Fiedler Cut Applied to Different Color Spaces: RGB & CMY

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Presentation Overview

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- Background
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Original Question

"Can the same image, represented in two different color spaces, lead to a different Fiedler cut?"

Motivation

- We talked about different distance functions, but not different color spaces for image segmentation.
- Distance is somewhat arbitrary, just like color spaces!
- Claim: RGB color space is conventional because of black screens.

Distance Functions	Color Spaces
Manhattan Cosine	Red, Green, Blue
Euclidean	Cyan, Magenta, Yellow
Haversine Hamming	Black & White
	UV Color Spaces

Background - RGB & CMY

- **RGB** = Red, Green, Blue. Black background (screen).
- **CMY** = Cyan, Magenta, Yellow. White background (paper).
- Both use values from 0.0 to 1.0.
- Have a "one to one" mapping with each other:

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Original Question

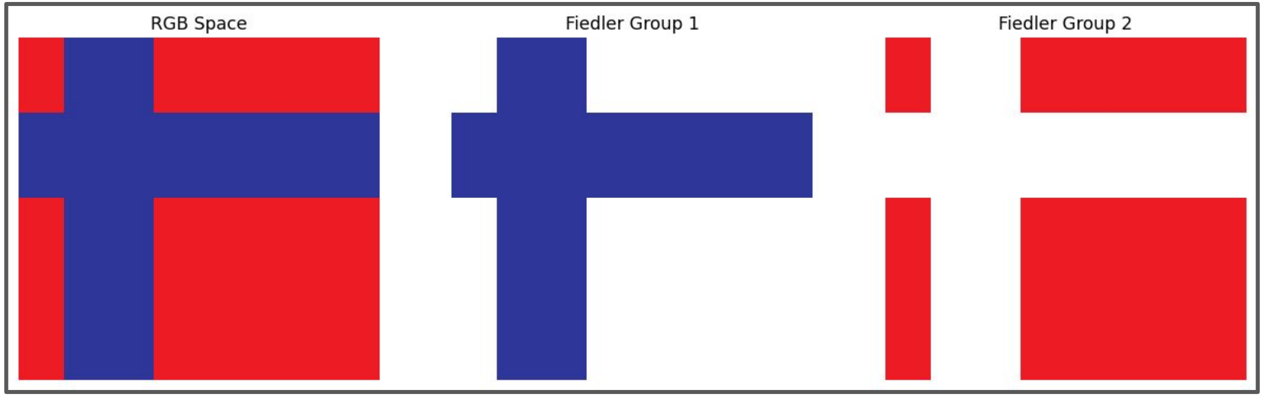
"Can the same image, represented in two different color spaces, lead to a different Fiedler cut?"

Refined Question

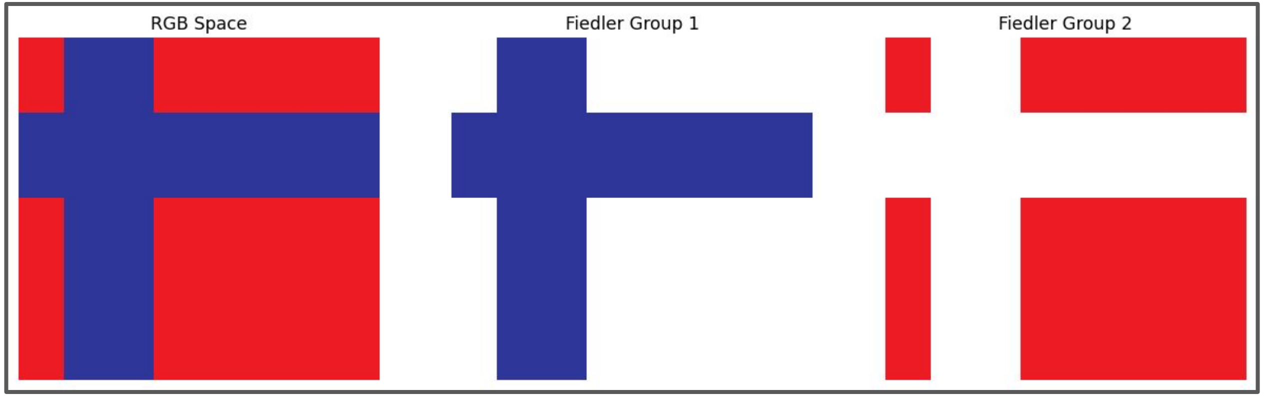
"Can the same image, represented in RGB and CMY color spaces, lead to a different Fiedler cut with the same distance function?"

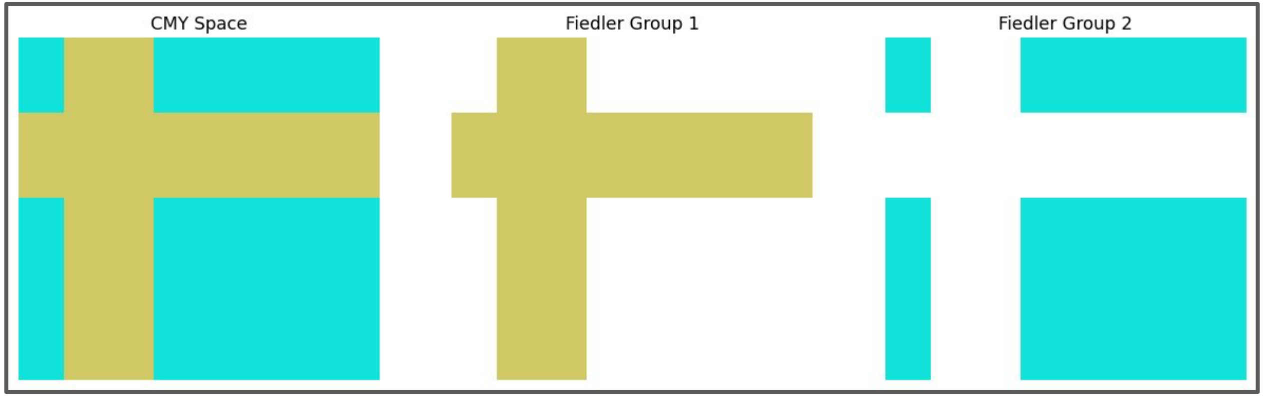
Experiment Setup

- Build a clique graph from Euclidean RGB distances.
- Apply a Gaussian Weighting Function, so that dissimilar pixels have the lowest edge weights. (Fiedler Cut = Minimum Cut).
- Repeat with RGB->CMY. Use Euclidean CMY distance!

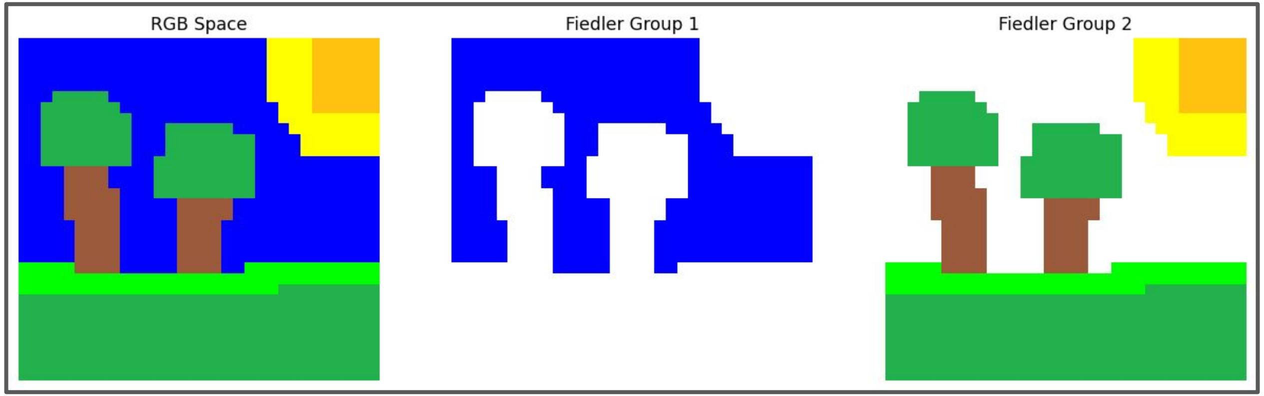


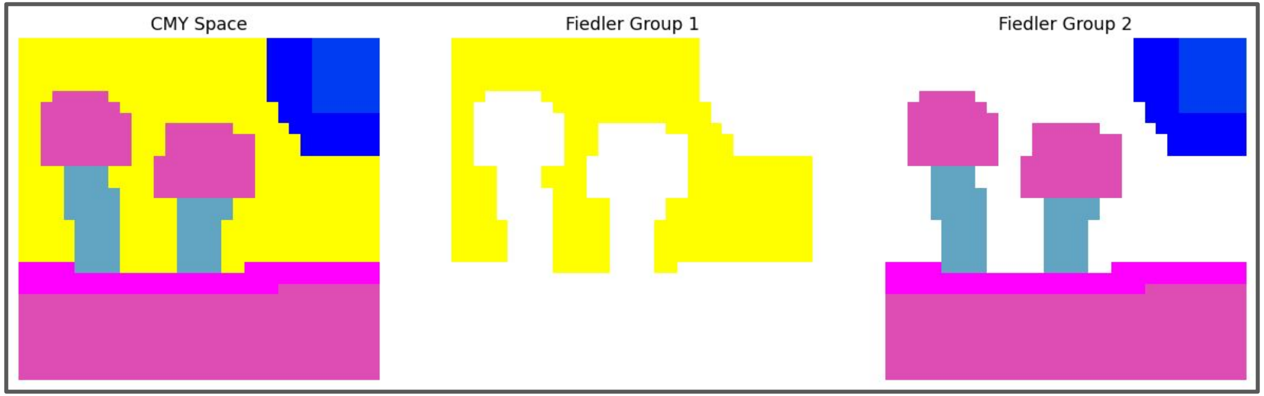
Experiment: (Black + RGB) = (White - CMY)



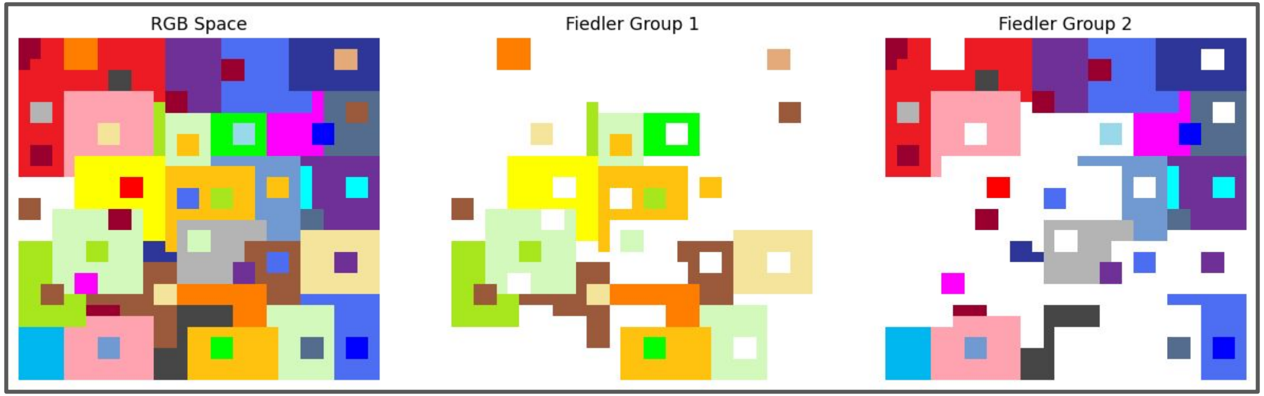


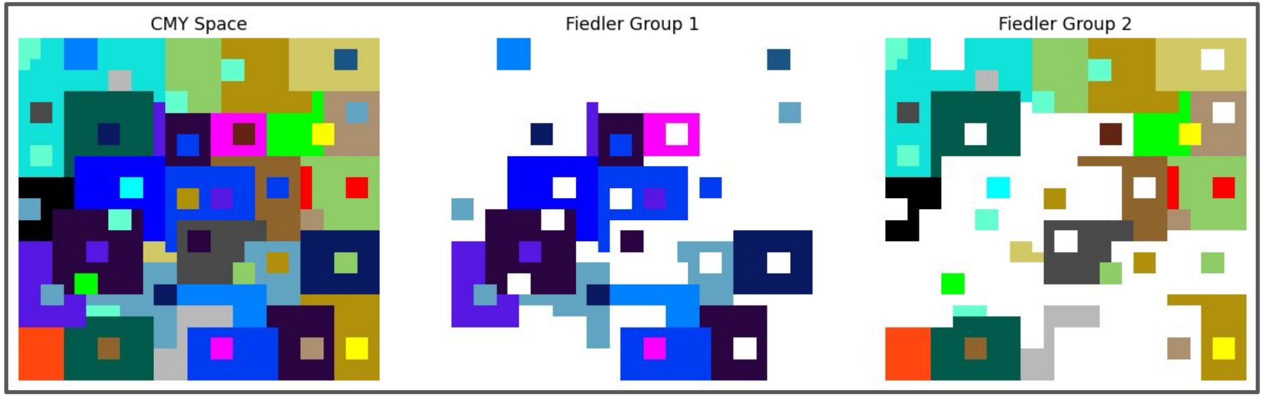
Experiment: (Black + RGB) = (White - CMY)



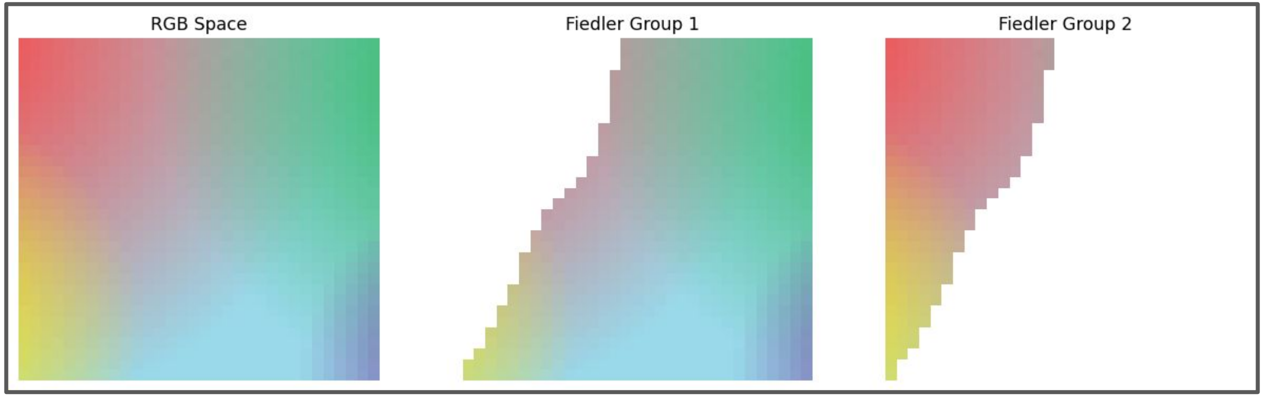


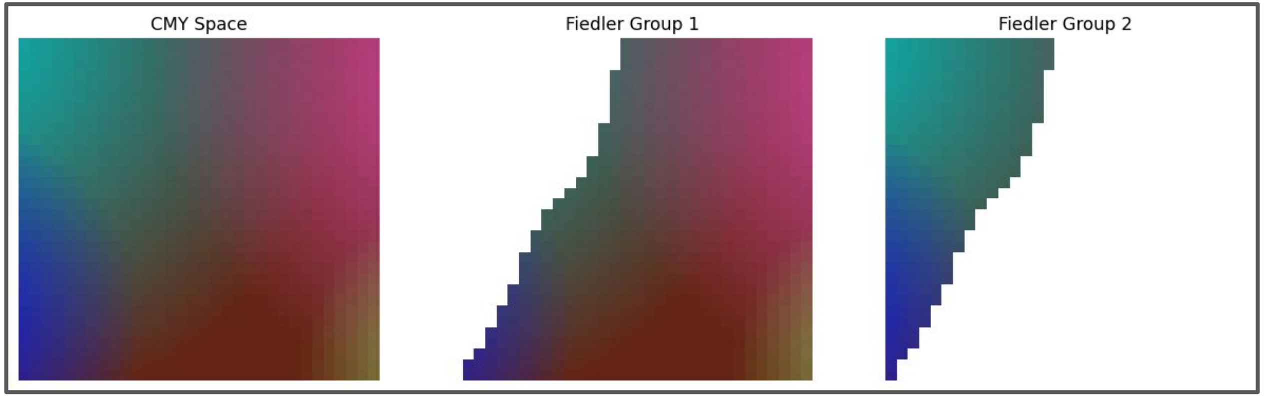
Experiment: (Black + RGB) = (White - CMY)





Experiment: (Black + RGB) = (White - CMY)





Results

- When using the Euclidean Distance Function, RGB and CMY have the same Fiedler Cut!

```
Distance(RGB_1, RGB_2) == Distance(CMY_1, CMY_2)

C = 1.0 - R

M = 1.0 - G

Y = 1.0 - B
```

Proof

$$
\begin{aligned}
d_{RGB} &= \sqrt{(r_1 - r_2)^2 + (g_1 - g_2)^2 + (b_1 - b_2)^2} \\
d_{CMY} &= \sqrt{(c_1 - c_2)^2 + (m_1 - m_2)^2 + (y_1 - y_2)^2} \\
&= \sqrt{\big((1 - r_1) - (1 - r_2)\big)^2 + \big((1 - g_1) - (1 - g_2)\big)^2 + \big((1 - b_1) - (1 - b_2)\big)^2} \\
&= \sqrt{(r_2 - r_1)^2 + (g_2 - g_1)^2 + (b_2 - b_1)^2} \\
&= \sqrt{(r_1 - r_2)^2 + (g_1 - g_2)^2 + (b_1 - b_2)^2} \\
d_{CMY} &= d_{RGB}
\end{aligned}
$$

Key Takeaways

- *"Can the same image, represented in two different color spaces, lead to a different Fiedler cut?"*
- RGB images and CMY images have the same Fiedler Cut when using a Euclidean distance function.
- A **different color space counter-example** could exist!
    - RGB -> black and white (not 1:1 mapping)
    - RGB -> alternate color space (1:1 mapping)