MATH 411 HW1

Problem 1.

Let $X \sim N(\mu, \sigma^2)$

From the definition of Normal Random Variable, we have:

$$P(X = x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Thus the expectation is:

$$E(X) = \int_{-\infty}^{\infty} xP(X = x)dx$$

Substitute the probability density function of the normal distribution:

$$E(X) = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Make a change of variable: let $u = \frac{x-\mu}{\sigma}$, hence $du = \frac{dx}{\sigma}$ and $dx = \sigma du$:

$$E(X) = \int_{-\infty}^{\infty} (\sigma u + \mu) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2}} \sigma du$$

Separate the integral into two parts:

$$E(X) = \int_{-\infty}^{\infty} \sigma u \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2}} \sigma du + \int_{-\infty}^{\infty} \mu \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2}} \sigma du$$

The first integral is zero because it is the integral of an odd function over a symmetric interval:

$$\int_{-\infty}^{\infty} \sigma u \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2}} \sigma du = 0$$

The second integral simplifies to:

$$\mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2}} \sigma du = \mu \cdot 1 = \mu$$

Thus, the expectation is:

$$E(X) = \mu$$

Thus the variance is:

$$Var(X) = E(X^2) - E(X)^2$$

We need to calculate $E(X^2)$:

$$E(X^2) = \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Using the same substitution as before: $u = \frac{x-\mu}{\sigma}$, $x = \sigma u + \mu$, and $dx = \sigma du$:

$$E(X^2) = \int_{-\infty}^{\infty} (\sigma u + \mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2}} \sigma du$$

Expanding the squared term:

$$E(X^2) = \int_{-\infty}^{\infty} (\sigma^2 u^2 + 2\sigma\mu u + \mu^2) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{u^2}{2}} \sigma du$$

Breaking this into three integrals:

$$E(X^2) = \sigma^2 \int_{-\infty}^{\infty} u^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du + 2\sigma\mu \int_{-\infty}^{\infty} u \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du + \mu^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

The middle integral is zero (odd function over symmetric interval). The last integral equals 1 (total probability). For the first integral, we know that $\int_{-\infty}^{\infty} u^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = 1$ (second moment of standard normal).

Therefore:

$$E(X^2) = \sigma^2 \cdot 1 + 0 + \mu^2 \cdot 1 = \sigma^2 + \mu^2$$

Now we can calculate the variance:

$$Var(X) = E(X^2) - E(X)^2 = \sigma^2 + \mu^2 - \mu^2 = \sigma^2$$

Therefore, for $X \sim N(\mu, \sigma^2)$:

- $E(X) = \mu$
- $Var(X) = \sigma^2$

Problem 2.

Consider a simple class of model of the form

$$y_t = r + y_{t-1} + c_0\epsilon_t + c_{-1}\epsilon_{t-1} + \cdots + c_{-p}\epsilon_{t-p},$$

where $\epsilon_t \sim N(0, \sigma^2)$. We want to use this model to mimic the stock market. Here, y_t represents the log-price, r represents the expected return, and ϵ_t represents the impact from the news and are independent from each other. We define the residual as $z_t = y_t - r - y_{t-1}$.

1. Calculate the expectation of log return $\mathbb{E}[y_t - y_{t-1}]$

The log return is defined as $y_t - y_{t-1}$. From the given model:

$$y_t = r + y_{t-1} + c_0\epsilon_t + c_{-1}\epsilon_{t-1} + \cdots + c_{-p}\epsilon_{t-p}$$

Therefore:

$$\begin{aligned} y_t - y_{t-1} &= r + y_{t-1} + c_0\epsilon_t + c_{-1}\epsilon_{t-1} + \cdots + c_{-p}\epsilon_{t-p} - y_{t-1} \\ &= r + c_0\epsilon_t + c_{-1}\epsilon_{t-1} + \cdots + c_{-p}\epsilon_{t-p} \end{aligned}$$

Taking the expectation:

$$\begin{aligned} \mathbb{E}[y_t - y_{t-1}] &= \mathbb{E}[r + c_0\epsilon_t + c_{-1}\epsilon_{t-1} + \cdots + c_{-p}\epsilon_{t-p}] \\ &= r + c_0\mathbb{E}[\epsilon_t] + c_{-1}\mathbb{E}[\epsilon_{t-1}] + \cdots + c_{-p}\mathbb{E}[\epsilon_{t-p}] \end{aligned}$$

Since $\epsilon_t \sim N(0, \sigma^2)$, we have $\mathbb{E}[\epsilon_t] = 0$ for all t. Therefore:

$$\mathbb{E}[y_t - y_{t-1}] = r$$

Thus, the expectation of the log return is r.

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2. Suppose $y_t = r + y_{t-1} + \epsilon_t$ and $y_0 = 0$. Suppose we are interested in the event when $y_1 > \eta$ for a threshold η to represent the scenario that the company has received good news. Calculate the conditional expectation $\mathbb{E}[z_2|y_1 > \eta]$.

First, let's identify what z_2 is in this context:

$z_t = y_t - r - y_{t-1}$

So $z_2 = y_2 - r - y_1$

From the given model for this part, $y_t = r + y_{t-1} + \epsilon_t$, we have:

$y_1 = r + y_0 + \epsilon_1 = r + 0 + \epsilon_1 = r + \epsilon_1$

$y_2 = r + y_1 + \epsilon_2 = r + (r + \epsilon_1) + \epsilon_2 = 2r + \epsilon_1 + \epsilon_2$

Now we can compute z_2:

$z_2 = y_2 - r - y_1 = 2r + \epsilon_1 + \epsilon_2 - r - (r + \epsilon_1) = \epsilon_2$

So $z_2 = \epsilon_2$

To calculate $\mathbb{E}[z_2|y_1 > \eta]$, we need to find $\mathbb{E}[\epsilon_2|y_1 > \eta]$.

Since $y_1 = r + \epsilon_1$, the condition $y_1 > \eta$ is equivalent to $\epsilon_1 > \eta - r$.

Given that ϵ_1 and ϵ_2 are independent (as stated in the problem), the condition $\epsilon_1 > \eta - r$ does not affect the distribution of ϵ_2. Therefore:

$\mathbb{E}[z_2|y_1 > \eta] = \mathbb{E}[\epsilon_2|y_1 > \eta] = \mathbb{E}[\epsilon_2|\epsilon_1 > \eta - r] = \mathbb{E}[\epsilon_2] = 0$

Thus, the conditional expectation $\mathbb{E}[z_2|y_1 > \eta] = 0$.

3. Suppose now $y_t = r + y_{t-1} + \epsilon_t + c_{-1}\epsilon_{t-1}$, $y_0 = 0$, and $\epsilon_0 = 0$. Calculate the conditional expectation $\mathbb{E}[z_2|y_1 > \eta]$. (You can use the $\phi(x)$ to denote the cumulative distribution function of a standard normal distribution in the answer)

First, let's identify what z_2 is in this context:

$z_t = y_t - r - y_{t-1}$

So $z_2 = y_2 - r - y_1$

From the given model for this part, $y_t = r + y_{t-1} + \epsilon_t + c_{-1}\epsilon_{t-1}$, we have:

$y_1 = r + y_0 + \epsilon_1 + c_{-1}\epsilon_0 = r + 0 + \epsilon_1 + c_{-1} \cdot 0 = r + \epsilon_1$

$y_2 = r + y_1 + \epsilon_2 + c_{-1}\epsilon_1 = r + (r + \epsilon_1) + \epsilon_2 + c_{-1}\epsilon_1 = 2r + \epsilon_2 + (1 + c_{-1})\epsilon_1$

Now we can compute z_2:

$z_2 = y_2 - r - y_1 = 2r + \epsilon_2 + (1 + c_{-1})\epsilon_1 - r - (r + \epsilon_1) = \epsilon_2 + c_{-1}\epsilon_1$

To calculate $\mathbb{E}[z_2|y_1 > \eta]$, we need to find $\mathbb{E}[\epsilon_2 + c_{-1}\epsilon_1|y_1 > \eta]$.

Since $y_1 = r + \epsilon_1$, the condition $y_1 > \eta$ is equivalent to $\epsilon_1 > \eta - r$.

We can split this expectation:
$\mathbb{E}[z_2|y_1 > \eta] = \mathbb{E}[\epsilon_2 + c_{-1}\epsilon_1|y_1 > \eta] = \mathbb{E}[\epsilon_2|y_1 > \eta] + c_{-1}\mathbb{E}[\epsilon_1|y_1 > \eta]$

Since ϵ_2 is independent of ϵ_1 (and thus of y_1), we have: $\mathbb{E}[\epsilon_2|y_1 > \eta] = \mathbb{E}[\epsilon_2] = 0$

For the second term, we need to calculate $\mathbb{E}[\epsilon_1|\epsilon_1 > \eta - r]$.

For a normal random variable $X \sim N(0, \sigma^2)$, the conditional expectation $\mathbb{E}[X|X > a]$ can be derived as follows:

Step 1: By definition of conditional expectation: $\mathbb{E}[X|X > a] = \frac{\int_a^\infty x f_X(x)dx}{P(X>a)}$

Step 2: For a normal distribution with PDF $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x)^2}{2\sigma^2}}$:

$\mathbb{E}[X|X > a] = \frac{\int_a^\infty x \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x^2}{2\sigma^2}}dx}{1-\phi(a/\sigma)}$

Step 3: Solving the integral in the numerator using integration by parts:

$\mathbb{E}[X|X > a] = \sigma\frac{\frac{1}{\sqrt{2\pi}}e^{-\frac{(a/\sigma)^2}{2}}}{1-\phi(a/\sigma)}$

where $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ is the standard normal probability density function and $\phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$ is the standard normal cumulative distribution function.

In our case, $\epsilon_1 \sim N(0, \sigma^2)$, so: $\mathbb{E}[\epsilon_1|\epsilon_1 > \eta - r] = \sigma \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{((\eta-r)/\sigma)^2}{2}}}{1-\phi((\eta-r)/\sigma)}$

Therefore: $\mathbb{E}[z_2|y_1 > \eta] = 0 + c_{-1} \cdot \sigma \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{((\eta-r)/\sigma)^2}{2}}}{1-\phi((\eta-r)/\sigma)} = c_{-1} \cdot \sigma \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{((\eta-r)/\sigma)^2}{2}}}{1-\phi((\eta-r)/\sigma)}$

Thus, the conditional expectation $\mathbb{E}[z_2|y_1 > \eta] = c_{-1} \cdot \sigma \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{((\eta-r)/\sigma)^2}{2}}}{1-\phi((\eta-r)/\sigma)}$.

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4. Suppose now $y_t = r + y_{t-1} + \epsilon_t + c_{-1}\epsilon_{t-1}$, $y_0 = 0$, and $\epsilon_0 = 0$. Calculate the conditional expectation of $\mathbb{E}[z_3|y_2 > \eta]$ (You can use the $\phi(x)$ to denote the cumulative distribution function of a standard normal distribution in the answer).

First, let's calculate z_3 and express it in terms of the error terms.

From the given model, we have:
$y_1 = r + y_0 + \epsilon_1 + c_{-1}\epsilon_0 = r + 0 + \epsilon_1 + c_{-1} \cdot 0 = r + \epsilon_1$

$y_2 = r + y_1 + \epsilon_2 + c_{-1}\epsilon_1 = r + (r + \epsilon_1) + \epsilon_2 + c_{-1}\epsilon_1 = 2r + \epsilon_2 + (1 + c_{-1})\epsilon_1$

$y_3 = r + y_2 + \epsilon_3 + c_{-1}\epsilon_2 = r + [2r + \epsilon_2 + (1 + c_{-1})\epsilon_1] + \epsilon_3 + c_{-1}\epsilon_2 = 3r + \epsilon_3 + ($

Now we can compute z_3:
$z_3 = y_3 - r - y_2 = 3r + \epsilon_3 + (1 + c_{-1})\epsilon_2 + (1 + c_{-1})\epsilon_1 - r - [2r + \epsilon_2 + (1 + c_{-1})$
$z_3 = \epsilon_3 + c_{-1}\epsilon_2$

To calculate $\mathbb{E}[z_3|y_2 > \eta]$, we need to find $\mathbb{E}[\epsilon_3 + c_{-1}\epsilon_2|y_2 > \eta]$.

Since $y_2 = 2r + \epsilon_2 + (1 + c_{-1})\epsilon_1$, the condition $y_2 > \eta$ is more complex than in the previous part.

We can split the expectation:
$\mathbb{E}[z_3|y_2 > \eta] = \mathbb{E}[\epsilon_3 + c_{-1}\epsilon_2|y_2 > \eta] = \mathbb{E}[\epsilon_3|y_2 > \eta] + c_{-1}\mathbb{E}[\epsilon_2|y_2 > \eta]$

Since ϵ_3 is independent of ϵ_2 and ϵ_1 (and thus of y_2), we have:
$\mathbb{E}[\epsilon_3|y_2 > \eta] = \mathbb{E}[\epsilon_3] = 0$

For the second term, we need to calculate $\mathbb{E}[\epsilon_2|y_2 > \eta]$.

The condition $y_2 > \eta$ can be rewritten as: $2r + \epsilon_2 + (1 + c_{-1})\epsilon_1 > \eta$,
$\epsilon_2 > \eta - 2r - (1 + c_{-1})\epsilon_1$

This is a conditional expectation where the condition itself depends on another random variable ϵ_1. However, since ϵ_2 is independent of ϵ_1, we can treat this as a simple truncated normal distribution.

For a normal random variable $X \sim N(0, \sigma^2)$, the conditional expectation $\mathbb{E}[X|X > a]$ is: $\mathbb{E}[X|X > a] = \sigma \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{(a/\sigma)^2}{2}}}{1-\phi(a/\sigma)}$

where $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ is the standard normal probability density function and $\phi(x)$ is the standard normal cumulative distribution function.

In our case, with $a = \eta - 2r - (1 + c_{-1})\epsilon_1$, we have:

$\mathbb{E}[\epsilon_2|y_2 > \eta] = \sigma \frac{\frac{1}{\sqrt{2\pi}}e^{-\frac{((\eta-2r-(1+c_{-1})\epsilon_1)/\sigma)^2}{2}}}{1-\phi((\eta-2r-(1+c_{-1})\epsilon_1)/\sigma)}$

However, this expression still contains ϵ_1, which is a random variable. The correct approach is to recognize that the condition $y_2 > \eta$ depends on both ϵ_1 and ϵ_2, so we cannot simply apply the formula for a truncated normal distribution directly.

Since $y_2 = 2r + \epsilon_2 + (1 + c_{-1})\epsilon_1$, the condition $y_2 > \eta$ is equivalent to $\epsilon_2 > \eta - 2r - (1 + c_{-1})\epsilon_1$.

Given that $\epsilon_2 \sim N(0, \sigma^2)$ and is independent of ϵ_1, we have:
$\mathbb{E}[\epsilon_2|y_2 > \eta] = \mathbb{E}[\epsilon_2|\epsilon_2 > \eta - 2r - (1 + c_{-1})\epsilon_1]$

For a standard normal random variable truncated at a point, we have the formula:
$\mathbb{E}[\epsilon_2|\epsilon_2 > \eta - 2r - (1 + c_{-1})\epsilon_1] = \sigma \frac{\phi'((\eta-2r-(1+c_{-1})\epsilon_1)/\sigma)}{1-\phi((\eta-2r-(1+c_{-1})\epsilon_1)/\sigma)}$

Where $\phi'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ is the standard normal PDF.

Therefore: $\mathbb{E}[z_3|y_2 > \eta] = 0 + c_{-1} \cdot \sigma \frac{\phi'((\eta-2r)/\sigma)}{1-\phi((\eta-2r)/\sigma)} = c_{-1} \cdot \sigma \frac{\phi'((\eta-2r)/\sigma)}{1-\phi((\eta-2r)/\sigma)}$

Thus, the conditional expectation $\mathbb{E}[z_3|y_2 > \eta] = c_{-1} \cdot \sigma \frac{\phi'((\eta-2r)/\sigma)}{1-\phi((\eta-2r)/\sigma)}$.

Problem 3.

Now consider we have a collection of log prices for different companies

$$y_{i,t} = r_i + y_{i,t-1} + c_0\epsilon_{i,t} + c_{-1}\epsilon_{i,t-1} + \cdots + c_{-p}\epsilon_{i,t-p}, \quad i = 1, \ldots, M,$$

where $y_{i,t}$ represents the log-price, r_i represents the expected return, and $\epsilon_{i,t} \sim N(0, \sigma_i^2)$ represents the impact from the news for the i th company, and these impacts are independent from each other. We define the residual as $z_{i,t} = y_{i,t} - r_i - y_{i,t-1}$. For r_i and σ_i, you can randomly select values as long as they are positive. (As an example for demonstration, I used $r_i \sim 0.05/252 * (1 + N(0, 1))$ and $\sigma_i \sim 0.05/252 * |N(0, 1)|$). Use the Monte Carlo simulation to generate a sequence of N (e.g., 10^4) log prices for M (e.g., 10^3) companies. Consider the event that the company keeps receiving good news, that is, given y_i, we find τ_i such that $y_{\tau_i} - y_0 > \eta$, where η is the threshold (e.g., $\log(2)$). We wish to estimate the impact of the news by calculating $w_k = \sum_{j|y_{j,\tau_i}>\eta} z_{j,k} / \sum_{j|y_{j,\tau_i}>\eta} 1$. Then we look at $v_l = \sum_{k=1}^{l} w_k$ to test the

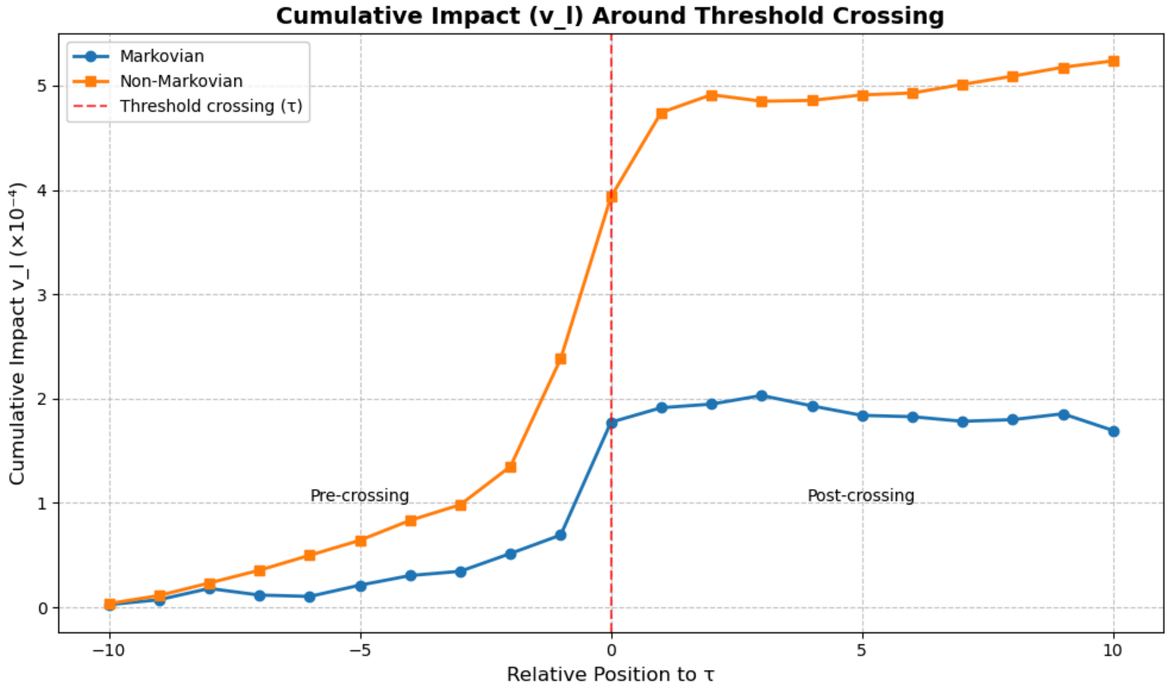
Markovian property. For convenience, we can plot v_l from $\tau - m$ to $\tau + m$ (e.g., $m = 10$).

1. Use this model together with Monte Carlo simulation to generate a result that indicates the Markovian behavior.

2. Use this model together with Monte Carlo simulation to generate a result that indicates the Non-Markovian behavior.

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
np.random.seed(42)

# Parameters
M = 1000  # Number of companies
N = 10000  # Number of time steps
eta = np.log(2)  # Threshold
m = 10  # Window size for plotting

r_i = 0.05/252 * (1 + np.random.randn(M))
sigma_i = 0.05/252 * np.abs(np.random.randn(M))
y = np.zeros((M, N))

# Generate innovations (news impacts)
epsilon = np.random.normal(0, 1, (M, N+10))  # Extra buffer for lagged te
for i in range(M):
    epsilon[i, :] *= sigma_i[i]  # Scale by company-specific volatility

# Case 1: Markovian behavior (p=0)
c_markov = [1, 0, 0]  # Only c_0 = 1, others are 0
y_markov = np.zeros((M, N))
for t in range(1, N):
    y_markov[:, t] = r_i + y_markov[:, t-1] + c_markov[0] * epsilon[:, t]
z_markov = np.zeros_like(y_markov)
for t in range(1, N):
    z_markov[:, t] = y_markov[:, t] - r_i - y_markov[:, t-1]
tau_markov = np.zeros(M, dtype=int)
for i in range(M):
    exceeds = np.where(y_markov[i, :] - y_markov[i, 0] > eta)[0]
    tau_markov[i] = exceeds[0] if len(exceeds) > 0 else N-1
companies_exceed = np.where(np.array([y_markov[i, tau_markov[i]] - y_mark
w_markov = np.zeros(2*m+1)
v_markov = np.zeros(2*m+1)
for l in range(2*m+1):
    k = l - m  # Relative position to tau
    w_sum = 0
    count = 0
    for j in companies_exceed:
        tau = tau_markov[j]
        if 0 <= tau+k < N:
            w_sum += z_markov[j, tau+k]
            count += 1
    w_markov[l] = w_sum / max(1, count)
    if l > 0:
        v_markov[l] = v_markov[l-1] + w_markov[l]
    else:
        v_markov[l] = w_markov[l]
```

```python
# Case 2: Non-Markovian behavior (p>0)
c_non_markov = [1, 0.7, 0.2]  # c_0 = 1, c_-1 = 0.7, c_-2 = 0.2
y_non_markov = np.zeros((M, N))
for t in range(1, N):
    y_non_markov[:, t] = r_i + y_non_markov[:, t-1] + c_non_markov[0] * e
    for p in range(1, len(c_non_markov)):
        if t-p >= 0:
            y_non_markov[:, t] += c_non_markov[p] * epsilon[:, t-p]
z_non_markov = np.zeros_like(y_non_markov)
for t in range(1, N):
    z_non_markov[:, t] = y_non_markov[:, t] - r_i - y_non_markov[:, t-1]
tau_non_markov = np.zeros(M, dtype=int)
for i in range(M):
    exceeds = np.where(y_non_markov[i, :] - y_non_markov[i, 0] > eta)[0]
    tau_non_markov[i] = exceeds[0] if len(exceeds) > 0 else N-1
companies_exceed = np.where(np.array([y_non_markov[i, tau_non_markov[i]]
w_non_markov = np.zeros(2*m+1)
v_non_markov = np.zeros(2*m+1)
for l in range(2*m+1):
    k = l - m  # Relative position to tau
    w_sum = 0
    count = 0
    for j in companies_exceed:
        tau = tau_non_markov[j]
        if 0 <= tau+k < N:
            w_sum += z_non_markov[j, tau+k]
            count += 1
    w_non_markov[l] = w_sum / max(1, count)
    if l > 0:
        v_non_markov[l] = v_non_markov[l-1] + w_non_markov[l]
    else:
        v_non_markov[l] = w_non_markov[l]

# Plot v_l for both cases
plt.figure(figsize=(10, 6))
# Use a more intuitive x-axis that shows relative position to tau
x_axis = np.arange(-m, m+1)
plt.plot(x_axis, v_markov * 10**4, 'o-', color='#1f77b4', linewidth=2, ma
plt.plot(x_axis, v_non_markov * 10**4, 's-', color='#ff7f0e', linewidth=2
plt.axvline(x=0, color='red', linestyle='--', alpha=0.7, label='Threshold
plt.title('Cumulative Impact (v_l) Around Threshold Crossing', fontsize=1
plt.xlabel('Relative Position to τ', fontsize=12)
plt.ylabel('Cumulative Impact v_l (×10⁻⁴)', fontsize=12)
plt.legend(fontsize=10, framealpha=0.9)
plt.grid(True, linestyle='--', alpha=0.7)
plt.annotate('Pre-crossing', xy=(-m/2, max(v_markov * 10**4)/2),
             xytext=(-m/2, max(v_markov * 10**4)/2), ha='center', fontsiz
plt.annotate('Post-crossing', xy=(m/2, max(v_markov * 10**4)/2),
             xytext=(m/2, max(v_markov * 10**4)/2), ha='center', fontsize
plt.xticks(np.arange(-m, m+1, 5))
plt.tight_layout()
plt.show()
```


Cumulative Impact (v_l) Around Threshold Crossing
Markovian
Non-Markovian
Threshold crossing (τ)
Cumulative Impact v_l (×10⁻⁴)
Pre-crossing
Post-crossing
Relative Position to τ
0
1
2
3
4
5
−10
−5
0
5
10

In [ ]: