Tugas 2

Qornain A
21/481767/TK/53170

Saturday, 21 August 2021 00.41

Bagian 1.6

29 Population Growth

Knoxvile is 500.000 people
$r = 3.75\%$ per year.

When the population will reach 1 million.

- The formula of the population growth from Knoxvile is express as

$$P = \text{People}\,(1 + \text{ratio})^x$$
$$10^6 = 5\cancel{00.000}\,(1 + 0{,}0375)^x$$
$$2\,\frac{\cancel{10}}{\cancel{8}} = (1{,}0375)^x$$
$$2 = (1{,}0375)^x$$
$$x = {}^{1{,}0375}\log 2$$
$$x = 18{,}8 \approx 19 \text{ year}$$

The approximet years to reach 1 million people are 19 years.

30 1890 in Silver Run was 6250. If the population increased at ratio 2.75% per year,

a. The population in 1915 and 1940

- Year 1915

Formula :
$$P = P_0\,(1 + \text{ratio})^{\text{year}}$$
$$= 6250\,(1{,}0275)^{25}$$
$$= 12.314{,}75$$
$$P \approx 12.314 \text{ people}$$

- Year 1940

Formula :
$$P = P_0\,(1 + \text{ratio})^{\text{year}}$$
$$= 6250\,(1{,}0275)^{50}$$
$$= 24.264{,}51$$
$$P \approx 24.264$$

The population in 1915 and 1940 are
1915 ⇝ 12.314 people
1940 ⇝ 24.264

b. When the population will hit 50.000 people ?

Formula :
$$P = P_0\,(1 + \text{ratio})^{\text{year}}$$
$$50.000 = 625\cancel{0}\,(1{,}0275)^{\text{year}}$$
$$\frac{5000}{625} = (1{,}0275)^{\text{year}}$$

$$\text{Year} = {}^{1{,}0275}\log 8$$
$$\text{year} = 76{,}6$$
$$\text{year} \approx 77$$

$$50.000 = 625 (1,0275)^{year}$$
$$\frac{5000}{625} = (1,0275)^{year}$$
$$\frac{1000}{125} = (1,0275)^{year}$$
$$\frac{40}{5} = (1,0275)^{year}$$
$$8 = (1,0275)^{year}$$

$$year = 76.6$$
$$year \approx 77$$

The population will hit 50.000 people after 77 years.

31 Radioactive decay

The half life of phosphorus = 14 days
There are 6.6 grams initially.

a. The function t

$$f(t) = \left(\frac{1}{2}\right)^{\frac{t}{T}} N_0$$
$$= \frac{1}{2}^{\frac{t}{14}} \cdot 6.6$$

$$\boxed{\begin{gathered} \frac{N_t}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T}} \\ N_t = \left(\frac{1}{2}\right)^{\frac{t}{T}} N_0 \end{gathered}}$$

N_t = atom after t years
N_0 = The initial amount of atom
t = number of days
T = half life durations.

b. If $f(t)$ = 1 gram

$$f(t) = 1 = \frac{1}{2}^{\frac{t}{14}} \cdot 6.6$$
$$\frac{10}{66} = \frac{1}{2}^{\frac{t}{14}}$$
$$\frac{5}{33} = \frac{1}{2}^{\frac{t}{14}}$$
$$\frac{t}{14} = {}^{\frac{1}{2}}\log \frac{5}{33}$$
$$t = -14 \ {}^{2}\log \frac{5}{33}$$
$$= 38.1$$

$$t \approx 38 \text{ days}$$

The durations for the phosphorus hal to take are about 38 days.

32 John initial money = \$ 2300
interest rate = 6 %
target balance = \$ 4150
how long will it takes ?

The equation for balance in the end of the years.

$$P = P_0 \cdot (1 + \text{interest rate})^x$$
$$4150 = 2300\,(1 + 0{,}06)^x$$
$$\frac{415}{230} = (1{,}06)^x$$
$$\frac{83}{46} = (1{,}06)^x$$
$$x = {}^{1{,}06}\log \frac{83}{46}$$
$$x = 10{,}12 \text{ years}$$
$$x \approx 10 \text{ years.}$$

The duration that Johnson has to takes to reach target balance are about 10 years.

33 The time for the money to double in total while the interest rate is 6,25% compounded annually can be express as

$$P = 2P_0$$

$$P = P_0\,(1+r)^t$$
$$2\cancel{P_0} = \cancel{P_0}\,(1+r)^t$$
$$2 = (1 + 0{,}0625)^t$$
$$2 = 1{,}0625^t$$
$$t = {}^{1{,}0625}\log 2$$
$$t = 11.43$$
$$t \approx 11 \text{ year.}$$

The time for the money to reach double in total is about 11 years.

34 The time for the money to triple in total while the interest rate is 5.75% compounded annually can be express as

$$P = 3P_0$$

$$P = P_0\,(1+r)^t$$
$$3\cancel{P_0} = \cancel{P_0}\,(1+r)^t$$
$$3 = (1 + 0{,}0575)^t$$
$$3 = (1{,}0575)^t$$
$$t = {}^{1{,}0575}\log 3$$
$$= 19{,}65$$
$$t \approx 20 \text{ years.}$$

The time for the money to reach double in total is about 11 years.

35 Cholera bacteria growth model express as

35 Cholera bacteria growth model express as

$$y = y_0 r^n \quad ; \quad y_0 = 1 \quad ; \quad r = 2$$

the growth rate is double the total of bacterium every half hour.

24 hours are equal to 720 minute, so the total times to grow double in size is 24 times. Now we can calculate the result.

$$y = 1(2)^{24}$$
$$= 16.777.216 \text{ bacterias.}$$

36 Eliminating a disease

initial cases = 10.000 cases
reduce rate = 20%

a. How many year will it take to reached 1000 case

The formula is : $y = y_0 (1-r)^x$; x = years.

$$1000 = 10.000 \, (0{,}8)^x$$
$$\frac{1}{10} = 0{,}8^x$$
$$\frac{1}{10} = \frac{4}{5}^x$$
$$x = {}^{\frac{4}{5}}\log \frac{1}{10}$$
$$x = 10{,}3 \text{ years}$$
$$x \approx 10 \text{ years to reach 1000 cases.}$$

b. Let's say we are going to reduce the case until it reaches 0,1 case.

$$y = y_0 (1-r)^x$$
$$0{,}1 = 10.000 \, (0{,}8)^x$$
$$10^{-6} = \left(\frac{4}{5}\right)^x$$
$$x = 51.59 \text{ years}$$
$$x \approx 52 \text{ years.}$$

Bagian 1.6

19 $f(x) = x^2 + 1 \quad ; \quad x \geq 0$

$f^{-1}(x) = ?$

$f(x) = y = x^2 + 1$

$$y - 1 = x^2$$
$$\pm\sqrt{y-1} \quad x$$

Now, switch the y and x !

$f^{-1}(x) = \pm\sqrt{y-1}$

$\boxed{38}$ $f(x) = -x+1$; find the inverse and draw the line together with the line $y = x$ and determine the angle of the intersect.

Find the inverse :

$f(x) = -x+1 = y$

$-x = y-1$

$x = 1-y$

switch the y and x variables.

$y = 1-x$

$f^{-1}(x) = 1-x$

x	-1	0	1	2	3
y	2	1	0	-1	-2

■ = $f^{-1}(x)$

$f(x) = -x+1$

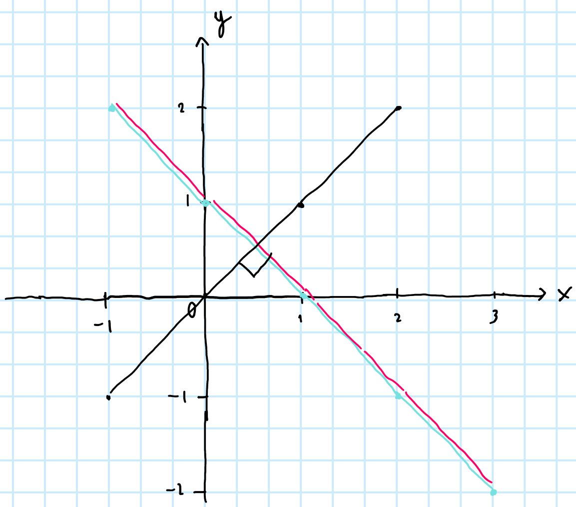
x	-1	0	1	2	3
y	2	1	0	-1	-2

■ = $f(x)$

$y = x$

x	-1	0	1	2	3
y	-1	0	1	2	3

□ = $y = x$



its appear that the angle of the intersect is 90°

43) Simplify the expressions.

a. $e^{\ln 7,2}$

$e^{\ln 7,2} = 7,2$

b. $e^{-\ln x^2} = e^{\ln x^{-2}}$

$= x^{-2}$

$= \dfrac{1}{x^2}$

c. $e^{\ln x - \ln y} = e^{\ln \frac{x}{y}}$

$= \dfrac{x}{y}$

*note

$$\boxed{e^{\ln x} = x}$$

47) Solve the y in terms of t or x as appropriate

$\ln y = 2t + 4$

$\ln y - 4 = 2t$

$t = \dfrac{\ln y - 4}{2} = \dfrac{\ln y}{2} - 2$

$t = 2^{-1} \ln y - 2$

$= 2 \ln \dfrac{1}{y} - 2 = 2\left(\ln \dfrac{1}{y} - 1\right)$

$= 2\left(\ln \dfrac{1}{y} - \ln e\right)$

$= 2\left(\ln \dfrac{1}{ye}\right)$

61) Simpify

a. $2^{\,^4\log x} = 2^{\,^{2^2}\log x}$

$= 2^{\,^2\log x^{\frac{1}{2}}}$

$= x^{\frac{1}{2}} = \sqrt{x}$

b. $9^{\,^3\log x} = 3^{2\,^3\log x}$

$= 3^{\,^3\log x^2}$

b. $9^{{}^3\log x} = 3^{2\,{}^3\log x}$

$= 3^{{}^3\log x^2}$

$= x^2$

c. ${}^2\log\left(e^{(\ln 2)(\sin x)}\right) = {}^2\log\left(2^{\sin x}\right)$

$= \sin x$

$\boxed{63}$ Express as natural log and simplify.

a. $\dfrac{{}^2\log x}{{}^3\log x} = \dfrac{\frac{\cancel{\ln x}}{\ln 2}}{\frac{\cancel{\ln x}}{\ln 3}} = \dfrac{\ln 3}{\ln 2}$ //

b. $\dfrac{{}^2\log x}{{}^8\log x} = \dfrac{{}^2\log x}{{}^{2^3}\log x} = \dfrac{\frac{\cancel{\ln x}}{\ln 2}}{\frac{\cancel{\ln x}}{\ln 2^3}} = \dfrac{\ln 2^3}{\ln 2} = 3$ //

c. $\dfrac{{}^x\log a}{{}^{x^2}\log a} = \dfrac{\frac{\cancel{\ln a}}{\ln x}}{\frac{\cancel{\ln a}}{\ln x^2}} = \dfrac{\ln x^2}{\ln x} = 2$ //