PHYS121 Integrated Science-Physics

W1T1 Kinematics

References:
[1] David Halliday, Jearl Walker, Resnick Jearl, ‘Fundamentals of Physics’, (Wiley, 2018)
[2] Doug Giancoli, ‘Physics for Scientists and Engineers with modern physics’, (Pearson, 2009)
[3] Hugh D. Young, Roger A. Freedman, ‘University Physics with Modern Physics’, (Pearson, 2012)
And others specified when needed.

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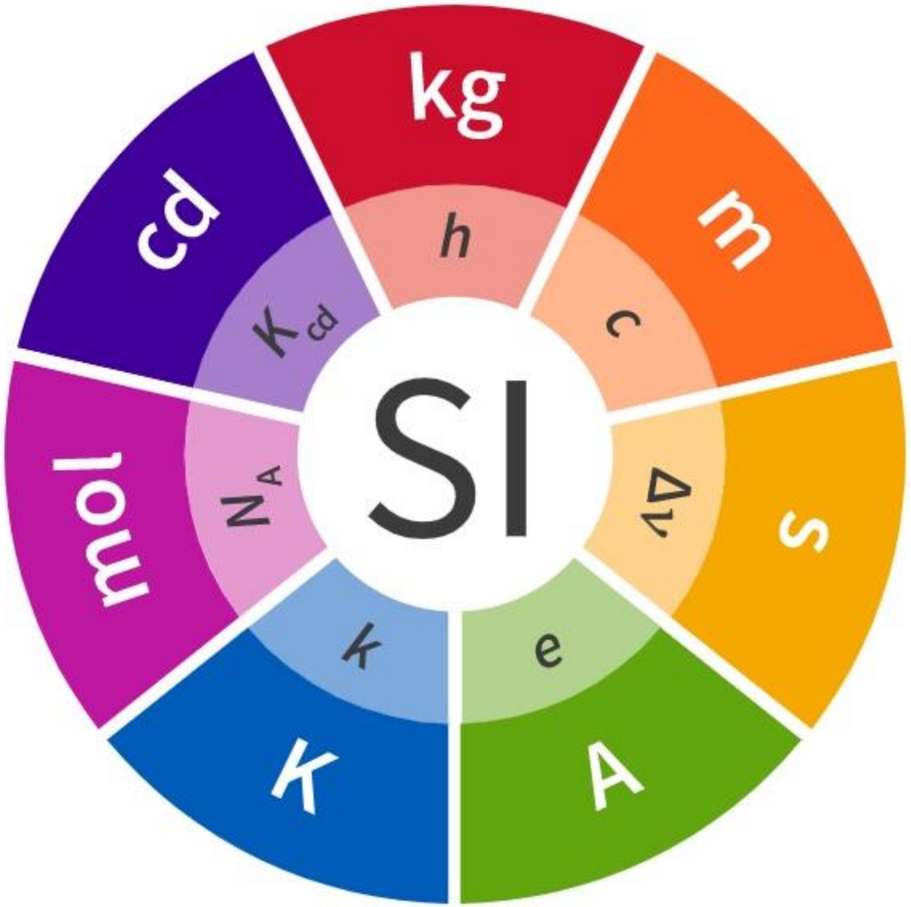
Learning outcomes

- Describe motions in three dimensions accurately (coordinates) using appropriate units.
- Perform quick estimation of quantities and dimensional analysis of equations.
- Apply calculus to solve problems with changing velocity/acceleration by taking into account of magnitude, direction, and unit.

SI units

- [Length - meter (m)]
- [Time - second (s)]
- [Amount of substance - mole (mole)]
- [Electric current - ampere (A)]
- [Temperature - kelvin (K)]
- [Luminous intensity - candela (cd)]
- [Mass - kilogram (kg)]



From nist.gov

Defining constant	Symbol	Numerical value	Unit
hyperfine transition frequency of Cs	$\Delta\nu_{\text{Cs}}$	9 192 631 770	Hz
speed of light in vacuum	c	299 792 458	m s^{-1}
Planck constant	h	$6.626\,070\,15 \times 10^{-34}$	J s
elementary charge	e	$1.602\,176\,634 \times 10^{-19}$	C
Boltzmann constant	k	$1.380\,649 \times 10^{-23}$	J K^{-1}
Avogadro constant	N_A	$6.022\,140\,76 \times 10^{23}$	mol^{-1}
luminous efficacy	K_{cd}	683	lm W^{-1}

From nist.gov

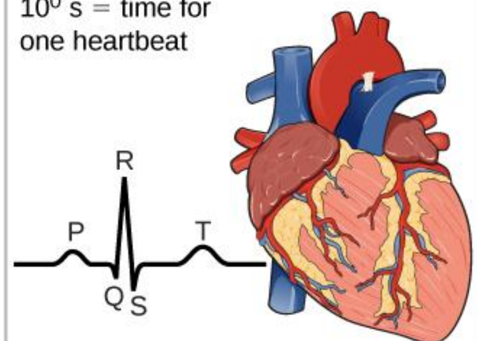
Length in Meters (m)	Masses in Kilograms (kg)	Time in Seconds (s)
10^{-15} m = diameter of proton	10^{-30} kg = mass of electron	10^{-22} s = mean lifetime of very unstable nucleus
10^{-14} m = diameter of large nucleus	10^{-27} kg = mass of proton	10^{-17} s = time for single floating-point operation in a supercomputer
10^{-10} m = diameter of hydrogen atom	10^{-15} kg = mass of bacterium	10^{-15} s = time for one oscillation of visible light
10^{-7} m = diameter of typical virus	10^{-5} kg = mass of mosquito	10^{-13} s = time for one vibration of an atom in a solid
10^{-2} m = pinky fingernail width	10^{-2} kg = mass of hummingbird	10^{-3} s = duration of a nerve impulse
10^{0} m = height of 4 year old child 	10^{0} kg = mass of liter of water 	10^{0} s = time for one heartbeat 
10^{2} m = length of football field	10^{2} kg = mass of person	10^{5} s = one day
10^{7} m = diameter of Earth	10^{19} kg = mass of atmosphere	10^{7} s = one year
10^{13} m = diameter of solar system	10^{22} kg = mass of Moon	10^{9} s = human lifetime
10^{16} m = distance light travels in a year (one light-year)	10^{25} kg = mass of Earth	10^{11} s = recorded human history
10^{21} m = Milky Way diameter	10^{30} kg = mass of Sun	10^{17} s = age of Earth
10^{26} m = distance to edge of observable universe	10^{53} kg = upper limit on mass of known universe	10^{18} s = age of the universe

TABLE 1–4 Metric (SI) Prefixes

Prefix	Abbreviation	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

† μ is the Greek letter "mu."

These are the standard SI prefixes for indicating powers of 10. Many are familiar; yotta, zetta, exa, hecto, deka, atto, zepto, and yocto are rarely used.

Slide based on Ref. [2]

Scientific notation and Significant figures

- **Significant figures** are meaningful digits
- Generally, round to the least number of significant figures of the given data
  - $25 \times 18 \rightarrow$ 2 significant figures; $25 \times 18975 \rightarrow$ still 2
  - Round up for 5 + ($13.5 \rightarrow 14$, but $13.4 \rightarrow 13$)
- Significant figures are not decimal places
  - 0.00356 has 5 decimal places, 3 significant figures
- In general, trailing zeros are not significant

In other words, 3000 may have 4 significant figures but usually 3000 will have only 1 significant figure!

When in doubt, use scientific notation 3.000×10^3 or 3×10^3

Do not keep too many significant figures to give a wrong impression of high accuracy in your calculation.

Slide based on Ref. [1]

Order of Magnitude: Rapid Estimating

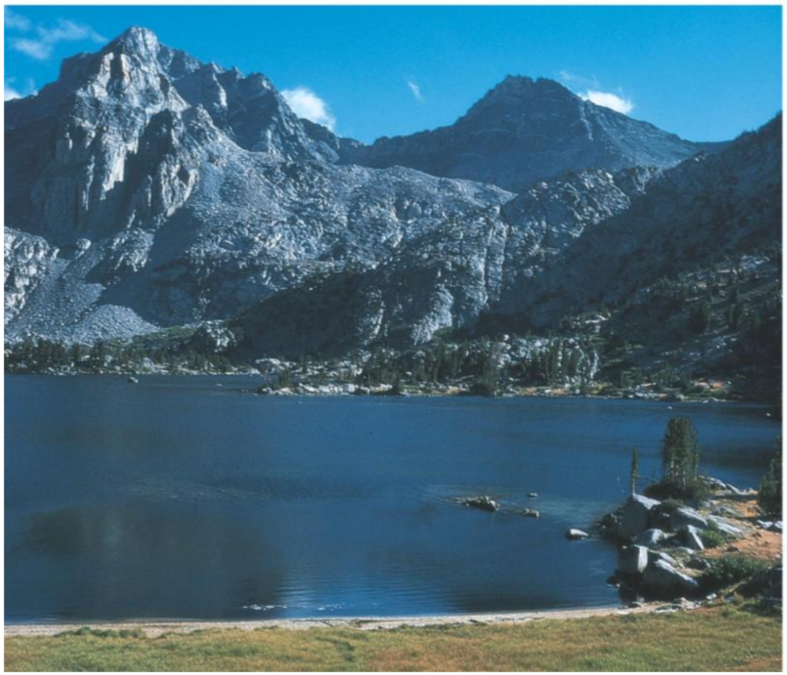
A quick way to **estimate** a calculated quantity is to round off all numbers to **one** significant figure and then calculate. Your result should at least be the right **order of magnitude 数量级**; this can be expressed by rounding it off to the nearest power of 10.

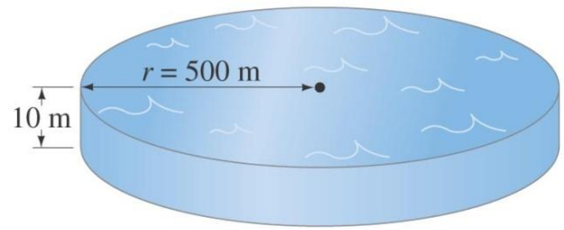
Diagrams are also very useful in making estimations.

Slide based on Ref. [2]

Order of Magnitude: Rapid Estimating

Example 1-5: Volume of a lake.

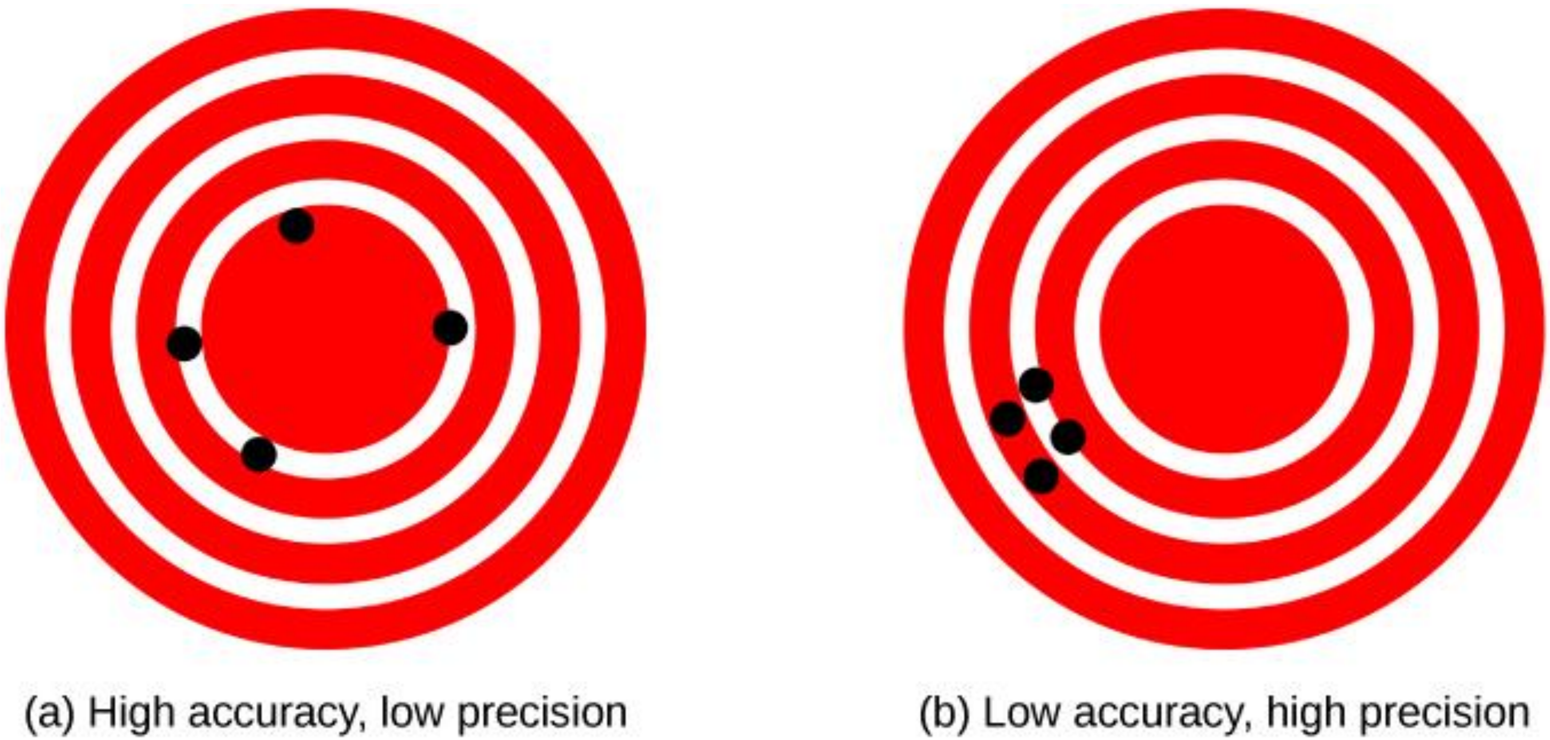




Estimate how much water there is in a particular lake, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

Slide based on Ref. [2]

FIGURE 1.12



A GPS attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant.

(a) The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy.

(b) The dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit a and credit b: modification of works by Dark Evil)

Dimensions and Dimensional Analysis

Dimensional analysis 量纲分析 is the checking of dimensions of all quantities in an equation to ensure that those which are added, subtracted, or equated have the **same** dimensions.

Example: Is this the **correct** equation for velocity? $v = v_0 + \frac{1}{2}at^2.$

Check the dimensions:

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right][T^2] = \left[\frac{L}{T}\right] + [L].$$

Wrong!

Slide based on Ref. [2]

Strategy

Strategy is the beginning stage of solving a problem. The idea is to figure out exactly what the problem is and then develop a strategy for solving it. Some general advice for this stage is as follows:

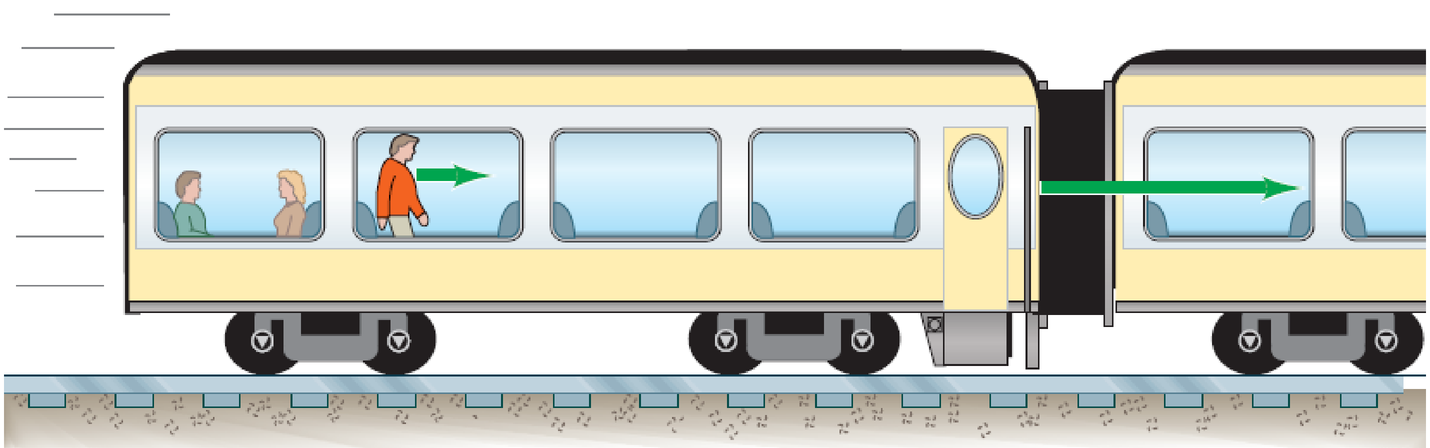
- *Examine the situation to determine which physical principles are involved*. It often helps to *draw a simple sketch* at the outset. You often need to decide which direction is positive and note that on your sketch. When you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.
- *Make a list of what is given or can be inferred from the problem as stated (identify the "knowns").* Many problems are stated very succinctly and require some inspection to determine what is known. Drawing a sketch can be very useful at this point as well. Formally identifying the knowns is of particular importance in applying physics to real-world situations. For example, the word *stopped* means the velocity is zero at that instant. Also, we can often take initial time and position as zero by the appropriate choice of coordinate system.
- *Identify exactly what needs to be determined in the problem (identify the unknowns).* In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help identify the unknowns.
- *Determine which physical principles can help you solve the problem*. Since physical principles tend to be expressed in the form of mathematical equations, a list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all the other variables are known—so you can solve for the unknown easily. If the equation contains more than one unknown, then additional equations are needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Page 33, online textbook.

Reference Frames and Displacement

Any **measurement** of position, distance, or speed must be made with respect to a **reference frame 参照系**.

For example, if you are sitting on a train and someone walks down the aisle, the person's speed with respect to the train is a few kilometers per hour, at most. The person's speed with respect to the ground is much higher.



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Position, Displacement, and Average Velocity

- Position is measured relative to a reference point:
  - **The origin, or zero point, of an axis** (reference frame)
- Position has a sign:
  - **Positive direction** is in the direction of increasing numbers
  - **Negative direction** is opposite the positive

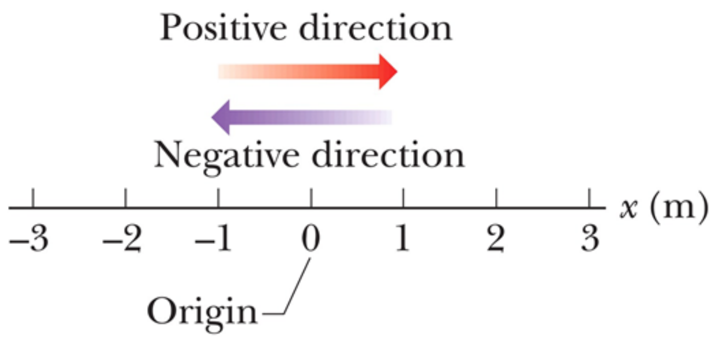


Figure 2-1

Displacement

- A change in position is called **displacement**
  - Δx is the change in x, (final position) − (initial position)

$$\Delta x = x_2 - x_1$$

Equation (2-1)

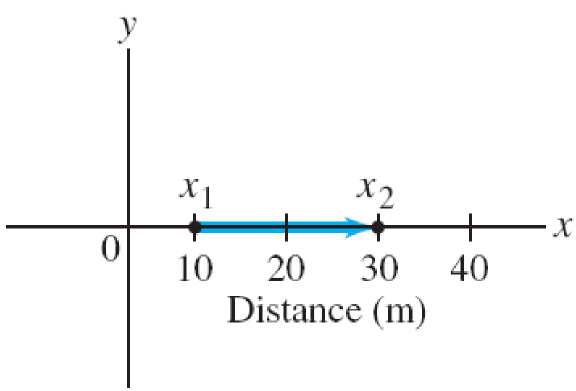
Examples A particle moves . . .

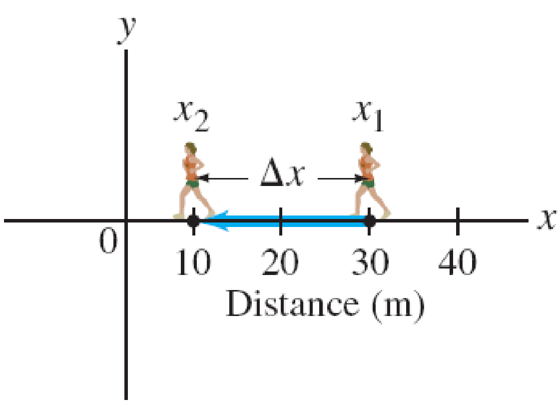
- From $x = 5$ m to $x = 12$ m: $\Delta x = 7 \text{ m} \left(\text{positive direction}\right)$
- From $x = 5$ m to $x = 1$ m: $\Delta x = -4 \text{ m} \left(\text{negative direction}\right)$
- From $x = 5$ m to $x = 200$ m to $x = 5$ m: $\Delta x = 0 \text{ m}$

- The actual distance covered is irrelevant/<u>different</u>

Displacement, a vector

- Displacement is therefore a **vector quantity**
  - Direction: along a single axis, given by sign (+ or −)
  - Magnitude: length or distance, in this case meters or feet
- Ignoring sign, we get its **magnitude** (absolute value)
  - The magnitude of $\Delta x = -20$ m is 20 m.





Average Velocity

What is the average speed?

- **Average velocity** is the ratio of:
  - A displacement, Δx
  - To the time interval in which the displacement occurred, Δt

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Equation (2-2)

- Average velocity has units of $\dfrac{(\text{displacement})}{(\text{time})}$
  - Meters per second, m/s

Average Velocity, a vector

- On a graph of x vs. t, the average velocity is the **slope** of the straight line that connects two points
- Average velocity is therefore a vector quantity
  - Positive slope means positive average velocity
  - Negative slope means negative average velocity

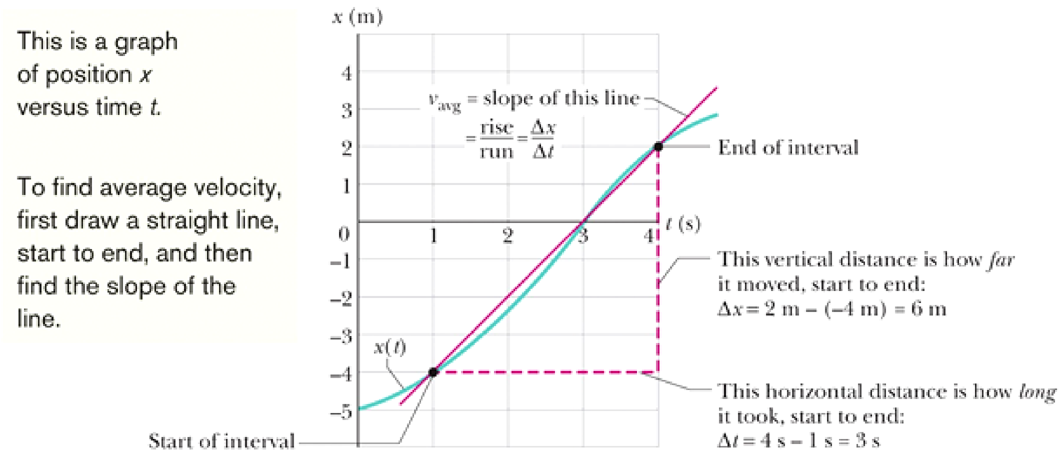
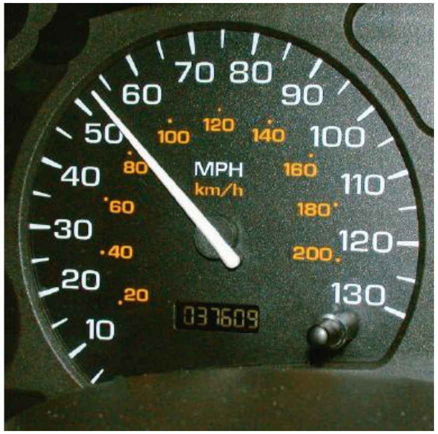


Figure 2-4

Instantaneous Velocity and Speed

- **Instantaneous velocity,** or just **velocity**, v, is:
  - At a single moment in time
  - Obtained from average velocity by shrinking Δt
  - The slope of the position-time curve for a particle at an instant (the derivative of position)
  - A vector quantity with units $\frac{(\text{displacement})}{(\text{time})}$
    - The sign of the velocity represents its direction



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$ **Equation (2-4)**

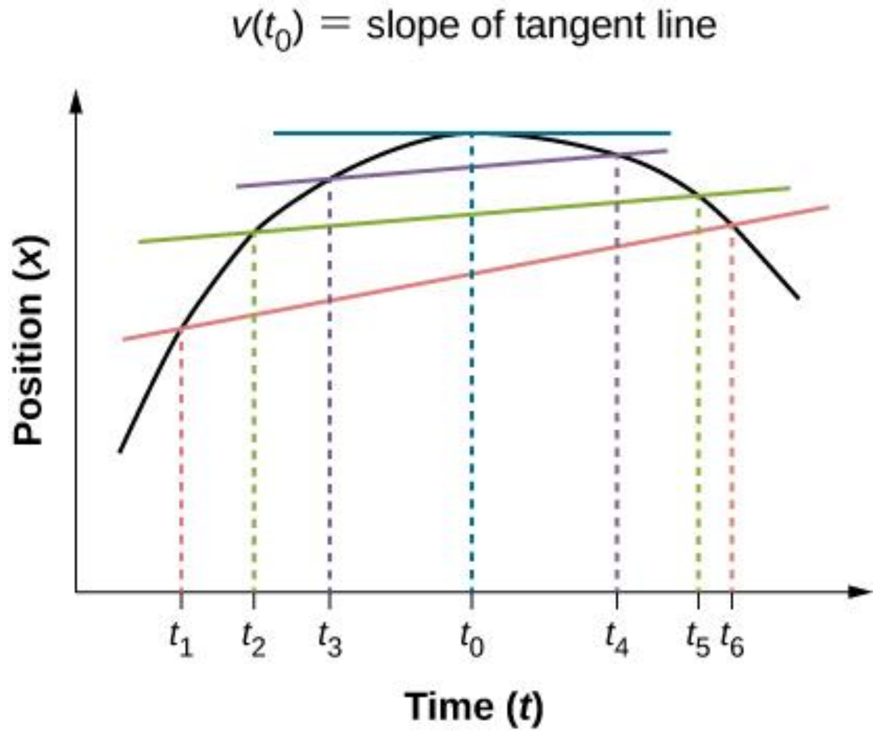
- **Speed** is the magnitude of (instantaneous) velocity

Example A velocity of 5 m/s and −5 m/s both have an associated speed of 5 m/s.

Checkpoint 2

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): $(1)\, x = 3t - 2$; $(2)\, x = -4t^2 - 2$; $(3)\, x = \frac{2}{t^2}$; and $(4)\, x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

FIGURE 3.6



In a graph of position versus time, the instantaneous velocity is the slope of the tangent line at a given point. The average velocities $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ between times $\Delta t = t_6 - t_1$, $\Delta t = t_5 - t_2$, and $\Delta t = t_4 - t_3$ are shown. When $\Delta t \to 0$, the average velocity approaches the instantaneous velocity at $t = t_0$.

Example

- The graph shows the position and velocity of an elevator cab over time.
- The slope of $x(t)$, and so also the velocity v, is zero from 0 to 1 s, and from 9s on.
- During the interval bc, the slope is constant and nonzero, so the cab moves with constant velocity (4 m/s).

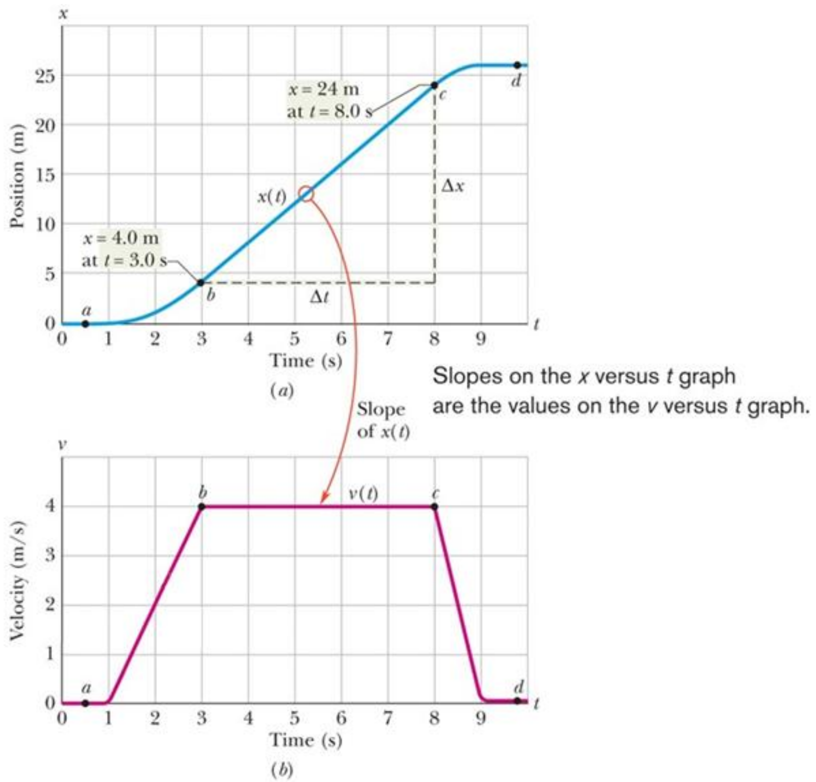


Figure 2-6

Acceleration

- A change in a particle's velocity is **acceleration**
- **Average acceleration** over a time interval Δt is

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Equation (2-7)

- **Instantaneous acceleration** (or just **acceleration**), *a*, for a single moment in time is:
  - Slope of velocity vs. time graph

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

$$a = \frac{dv}{dt}$$

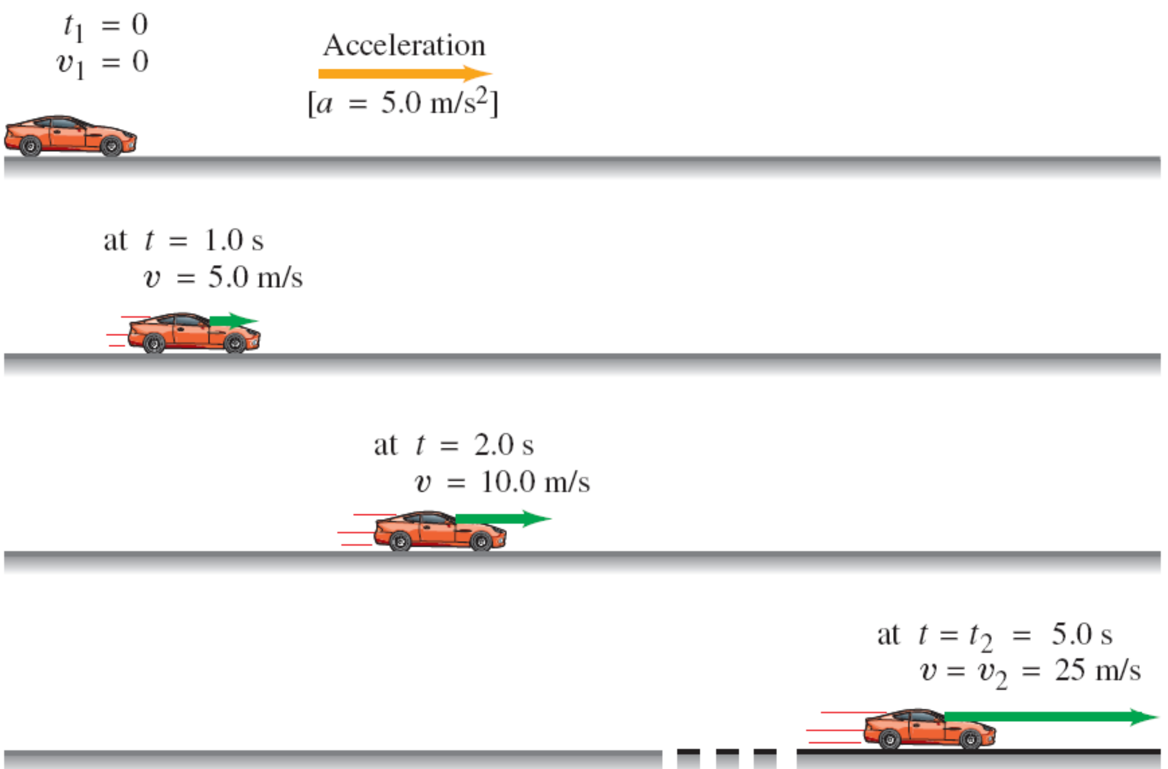
Equation (2-8)

Acceleration is the rate of change of velocity.

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

Example 2-4: Average acceleration.

A car accelerates along a straight road from rest to 90 km/h in 5.0 s. What is the magnitude of its average acceleration?



Slide based on Ref. [2]

Acceleration

- Combining Equations. 2-8 and 2-4:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$ **Equation (2-9)**

- Acceleration is a vector quantity:
  - Positive sign means in the positive coordinate direction
  - Negative sign means the opposite
  - Units of $\frac{(\text{displacement})}{(\text{time squared})}$

Checkpoint 3

A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Example

- The graph shows the velocity and acceleration of an elevator cab over time.
- When acceleration is 0 (e.g. interval *bc*) velocity is constant.
- When acceleration is positive (*ab*) upward velocity increases.
- When acceleration is negative (*cd*) upward velocity decreases.
- Steeper slope of the velocity-time graph indicates a larger magnitude of acceleration: the cab stops in half the time it takes to get up to speed.

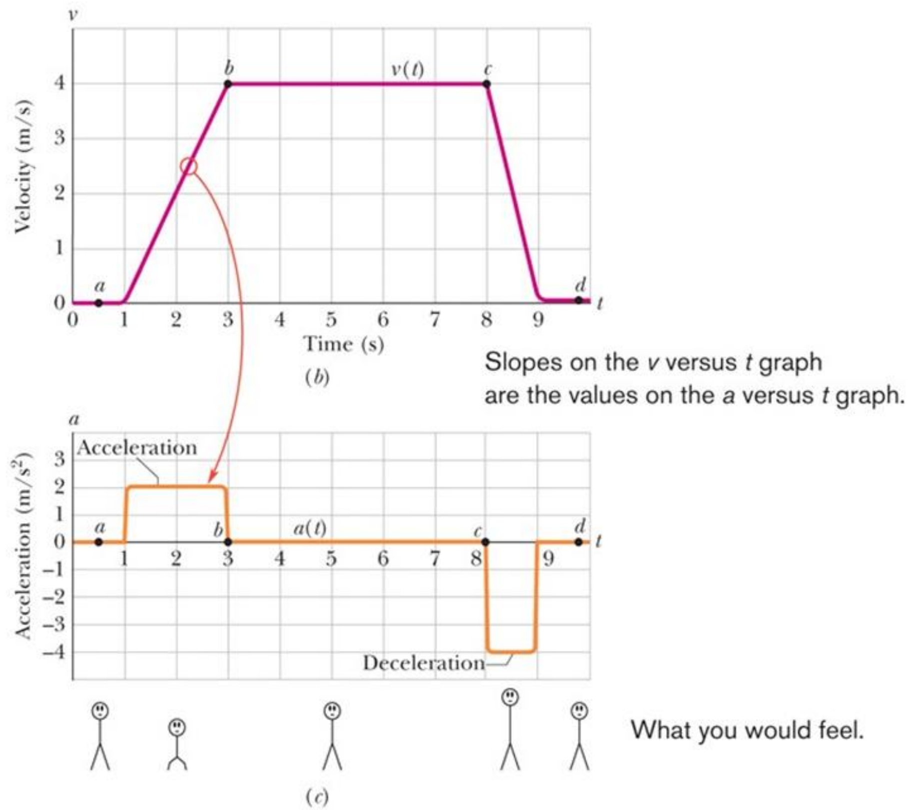


Figure 2-6

Constant Acceleration

- In many cases acceleration is constant, or nearly so.
- For these cases, **5 special equations** can be used.
- Note that constant acceleration means a velocity with a constant slope, and a position with varying slope (unless $a = 0$).

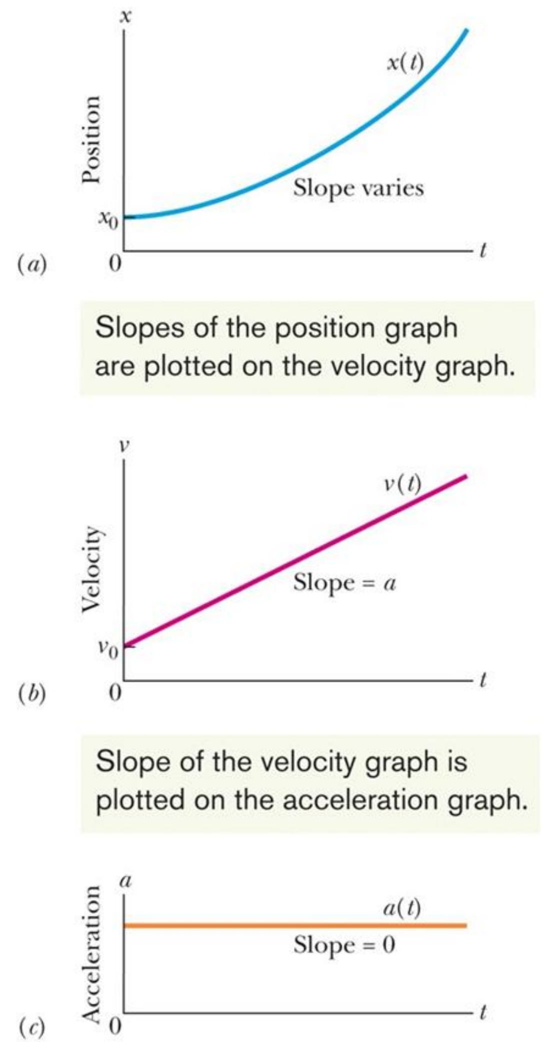


Figure 2-9

Table 2-1 Equations for Motion with Constant Acceleration a

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Make sure that the acceleration is indeed constant before using the equations in this table.

No need to memorize these equations, it is easy to derive them from calculus.

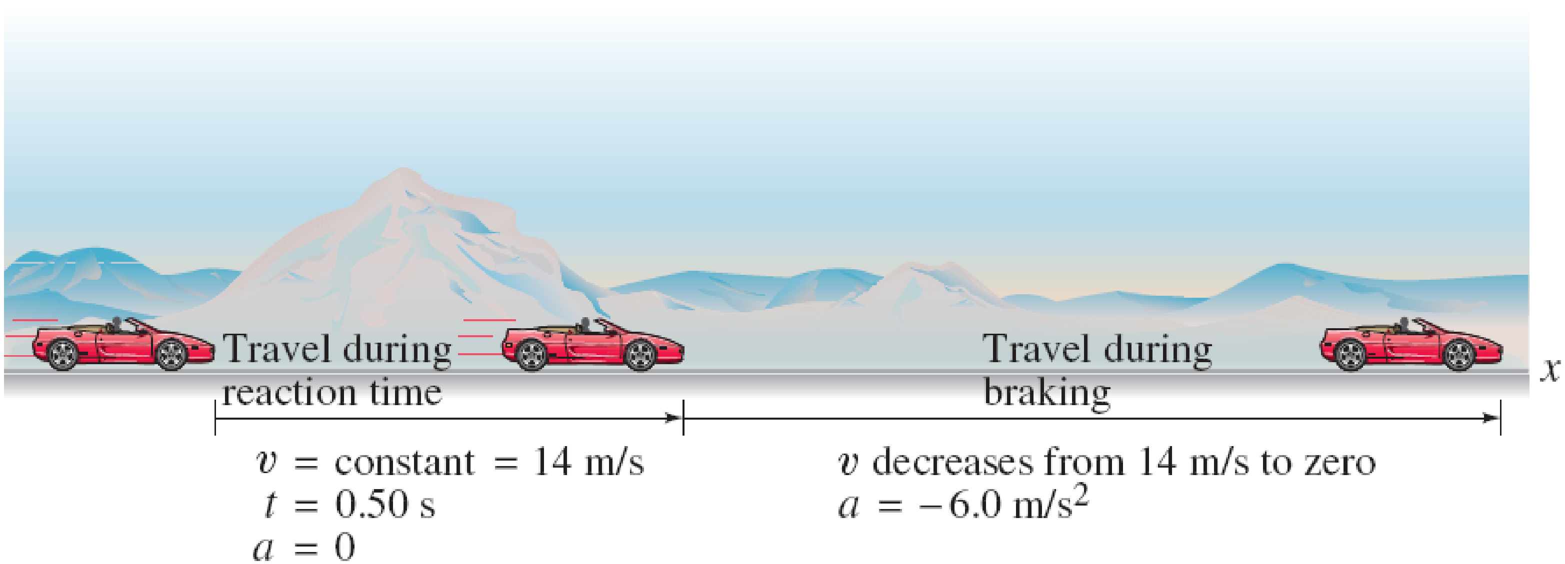
Checkpoint 4

The following equations give the position $x(t)$ of a particle in four situations: $(1)\,x=3t-4;\,(2)\,x=-5t^3+4t^2+6;\,(3)\,x=\frac{2}{t^2}-\frac{4}{t};\,(4)\,x=5t^2-3.$

To which of these situations do the equations of Table 2-1 apply?

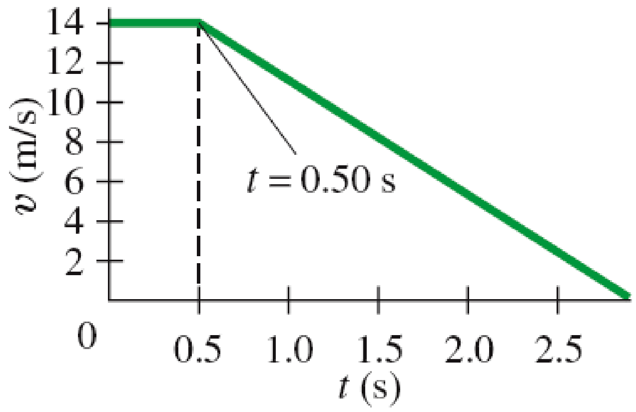
Braking distances.

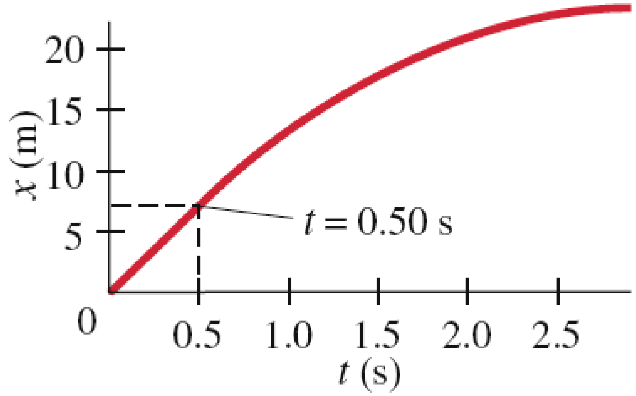
Estimate the minimum stopping distance for a car. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time," about 0.50 s, during which the speed is constant, so $a = 0$.



Slide based on Ref. [2]

(2) The second time interval is the actual braking period when the vehicle slows down ($a \neq 0$) and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. Calculate the total stopping distance for an initial velocity of 50 km/h (= 14 m/s ≈ 31 mi/h) and assume the acceleration of the car is -6.0 m/s^2 (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing).





Slide based on Ref. [2]

Learning outcomes

- ✓ Describe motions in three dimensions accurately (coordinates) using appropriate units.
- ✓ Perform quick estimation of quantities and dimensional analysis of equations.
- ✓ Apply calculus to solve problems with changing velocity/acceleration by taking into account of magnitude, direction, and unit. (to be revisited)

Questions