PR KVT

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1) Selesaikan integral dibawah !

A $\int \frac{\cos x}{\sin^3 x + \sin x} \, dx = \int (\underbrace{\sin^3 x + \sin x}_{u})^{-1} \cos x \, dx$

$= \int (\underbrace{\sin^3 x}_{u^3} + \underbrace{\sin x}_{u})^{-1} (\underbrace{d(\sin x)}_{du})$

$= \int (u^3 + u)^{-1} \, du$

$= \int u^{-1} \, du - \int u (u^2+1)^{-1} \, du$

$= \ln u + \ln \frac{(u^2+1)}{2} + C$

$= \ln(\sin x) + \ln \frac{(\sin^2 x + 1)}{2} + C$ //

B. $\int \sin^2(3x) \, dx$?

$= \int \frac{1}{2} - \frac{\cos 6x}{2} \, dx$

$= \int \frac{1}{2} \, dx - \int \frac{\cos 6x}{2} \, dx$

$= \frac{1}{2} x - \frac{1}{2} \int \cos 6x \, dx$

$= \frac{1}{2} x - \frac{1}{2} \cdot \frac{1}{6} \cdot \sin 6x + C$

$= \frac{1}{2} x - \frac{\sin 6x}{12} + C$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 2x$

$1 - \cos 6x = 2\sin^2 2x$

$\frac{1 - \cos 6x}{2} = \sin^2 2x$

$$= \frac{1}{2}x - \frac{\sin 6x}{12} + C$$

2) Selesaikan Integral dibawah

A. Jika $f(x) = 1 + \sin x + \sin x + \sin x + \ldots$

$0 \leq x \leq \frac{\pi}{4}$; maka $\int_0^{\frac{\pi}{4}} f(x)\, dx = ?$

$$\int_0^{\frac{\pi}{4}} 1 + \sin x + \sin x + \ldots\, dx = x - \cos x - \cos x - \ldots \Big]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{\pi}{4} - \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} \ldots + n\right) - \left(0 - 1 - 1 - 1 \ldots + n\right)$$

$$\frac{\pi}{4} - \frac{n\sqrt{2}}{2} + n$$

B. $\int \underset{u}{x^2} \underbrace{\cos x\, dx}_{dv}$?

$u = x^2 \quad dv = \cos x\, dx$
$du = 2x\, dx \quad v = \sin x$

$$\int u\, dv = uv - \int v\, du$$

$$= x^2 \sin x - \int \sin x (2x)\, dx$$

$$= x^2 \sin x - 2\int \underset{u}{x} \underset{dv}{\sin x\, dx}$$

$u = x \quad dv = \sin x\, dx$
$du = 1\, dx \quad v = -\cos x$

$$= x^2 \sin x - \left(2\left(-x\cos x\right) - 2\int\left(-\cos x\, dx\right)\right)$$

ingat.

$$= x^2 \sin x + 2x\cos x - 2\sin x + C$$