Math 302 HW5
Section IV 2024
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Problem 1 (Textbook 4.12, Analytical Problem)**.** *Find the linear least squares approximation to* $f(x) = \ln(x)$ *on* $[1, 2]$.

Problem 2 (Analytical Problem)**.** *Consider the Chebyshev series* $f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k T_k(x)$*. Show that* $2T_n(x) = \frac{T'_{n+1}(x)}{n+1} - \frac{T'_{n-1}(x)}{n-1}$.

Problem 3. *Consider the derivative of the Chebyshev series* $f'(x) = \frac{b_0}{2} + \sum_{k=1}^{\infty} b_k T_k(x)$*. Show that*

$$b_k = 2 \sum_{\substack{j=k+1 \\ j+k \ odd}}^{\infty} j \cdot a_j.$$

(Hint: Use the result of the Problem 2)

Problem 4. *Consider the integral of the Chebyshev series* $\int_{-1}^{x} f(t)\ dt = \frac{c_0}{2} + \sum_{k=1}^{\infty} c_k T_k(x)$*. Show that*

$$c_k = \frac{1}{2k}(a_{k-1} - a_{k+1}),\ k \geq 1,$$
$$c_0 = 2(c_1 - c_2 + c_3 - \dots)$$

(Hint: Use the result of problem 2)

Problem 5 (Coding Problem)**.** *Write the code using polynomial interpolation using Chebyshev points. More specifically, interpolate the function* $f(x) = \frac{1}{1+x^2}$ *for* $-5 \leq x \leq 5$ *and increase the degree of the polynomial.* ***Compare the results using equispace points (Problem 4 in HW4) and comment.***

Chebyshev points:

- *For* $k = 1, \dots, n,\ x_k = \cos\left(\frac{\pi\left(k - \frac{1}{2}\right)}{n}\right)$
- *For* $k = 0, \dots, n,\ x_k = \cos\left(\frac{\pi(2k+1)}{2n+2}\right)$

Problem 6 (Coding Problem)**.** *Consider the function* $f(x) = \exp(x)$ *in* $[0, 1]$*. Calculate 6 Chebyshev points for the interpolation.* ***Compare your results with the case using equispaced points (Problem 1 in HW3) and comment.***

- *Calculate the error* $\max_i |f(y_i) - p_n(y_i)|$ *using 101 points of equispaced* y_i.
- *Calculate* $p'_n(x)$ *at* $\{x_i\}$ *using a differentiation matrix.* ***Calculate the inf norm of the error.***
- *Calculate* $\int_0^{x_i} p_n(y)\ dy$ *using an integral matrix.* ***Calculate the inf norm of the error.***