

Engineering Electromagnetics - Experiment 3

Magnetic Field of Two Current Loops

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Abstract—This article describes the magnetic field of two current loops with same radius and center axis. By using MATLAB to simulate the magnetic field and draw the pictures; Using Biot-Savart and superposition principle divide two current loop into 50 segments, and calculate the magnetic field intensity vector at each point of yz plane which is perpendicular to the current loops. We can find that if two current directions of loops are same, the magnetic field distribution between two loops is uniform; and if the directions are different, the magnetic fields for each loop are cancelled out in the space between loops, so

I. INTRODUCTION

THIS experiment is to analyze the magnetic field of the two current loops in a free space. And the objectives of this experiment is:

- i) Get familiar with the spatial magnetic field that is produced by an electric current loop.
- ii) Calculate the magnetic field distribution and plot the relevant graphs through MATLAB.

The tasks of this experiment is that using MATLAB to analyze the magnetic field distribution of the following two cases:

- Case 1: Two current loops with same radius $a = 2m$, and the current in both of the loops is 500A. The two loops are parallel to the xy plane, and the loop centers are located at $O_1(0, 0, -1)$, $O_2(0, 0, 1)$ respectively. The direction of the current are the same.
- Case 2: Set the current directions of the two loops in case 1 to be opposite.

The experiment of requirements are:

- i) Calculate and plot the magnetic field intensity vector distribution (represented by arrows).
- ii) Calculate and plot the magnetic field intensity magnitude distribution.

For simplicity, only analyze the magnetic field distribution on the yz plane.

II. RELATED KNOWLEDGE

A. Biot-Savart Law

In vacuum, the magnetic field intensity vector (\mathbf{H}) of a short current segment ($d\mathbf{L}$) can be expressed as:

$$d\mathbf{H} = \frac{Id\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{Id\mathbf{L} \times \mathbf{R}}{4\pi R^3} \quad (1)$$

Where the coefficient $Id\mathbf{L}$ is elementary current vector, \mathbf{R} is the vector that point from the elementary current $Id\mathbf{L}$ to a field point P and its magnitude is R .

B. Magnetic field along the center axis of a current loop

As for the magnetic field created by an electric current loop, we can use the Biot-Savart law to derive the distribution of the magnetic field intensity along the center axis of the current loop:

$$\mathbf{H} = \frac{I(\pi a^2)\mathbf{a}_z}{2\pi(a^2 + z_0^2)^{3/2}} \quad (2)$$

Where a is the radius of the current loop, I is the current in the current loop, z_0 is the coordinate of the field point (located on the center axis, i.e. z axis). However, for the field points that are not on the center axis, it would be hard to deploy Biot-Savart law to derive an analytical solution for the magnetic field intensity.

C. Superposition principle

Similarly, magnetic field also follows the superposition principle. Therefore, we can divide the current-carrying conductor into many elementary currents. Thus, the magnetic field created by the current-carrying conductor is equal to the superposition of the magnetic fields created by all elementary currents.

D. Helmholtz coils

For two parallelly placed current loops, which currents are same at magnitude and direction, when the distance between them is equal to their radius, this two-current-loops system is usually called Helmholtz coils. One characteristic of the Helmholtz coils is that the spatial magnetic field distribution between these two current loops is very uniform.

III. SAME CURRENT DIRECTION LOOPS

By using MATLAB to simulate case 1, code and graphs are follow:

Firstly, initialize and set a , I and coordinates, etc.

```
1 clear; % clear memory
2 clc; % clear comand window
3 a = 2; % set radius of the current loops
4 I = 500; % set the current of the loop
5 C = I / (4 * pi); % merge the constants
6 zc1 = -1;
7 zc2 = 1; % set location of the center of ...
           each loop(x and y are 0)
8 ym = 5;
9 zm = 5; % set range
10 pn = 60; % set accuracy of coordinates
11 y = linspace(-ym, ym, pn);
12 z = linspace(-zm, zm, pn);
13 [Y, Z] = meshgrid(y, z);
```

Then depart current loop to N segments, $N = 50$;

```

1 N = 50; % set the segments number
2 dphi = 2 * pi / N; % set angle of each ...
  segment of the loop
3 phi = linspace(dphi, 2 * pi - dphi, N); % ...
  set angle - coordinates of segments
4
5 xc = a * cos(phi);
6 yc = a * sin(phi); % transform angle to xy ...
  coordiantes
7 dl = a * dphi; % set length of each segment

```

Calculate the magnetic field intensity of each point.

```

1 Hx = zeros(pn);
2 Hy = zeros(pn);
3 Hz = zeros(pn);
4 H = zeros(pn); % construct matrixes of ...
  magnetic field
5 ci = 1; % ci is the z coordinate of matrixes
6 for bi = y % each point of bi as y coordinate
7   cj = 1; % cj is the y coordinate of matrixes
8   for bj = z % each point of bj as z coordinate
9     dHx = 0;
10    dHy = 0;
11    dHz = 0;
12    dH = 0;
13    for li = 1:N % calculate magnetic ...
      field intensity vector at point ...
      B (bi, bj) of loop1
14      R = sqrt((0 - xc(li))^2 + (bi - ...
        yc(li))^2 + (bj - zc2)^2); % ...
        calculate the distance ...
        between segment and B
15      v_l = dl * [-sin(phi(li)), ...
        cos(phi(li)), 0]; % set the ...
        vector of dl
16      v_r = [0 - xc(li), bi - yc(li), ...
        bj - zc2]; % set vector of R
17      v_H = cross(v_l, v_r); % ...
        calculate value of cross product
18      dHx = dHx + C * v_H(1) / (R^3);
19      dHy = dHy + C * v_H(2) / (R^3);
20      dHz = dHz + C * v_H(3) / (R^3); ...
        % accumalte magnitude of ...
        vectors on each axis
21    end
22
23    for li = 1:N % calculate magnetic ...
      field intensity vector at (bi, ...
      bj) of loop2
24      R = sqrt((0 - xc(li))^2 + (bi - ...
        yc(li))^2 + (bj - zc2)^2);
25      v_l = dl * [-sin(phi(li)), ...
        cos(phi(li)), 0];
26      v_r = [0 - xc(li), bi - yc(li), ...
        bj - zc2];
27      v_H = cross(v_l, v_r);
28      dHx = dHx + C * v_H(1) / (R^3);
29      dHy = dHy + C * v_H(2) / (R^3);
30      dHz = dHz + C * v_H(3) / (R^3);
31      dH = dH + sqrt(dHy.^2 + dHz.^2);
32    end
33    Hx(cj, ci) = dHx;
34    Hy(cj, ci) = dHy;
35    Hz(cj, ci) = dHz;
36    cj = cj + 1;
37  end
38  ci = ci + 1;
39 end
40
41 H = sqrt(Hy.^2 + Hz.^2); % calculate the ...
  magnitude on yz plane

```

Draw the magnetic field intensity magnitude distribution.

```

1 figure(1);
2 quiver(Y, Z, Hy, Hz); % plot the vector ...
  graph of magnetic field intensity
3 hold on
4 plot(-2, -1, 'bo');
5 plot(2, -1, 'ro');
6 plot(-2, 1, 'bo');
7 plot(2, 1, 'ro'); % plot the point of coils
8 xlabel("Y (units: m)");
9 ylabel("Z (units: m)"); % label the axis
10 axis([-2, 2, -1, 1]);
11 hold off;

```

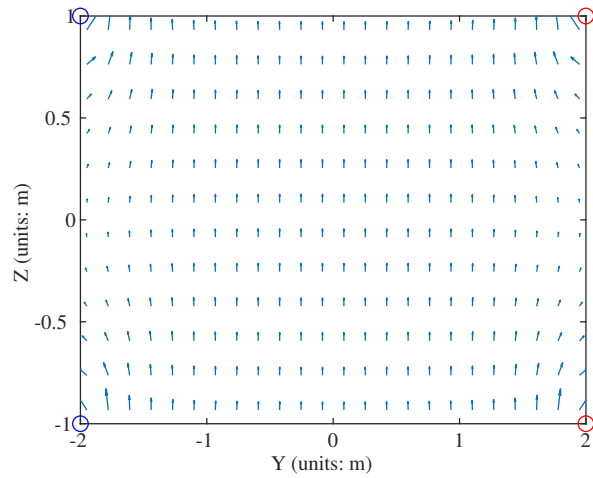


Fig. 1. Magnetic field intensity magnitude distribution

As Figure 1 shows

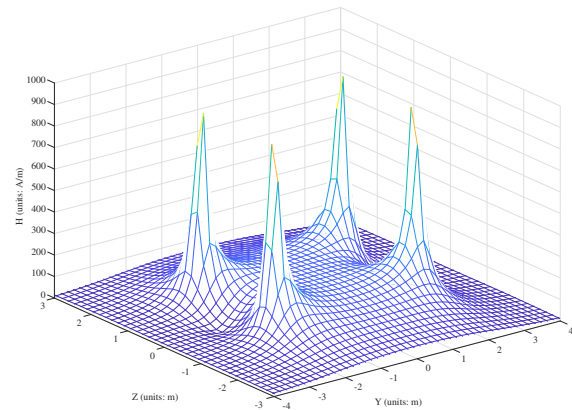


Fig. 2. Magnetic field intensity magnitude distribution

IV. DIFFERENT CURRENT DIRECTION LOOPS

```

1 clear; % clear memory
2 clc; % clear comand window
3 a = 2; % set radius of the current loops
4 I = 500; % set the current of the loop

```

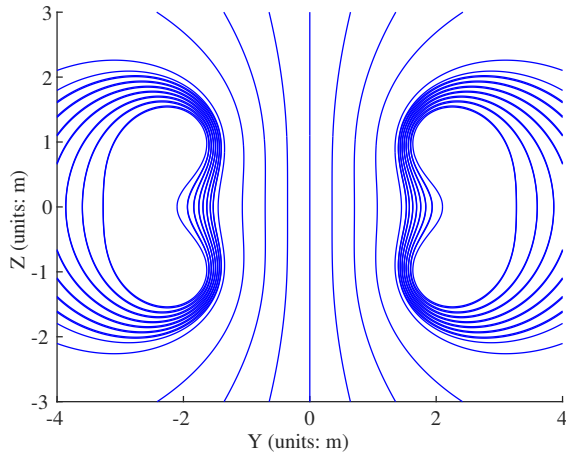


Fig. 3. Magnetic field intensity magnitude distribution

```

5 C = I / (4 * pi);
6
7 zc1 = -1;
8 zc2 = 1; % set location of the center of ...
   each loop(x and y are 0)
9
10 N = 50; % set the segments number
11 ym = 5;
12 zm = 5;
13 pn = 60; % set the of coordinate
14
15 dphi = 2 * pi / N; % set angle of each ...
   segment of the loop
16 phi = linspace(dphi, 2 * pi - dphi, N); % ...
   set angle - coordinates of segments
17
18 y = linspace(-ym, ym, pn);
19 z = linspace(-zm, zm, pn);
20 [Y, Z] = meshgrid(y, z);
21
22 xc = a * cos(phi);
23 yc = a * sin(phi); % transform angle to xy ...
   coordiantes
24
25 dl = a * dphi; % set length of each segment
26
27 Hx = zeros(pn);
28 Hy = zeros(pn);
29 Hz = zeros(pn);
30 H = zeros(pn);
31
32 ci = 1;
33
34 for bi = y
35
36     cj = 1;
37
38     for bj = z
39
40         dHx = 0;
41         dHy = 0;
42         dHz = 0;
43         dH = 0;
44
45         for li = 1:N% calculate magnetic ...
           field intensity vector at (bi, ...
           bj) of loop1
46             R = sqrt((0 - xc(li))^2 + (bi - ...
               yc(li))^2 + (bj - zc1)^2);
47
48             v_l = dl * [-sin(phi(li)), ...
               cos(phi(li)), 0]; % set the ...

```

```

           vector of dL
49             v_r = [0 - xc(li), bi - yc(li), ...
               bj - zc1]; % set vector of R
50             v_H = cross(v_l, v_r); % ...
               calculate value of cross product
51             dHx = dHx + C * v_H(1) / (R^3);
52             dHy = dHy + C * v_H(2) / (R^3);
53             dHz = dHz + C * v_H(3) / (R^3); ...
               % accumalte magnitude of ...
               vectors on each axis
54         end
55
56         for li = 1:N% calculate magnetic ...
           field intensity vector at (bi, ...
           bj) of loop2
57             R = sqrt((0 - xc(li))^2 + (bi - ...
               yc(li))^2 + (bj - zc2)^2);
58
59             v_l = -dl * [-sin(phi(li)), ...
               cos(phi(li)), 0]; % the ...
               direction of elementary is ...
               opposite
60             v_r = [0 - xc(li), bi - yc(li), ...
               bj - zc2];
61             v_H = cross(v_l, v_r);
62             dHx = dHx + C * v_H(1) / (R^3);
63             dHy = dHy + C * v_H(2) / (R^3);
64             dHz = dHz + C * v_H(3) / (R^3);
65         end
66
67         Hx(cj, ci) = dHx;
68         Hy(cj, ci) = dHy;
69         Hz(cj, ci) = dHz;
70
71         cj = cj + 1;
72     end
73
74     ci = ci + 1;
75 end
76
77 H = sqrt(Hy.^2 + Hz.^2); % calculate the ...
   magnitude on yz plane
78
79 figure(1);
80 quiver(Y, Z, Hy, Hz); % plot the vector ...
   graph of magnetic field intensity
81 hold on
82 plot(-2, -1, 'bo');
83 plot(2, -1, 'ro');
84 plot(-2, 1, 'ro');
85 plot(2, 1, 'bo');
86 xlabel("Y (units: m)");
87 ylabel("Z (units: m)"); % label the axis
88 axis([-3, 3, -2, 2]);
89 hold off;
90
91 figure(2);
92 mesh(Y, Z, H); % plot graph of magnetic ...
   field intensity
93 hold on;
94 xlabel("Y (units: m)");
95 ylabel("Z (units: m)");
96 zlabel("H (units: A/m)"); % label the axis
97 axis([-4, 4, -3, 3]);
98 hold off;
99
100 figure(3);
101 ys = linspace(-1.5, 1.5, 10);
102 zs = linspace(-0.5, 0.5, 10);
103 streamline(Y, Z, Hy, Hz, ys, zs); % plot the ...
   vector graph of magnetic field intensity
104 hold on;
105 streamline(Y, Z, -Hy, -Hz, ys, zs);
106 streamline(Y, Z, Hy, Hz, -ys, zs);
107 streamline(Y, Z, -Hy, -Hz, -ys, zs);
108
109

```

```

110 xlabel("Y (units: m)");
111 ylabel("Z (units: m)"); % label the axis
112 axis([-4, 4, -3, 3]);
113
114 hold off;

```

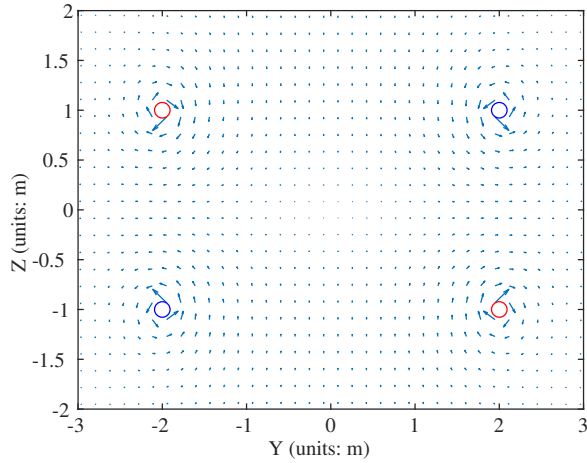


Fig. 4. Magnetic field intensity magnitude distribution

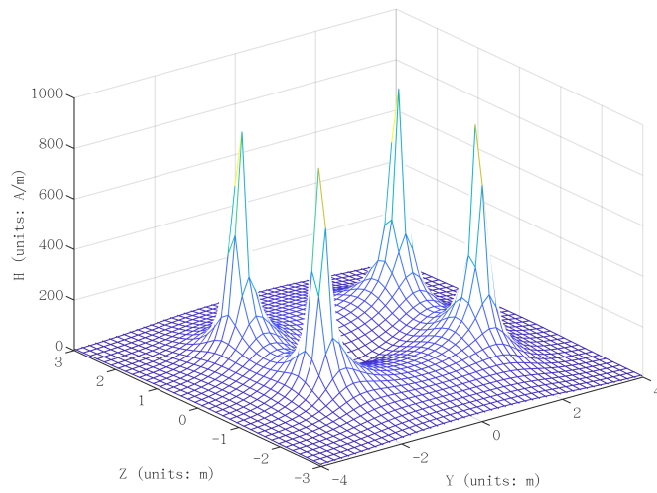


Fig. 5. Magnetic field intensity magnitude distribution

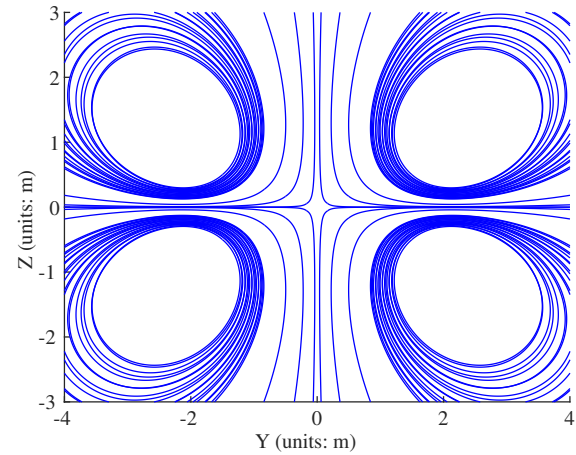


Fig. 6. Magnetic field intensity magnitude distribution

V. INSPIRATION

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Thanks to Youwei Jia, the teacher who teach me the knowledge about Electromagnetics and the using methods of MATLAB. He gives amounts of help to me to finish this article.