Math 302 HW4
Section IV 2024
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Problem 1 (Analytical Problem). *Suppose we calculate the integration matrix* $\mathbf{S}$ *for the degree* n *polynomial* $p_n(x)$ *with* $x \in [0,1]$ *using equispaced points* $x_i = \frac{i}{n}$ *such that* $\mathbf{S}\begin{bmatrix} p_n(x_0) \\ p_n(x_1) \\ \vdots \\ p_n(x_n) \end{bmatrix} = \begin{bmatrix} \int_0^{x_0} p_n(t)\ dt \\ \int_0^{x_1} p_n(t)\ dt \\ \vdots \\ \int_0^{x_n} p_n(t)\ dt \end{bmatrix}$. *What is the rank of the matrix* $\mathbf{S}$*? If we calculate the integration matrix* $\mathbf{S}$ *using* $x_1, \ldots, x_n$ *without* $x_0 = 0$*, would the rank changes?*

Problem 2 (Textbook 3.24, Analytical Problem). *Textbook 3.24.*

Problem 3 (Textbook 4.10, Analytical Problem).

1. *Produce the linear Taylor polynomials to* $f(x) = \ln(x)$ *on* $1 \le x \le 2$*, expanding about* $x_0 = \frac{3}{2}$.
2. *Produce the linear minimax approximation to* $f(x) = \ln(x)$ *on* $[1,2]$*. Compare it with the Taylor approximation. (See the first example in Sec 4.2)*

Problem 4 (Textbook 3.23, Coding Problem). *Write the code using polynomial interpolation using equispaced points to visualize the Runge Phenomenon (Similar to page 58 in the slide). More specifically, interpolate the function* $f(x) = \frac{1}{1+x^2}$ *for* $-5 \le x \le 5$ *and increase the degree of the polynomial and comment your observations.*

Problem 5 (Coding Problem). *Consider the function* $f(x) = e^{\cos(x)}$ *in* $[-\pi, \pi]$*. Construct the Fourier series with* 10 *points to approximate* $f(x)$ *using these points. Compare the results by using interpolation (setting up the linear system and match function values at equispaced points) and integrations (calculate coefficients using the Trapezoidal's rule). Details are included in the slides (summaries on page 43 and 44). Calculate the maximum error using 1001 equispaced points.*