Quiz 2

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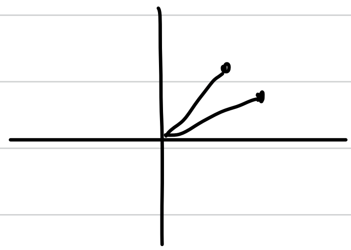
21/481767/TK/53170

1) $\dfrac{(\sqrt{3}+i)^7}{(1+i)^3}$



$\dfrac{\pi}{6}$ $\dfrac{1}{\sqrt{3}}$

Jawab : $(\sqrt{3}+i)^7 = \left[\sqrt{3+1}\ \text{cis}\left(\frac{\pi}{6}\right)\right]^7$

$= \left[2\ \text{cis}\left(\frac{\pi}{6}\right)\right]^7$

$= 128\ \text{cis}\ \frac{7\pi}{6}$

$(1+i)^3 = \left[\sqrt{2}\ \text{cis}\left(\frac{\pi}{4}\right)\right]^3$

$= 2\sqrt{2}\ \text{cis}\ \frac{3\pi}{4}$

Sehingga $\dfrac{(\sqrt{3}+i)^7}{(1+i)^3} = \dfrac{128\ \text{cis}\ \frac{7}{6}\pi}{2\sqrt{2}\ \text{cis}\ \frac{3}{4}\pi}$

$= \dfrac{128\sqrt{2}}{4}\ \text{cis}\left(\frac{7}{6}\pi - \frac{3}{4}\pi\right)$

$= 32\sqrt{2}\ \text{cis}\left(\dfrac{14\pi - 9\pi}{12}\right)$

$$= 32\sqrt{2}\ \text{cis}\ \frac{5}{12}\pi = 32\sqrt{2}\left(\cos\left(\frac{5}{12}\pi\right) + i\sin\left(\frac{5}{12}\pi\right)\right)$$

$$\text{Re}(z) = 32\sqrt{2}\cos\frac{5}{12}\pi = 32\sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

$$= 8\sqrt{2}\left(\sqrt{6}-\sqrt{2}\right)$$

$$= 8\sqrt{12} - 16$$

$$= 16\sqrt{3} - 16$$

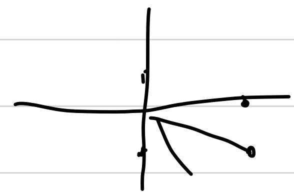
$$\text{Im}(z) = 32\sqrt{2}\sin\left(\frac{5}{12}\pi\right)$$

$$= 32\sqrt{2}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)$$

$$= 8\sqrt{2}\left(\sqrt{2}+\sqrt{6}\right)$$

$$= 16 + 8\sqrt{12} = 16 + 16\sqrt{3}$$

2) Tentukan konjugat $(3 - i\sqrt{3})^5(-1+i)^7$



$$(3 - i\sqrt{3})^5 = \left(\sqrt{9+3}\ \text{cis}\left(-\frac{\pi}{6}\right)\right)^5$$

$$= \sqrt{12}^5\ \text{cis}\left(-\frac{5\pi}{6}\right)$$

$$= 288\sqrt{3}\ \text{cis}\left(-\frac{5\pi}{6}\right)$$

$$(-1+i)^7 = \left(\sqrt{2}\ \text{cis}\left(-\frac{\pi}{4}\right)\right)^7$$

$$= 8\sqrt{2}\ \text{cis}\left(-\frac{7\pi}{4}\right)$$

$$(3-i\sqrt{3})^5(-1+i)^7 = \left(288\sqrt{3}\ \text{cis}\left(\frac{-5}{6}\pi\right)\right)\left(8\sqrt{2}\ \text{cis}\left(\frac{-7\pi}{4}\right)\right)$$

$$= 2304\sqrt{6}\ \text{cis}\left(-\frac{5}{6}\pi - \frac{7}{4}\pi\right)$$

$$= 2304\sqrt{6}\ \text{cis}\left(\frac{-10\pi - 21\pi}{12}\right)$$

$$= 2304\sqrt{6}\ \text{cis}\left(\frac{-31\pi}{12}\right)$$

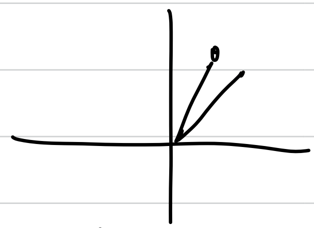
$$= 2304\sqrt{6}\ \text{cis}\left(-\frac{7}{12}\pi\right)$$

$$2304\sqrt{6}\left(\cos\left(-\frac{7}{12}\pi\right)+i\sin\left(-\frac{7}{12}\pi\right)\right)$$

konjugat $= 2304\sqrt{6}\left(\cos\left(\frac{7}{12}\pi\right)-i\left(-\sin\left(\frac{7}{12}\pi\right)\right)\right)$

$$= 2304\sqrt{6}\left(\cos\left(\frac{7}{12}\pi\right)+i\sin\left(\frac{7}{12}\pi\right)\right)$$

dan $\dfrac{(\sqrt{3}+3i)^4}{(1+i)^{10}}$



$\frac{\pi}{3}$ $\frac{\pi}{4}$

$$(\sqrt{3}+3i)^4 = \left(\sqrt{12}\ \text{cis}\left(\frac{\pi}{3}\right)\right)^4$$
$$= 144\ \text{cis}\left(\frac{4\pi}{3}\right)$$

$$(1+i)^{10} = \left(\sqrt{2}\ \text{cis}\left(\frac{\pi}{4}\right)\right)^{10}$$
$$= 32\ \text{cis}\left(\frac{10\pi}{4}\right)$$
$$= 32\ \text{cis}\left(\frac{5}{2}\pi\right)$$

sehingga $\dfrac{(\sqrt{3}+3i)^4}{(1+i)^{10}} = \dfrac{144 \text{ cis } \left(\frac{4}{3}\pi\right)}{32 \text{ cis } \left(\frac{5}{2}\pi\right)}$

$= \frac{9}{2} \text{ cis } \left(\frac{4}{3}\pi - \frac{5}{2}\pi\right)$

$= \frac{9}{2} \text{ cis} \left(\frac{8\pi - 15\pi}{6}\right) = \frac{9}{2} \text{ cis } \left(\frac{-7}{6}\pi\right)$

$= \frac{9}{2}\left(\cos\left(-\frac{7}{6}\pi\right) + i \sin\left(-\frac{7}{6}\pi\right)\right)$

$= \frac{9}{2}\left(\cos\left(\frac{7}{6}\pi\right) - i \sin\left(\frac{7}{6}\pi\right)\right)$

konjugat $= \frac{9}{2}\left(\cos\left(\frac{7}{6}\pi\right) + i \sin\left(\frac{7}{6}\pi\right)\right)$ //

4) Tentukan nilai z

$z^4 + 8(1 + i\sqrt{3}) = 0$

$z^4 = -8(1 + i\sqrt{3})$

$z^4 = (-8 - i8\sqrt{3})$

$$z^n = r^n \operatorname{cis} n\theta$$

$$(-8 - i8\sqrt{3}) = \sqrt{64 + 64(3)} \operatorname{cis} \theta$$

$$\tan \theta = \frac{8\sqrt{3}}{8} = \frac{\sqrt{3}}{1} = \frac{\pi}{3}$$