PHYS 121 — HW2 Solutions

October 31, 2025

Q1. Spider-Man Swing (Tension Strength)

Bottom of arc (radius L). Forces: tension T up along string, weight mg down. Centripetal: $T - mg = mv^2/L$. Hence

$$T = m\,\frac{v^2}{L} + mg.$$

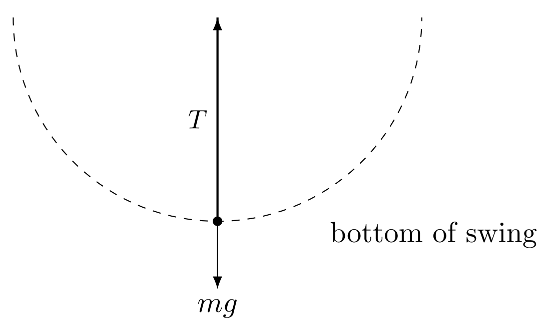
Assume traffic speed $\approx 13\,\mathrm{m/s}$, so $v \approx 26\,\mathrm{m/s}$; take $m = 75\,\mathrm{kg}$, $L \approx 75\,\mathrm{m}$ (25 floors). Then

$$T \approx 75 \cdot \frac{26^2}{75} + 75 \cdot 9.8 \approx 1410\,\mathrm{N}.$$

Minimum silk diameter from allowable stress σ (take dragline silk $\sigma \approx 1.0 \times 10^9\,\mathrm{Pa}$): area $A = T/\sigma$, $d = 2\sqrt{A/\pi}$:

$$A \approx 1.41 \times 10^{-6}\,\mathrm{m}^2, \quad \boxed{d_{\min} \approx 1.34\,\mathrm{mm}}.$$

For equal safety, the better cord is the one with higher allowable stress (smaller required diameter). Dragline silk ($\sim$1 GPa) outperforms typical nylon rope ($\sim$0.08 −−0.10 GPa).



Q2. Motorboat Terminal Speed

Given $m = 190\,\mathrm{kg}$, thrust $F_T = 40.0\,\mathrm{N}$, linear drag $b\,v$ with $b = 2\,\mathrm{N\,s/m}$. Newton's 2nd law gives

$$m\,\frac{dv}{dt} = F_T - bv.$$

(i) Rearrange to isolate v and t: $\dfrac{dv}{dt} = \dfrac{F_T}{m} - \dfrac{b}{m}v$.

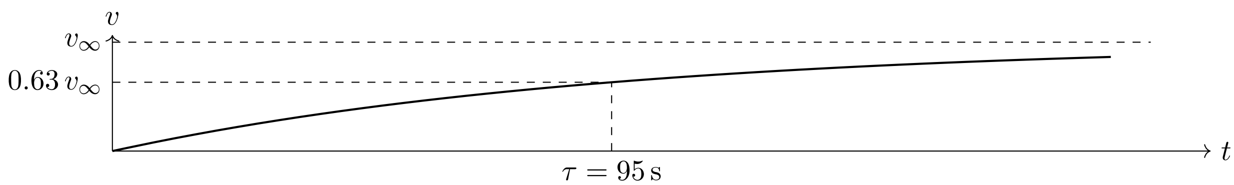
(ii) Define the terminal speed $v_\infty = \dfrac{F_T}{b}$ so that $\frac{dv}{dt} = \frac{b}{m}(v_\infty - v)$.

(iii) Separate variables: $\dfrac{dv}{v_\infty - v} = \dfrac{b}{m}\,dt$ and integrate from 0 to t (start from rest):

$$-\ln\!\Big(1 - \frac{v}{v_\infty}\Big) = \frac{b}{m}\,t.$$

(iv) Solve for $v(t)$: $v(t) = v_\infty\Big(1 - e^{-(b/m)\,t}\Big)$.

Numerics: $v_\infty = \dfrac{40.0}{2} = \boxed{20\,\mathrm{m/s}}$, and $\dfrac{b}{m} = \dfrac{2}{190} = \dfrac{1}{95}$. The time when $v = 0.63\,v_\infty$ is $t \approx \boxed{95\,\mathrm{s}}$ (since $e^{-1} = 0.37$). Checks: (1) As $t \to \infty$, $e^{-(b/m)t} \to 0$, so $v \to v_\infty$. (2) At $t \to 0$, $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{b}{m}v_\infty = \dfrac{F_T}{m}$, equal to $F_{\rm net}/m$ initially (drag = 0 at rest). Initial acceleration = $\boxed{0.211\,\mathrm{m/s^2}}$.



Q3. Box up an Incline (constant speed)

Incline 30°, $m = 45\,\mathrm{kg}$, $\mu_k = 0.35$, horizontal F. Along the plane (up positive):

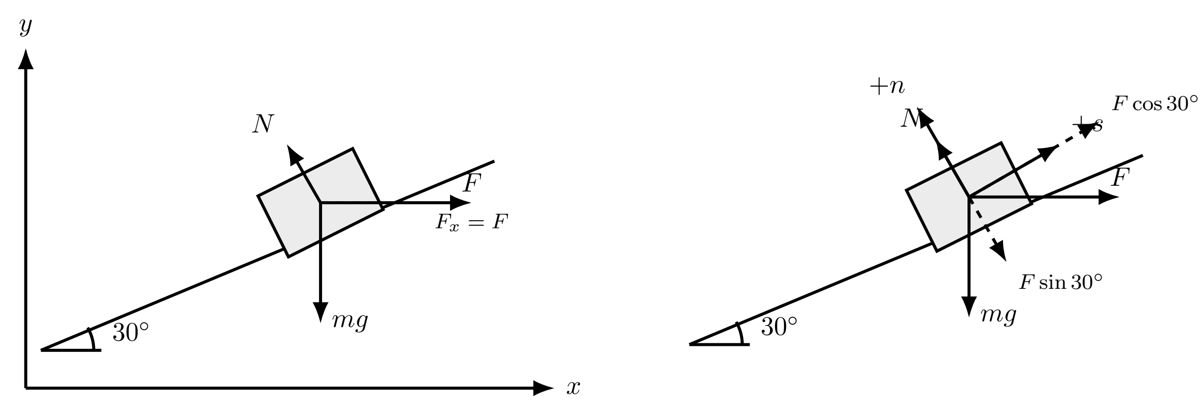
$$F\cos 30^\circ = mg\sin 30^\circ + \mu_k\big(mg\cos 30^\circ + F\sin 30^\circ\big).$$

Solve: $F = \dfrac{mg(\sin 30^\circ + \mu_k\cos 30^\circ)}{\cos 30^\circ - \mu_k \sin 30^\circ} = \boxed{5.13 \times 10^2\,\mathrm{N}}$. Distance $s = 8.0\,\mathrm{m}$. Work:

$$W_F = Fs\cos 30^\circ = \boxed{3.55\times 10^3\,\mathrm{J}}, \quad N = mg\cos 30^\circ + F\sin 30^\circ \Rightarrow f_k = \mu_k N \approx 224\,\mathrm{N},$$

$$W_f = -f_k s = \boxed{-1.79\times 10^3\,\mathrm{J}}, \quad W_g = -mg(s\sin 30^\circ) = \boxed{-1.76\times 10^3\,\mathrm{J}}.$$

Signs: $W_F > 0$ because $\vec{F}$ has a component along the displacement up the plane; $W_f < 0$ since kinetic friction opposes motion; $W_g < 0$ since gravity's component is down the plane while displacement is up the plane; $W_N = 0$ because $\vec{N} \perp$ displacement. *Check (Work–Energy):* $W_{\rm net} = \Delta K$. Constant speed $\Rightarrow \Delta K = 0 \Rightarrow W_{\rm net} = 0$, which our totals satisfy. Thus $W_{\rm net} \approx 0$ consistent with constant speed.



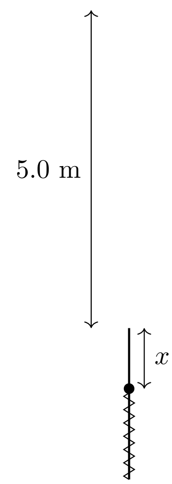
Incline-aligned view is better for writing component equations; world-axes view is good for visualization.

Q4. Vertical Spring Gun

$k = 14\,\mathrm{N/cm} = 1400\,\mathrm{N/m}$, $m = 15\,\mathrm{g}$, top is $5.0\,\mathrm{m}$ above the uncompressed end. Energy from compressed to top (include rise by x while leaving):

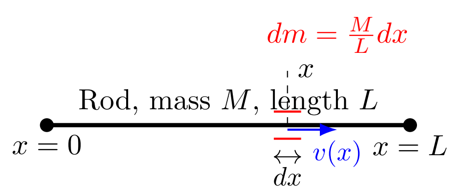
$$\tfrac{1}{2}kx^2 = mg(5.0 + x).$$

Quadratic $700x^2 - 0.147x - 0.735 = 0$ gives physical root $\boxed{x \approx 3.25\,\mathrm{cm}}$.



Q5. Kinetic Energy of a Massive Spring

Rod model: Consider a rod of length L and total mass M. At position x (measured from one end, $0 \le x \le L$), take a slice of length dx. The mass of this slice is $dm = (M/L)dx$, where M/L is the linear mass density. If the rod has a linear speed profile $v(x) = (x/L)v$ (where v is the speed at the far end $x = L$), then each slice moves with speed $v(x)$.



Kinetic energy element: The kinetic energy of the slice at position x is

$$dK = \tfrac{1}{2}dm\,v(x)^2 = \tfrac{1}{2}\frac{M}{L}\left(\frac{x}{L}v\right)^2 dx = \tfrac{1}{2}\frac{Mv^2}{L^3}x^2\,dx.$$

Integration: The total kinetic energy is obtained by integrating dK from $x = 0$ to $x = L$:

$$K = \int_0^L dK = \frac{Mv^2}{2L^3}\int_0^L x^2\,dx = \frac{Mv^2}{2L^3}\cdot\frac{L^3}{3} = \boxed{\tfrac{1}{6}Mv^2}.$$

Comparison with point-mass result: For a point mass of mass M moving at speed v, the kinetic energy would be $K_{\text{point}} = \frac{1}{2}Mv^2$. Our result is $\frac{1}{6}Mv^2$, which is exactly *one-third* of the point-mass result.

Explanation: The difference arises because the rod has a *distribution of speeds.* In our model, only the far end ($x = L$) moves at speed v; parts closer to $x = 0$ move slower (e.g., the center at $x = L/2$ moves at $v/2$). Since kinetic energy depends on v^2, the slower-moving parts contribute less energy than if the entire mass were moving at speed v. The factor of 1/3 reflects the quadratic averaging over the linear speed profile: $\langle v^2 \rangle = \frac{1}{L}\int_0^L \left(\frac{x}{L}v\right)^2 dx = \frac{v^2}{3}$.

Q6. Power, Drag, and Fuel

At $v = 15\,\mathrm{m/s}$, power to wheels $P_{15} = 20\,\mathrm{hp} = 1.492 \times 10^4\,\mathrm{W}$. (i) Constant drag F_d:

$$F_d = \frac{P_{15}}{v} = \boxed{9.95 \times 10^2\,\mathrm{N}}, \quad P_{30} = F_d\,(30\,\mathrm{m/s}) = \boxed{2.98 \times 10^4\,\mathrm{W}}\ (\approx 40\,\mathrm{hp}).$$

Energy for 10 km: $W = F_d d = \boxed{9.95 \times 10^6\,\mathrm{J}}$ to wheels; fuel energy (25% efficiency): $\boxed{3.98 \times 10^7\,\mathrm{J}}$ at either speed.

(ii) Linear drag $F_d = kv$ matched at 15 m/s: $k = F_d/v = \boxed{66.3\,\mathrm{N\,s/m}}$. Then

$$P(v) = kv^2,\ P_{30} = k(30)^2 = \boxed{5.97 \times 10^4\,\mathrm{W}}\ (\approx 80\,\mathrm{hp}).$$

Energy for 10 km: $W = kvd$ gives $W_{15} = \boxed{9.95 \times 10^6\,\mathrm{J}}$, $W_{30} = \boxed{1.99 \times 10^7\,\mathrm{J}}$; fuel energies are $4\times$ these.

Reflection: In (i) doubling speed doubles power and leaves energy per distance unchanged; in (ii) doubling speed quadruples power and doubles energy per distance. Real driving matches the trend that higher speed needs much more power and fuel per mile. Aerodynamic drag is approximately quadratic, $F \propto v^2$, so realistically $P \propto v^3$ and energy per distance $\propto v^2$, which increases even faster with speed than our linear-drag model predicts.

Q7. Peg and Complete Loop

Release from horizontal, length a. After catching the peg, the small-circle radius is $a - h$.

- Energy (release $\to$ bottom of big circle, drop a):

$$mg\,a = \tfrac{1}{2}mv_b^2 \quad \Rightarrow \quad v_b^2 = 2ga, \quad v_b = \sqrt{2ga}.$$

- Energy (bottom $\to$ top of small circle, rise $2(a-h)$):

$$\tfrac{1}{2}mv_b^2 = \tfrac{1}{2}mv_t^2 + mg\,2(a-h) \quad \Rightarrow \quad v_t^2 = v_b^2 - 4g(a-h) = 2ga - 4g(a-h), \quad v_t = \sqrt{2ga - 4g(a-h)}.$$

Non-slack at the top requires a strictly positive tension. At the top, taking inward as positive, the radial balance is

$$T + mg = m\,\frac{v_t^2}{a-h} \quad \Rightarrow \quad T = m\,\frac{v_t^2}{a-h} - mg.$$

Requiring $T > 0$ gives

$$v_t^2 > g\,(a-h).$$

Using $v_t^2 = 2ga - 4g(a-h)$ gives $2a - 4(a-h) > (a-h)$, i.e., $-2a + 4h > a - h$. Hence $5h > 3a$, so $\boxed{h > \frac{3}{5}a}$.

Q8. Bungee Jump Estimate

Mass 72 kg, free fall 15 m before stretch; spring $k = 50\,\mathrm{N/m}$. Energy from platform to lowest point:

$$mg(15 + x) = \tfrac{1}{2}kx^2.$$

Solve $25x^2 - 705.6x - 10584 = 0$ for $x > 0$: $\boxed{x \approx 39.1\,\mathrm{m}}$. Total drop $\boxed{15 + x \approx 54.1\,\mathrm{m}}$.

Q9. Bullet + Block (inelastic)

$m_b = 0.220\,\mathrm{kg}$ at $v_i = 400\,\mathrm{m/s}$ embeds into $M = 1.30\,\mathrm{kg}$ on a frictionless table. Momentum conservation:

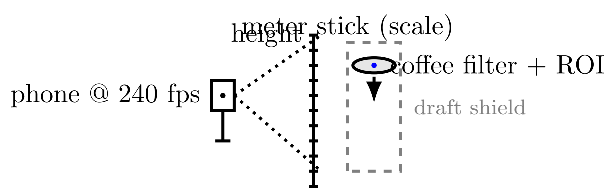
$$v_f = \frac{m_b v_i}{m_b + M} = \boxed{57.9\,\mathrm{m/s}}\ \text{(east)}.$$

Impulses: $J_b = m_b(v_f - v_i) = \boxed{-75.3\,\mathrm{N\,s}}$ (on bullet, west), $J_M = M v_f = \boxed{75.3\,\mathrm{N\,s}}$ (on block, east). Over 3 ms: $\boxed{\overline{F} \approx 2.51 \times 10^4\,\mathrm{N}}$.

Vectors drawn to scale: $p_b = p_f = 88\,\mathrm{N\,s}$ east. With $m_b = 0.220\,\mathrm{kg}$, $v_i = 400\,\mathrm{m/s}$, $(m_b + M)v_f = 88\,\mathrm{N\,s} \Rightarrow v_f = 57.9\,\mathrm{m/s}$.

Note: Q10 omitted per course announcement.

Bonus. Terminal-speed experiment (0–2 pts)



- **Setup**: Indoors, drop a coffee filter beside a vertical meter stick; phone on tripod ≥240 fps, optical axis perpendicular, use a clear bin as a draft shield and a colored ROI dot on the filter.
- **Data**: Track $y(t)$ of the ROI every 2–4 frames across 3–5 drops; repeat with 1–4 stacked filters to vary mass and confirm v_t scaling.

- **Model/fit**: Quadratic drag gives $v(t) = v_t \tanh(gt/v_t)$ and the position model $y(t) = y_0 + \frac{v_t^2}{g} \ln\cosh(gt/v_t)$; fit $y(t)$ directly (less noisy) to extract v_t. Linear-drag cross-check: $v(t) = v_t(1 - e^{-t/\tau})$.
- **Uncertainty**: Systematic (parallax, scale skew, timing) and random (drafts, tilt). Mitigate via long camera distance, careful alignment, averaging trials, and verifying that late-time $v(t)$ plateaus.

Assitance notes:

LLMs were used as translators and concept explainers. No direct solutions were provided.