Engineering Electromagnetics - Experiment 2

1. Objectives:

A. Calculate the distribution of electric field built by continuous line charge, and plot the relevant figures on MATLAB environment;

B. Study the difference between integration and infinitesimal methods on analyzing electric field.

2. Related knowledge:

In vacuum, the electric field intensity (**E**) of a point charge can be expressed as:

$$\mathrm{E} = \mathrm{k}\frac{Q}{R^2}a_R \tag{1}$$

Where the coefficient $\mathrm{k} = 9 \times 10^9 F/m$ is the electrostatic constant. Q represents the total amount of charge. R denotes the distance between the point in the electric field and the source charge.

If we take the reference point as the infinite distance, then the electric potential at a point in the field is expressed as:

$$V = k\frac{Q}{R} \tag{2}$$

The electric field intensity can be expressed as the negtive gradient of the electric potential

$$\mathbf{E} = -\nabla \mathrm{V} \tag{3}$$

The electric field generated by N point charge in the vacuum is expressed as:

$$V = \sum_{i=1}^{N} k\frac{Qi}{Ri} \tag{4}$$

Similarly, the field magnitude generated by N point charges in the vacuum can be obtained through equation (3).

When the field source is continuous charge, e.g. line charge, we can readily resolve it by using infinitesimal or integral method. The procedure of applying this method is listed as follows:

1) Divide the line charge into small segments of charges (usually being divided evenly).

2) Treat each small segment of charges as a point charge and calculate the electric potential through equation (2).

3) Sum up all the electric potential by using equation (4) to obtain the electric potential.

4) Calculate the electric field intensity generated by this line charge through equation (3).

There exists a difference between the obtained result by infinitesimal method and the real value. The difference is determined by the number of segments we divided in step 1. Generally, the more segments we take, the smaller the deviation will be. In some cases, we can also apply integration method to calculate the real distribution of the electric field, which makes it possible to study the relationship between the number of segments we divide and the deviations caused by the infinitesimal method.

3. Experiment Contents

Analyze the electric field distribution of line charge in a 2-D rectangular coordinate through MATLAB.

Suppose there is a uniformly distributed line charge between point A(-1,0) and point B(1,0), with line charge density of $\rho=1\times10^{-9}$ C/m. (The unit for the coordinate is m)

Procedures and Requirements:

1. Using integration method to calculate the distribution of electric potential at each point of the coordinate, namely, the real distribution. The procedure is given below:

Given a point (X_0,Y_0)

$$\mathbf{V} = \mathbf{k}\int_{-1}^{1}\frac{\rho dx}{\boldsymbol{R}}$$

$$= \boldsymbol{k}\int_{-1}^{1} \frac{\rho dx}{\sqrt{(x-X_0)^2+{Y_0}^2}}$$

$$= \mathbf{k}\,\rho \mathrm{dx}\,\mathbf{ln}\left|(x-X_0)+\sqrt{(x-X_0)^2+{Y_0}^2}\right|\Big|_{-1}^{1}$$

$$= \mathbf{k}\rho\mathbf{ln}\left(\frac{1-X_0+\sqrt{(1-X_0)^2+{Y_0}^2}}{-1-X_0+\sqrt{(-1-X_0)^2+{Y_0}^2}}\right)$$

Choose a proper range for the electric field, based on the result, using Matlab to i) program and calculate the distribution of electric field for each point; ii) find the equipotential lines (The electric potential of the equipotential line can be selected according to the real situation) and iii) distribution of electric field lines(represented by continuous lines).

2. By using infinitesimal method, divide the line charge into 20, 50 and 100 segments respectively. Choose the same field range as before to repeat the procedure in (1).

3. Quantitatively compare the results of the infinitesimal method and the real value. Study the relationship between the deviation from the real value and the number of segments we take.

4. Write a comprehensive experiment report. (The MATLAB code should be included in the report. Name and student ID should be included in the title of each figure)