Engineering Electromagnetics - Experiment 2 Electric Field of Line Charge

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***Abstract*—This article describes the electric field of the line charge which lenth is 2. By using MATLAB to simulate the electric field and draw the pictures; Using integration method to calculate the distribution of electric potential, the shape of electric field distribution is similar to the fin. Using infinitesimal method, divide line charge into 20, 50 and 100 segments, think of them as point charges, when increasing the degree of seperation, the electric field distribution is close to integration method. To analyze the difference between two methods, the difference caused by degree of seperation.**

I. Introduction

THIS experiment is to analyze the electric field of the line charge in a free space. And the objectives of this experiment is:

- Calculate the distribution of electric field built by continuous line charge
- Plot the relevant figures on MATLAB environment
- Study the difference betweenintegration and infinitesimal methodson on analyzing electric field.

By using the scientific analysis software MATLAB, simulated the electric field distribution of line charge in a 2-D rectangular coordinate can help us to understand the electric field in a visualized way.

Suppose there is a uniformly distributed line charge between point A(-1,0) and point B(1,0), with line charge density of $\rho = 1 \times 10^{-9}$ C/m. (The unit for the coordinate is m)

Using integration method in II-A to calculate the electric potential at each point of the coordinate, we can get the real distribution in theory. And using infinitesimal method in II-B to do a approximate simulation.

To describe the distribution, I use three graph included for each method. They include:

- i) the distribution of electric field for each point;
- ii) equipotential lines;
- iii) distribution of electric field lines(represented by continuous lines).

To analyze the difference, I calculate the difference of electric potential between two ways at three different degree of seperation(20, 50 and 100). Draw the graph of difference distribution. And analyzed the difference at line: $y = 0.1$, using knowledge of calculus to describe the difference.

II. Related Knowledge and Functions

In vaccuum, the electric field intensity ($\mathbf{E}$) of a point charge can be expressed as:

$$\mathbf{E} = k\frac{Q}{R^2}\mathbf{a}_R \tag{1}$$

Where the coefficient $k = 9 \times 10^9$ F/m is the electrostatic constant. Q represent the total amount of charges. R denotes the distance between the point in the electric field and the source charge.

If we take reference point as the infinite distance, then the electric potential at a point in the field is expressed as:

$$V = k\frac{Q}{R} \tag{2}$$

The electric field intensity can be expressed as the negative gradient of the electric potential:

$$\mathbf{E} = -\nabla V \tag{3}$$

The electric field generated by N point charge in the vaccuum is expressed as:

$$V = \sum_{i=1}^{N} k\frac{Q_i}{R_i} \tag{4}$$

Similarly, the field magnitude generated by N point charges in the vaccuum can be obtained through equation (3).

When the field source is continuous charge, e.g. line charge, we can readily resolve it by using infinitesimal or integral method. the procedure of applying this method is listed as follows:

- 1) Divide the line charge into small segments of charges (usuallybeingdivided evenly).
- 2) Treat eachsmall segment of charges as a point charge and calculate the electric potential through equation (2).
- 3) Sum up all the electric potential by using equation (4) to obtain the electric potential.
- 4) Calculatethe electric field intensity generated by this line charge through equation (3).

A. *integration method*

Using integration method to calculate the distribution of electric potential at each point of the coordinate, namely, the

real distribution. The procedure is given below:
Given a point (X_0, Y_0):

$$\begin{aligned} V &= k\int_{-1}^{1}\frac{\rho dx}{R} \\ &= k\int_{-1}^{1}\frac{\rho dx}{\sqrt{(x-X_0)^2+Y_0^2}} \\ &= k\rho\ln\left|(x-X_0)+\sqrt{(x-X_0)^2+Y_0^2}\right|\Bigg|_{-1}^{1} \\ &= k\rho\ln\left(\frac{1-X_0+\sqrt{(1-X_0)^2+Y_0^2}}{-1-X_0+\sqrt{(-1-X_0)^2+Y_0^2}}\right) \end{aligned} \tag{5}$$

If we calculate each point of the coordinate.

$$V(x,y) = k\rho\ln\left(\frac{1-x+\sqrt{(1-x)^2+y^2}}{-1-x+\sqrt{(-1-x)^2+y^2}}\right) \tag{6}$$

Easy to prove, when calculated point is on the line charge, namely, $x \in [-1, 1]$ and $y = 0$, the electric potential is ∞.

B. infinitesimal method

Using infinitesimal method to calculate the distribution of electric potential at each point of the coordinate, however, we cannot have infinitesimal in MATLAB, so we use small segments as infinitesimal.

Set the approximative infinitesimal of distance is $\Delta x = \frac{l}{N}$. Where l is the length of line charge $l = x_A - x_B = 2$, N is number of segments. Using point charge to replace the segments, so the x-coordinates are the midpoints of the segments. Each x-coordinate is $x_i = i\frac{\Delta x}{2} + x_A = \frac{i}{N} - 1$ So, the approximate infinitesimal of charge is $\Delta Q = \rho\Delta x$. According to equation (4), given a point (X_0, Y_0):

$$\begin{aligned} V &= \sum_{i=1}^{N} k\frac{\Delta Q}{R_i} \\ &= k\rho\Delta x\sum_{i=1}^{N}\frac{1}{\sqrt{(x_i-X_0)^2+Y_0^2}} \\ &= k\rho\frac{l}{N}\sum_{i=1}^{N}\frac{1}{\sqrt{(\frac{i}{N}-1-X_0)^2+Y_0^2}} \\ &= \frac{2k\rho}{N}\sum_{i=1}^{N}\frac{1}{\sqrt{(\frac{i}{N}-X_0-1)^2+Y_0^2}} \end{aligned} \tag{7}$$

If we calculate each point of the coordinate.

$$V(x,y) = \frac{2k\rho}{N}\sum_{i=1}^{N}\frac{1}{\sqrt{(\frac{i}{N}-x-1)^2+y^2}} \tag{8}$$

C. MATLAB Functions

Set functions can help to reduced the code. Functions included:

- 1) function: V_con
- 2) function: V_dis

V_con : To calculate the electric field distribution by using integration method, namely, it use equation (6) to calculate.

```
function [V] = V_con(ro,X,Y)
%V_CON is the function to calculte the ...
    electric field distribution by ...
    continuous function
%   ro is the charge density
%   (X, Y) is the coordinates of the space

k = 9e9; % the electrostatic constant

lo = (1-X+sqrt((1-X).^2+Y.^2));
lo = lo./(-1-X+sqrt((-1-X).^2+Y.^2)); % ...
    calculte the part in ln() function
V = k * ro * log(lo); % calculte electric ...
    potential distribution

end
```

V_dis : To calculate the electric field distribution by using infinitesimal method, namely, it use equation (8) to calculate.

```
function [V] = V_dis(ro,xa,xb,N,n,X,Y)
% V_DIS is the function to calculte the ...
    electric feild distribution by deviding ...
    line charge to segments.
%   ro is the line charge density
%   xa and xb is the x-coordinate of the ...
    endpoints
%   N is the number of depart segments
%   n is the number of coordinates
%   (X, Y) is the coordinates of the space

k = 9e9; % electrostatic constant
l = 2; % length of line charge
dx = l / N; % length of each segments

x0 = xa + dx/2; % the first x-coordinate
xn = xb - dx/2; % the last x-coordinate
qx = x0: dx :xn;    % the x-coordinate of ...
    charge segments

V = zeros(N,n,n);          % create the ...
    coordinates space for N point charges

i = 1;
for qxi = qx
    r = sqrt((X-qxi).^2+(Y.^2)); % calculate ...
        the distances for coordinates to ...
        each charge point
    V(i,:,:) = 1 ./ r; % storage reciprocal ...
        of the distances
    i = i + 1;
end

V = sum(V);  % calculate the sum of ...
    reciprocal of the distances
V = reshape(V,n,n); % reshape matrix to two ...
    demension
dq = ro * l / N; %calculate dq
V = k*dq.*V; % times the k and dq

end
```

III. Simulation of Integration Method

Using MATLAB to describe electric field and draw pictures simulated by integration methods.
1. Initialization. Set the coordinates of 4m × 4m. The number of coordinates is 60 × 60.

```
clear; % clear memory
clc; % clear command window
l = 2; % set length
ro = 1e-9; % set charge density
k=9e9; % set electrostatic constant
pn = 60; % set accuarcy of coordinates
xa = -1; % set x-coordinate of point A
xb = 1; % set x-coordinate of point A

xm = 4; % set max value of x
ym = 4; % set max value of y
x = linspace(-xm,xm,pn); % devide the x-axis ...
    into pn segments
y = linspace(-ym,ym,pn); % devide the x-axis ...
    into pn segments
[X,Y] = meshgrid(x,y); % to form the coordinates
```

2. Calculate the electric potential distributionby using II-C: *V_con*. And draw picture of the distribution of electric field to Figure 1;

```
figure(11); % plot at figure 11
mesh(X,Y,V); % plot the distribustion of ...
    electric potential
hold on;
xlabel('X axis(Unit: m)','fontsize',15); % ...
    label X axis
ylabel('Y axis(Unit: m)','fontsize',15); % ...
    label Y axis
zlabel('V(Unit: F/m)','fontsize',15) % label ...
    Z axis
title({'Distribustion of electric potential ...
    of a line charge';'by integration method ...
    (by 11910103 Qingfu ...
    Qin)'},'fontsize',20) % title figure
```

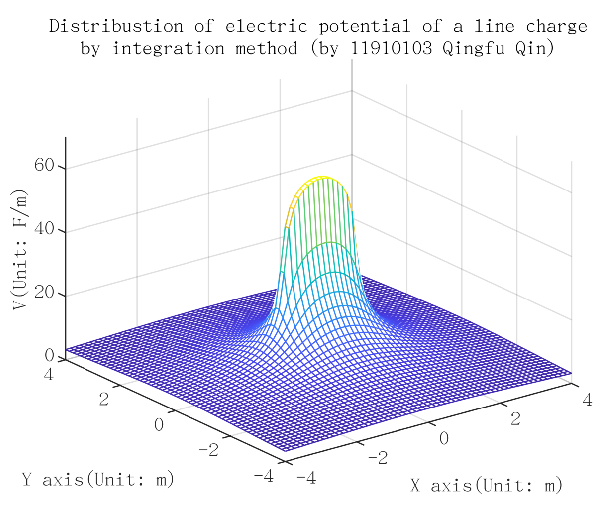


Fig. 1. Distribution of electric potential simulated by integration method

As Figure 1 shows that, the distribution of electric potential is similar to shape of fins. But for simulated by matlab, the electric potential on the line charge is infinite in fact.

3. Draw picture of Isopotential lines to Figure 2. Set range of potential is (0, 60) V.

```
Vmin=0; % set minimum potential
Vmax=60; % set maximum potential
Veq=linspace(Vmin,Vmax,40); % set 40 ...
    potential of isopotential lines
figure(12); % plot at figure 12
contour(X,Y,V,Veq); % plot 40 lines
grid on;
hold on;
title({'Isopotential lines of a line ...
    charge';'by integration method (by ...
    11910103 Qingfu Qin)'},'fontsize',20); % ...
    title figure
xlabel('X axis(Unitm)','fontsize',15); % ...
    label X axis
ylabel('Y axis(Unitm)','fontsize',15); % ...
    label Y axis
hold off;
```

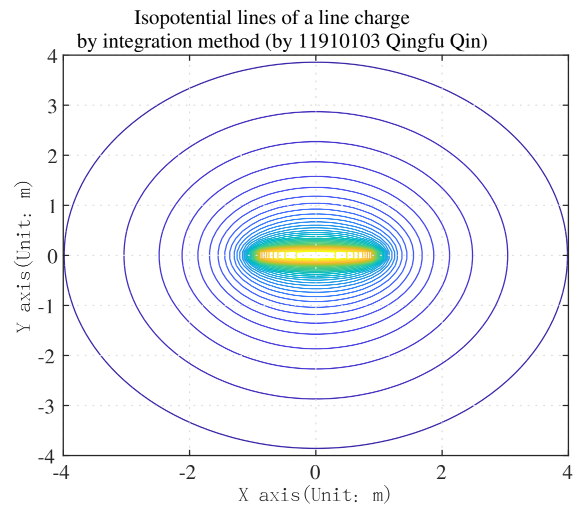


Fig. 2. Isopotential lines simulated by integration method

As Figure 2 shows that, the Isopotential lines are ellipse. And where the coordinates is far from line charge, the isopotential lines is being more round. It is means that the potential distribution of a short line charge is close to point charge in some ways. And It also prove that if the line charge is short enough, it similar to point charge

4. Draw picture of power lines to Figure 3.

```

[Ex,Ey]=gradient(-V); % calculation of ...
    electric field intensity at each point
del_theta=15; % set angular difference
theta=(0:del_theta:360).*pi/180; % express ...
    the angle into radian
xs1=1.1*cos(theta); % generate the x ...
    coordinate for the start of the field (a ...
    oval coordinate)
ys=sqrt(0.21)*sin(theta); % generate the x ...
    coordinate for the start of the field (a ...
    oval coordinate)
figure(13); % plot at figure 13
streamline(X,Y,Ex,Ey,xs1,ys); % generate the ...
    field lines
grid on;
hold on;
contour(X,Y,V,Veq); % plot eauipotential lines
title({'Isopotential lines and Power lins of ...
    a line charge';'by integration method ...
    (by 11910103 Qingfu ...
    Qin)'},'fontsize',20);    %
```

```
xlabel('X axis(Unitm)','fontsize',15); % ...
    label X axis
ylabel('Y axis(Unitm)','fontsize',15); % ...
    label Y axis
hold off;
```

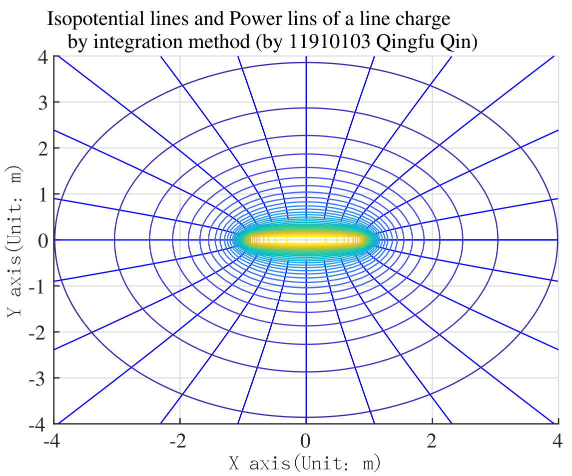


Fig. 3. Isopotential lines simulated by integration method

As Figure 3 shows that, power lines is flow outward from line charge.

IV. Simulation of Infinitesimal Method

Using MATLAB to describe electric field and draw pictures simulated by infinitesimal methods.
1. Initialization. Set the coordinates of 4m × 4m. The number of coordinates is 60 × 60, just similar to III.

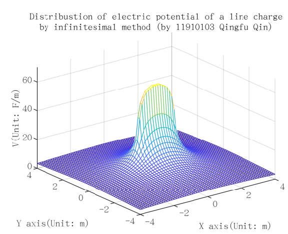
```
clear; % clear memory
clc; % clear command window
l = 2; % set length
ro = 1e-9; % set charge density
k=9e9; % set electrostatic constant
n = [20, 20, 20]; % set three degree of ...
    seperation
pn = 60; % set accuarcy of coordinates
xa = -1; % set x-coordinate of point A
xb = 1; % set x-coordinate of point A

xm = 4; % set max value of x
ym = 4; % set max value of y
x = linspace(-xm,xm,pn); % devide the x-axis ...
    into pn segments
y = linspace(-ym,ym,pn); % devide the x-axis ...
    into pn segments
[X,Y] = meshgrid(x,y); % to form the coordinates
```

2. Draw three times pictures for different N (20, 50, 100)

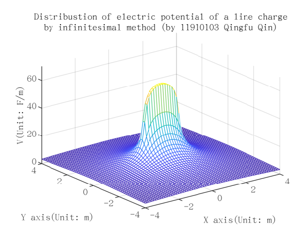
```
    li = 1;
    for ni = n % darw 3 times picture for ...
        different n
        ... part 1 ...
        ... part 2 ...
        ... part 3 ...
        li = li + 1;
    end
```

part 1. Calculate the electric potential distributionby using II-C: *V_dis*. And draw picture of the distribution of electric to Figure 4.

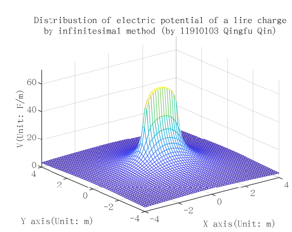
```
    V = V_dis(ro,xa,xb,ni,pn,X,Y);

    figure(20 + li); % plot at figure 21, ...
        22, 23
    mesh(X,Y,V); % plot the distribusion of ...
        electric potential
    hold on;
    xlabel('X axis(Unit: m)','fontsize',15); ...
        % label X axis
    ylabel('Y axis(Unit: m)','fontsize',15); ...
        % label Y axis
    zlabel('V(Unit: F/m)','fontsize',15) % ...
        label Z axis
    title({'Distribution of electric ...
        potential of a line charge';'by ...
        infinitesimal method (by 11910103 ...
        Qingfu Qin)'},'fontsize',20) % title ...
        figure
    hold off;
```



(a) N = 20



(b) N = 50



(c) N = 100

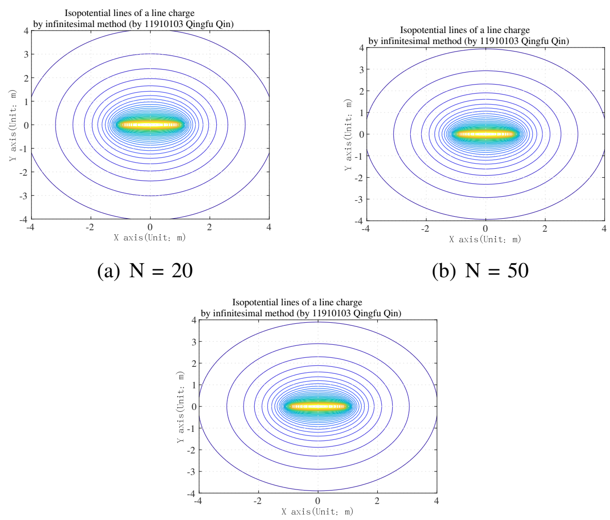
Fig. 4. Distribution of electric potential simulated by infinitesimal method

As three figures show, the distribution of electric potential simulated by infinitesimal method.

part 2. Draw picture of Isopotential lines to Figure 5. Set range of potential is (0, 60) V.

```
    Vmin=0; % set minimum potential
    Vmax=60; % set maximum potential
    Veq=linspace(Vmin,Vmax,40); % set 40 ...
        potential of isopotential lines
    figure(23 + li); % plot at figure 24, ...
        25, 26
    contour(X,Y,V,Veq); % plot 40 lines
    grid on;
    hold on;
    title({'Isopotential lines of a line ...
        charge';'by infinitesimal method (by ...
        11910103 Qingfu ...
        Qin)'},'fontsize',20); % title figure
    xlabel('X axis(Unitm)','fontsize',15); % ...
        label X axis
```

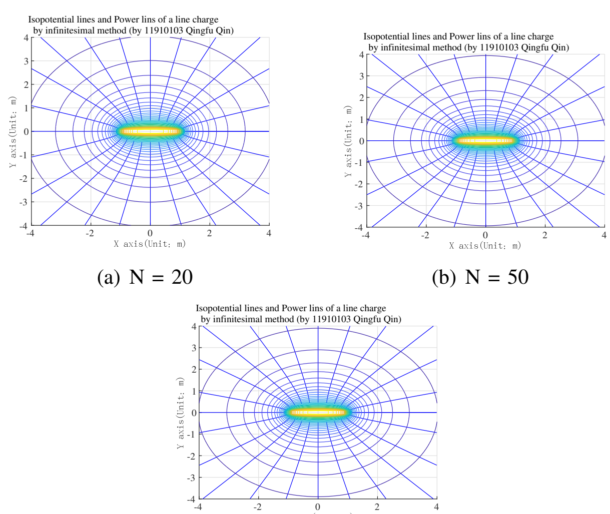
```
    ylabel('Y axis(Unitm)','fontsize',15); % ...
        label Y axis
    hold off;
```



(a) N = 20 (b) N = 50

(c) N = 100

Fig. 5. Distribution of electric potential simulated by infinitesimal method



(a) N = 20 (b) N = 50

(c) N = 100

Fig. 6. Distribution of electric potential simulated by infinitesimal method

part 3. Draw picture of power lines to Figure 6

```

    [Ex,Ey]=gradient(-V); % calculation of ...
        electric field intensity at each point
    del_theta=15; % set angular difference
    theta=(0:del_theta:360).*pi/180; % ...
        express the angle into radian
    xs1=1.1*cos(theta); % generate the x ...
        coordinate for the start of the ...
        field (a oval coordinate)
    ys=sqrt(0.21)*sin(theta); % generate the ...
        x coordinate for the start of the ...
        field (a oval coordinate)
    figure(26 + li); % plot at figure 27, ...
        28, 29
    streamline(X,Y,Ex,Ey,xs1,ys); % generate ...
        the field lines
    grid on;
    hold on;
    contour(X,Y,V,Veq); % plot eauipotential ...
        lines
    title({'Isopotential lines and Power ...
        lins of a line charge';'by ...
        infinitesimal method (by 11910103 ...
        Qingfu Qin)'},'fontsize',20);    % ...
        title figure
    xlabel('X axis(Unitm)','fontsize',15); % ...
        label X axis
    ylabel('Y axis(Unitm)','fontsize',15); % ...
        label Y axis
    hold off;
```

As Figure 5 and Figure 6 show that, simulation by using infinitesimal method is very close to integration method

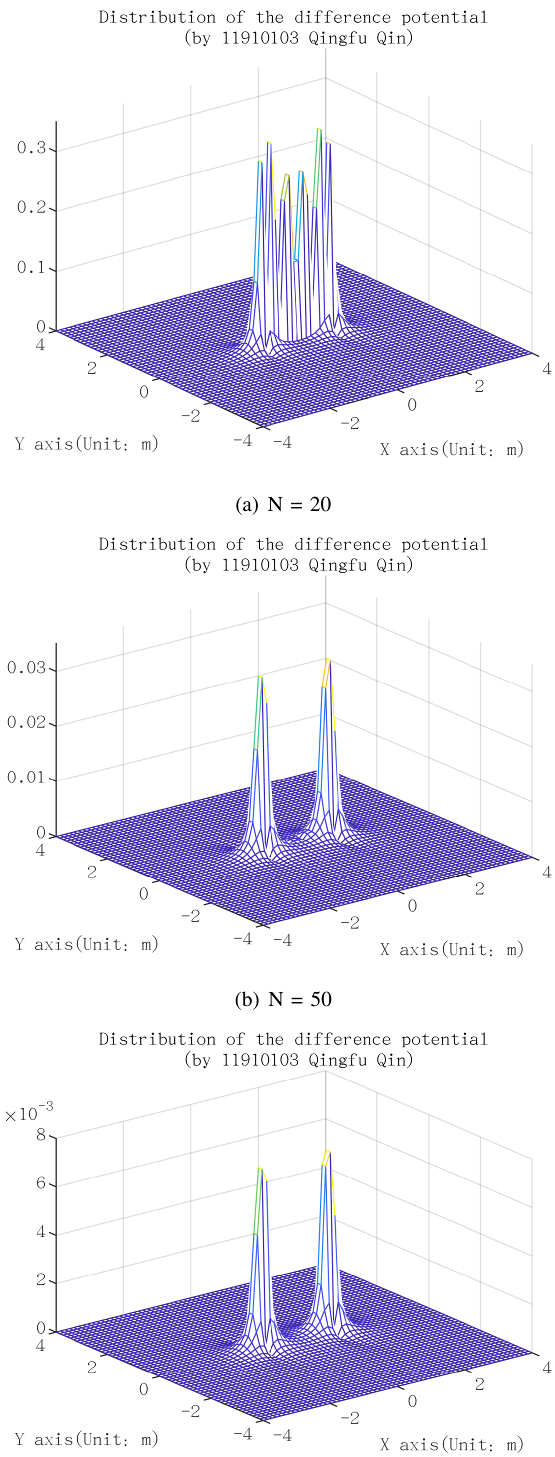
V. Analyze the Difference between two Methods

1. Initialization. Set coordinates as III and IV. And the real distribution of electric distribution V_1 will not change.

```
clear; % clear memory
clc; % clear command window
l = 2; % set length
ro = 1e-9; % set charge density
k=9e9; % set electrostatic constant
n = [20, 50, 100]; % set three degree of ...
    seperation
pn = 60; % set accuarcy of coordinates
xa = -1; % set x-coordinate of point A
xb = 1; % set x-coordinate of point A

xm = 4; % set max value of x
ym = 4; % set max value of y
x = linspace(-xm,xm,pn); % devide the x-axis ...
    into pn segments
y = linspace(-ym,ym,pn); % devide the x-axis ...
    into pn segments
[X,Y] = meshgrid(x,y); % to form the coordinates
V1 = V_con(ro,X,Y); % the real electric ...
    potential distribustion of line charge
```

2. Draw difference of electric potential distribution between two methods to Figure 7.

```
li = 1;
for ni = n % draw figure three times fo ...
    different N (20, 50, 100)
    V2 = V_dis(ro,xa,xb,ni,pn,X,Y); % the ...
        electric potential distribution of ...
        the infinitemal method
    dV = abs(V1 - V2); % calculate the ...
        difference between two method

    figure(30 + li); % plot at figure 31, ...
        32, 33
    mesh(X, Y, dV); % plot distribution of ...
        the difference
    hold on;
    title({'Distribution of the difference ...
        potential';' (by 11910103 Qingfu ...
        Qin)'},'fontsize',20);    % title figure
    xlabel('X axis(Unitm)','fontsize',15); % ...
        label X axis
    ylabel('Y axis(Unitm)','fontsize',15); % ...
        label Y axis
    hold off;
    li = li +1;
end
```



(a) N = 20

(b) N = 50

(c) N = 100

Fig. 7. Difference between two distributions of electric potential

According to Figure 7, the bigger N is, the smaller difference is. And the biggest difference is around the endpoints of the line charge.

For the electric potential on the line charge is infinite, the line near the line charge to analyze the difference is a great choice, specifically, let $x \in [-4, 4]$, $y = 0.1$. When $N = 20$, the difference is bigger, so come from the situation of N = 20 for infinitesimal method.

```
N = 20; % set the number of segments
y = [0.1]; % set y = 0.1;
[X,Y] = meshgrid(x,y); % to form the coordinates
V1 = V_con(ro,X,Y); % calculate the real ...
    distribustion at y = 0.1

V2 = zeros(N,1,pn);          % create the ...
    coordinates space for N point charges
dx = 2/N;
x0 = xa + dx/2; % the first x-coordinate
xn = xb - dx/2; % the last x-coordinate
qx = x0: dx :xn;    % the x-coordinate of ...
    charge segments
i = 1;
for qxi = qx
    r = sqrt((X-qxi).^2+(Y.^2)); % calculate ...
        the distances for coordinates to ...
        each charge point
    V2(i,:,:) = 1 ./ r; % storage reciprocal ...
        of the distances
    i = i + 1;
end

V2 = sum(V2);  % calculate the sum of ...
    reciprocal of the distances
V2 = reshape(V2,1,pn); % reshape matrix to ...
    two demension
dq = ro * l / N; %calculate dq
figure(34); % plot at figure 34
plot(x, V1, 'r'); % plot V1-a
hold on;
plot(x, V2, 'b'); % plot V2-a
xlabel('X axis(Unitm)','fontsize',15); % ...
    label X axis
ylabel('V(UnitF/m)','fontsize',15); % label ...
    Y axis (Y axis is potential level when y ...
    = 0.1)
hold off;
```

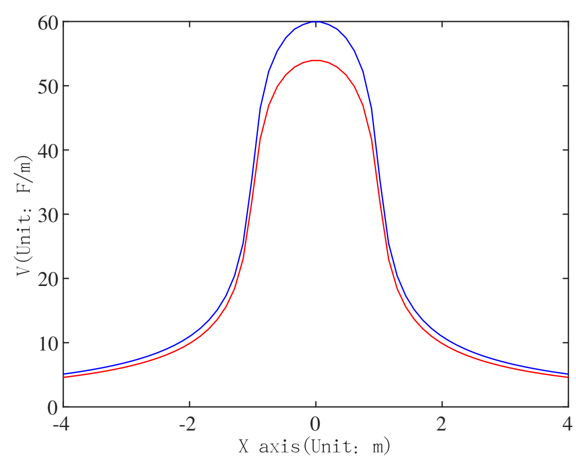


Fig. 8. Simulation at y = 0.1

If we calculate the equation of the distance to the line charge at point P $(0, 0.1)$, According to equation (5) we can get dV/dx of P is:

$$f(x) = \frac{dV}{dx} = \frac{k\rho}{\sqrt{x^2 + 0.1^2}} \tag{9}$$

$$V(x) = \int \frac{k\rho}{\sqrt{x^2 + 0.1^2}} = \ln(\left|sqrt(100x^2 + 1) + 10x\right|) + C \tag{10}$$

Draw the figure of dV/dx and the rectangular areas formed depart x axis to N segments, using the middle point's value as height, make rectangular area (width id dx ,the length of segments), to replace the area of function and x-axis in the same width.

```
x = [-1:0.05/100:1];
dVm = k * ro ./ (x.^2+0.1^2);
figure(35); % plot at figure 34
plot(x, dVm, 'b');
hold on;
qx = xa + 0.05: dx :xb;   % get x-coordinate ...
    of left point of each rectangular area.
dVq = k * ro ./ (qx.^2+0.1^2); % calculate ...
    the each area's height.
plot(qx,dVq, 'ro'); % draw the point of each ...
    height at the function

qx = qx - 2/40; % move middle point of each ...
    area to fit the value of the function

stem(qx,dVq,'r','Marker','None');
stairs(qx,dVq, 'r'); % draw the areas.
hold off;
```

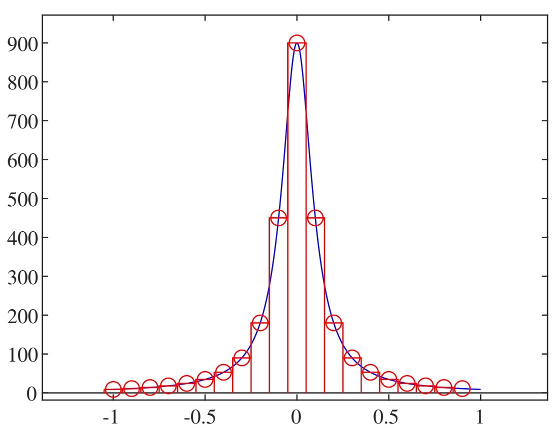


Fig. 9. Potential density distribution at point(0, 0.1)

According to Figure **??** bule function is the real value of Can be proved, when $dx = l/N$ approximate to 0, N approximate infinity, the integration between real value and infinitesimal will be small.

The error (ϵ) from real value to approximative value is:

$$\epsilon_i = \left| k \int_t^{t+\Delta x} \frac{\rho dx}{R} - \frac{k\rho\Delta x}{\sqrt{(t - \frac{\Delta x}{2})^2 + 0.01}} \right| \tag{11}$$

$$\epsilon = \sum_{i=1}^{N} \epsilon_i \tag{12}$$

Find the error at point p for each segment.

```
Sr = dVq * dx; % area of each rectangular block

qx = xa: dx :xb;    % get x-coordinate of ...
    left point of each rectangular area.

Fi = k * ro .* ...
    log(abs(sqrt(100.*qx.^2+1)+10.*qx));
p = 2;
Si = linspace(0,0,20);
while p < 22
Si(p-1) = Fi(p) - Fi(p-1);
    p= p + 1;
end

ei = abs(Si - Sr);
e = sum(ei);
```

TABLE I
ERROR FROM INFINITESIMAL METHOD

i	1	2	3	4	5
ϵ_i	0.043	0.176	0.381	0.710	1.27
i	6	7	8	9	10
ϵ_i	2.28	4.31	9.04	22.6	64.1
i	11	12	13	14	15
ϵ_i	64.1	22.6	9.04	4.31	2.28
i	16	17	18	19	20
ϵ_i	1.27	0.710	0.381	0.176	0.0433

$\epsilon \approx 210$;

From Table **??**, each rectangular area has difference of the trapezoid with curve side, so if the curve is bended more seriously, then the error will be increase. That is affected by the degree of seperation, because for a smaller distance, the bend level is lower.

VI. Conclusion

The distribution of electric potential of line charge is similar to shape of fins. The potential distribution of a short line chargeis close to point charge in some ways. The difference between two methods, the difference caused by effeacted by degree of seperation.

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