Math 302 HW6
Section IV 2022
Duke Kunshan University

Problem 1 (Textbook 4.20(c), Analytical Problem)**.** *Find orthogonal polynomials ψ_0, ψ_1, ψ_2 for the weight function $w(x) = \sqrt{1-x^2}$ on $[-1, 1]$.*

Problem 2 (Textbook 5.9, Analytical Problem)**.** *Let $p_2(x)$ be the quadratic polynomial interpolating $f(x)$ at $x = 0, h, 2h$. Use this to derive a numerical integration formula I_h for $I = \int_0^{3h} f(x)\,dx$. Use a Taylor series expansion of $f(x)$ to show*

$$I - I_h = \frac{3}{8}h^4 f^{(3)}(0) + \mathcal{O}(h^5).$$

Problem 3 (Textbook 5.10, Analytical Problem)**.** *Provide the error formula for the midpoint formula. You can either follow HW 5.10 or you can provide other derivations with similar results (e.g., assume $\max_{x\in(a,b)} |f''(x)| \le K$).*

Problem 4 (Analytical Problem)**.** *For a function of two variables $f(x, y)$, suppose we approximate its partial derivative using the formula $f_{xy}(x, y) \approx af(x+\Delta x, y+\Delta y) + bf(x+\Delta x, y-\Delta y) + cf(x-\Delta x, y+\Delta y) + df(x-\Delta x, y-\Delta y)$. Determine a, b, c, d such that the approximation is exact for $f(x, y) = 1, x, y, x^2, y^2, xy$.*

Problem 5 (Textbook 5.1, Coding Problem)**.** *Write a program to evaluate $I = \int_a^b f(x)\,dx$ using the trapezoidal rule with n subdivisions, calling the result I_n. Use the program to calculate the following integrals with $n = 4, 8, 16$.*

1. $\int_0^1 e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}erf(1)$.
2. $\int_0^{\pi} e^x \cos(4x)\,dx = \frac{1}{17}(-1 + 4e\sin(4) + e\cos(4))$.

Verify the rate of convergence of I_n to I.

Problem 6 (Textbook 5.14, Coding Problem)**.** *Apply Gauss-Legendre quadrature to the integrals in Problem 5. Compare the results with those for the Trapezoidal methods. (no need to compute the Simpson methods, use the Gaussian-Legendre results from the textbook or online.)*