Qornain Aji
21/481767/TK/53170

Tugas 1

Vektor dan Matriks
Kalkulus Variabel Jamak

1

$A_{3\times 2}$

$$A = \begin{pmatrix} 6 & -3 \\ 6 & -2 \\ 6 & -1 \end{pmatrix}$$

Tentukan hasil dari :
$A^T \times A$, $A \times A^T$, dan $|A \times A^T|$

※ $A^T \times A$

$\therefore A^T = \begin{pmatrix} 6 & 6 & 6 \\ -3 & -2 & -1 \end{pmatrix}$

$$A^T \times A = \begin{pmatrix} 6 & 6 & 6 \\ -3 & -2 & -1 \end{pmatrix} \times \begin{pmatrix} 6 & -3 \\ 6 & -2 \\ 6 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 36 & -36 \\ -36 & 14 \end{pmatrix}$$

※ $A \times A^T = \begin{pmatrix} 6 & -3 \\ 6 & -2 \\ 6 & -1 \end{pmatrix} \times \begin{pmatrix} 6 & 6 & 6 \\ -3 & -2 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 45 & 42 & 39 \\ 42 & 40 & 38 \\ 39 & 38 & 37 \end{pmatrix}$$

※ $|A \times A^T|$

Dengan menggunakan cara sarrus.

$$\left|\begin{array}{ccc|cc} 45 & 42 & 39 & 45 & 42 \\ 42 & 40 & 38 & 42 & 40 \\ 39 & 38 & 37 & 39 & 38 \end{array}\right|$$

$66600 + 62244 + 62244 - 60840 - 64980 - 65268 = 0$

2

$$\begin{aligned} 2x + 3y + 4z &= 10 \\ x - 21y + 3z &= -2 \\ x - y - z &= 26 \end{aligned}$$

$$\underbrace{\begin{pmatrix} 2 & 3 & 4 \\ 1 & -21 & 3 \\ 1 & -1 & -1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 10 \\ -2 \\ 26 \end{pmatrix}}_{B}$$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 3 & 4 \\ 1 & -21 & 3 \\ 1 & -1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 10 \\ -2 \\ 26 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{Adj}(A)$$

$$\therefore |A| = \left|\begin{array}{ccc|cc} 2 & 3 & 4 & 2 & 3 \\ 1 & -21 & 3 & 1 & -21 \\ 1 & -1 & -1 & 1 & -1 \end{array}\right|$$

$42 + 9 + (-4) - (-84) - (-6) - (-3)$
$= 42 + 9 - 4 + 84 + 6 + 3$
$= 42 + 18 + 80 = 60 + 80 = 140$

$\therefore \operatorname{Adj}(A) = (\text{Kof } A)^T$

$$M = \begin{pmatrix} 24 & -4 & 20 \\ 1 & -6 & -5 \\ 93 & 2 & -45 \end{pmatrix}; \quad \text{Kof } A = \begin{pmatrix} 24 & 4 & 20 \\ -1 & -6 & 5 \\ 93 & -2 & -45 \end{pmatrix}$$

$$(\text{Kof } A)^T = \begin{pmatrix} 24 & -1 & 93 \\ 4 & -6 & -2 \\ 20 & 5 & -45 \end{pmatrix} = \operatorname{Adj}(A)$$

$$A^{-1} = \frac{1}{140} \begin{pmatrix} 24 & -1 & 93 \\ 4 & -6 & -2 \\ 20 & 5 & -45 \end{pmatrix}$$

$$X = A^{-1} B$$

$$= \frac{1}{140}\begin{pmatrix} 24 & -1 & 93 \\ 4 & -6 & -2 \\ 20 & 5 & -45 \end{pmatrix}\begin{pmatrix} 10 \\ -2 \\ 26 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{140}\begin{pmatrix} 24 & -1 & 93 \\ 4 & -6 & -2 \\ 20 & 5 & -45 \end{pmatrix}\begin{pmatrix} 10 \\ -2 \\ 26 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{140}\begin{pmatrix} 2660 \\ 0 \\ -980 \end{pmatrix} = \begin{pmatrix} 19 \\ 0 \\ -7 \end{pmatrix}$$

$x = 19$, $y = 0$, $z = -7$ //

3. Diketahui bidang α sejajar garis $\frac{2}{3} - x = \frac{y-7}{3} = 11 - z$ dan garis $x - 1 = \frac{y-2}{2} = \frac{z-3}{3}$ terletak pada bidang α. Tentukan persamaan bidang α!

Jawab:

∴ garis P	∴ garis Q
$\frac{2}{3} - x = t$	$x - 1 = t$
$\frac{y-7}{3} = t$	$\frac{y-2}{2} = t$
$11 - z = t$	$\frac{z-3}{3} = t$

$x = \frac{2}{3} - t$	$x = 1 + t$
$y = 7 + 3t$	$y = 2 + 2t$
$z = 11 - t$	$z = 3 + 3t$

$\bar{V}_P = -\bar{i} + 3\bar{j} - \bar{k}$

$\bar{V}_Q = \bar{i} + 2\bar{j} + 3\bar{k}$

$\therefore P_0 = \left(\frac{2}{3}, 7, 11\right)$

$P_0P = (-1, 3, -1)$

$\therefore Q_0 = (1, 2, 3)$

$Q_0Q = (1, 2, 3)$

Karena Vektor PoP berada diatas bidang α, akan titik yang merupakan jarak terdekat dari PoP dengan bidang α, sebut saja R dengan koordinat (x, y, z) sehingga

$RP_0 = \bar{N}$ bidang α. Disimpulkan juga bahwa:

$Q_0Q \times Q_0R = \bar{N}$ bidang α.

※ $RP_0 = \bar{N} = \left(\frac{2}{3} - x, 7 - y, 11 - z\right)$

※ $Q_0R = (x-1, y-2, z-3)$

$$Q_0Q \times Q_0R : \begin{vmatrix} i & \bar{j} & \bar{k} \\ 1 & 2 & 3 \\ x-1 & y-2 & z-3 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 \\ y-2 & z-3 \end{vmatrix} i - \begin{vmatrix} 1 & 3 \\ x-1 & z-3 \end{vmatrix} \bar{j} + \begin{vmatrix} 1 & 2 \\ x-1 & y-2 \end{vmatrix} \bar{k} = \bar{N}$$

$\left(2(z-3) - 3(y-2)\right)i - \left(z-3-3(x-1)\right)\bar{j} + \left(y-2-2(x-1)\right)\bar{k}$

$(2z-6-3y+6)i - (z-3-3x+3)\bar{j} + (y-2-2x+2)\bar{k}$

$(2z-3y)i - (z-3x)\bar{j} + (y-2x)\bar{k} = \bar{N}$

※ $\bar{N} = \bar{N}$

∘) $\frac{2}{3} - x = 2z - 3y$: $2 = 3x + 6z - 9y$

∘) $7 - y = 3x - z$: $7 = y + 3x - z$

∘) $11 - z = y - 2x$: $11 = z + y - 2x$

$2 = 3x + 6z - 9y$
$7 = y + 3x - z$
$11 = z + y - 2x$

i) $$\begin{array}{r} 3x + 6z - 9y = 2 \\ 3x - z + y = 7 \\ \hline 7z - 10y = 5 \end{array} \; -$$

ii) $$\left.\begin{array}{r} y + 3x - z = 7 \\ y + z - 2x = 11 \end{array}\right| \begin{array}{c} 2 \\ 3 \end{array} \left| \begin{array}{r} 2y + 6x - 2z = 14 \\ 3y + 3z - 6x = 33 \\ \hline 5y + z = 47 \end{array} \right. +$$

iii) $$\left.\begin{array}{r} 7z - 10y = 5 \\ 5y + z = 47 \end{array}\right| \begin{array}{c} 1 \\ 2 \end{array} \left| \begin{array}{r} 7z - 10y = 5 \\ 10y + 2z = 94 \\ \hline 9z = 99 \\ z = 11 \end{array} \right. +$$

$5y + 11 = 47$
$5y = 36$
$y = \frac{36}{5}$

$7 = y + 3x - z$
$7 = \frac{36}{5} + 3x - 11$
$x = \frac{18}{5}$

Diketahui $R\left(\frac{18}{5}, \frac{36}{5}, 11\right)$ sehingga

$RP_0 = \left(\frac{2}{3} - \frac{18}{5}, 7 - \frac{36}{5}, 11 - 11\right) = \bar{N}$

$= \left(-\frac{44}{15}, -\frac{1}{5}, 0\right) = \bar{N}$

$\bar{N} = \frac{-44}{15} i - \frac{1}{5} j$

∴ Pers bidang $\alpha = \frac{-44}{15} x - \frac{1}{5} y = d$

$d = \frac{-44}{15}(Q_0x) - \frac{1}{5}(G_0y) + 0$

$= \frac{-44}{15}(1) - \frac{1}{5}(2) = \frac{-44}{15} - \frac{6}{15} = \frac{-50}{15}$

$= \frac{-10}{3}$

$\alpha = \frac{-44x}{15} - \frac{1}{5} y = \frac{-10}{3}$

Bidang α melalui $(2,1,3)$ memuat garis g dngn persamaan $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{5}$. Tentukan Jarak $(1,1,1)$ ke bidang α!

Jawab :

$x = 1 + 2t$ $\quad P_0 = (1, 2, 4)$
$y = 2 + 3t$ $\quad \bar{V} = (2, 3, 5)$
$z = 4 + 5t$

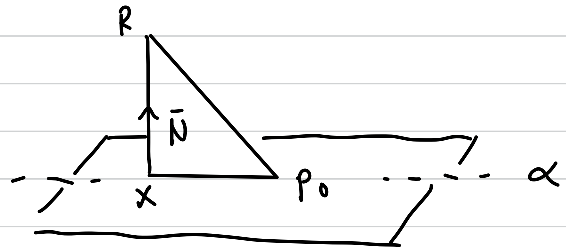
Anggap titik $Q = (2, 1, 3)$ dan $\bar{V} = P_0P = (2\ 3\ 5)$.

Didapatkan titik $P_0Q = (1, -1, -1)$ sehingga kita dapat mencari $\bar{N}$:

$$P_0P \times P_0Q = \begin{vmatrix} i & \bar{j} & \bar{k} \\ 2 & 3 & 5 \\ 1 & -1 & -1 \end{vmatrix}$$

$(-3+5)i - (-2-5)\bar{j} + (-2-3)\bar{k} = \bar{N}$
$2i + 7\bar{j} - 5\bar{k} = \bar{N}$

Anggap titik $(1,1,1)$ sebagai titik R



$\|Rx\| = \|RP_0\| \cos\theta$

$= \left| \frac{RP_0 \cdot \bar{N}}{\|\bar{N}\|} \right|$

$= \left| \frac{(0,1,3) \cdot (2,7,-5)}{\sqrt{4 + 49 + 25}} \right|$

$= \left| \frac{-8}{\sqrt{78}} \right| = \frac{8}{\sqrt{78}}$