Math 411 HW1
Section IV 2025
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Problem 1. *Calculate the expectation and variance of a normal random variable.*

Problem 2. *Consider a simple class of model of the form*

$$y_t = r + y_{t-1} + c_0\epsilon_t + c_{-1}\epsilon_{t-1} + \cdots + c_{-p}\epsilon_{t-p}, \tag{1}$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$. We want to use this model to mimic the stock market. Here, y_t represent the log-price, r represents the expected return, and ϵ_t represents the impact from the news and are independent from each other. We define the residual as $z_t = y_t - r - y_{t-1}$.

1. *Calculate the expectation of log return $\mathbb{E}[y_t - y_{t-1}]$.*
2. *Suppose $y_t = r + y_{t-1} + \epsilon_t$ and $y_0 = 0$. Suppose we are interested in the event when $y_1 > \eta$ for a threshold η to represent the scenario that the company has received good news. Calculate the conditional expectation of $\mathbb{E}[z_2|y_1 > \eta]$.*
3. *Suppose now $y_t = r + y_{t-1} + \epsilon_t + c_{-1}\epsilon_{t-1}$, $y_0 = 0$, and $\epsilon_0 = 0$. Calculate the conditional expectation of $\mathbb{E}[z_2|y_1 > \eta]$ (You can use the $\phi(x)$ to denote the cumulative distribution function of a standard normal distribution in the answer).*
4. *(Optional) Suppose now $y_t = r + y_{t-1} + \epsilon_t + c_{-1}\epsilon_{t-1}$, $y_0 = 0$, and $\epsilon_0 = 0$. Calculate the conditional expectation of $\mathbb{E}[z_3|y_2 > \eta]$ (You can use the $\phi(x)$ to denote the cumulative distribution function of a standard normal distribution in the answer).*

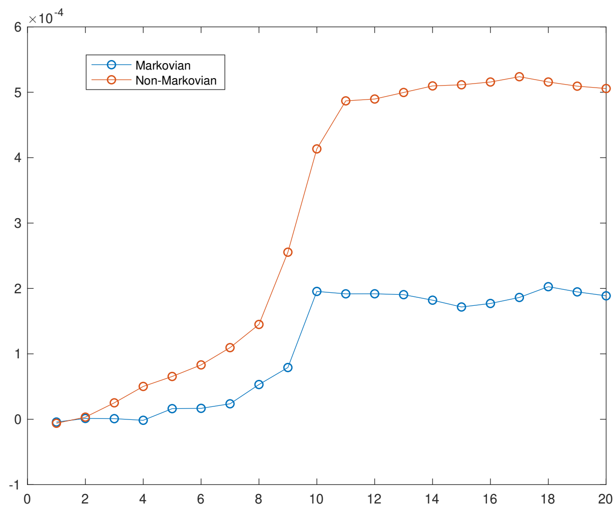
Problem 3 (Coding)**.** *Now consider we have a collection of log prices for different companies*

$$y_{i,t} = r_i + y_{i,t-1} + c_0\epsilon_{i,t} + c_{-1}\epsilon_{i,t} + \cdots + c_{-p}\epsilon_{i,t-p}, \ i = 1, \ldots, M, \tag{2}$$

whereas $y_{i,t}$ represent the log-price, $r_{i,t}$ represents the expected return, and $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$ represents the impact from the news for the ith company and are independent from each other. We define the residual as $z_{i,t} = y_{i,t} - r_i - y_{i,t-1}$. For r_i and σ_i, you can randomly select values as long as they are positive. (As an example of demonstration, I used $r_i \sim \frac{0.05}{252}(1+\mathcal{N}(0,1))$ and $\sigma_i \sim \frac{0.05}{252} \cdot |\mathcal{N}(0,1)|$). Use the Monte Carlo simulation to generate a sequence of N (e.g., 10^4) log prices for M (e.g., 10^3) companies. Consider the event that the company keep receives the good news, that is given y_i, we find τ_i s.t. $y_{\tau_i} - y_0 > \eta$, whereas η is the threshold (e.g., $\log(2)$). We wish to estimate the impact of the news by calculating $w_k = \frac{\sum_{j|y_{j,\tau_i}>\eta} z_{j,k}}{\sum_{j|y_{j,\tau_i}>\eta} 1}$. Then we look at $v_l = \sum_{k=1}^{l} w_k$ to test the Markovian. For convenience, we can plot v_l from $\tau - m$ to $\tau + m$ (e.g., $m = 10$).

1. *Use this model together with Monte Carlo simulation to generate a result that indicates the Markovian behavior. .*
2. *Use this model together with Monte Carlo simulation to generate a result that indicates the Non-Markovian behavior.*

Here is an example of what we should observe


×10^-4
Markovian
Non-Markovian
6
5
4
3
2
1
0
-1
0
2
4
6
8
10
12
14
16
18
20