PHYS 121 — HW3 (Redesigned): Gravity, Oscillations & Waves

Learning-by-doing version with scaffolds, required sketches, and reflective checks.

Student Name: ____________________________. NetID: _________________.

Q1. Gravitation from a ball (8 pts)

A small test mass *m* is placed a distance r from the center of a uniform solid sphere (radius R, mass M). 1) Drawing: sphere with center O, radial axis; mark cases $r<R$ and $r\geq R$. 2) Using the shell theorem (state it), write g(r) inside and outside the sphere. 3) Compute the gravitational potential energy U(r) in each region with $U(\infty)=0$; ensure continuity at $r=R$. 4) Evaluate magnitudes of U(r) at $r=R/2$, $r=R$, and $r=2R$; comment which is largest/smallest and why. 5) (i) Does g(r) vary linearly with r inside? (ii) As $r\to\infty$, does $U\to 0^{-}$?

Q2. Explore Vesta (10 pts)

Assume Vesta is a sphere of diameter 520 km and mass 2.67×10^{20} kg. 1) Compute escape speed from the surface. 2) Find the orbital period for a circular orbit 15 km above the surface. 3) Reflection: Why is a spherical model only marginally useful? What features of Vesta [check the image of Vesta] make real orbits tricky?

Q3. Kepler's law—Pluto's small moons (8 pts)

Charon orbits Pluto at 19,600 km with a period of 6.39 d. Two small satellites are at 48,000 km and 64,000 km. Neglect mutual interactions between these two. 1) Drawing: top view with three circular orbits labeled by radius. 2) Apply Kepler's third law to compute both periods by scaling from Charon; show the 3/2 power clearly. 4) Report both in days and hours.

Q4. Gravity and SHM in a straight tunnel (14 pts)

A narrow tunnel passes through the center of a uniform, airless spherical planet (radius R, density ρ). A ball with mass *m* is released from rest at the surface into the tunnel. Suppose the size of the tunnel (larger than the ball) is small and does not have a noticeable effect on the overall gravitation from the planet. 1) Drawing: cross-section with tunnel and a point at distance r from the center. 2) Use the mass-inside-r idea to find F(r) the gravitational force on the ball for $r \le R$. 3) Show that the ball will execute a simple harmonic motion and identify ω. 5) Deduce the period T and show this equals the period of a circular orbit skimming the surface/equator. 6) Check: If density doubled (same R), how does T change?

Q5. Pendulum in an elevator (10 pts)

A small-angle pendulum (length ℓ) hangs in an elevator. Consider: rest; accelerating upward at a; accelerating downward at a (with $a<g$). 1) Drawing (required): each case with an effective gravity vector. 2) Find the period of the pendulum for each. 3) For $\ell=0.90$ m, $a=2.0$ m/s^2, compute all three periods and rank them. 4) Check: What happens in free fall?

Q6. Van der Waals ≈ Hooke near the minimum (8 pts)

Take $\mathrm{U(r)} = U_0\left[\left(\frac{\mathrm{R_0}}{\mathrm{r}}\right)^{12} - 2\left(\frac{\mathrm{R_0}}{\mathrm{r}}\right)^{6}\right]$) with minimum at $\mathrm{r = R_0}$. 1) Drawing: sketch U vs r; label $\mathrm{R_0}$ and curvature at r=$\mathrm{R_0}$. 2) Compute the conservative force, F(r), associated with this potential energy. 3) Let $(\mathrm{r = R_0 + r'})$ with $|\mathrm{r'}| \ll \mathrm{R_0}$. Expand F(r) to the first order in r', show $\mathrm{F} \approx -\mathrm{kr'}$ and identify k in terms of U_0 and $\mathrm{R_0}$. 4) Check: Units of *k*; is the force restoring (sign)?

Q7. A moving pulse (9 pts)

The pulse along +x has the wavefunction given $y(x,t) = 4.20m\ e^{-\left(\frac{x+(2.00\ \mathrm{m/s})t}{1.20m}\right)^2}$. 1) Drawing: y vs x at t=0 showing width/center; indicate predicted direction. 2) Rewrite as $y(x,t) = f(x \pm vt)$. From the sign, state the wave direction (±x) and speed v (with units). 3) How far does the center move in 3.00 s? Sketch at t=0 and t=3.00 s.

Q8. Wave on a string (10 pts)

A wave is $y(x,t) = A\,cos(kx - \omega\, t + \varphi) = 0.15m\,cos(0.15m^{-1}x + 1.50s^{-1}t + 0.25)$. 1) Wave speed and direction? 2) (b) Find the position in the y-direction, the velocity perpendicular to the motion of the wave, and the acceleration perpendicular to the motion of the wave, of a small segment of the string centered at $x = 0.40$ m at time $t = 5.00$ s.

Bonus (0–2 pts): Wave in different mediums

Search and/or read the characteristics of sound wave. Briefly describe what happens if you shout loudly near a pond, how your sound will transmit into the water (suppose there is a sound detector in water). [less than 150 words]

Assistance notes: