Pr KVT 2

Tuesday, 21 September 2021 13.55

1. Evaluasi integral dibawah ini

a) $\int x\,dx = \frac{1}{2}x^2 + c$

b) $\int \frac{3x^5}{\sqrt{5-9x^6}}\,dx = \int 3x^5 (5-9x^6)^{-\frac{1}{2}}\,dx$

$u = 5-9x^6$

$du = -54x^5\,dx$

$dx = \frac{du}{-54x^5}$

$\int 3x^5 (u)^{-\frac{1}{2}} \frac{du}{-54x^5} = \int -\frac{1}{18} u^{-\frac{1}{2}}\,du$

$= -\frac{1}{18}\int u^{-\frac{1}{2}}\,du = -\frac{1}{18}\, 2\, u^{\frac{1}{2}} = -\frac{1}{9} u^{\frac{1}{2}}$

$= -\frac{1}{9}(5-9x^6)^{\frac{1}{2}} + C = -\frac{1}{9}\sqrt{5-9x^6} + C$

c) $\int \frac{5x-10}{2x^2+x-1}\,dx = \int \frac{5x-10}{(x+1)(2x-1)}\,dx$

$= \int \frac{5x-10}{(x+1)(2x-1)}\,dx = \int \frac{A}{(x+1)} + \frac{B}{(2x-1)}\,dx$

$= \int \frac{5x-10}{(x+1)(2x-1)}\,dx = \int \frac{2xA - A + Bx + B}{(x+1)(2x-1)}\,dx$

$5x-10 = 2Ax - A + Bx + B$

$5x-10 = 2Ax + Bx - A + B$

$5x-10 = (2A+B)x - (A-B)$

$2A + B = 5$

$A - B = 10$ +

$3A = 15$

$A = 5$

$5 - B = 10$

$B = -5$

$\ln(x) = \frac{1}{x} \cdot 1$

$\ln(2x) = \frac{1}{2x}\, 2$

$= \int \frac{5x-10}{(x+1)(2x-1)}\,dx = \int \frac{5}{(x+1)} + \frac{-5}{(2x-1)}\,dx$

$= \int \frac{5}{x+1}\,dx + \int \frac{-5}{2x-1}\,dx = \int 5(x+1)^{-1}\,dx + \int -5(2x-1)^{-1}\,dx$

$= 5\ln(x+1) - \frac{5}{2}\ln(2x-1) + C$

$= 5\ln(x+1) - \frac{5}{2}\ln(2x-1) + C$

$\boxed{\ln(2x-1) = \frac{1}{2x-1}\cdot 2}$

d) $\int_3^4 \frac{x^4-2x^2-10}{x-5}\,dx$

$115 \times 5 = 575$

$$\begin{array}{r l}
 & x^3 + 5x^2 + 23x + 115 \\
x-5 \,\big) & x^4 - 2x^2 - 10 \\
 & \underline{x^4 - 5x^3} \; - \\
 & 5x^3 - 2x^2 \\
 & \underline{5x^3 - 25x^2} \; - \\
 & 23x^2 - 10 \\
 & \underline{23x^2 - 115x} \; - \\
 & 115x - 10 \\
 & \underline{115x - 575} \; - \\
 & 565
\end{array}$$

$= \int_3^4 x^3 + 5x^2 + 23x + 115 + \frac{565}{(x-5)}\,dx$

$= \left.\frac{1}{4}x^4 + \frac{5}{3}x^3 + \frac{23}{2}x^2 + 115x + 565\ln(x-5)\right]_3^4$

$= \frac{1}{4}(4)^4 + \frac{5}{3}(4)^3 + \frac{23}{2}(4)^2 + 115(4) + 565\ln(-1) - \frac{1}{4}(3)^4 - \frac{5}{3}(3)^3 - \frac{23}{2}(3)^2 - 115(3) - 565\ln(-2)$

$= \frac{3611}{12} + 565\left(\ln(-1) - \ln(-2)\right)$

$= \frac{3611}{12} + 565\ln\frac{(-1)}{(-2)} = \frac{3611}{12} + 565\ln\left(\frac{1}{2}\right) = \frac{3611}{12} + 565\ln 2^{-1} = \frac{3611}{12} - 565\ln 2$ //

e) $\int_2^4 \frac{x^2-4x-5}{x^4-2x^3+2x-1}\,dx = \int_2^4 \frac{(x-5)(x+1)}{x^4-2x^3+2x-1}\,dx$

$= \int_2^4 \frac{(x-5)(x+1)}{x^4-1-2x^3+2x}\,dx = \int_2^4 \frac{(x-5)(x+1)}{(x^2-1)(x^2+1)-2x(x^2-1)}\,dx$

$= \int^4 \frac{(x-5)\cancel{(x+1)}}{\quad}\,dx = \int^4 \frac{x-5}{\quad}\,dx$

$$= \int_2^4 \frac{(x-5)\cancel{(x+1)}}{(x-1)\cancel{(x+x)}(x^2+1) - 2x(x-1)\cancel{(x+1)}}\,dx = \int_2^4 \frac{x-5}{(x-1)\,(x^2-2x+1)}\,dx$$

$$= \int_2^4 \frac{x-5}{(x-1)\,(x-1)(x-1)}\,dx = \int_2^4 \frac{x-5}{(x-1)^3}\,dx$$

$u = x-1$

$du = 1\,dx$

$dx = du$

$$\int \frac{x-1-4}{(u)^3}\,du = \int \frac{u-4}{u^3}\,du = \int \frac{u}{u^3}\,du - \frac{4}{u^3}\,du$$

$$= \int u^{-2}\,du - \int 4\,(u^{-3})\,du = -u^{-1} - \overset{2}{\cancel{4}}\left(\frac{-1}{\cancel{2}}\right)u^{-2} \Big]$$

$$= \frac{1}{1-x} + \frac{2}{(x-1)^2}\Big]_2^4$$

$$= \frac{-1}{3} + \frac{2}{9} - (-1 + 2) = \frac{-3+2}{9} - 1 = \frac{-1-9}{9}$$

$$= \frac{-10}{9} \,//$$