Engineering Electromagnetics - Experiment 2 Electric Field of Line Charge

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***Abstract*—This article describes the electric field of the line charge which lenth is 2. By using MATLAB to simulate the electric field and draw the pictures; Using integration method to calculate the distribution of electric potential, the shape of electric field distribution is similar to the fin. Using infinitesimal method, divide line charge into 20, 50 and 100 segments, think of them as point charges, when increasing the degree of seperation, the electric field distribution is close to integration method. To analyze the difference between two methods, the difference caused by degree of seperation and the coordinate precision.**

I. Introduction

THIS experiment is to analyze the electric field of the linecharge in a free space. And the objectives of this experiment is:

- Calculate the distribution of electric field built by continuous line charge
- Plot the relevant figures on MATLAB environment
- Study the difference betweenintegration and infinitesimal methodson on analyzing electric field.

By using the scientific analysis software MATLAB, simulated the electric field distribution of line charge in a 2-D rectangular coordinate can help us to understand the electric field in a visualized way.

Suppose there is a uniformly distributed line charge between point A(-1,0) and point B(1,0), with linechargedensity of $\rho = 1 \times 10^{-9}$ C/m. (The unit for the coordinate is m)

Using integration method in II-A to calculate the electric potential at each point of the coordinate, we can get the real distribution in theory. And using infinitesimal method in II-B to do a approximate simulation.

To describe the distribution, I use three graph included for each method. They include:

- i) the distribution of electric field for each point;
- ii) equipotential lines;
- iii) distribution of electric field lines(represented by continuous lines).

To analyze the difference, I calculate the difference of electric potential between two ways at three different degree of seperation(20, 50 and 100). Draw the graph of difference distribution. And analyzed the difference at line: $y = 0.5$, using knowledge of calculus to describe the difference.

II. Related Knowledge and MATLAB Code

In vaccuum, the electric field intensity ($\mathbf{E}$) of a point charge can be expressed as:

$$\mathbf{E} = k\frac{Q}{R^2}\mathbf{a}_R \tag{1}$$

Where the coefficient $k = 9 \times 10^9$ F/m is the electrostatic constant. Q represent the total amount of charges. R denotes the distance between the point in the electric field and the source charge.

If we take reference point as the infinite distance, then the electric potential at a point in the field is expressed as:

$$V = k\frac{Q}{R} \tag{2}$$

The electric field intensity can be expressed as the negative gradient of the electric potential:

$$\mathbf{E} = -\nabla V \tag{3}$$

The electric field generated by N point charge in the vaccuum is expressed as:

$$V = \sum_{i=1}^{N} k\frac{Q_i}{R_i} \tag{4}$$

Similarly, the field magnitude generated by N point charges in the vaccuum can be obtained through equation (3).

When the field source is continuous charge, e.g. line charge, we can readily resolve it by using infinitesimal or integral method. the procedure of applying this method is listed as follows:

- 1) Divide the line charge into small segments of charges (usuallybeingdivided evenly).
- 2) Treat eachsmall segment of charges as a point charge and calculate the electric potential through equation (2).
- 3) Sum up all the electric potential by using equation (4) to obtain the electric potential.
- 4) Calculatethe electric field intensity generated by this line charge through equation (3).

A. integration method

Using integration method to calculate the distribution of electric potential at each point of the coordinate, namely, the

real distribution. The procedure is given below:
Given a point (X_0, Y_0):

$$\begin{aligned} V &= k\int_{-1}^{1}\frac{\rho dx}{R} \\ &= k\int_{-1}^{1}\frac{\rho dx}{\sqrt{(x-X_0)^2+Y_0^2}} \\ &= k\rho\ln\left|(x-X_0)+\sqrt{(x-X_0)^2+Y_0^2}\right|\Bigg|_{-1}^{1} \\ &= k\rho\ln\left(\frac{1-X_0+\sqrt{(1-X_0)^2+Y_0^2}}{-1-X_0+\sqrt{(-1-X_0)^2+Y_0^2}}\right) \end{aligned} \tag{5}$$

If we calculate each point of the coordinate.

$$V(x,y) = k\rho\ln\left(\frac{1-x+\sqrt{(1-x)^2+y^2}}{-1-x+\sqrt{(-1-x)^2+y^2}}\right) \tag{6}$$

Easy to prove, when calculated point is on the line charge, namely, $x \in [-1, 1]$ and $y = 0$, the electric potential is ∞.

B. infinitesimal method

Using infinitesimal method to calculate the distribution of electric potential at each point of the coordinate, however, we cannot have infinitesimal in MATLAB, so we use small segments as infinitesimal.

Set the approximative infinitesimal of distance is $\Delta x = \frac{l}{N}$. Where l is the length of line charge $l = x_A - x_B = 2$, N is number of segments. Using point charge to replace the segments, so the x-coordinates are the midpoints of the segments. Each x-coordinate is $x_i = i\Delta x + x_A = \frac{2i}{N} - 1$ So, the approximate infinitesimal of charge is $\Delta Q = \rho\Delta x$. According to equation (4), given a point (X_0, Y_0):

$$\begin{aligned} V &= \sum_{i=1}^{N} k\frac{\Delta Q}{R_i} \\ &= k\rho\Delta x\sum_{i=1}^{N}\frac{1}{\sqrt{(x_i-X_0)^2+Y_0^2}} \\ &= k\rho\frac{l}{N}\sum_{i=1}^{N}\frac{1}{\sqrt{(i\frac{l}{N}-1-X_0)^2+Y_0^2}} \\ &= \frac{2k\rho}{N}\sum_{i=1}^{N}\frac{1}{\sqrt{(\frac{2i}{N}-X_0-1)^2+Y_0^2}} \end{aligned} \tag{7}$$

If we calculate each point of the coordinate.

$$V(x,y) = \frac{2k\rho}{N}\sum_{i=1}^{N}\frac{1}{\sqrt{(\frac{2i}{N}-x-1)^2+y^2}} \tag{8}$$

C. MATLAB code

All prgramming code included:

- 1) function: V_con
- 2) function: V_det
- 3) script: work1
- 4) script: work2
- 5) script: work3

V_con : Using to calculate the electric field distribution by using integration method, namely, it use equation (6) to calculate.

```
function [V] = V_con(ro,X,Y)
%V_CON is the function to calculte the ...
    electric field distribution
%
k = 9e9;
lo = (1-X+sqrt((1-X).^2+Y.^2));
lo = lo./(-1-X+sqrt((-1-X).^2+Y.^2));
V = k * ro * log(lo);
end
```

V_det : Using to calculate the electric field distribution by using infinitesimal method, namely, it use equation (8) to calculate.

```
function [V] = V_det(ro,xa,xb,N,n,X,Y)
% V_DET is the function to calculte the ...
    electric feild distribution .
%   ro is the line charge density
%   xa and xb is the x-coordinate of the ...
    endpoints
k = 9e9;                    % set the
l = 2; %set ht
dq = ro * l / N;
qx = linspace(xa,xb,N);   % set the ...
    x-coordinate of charge segments


V = zeros(N,n,n);         % V

i = 1;
for qxi = qx
    r = sqrt((X-qxi).^2+(Y.^2)); % calculate ...
        the distace to each charge point
    V(i,:,:) = 1 ./ r;
    i = i +1;
end

V = sum(V);  % calculate the sum of ...
    reciprocal of the distances
V = reshape(V,n,n); % reshape matrix to two ...
    demension
V = k*dq.*V;

end
```

D. rs

III. Simulation of Integration Method

IV. Simulation of Infinitesimal Method

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