**Swimming Problem**

**Description**

A person is swimming across several rivers.

The widths of rivers are: ***S1, S2, ..., Sn***.

Speeds of those rivers are different: ***v1, v2, ..., vn***. To simplify this problem, only consider the speed in vertical direction.

The person's swimming speed is a constant ***v***. The angle of the person's velocity to ho horizontal line is ***a1, a2, ..., an***.

The total time for swimming is ***T***. And the person must pass those rivers.

**Task**

Find out an equation to determine by choosing what angles (***a1, a2, ..., an***) the person can get maxmimum distance in vertical direction. That is to say, please maximize ***dh*** by determine ***a1, a2, ..., an***) under the total time T.

You are not required to give out concrete angle numbers, a ***cost function*** that can be derived from is enough.

Tips: A mathematical tool you may need is called ***Lagrangian Multiplier***, which means, when you provide a formula, say ***E***, which still need to satisfy some more conditions, say ***a>1***, for the convenience of calculating, we can write those 2 parts (formula ***E*** and condition ***a>1***) together as one new formula. Here the new formula will be:

**Solution**

When the person swims cross the river,

Horizontal speed:

Time:

Vertical speed:

Vertical distance:

Total time:

Total vertical distance:

The question becomes:

Build Lagrangian Multiplier:

Let the first-order partial derivative be equal to :

There are unknown numbers: for and 1 for .

There are equations: for and 1 for .

So the equations are resoluble.