HW5

Problem 1 (Analytical problem). *3.2*

Problem 2 (Analytical problem). *3.3*

Problem 3 (Analytical problem,optional). *3.6(i)*

Problem 4 (Analytical problem). *Consider a normal random variable $X \sim \mathcal{N}(0,t)$ and a scaled random walk $W^{(n)}(t) = \frac{1}{\sqrt{n}} \sum_{j=1}^{nt} X_j$, whereas $X_j = \begin{cases} 1, & p = 0.5, \\ -1, & p = 0.5 \end{cases}$. Show the limit of the scaled random walk is the normal by comparing their moment-generating function.*

Problem 5 (Analytical problem). *Suppose the log price of a stock follows the Brownian motion $X_t = \mu t + \sigma dW_t$. We know the standard deviation of daily log returns is 0.01. Based on the previous data, we calculated the daily mean as 0.001. However, the market has been really bad this year, so the total log return is 0. In this year's log return, we think it's decayed. Answer whether this year's mean daily log return has decayed using hypothesis testing with the significance level $\alpha = 5\%$.*

Problem 6 (Coding problem). *Download the S&P 500 data of 2021. Use the hypothesis testing based on the Kolmogorov-Smirnov test to conclude whether the drifted Brownian motion is a goodfit for the log return dynamics.*