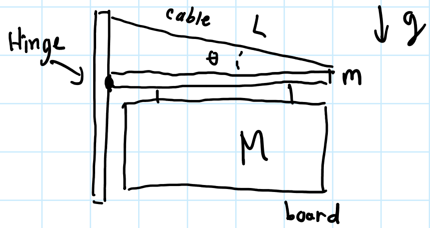
Home work ※ 2

Monday, 11 October 2021 01.35

Rotation of Rigid Body

[1] Problem ※ 1 : Advertisement Board



We know : M, m, L, θ_{crit}, θ, T_{crit}

A. Prove that θ should be $\theta > \theta_{crit}$ for the rope to not break! Determine the value of θ_{crit}!

$$\tau = 0 = -\frac{1}{2} M g \cancel{L} \cos(\theta_{crit}) - \frac{1}{2} m \cancel{L} \cos(\theta_{crit}) + T_{crit} \cdot \cancel{L} \sin\theta_{crit} \cos\theta_{crit} = 0$$

$$-\frac{1}{2} g \cancel{\cos(\theta_{crit})} (M + m) + T_{crit} \cdot \sin(\theta_{crit}) \cancel{\cos(\theta_{crit})} = 0$$

$$-\frac{1}{2} g (M+m) = -T_{crit} \sin\theta_{crit}$$

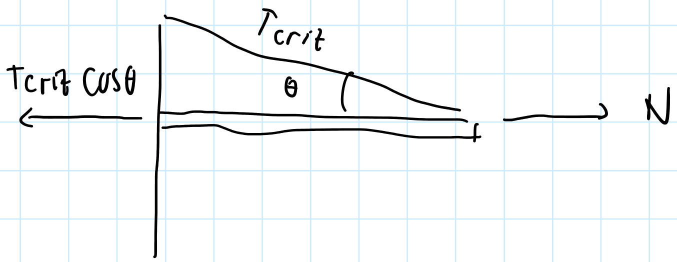
$$\sin\theta_{crit} = \frac{(M+m) g}{T_{crit}}$$

$$\sin^{-1}\left(\frac{(M+m) g}{T_{crit}}\right) = \theta_{crit}$$

In order to prevent the rope from breaking, the value of θ has to be greater than θ_{crit}. When the anggel is θ, it create a new tension call T. The expression is

$$\sin^{-1}\left(\frac{(M+m) g}{T_{crit}}\right) = \theta_{crit} < \theta = \sin^{-1}\left(\frac{(M+m) g}{T_{crit}}\right)$$

B. The normal force (N) that acting in the hinge when $\theta = \theta$ hinge!



Normal $= T_{crit} \cos \theta_{crit}$

Problem #2 : Rolling Yo-yo

We know that : $I = KMR^2$

$r = \alpha R$

μ_s

μ_k

∴ To find the translation accelaration, we can use this equation

First configuration

Rotation =

$$\tau_{net} = \alpha_{rad} I$$

$$F\alpha R - fR = a_{rad} I$$

$$F\alpha \cancel{R} + f\cancel{R} = KM\cancel{R^2} \cdot \frac{a}{\cancel{R}} \cancel{R}$$

$$F\alpha \cancel{R} + f\cancel{R} = KM\cancel{R^2} \cdot \frac{a}{\cancel{R}}$$

$$f = -F\alpha \pm KMa$$

Translation: $\Sigma F = m \cdot a$

$$F + f = m \cdot a$$

$$F - F\alpha + KMa = Ma$$

$$F(1-\alpha) = Ma(1-k)$$

$$a = \frac{F}{M}\frac{(1-\alpha)}{(1-k)}$$

: Second configuration

Rotation : $\tau_{net} = a_{rad}\, I$

$$F\alpha \cancel{R} - f\cancel{R} = KM\cancel{R^2} \cdot \frac{a}{\cancel{R}}$$

$$f = F\alpha - KMa$$

Translation : $\Sigma F = m \cdot a$

$$F - f = M \cdot a$$

$$F - F\alpha + KMa = Ma$$

$$F(1-\alpha) = Ma(1-k)$$

$$a = F(1-\alpha)$$

$$F(1-\alpha) = Ma(1-k)$$
$$a = \frac{F}{M}\frac{(1-\alpha)}{(1-k)}$$

B. Prove that $F \leq F_{max}$, the yo-yo can roll without slip. Determine F_{max}!

We know that

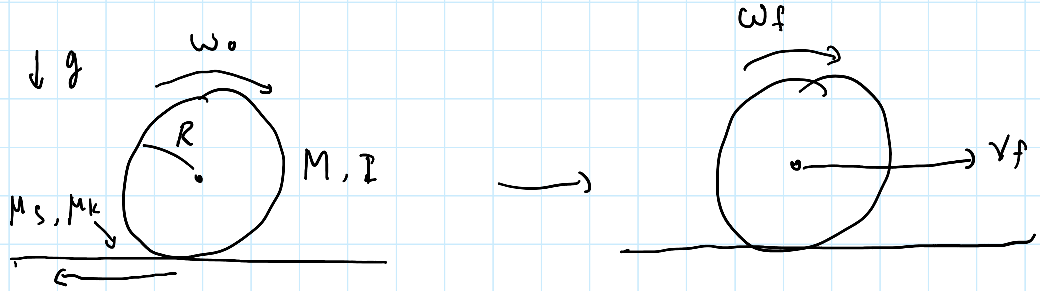
$$f = \mu_s \cdot M \cdot g \leq F_{max}$$

$$F\alpha - KMa = \mu_s M g \leq F_{max}$$
$$Ma\alpha - KMa = \mu_s M g \leq F_{max}$$
$$\frac{Ma(\alpha - k)}{\mu_s M g} \leq F_{max}$$

When $F > f$, the static friction is removed and change into kinetik friction.

Hence, $F \leq F_{max}$, the yo-yo can roll without slipping

Problem #3 : Rolling Cylinder



A. Determine the value of V_f, ω_f, and Δt!

$$\tau_{net} = a_{rad} \cdot I$$

$$F \cdot R = KM\cancel{R^2} \cdot \frac{a}{\cancel{R}}$$

$$f_g \cdot \cancel{R} = KM\cancel{R}\, a$$

$$\mu_s \cdot M \cdot g = KMa$$

$$\therefore a = \frac{\mu_s g}{K}$$

$$\therefore a_{rad} = \frac{\mu_s g}{KR}$$

∴ Determine the value of V_f

$$V_f = a \cdot \Delta t$$
$$= \frac{\mu_k g \Delta t}{K}$$

∴ Determine the value of ω_f

$$\omega_f = \frac{\mu_k g \Delta t}{KR}$$

∴ Determine the value of Δt

$$\Delta t = \frac{\omega_f KR}{\mu_k g} \quad \text{or} \quad \Delta t = \frac{V_f k}{\mu_k g}$$

B. Prove that ΔE_k can be express as

$$\Delta E_k = \eta \left(\frac{M \omega_0^2 R^2}{2} \right)$$ and determine the value of η!

$$\Delta E_k = \eta \left(\frac{M \omega_0^2 R^2}{2} \right)$$ and determine the value of η !

∴ Energy lost can also be express as

$$\Delta E_k = \frac{1}{2} m \left(v_f^2 - v_i^2 \right)$$

So we need to get some equality.

$$\frac{1}{2} m \left(\frac{\mu_k^2 g^2 \delta t^2}{k^2} - \frac{\mu_k^2 g^2 \delta t^2}{k^2} \right) = \eta \left(M \omega_0^2 R^2 \right)$$

"Sorry sir, I cannot proof the equation sir."

I gense I have to answer it anyway.

$$\eta = \frac{k}{k-1}$$