

# 1 Notation

Variable	Meaning
$\rho$	Density matrix
$ \psi\rangle$	State vector
$D(\alpha)$	Displacement operator
$\alpha$	A complex number or a vector of complex numbers $\alpha = \begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{pmatrix}^T$
$I$	Identity matrix, note that the dimension of $I$ will vary with context
$\int_{\mathbb{C}} d^2\alpha$	Integral over the complex plane. Can for example be done in the real and imaginary part of $\alpha$ , or polar coordinates.
$\chi(\alpha, \alpha^*), \chi_\rho(\alpha)$	(Wigner) Characteristic function. The subscript indicates the associated operator.
$a, a^\dagger$	Ladder operators. Often vectors over $n$ modes $a = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}^T$
$q, p$	Quadrature operators. Often vectors over $n$ modes.
$C_\alpha, C_a$	Vector of complex numbers or operators, $C_\alpha = \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix}, C_a = \begin{pmatrix} a \\ a^\dagger \end{pmatrix}$
$\text{Tr}\{A\}$	Has two possible meanings depending on context. If $A$ is matrix of numbers, then it is the sum of the diagonal of $A$ . If $A$ is a matrix of operators, e.g. $A = \rho \begin{pmatrix} a & a^\dagger \\ a^\dagger & a \end{pmatrix} = \begin{pmatrix} \rho a & \rho a^\dagger \\ \rho a^\dagger & \rho a \end{pmatrix}$ , then the trace is over Hilbert space, $\text{Tr}\{A\} = \begin{pmatrix} \text{Tr}\{\rho a\} & \text{Tr}\{\rho a^\dagger\} \\ \text{Tr}\{\rho a^\dagger\} & \text{Tr}\{\rho a\} \end{pmatrix}$ .
$\bar{C}$	Mean amplitude, or displacement, $\bar{C} = \text{Tr}\{\rho C_a\}$ .
$\Sigma$	Covariance matrix for the amplitudes, $\Sigma = \frac{1}{2}\text{Tr}\{\rho(C_a \otimes C_a^T + C_a^T \otimes C_a)\} - \text{Tr}\{\rho C_a^T\} \otimes \text{Tr}\{\rho C_a\}$
$\Omega$	The symplectic form $\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$
$H_G$	Hamiltonian that is at most quadratic in the ladder operators
$M_+, D_+$	Bogoliubov transformation and displacement associated to the unitary transformation $e^{itH_G} C_a e^{-itH_G} = M_+ C_a + D_+$
$V, J$	Submatrices of the $M_+$ , i.e. $M_+ = \begin{pmatrix} V & J \\ J^* & V^* \end{pmatrix}$
$z, z^*$	Subvectors of $D_+$ , i.e. $D_+ = \begin{pmatrix} z \\ z^* \end{pmatrix}$

Variable	Meaning
$M_-, D_-$	Bogoliubov transformation and displacement associated to the unitary transformation, $e^{-itH_G} C_a e^{itH_G} = M_- C_a + D_-$ .
$x, y$	Vectors of real numbers.
$k_c$	Constant characterizing quadrature convention, $q = \frac{k_c}{2} (a^\dagger + a), p = \frac{k_c}{2} i (a^\dagger - a)$
$T, T_k$	Matrices that converts $\alpha, \alpha^*$ to real and imaginary parts, $T = \begin{pmatrix} \frac{1}{2}I & \frac{1}{2}I \\ -\frac{1}{2}iI & \frac{1}{2}iI \end{pmatrix}, T_k = k_c T$
$R_Q$	Vector of quadrature operators $R_Q = \begin{pmatrix} q \\ p \end{pmatrix}$
$S_+, \mu_+$	Symplectic matrix and displacement associated with the transformation $e^{itH_G} R_Q e^{-itH_G}$
$S_-, \mu_-$	Symplectic matrix and displacement associated with the transformation $e^{-itH_G} R_Q e^{itH_G}$
$R_X, R_Y, R_\Lambda$	Vectors of real numbers put into a bi-partite structure, $R_\Lambda = \begin{pmatrix} \Lambda_q \\ \Lambda_p \end{pmatrix}$
$\chi_\rho^{(Q)}(R_\Lambda)$	Quadrature characteristic function associated with state $\rho$
$\bar{R}$	Quadrature average $\bar{R} = \text{Tr} \{ \rho R_Q \}$
$Q$	Quadrature covariance matrix $Q = \frac{1}{2} \text{Tr} \{ \rho (R_Q \otimes R_Q^T + R_Q^T \otimes R_Q) \} - \text{Tr} \{ \rho R_Q^T \} \otimes \text{Tr} \{ \rho R_Q \}$
$W_\rho(R_X)$	Wigner function of the state $\rho$ .
$ A $	Absolute value of scalar $A$ and determinant of matrix $A$
$\langle A \rangle$	Expectation value of the operator $A = \text{Tr} \{ \rho A \}$
$\otimes$	The Kronecker product of two arrays $A \otimes B$
$A^H$	Superscript $H$ mean the conjugate transpose of $A$ .
$A^*$	Elementwise complex conjugation of $A$ .