



# PHYS121 Integrated Science-Physics

## W2D4 Gravitation

### References:

- [1] David Halliday, Jearl Walker, Resnick Jearl, 'Fundamentals of Physics', (Wiley, 2018)
  - [2] Doug Giancoli, 'Physics for Scientists and Engineers with modern physics', (Pearson, 2009)
  - [3] Hugh D. Young, Roger A. Freedman, 'University Physics with Modern Physics', (Pearson, 2012)
- And others specified when needed.

8.7.2. A rubber ball is dropped from rest from a height  $h$ . The ball bounces off the floor and reaches a height of  $2h/3$ . How can we use the principle of the conservation of mechanical energy to interpret this observation?

- a) During the collision with the floor, the floor did not push hard enough on the ball for it to reach its original height.
- b) Some of the ball's potential energy was lost in accelerating it toward the floor.
- c) The force of the earth's gravity on the ball prevented it from returning to its original height.
- d) Work was done on the ball by the gravitational force that reduced the ball's kinetic energy.
- e) Work was done on the ball by non-conservative forces that resulted in the ball having less total mechanical energy after the bounce.



# Learning outcomes

- Calculate the gravitation from Newton's law
- Compare the gravitation near the surface of Earth (gravity), inside/outside of Earth.
- Analyze the work done by gravitation.
- Explain and apply Kepler's laws of planetary motions.
- Describe Einstein's theory of gravitation.

# Newton's Law of Gravitation

- The gravitational force
  - Holds us to the Earth
  - Holds Earth in orbit around the Sun
  - Holds the Sun together with the stars in our galaxy
  - Reaches out across intergalactic space to hold together the Local Group of galaxies
  - Holds together the Local Supercluster of galaxies
  - Pulls the supercluster toward the Great Attractor
  - Attempts to slow the expansion of the universe
  - Is responsible for black holes
- Gravity is far-reaching and very important!

- Gravitational attraction depends on **the amount of “stuff”** an object is made of
- Earth has lots of “stuff” and produces a large attraction
- The force is always attractive, never repulsive
- **Gravitation** is the tendency for bodies to attract each other
- Newton realized this attraction was responsible for maintaining the orbits of celestial bodies
- Newton's **law of gravitation** defines (*from reasoning*) the strength of this attractive force between particles
- For apple & Earth: 0.8 N; for 2 people:  $< 1 \mu\text{N}$

- The magnitude of the force is given by:

$$F = G \frac{m_1 m_2}{r^2} \quad (\text{Newton's law of gravitation}). \quad \text{Equation (13-1)}$$

- Where  $G$  is the **gravitational constant**:

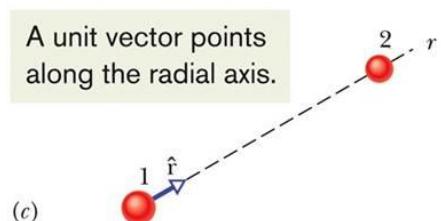
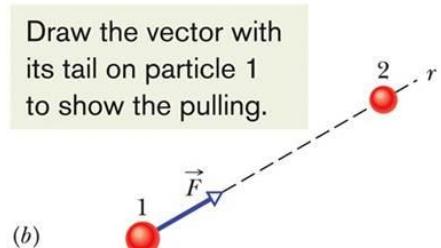
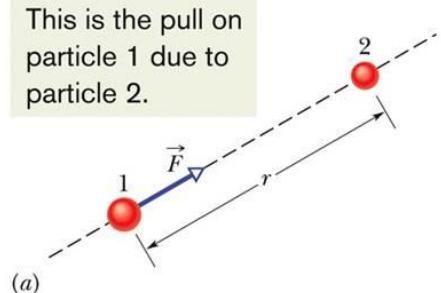
$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \quad \text{Equation (13-2)}$$

- The force always points from one particle to the other, so this equation can be written in vector form:

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}. \quad \text{Equation (13-3)}$$

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center (*proved from calculus*).

- The shell theorem describes gravitational attraction for objects
- Earth is a nesting of shells, so we feel Earth's mass as if it were all located at its center
- Gravitational force forms third-law force pairs
- E.g., Earth-apple and apple-Earth forces are both 0.8 N

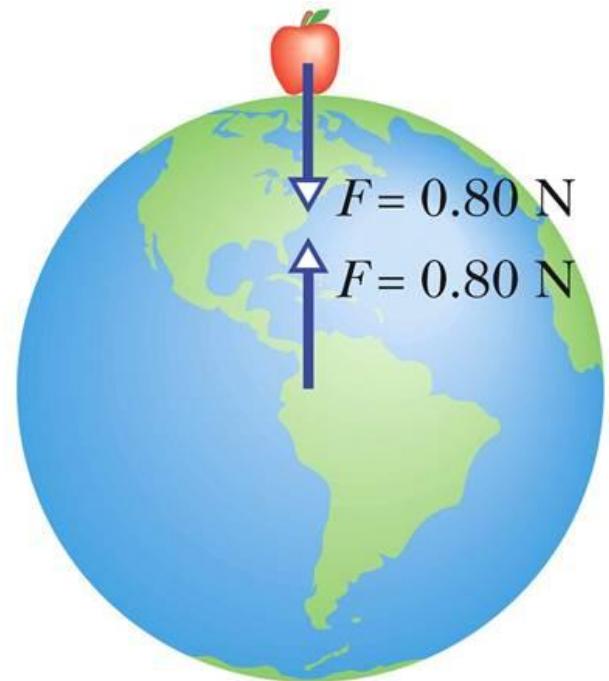


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**Figure 13-2**

The difference in mass causes the difference in accelerations of the apple and Earth (Newton's 3<sup>rd</sup> law of motion):

$\sim 10 \text{ m/s}^2$  vs.  $\sim 10^{-25} \text{ m/s}^2$



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**Figure 13-3**

## Checkpoint 1

A particle is to be placed, in turn, outside four objects, each of mass  $m$ : (1) a large uniform solid sphere, (2) a large uniform spherical shell, (3) a small uniform solid sphere, and (4) a small uniform shell. In each situation, the distance between the particle and the center of the object is  $d$ . Rank the objects according to the magnitude of the gravitational force they exert on the particle, greatest first.

**Answer:** All exert equal forces on the particle

# the Principle of Superposition

- Find the net gravitational force by the **principle of superposition**: the net is the sum of individual effects
- Add the individual forces as vectors:

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^n \vec{F}_{1i}. \quad \text{Equation (13-5)}$$

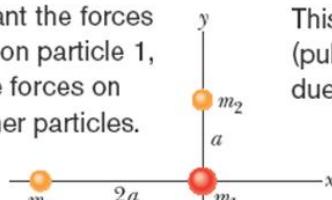
- For a real (extended) object, this becomes an integral:

$$\vec{F}_1 = \int d\vec{F}, \quad \text{Equation (13-6)}$$

- If the object is a uniform sphere or shell we can treat its mass as being at its center instead (*prove from calculus*).

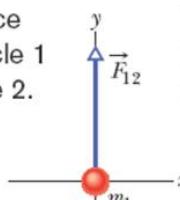
# Example Summing two forces:

We want the forces (pulls) on particle 1, *not* the forces on the other particles.



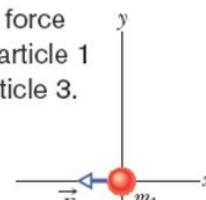
(a)

This is the force (pull) on particle 1 due to particle 2.

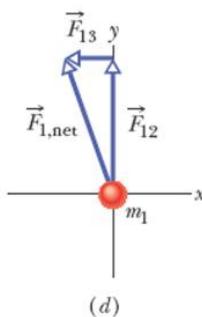


(b)

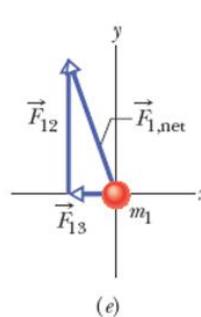
This is the force (pull) on particle 1 due to particle 3.



(c)



(d)



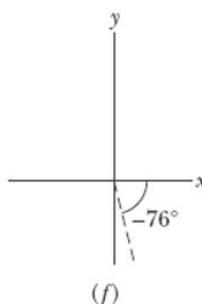
(e)

This is one way to show the net force on particle 1. Note the head-to-tail arrangement.

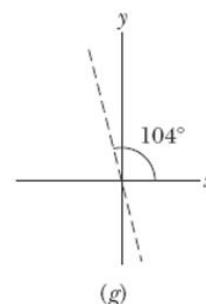
This is another way, also a head-to-tail arrangement.

A calculator's inverse tangent can give this for the angle.

But this is the correct angle.



(f)



(g)

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Figure 13-4

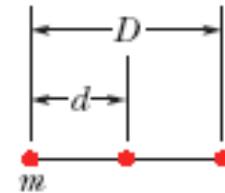
## Checkpoint 2

The figure shows four arrangements of three particles of equal masses. (a) Rank the arrangements according to the magnitude of the net gravitational force on the particle labeled  $m$ , greatest first. (b) In arrangement 2, is the direction of the net force closer to the line of length  $d$  or to the line of length  $D$ ?

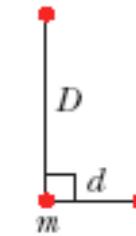
### Answer:

- (a) 1, 2 & 4, 3  
(b) line of length  $d$

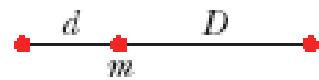
(1)



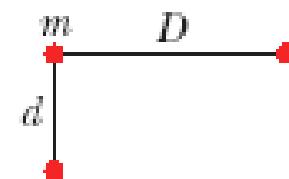
(2)



(3)



(4)



# Gravitation Near Earth's Surface (gravity)

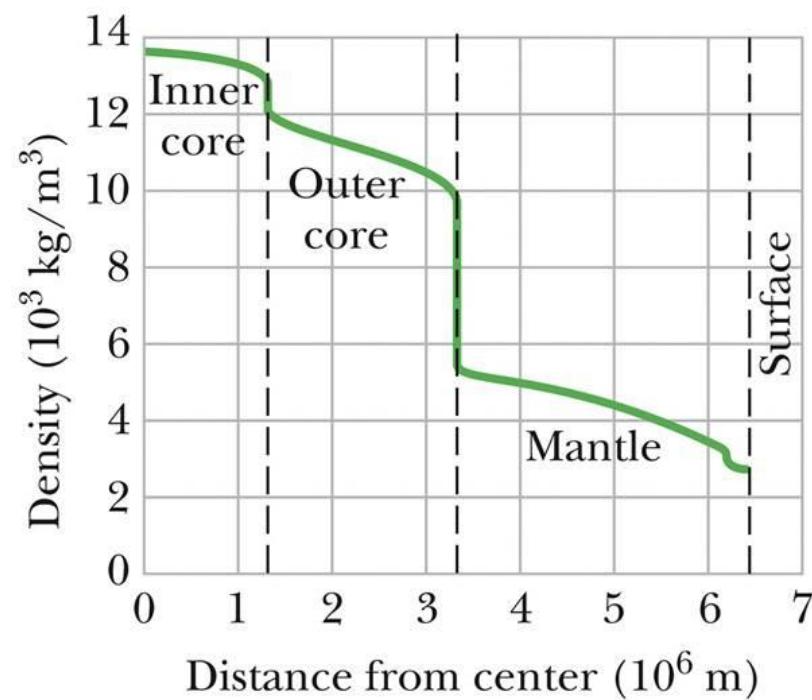
- Combine  $F = \frac{GMm}{r^2}$  and  $F = ma_g$ :

$$a_g = \frac{GM}{r^2}. \quad \text{Equation (13-11)}$$

- This gives the magnitude of the gravitational acceleration at a given distance from the center of the Earth.

Altitude (km)		Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

- The calculated  $a_g$  will differ slightly from the measured  $g$  at any location
- Therefore the calculated gravitational force on an object will not match its weight for the same 3 reasons:
  1. Earth's mass is not uniformly distributed, Figure 13-5
  2. Earth is not a sphere
  3. Earth rotates

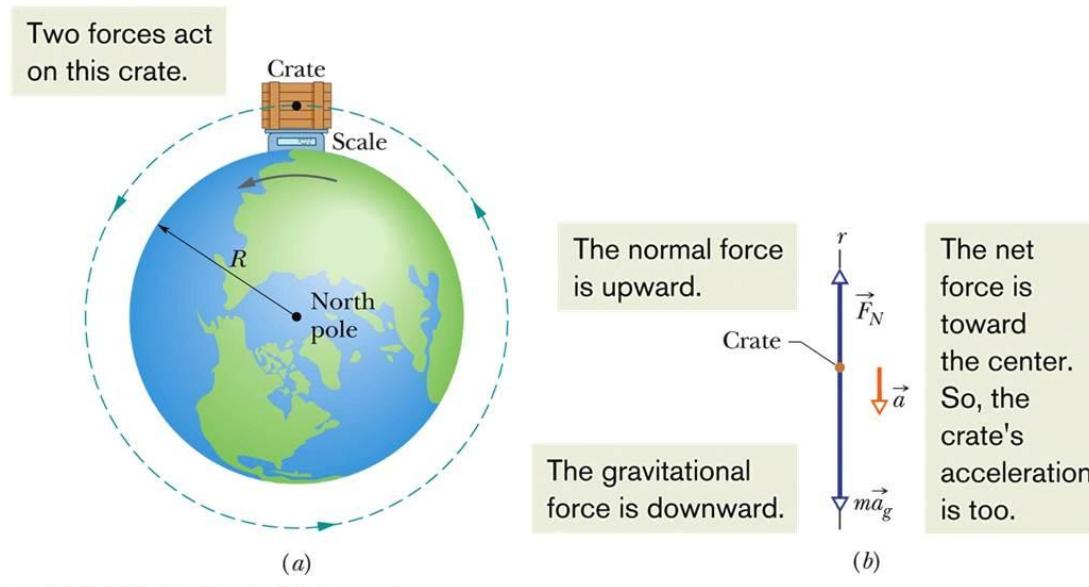


**Figure 13-5**

**Example** Difference in gravitational force and weight due to rotation at the equator:

$$F_N - ma_g = m(-\omega^2 R).$$

**Equation (13-12)**



**Figure 13-6**

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# Gravitational Potential Energy

- Note that gravitational potential energy is a property of a pair of particles/objects
- We cannot divide it up to say how much of it “belongs” to each particle in the pair
- We often speak as of the “gravitational potential energy of an baseball” in the ball-Earth system
- We get away with this because the energy change appears almost entirely as kinetic energy of the ball
- *This is only true for systems where one object is much less massive than the other*

- Gravitational potential energy for a two-particle system is written (negative of the work done by gravitation):

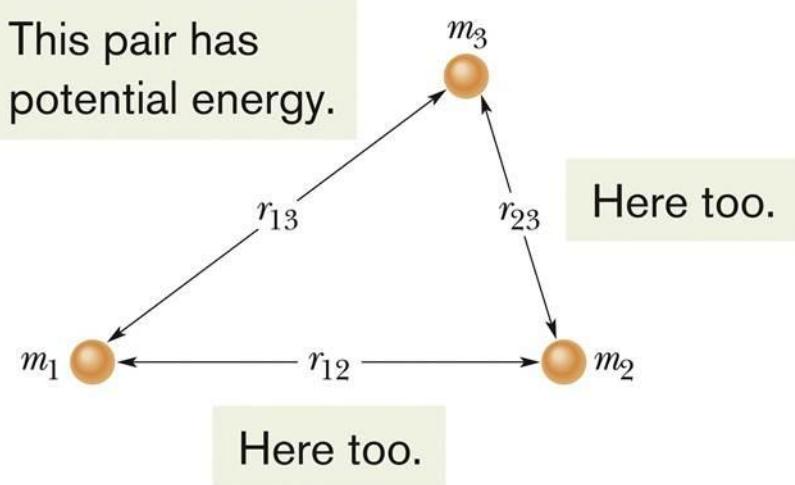
$$U = -\frac{GMm}{r} \quad (\text{gravitational potential energy}). \quad \text{Equation (13-21)}$$

- *Note this value is negative and approaches 0 for  $r \rightarrow \infty$*
- The gravitational potential energy of a system is the sum of potential energies for all pairs of particles

$$U = -\left( \frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}} \right).$$

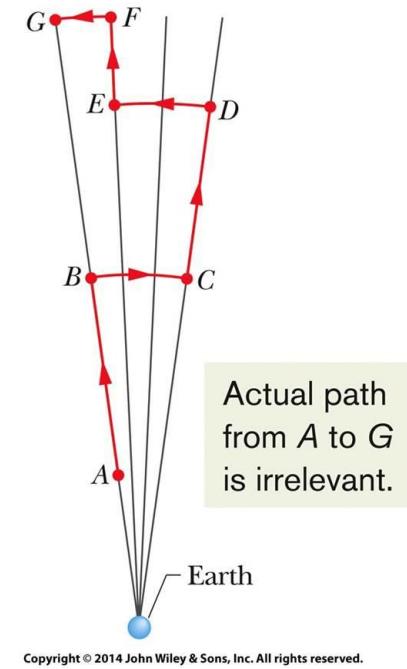
**Equation (13-22)**

**Figure 13-8**



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- The gravitational force is conservative
- The work done by this force does not depend on the path followed by the particles, only the difference in the initial and final positions of the particles
- Since the work done is independent of path, so is the gravitational potential energy change



**Figure 13-10**

$$\Delta U = U_f - U_i = -W. \quad \text{Equation (13-26)}$$

- Newton's law of gravitation can be derived from the potential energy formula by taking the derivative (*or vice versa*)
- For a projectile to *escape the gravitational pull* of a body, it must come to rest only at infinity, if at all
- (if) *At rest at infinity*:  $K = 0$  and  $U = 0$  (*because  $r \rightarrow \infty$* )  
‘reference point’
- So  $K + U$  must be at least 0 at the surface of the body (*why?*):

$$K + U = \frac{1}{2}mv^2 + \left( -\frac{GMm}{R} \right) = 0. \quad \text{Equation (13-28)}$$

$$v = \sqrt{\frac{2GM}{R}}.$$

- Rockets launch eastward to take advantage of Earth's rotational speed, to reach  $v$  more easily

**Table 13-2** Some Escape Speeds

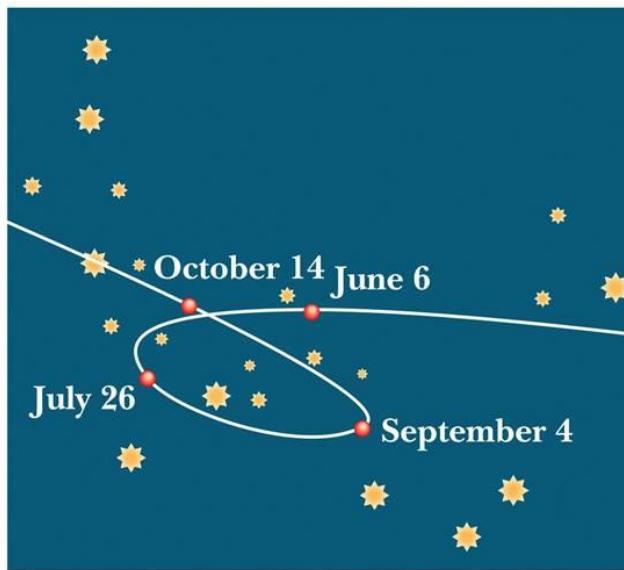
Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres <sup>a</sup>	$1.17 \times 10^{21}$	$3.8 \times 10^5$	0.64
Earth's moon <sup>a</sup>	$7.36 \times 10^{22}$	$1.74 \times 10^6$	2.38
Earth	$5.98 \times 10^{24}$	$6.37 \times 10^6$	11.2
Jupiter	$1.90 \times 10^{27}$	$7.15 \times 10^7$	59.5
Sun	$1.99 \times 10^{30}$	$6.96 \times 10^8$	618
Sirius B <sup>b</sup>	$2 \times 10^{30}$	$1 \times 10^7$	5200
Neutron star <sup>c</sup>	$2 \times 10^{30}$	$1 \times 10^4$	$2 \times 10^5$

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**Table 13-2**

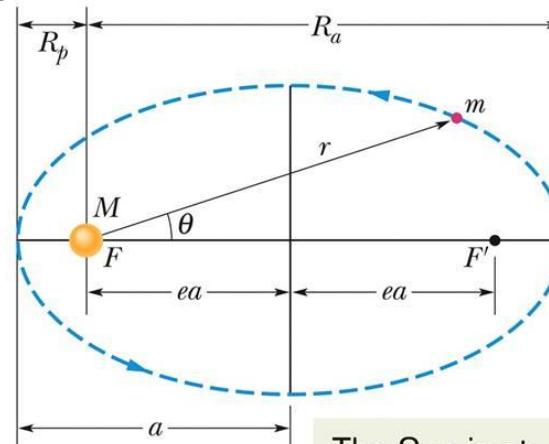
# Planets and Satellites: Kepler's Laws

- The motion of planets in the solar system was a puzzle for astronomers, especially curious motions such as in Figure 13-11
- Johannes Kepler (1571-1630) derived laws of motion using Tycho Brahe's (1546-1601) measurements



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Figure 13-11

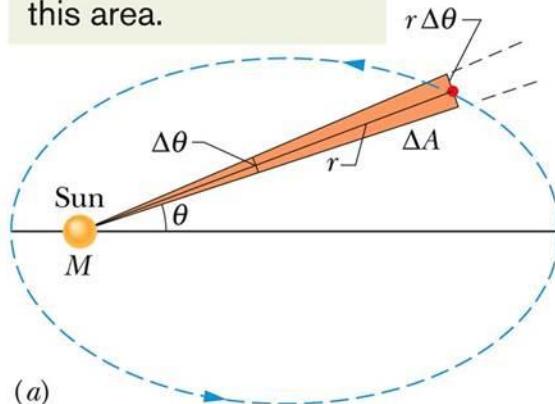


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Figure 13-12

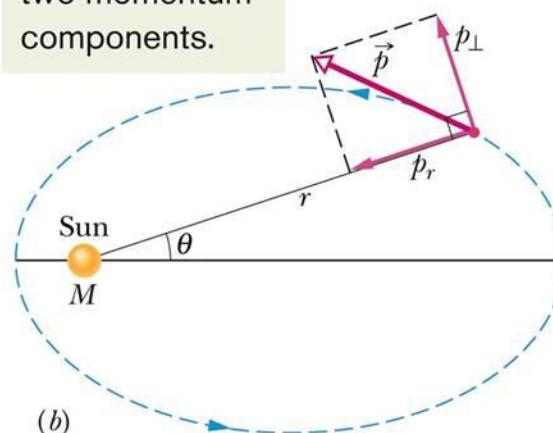
- 1. The Law of Orbits:** All planets move in elliptical orbits, with the Sun at one focus.
  - The orbit is defined by its **semimajor axis**  $a$  and its **eccentricity**  $e$
  - An eccentricity of zero corresponds to a circle
  - Eccentricity of Earth's orbit is 0.0167
- 2. The Law of Areas:** A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate  $\frac{dA}{dt}$  at which it sweeps out area  $A$  is constant.
  - Equivalent to the law of conservation of angular momentum
- 3. The Law of Periods:** The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

The planet sweeps out  
this area.



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These are the  
two momentum  
components.



**Figure 13-13**

- The law of periods can be written mathematically as:

$$T^2 = \left( \frac{4\pi^2}{GM} \right) r^3 \quad (\text{law of periods}). \qquad \text{Equation (13-34)}$$

- Holds for elliptical orbits if we replace  $r$  with  $a$

**Table 13-3** Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis $a(10^{10} \text{ m})$	Period $T(\text{y})$	$\frac{T^2}{a^3} \left( 10^{-34} \frac{y^2}{m^3} \right)$
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

# Satellites: Orbits and Energy

- Relating the centripetal acceleration of a satellite to the gravitational force, we can rewrite as energies:

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}, \quad \text{Equation (13-38)}$$

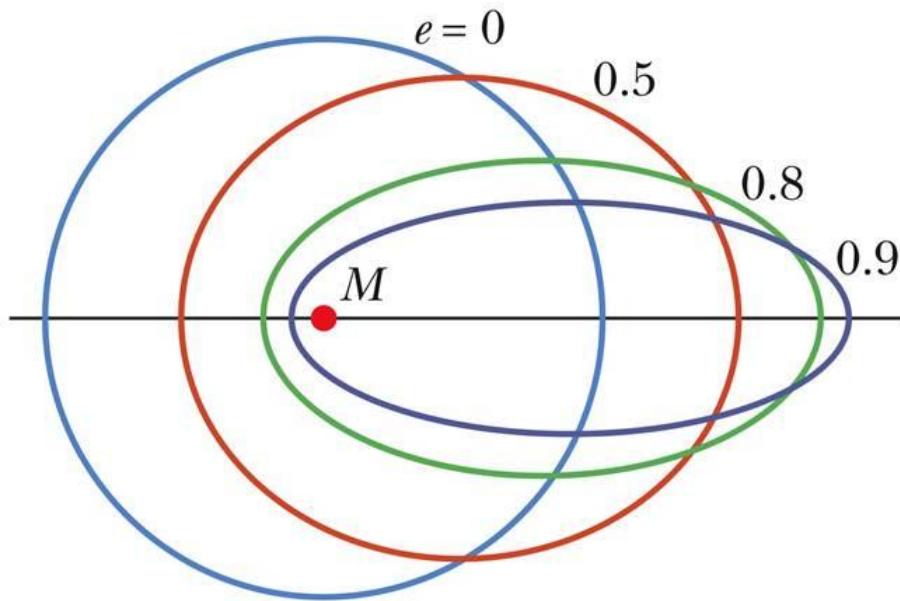
- Meaning that:

$$K = -\frac{U}{2} \quad (\text{circular orbit}). \quad \text{Equation (13-39)}$$

- Therefore the total mechanical energy is:

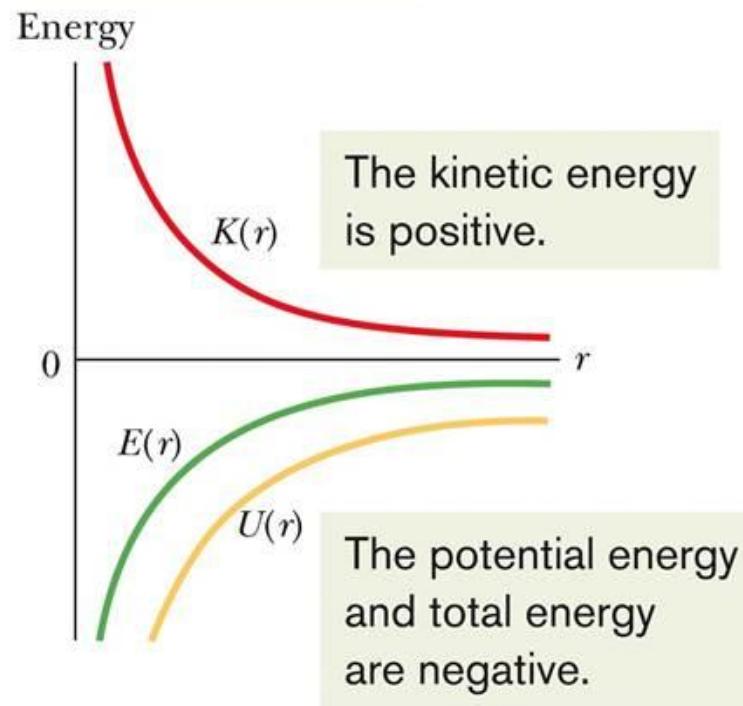
$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$
$$E = -\frac{GMm}{2r} \quad (\text{circular orbit}). \quad \text{Equation (13-40)}$$

- Total energy  $E$  is the negative of the kinetic energy
- For an ellipse, we substitute  $a$  for  $r$
- Therefore the energy of an orbit depends only on its semimajor axis, not its eccentricity
- All orbits in Figure 13-15 have the same energy



**Figure 13-15**

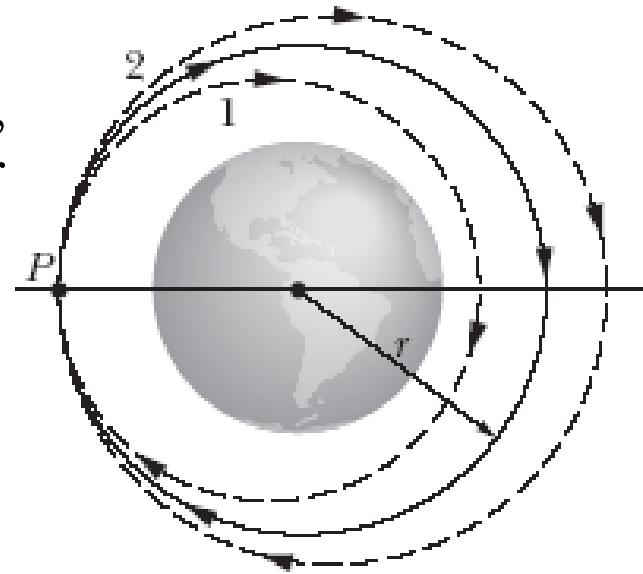
This is a plot of a satellite's energies versus orbit radius.



**Figure 13-16**

## Checkpoint 5

In the figure here, a space shuttle is initially in a circular orbit of radius  $r$  about Earth. At point  $P$ , the pilot briefly fires a forward-pointing thruster to decrease the shuttle's kinetic energy  $K$  and mechanical energy  $E$ . (a) Which of the dashed elliptical orbits shown in the figure will the shuttle then take? (b) Is the orbital period  $T$  of the shuttle (the time to return to  $P$ ) then greater than, less than, or the same as in the circular orbit?

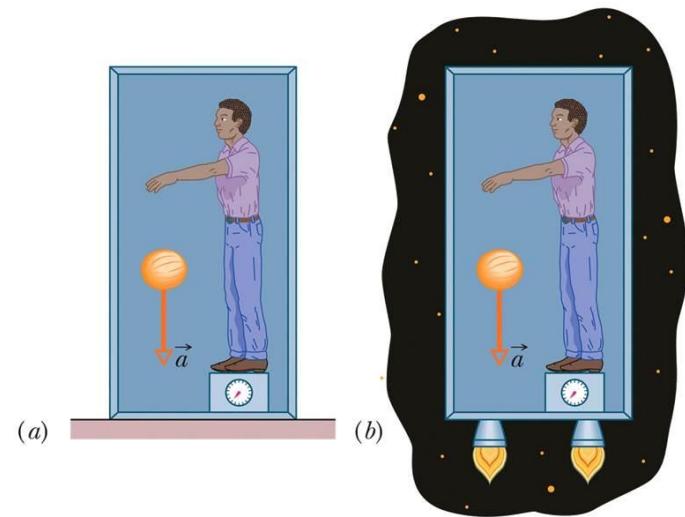


### Answer:

- (a) orbit 1, since the energy has decreased
- (b) the semimajor axis has decreased, so the period decreases

# Einstein and Gravitation

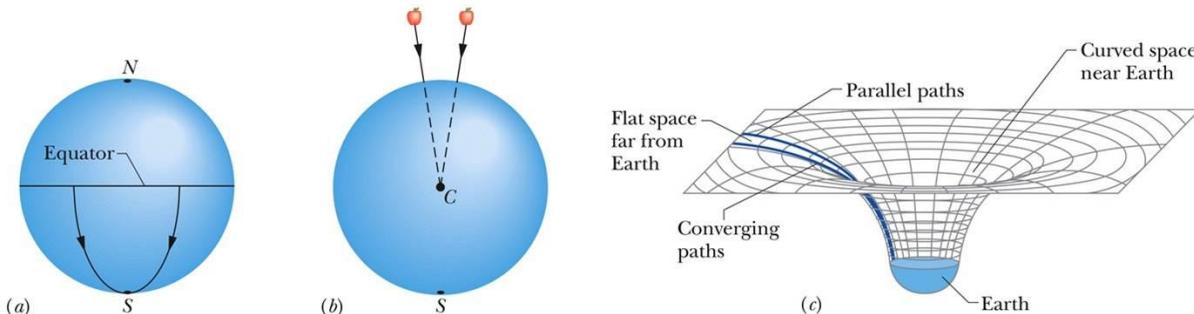
- The **general theory of relativity** describes gravitation
- Its fundamental postulate is the **principle of equivalence**
- *Gravitation and acceleration are equivalent*
- The experimenter inside this box is unable to tell whether he is on Earth experiencing  $g = 9.8 \text{ m/s}^2$ , or in free space accelerating at  $9.8 \text{ m/s}^2$ .



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**Figure 13-18**

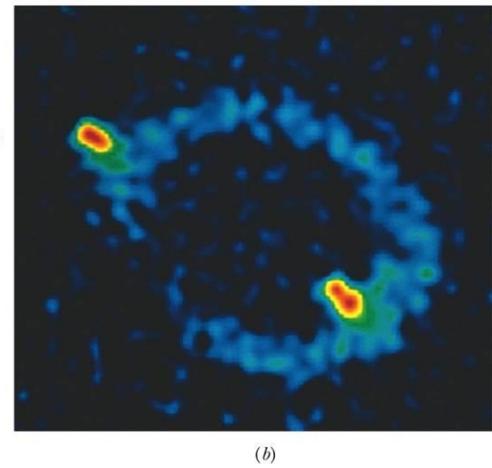
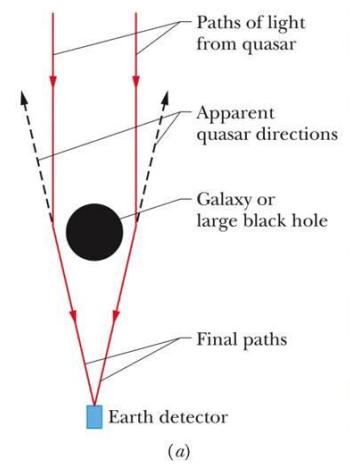
- Space (space-time) is curved
- Analogies: In (a) and (b), paths that appear to be parallel, along the surface of the Earth or falling toward the Earth's center, actually converge
- We can see why by stepping “outside” the curved Earth, but we can't step “outside” of curved space



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**Figure 13-19**

- But we can observe the curvature of space
- Light bends as it passes by massive objects: an effect called *gravitational lensing*
- In extreme cases we observe the light coming from multiple places, or bent into an Einstein ring



Courtesy National Radio Astronomy Observatory

**Figure 13-20**



# Learning outcomes

- ✓ Calculate the gravitation from Newton's law
- ✓ Compare the gravitation near the surface of Earth (gravity), inside/outside of Earth.
- ✓ Analyze the work done by gravitation.
- ✓ Explain and apply Kepler's laws of planetary motions.
- ✓ Describe Einstein's theory of gravitation.

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}.$$

$$U = -\frac{GMm}{r} \quad (\text{gravitational potential energy}).$$

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

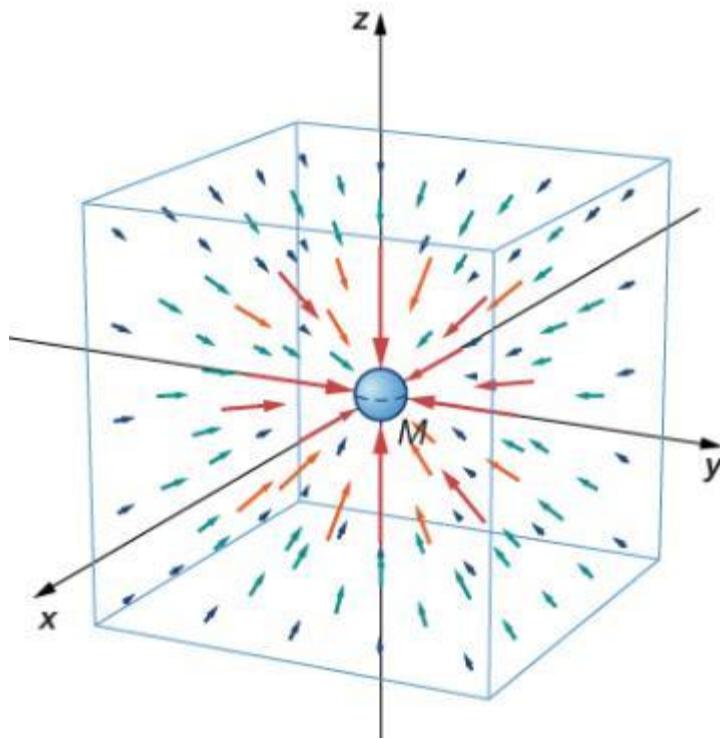
# CHAPTER REVIEW AND EXAMPLES, DEMOS

<https://openstax.org/books/university-physics-volume-1/pages/13-summary>

Note two important items with this definition. First,  $U \rightarrow 0$  as  $r \rightarrow \infty$ . The potential energy is zero when the two masses are infinitely far apart. Only the difference in  $U$  is important, so the choice of  $U=0$  for  $r=\infty$  is merely one of convenience. (Recall that in earlier gravity problems, you were free to take  $U=0$  at the top or bottom of a building, or anywhere.) Second, note that  $U$  becomes increasingly more negative as the masses get closer. That is consistent with what you learned about potential energy in [Potential Energy and Conservation of Energy](#). As the two masses are separated, positive work must be done against the force of gravity, and hence,  $U$  increases (becomes less negative). All masses naturally fall together under the influence of gravity, falling from a higher to a lower potential energy.

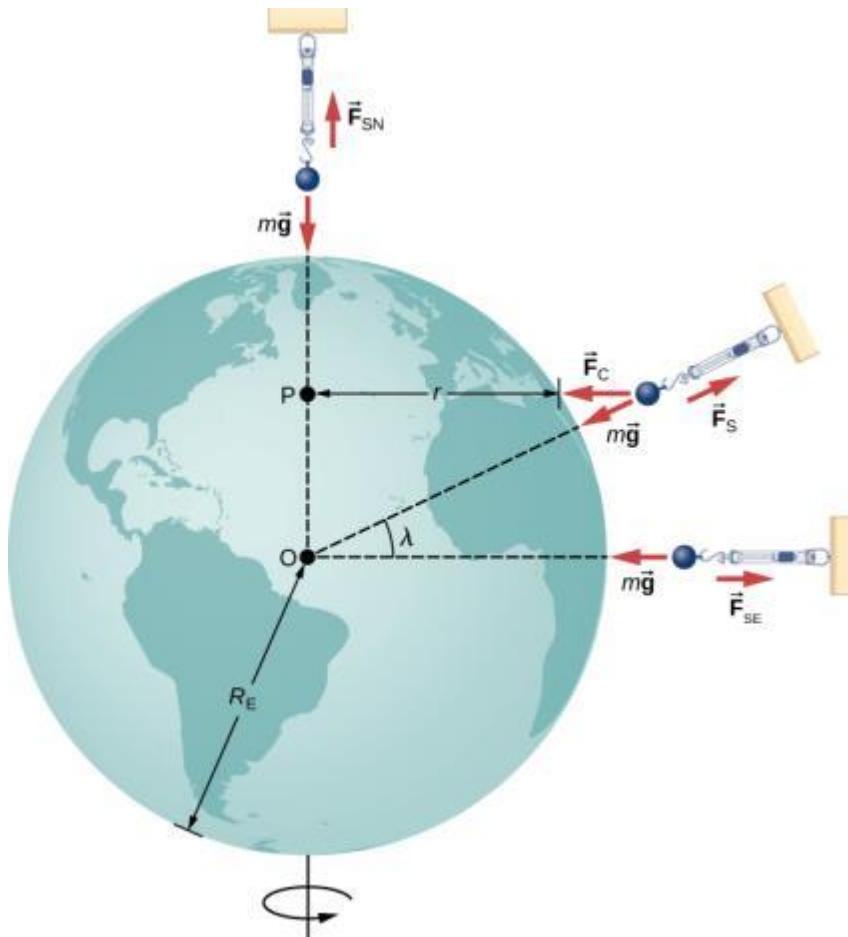
<https://openstax.org/books/university-physics-volume-1/pages/13-3-gravitational-potential-energy-and-total-energy>

## FIGURE 13.8

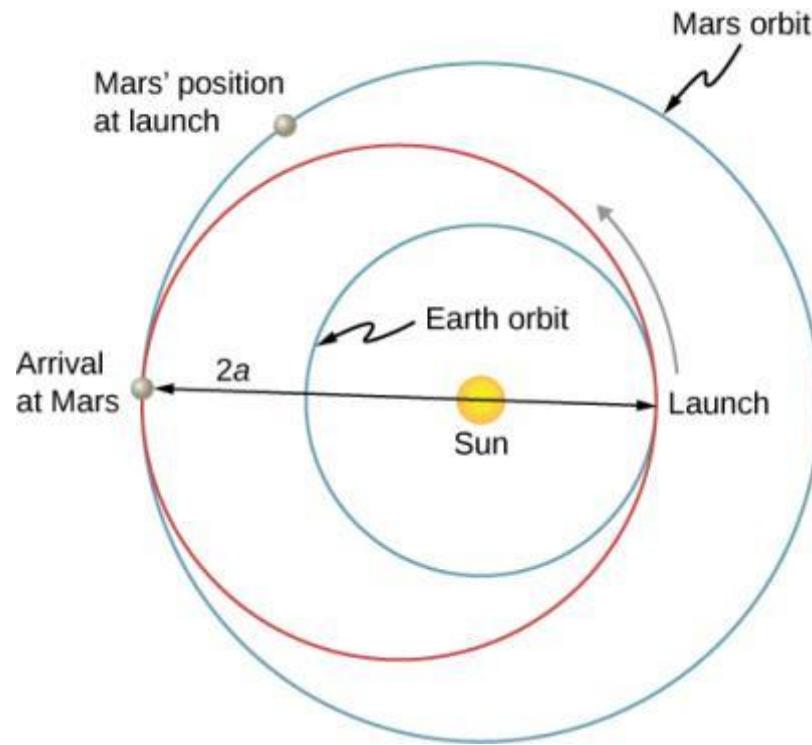


A three-dimensional representation of the gravitational field created by mass  $M$ . Note that the lines are uniformly distributed in all directions. (The box has been added only to aid in visualization.)

## FIGURE 13.9



For a person standing at the equator, the centripetal acceleration ( $a_c$ ) is in the same direction as the force of gravity. At latitude  $\lambda$ , the angle between  $a_c$  and the force of gravity is  $\lambda$  and the magnitude of  $a_c$  decreases with  $\cos\lambda$ .

**FIGURE 13.19**

The transfer ellipse has its perihelion at Earth's orbit and aphelion at Mars' orbit.

## EXAMPLE 13. 6

### Lifting a Payload

How much energy is required to lift the 9000–kg *Soyuz* vehicle from Earth’s surface to the height of the ISS, 400 km above the surface?

## EXAMPLE 13. 12

### Energy Required to Orbit

In [Example 13. 6](#), we calculated the energy required to simply lift the 9000–kg *Soyuz* vehicle from Earth’s surface to the height of the ISS, 400 km above the surface. In other words, we found its *change* in potential energy. We now ask, what total energy change in the *Soyuz* vehicle is required to take it from Earth’s surface and put it in orbit with the ISS for a rendezvous ([Figure 13. 15](#))? How much of that total energy is kinetic energy?

# Questions

