Engineering Electromagnetics - Experiment 3 Magnetic Field of Two Current Loops

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***Abstract*—This article describes the magnetic feild of two current loops with same radius and center axis. By using MATLAB to simulate the magnetic field and draw the pictures; Using Biot-Savart and superposition principle divide two current loop into 50 segments, and calculate the magnetic feild intensity vector at each point of yz plane which is perpendicular to the current loops. We can find that if two current directions of loops are same, the magnetic field distribution between two loops is uniform; and if the directions are different, the magnetic fields for each loop are cancelled out, so**

I. Introduction

THIS experiment is to analyze the magnetic field of the two current loops in a free space. And the objectives of this experiment is:

- i) Get familiar with the spatial magnetic field that is produced by an electric current loop.
- ii) Calculate the magnetic field distribution and plot the relevant graphs through MATLAB.

The tasks of this experiment is that using MATLAB to analyze the magnetic field distribution of the following two cases:

- Case 1: Two current loops with same radius $a = 2m$, and the current in both of the loops is 500A The two loops are parallel to the xy plane, and the loop centers are located at $O_1(0, 0, -1)$, $O_2(0, 0, 1)$ respectively. The direction of the current are the same.
- Case 2: Set the current directions of the two loops in case 1 to be opposite.

The experiment of requirements are:

- i) Calculate and plot the magnetic field intensity vector distribution (represented by arrows).
- ii) Calculate and plot the magnetic field intensity magnitude distribution.

For simplicity, only analyze the magnetic field distribution on the yz plane.

II. Related Knowledge

A. Biot-Savart Law

In vaccuum, the magnetic field intensity vector ($\mathbf{H}$) of a short current segment ($d\mathbf{H}$) can be expressed as:

$$d\mathbf{H} = \frac{Id\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{Id\mathbf{L} \times \mathbf{R}}{4\pi R^3} \tag{1}$$

Where the coefficient $Id\mathbf{L}$ is elementary current vector, $\mathbf{R}$ is the vector that point from the elementary current $Id\mathbf{L}$ to a field point P and its magnitude is R.

B. Magnetic field along the center axis of a current loop

As for the magnetic field created by an electric current loop, we can used the Biot-Savart law to derive the distribution of the magnetic field intensity along the center axis of the current loop:

$$\mathbf{H} = \frac{I(\pi a^2)\mathbf{a}_z}{2\pi(a^2 + z_0^2)^{3/2}} \tag{2}$$

Where a is the radius of the current loop, I is the current in the current loop, z_0 is the coordinate of the field point (located on the center axis, i.e. z axis). However, for the field points that are not on the center axis, it would be hard to deploy Biot-Savart law to derive an analytical solution for the magnetic field intensity.

C. Superposition principle

Similarly, magnetic field also follows the superposition principle. Therefore, we can divide the current-carrying conductor into many elementary currents. Thus, the magnetic field created by the current-carrying conductor is equal to the superposition of the magnetic fields created by all elementary currents.

D. Helmholtz coils

For two parallelly placed current loops, which currents is same at magnitude and direction, when the distance between them is equal to their radius, this two-current-loops system is usually called Helmholtz coils. One characteristic of the Helmholtz coils is that the spatial magnetic field distribution between these two current loops is very uniform.

III. Same Current Direction Loops

```
clear; % clear memory
clc; % clear comand window
a = 2; % set radius of the current loops
I = 500; % set the current of the loop
C = I / (4 * pi);

zc1 = -1;
zc2 = 1; % set location of the center of ...
    each loop(x and y are 0)

N = 100; % set the segments number
ym = 5;
zm = 5;
pn = 60; % set the of coordinate

dphi = 2 * pi / N; % set angle of each ...
    segment of the loop
```

```
phi = linspace(dphi, 2 * pi - dphi, N); % ...
    set angle - coordinates of segments

y = linspace(-ym, ym, pn);
z = linspace(-zm, zm, pn);
[Y, Z] = meshgrid(y, z);

xc = a * cos(phi);
yc = a * sin(phi); % transform angle to xy ...
    coordiantes

dl = a * dphi; % set length of each segment

Hx = zeros(pn);
Hy = zeros(pn);
Hz = zeros(pn);
H = zeros(pn); % construct matrixes of ...
    magnetic field

ci = 1; % ci is the z coordinate of matrixes

for bi = y% each point of bi as y coordinate

    cj = 1; % cj is the y coordinate of matrixes

    for bj = z% each point of bj as z coordinate

        dHx = 0;
        dHy = 0;
        dHz = 0;
        dH = 0;

        for li = 1:N% calculate magnetic ...
            field intensity vector at (bi, ...
            bj) of loop1
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc1)^2);

            v_l = dl * [-sin(phi(li)), ...
                cos(phi(li)), 0]; % set the ...
                vector of dL
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc1]; % set vector of R
            v_H = cross(v_l, v_r); % ...
                calculate value of cross product
            dHx = dHx + C * v_H(1) / (R^3);
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3); ...
                % accumalte magnitude of ...
                vectors on each axis
        end

        for li = 1:N% calculate magnetic ...
            field intensity vector at (bi, ...
            bj) of loop2
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc2)^2);

            v_l = dl * [-sin(phi(li)), ...
                cos(phi(li)), 0];
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc2];
            v_H = cross(v_l, v_r);
            dHx = dHx + C * v_H(1) / (R^3);
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3);
            dH = dH + sqrt(dHy.^2 + dHz.^2);
        end

        Hx(cj, ci) = dHx;
        Hy(cj, ci) = dHy;
        Hz(cj, ci) = dHz;

        cj = cj + 1;
    end

    ci = ci + 1;
end

H = sqrt(Hy.^2 + Hz.^2); % calculate the ...
    magnitude on yz plane

figure(1);
mesh(Y, Z, H); % plot graph of magnetic ...
    field intensity
hold on;
xlabel("Y (units: m)");
ylabel("Z (units: m)");
zlabel("H (units: A/m)")% label the axis
axis([-4, 4, -3, 3]);
hold off;

figure(2);
quiver(Y, Z, Hy, Hz); % plot the vector ...
    graph of magnetic field intensity
hold on
plot(-2, -1, 'bo');
plot(2, -1, 'ro');
plot(-2, 1, 'bo');
plot(2, 1, 'ro'); % plot the point of coils
xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
axis([-2, 2, -1, 1]);
hold off;

figure(3);
theta = [0 50 60 70 80 90 100 110 120 130 ...
    180] .* pi / 180; % Set the radian value ...
    of the streamlines
ys = 2.1 * cos(theta);
zs = 1.1 * sin(theta);
streamline(Y, Z, Hy, Hz, ys, zs); % plot the ...
    vector graph of magnetic field intensity
axis([-4, 4, -3, 3]);

hold on;
streamline(Y, Z, -Hy, -Hz, ys, zs);
xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
hold off;
```

IV. Different Current Direction Loops

```
clear; % clear memory
clc; % clear comand window
a = 2; % set radius of the current loops
I = 500; % set the current of the loop
C = I / (4 * pi);

zc1 = -1;
zc2 = 1; % set location of the center of ...
    each loop(x and y are 0)

N = 50; % set the segments number
ym = 5;
zm = 5;
pn = 60; % set the of coordinate

dphi = 2 * pi / N; % set angle of each ...
    segment of the loop
phi = linspace(dphi, 2 * pi - dphi, N); % ...
    set angle - coordinates of segments

y = linspace(-ym, ym, pn);
z = linspace(-zm, zm, pn);
[Y, Z] = meshgrid(y, z);

xc = a * cos(phi);
yc = a * sin(phi); % transform angle to xy ...
    coordiantes
```

```
dl = a * dphi; % set length of each segment

Hx = zeros(pn);
Hy = zeros(pn);
Hz = zeros(pn);
H = zeros(pn);

ci = 1;

for bi = y

    cj = 1;

    for bj = z

        dHx = 0;
        dHy = 0;
        dHz = 0;
        dH = 0;

        for li = 1:N% calculate magnetic ...
            field intensity vector at (bi, ...
            bj) of loop1
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc1)^2);

            v_l = dl * [-sin(phi(li)), ...
                cos(phi(li)), 0]; % set the ...
                vector of dL
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc1]; % set vector of R
            v_H = cross(v_l, v_r); % ...
                calculate value of cross product
            dHx = dHx + C * v_H(1) / (R^3);
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3); ...
                % accumalte magnitude of ...
                vectors on each axis
        end

        for li = 1:N% calculate magnetic ...
            field intensity vector at (bi, ...
            bj) of loop2
            R = sqrt((0 - xc(li))^2 + (bi - ...
                yc(li))^2 + (bj - zc2)^2);

            v_l = -dl * [-sin(phi(li)), ...
                cos(phi(li)), 0]; % the ...
                direction of elementary is ...
                opposite
            v_r = [0 - xc(li), bi - yc(li), ...
                bj - zc2];
            v_H = cross(v_l, v_r);
            dHx = dHx + C * v_H(1) / (R^3);
            dHy = dHy + C * v_H(2) / (R^3);
            dHz = dHz + C * v_H(3) / (R^3);
        end

        Hx(cj, ci) = dHx;
        Hy(cj, ci) = dHy;
        Hz(cj, ci) = dHz;

        cj = cj + 1;
    end

    ci = ci + 1;
end

H = sqrt(Hy.^2 + Hz.^2); % calculate the ...
    magnitude on yz plane

figure(1);
mesh(Y, Z, H); % plot graph of magnetic ...
    field intensity
hold on;
xlabel("Y (units: m)");
ylabel("Z (units: m)");
zlabel("H (units: A/m)")% label the axis
axis([-4, 4, -3, 3]);
hold off;

figure(2);
quiver(Y, Z, Hy, Hz); % plot the vector ...
    graph of magnetic field intensity
hold on
plot(-2, -1, 'bo');
plot(2, -1, 'ro');
plot(-2, 1, 'ro');
plot(2, 1, 'bo');
xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
axis([-4, 4, -3, 3]);
hold off;

figure(3);

theta = [0 45 60 75 90 105 120 135 180] .* ...
    pi / 180; % Set the radian value of the ...
    streamlines
ysu = (sqrt(5) + 0.1) * cos(theta);
zsu = (sqrt(5) + 0.1) * sin(theta);
ysd = (sqrt(5) + 0.1) * cos(-theta);
zsd = (sqrt(5) + 0.1) * sin(-theta);
streamline(Y, Z, Hy, Hz, ysu, zsu); % plot ...
    the vector graph of magnetic field intensity
hold on;
streamline(Y, Z, -Hy, -Hz, ysu, zsu);
streamline(Y, Z, Hy, Hz, ysd, zsd);
streamline(Y, Z, -Hy, -Hz, ysd, zsd);

xlabel("Y (units: m)");
ylabel("Z (units: m)"); % label the axis
axis([-4, 4, -3, 3]);

hold off;
```

V. Inspiration

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Thanks to Youwei Jia, the teacher who teach me the knowledge about Electromagnetics and the using methods of MATLAB. He gives amounts of help to me to finish this article.