

Math 302 HW3
Section IV 2024
Duke Kunshan University

Problem 1. Consider the function $f(x) = \exp(x)$ in $[0, 1]$. Calculate 6 equispaced points $(x_0, y_0), \dots, (x_5, y_5)$. Use the polynomial as the approximation to do followings.

- Calculate the error $\max_i |f(y_i) - p_n(y_i)|$ using 101 points of equispaced y_i .
- Calculate $p'_n(x)$ at $\{x_i\}$ using a differentiation matrix.
- Calculate $\int_0^{x_i} p_n(y) dy$ using an integral matrix.

Problem 2. As we did in class, we can construct the Taylor series by assuming the polynomial and matching all coefficients. Rewrite the entire process in matrix language in a similar manner to polynomial interpolation, differentiation, and integration processes. Use the Taylor series to solve Problem 1 and compare the results with those obtained by polynomial interpolation. Comment on the accuracy.

Problem 3. Numerically verify problem 3.42 in the textbook. You don't have to prove them.