Engineering Electromagnetics - Experiment 3 Magnetic Field of Two Current Loops

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***Abstract*—This article describes the magnetic feild of two current loops with same radius and center axis. By using MATLAB to simulate the magnetic field and draw the pictures; Using Biot-Savart and superposition principle divide two current loop into 50 segments, and calculate the magnetic feild intensity vector at each point of yz plane which is perpendicular to the current loops. We can find that if two current directions of loops are same, the magnetic field distribution between two loops is uniform; and if the directions are different, the magnetic fields for each loop are cancelled out in the space between loops, so the magnetic field intensity at the center between two loops is small. and the magnetic lines are all closed.**

I. Introduction

THIS experiment is to analyze the magnetic field of the two current loops in a free space. And the objectives of this experiment is:

- i) Get familiar with the spatial magnetic field that is produced by an electric current loop.
- ii) Calculate the magnetic field distribution and plot the relevant graphs through MATLAB.

The tasks of this experiment is that using MATLAB to analyze the magnetic field distribution of the following two cases:

- Case 1: Two current loops with same radius $a = 2m$, and the current in both of the loops is 500A The two loops are parallel to the xy plane, and the loop centers are located at $O_1(0, 0, -1)$, $O_2(0, 0, 1)$ respectively. The direction of the current are the same.
- Case 2: Set the current directions of the two loops in case 1 to be opposite.

The experiment of requirements are:

- i) Calculate and plot the magnetic field intensity vector distribution (represented by arrows).
- ii) Calculate and plot the magnetic field intensity magnitude distribution.

For simplicity, only analyze the magnetic field distribution on the yz plane.

II. Related Knowledge

A. Biot-Savart Law

In vaccuum, the magnetic field intensity vector ($\mathbf{H}$) of a short current segment ($d\mathbf{H}$) can be expressed as:

$$d\mathbf{H} = \frac{Id\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{Id\mathbf{L} \times \mathbf{R}}{4\pi R^3} \tag{1}$$

Where the coefficient $Id\mathbf{L}$ is elementary current vector, $\mathbf{R}$ is the vector that point from the elementary current $Id\mathbf{L}$ to a field point P and its magnitude is R.

B. Magnetic field along the center axis of a current loop

As for the magnetic field created by an electric current loop, we can used the Biot-Savart law to derive the distribution of the magnetic field intensity along the center axis of the current loop:

$$\mathbf{H} = \frac{I(\pi a^2)\mathbf{a}_z}{2\pi(a^2 + z_0^2)^{3/2}} \tag{2}$$

Where a is the radius of the current loop, I is the current in the current loop, z_0 is the coordinate of the field point (located on the center axis, i.e. z axis). However, for the field points that are not on the center axis, it would be hard to deploy Biot-Savart law to derive an analytical solution for the magnetic field intensity.

C. Superposition principle

Similarly, magnetic field also follows the superposition principle. Therefore, we can divide the current-carrying conductor into many elementary currents. Thus, the magnetic field created by the current-carrying conductor is equal to the superposition of the magnetic fields created by all elementary currents.

D. Helmholtz coils

For two parallelly placed current loops, which currents is same at magnitude and direction, when the distance between them is equal to their radius, this two-current-loops system is usually called Helmholtz coils. One characteristic of the Helmholtz coils is that the spatial magnetic field distribution between these two current loops is very uniform.

III. Same Current Direction Loops

By using MATLAB to simulate case 1, code and graphs are follow:

Firstly, initialize and set a, I and coordinates, etc.

```
clear; % clear memory
clc; % clear comand window
a = 2; % set radius of the current loops
I = 500; % set the current of the loop
C = I / (4 * pi); % merge the constants
zc1 = -1;
```

```
zc2 = 1; % set location of the center of ...
    each loop(x and y are 0)
ym = 5;
zm = 5; % set range
pn = 60; % set accuarcy of coordinates
y = linspace(-ym, ym, pn);
z = linspace(-zm, zm, pn);
[Y, Z] = meshgrid(y, z);
```

Then depart current loop to N segments, N = 50; And calculate the coordinates of each segment center.

```
N = 50; % set the segments number
dphi = 2 * pi / N; % set angle of each ...
    segment of the loop
phi = linspace(dphi, 2 * pi - dphi, N); % ...
    set angle - coordinates of segments

xc = a * cos(phi);
yc = a * sin(phi); % transform angle to xy ...
    coordiantes
dl = a * dphi; % set length of each segment
```

Calculate the magnetic field intensity of each point by using equation in II-A.

```
Hx = zeros(pn);
Hy = zeros(pn);
Hz = zeros(pn);
H = zeros(pn); % construct matrixes of ...
    magnetic field
ci = 1; % ci is the z coordinate of matrixes
for bi = y% each point of bi as y coordinate
    cj = 1; % cj is the y coordinate of matrixes
    for bj = z% each point of bj as z coordinate
        dHx = 0;
        dHy = 0;
        dHz = 0;
        dH = 0;
        for li = 1:N% calculate magnetic ...
            field intensity vector at point ...
            B (bi, bj) of loop1
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc1)^2); % ...
                calculate the distance ...
                between segment and B
            v_l = dl * [-sin(phi(li)), ...
                cos(phi(li)), 0]; % set the ...
                vector of dL
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc1]; % set vector of R
            v_H = cross(v_l, v_r); % ...
                calculate value of cross product
            dHx = dHx + C * v_H(1) / (R^3);
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3); ...
                % accumalte magnitude of ...
                vectors on each axis
        end

        for li = 1:N% calculate magnetic ...
            field intensity vector at (bi, ...
            bj) of loop2
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc2)^2);
            v_l = dl * [-sin(phi(li)), ...
                cos(phi(li)), 0];
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc2];
            v_H = cross(v_l, v_r);
            dHx = dHx + C * v_H(1) / (R^3);
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3);
            dH = dH + sqrt(dHy.^2 + dHz.^2);
        end
        Hx(cj, ci) = dHx;
        Hy(cj, ci) = dHy;
        Hz(cj, ci) = dHz;
        cj = cj + 1;
    end
    ci = ci + 1;
end

H = sqrt(Hy.^2 + Hz.^2); % calculate the ...
    magnitude on yz plane
```

Draw the magnetic field intensity vector distribution at the region between two loops.

```
figure(1);
quiver(Y, Z, Hy, Hz); % plot the vector ...
    graph of magnetic field intensity
hold on
plot(-2, -1, 'bo');
plot(2, -1, 'ro');
plot(-2, 1, 'bo');
plot(2, 1, 'ro'); % plot the point of coils
xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
axis([-2, 2, -1, 1]);
hold off;
```

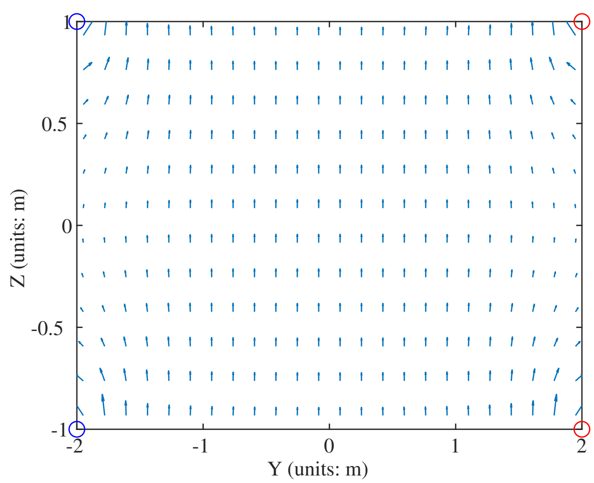

Fig. 1. Magnetic field intensity vector distribution in case 1

As Figure 1 shows the intensity vector field distribution is uniform. and accoring to right hand rule (blue point is the point of the current flow out the screen and the red is flow in), the vectors between two loops are along the direction of $\mathbf{a}_z$.

Draw the magnetic field intensity magnitude distribution in the space.

```
figure(2);
mesh(Y, Z, H); % plot graph of magnetic ...
    field intensity
hold on;
xlabel("Y (units: m)");
ylabel("Z (units: m)");
zlabel("H (units: A/m)")% label the axis
axis([-4, 4, -3, 3]);
hold off;
```

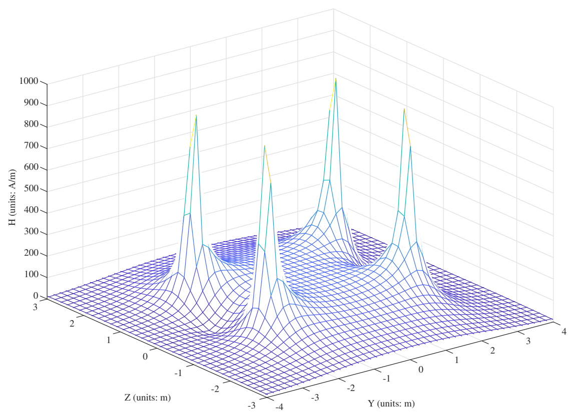

Fig. 2. Magnetic field intensity magnitude distribution in case 1

As Figure 2 shows that the magnetic field distribution concentrate near the coils. Between two loops, there is a flat area with uniform magnetic field distribution.

Draw the magnetic lines in the space, and to avoid the lines at the area near the coils are too intensive, the graph cancell some lines.

```
figure(3);
theta = [0 50 60 70 80 90 100 110 120 130 ...
    180] .* pi / 180; % Set the radian value ...
    of the streamlines
ys = 2.1 * cos(theta);
zs = 1.1 * sin(theta); % set the coordinates ...
    of start points
streamline(Y, Z, Hy, Hz, ys, zs); % plot the ...
    magnentic line of force
hold on;
streamline(Y, Z, -Hy, -Hz, ys, zs); % plot ...
    negative direction lines
xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
axis([-4, 4, -3, 3]);
hold off;
```

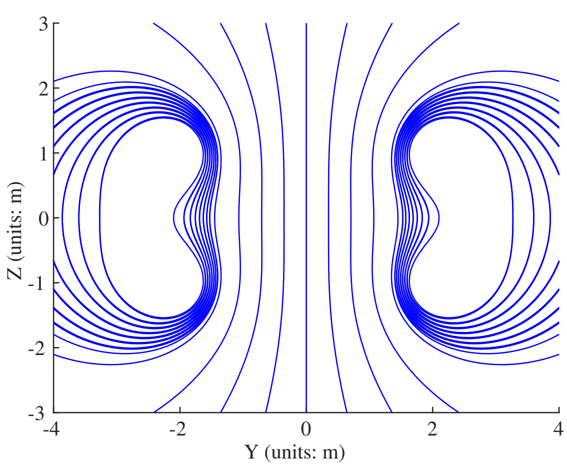

Fig. 3. Magnetic lines distribution in case 1

As Figure 3 shows that the magnetic lines is around the coils and close to the single current loop in the space when far away form the coils. It can be proved from the graph that each magnetic line is closed. And it is axisymmetric.

IV. Different Current Direction Loops

By using MATLAB to simulate the case 2, code and graphs are follow:

Firstly, initialize and set a, I and coordinates as case 1.

```
clear; % clear memory
clc; % clear comand window
a = 2; % set radius of the current loops
I = 500; % set the current of the loop
C = I / (4 * pi); % merge the constants
zc1 = -1;
zc2 = 1; % set location of the center of ...
    each loop(x and y are 0)
ym = 5;
zm = 5; % set range
pn = 60; % set accuary of coordinates
y = linspace(-ym, ym, pn);
z = linspace(-zm, zm, pn);
[Y, Z] = meshgrid(y, z);
```

Then depart current loop to N segments, N = 50; And calculate the coordinates of each segment center.

```
N = 50; % set the segments number
dphi = 2 * pi / N; % set angle of each ...
    segment of the loop
phi = linspace(dphi, 2 * pi - dphi, N); % ...
    set angle - coordinates of segments

xc = a * cos(phi);
yc = a * sin(phi); % transform angle to xy ...
    coordiantes
dl = a * dphi; % set length of each segment
```

Calculate the magnetic field intensity of each point by using equation in II-A. And change the current direction of the loop which center is at O_2

```
Hx = zeros(pn);
Hy = zeros(pn);
Hz = zeros(pn);
H = zeros(pn);

ci = 1;
for bi = y
    cj = 1;
    for bj = z
        dHx = 0;
        dHy = 0;
        dHz = 0;
        dH = 0;
        for li = 1:N% calculate magnetic ...
            field intensity vector at (bi, ...
            bj) of loop1
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc1)^2);
            v_l = dl * [-sin(phi(li)), ...
                cos(phi(li)), 0]; % set the ...
                vector of dL
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc1]; % set vector of R
            v_H = cross(v_l, v_r); % ...
                calculate value of cross product
            dHx = dHx + C * v_H(1) / (R^3);
```

```
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3); ...
                % accumalte magnitude of ...
                vectors on each axis
        end

        for li = 1:N% calculate magnetic ...
            field intensity vector at (bi, ...
            bj) of loop2
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc2)^2);
            v_l = -dl * [-sin(phi(li)), ...
                cos(phi(li)), 0]; % the ...
                direction of elementary is ...
                opposite
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc2];
            v_H = cross(v_l, v_r);
            dHx = dHx + C * v_H(1) / (R^3);
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3);
        end
        Hx(cj, ci) = dHx;
        Hy(cj, ci) = dHy;
        Hz(cj, ci) = dHz;
        cj = cj + 1;
    end
    ci = ci + 1;
end

H = sqrt(Hy.^2 + Hz.^2); % calculate the ...
    magnitude on yz plane
```

Draw the magnetic field intensity vector distribution.

```
figure(1);
quiver(Y, Z, Hy, Hz); % plot the vector ...
    graph of magnetic field intensity
hold on
plot(-2, -1, 'bo');
plot(2, -1, 'ro');
plot(-2, 1, 'ro');
plot(2, 1, 'bo');
xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
axis([-3, 3, -2, 2]);
hold off;
```

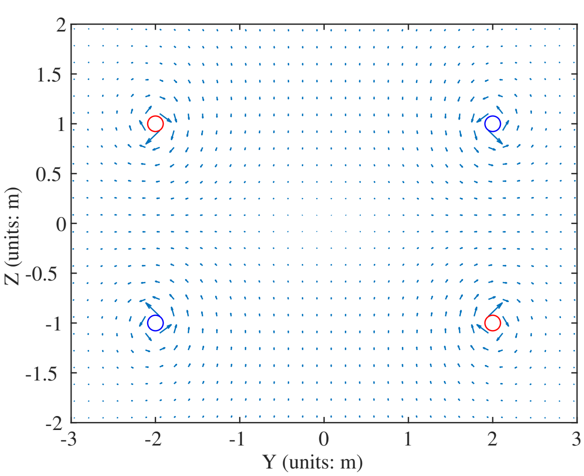

Fig. 4. Magnetic field intensity vector distribution in case 2

As Figure 4 shows that the magnetic field intensity vector distribution between two loops are small, because two loops has different current direction, so that the magnetic field vector on yz plane is also opposite, so they cancelled out.

Draw the magnetic field intensity magnitude distribution in the space.

```
figure(2);
mesh(Y, Z, H); % plot graph of magnetic ...
    field intensity
hold on;
xlabel("Y (units: m)");
ylabel("Z (units: m)");
zlabel("H (units: A/m)")% label the axis
axis([-4, 4, -3, 3]);
hold off;
```

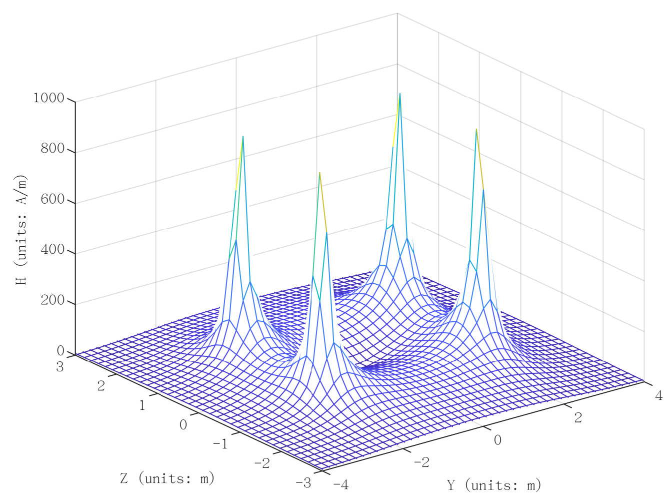

Fig. 5. Magnetic field intensity magnitude distribution in case 2

As Figure 5 shows that the magnetic field intensity distribution concentrate near the coils just like case 1. However, there is not a flat area with uniform magnetic field distribution between two coils.

Draw the magnetic line distribution.

```
figure(3);
ys = linspace(-1.5, 1.5, 10);
zs = linspace(-0.5, 0.5, 10); % set the ...
    coordinates of start points
streamline(Y, Z, Hy, Hz, ys, zs);
hold on;
streamline(Y, Z, -Hy, -Hz, ys, zs); % plot ...
    magnetic lines at 1, 3 quadrant
streamline(Y, Z, Hy, Hz, -ys, zs);
streamline(Y, Z, -Hy, -Hz, -ys, zs); % plot ...
    at 2, 4 quadrant


xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
axis([-4, 4, -3, 3]);

hold off;
```

As Figure 6 shows that the magnetic lines is centrosymmetric. And It is not like the first case, the lines near the coils are separate, combine with Figure 5, the magnetic feild intensity

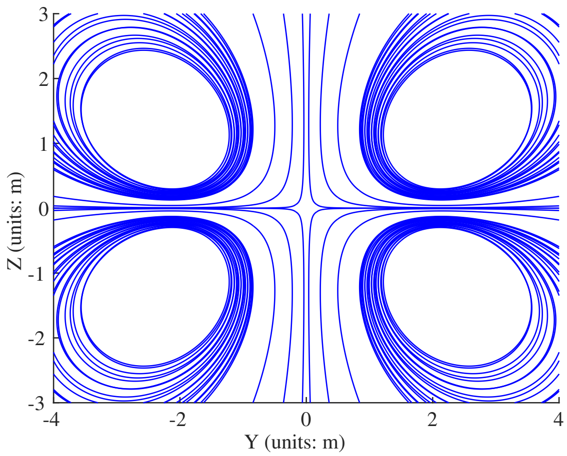

Fig. 6. Magnetic lines distribution in case 2

near coils in case 2 should at different degree (when $z > 0$, the magnitude should be negtive to the area when $z < 0$).

V. Inspiration

To analyze the two cases, they are different with the single current loop. When the current directions are same, the magnetic field is superposed, and at different directions, the magnetic field of each current loop are cancelled out at the area between two loops.

For each magnetic line, it will closed without start point and end point, and they are around the points of the current loops accorss the plane.

In simulation, MATLAB can do the cross product automatic, but to do accumulation, it still depart current loop to small segments.

To analyze the vector on yz plane, the magnitude at x coordinate should be ignored. For better graph, some lines too concentrated should be ignored and the start points of magnetic lines should be selected careful, espacially in case 2.

Acknowledgment

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