CS 402 Assignment 1 - Search Algorithms

Total Points: 8 points
Scoring: Each question = 0.5 points, blank = 0 points, incorrect = -0.5 points

Q1a: Search Algorithm Properties (3.5 points)

Context: Graph search with consistent heuristics and positive edge costs (ε > 0).

Answers:

1. **(i) Depth-first optimal:** $\boxed{\text{FALSE}}$
    - *Justification:* DFS explores one path completely before backtracking. Even with consistent heuristics, it doesn't guarantee optimality.
2. **(ii) Breadth-first optimal:** $\boxed{\text{FALSE}}$
    - *Justification:* BFS is optimal only for unit costs. With arbitrary positive costs (ε > 0), it's not guaranteed optimal.
3. **(iii) Uniform-cost optimal:** $\boxed{\text{TRUE}}$
    - *Justification:* UCS explores nodes in order of increasing g(n). It's optimal for any cost function.
4. **(iv) Greedy optimal:** $\boxed{\text{FALSE}}$
    - *Justification:* Greedy uses only h(n), ignoring g(n). Even with consistent heuristics, it's not optimal.
5. **(v) A* optimal:** $\boxed{\text{TRUE}}$
    - *Justification:* A* uses f(n) = g(n) + h(n). With consistent (and thus admissible) heuristics, A* is guaranteed optimal.
6. **(vi) A* ≤ DFS nodes:** $\boxed{\text{FALSE}}$
    - *Justification:* DFS may find a solution by expanding fewer nodes than A, *but that solution might be suboptimal. A* guarantees optimality but may need to expand more nodes.
7. **(vii) A* ≤ UCS nodes:** $\boxed{\text{TRUE}}$
    - *Justification:* Both A* and UCS are optimal. A* uses f(n) = g(n) + h(n), UCS uses only g(n). With admissible heuristics, A* is more informed.

Q1b: Heuristic Scaling Effects (1.5 points)

Context: h1(s) is an admissible A* heuristic. h2(s) = 2 × h1(s).

Answers:

1. **(i) h2 optimal solution:** $\boxed{\text{FALSE}}$
   - *Justification:* h2 = 2h1 is NOT admissible because it overestimates by a factor of 2. A* with non-admissible heuristics is not guaranteed optimal.
2. **(ii) h2 ≤ 2×optimal:** $\boxed{\text{FALSE}}$
   - *Justification:* Since h2 = 2h1 is not admissible, A* with h2 is not guaranteed to find optimal solutions. The solution cost is not bounded by 2×optimal and could be much worse.
3. **(iii) h2 graph search optimal:** $\boxed{\text{FALSE}}$
   - *Justification:* Graph search vs tree search doesn't change the admissibility requirement. Since h2 is not admissible, the solution is not guaranteed optimal.

Q1c: Making Heuristics Admissible and Consistent (1 point)

Graph: S(h=6)→A(h=5)→B(h=1)→C(h=2)→G(h=0), B→D(h=0)→G, C→G
Edges: S→A(1), A→B(3), B→C(1), B→D(5), C→G(4), D→G(2)

Answer:

State to change: $\boxed{B}$
Range: $\boxed{2 \leq h(B) \leq 3}$

Justification: Check consistency for each edge:

- S→A: 6 ≤ 1+5 ✓
- A→B: 5 ≤ 3+1 ❌ (5 ≤ 4 is false)
- B→C: 1 ≤ 1+2 ✓
- B→D: 1 ≤ 5+0 ✓
- C→G: 2 ≤ 4+0 ✓
- D→G: 0 ≤ 2+0 ✓

The constraint A→B is violated. To fix:

- From A→B: h(B) ≥ h(A) - c(A,B) = 5 - 3 = 2
- From B→C: h(B) ≤ c(B,C) + h(C) = 1 + 2 = 3
- Therefore: 2 ≤ h(B) ≤ 3

Q2: Practical A* Implementation (2 points)

Graph: S(h=7)→A(h=5), S→B(h=7), A→B, A→C(h=4), B→C, C→D(h=1)→G(h=0), C→G
Edges: S→A(3), S→B(1), A→B(2), A→C(2), B→C(3), C→D(4), C→G(4), D→G(1)

Q2a: Heuristic Properties (0.5 points)

Answer: $\boxed{\text{Both consistent and admissible}}$

Justification: Check consistency for each edge:

- S→A: 7 ≤ 3+5 ✓
- S→B: 7 ≤ 1+7 ✓
- A→B: 5 ≤ 2+7 ✓
- A→C: 5 ≤ 2+4 ✓
- B→C: 7 ≤ 3+4 ✓
- C→D: 4 ≤ 4+1 ✓
- C→G: 4 ≤ 4+0 ✓
- D→G: 1 ≤ 1+0 ✓

All consistency constraints satisfied. Since consistency implies admissibility, the heuristic is both consistent and admissible.

Q2b: A* Expansion Order (0.5 points)

Answer: $\boxed{S(1), A(2), B(3), C(4), D(\text{not expanded}), G(5)}$

Justification: A* execution with f(n) = g(n) + h(n):

1. Start: S (f=0+7=7)
2. Expand S: Add A(f=3+5=8), B(f=1+7=8) → Expand A (alphabetical tie-break: A < B)
3. Expand A: Add C(f=5+4=9), B already in frontier with f=8 → Expand B (f=8)
4. Expand B: Add C(f=4+4=8) → Expand C (f=8)
5. Expand C: Add D(f=8+1=9), G(f=8+0=8) → Expand G (f=8)

Q2c: Path Returned (0.5 points)

Answer: $\boxed{S \to B \to C \to G}$

Justification: Following the A* execution above, the path to goal G is S→B→C→G.

Q2d: Admissible Heuristic Combinations (0.5 points)

Answer: $\boxed{\frac{1}{2}(h_A), \frac{1}{2}(h_B), \frac{1}{2}(h_A + h_B), \max(h_A, h_B), \min(h_A, h_B)}$

Justification:

- **½(hA), ½(hB):** Scaling by factor ≤ 1 preserves admissibility
- **½(hA+hB):** Average of admissible heuristics is admissible
- **max(hA,hB), min(hA,hB):** Maximum and minimum of admissible heuristics are admissible
- **hA+hB:** NOT admissible (overestimates)
- **hA×hB:** NOT admissible (product can overestimate)

🎯 FINAL ANSWER SUMMARY

📦 SUBMISSION ANSWERS

Q1a (3.5 points):

❌ ❌ ✅ ❌ ✅ ❌ ✅

Q1b (1.5 points):

❌ ❌ ❌

Q1c (1 point):

State: B
Range: 2 ≤ h(B) ≤ 3

Q2a:

✅ **Both consistent and admissible**

Q2b:

S(1), A(2), B(3), C(4), D(not expanded), G(5)

Q2c:

✅ **S→B→C→G**

Q2d:

✅ **½(hA), ½(hB), ½(hA+hB), max(hA,hB), min(hA,hB)**
