Tugas 3

Qomain Aji
21/481767/TK/53170

Thursday, 26 August 2021 22.57

Exercise 2.2

Use Example 3 to find
a) the slope of the curve at given point P
b) an equation of tangent line at P

$\boxed{7}$ $y = x^2 - 3$, $P(2,1) \equiv P(x_1, y_1)$

a)

※ Secant slope : $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1+h) - f(x_1)}{h}$

$= \frac{f(2+h) - 2}{h} = \frac{(2+h)^2 - 3 - 1}{h}$

$\boxed{\begin{array}{l} x_2 - x_1 = h \\ x_2 = h + x_1 \end{array}}$

$= \frac{4 + 4h + h^2 - 4}{h} = \frac{4h + h^2}{h}$

$= 4 + h$: Secant slope

※ the slope of P : If the change of rate from point P to some unknown point called Q is near to or same as "0" than

$h = 0 \longrightarrow$ secant slope $= 4 + 0 = 4$ = gradient of P point.

So, the slope of the curved at given point P is equal to 4

b)

※ The equation of tangent line at P

$y - y_1 = m(x - x_1)$
$y - 1 = 4(x - 2)$
$y = 4x - 8 + 1$
$y = 4x - 7$ //

$\boxed{8}$ $y = 5 - x^2$; $P(1,4) \equiv P(x_1, y_1)$

a)

※ Secant slope : $\frac{f(x_1+h) - f(x_1)}{h} = \frac{f(1+h) - f(1)}{h} = \frac{5 - (1+h)^2 - 4}{h}$

$= \frac{5 - 1 - 2h - h^2 - 4}{h} = \frac{-(h^2 + 2h)}{h} = -(2+h)$ = secant slope

※ the slope from point P : $m = -(2+0) = -2$
So the slope of the given point P is equal to -2

b)

※ $y - y_1 = m(x - x_1)$
$y - 4 = -2(x - 1)$
$y = -2x + 2 + 4$
$= -2x + 6$ //

$\boxed{9}$ $y = x^2 - 2x - 3$, $P(2, -3) \equiv P(x_1, y_1)$

a)

※ secant slope : $\dfrac{f(x_1+h) - f(x_1)}{h} = \dfrac{f(2+h) - (-3)}{h} = \dfrac{(2+h)^2 - 2(2+h) - 3 + 3}{h}$

$= \dfrac{4 + 4h + h^2 - 4 - 2h}{h} = \dfrac{h^2 + 2h}{h} = h + 2$

$h + 2$ = secant slope

※ The slope from point P : $m = 0 + 2 = 2$

So the slope of the given point P is equal to 2

b)

※ $y - y_1 = m(x - x_1)$

$y - (-3) = 2(x - 2)$

$y = 2x - 4 - 3$

$y = 2x - 7$ // : So the tangent line equation is $y = 2x - 7$

$\boxed{10}$ $y = x^2 - 4x$; $P(1, -3) \equiv P(x_1, y_1)$

a)

※ secant slope : $\dfrac{f(x_1+h) - f(x_1)}{h} = \dfrac{f(1+h) - (-3)}{h} = \dfrac{(1+h)^2 - 4(1+h) + 3}{h}$

$= \dfrac{1 + 2h + h^2 - 4 - 4h + 3}{h} = \dfrac{h^2 - 2h}{h}$

secant slope = $h - 2$

※ The slope frome point P : $m = 0 - 2$

$= -2$

So the slope of the given point P is equal to -2

b)

※ $y - y_1 = m(x - x_1)$

$y - (-3) = -2(x - 1)$

$y = -2x + 2 - 3$

$y = -(2x + 1)$ // : So, the tangent line equation is $y = -(2x+1)$ //

$\boxed{20}$ Let $f(t) = \dfrac{1}{t}$; $t \neq 0$

a) Find the average change of f in

i) $t_1 = 2$ to $t_2 = 3$

ii) $t_1 = 2$ to $t_2 = T$

i) The average change : secant slope : $\dfrac{\Delta y}{\Delta x} = \dfrac{f(t_2) - f(t_1)}{t_2 - t_1}$

$\dfrac{f(t_2) - f(t_1)}{t_2 - t_1} = \dfrac{f(3) - f(2)}{3 - 2} = \dfrac{f(3) - f(2)}{1}$

※ $f(3) = \dfrac{1}{3}$; $f(2) = \dfrac{1}{2}$

$\therefore \dfrac{f(3) - f(2)}{1} = \dfrac{1}{3} - \dfrac{1}{2} = \dfrac{2 - 3}{6} = -\dfrac{1}{6}$

So the average change of $f(t)$ is $-\dfrac{1}{6}$

ii) The average change from $t_1 = 2$ to $t_2 = T$

※ secant slope :

$$\frac{f(t_2) - f(t_1)}{t_2 - t_1} = \frac{f(T) - f(2)}{T - 2}$$

$$= \frac{\frac{1}{T} - \frac{1}{2}}{T - 2} = \frac{\frac{2-T}{2T}}{T-2} = \frac{2-T}{2T(T-2)} = \frac{\cancel{2-T}}{-2T\cancel{(2-T)}}$$

$$= \frac{1}{-2T} = \text{secant slope.}$$

so the average change of $f(t)$ from $t_1 = 2$ to $t_2 = T$ is $\frac{1}{-2T}$

b) Make a table of values of the average change $f(t)$ with respect to t over the interval $[2, T]$

$T = 2.1, 2.01, 2.001, 2.0001, 2.00001, 2.000001$

T	average change value funtion : $\frac{1}{-2T}$
2,1	-0,23809
2,01	-0,24875
2,001	-0,24987
2,0001	-0,24998
2,00001	-0,24999
2,000001	-0,25

c) The table indicate the rate of change of f with respect to t at $t = 2$ is equal to $-0,25$.

d) if $T = 2$ that the rate of change would be

$$m = \frac{1}{-2(2)} = -\frac{1}{4}$$

$\boxed{22}$

a) Estimate gasoline consumption over the time interval $[0,3]$, $[0,5]$, and $[7,10]$

※ interval $[0,3]$: the coordinates are $\underset{x_1\ y_1}{(0, 15)}$ and $\underset{x_2\ y_2}{(3, 8)}$

average of change : secant slope $= \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{8 - 15}{3 - 0} = \frac{-7}{3}$ gals/days

※ interval $[0,5]$: the coordinates are $\underset{x_1\ y_1}{(0, 15)}$ and $\underset{x_2\ y_2}{(5, 3)}$

secant slope $= \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{3 - 15}{5 - 0} = \frac{-12}{5}$ gals/day

※ interval $[7,10]$: the coordinates are $\underset{x_1\ y_1}{(7, 1)}$ and $\underset{x_2\ y_2}{(10, 0)}$

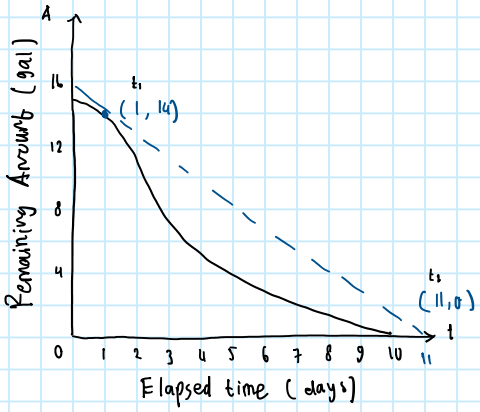
secant slope $= \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0 - 1}{10 - 7} = \frac{-1}{3}$ gals/day

$x_1 \; y_1$ $\qquad$ $x_2 \; y_2$

$$\text{secant slope} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0-1}{10-7} = \frac{-1}{3} \text{ gal}/\text{day}$$

b) Instantaneous rate of gasoline :

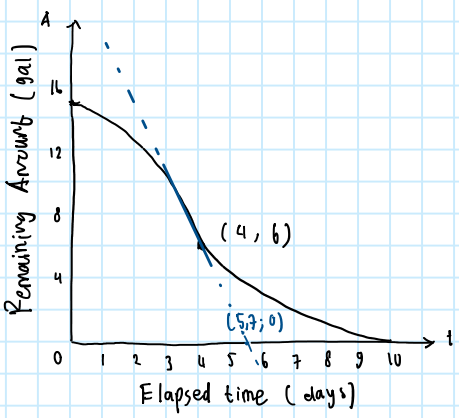
※ $t = 1$:



The approximate point that is in line with $t = 1$ is $(11, 0)$. So, we can calculate tangent line of the rate of change of gasoline consumption.

$$\frac{\Delta y}{\Delta x} = \frac{f(t_2) - f(t_1)}{t_2 - t_1} = \frac{0 - 14}{11 - 1} = -\frac{14}{10} = -1{,}4 \; \frac{\text{gal}}{\text{day}}$$

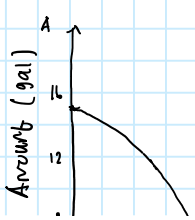
※ $t = 4$

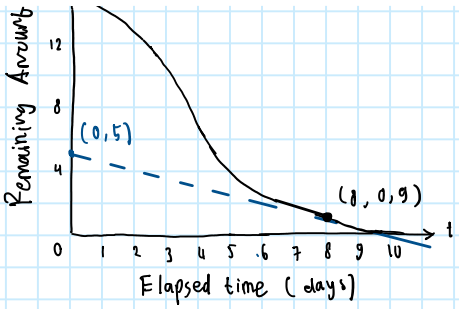


The approximate point that is in line with $t = 4$ is $(5{,}7; 0)$. So, we can calculate tangent line of the rate of change of gasoline consumption.

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(5{,}5)}{4 - 5{,}7} = \frac{6 - 0}{4 - 5{,}7} = -\frac{6}{1{,}7} = -\frac{60}{17} = -3{,}52 \; \frac{\text{gal}}{\text{day}}$$

※ $t = 8$





The approximate point that is in line with $t = 8$ is $(0, 5)$. So, we can calculate tangent line of the rate of change of gasoline consumption.

$$\frac{\Delta y}{\Delta x} = \frac{f(0) - f(8)}{0 - 8} = \frac{5 - 0{,}9}{-8} = \frac{4{,}1}{-8} = -0{,}51 \ \frac{gal}{day}$$

c) The maximum rate of gasoline consumption occurs at interval between day 3 – 4. to know the specific measure, we need to use table to identify the precise measurement

We will use the point $(3{,}1 ; 9{,}8)$, $(3{,}2 ; 9{,}6)$, $(3{,}5 ; 8)$, $(3{,}8 ; 6{,}8)$ and $(3{,}9 ; 6{,}2)$
we will use $(3{,}5 ; 8)$ as the measure point

Point	the slope : $\frac{\Delta y}{\Delta x}$
$(3{,}1 ; 9{,}8)$	$\frac{9{,}8 - 8}{3{,}1 - 3{,}5} = -4{,}5$
$(3{,}2 ; 9{,}6)$	$\frac{9{,}2 - 8}{3{,}2 - 3{,}5} = -5{,}3$
...	
$(3{,}8 ; 6{,}5)$	$\frac{6{,}5 - 8}{3{,}8 - 3{,}5} = -5$
$(3{,}9 ; 6{,}2)$	$\frac{6{,}2 - 8}{3{,}9 - 3{,}5} = -4{,}5$

Base on the table we can estimate that the maximum rate of gasoline consumption occurs around day 3,5 and the value are around $-5{,}5 \ \frac{gal}{day}$

Exercise 2.2

Find the limits

15 $\lim_{x \to 2} \frac{x+3}{x+6} = \frac{2+3}{2+6} = \frac{5}{8}$

27 $\lim_{t \to 1} \frac{t^2 + t - 2}{t^2 - 1} = \lim_{t \to 1} \frac{(t+2)\cancel{(t-1)}}{(t+1)\cancel{(t-1)}}$

$$= \frac{1+2}{1+1} = \frac{3}{2}$$

47 $\lim_{x \to 0} \frac{1 + x + \sin x}{3 \cos x} = \frac{1 + 0 + \sin 0}{3 \cos 0} = \frac{1 + 0 + 0}{3(1)} = \frac{1}{3}$

Exercise 2.3

55

$A = \pi (x/2)^2$

$= \pi \frac{x^2}{4}$; $|A - 9| \leq 0{,}01$

$\left| \pi \frac{x^2}{4} - 9 \right| \leq 0{,}01$

$-0{,}01 \leq \pi \frac{x^2}{4} - 9 \leq 0{,}01$

$8{,}99 \leq \pi \frac{x^2}{4} \leq 9{,}01$

$\sqrt{8{,}99 \cdot \frac{4}{\pi}} \leq x \leq \sqrt{9{,}01 \cdot \frac{4}{\pi}}$

$3{,}38326 \leq x \leq 3{,}38702$

The interval of the cylinder is $3{,}38326 \leq x \leq 3{,}38702$

56 If the current $I_0 = 5$ amp, then R is

$V = I_0 R$ $\boxed{V = 120}$

$120 = 5R$

$24 = R$

I is $5 \pm 0{,}1$ amp, then the interval is

※ $V = IR$; $I = 5{,}1$

$120 = 5{,}1 \cdot R$

$R = 23.529$

※ $V = IR$

$120 = 4.9 \cdot R$

$R = 24{,}489$

The interval of R is

$23{,}529 \leq R \leq 24{,}489$

Chapter 2 Practice Exercise

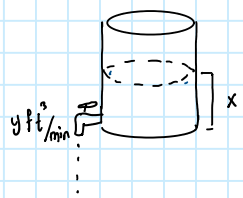
44 $\lim_{x \to \infty} \frac{1}{x^2 - 7x + 1} = \lim_{x \to \infty} \frac{\cancel{x^2}\left(\frac{1}{x^2}\right)}{\cancel{x^2}\left(1 - \frac{7}{x} + \frac{1}{x^2}\right)} = \frac{\frac{1}{\infty^2}}{1 - \frac{7}{\infty} + \frac{1}{\infty^2}} = \frac{0}{1 - 0 + 0} = 0$

46 $\lim_{x \to \infty} \frac{x^4 + x^3}{10x^3 + 10} = \frac{\cancel{x^4}\left(1 + \frac{1}{x}\right)}{\cancel{x^4}\left(\frac{10}{x} + \frac{10}{x^3}\right)} = \infty$

$\boxed{46}$ $\lim_{x \to \infty} \dfrac{x^4 + x^3}{12x^3 + 128} = \dfrac{\cancel{x^4}\left(1 + \frac{1}{x}\right)}{\cancel{x^4}\left(\frac{12}{x} + \frac{128}{x^4}\right)} = \infty$

Chapter 2 Additional and Advanced Exercises

$\boxed{4}$ Controlling the flow from draining water



Suppose that $y = \sqrt{\frac{x}{2}}$; How deep must you keep the water if you want to maintain the exit rate?

※ within 0,2 ft^3/min of the rate $y_0 = 1\ ft^3/min$

$0{,}8 \leq \sqrt{\frac{x}{2}} \leq 1{,}2$

$0{,}64 \leq \frac{x}{2} \leq 1{,}44$

$1{,}28 \leq x \leq 2{,}88$

So the depth of the water should be around 1,28 until 2,88

※ within 0,1 ft^3/min of the rate $y_0 = 1\ ft^3/min$

$0{,}9 \leq \sqrt{\frac{x}{2}} \leq 1{,}1$

$0{,}81 \leq \frac{x}{2} \leq 1{,}21$

$1{,}62 \leq x \leq 2{,}42$

So the dept of the water should be around 1,62 until 2,42

$\boxed{5}$ Thermal expancion in precise equipment.
the function of t is related to the expand of the metal can be express as
$y = 10 + (t - 70) \times 10^{-4}$ cm

※ within 0,0005 of ideal 10 cm

$9{,}9995 \leq y \leq 10{,}0005$
$9{,}9995 \leq 10 + (t - 70) \times 10^{-4} \leq 10{,}0005$

$-0{,}0005 \leq (t - 70) \cdot 10^{-4} \leq 0{,}0005$
$-5 \leq t - 70 \leq 5$
$65\ F^\circ \leq t \leq 75\ F^\circ$

So, the interval of the temperature that needs to be maintaine to ensure is around 65 F° until 75 F°