PHYS121 Integrated Science-Physics

W1T3 Motion in Two and Three Dimensions

References:
[1] David Halliday, Jearl Walker, Resnick Jearl, ‘Fundamentals of Physics’, (Wiley, 2018)
[2] Doug Giancoli, ‘Physics for Scientists and Engineers with modern physics’, (Pearson, 2009)
[3] Hugh D. Young, Roger A. Freedman, ‘University Physics with Modern Physics’, (Pearson, 2012)
And others specified when needed.

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3.8.2. What is the scalar product, $\vec{A} \bullet \vec{B}$, if $\vec{A} = 1.1\hat{\mathrm{i}} + 2.0\hat{\mathrm{j}}$ and $\vec{B} = 1.0\hat{\mathrm{i}} - 1.0\hat{\mathrm{j}}$?

a) zero

b) –0.9

c) $1.1\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}}$

d) 3.1

e) $0.1\hat{\mathrm{i}} + 1.0\hat{\mathrm{j}}$

3.8.3. What is the vector product, $\vec{A} \times \vec{B}$, if

$$\vec{A} = 2.2\hat{\text{i}} + 3.4\hat{\text{j}} \text{ and } \vec{B} = 4.4\hat{\text{i}} + 2.0\hat{\text{j}}?$$

a) zero

b) $-10.6\hat{\text{k}}$

c) $4.4\hat{\text{i}} - 15.0\hat{\text{j}}$

d) $19.4\hat{\text{k}}$

e) 8.3

Learning outcomes

- Explain 2- and 3-D motions by ‘components’ in separated directions.
- Solve problems with 2- and 3-D motions using vectors (components, projectile motion).
- Describe relative motion and perform calculation in different reference frames.

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Position vector

- A **position vector** locates a particle in space
  - Extends from a reference point (origin) to the particle

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}},$$ **Equation (4-1)**

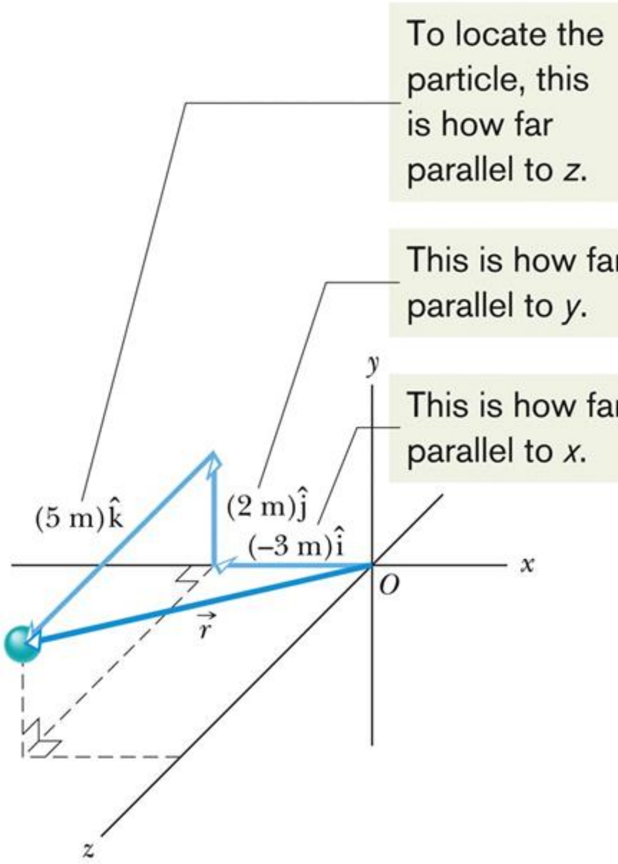


Figure 4-1

Example

Position vector (−3m, 2m, 5m)

$$\vec{r} = (-3\text{ m})\hat{\mathrm{i}} + (2\text{ m})\hat{\mathrm{j}} + (5\text{ m})\hat{\mathrm{k}}$$

Displacement

- Change in position vector is a **displacement**

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}.$$ **Equation (4-2)**

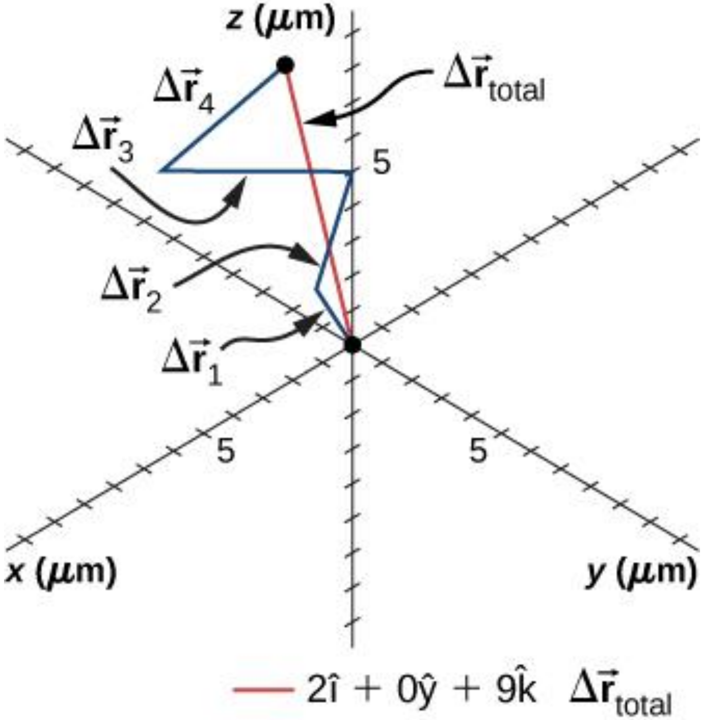
- We can rewrite this as:

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\mathrm{k},$$ **Equation (4-3)**

- Or express it in terms of changes in each coordinate:

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\mathrm{k}.$$ **Equation (4-4)**

FIGURE 4.6



Trajectory of a particle undergoing random displacements of Brownian motion. The total displacement is shown in red.

Average Velocity

- **Average velocity** is
  - A displacement divided by its time interval

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$

Equation (4-8)

- We can write this in component form:

$$\vec{v}_{\text{avg}} = \frac{\Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\mathrm{k}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathrm{i}} + \frac{\Delta y}{\Delta t}\hat{\mathrm{j}} + \frac{\Delta z}{\Delta t}\mathrm{k}.$$

Equation (4-9)

A particle moves through displacement (12 m)i + (3.0 m)k in 2.0 s:

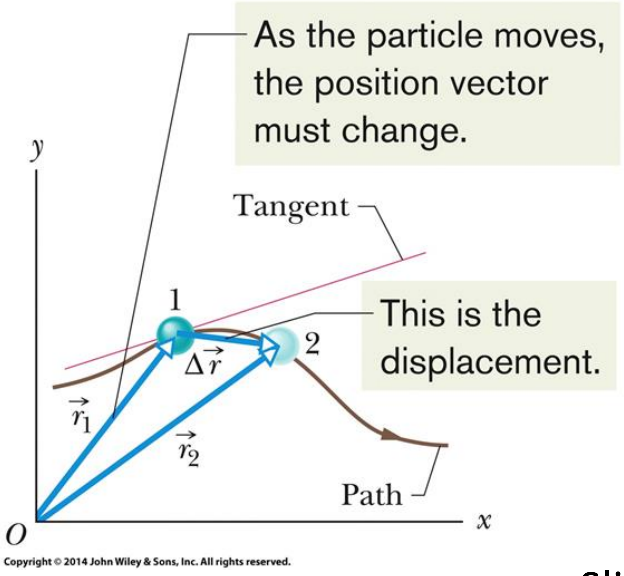
$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(12\ \mathrm{m})\hat{\mathrm{i}} + (3.0\ \mathrm{m})\mathrm{k}}{2.0\mathrm{s}} = (6.0\,\mathrm{m/s})\hat{\mathrm{i}} + (1.5\,\mathrm{m/s})\mathrm{k}.$$

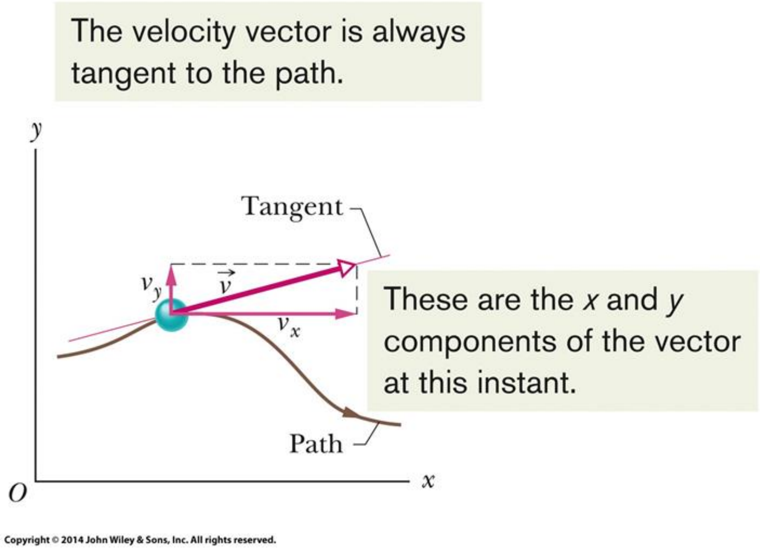
Instantaneous Velocity

- **Instantaneous velocity** is
  - The velocity of a particle at a single point in time
  - The limit of avg. velocity as the time interval shrinks to 0

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

Equation (4-10)





Slide based on Ref. [1]

Figure 4-3

Figure 4-4

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

- In unit-vector form, we write:

$$\vec{v} = \frac{d}{dt}\left(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}\right) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

- Which can also be written:

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$

Equation (4-11)

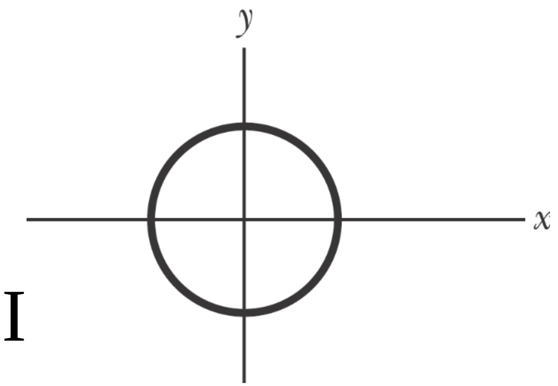
$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}.$$

Equation (4-12)

- Note: a velocity vector does not extend from one point to another, only shows direction and magnitude

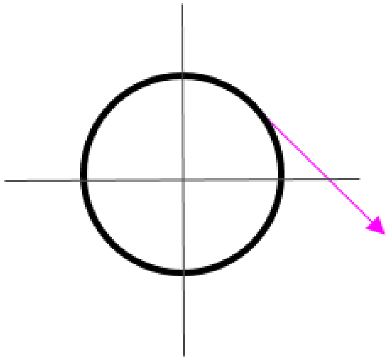
Checkpoint 1

The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2\text{ m/s})\hat{\mathrm{i}} - (2\text{ m/s})\hat{\mathrm{j}}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.

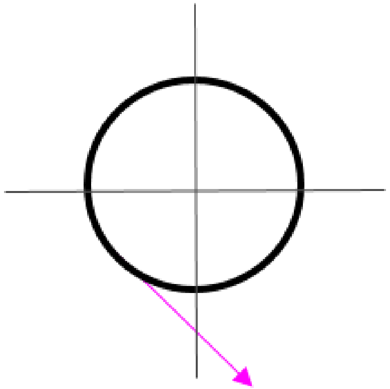


Answer:

(a) Quadrant I



(b) Quadrant III



Average Acceleration and Instantaneous Acceleration

- **Average acceleration** is
  - A change in velocity divided by its time interval

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}. \qquad \textbf{Equation (4-15)}$$

- **Instantaneous acceleration** is again the limit $t \longrightarrow 0$:

$$\vec{a} = \frac{d\vec{v}}{dt}. \qquad \textbf{Equation (4-16)}$$

$$\begin{aligned} \vec{a} &= \frac{d}{dt}\left(v_x \hat{\mathrm{i}} + v_y \hat{\mathrm{j}} + v_z \hat{\mathrm{k}}\right) \\ &= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}. \end{aligned}$$

- We can rewrite as:

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\mathrm{k},$$ **Equation (4-17)**

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}.$$ **Equation (4-18)**

- To get the components of acceleration, we differentiate the components of velocity with respect to time

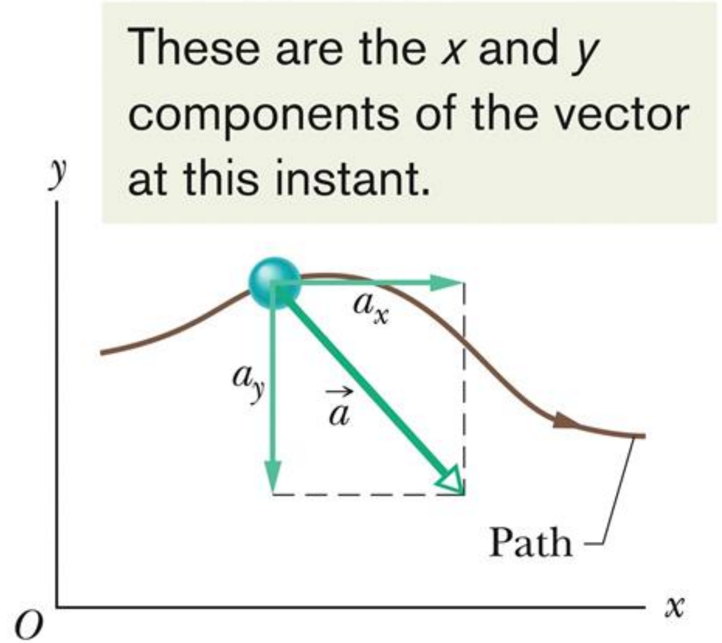


Figure 4-6

- Note: as with velocity, an acceleration vector does not extend from one point to another, only shows direction and magnitude

Checkpoint 2

Here are four descriptions of the position (in meters) of a puck as it moves in an *xy* plane:

1. $x = -3t^2 + 4t - 2 \quad \text{and} \quad y = 6t^2 - 4t$
2. $x = -3t^3 - 4t \quad \text{and} \quad y = -5t^2 + 6$
3. $\vec{r} = 2t^2\hat{\mathrm{i}} - (4t + 3)\hat{\mathrm{j}}$
4. $\vec{r} = (4t^3 - 2t)\hat{\mathrm{i}} + 3\hat{\mathrm{j}}$

Are the *x* and *y* acceleration components constant? Is acceleration $\vec{a}$ constant?

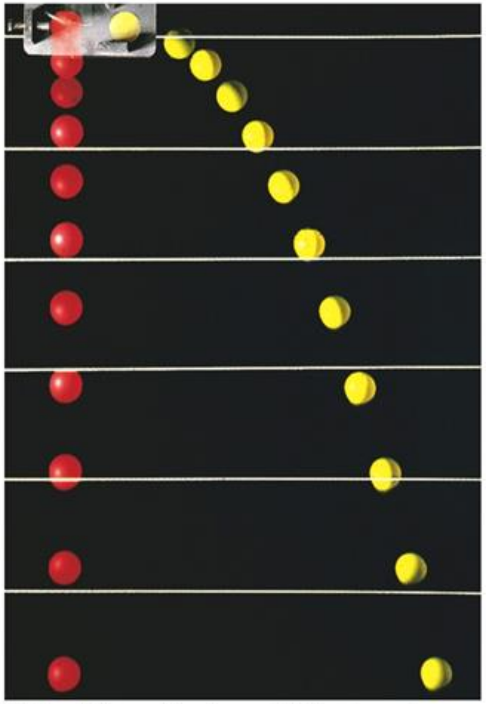
Projectile Motion

- The motion of a projectile is **projectile motion**
- Launched with an initial velocity v_0

$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}.$$ **Equation (4-19)**

$$v_{0x} = v_0 \cos\theta_0;\ \ v_{0y} = v_0 \sin\theta_0.$$ **Equation (4-20)**

$$a_x = 0;\ \ a_y = \vec{g}.$$



Richard Megna/Fundamental Photographs

Figure 4-10

- We can decompose two-dimensional motion into 2 one-dimensional problems

Demo

Projectile motion is motion with constant acceleration in two dimensions, where the acceleration is *g* and is down.

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0,\ a_y = -g = -9.80\ \text{m/s}^2$)

Horizontal Motion ($a_x = 0,\ v_x =$ constant)		Vertical Motion† ($a_y = -g =$ constant)
$v_x = v_{x0}$	(Eq. 2–12a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	(Eq. 2–12b)	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	(Eq. 2–12c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

† If y is taken positive downward, the minus (−) signs in front of g become plus (+) signs.

- The projectile's **trajectory** is a parabola

$$y = \left(\tan\theta_0\right)x - \frac{gx^2}{2\left(v_0\cos\theta_0\right)^2}$$

Equation (4-25)

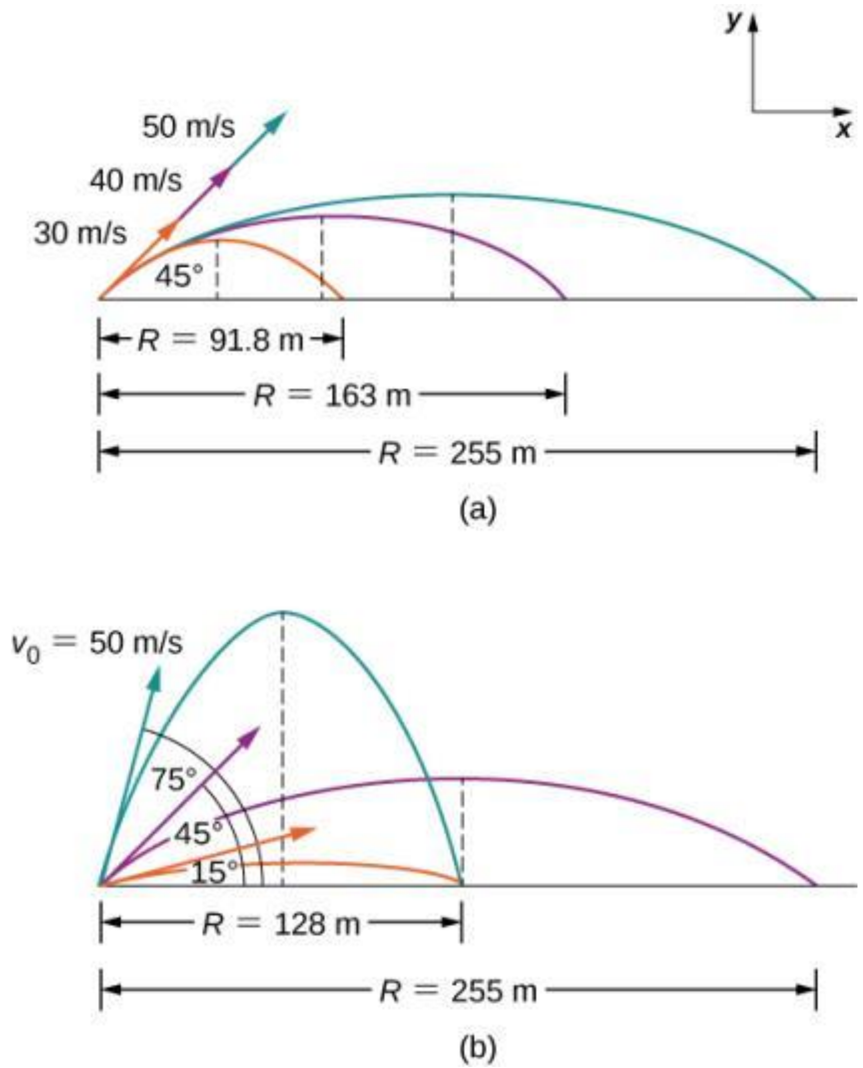
Slide based on Ref. [2]

FIGURE 4.15

Trajectories of projectiles on level ground.

(a) The greater the initial speed v_0, the greater the range for a given initial angle.

(b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for initial angles of 15° and 75°, although the maximum heights of those paths are different.



Solving Problems Involving Projectile Motion

1. **Read** the problem carefully, and choose the object(s) you are going to analyze.

2. **Draw** a diagram.

3. **Choose** an origin and a coordinate system.

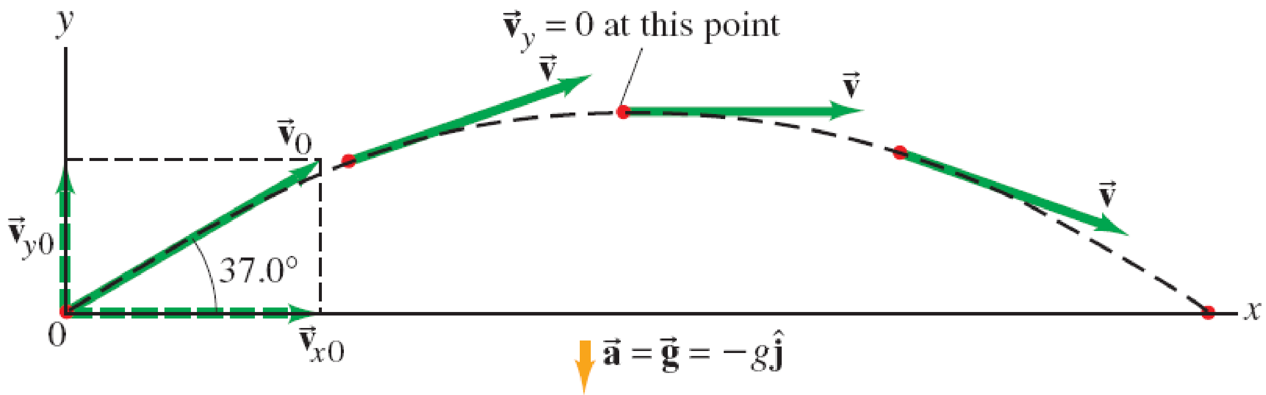
4. **Decide** on the **time** interval; this is the same in both directions, and includes only the time the object is moving with constant acceleration g.

5. **Examine** the x and y motions separately.

6. **List** known and unknown quantities. Remember that v_x never changes, and that $v_y = 0$ at the highest point.

7. **Plan** how you will proceed. Use the appropriate equations; you may have to combine some of them.

Slide based on Ref. [2]



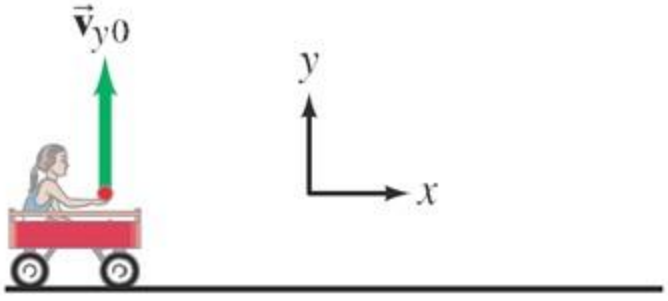
Example 3-7: A kicked football.

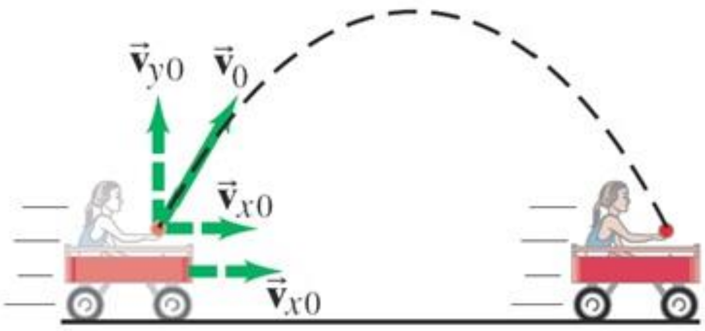
A football is kicked at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown. Calculate (a) the maximum height, (b) the time of travel before the football hits the ground, (c) how far away it hits the ground, (d) the velocity vector at the maximum height, and (e) the acceleration vector at maximum height. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

Slide based on Ref. [2]

Conceptual Example 3-8: Where does the apple land?

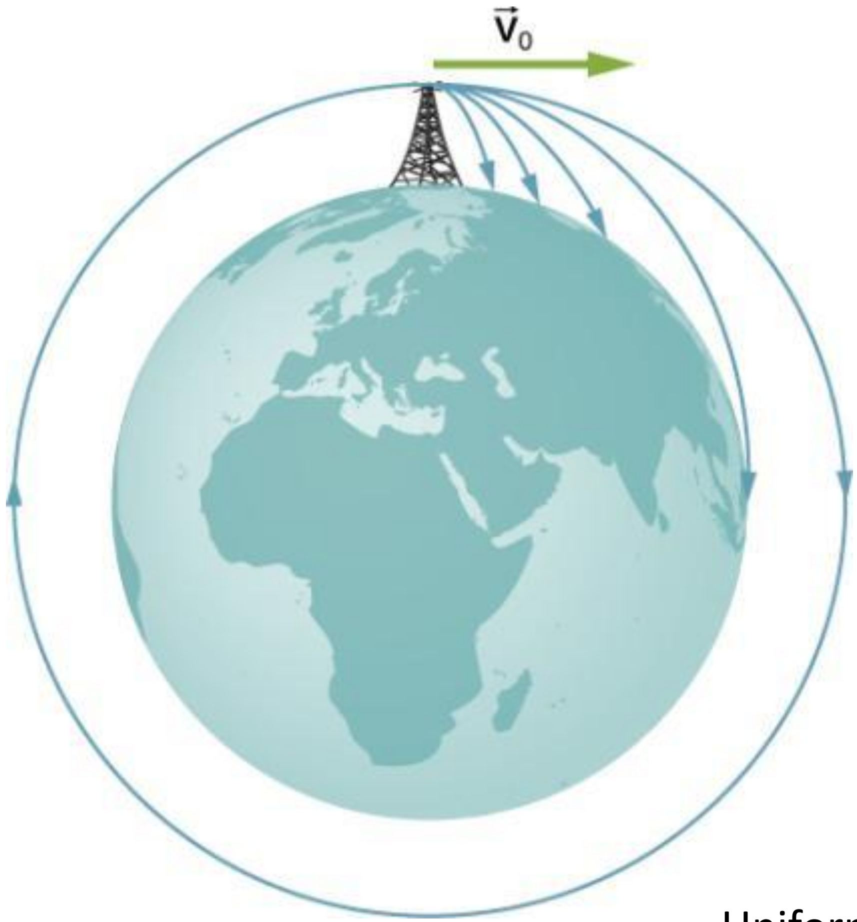
A child sits upright in a wagon which is moving to the right at constant speed as shown. The child extends her hand and throws an apple straight upward (from her own point of view), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (a) behind the wagon, (b) in the wagon, or (c) in front of the wagon?



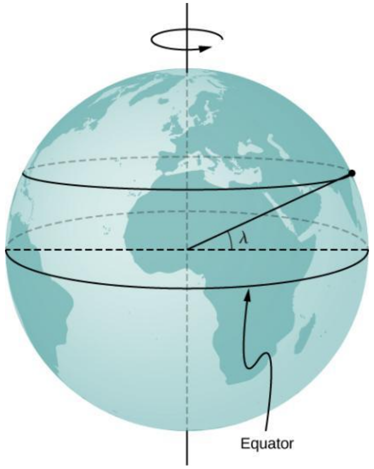


Slide based on Ref. [2]

FIGURE 4.17



Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because Earth curves away beneath its path. With a speed of 8000 m/s, orbit is achieved.



Uniform Circular Motion will be discussed next week.

Relative Motion in One Dimension

- Measures of position and velocity depend on the **reference frame** of the measurer
  - How is the observer moving?
  - Our usual reference frame is that of the ground
- Read subscripts "*PA*", "*PB*", and "*BA*" as "*P* as measured by *A*", "*P* as measured by *B*", and "*B* as measured by *A*"
- Frames *A* and *B* are each watching the movement of object *P*

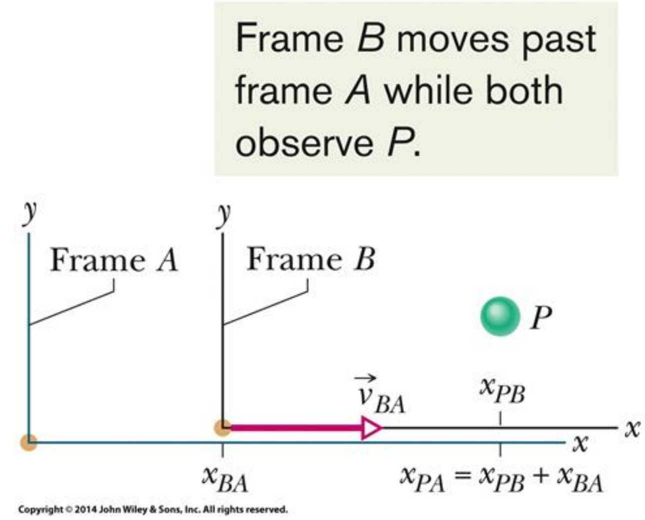


Figure 4-18

- Positions in different frames are related by:

$$x_{PA} = x_{PB} + x_{BA}. \qquad \textbf{Equation (4-40)}$$

- Taking the derivative, we see velocities are related by:

$$\frac{d}{dt}\left(x_{PA}\right) = \frac{d}{dt}\left(x_{PB}\right) + \frac{d}{dt}\left(x_{BA}\right).$$

$$v_{PA} = v_{PB} + v_{BA}. \qquad \textbf{Equation (4-41)}$$

- But accelerations (for *non-accelerating reference frames*, $a_{BA} = 0$) are related by

$$\frac{d}{dt}\left(v_{PA}\right) = \frac{d}{dt}\left(v_{PB}\right) + \frac{d}{dt}\left(v_{BA}\right).$$

$$a_{PA} = a_{PB}. \qquad \textbf{Equation (4-42)}$$

Relative Motion in Two Dimensions

- The same as in one dimension, but now with vectors:
- Positions in different frames are related by:

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}.$$ **Equation (4-43)**

- Velocities:

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}.$$ **Equation (4-44)**

- Accelerations (<u>for non-accelerating reference frames</u>):

$$\vec{a}_{PA} = \vec{a}_{PB}.$$ **Equation (4-45)**

- Again, observers in different frames will see the same acceleration

- Frames *A* and *B* are both observing the motion of *P*

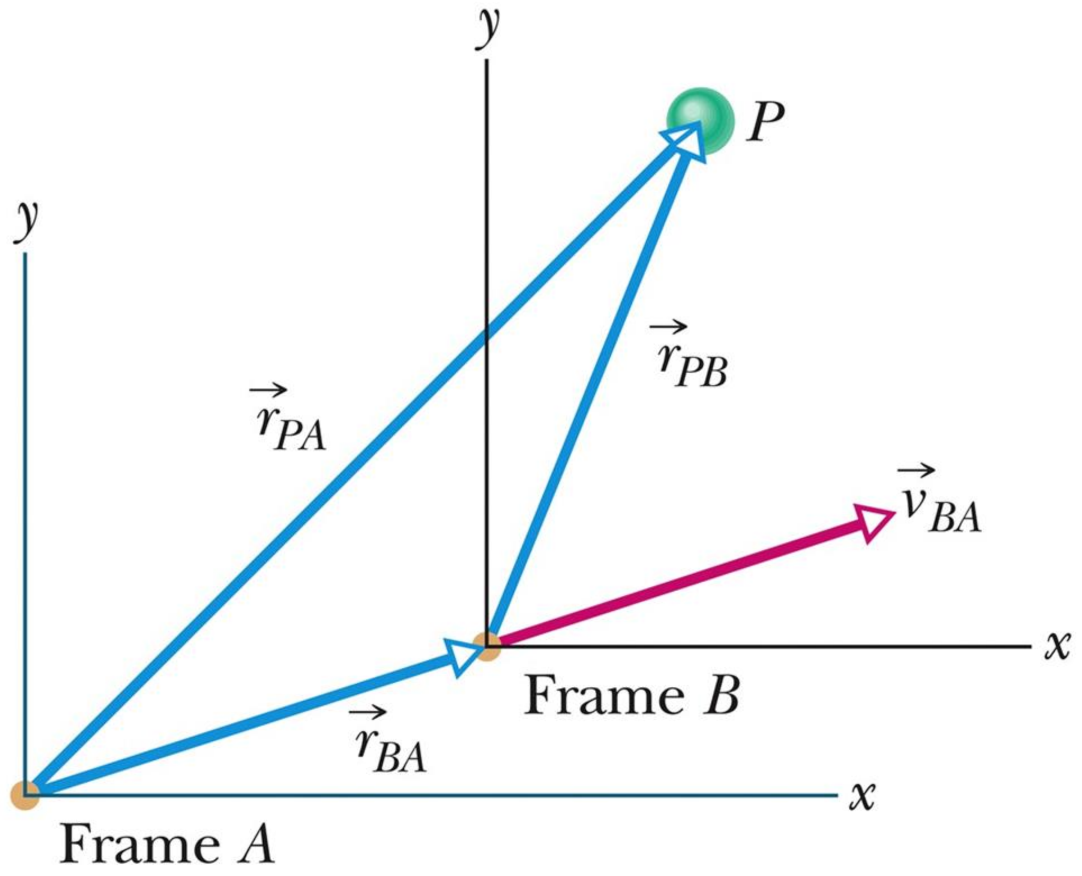
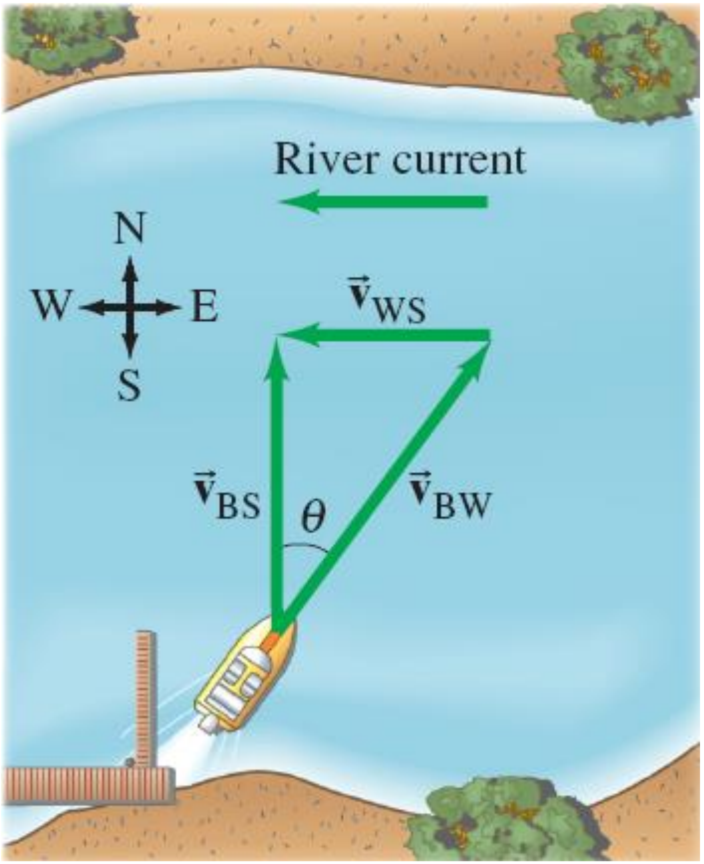


Figure 4-19

Relative Velocity



Here, v_{WS} is the velocity of the water in the shore frame, v_{BS} is the velocity of the boat in the shore frame, and v_{BW} is the velocity of the boat in the water frame.

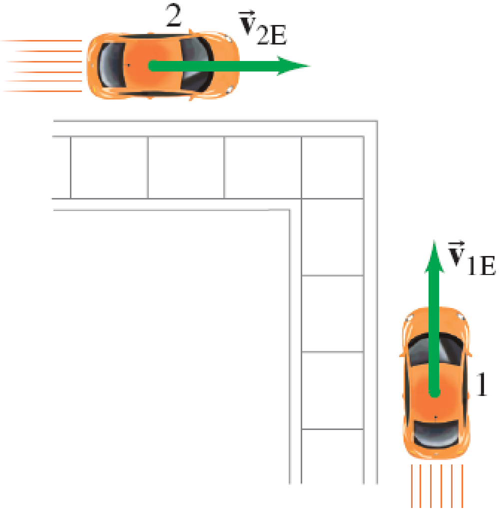
The relationship between the three velocities is:

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}.$$

Relative Velocity

Example 3-16: Car velocities at 90° .

Two automobiles approach a street corner at right angles to each other with the same speed of 40.0 km/h (= 11.1 m/s), as shown. What is the relative velocity of one car with respect to the other? That is, determine the velocity of car 1 as seen by car 2.



Learning outcomes

- ✓ Explain 2- and 3-D motions by 'components' in separated directions.
- ✓ Solve problems with 2- and 3-D motions using vectors (components, projectile motion).
- ✓ Describe relative motion and perform calculation in different reference frames.

$$\vec{a} = a_x \hat{\mathrm{i}} + a_y \hat{\mathrm{j}} + a_z \mathrm{k},$$

$$\vec{\mathbf{v}}_{\mathrm{BS}} = \vec{\mathbf{v}}_{\mathrm{BW}} + \vec{\mathbf{v}}_{\mathrm{WS}} \cdot$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}.$$

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Questions