PHYS121 Integrated Science-Physics

W2T2 Kinetic Energy and Work

References:
[1] David Halliday, Jearl Walker, Resnick Jearl, 'Fundamentals of Physics', (Wiley, 2018)
[2] Doug Giancoli, 'Physics for Scientists and Engineers with modern physics', (Pearson, 2009)
[3] Hugh D. Young, Roger A. Freedman, 'University Physics with Modern Physics', (Pearson, 2012)
And others specified when needed.

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Learning outcomes

- Calculate the kinetic energy of moving objects.
- Find out the work done by forces (gravity, friction force, time/position dependent forces).
- Use the work-kinetic energy theorem to solve problems.
- Solve problems involving power.

Kinetic Energy

- Energy is required for any sort of motion
- Energy:
  - Is a scalar quantity assigned to an object or a system of objects
  - Can be changed from one form to another
  - Is *conserved* in a closed system, that is the total amount of energy of all types is always the same
- In this chapter we discuss one type of energy (kinetic energy)
- We also discuss one method of transferring energy (work)

- **Kinetic energy:**
  - The faster an object moves, the greater its kinetic energy
  - Kinetic energy is zero for a stationary object
- For an object with v well <u>below the speed of light</u>:

$$K = \frac{1}{2}mv^2$$

Equation (7-1)

- The unit of kinetic energy is a **joule** (J)

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2.$$

Equation (7-2)

Work and Kinetic Energy

- Account for changes in kinetic energy by saying energy has been transferred to or from the object
- In a transfer of energy via a force, **work** is:
  - Done on the object by the force

Work W is energy transferred to or from an object by means of a force acting on the object. *Energy transferred to the object is positive work, and energy transferred from the object is negative work.*

- Start from force equation and 1-dimensional velocity (assume constant acceleration):

$$F_x = ma_x, \qquad \textbf{Equation (7-3)}$$

$$v^2 = v_0^2 + 2a_x d. \qquad \textbf{Equation (7-4)}$$

- Rearrange into kinetic energies (work-K_E theorem):

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d. \qquad \textbf{Equation (7-5)}$$

- The left side is now the change in energy
- Therefore work is:

$$W = F_x d. \qquad \textbf{Equation (7-6)}$$

To calculate the work a force does on an object as the object moves through some displacement, we use *only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.*

- For an angle ϕ between force and displacement:

$$W = Fd \cos\phi$$ **Equation (7-7)**

- As vectors we can write:

$$W = \vec{F} \cdot \vec{d}$$ **Equation (7-8)**

- *Notes on these equations:*
  - *Force is constant*
  - *Object is particle-like (rigid)*
  - *Work can be positive or negative*

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta.$$

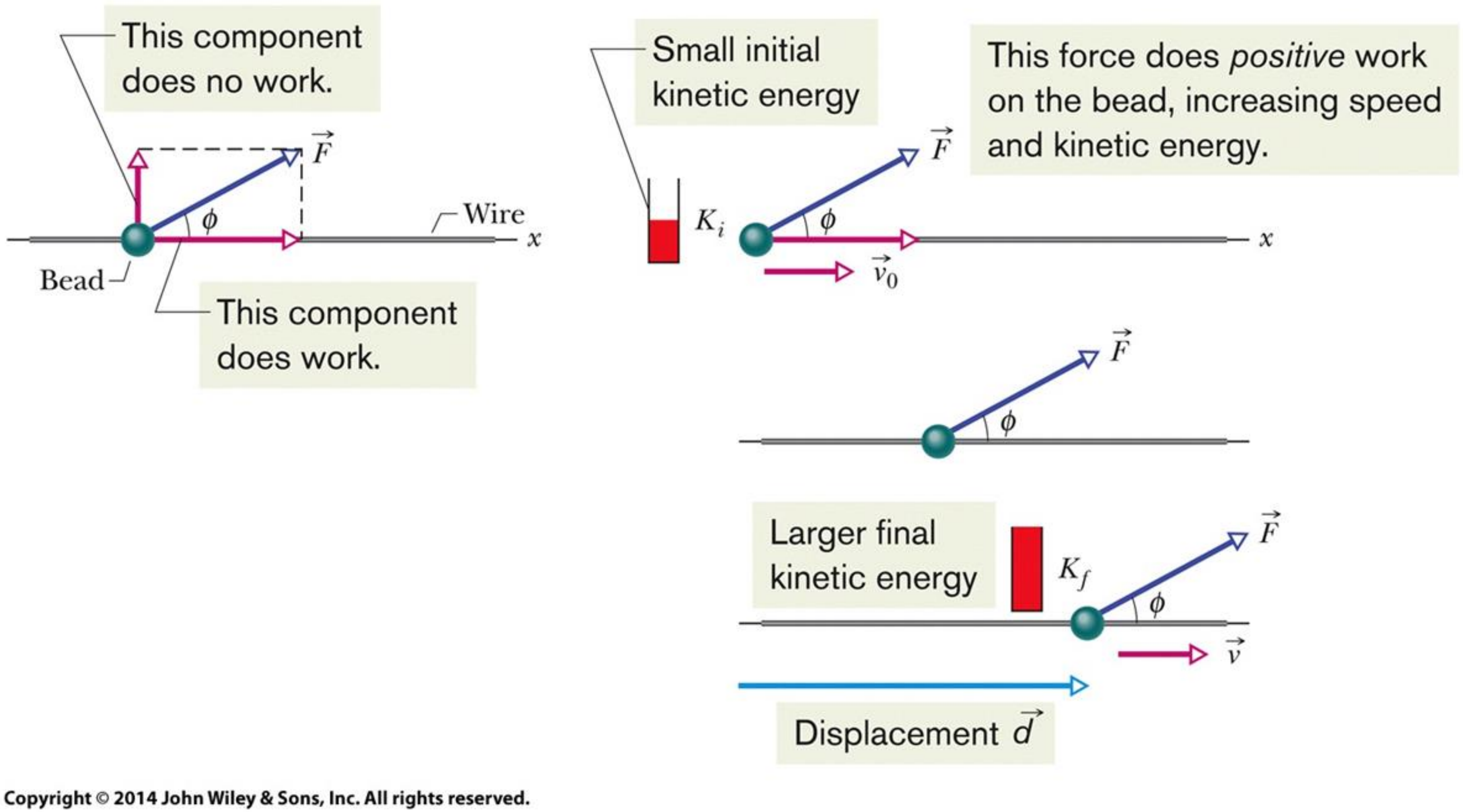


Figure 7-2

- Work has the SI unit of joules (J), the same as energy

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

- For two or more forces, the **net work** is the sum of the works done by all the individual forces
- Two methods to calculate net work:
  - We can find all the works and sum the individual work terms.
  - We can take the vector sum of forces (F_{net}) and calculate the net work once
- The **work-kinetic energy theorem** states:

$$\Delta K = K_f - K_i = W,$$ **Equation (7-11)**

- (change in kinetic energy) = (the net work done)

Checkpoint 1

A particle moves along an *x* axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from −3 m/s to −2 m/s and (b) from −2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

Answer:

(a) energy decreases

(b) energy remains the same

(c) work is negative for (a), and work is zero for (b)

Work Done by the Gravity

- We calculate the work as we would for any force
- Our equation is:

$$W_g = mgd\cos\phi$$ **Equation (7-12)**

- For a rising object:

$$W_g = mgd\ \cos 180^\circ = mgd\left(-1\right) = -mgd.$$ **Equation (7-13)**

- For a falling object:

$$W_g = mgd\ \cos 0^\circ = mgd\left(+1\right) = +mgd.$$ **Equation (7-14)**

- *Applies regardless of path*

- Figure 7-7 shows the orientations of forces and their associated works for upward and downward displacement
- Note that the works (in 7-16) need not be equal, they are only equal if the initial and final kinetic energies are equal (page 156 on the textbook)
- If the works are unequal, you will need to know the difference between initial and final kinetic energy to solve for the work

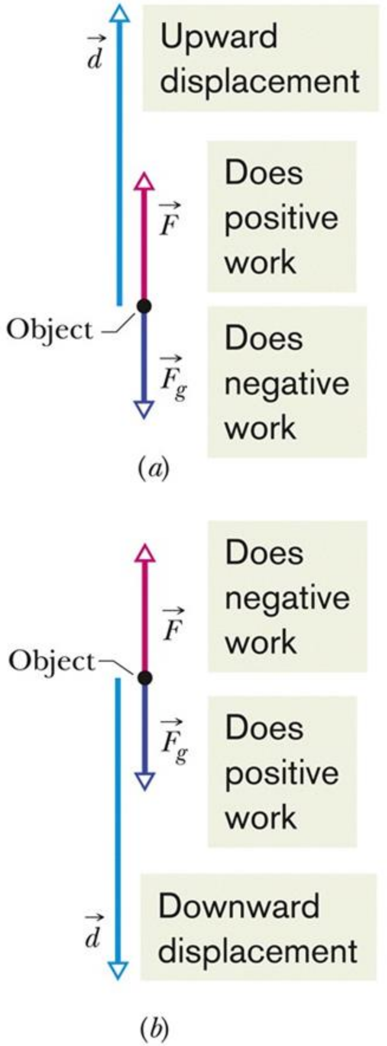


Figure 7-7

Examples You are a passenger:

- Being pulled up a ski-slope
  - Tension does positive work, gravity does negative work

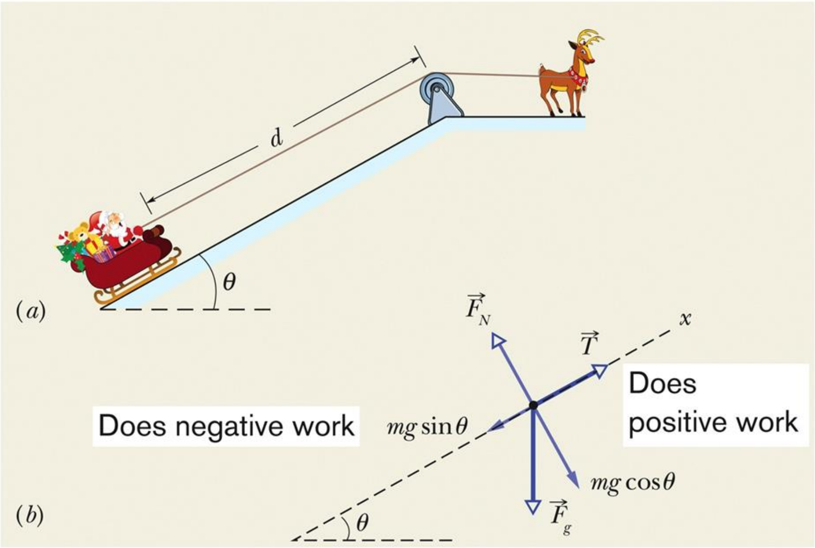


Figure 7-8

Examples You are a passenger:

- Being lowered down in an elevator
  - Tension does negative work, gravity does positive work

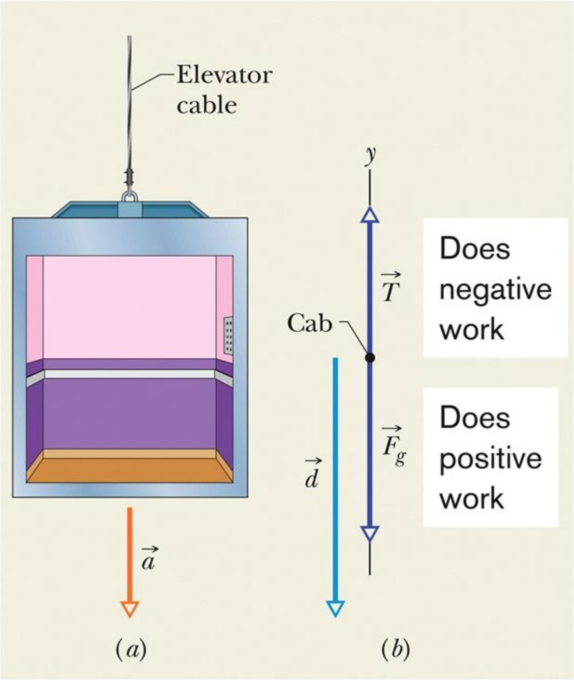


Figure 7-9

Work Done by a Spring Force

- A **spring force** is the variable force from a spring
  - A spring force has a particular mathematical form
  - Many forces in nature have this form
- Figure (a) shows the spring in its **relaxed state**: since it is neither compressed nor extended, no force is applied
- If we stretch or extend the spring it resists, and exerts *a restoring force* that attempts to return the spring to its relaxed state

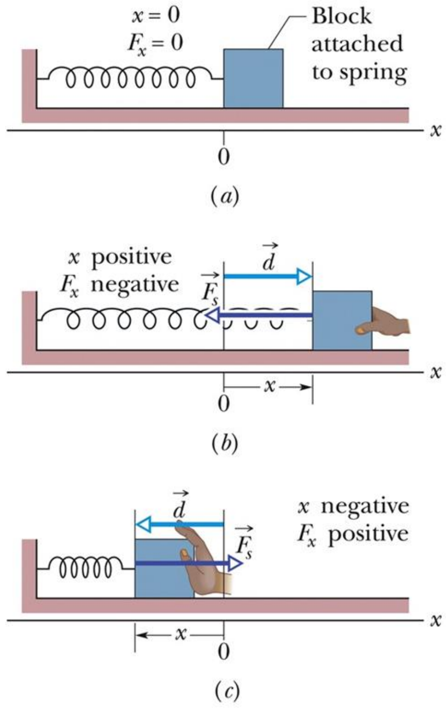


Figure 7-10

- The spring force is given by **Hooke's law:**

$$\vec{F}_s = -k\vec{d}$$ **Equation (7-20)**

- The *negative sign* represents that the force always opposes the displacement (*calculated with respect to the unstretched position*)
- The **spring constant** k is a is a measure of the stiffness of the spring
- This is a variable force (function of position) and it exhibits a linear relationship between F and d
- For a spring along the x-axis we can write:

$$F_x = -kx$$ **Equation (7-21)**

- We can find the work by integrating:

$$W_s = \int_{x_i}^{x_f} \vec{F}_x d\vec{x} = \int_{x_i}^{x_f} -kx\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

- For an initial position of $x = 0$:

$$W_s = -\frac{1}{2}kx^2$$

Equation (7-26)

- *For an applied force where the initial and final kinetic energies are zero*:

$$W_a = -W_s.$$

Equation (7-28)

- If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

Example: Work done on a spring.

(a) A person pulls on a spring, stretching it 3.0 cm, which requires a maximum force of 75 N. How much work does the person do? (b) If, instead, the person compresses the spring 3.0 cm, how much work does the person do?

Slide based on Ref. [2]

Work Done by a General Variable Force

- We take a one-dimensional example
- We need to integrate the work equation (which normally applies only for a constant force) over the change in position
- We can show this process by an approximation with rectangles under the curve (numerical integration)

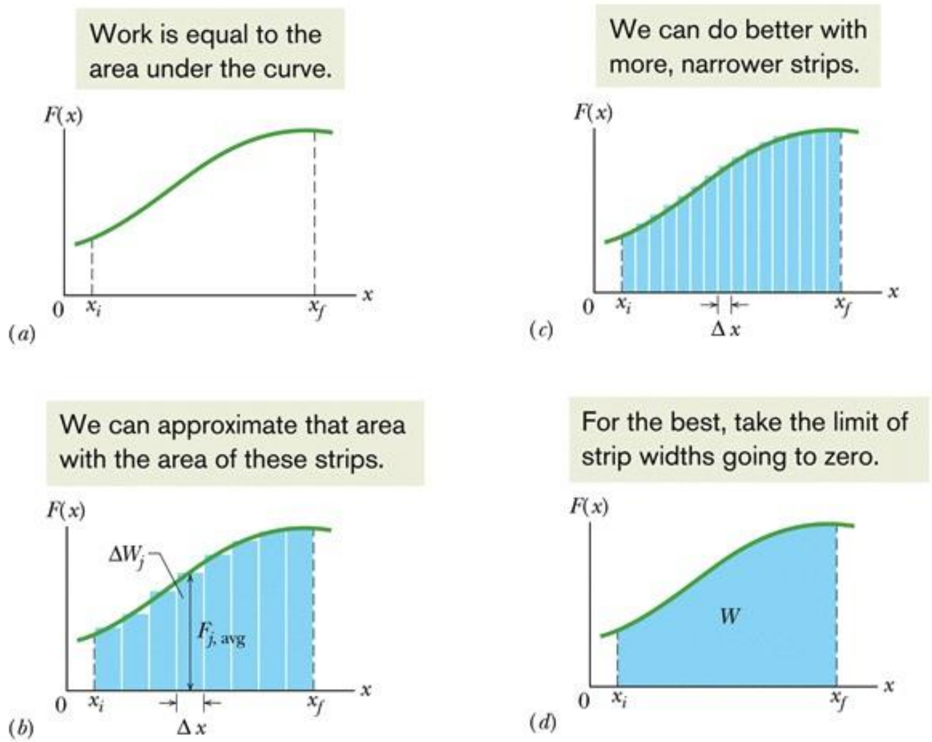


Figure 7-12

- Our sum of rectangles would be:

$$W = \lim_{\Delta x \to 0} \sum F_{j,\,\mathrm{avg}} \Delta x. \qquad \textbf{Equation (7-31)}$$

- As an integral this is:

$$W = \int_{x_i}^{x_f} F(x)\,dx \qquad \textbf{Equation (7-32)}$$

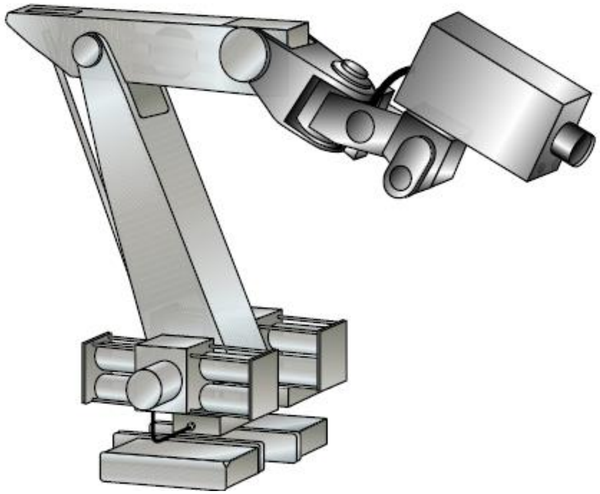
- In three dimensions, we integrate each separately (by components):

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz. \qquad \textbf{Equation (7-36)}$$

- The work-kinetic energy theorem still applies!

Example: Force as a function of x.

A robot arm that controls the position of a video camera in an automated surveillance system is manipulated by a motor that exerts a force on the arm. The force is given by



$$F(x) = F_0\left(1 + \frac{1}{6}\frac{x^2}{x_0^2}\right),$$

where F_0 = 2.0 N, x_0 = 0.0070 m, and x is the position of the end of the arm. If the arm moves from x_1 = 0.010 m to x_2 = 0.050 m, how much work did the motor do?

Power

- **Power** is the time rate at which a force does work
- A force does W work in a time Δt; the **average power** due to the force is:

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Equation (7-42)

- The **instantaneous power** at a particular time is:

$$P = \frac{dW}{dt}$$

Equation (7-43)

- The SI unit for power is the watt (W): 1 W = 1 J/s
- Therefore work-energy can be written as (power) × (time) e.g. kWh, the kilowatt-hour (your electric bill)

- Solve for the instantaneous power using the definition of work:

$$P = \frac{dW}{dt} = \frac{F\ \cos\phi\ dx}{dt} = F\ \cos\phi\left(\frac{dx}{dt}\right),$$

$$P = Fv\cos\phi. \qquad \textbf{Equation (7-47)}$$

- Or: $$P = \vec{F}\cdot\vec{v} \qquad \textbf{Equation (7-48)}$$

Checkpoint 3

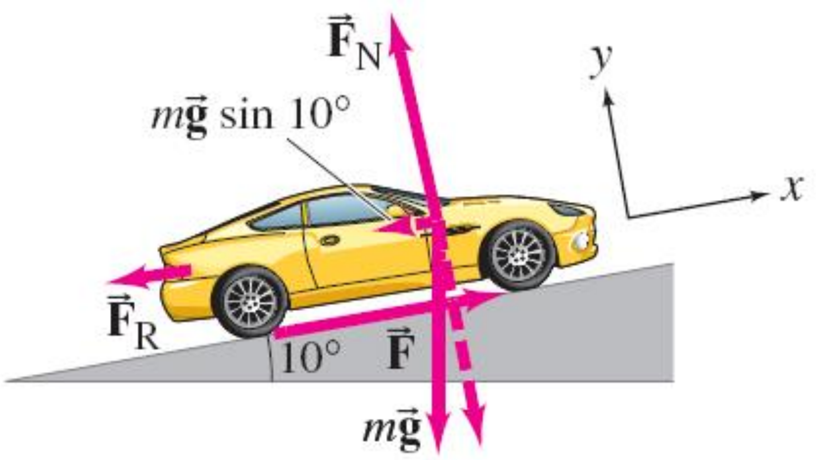
A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Answer:

$$\text{Zero}\left(\text{consider } P = Fv\cos\phi, \text{ and note that } \phi = 90^\circ\right)$$

Example: Power needs of a car.

Calculate the power required of a 1400-kg car under the following circumstances: (a) the car climbs a 10° hill (a fairly steep hill) at a steady 80 km/h; and (b) the car accelerates along a level road from 90 to 110 km/h in 6.0 s to pass another car. Assume that the average retarding force 阻力 on the car is F_R = 700 N throughout.



Learning outcomes

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CHAPTER REVIEW AND EXAMPLES, DEMOS

https://openstax.org/books/university-physics-volume-1/pages/7-summary

Unlike friction or other dissipative forces, described in Example 7.2, the total work done against gravity, over any closed path, is zero. Positive work is done against gravity on the upward parts of a closed path, but an equal amount of negative work is done against gravity on the downward parts. In other words, work done *against* gravity, lifting an object *up*, is “given back” when the object comes back down. Forces like gravity (those that do zero work over any closed path) are classified as conservative forces and play an important role in physics.

https://openstax.org/books/university-physics-volume-1/pages/7-1-work

Because velocity is a relative quantity, you can see that the value of kinetic energy must depend on your frame of reference. You can generally choose a frame of reference that is suited to the purpose of your analysis and that simplifies your calculations. One such frame of reference is the one in which the observations of the system are made (likely an external frame). Another choice is a frame that is attached to, or moves with, the system (likely an internal frame). The

https://openstax.org/books/university-physics-volume-1/pages/7-2-kinetic-energy

Example 7.8

Special Names for Kinetic Energy

(a) A player lobs a mid-court pass with a 624-g basketball, which covers 15 m in 2 s. What is the basketball's horizontal translational kinetic energy while in flight? (b) An average molecule of air, in the basketball in part (a), has a mass of 29 u, and an average speed of 500 m/s, relative to the basketball. There are about 3×10^{23} molecules inside it, moving in random directions, when the ball is properly inflated. What is the average translational kinetic energy of the random motion of all the molecules inside, relative to the basketball? (c) How fast would the basketball have to travel relative to the court, as in part (a), so as to have a kinetic energy equal to the amount in part (b)?

Example 7.11

Pull-Up Power

An 80-kg army trainee does 10 pull-ups in 10 s (**Figure 7.14**). How much average power do the trainee's muscles supply moving his body? (*Hint:* Make reasonable estimates for any quantities needed.)

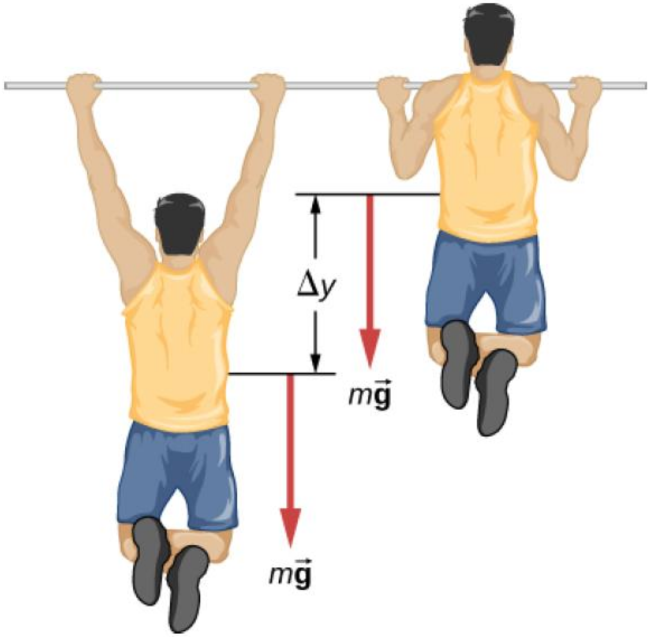

Figure 7.14 What is the power expended in doing ten pull-ups in ten seconds?

Questions