PHYS121 Integrated Science-Physics

W3T2 Mechanical Wave

References:
[1] David Halliday, Jearl Walker, Resnick Jearl, 'Fundamentals of Physics', (Wiley, 2018)
[2] Doug Giancoli, 'Physics for Scientists and Engineers with modern physics', (Pearson, 2009)
[3] Hugh D. Young, Roger A. Freedman, 'University Physics with Modern Physics', (Pearson, 2012)
And others specified when needed.

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15.2.4. Object A is attached to ideal spring A and is moving in simple harmonic motion. Object B is attached to ideal spring B and is moving in simple harmonic motion. The period and the amplitude of object B are both two times the corresponding values for object A. How do the maximum speeds of the two objects compare?
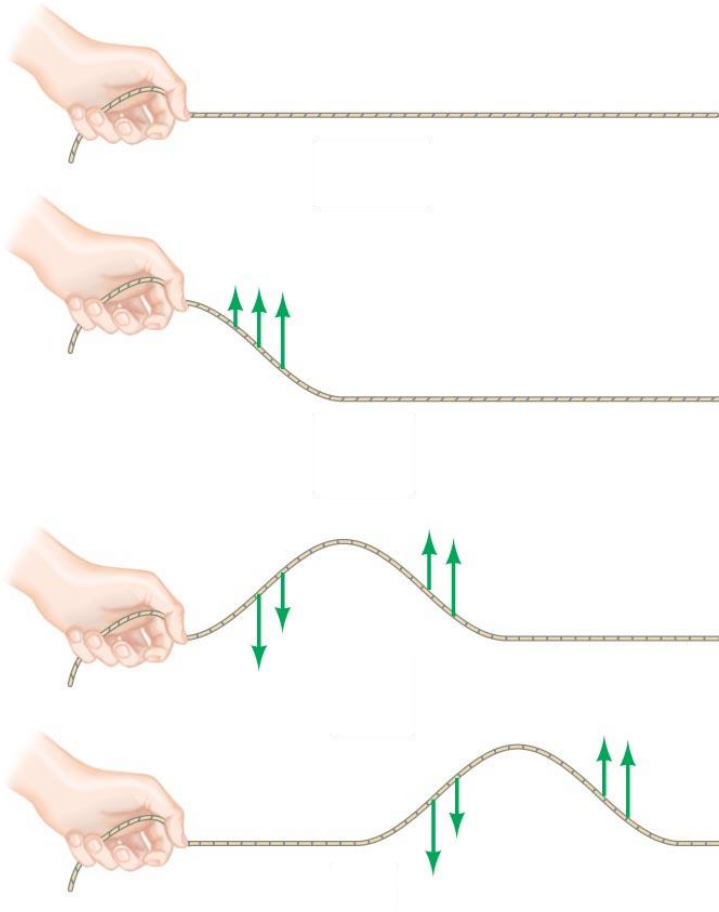
a) The maximum speed of A is one fourth that of object B.

b) The maximum speed of A is one half that of object B.

c) The maximum speed of A is the same as that of object B.

d) The maximum speed of A is two times that of object B.

e) The maximum speed of A is four times that of object B.

Learning Outcomes

- Describe the wave motion (transverse/longitudinal).

- Analyze the motion of sinusoidal waves (wave transport, take the wave on a string for example).

- Explain the superposition of waves (i.e., interference).

- Analyze the characteristics of standing waves.

All types of traveling waves transport energy.

Study of a single wave pulse shows that it is begun with a vibration and is transmitted through internal forces in the medium.

Continuous waves start with vibrations, too. If the vibration is SHM, then the wave will be sinusoidal.
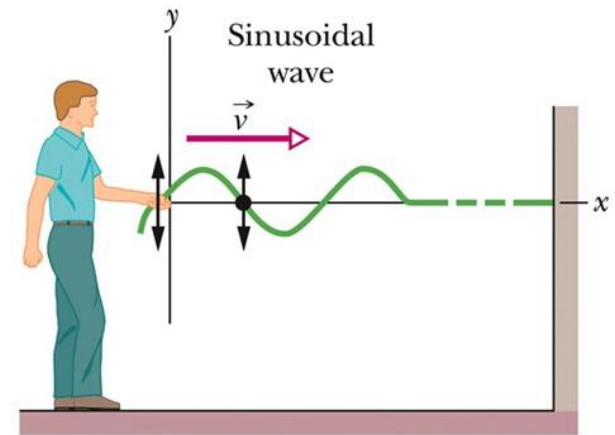
Transverse Waves

Types of Waves

1. **Mechanical Waves:** They are governed by Newton's laws, and they can exist only within a material medium, such as water, air, and rock. Examples: water waves, sound waves, and seismic waves.

2. **Electromagnetic waves:** These waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed $c = 299\ 792\ 458$ m/s.

3. **Matter waves:** These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

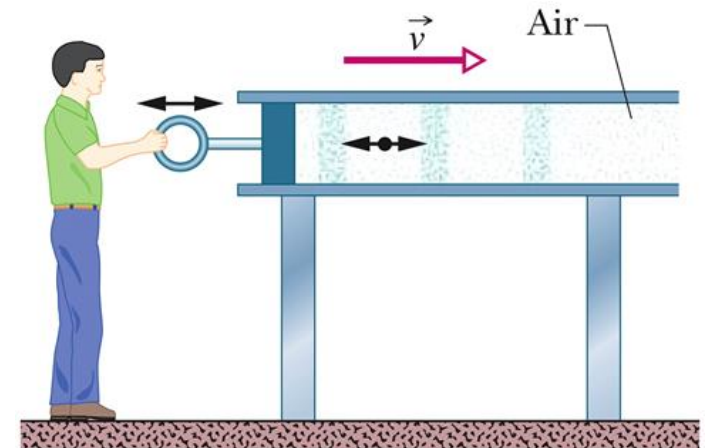# Transverse and Longitudinal Waves

A sinusoidal wave is sent along the string (Figure (a)). A typical string element moves up and down continuously as the wave passes. This is **transverse wave.**

A sound wave is set up in an air-filled pipe by moving a piston back and forth (Figure (b)). Because the oscillations of an element of the air (represented by the dot) are parallel to the direction in which the wave travels, the wave is a **longitudinal wave.**

(a) Transverse Wave

(b) Longitudinal Wave

The Wave Equation

Look at a segment of string under tension:

Newton's second law gives:

$$\Sigma F_y = ma_y$$

$$F_T \sin \theta_2 - F_T \sin \theta_1 = (\mu \, \Delta x) \frac{\partial^2 D}{\partial t^2}.$$

Assuming small angles, and taking the limit $\Delta x \to 0$, gives (_after some manipulation_):

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}.$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \left(\text{wave equation}\right).$$

This is the one-dimensional wave equation; it is a linear second-order partial differential equation in x and t. Its solutions are sinusoidal waves.

Sinusoidal Function

Five "snapshots" (y vs x each at a constant time) of a string wave traveling in the positive direction along an x axis. The amplitude y_m is indicated. A typical wavelength λ, measured from an arbitrary position x_1, is also indicated.

Watch this spot in this series of snapshots.

Amplitude

Displacement

Oscillating term

Phase

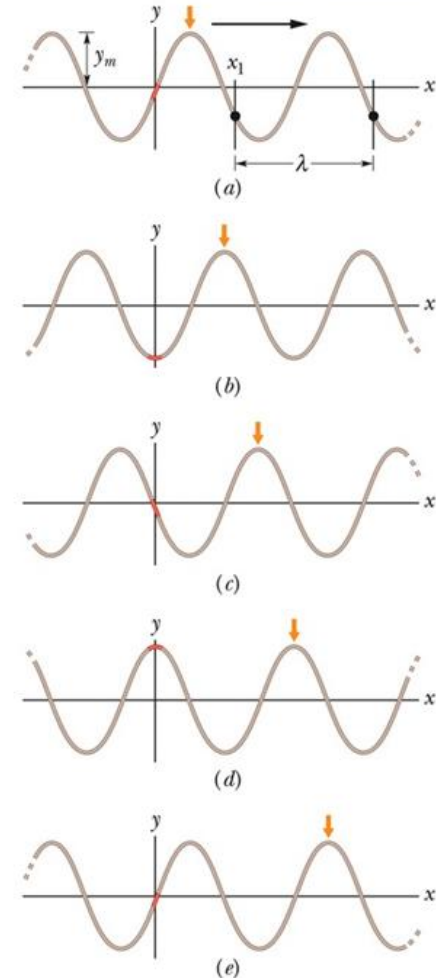$$y(x,t) = y_m \sin (kx - \omega t)$$

Angular wave number

Position

Time

Angular frequency

The sine function describes the shape of the wave

Period, Wave Number, Angular Frequency and Frequency
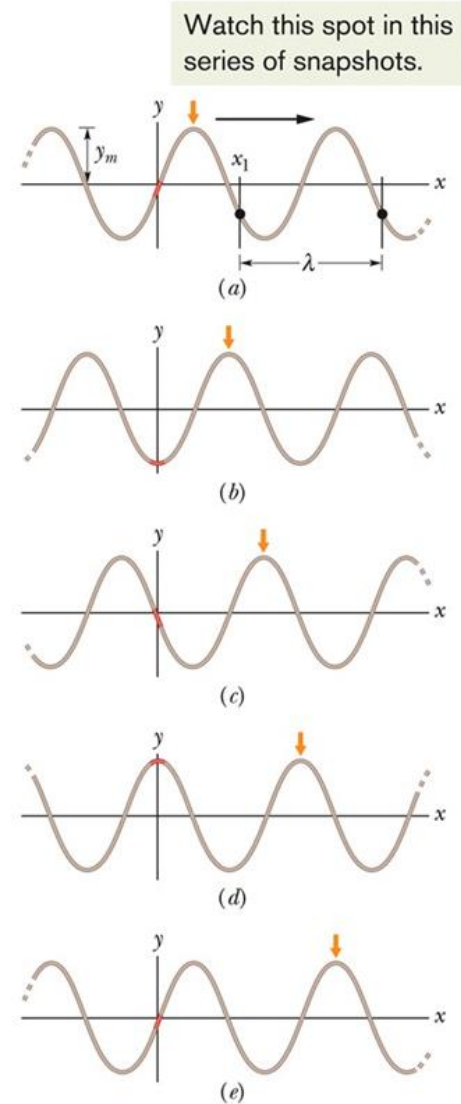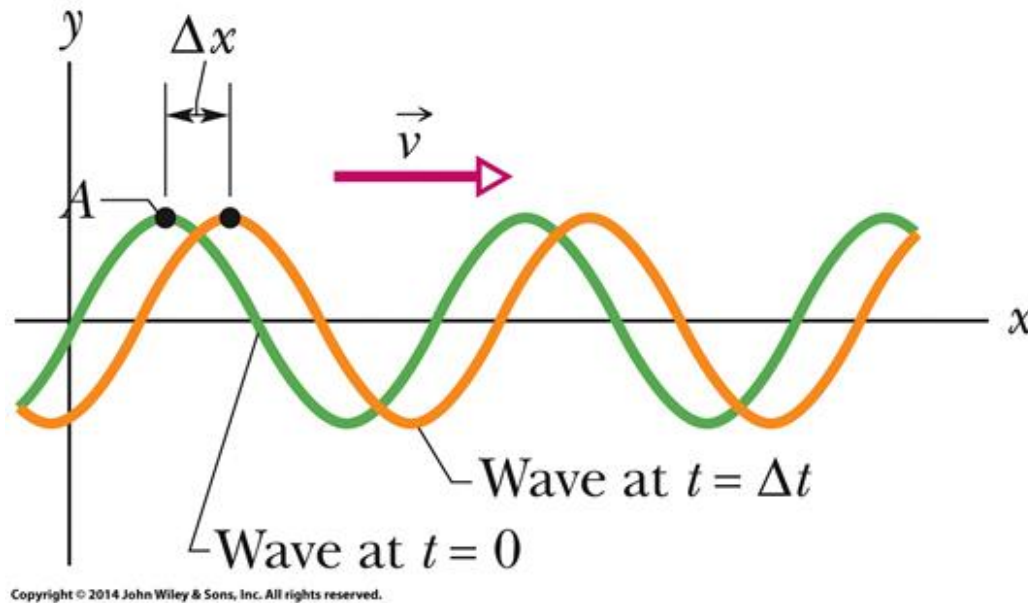
$$k = \frac{2\pi}{\lambda} \quad (\text{angular wave number}).$$

$$\omega = \frac{2\pi}{T} \quad (\text{angular frequency}).$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad (\text{frequency}).$$

Watch this spot in this series of snapshots.

(a)

(b)

(c)

(d)

(e)

The Speed of a Traveling Wave

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Two snapshots of the wave: at time $t = 0$, and then at time $t = \Delta t$. As the wave moves to the right at velocity v, the entire curve shifts a distance Δx during Δt.

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \left(\text{wave speed}\right).$$

Checkpoint 2

Here are the equations of the three waves:

$$(1)\ y(x,t) = 2\sin(4x - 2t),$$

$$(2)\ y(x,t) = \sin(3x - 4t),$$

$$(3)\ y(x,t) = 2\sin(3x - 3t),$$

Rank the waves according to their (a) wave speed and (b) maximum speed perpendicular to the wave's direction of travel (the transverse speed), greatest first.

Answer:

(a) (2), (3), (1)

(b) (3), (1) and (2)

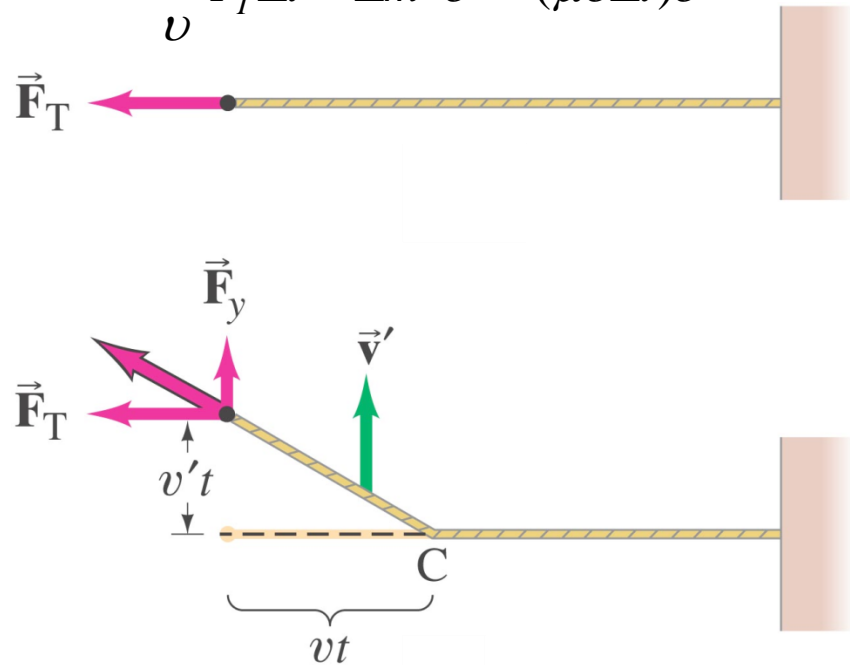**The velocity of a transverse wave on a cord/string is given by:**

$$\frac{F_T}{F_y} = \frac{v \Delta t}{v' \Delta t} = \frac{v}{v'}$$

$$F_y \Delta t = \Delta p \quad (impluse)$$

$$\frac{v'}{v} F_T \Delta t = \Delta m \cdot v' = (\mu v \Delta t) v'$$

$$v = \sqrt{\frac{F_T}{\mu}}.$$

As expected, the velocity increases when the tension increases, and decreases when the mass (*linear mass density*) increases.

Energy and Power of a Wave Traveling along a String

- When we set up a wave on a stretched string, we _provide energy for the motion of the string_. As the wave moves away from us, it transports that energy as both kinetic energy and elastic potential energy.

- **The Rate of Energy Transmission** The kinetic energy dK associated with a string element of mass dm is given by

$$dK = \frac{1}{2} dm \, u^2,$$

where u is the _transverse speed_ of the _oscillating string element_

$$u = \frac{\partial y}{\partial t} = -\omega y_m \, \cos(kx - \omega t).$$

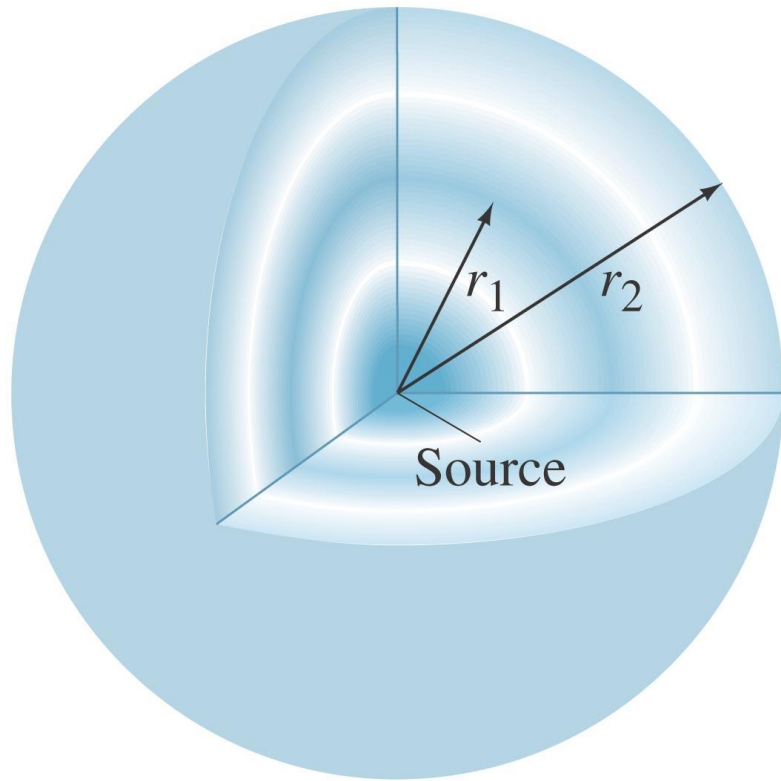$$dK = \tfrac{1}{2}(\mu \, dx)(-\omega y_m)^2 \cos^2(kx - \omega t).$$

$$\left(\frac{dK}{dt}\right)_{\mathrm{avg}} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \left[\cos^2(kx - \omega t)\right]_{\mathrm{avg}}$$

$$= \frac{1}{4}\mu v \omega^2 y_m^2.$$

- The **average power** of, or average rate at which energy is transmitted (kinetic + potential) by, a sinusoidal wave on a stretched string is given by

$$P_{\mathrm{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2.$$

The factors μ and v in this equation depend on the material and tension of the string. The factors ω and y_m depend on the process that generates the wave (plane wave).

If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is spherical.
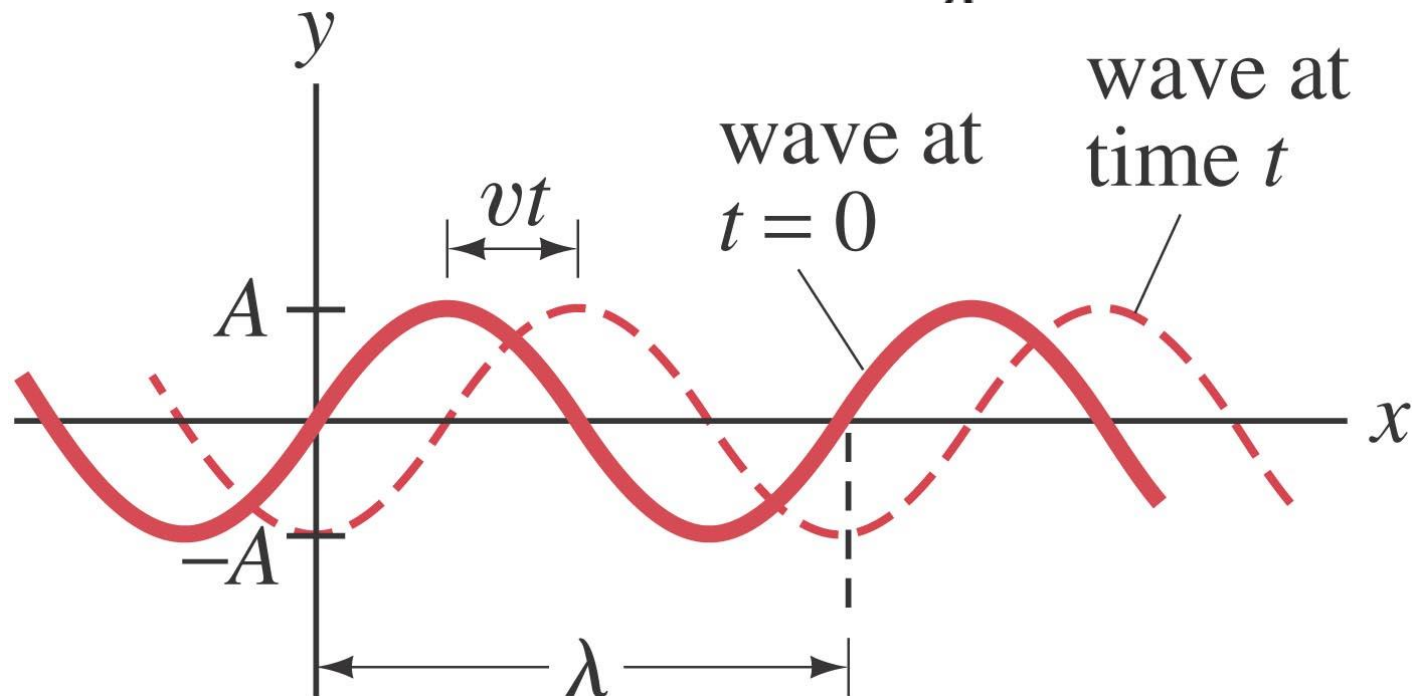

Source

Just from geometrical considerations, as long as the power output is constant, we see the intensity (power across unit area):

$$I \propto \frac{1}{r^2}.$$

15-4 Mathematical Representation of a Traveling Wave

Suppose the shape of a wave (at a certain time) is given by:

$$D(x) = A \sin \frac{2\pi}{\lambda} x.$$

After a time t, the wave crest has traveled a distance vt (to the right, *think about how to shift a curve to the positive x-axis direction*), so we write:

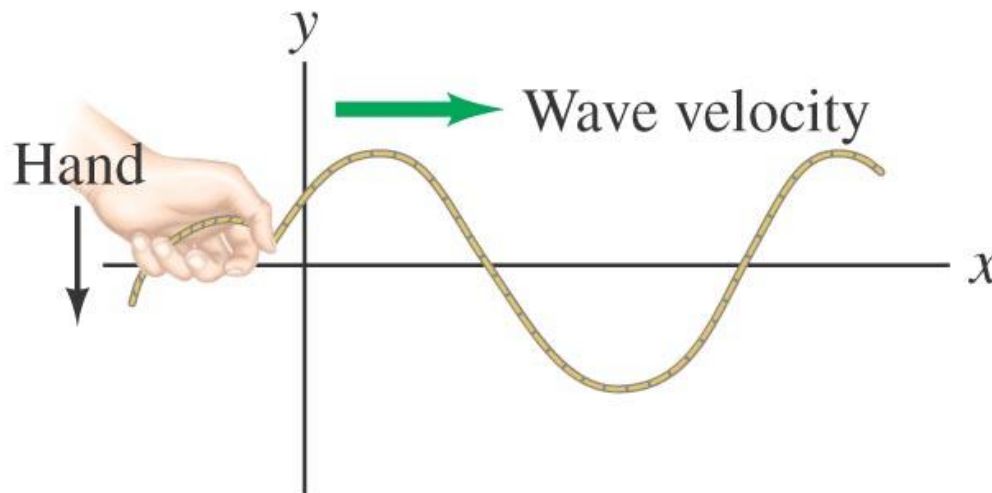$$D(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

Or: $D(x, t) = A \sin(kx - \omega t),$

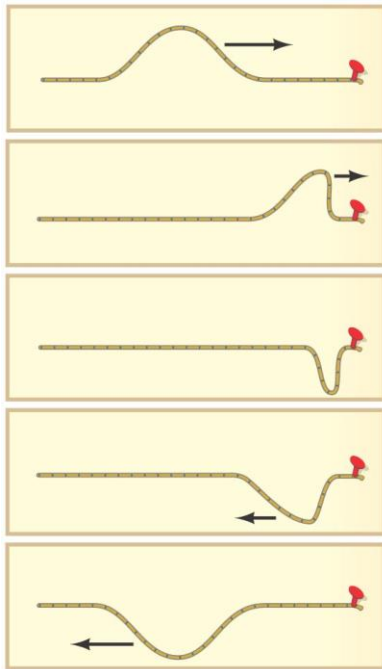**with** $\omega = 2\pi f, \quad k = \frac{2\pi}{\lambda}.$

$$f = v/\lambda$$
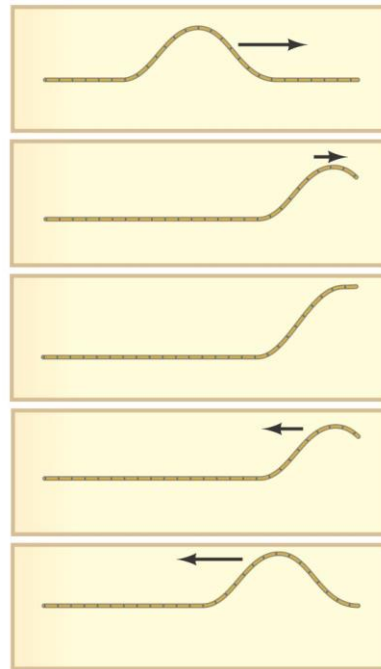
Example 15-5: A traveling wave.

The left-hand end of a long horizontal stretched cord oscillates transversely in SHM with frequency $f = 250$ Hz and amplitude 2.6 cm. The cord is under a tension of 140 N and has a linear density $\mu = 0.12$ kg/m. At $t = 0$, the end of the cord has an upward displacement of 1.6 cm and is falling. Determine (a) the wavelength of waves produced and (b) the equation for the traveling wave.
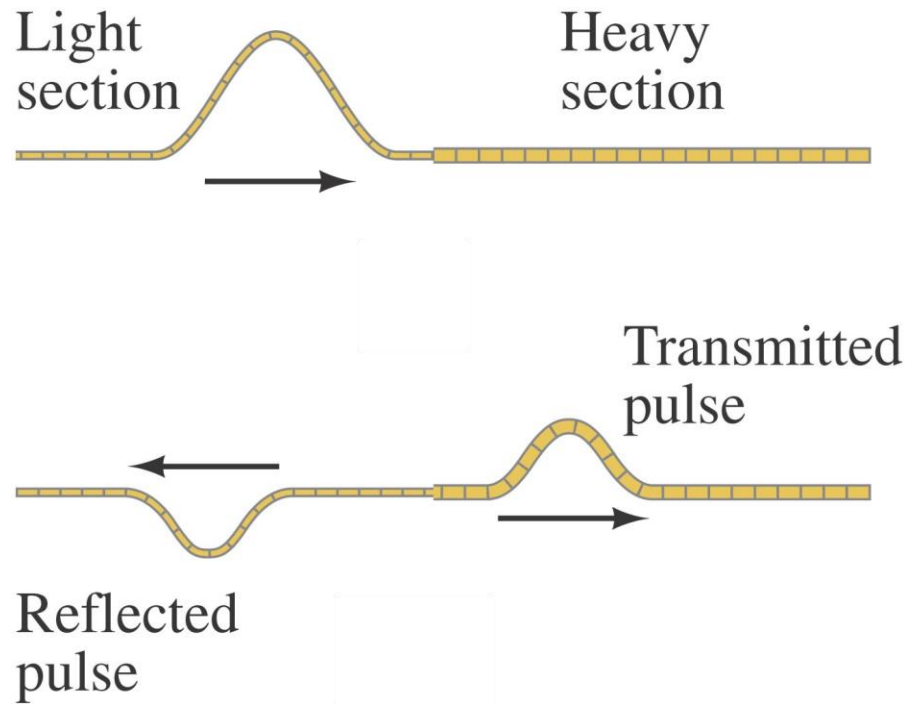
Reflection

(a)

(b)

A wave reaching the end of its medium, but where the medium is still **free** to move, will be **reflected** (b), and its reflection will be **upright.**
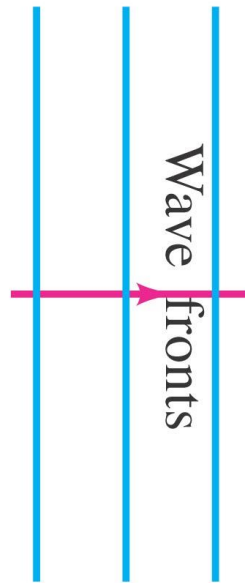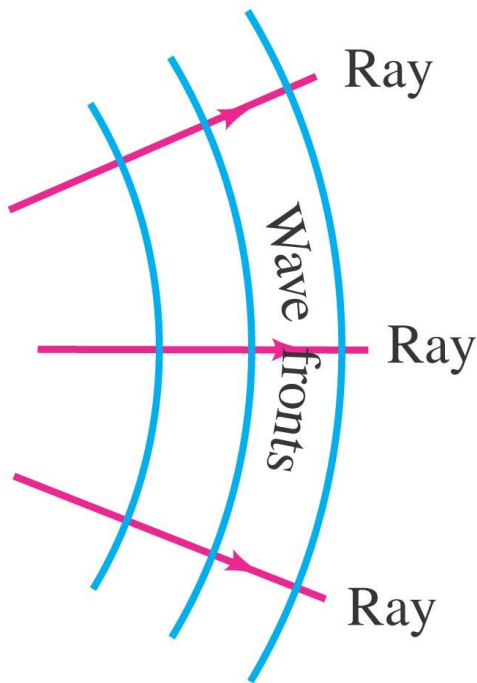
A wave hitting an **obstacle** will be **reflected** (a), and its reflection will be **inverted.**

Transmission
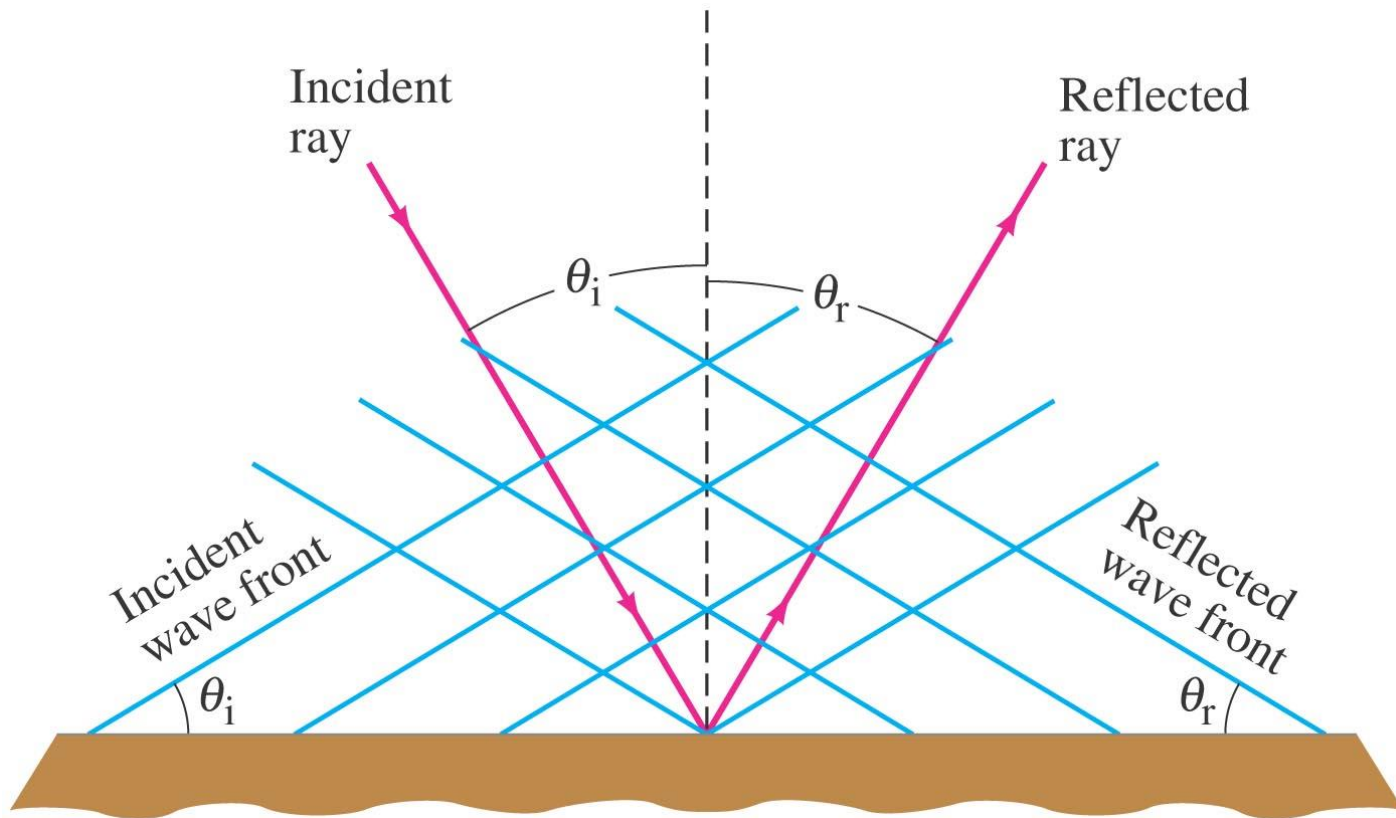

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A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter.

Two- or three-dimensional waves can be represented by wave fronts, which are curves of surfaces where all the waves have the same phase.

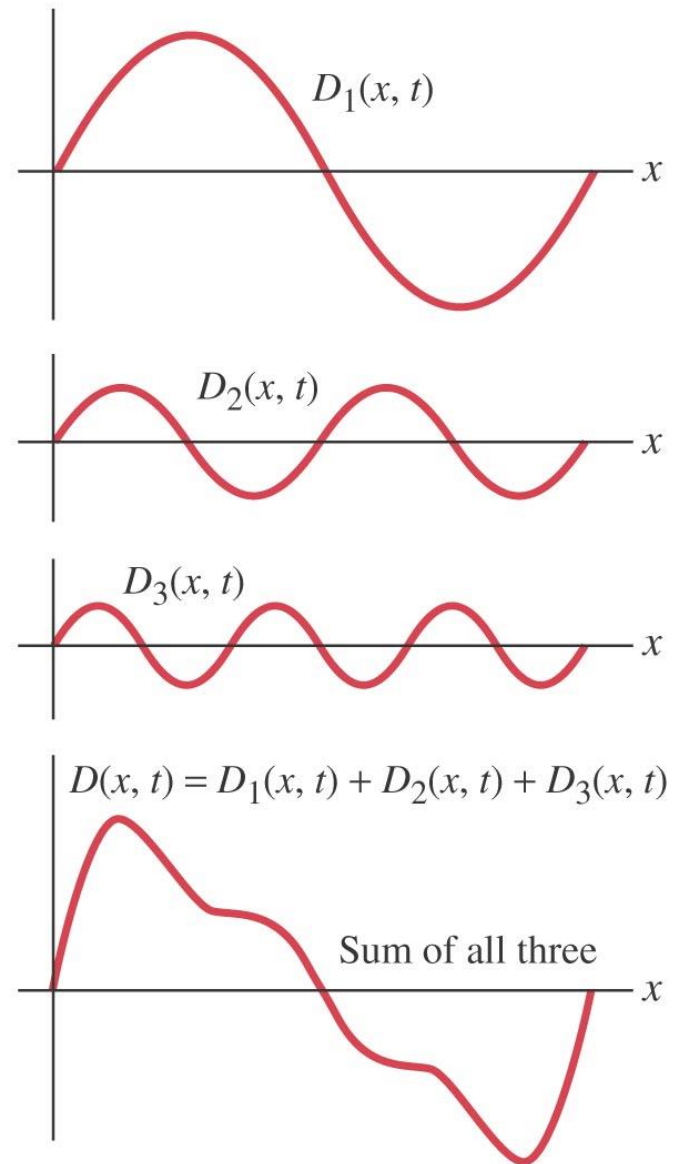**Lines perpendicular to the wave fronts are called rays; they point in the direction of propagation of the wave.**

The law of reflection: the angle of incidence equals the angle of reflection.
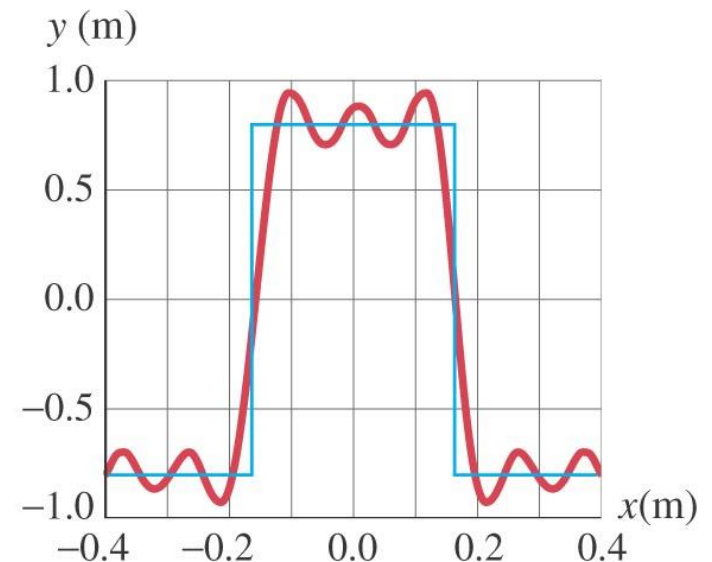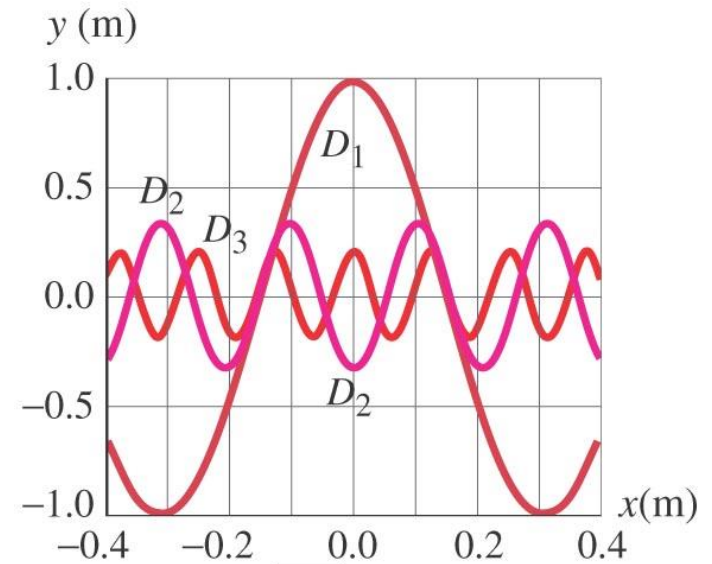
The Principle of Superposition

Superposition: The displacement at any point is the vector sum of the displacements of all waves passing through that point at that instant.

Fourier's theorem: Any complex periodic wave can be written as the sum of sinusoidal waves of different amplitudes, frequencies, and phases.

$D_1(x, t)$

$D_2(x, t)$

$D_3(x, t)$

$D(x, t) = D_1(x, t) + D_2(x, t) + D_3(x, t)$

Sum of all three

Conceptual Example 15-7: Making a square wave.

At $t = 0$, three waves are given by $D_1 = A \cos kx$, $D_2 = -\frac{1}{3}A \cos 3kx$, and $D_3 = \frac{1}{5}A \cos 5kx$, where $A = 1.0$ m and $k = 10$ m^{-1}. Plot the sum of the three waves from $x = $ -0.4 m to +0.4 m. (These three waves are the first three Fourier components of a "square wave.")

Interference of Waves
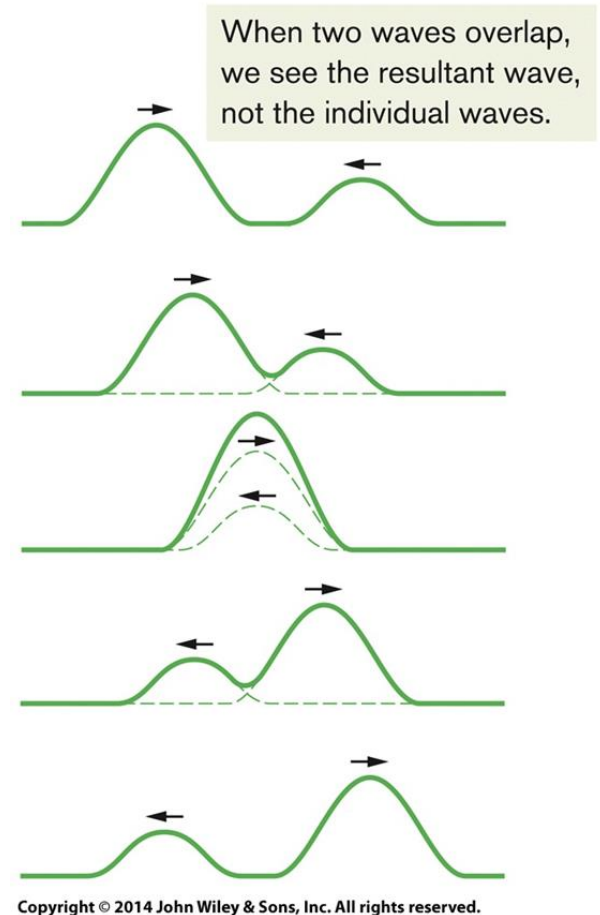
Principle of Superposition of waves

Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum

$$y'(x, t) = y_1(x, t) + y_2(x, t).$$

This summation of displacements along the string means that

Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

Overlapping waves do not in any way alter the travel of each other.

When two waves overlap, we see the resultant wave, not the individual waves.

The resultant wave due to the interference of two sinusoidal transverse waves (*travel to the same direction with a phase difference*), is also a sinusoidal transverse wave, with an amplitude and an oscillating term.

Displacement

$$y'(x,t) = \left[2y_m \cos \tfrac{1}{2}\phi \right] \sin\left(kx - \omega t + \tfrac{1}{2}\phi\right)$$

Magnitude gives amplitude

Oscillating term

Constructive and Destructive Interference

$$y'(x, t) = y_1(x, t) + y_2(x, t).$$

Two identical sinusoidal waves, $y_1(x, t)$ and $y_2(x, t)$, travel along a string in the positive direction of an x axis. They interfere to give a resultant wave $y'(x, t)$. The resultant wave is

what is actually seen on the string. The phase difference ϕ between the two interfering waves is (a) 0 rad or 0°, (b) π rad or 180°, and (c) $\frac{2}{3}\pi$ rad or 120°. The corresponding resultant waves are shown in (d), (e), and (f).
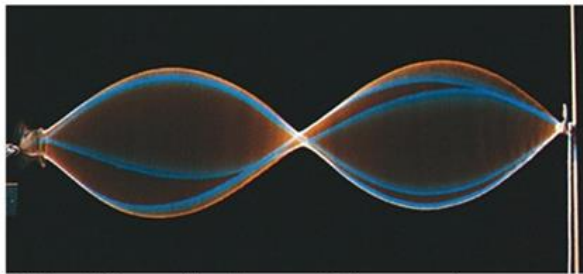
Standing Waves and Resonance

Standing Waves

The *interference* of two identical sinusoidal waves moving in *opposite directions* produces standing waves. For a string with fixed ends, the standing wave is given by
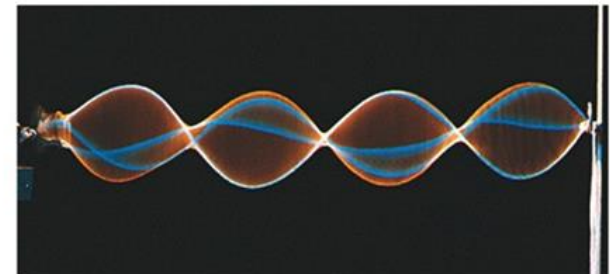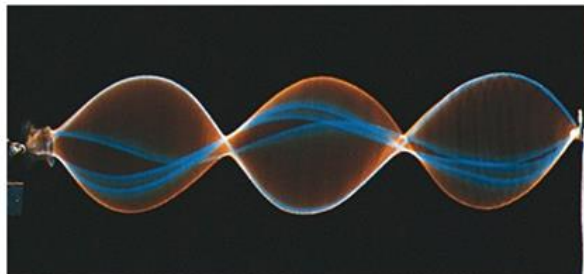
Displacement

$$y'(x,t) = [2y_m \sin kx]\cos \omega t$$

Magnitude gives amplitude at position x
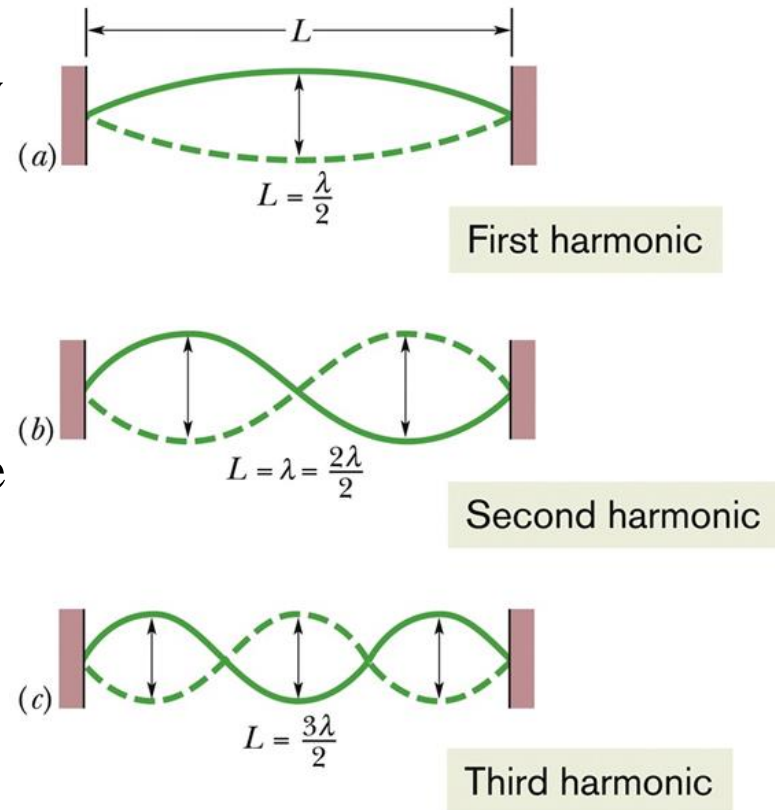
Oscillating term

Richard Megna/Fundamental Photographs

Harmonics

Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a **resonant frequency**, and the corresponding standing wave pattern is an oscillation mode. For a stretched string of length L with fixed ends, the resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \dots$$

(a) $L = \frac{\lambda}{2}$

First harmonic

(b) $L = \lambda = \frac{2\lambda}{2}$

Second harmonic

(c) $L = \frac{3\lambda}{2}$

Third harmonic

Learning Outcomes

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f.$$

✓ Describe the wave motion (transverse/longitudinal).

✓ Analyze the motion of sinusoidal waves (wave transport, take the wave on a string for example).

$$\upsilon = \sqrt{\frac{\tau}{\mu}}$$

✓ Explain the superposition of waves (i.e., interference).

✓ Analyze the characteristics of standing waves.

Amplitude
Oscillating term
Displacement
Phase
$y(x,t) = y_m \sin(kx - \omega t)$
Angular wave number
Position
Time
Angular frequency

Displacement
$y'(x,t) = [2y_m \sin kx] \cos \omega t$
Magnitude gives amplitude at position x
Oscillating term

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CHAPTER REVIEW AND EXAMPLES, DEMOS

https://openstax.org/books/university-physics-volume-1/pages/16-summary

Basic **mechanical waves** are governed by Newton's laws and require a medium. A medium is the substance a mechanical waves propagates through, and the medium produces an elastic restoring force when it is deformed. Mechanical waves transfer energy and momentum, without transferring mass.
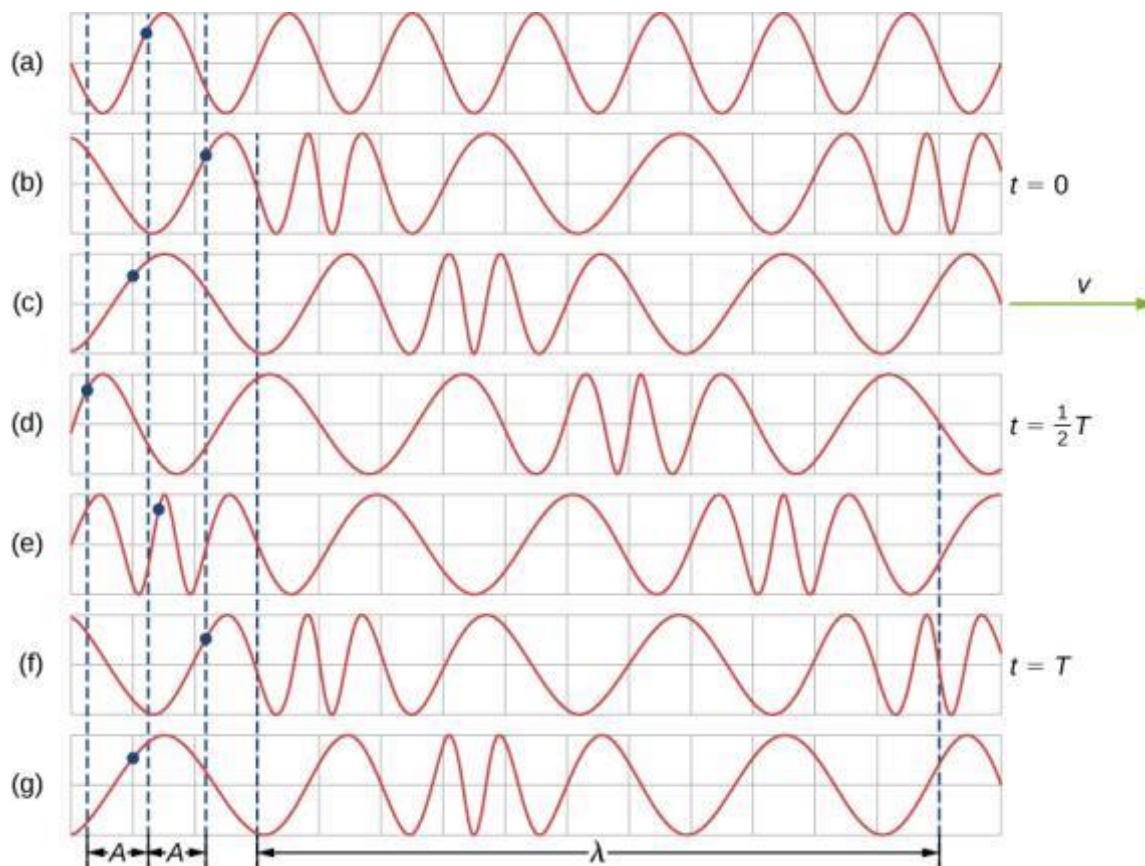
At this point, it is useful to recall from your study of algebra that if $f(x)$ is some function, then $f(x - d)$ is the same function translated in the positive x-direction by a distance d. The function $f(x + d)$ is the same function translated in the negative x-direction by a distance d. We want to define a wave function that will give the y-position of each segment of the string for every position x along the string for every time t.

A wave function is any function such that $f(x, t) = f(x - vt)$.

The velocity of the medium, which is perpendicular to the wave velocity in a transverse wave, can be found by taking the partial derivative of the position equation with respect to time.

$$|v| = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

FIGURE 16.5

(a) This is a simple, graphical representation of a section of the stretched spring shown in **Figure 16.4** (b), representing the spring's equilibrium position before any waves are induced on the spring. A point on the spring is marked by a blue dot. (b–g) Longitudinal waves are created by oscillating the end of the spring (not shown) back and forth along the x-axis. The longitudinal wave, with a wavelength λ , moves along the spring in the +x-direction with a wave speed v. For convenience, the wavelength is measured in (d). Note that the point on the spring that was marked with the blue dot moves back and forth a distance A from the equilibrium position, oscillating around the equilibrium position of the point.

Questions