = A_0\sin(k(x+\Delta x/2) - \omega t + \phi_0/2)$

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$D_1(x,t) = A_0 \sin\left(k(x-\Delta x/2) - \omega t - \phi_0/2\right)$
$D_2(x,t) = A_0 \sin\left(k(x+\Delta x/2) - \omega t + \phi_0/2\right) +$

$D_1(x,t) + D_2(x,t) = A_0\left(\sin\left(k(x-\Delta x/2) - \omega t - \phi_0/2\right) + \sin\left(k(x+\Delta x/2) - \omega t + \phi_0/2\right)\right)$

$= A_0\left(\sin(\underbrace{kx-\omega t}_{x} - \underbrace{(\phi_0/2 + k\Delta x/2)}_{y}) + \sin(\underbrace{kx-\omega t}_{x} + \underbrace{\phi_0/2 + k\Delta x/2}_{y})\right)$

$= A_0(\sin x \cos y - \cos x \sin y + \sin x \cos y + \cos x \sin y)$
$= 2A_0 \sin x \cos y$
$= \underbrace{2A_0 \cos(k\Delta x/2 + \phi_0/2)}_{A(x)} \sin(kx - \omega t)$

$I = c(A(x))^2 = c\left(A_0^2 \cos^2(k\Delta x/2 + \phi_0/2)\right)$

- $I_{max} \Rightarrow \cos\left(\frac{k\Delta x + \phi_0}{2}\right) = 1 = \cos m\pi$

Δx max is: $\lambda\left(m - \frac{\phi_0}{2\pi}\right)$

$\frac{k\Delta x + \phi_0}{2} = m\pi$

$k\Delta x + \phi_0 = 2m\pi$

$\Delta x = \frac{2m\pi - \phi_0}{k}$

$\Delta x = \frac{2m\pi - \phi_0}{\frac{2\pi}{\lambda}}$

$\Delta x_{max} = \lambda\left(m - \frac{\phi_0}{2\pi}\right)$ //

- $I_{min} \rightarrow \cos\left(\frac{k\Delta x + \phi_0}{2}\right) = 0 = \cos m + \frac{\pi}{2}$; $m = 0, 1, 2 \ldots$

Δx min is: $\lambda\left(\frac{m}{\pi} + \frac{1}{2} - \frac{\phi_0}{2\pi}\right)$

$\frac{k\Delta x + \phi_0}{2} = m + \frac{\pi}{2}$

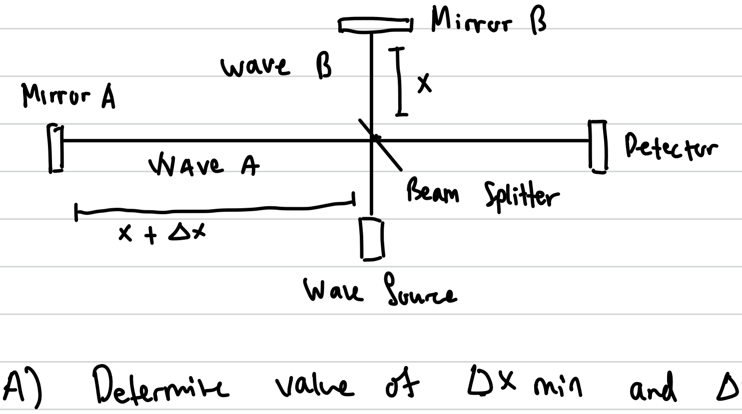
$k\Delta x + \phi_0 = 2m + \pi$

$\Delta x = \frac{2m + \pi - \phi_0}{k}$

$\Delta x = \frac{2m + \pi - \phi_0}{\frac{2\pi}{\lambda}}$

$\Delta x_{min} = \lambda\left(\frac{m}{\pi} + \frac{1}{2} - \frac{\phi_0}{2\pi}\right)$ //

2. Problem #2 LIGO Interferometer



A) Determine value of Δx min and Δx max !

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$D_1(x,t) = A\sin(k(x+\Delta x) - \omega t)$: wave A hitting mirror.
$D_2(x,t) = A\sin(kx - \omega t)$: wave B hitting mirror.
+

$= A(\sin(k(x+\Delta x) - \omega t) + \sin(kx - \omega t))$

$= A(\sin(\underbrace{kx - \omega t}_{x} + \underbrace{k\Delta x}_{y}) + \sin(kx - \omega t))$

$= A\left(2\sin\left(\frac{x+y+x}{2}\right)\cos\left(\frac{x+y-x}{2}\right)\right)$

$= \underbrace{2A\sin\left(\frac{2(kx-\omega t) + k\Delta x}{2}\right)}_{A(x)}\cos\left(\frac{k\Delta x}{2}\right)$

$I = c(A(x))^2$
$= c\left(4A^2\cos^2\left(\frac{k\Delta x}{2}\right)\right)$

- $I_{max} \longrightarrow \cos\left(\frac{k\Delta x}{2}\right) = 1 = \cos m\pi$; $m = 0, 1, 2, \ldots$

$\frac{k\Delta x}{2} = m\pi$

Therefore, $\Delta x_{max} = \frac{2m\pi}{k} = \frac{2m\pi}{\frac{2\pi}{\lambda}} = m\lambda$

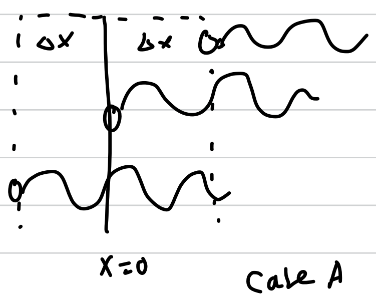
- $I_{min} \longrightarrow \cos\left(\frac{k\Delta x}{2}\right) = 0 = \cos m + \frac{\pi}{2}$; $m = 0, 1, 2, \ldots$

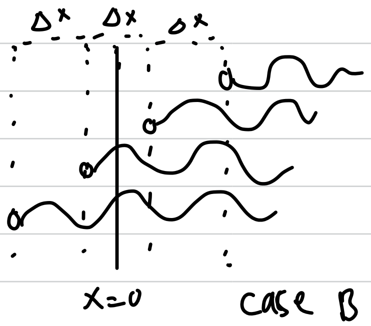
$\frac{k\Delta x}{2} = m + \frac{\pi}{2}$

$\Delta x = \frac{2m + \pi}{\frac{2\pi}{\lambda}} = \lambda\left(\frac{m}{\pi} + \frac{1}{2}\right)$

Therefore, $\Delta x_{min} = \lambda\left(\frac{m}{\pi} + \frac{1}{2}\right)$

3. Problem #3 Interference of Multiple Wave Sources





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A) Determine the amplitude and intensity of the resulting wave as a function of Δx ($A(\Delta x)$ and $I(\Delta x) = |A(\Delta x)|^2$)

CASE A

$D(x,t) = D_1(x,t) + D_2(x,t) + D_3(x,t)$

$= A(\sin(k(x-\Delta x)-\omega t) + \sin(kx-\omega t) + \sin(k(x+\Delta x)-\omega t))$

$= A(\sin(\underbrace{kx-\omega t}_{x} \underbrace{-\Delta x}_{y}) + \sin(kx-\omega t) + \sin(kx-\omega t + k\Delta x))$

$= A(\sin x\cos y - \cancel{\cos x \sin y} + \sin x + \sin x \cos y + \cancel{\cos x \sin y})$

$= A(2\sin x\cos y + \sin x)$

$= A(\sin x(2\cos y + 1)) = A\sin(kx-\omega t)(2\cos(k\Delta x)+1)$

$A(\Delta x)\sin(kx-\omega t)$

◦) Find the amplitude $A(x)$

$A(\Delta x) = A \cdot 2\cos(k\Delta x) + A$

$= 2A\cos(k\Delta x) + A$

$= 2A\cos\left(\frac{2\pi}{\lambda}\Delta x\right) + A$ //

◦) Find the intensity

$I(\Delta x) = c\left(A(\Delta x)\right)^2 = cA^2\left(2\cos\left(\frac{2\pi}{\lambda}\Delta x\right)+1\right)^2$ //

CASE B

$D(x,t) = D_1 + D_2 + D_3 + D_4$

$= A(\sin(k(x-\frac{3}{2}\Delta x)-\omega t) + \sin(k(x-\frac{\Delta x}{2})-\omega t) + \sin(k(x+\frac{\Delta x}{2})-\omega t) + \sin(k(x+\frac{3}{2}\Delta x)-\omega t))$

$= A(\sin(kx-\omega t - k\frac{3}{2}\Delta x) + \sin(kx-\omega t - k\frac{\Delta x}{2}) + \sin(kx-\omega t + k\frac{\Delta x}{2}) + \sin(kx-\omega t + k\frac{3}{2}\Delta x))$

$= A(\sin(kx-\omega t - k\frac{3}{2}\Delta x) + \sin(kx-\omega t + k\frac{3}{2}\Delta x) + \sin(kx-\omega t - \frac{k\Delta x}{2}) + \sin(kx-\omega t + \frac{k\Delta x}{2}))$

$= A(2\sin(kx-\omega t)\cos(-\frac{k3\Delta x}{2}) + 2\sin(kx-\omega t)\cos(-\frac{k\Delta x}{2}))$

$= 2A\sin(kx-\omega t)(\cos(k\frac{3}{2}\Delta x) + \cos(k\frac{\Delta x}{2}))$

$= 2A\sin(kx-\omega t)(2\cos(k\Delta x)\cos^2(k\frac{\Delta x}{2}))$

$= \underbrace{4A\cos(k\Delta x)\cos(k\frac{\Delta x}{2})}_{A(\Delta x)}\sin(kx-\omega t)$

$A(\Delta x) = 4A\cos(k\Delta x)\cos(k\frac{\Delta x}{2})$ //

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The value of $I(\Delta x)$ is:

$$I(\Delta x) = c\left(16A^2 \cos^2(k\Delta x)\cos^2\left(\frac{k\Delta x}{2}\right)\right)$$
$$= c\left(16A^2 \cos^2\left(\frac{2\pi}{\lambda}\Delta x\right)\cos^2\left(\frac{\pi\Delta x}{\lambda}\right)\right)$$

B) Plot a graph $-3\lambda \leq \Delta x \leq 3\lambda$.

Case A

$$I(\Delta x) = A^2\left(2\cos\left(\frac{2\pi}{\lambda}\Delta x\right)+1\right)^2$$

- $I(-3\lambda) = A^2\left(2\cos\left(\frac{2\pi}{\lambda}(-3\lambda)\right)+1\right)^2$
  $= A^2(2\cos(-6\pi)+1)^2$
  $= A^2(3)^2$
  $= A^2\, 9 = 9A^2$

- $I(3\lambda) = A^2(2\cos(6\pi)+1)^2$
  $= 9A^2$

- $I(2\lambda) = A^2(2\cos(4\pi)+1)^2$
  $= A^2(3)^2 = A^2 9 = 9A^2$

So, for every $m\lambda$ with $m = [Z^-, Z^+]$ the value is $9A^2$.

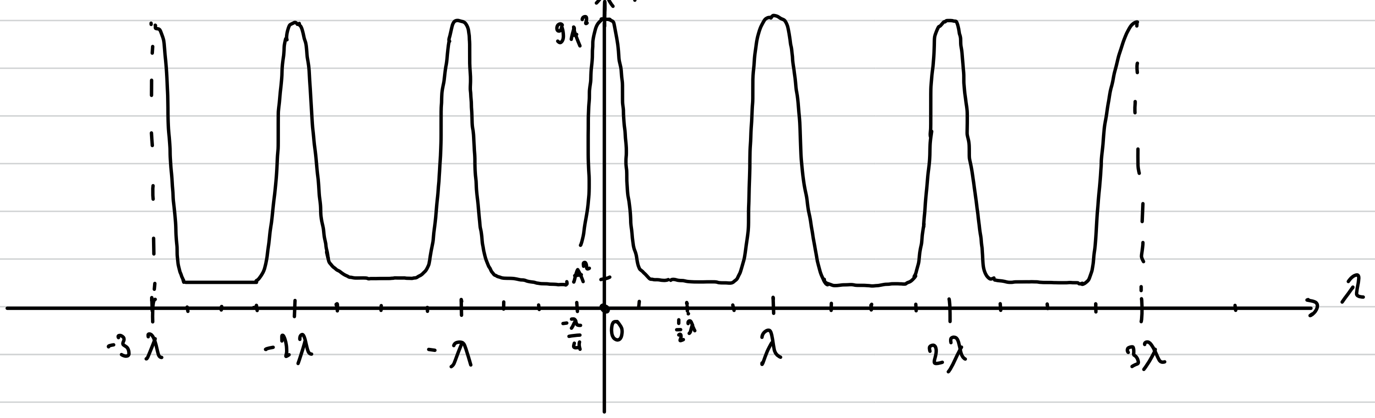
- $I\left(\frac{\lambda}{4}\right) = A^2\left(2\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)+1\right)^2$
  $= A^2\left(2\cos\left(\frac{\pi}{2}\right)+1\right)^2$
  $= A^2(1)$

For every $(m+\frac{1}{4})\lambda$ and $(\frac{m}{2}+\frac{1}{4})\lambda$, $m = [Z^-, Z^+]$ the value is A^2 because $\cos(-\lambda) = \cos\lambda$.

- $I\left(\frac{\lambda}{2}\right) = A^2\left(2\cos\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\right)+1\right)^2$
  $= A^2(2\cos\pi+1)^2 = A^2(-2+1)^2$
  $= A^2(-1)^2 = A^2$

For every $(m+\frac{1}{2})\lambda$ with $m = [Z^-, Z^+]$ the value is A^2 because $\cos(-\lambda) = \cos(\lambda)$

Therefore, the graphics is:



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case B

$I(\Delta x) = \left(16A^2 \cos^2\left(\frac{2\pi}{\lambda}\Delta x\right)\cos^2\left(\frac{\pi \Delta x}{\lambda}\right)\right)$, $-3\lambda \leq \Delta x \leq 3\lambda$.

• $I(-3\lambda) = 16A^2 \cos^2\left(\frac{2\pi}{\lambda}(-3\lambda)\right)\cos^2\left(\frac{\pi(-3\lambda)}{\lambda}\right)$

$= 16A^2 (1)^2 (-1)^2 = 16A^2$

So, for every $m\lambda$ with $m ; [Z^- \; Z^+]$ the value is $16A^2$. This applies becase $\cos(-\lambda) = \cos\lambda$

• $I\left(\frac{\lambda}{4}\right) = 16A^2 \cos^2\left(\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right)\right)\cos^2\left(\frac{\pi}{\lambda}\left(\frac{\lambda}{4}\right)\right)$

$= 16A^2 (0)^2 \left(\frac{\sqrt{2}}{2}\right)^2$

$= 0$

So, for every $(m+\frac{1}{4})\lambda$ and $(\frac{m}{2}+\frac{1}{4})\lambda$, $m ; [Z^- \; Z^+]$ the value is 0. This applies because $\cos(-\lambda) = \cos\lambda$.

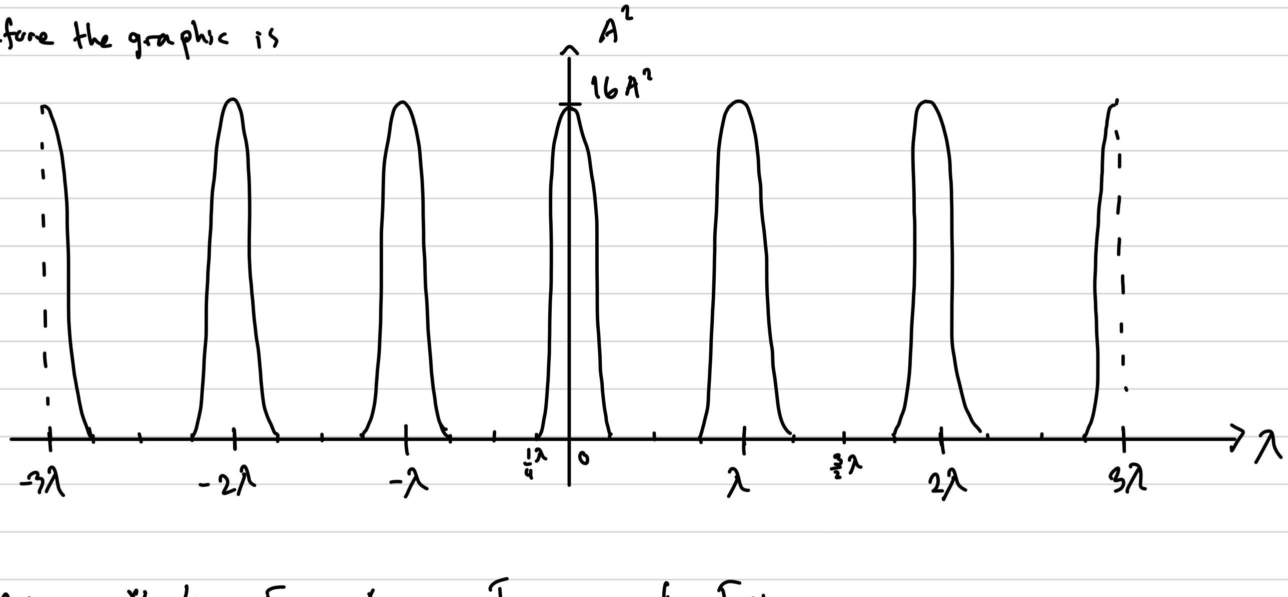
• $I\left(\frac{\lambda}{2}\right) = 16A^2 \cos^2\left(\frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right)\right)\cos^2\left(\frac{\pi}{\lambda}\left(\frac{\lambda}{2}\right)\right)$

$= 16A^2 (-1)^2 (0)^2$

$= 0$

So, for every $(m+\frac{1}{4})\lambda$ with $m = [Z^-, Z^+]$, the value is 0. This applies because $\cos(-\lambda) = \cos\lambda$

Therefore the graphic is



4) Problem # 4 : Far Away Train and Fatamorgana

A) Please explain why the student can hear train in the evening than in the morning or afternoon!

The logical explanation is that the sound wave it self is dependent on the temperature of the air. Different temperature can change the sound speed. It can be shown with this equation:

$$V_{wave} = 331\sqrt{\frac{\text{current temp} + 273}{273}}$$ while temperature is in kelvin

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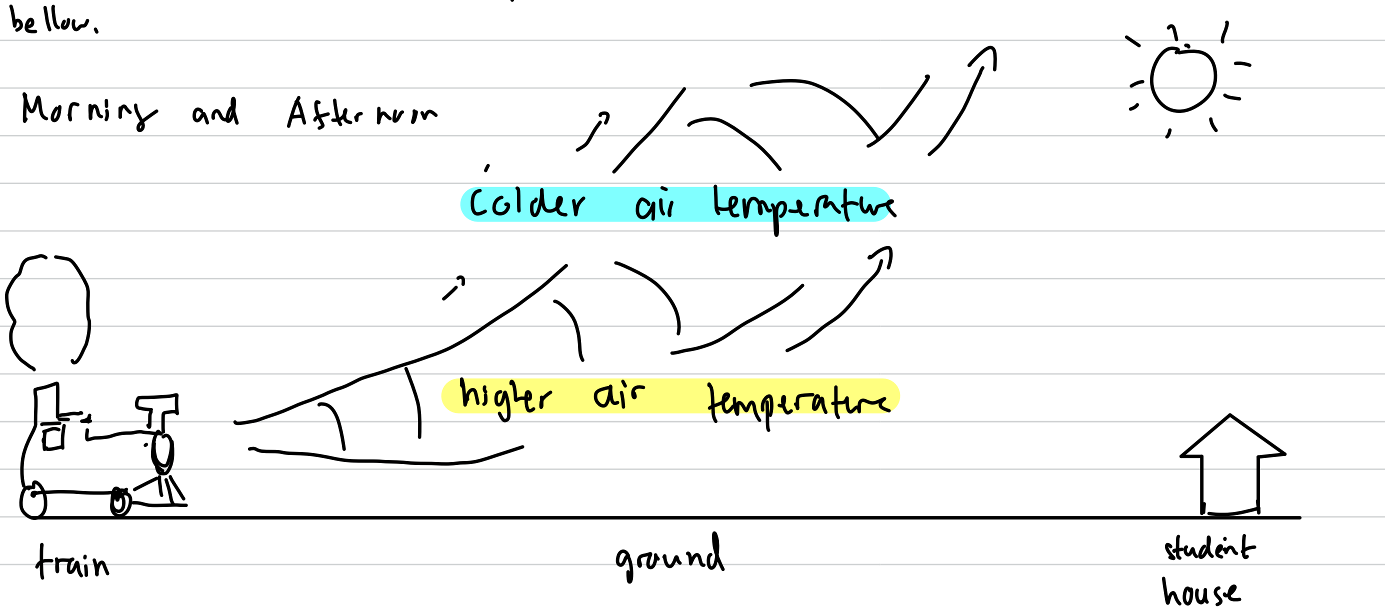
From the equation, we know that sound wave travel faster when the temperature is higher such like in the morning or afternoon. In evening however, the temperature is lower due to the sun that dont send enough heat and making the wave to travel slower. So how that effect on the train sound that can only be heard in the evening than in the morning or afternoon ? Even the sound travel slower, eventually it will arrive too isn't it ?

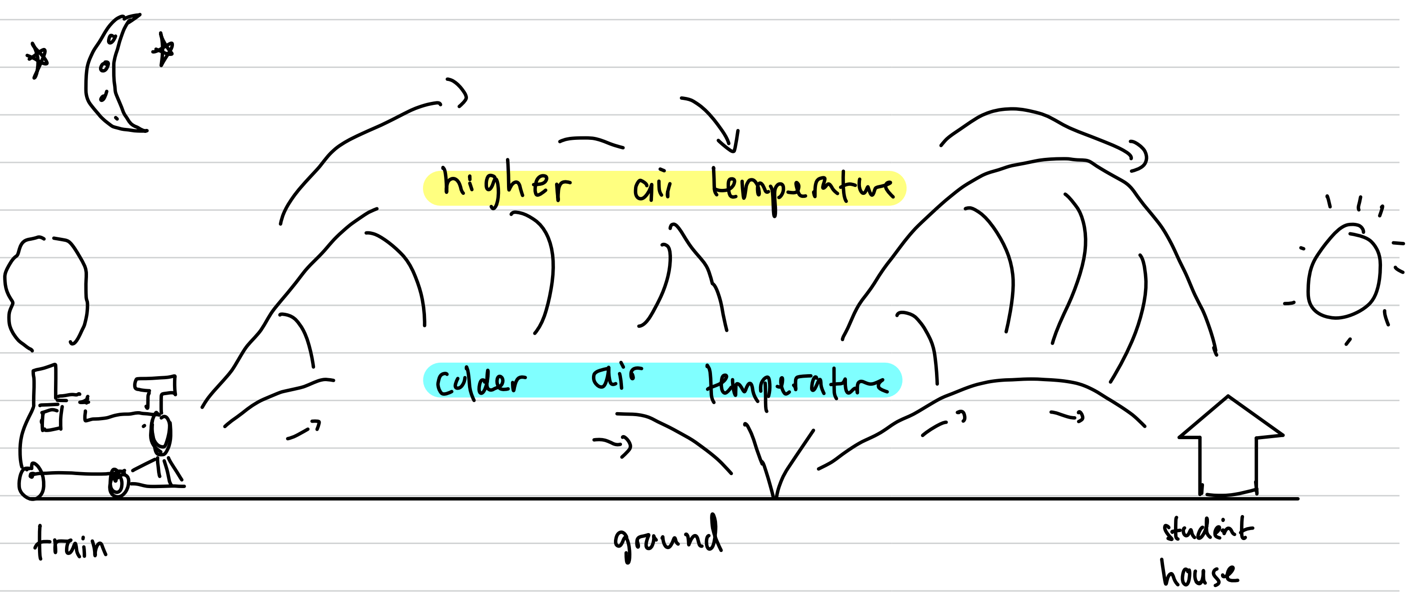
It has to do with the phenomenon called temperature inversion. Look at the demonstration bellow.

Morning and Afternoon



Because of higher temperature in air near to the ground then in the higher part of the air, the lower sound wave moves faster than the upper part of wave. Thus, the air acts like a lens that bends the wave upwards. That's why the student could never hear the train sound in the morning or afternoon becase the wave simply doesn't reached to the student house.

When the evening comes, look at the demonstration bellow



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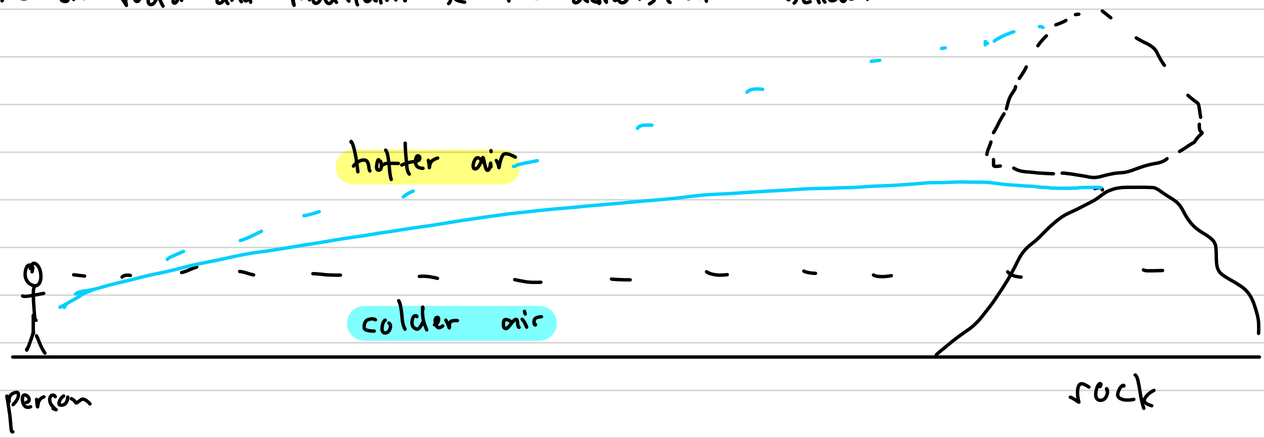
Because of the sun doesn't produce any heat to the grond, it released the heat.

Instead of moving upward, the sound wave moves downward because of the lower air temperature near to the ground and higher in the upper part. Thus, the upper part wave moves faster than the lower part so the wave moves downward and hit the ground and then bounced back until it reached the student house. Thats why the student can hear the train sound in the evening

Many animals evolved to use this evening and night advantages to hear better and to comunicate further. The example is wolf. It howl in night so they can comunicate further and hunt down the prey better.

B) The same phenomenon applies to fata morgana. I has to do with the temperature inversion. When the lower air area get colder and higher temperature at the upper area, the light get bends and creating the watery or floating effect on road and mountain. See the demonstration bellow.



As it seen, the light bends due to faster light speed in hotter air and lower speed near to the ground. The top of the rock seems come from higher ground level. Thus the rock appear to be floating in the human eyes. The road can also appear to be watery because the road seems to be floating in the human eyes.