PHYS121 Integrated Science-Physics

W2T3 Potential Energy and Conservation of Energy

References:
[1] David Halliday, Jearl Walker, Resnick Jearl, 'Fundamentals of Physics', (Wiley, 2018)
[2] Doug Giancoli, 'Physics for Scientists and Engineers with modern physics', (Pearson, 2009)
[3] Hugh D. Young, Roger A. Freedman, 'University Physics with Modern Physics', (Pearson, 2012)
And others specified when needed.

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7.6.3. Two balls of equal size are dropped from the same height from the roof of a building. One ball has twice the mass of the other. When the balls reach the ground, how do the kinetic energies of the two balls compare?

a) The lighter one has one fourth as much kinetic energy as the other does.

b) The lighter one has one half as much kinetic energy as the other does.

c) The lighter one has the same kinetic energy as the other does.

d) The lighter one has twice as much kinetic energy as the other does.

e) The lighter one has four times as much kinetic energy as the other does.

Learning outcomes

- Calculate the potential energy of a conservative force, such as gravitational force and elastic force.
- Find the mechanical energy of objects.
- Apply conservation of mechanical energy.
- Explain the status of motion with regarding to potential energy curves.
- Apply the general conservation of energy.

Potential Energy

- **Potential energy** *U* is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper

    Gravitational potential energy accounts for kinetic energy increase during the fall

    Elastic potential energy accounts for deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy

- For an object being raised or lowered:

$$\Delta U = -W.$$

Equation (8-1)

- *<u>The change in gravitational potential energy is the negative of the work done</u>*
- This also applies to an elastic block-spring system

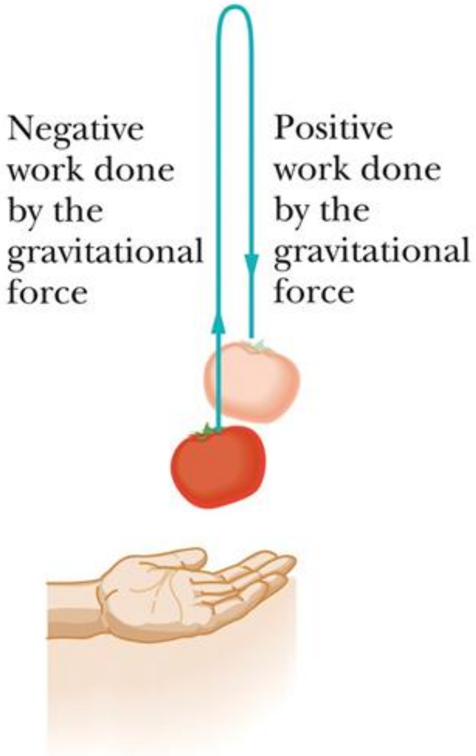


Figure 8-2

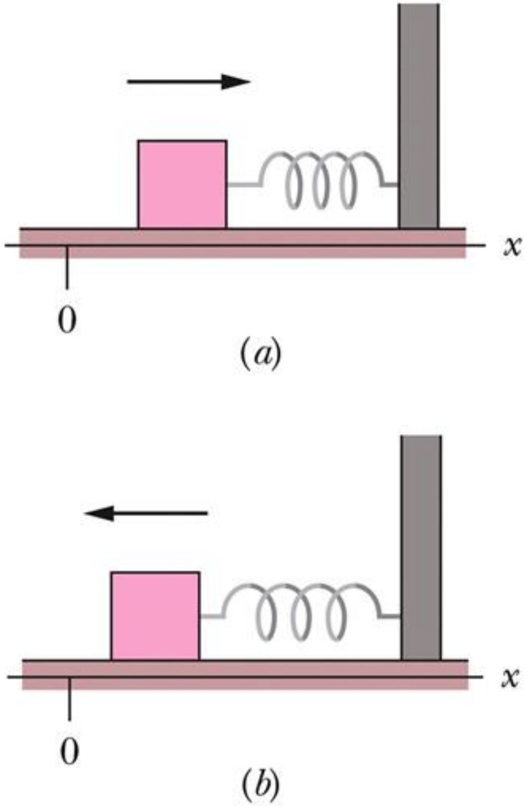


Figure 8-3

- Key points:

1. The system consists of two or more objects
2. A force acts between a particle (tomato/block) and the rest of the system
3. When the configuration changes, the force does work W_1, changing kinetic energy to another form
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2

- Thus the kinetic energy of the tomato/block becomes potential energy, and then kinetic energy again

- **Conservative forces** are forces for which $W_1 = -W_2$ is always true (*the work done by the force is independent of the path taken*)
  - Examples: gravitational force, spring force
  - Otherwise we could not speak of their potential energies
- **Nonconservative forces** are those for which it is false
  - Examples: kinetic friction force, drag force
  - *Kinetic energy of a moving particle is transferred to heat by friction*
  - Thermal energy *cannot* be recovered back into kinetic energy of the object via the friction force
  - Therefore the force is not conservative, thermal energy is not a potential energy

- When only conservative forces act on a particle, we find many problems can be simplified:

  The net work done by a conservative force on a particle moving around any closed path is zero.

- A result of this is that: $W_{ab,1} = W_{ab,2}$, **Equation (8-2)**

  The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

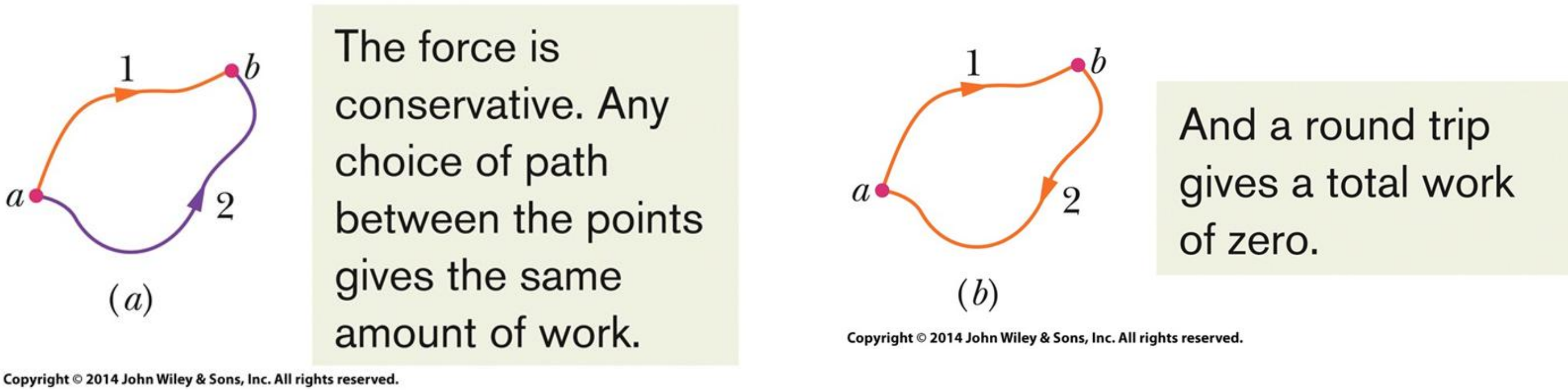
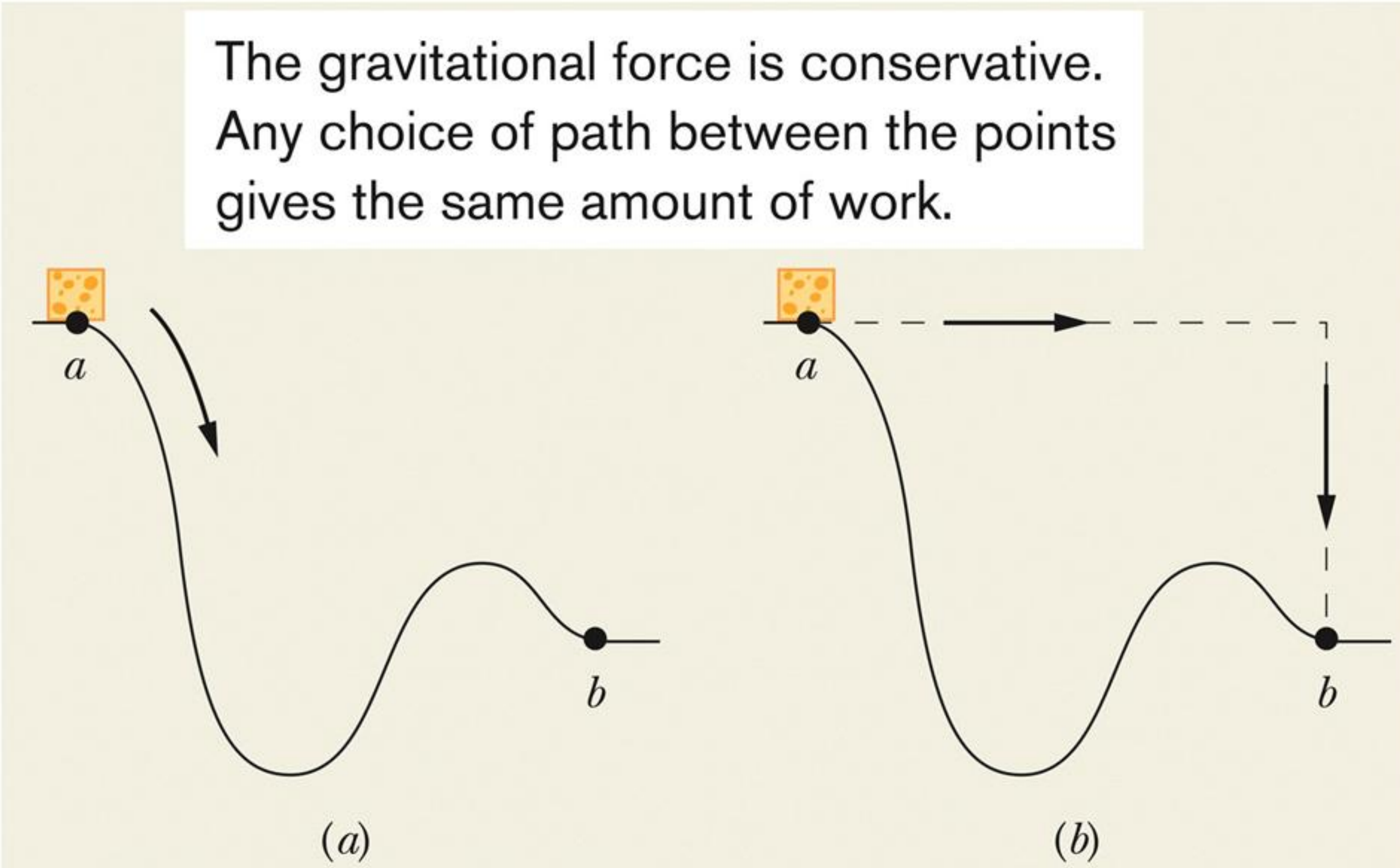


Figure 8-4



The gravitational force is conservative.
Any choice of path between the points
gives the same amount of work.
a
b
(a)
a
b
(b)

Figure 8-5

- For the general case, we calculate work as:

$$W = \int_{x_i}^{x_f} \vec{F}(x) \cdot d\vec{x}.$$ **Equation (8-5)**

- So we calculate potential energy as:

$$\Delta U = -\int_{x_i}^{x_f} \vec{F}(x) \cdot d\vec{x}.$$ **Equation (8-6)**

- Using this to calculate gravitational PE, relative to a **reference configuration** with **reference point** $y_i = 0$:

$$U(y) = mgy$$ **Equation (8-9)**

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

- Use the same process to calculate spring PE:

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\,dx = k\int_{x_i}^{x_f} x\,dx = \frac{1}{2}k\left[x^2\right]_{x_i}^{x_f},$$ **Equation (8-10)**

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2.$$

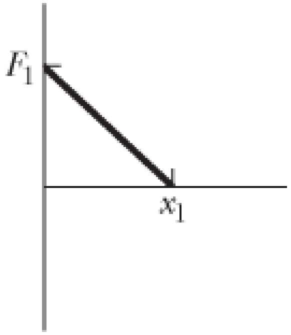
- With reference point $x_i = 0$ for a relaxed spring:

$$U(x) = \frac{1}{2}kx^2$$ **Equation (8-11)**

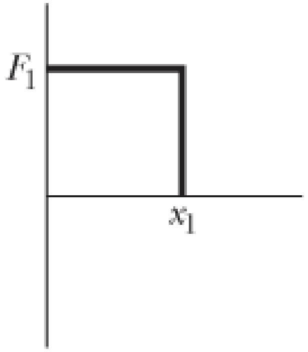
Checkpoint 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x. The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.

(1)



(2)



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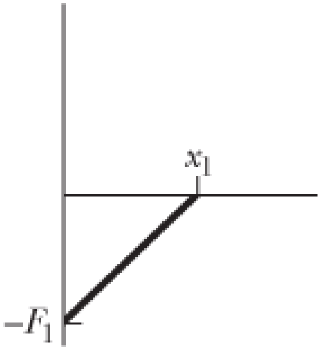


Answer:

(3), (1), (2); a positive force does positive work, decreasing the PE; a negative force (e.g., 3) does negative work, increasing the PE

Conservation of Mechanical Energy

- The mechanical energy of a system is the sum of its potential energy U and kinetic energy K:

$$E_{\text{mec}} = K + U \qquad \textbf{Equation (8-12)}$$

- Work done by conservative forces increases K and decreases U by that amount, so:

$$\Delta K = -\Delta U. \qquad \textbf{Equation (8-15)}$$

- Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1 \qquad \textbf{Equation (8-17)}$$

In an isolated system where *only conservative forces cause energy changes*, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

- This is the principle of the **conservation of mechanical energy:**

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \qquad \textbf{Equation (8-18)}$$

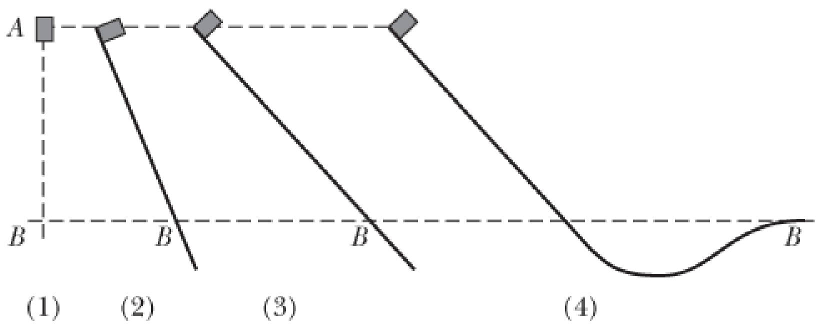
- This is very powerful tool:

When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy *at one instant to that at another instant without considering the intermediate motion and without finding the work one by the forces involved.*

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point $U = \text{max}$, and $K = \text{min}$

Checkpoint 3

The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point *B*, greatest first. (b) Rank them according to the speed of the block at point *B*, greatest first.



Answer:

Since there are no nonconservative forces, all of the difference in potential energy must go to kinetic energy. Therefore all are equal in (a). Because of this fact, they are also all equal in (b).

Reading a Potential Energy Curve

- For *one dimension*, force and potential energy are related (by work) as:

$$F(x) = -\frac{dU(x)}{dx}$$ **Equation (8-22)**

$$\vec{\mathbf{F}}(x, y, z) = -\hat{\mathbf{i}}\frac{\partial U}{\partial x} - \hat{\mathbf{j}}\frac{\partial U}{\partial y} - \hat{\mathbf{k}}\frac{\partial U}{\partial z}.$$

- Therefore we can find the force $F(x)$ from a plot of the potential energy $U(x)$, by taking the derivative (slope)
- If we write the mechanical energy out:

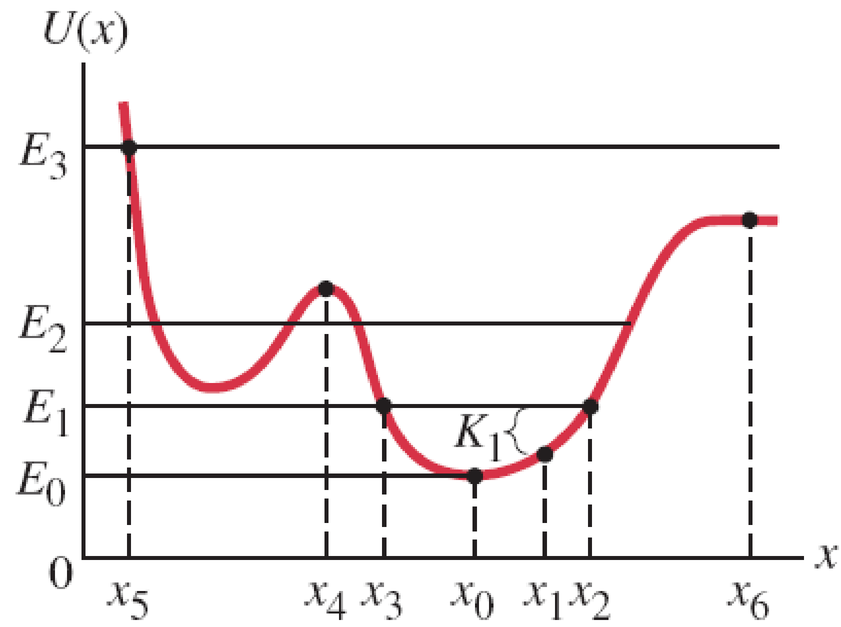
$$U(x) + K(x) = E_{\text{mec}}.$$ **Equation (8-23)**

- We see how $K(x)$ varies with $U(x)$:

$$K(x) = E_{\text{mec}} - U(x).$$ **Equation (8-24)**

- To find $K(x)$ at any place, take the total mechanical energy (constant) and subtract $U(x)$
- Places where $K = 0$ are **turning points**
  - There, the particle changes direction (K cannot be negative)
- *At equilibrium points, the slope of U(x) is 0*
- A particle in **neutral equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it would remain in its new position
  - Example: a marble on a flat tabletop

- A particle in **unstable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced slightly to one direction, it will feel a force propelling it in that direction
- A particle in **stable equilibrium** is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it will feel a force returning it to its original position



- Plot (a) shows the potential $U(x)$
- Plot (b) shows the force $F(x)$
- If we draw a horizontal line, (c) or (f) for example, we can see the range of possible positions
- $x < x_1$ is forbidden for the E_{mec} in (c): the particle does not have the energy to reach those points

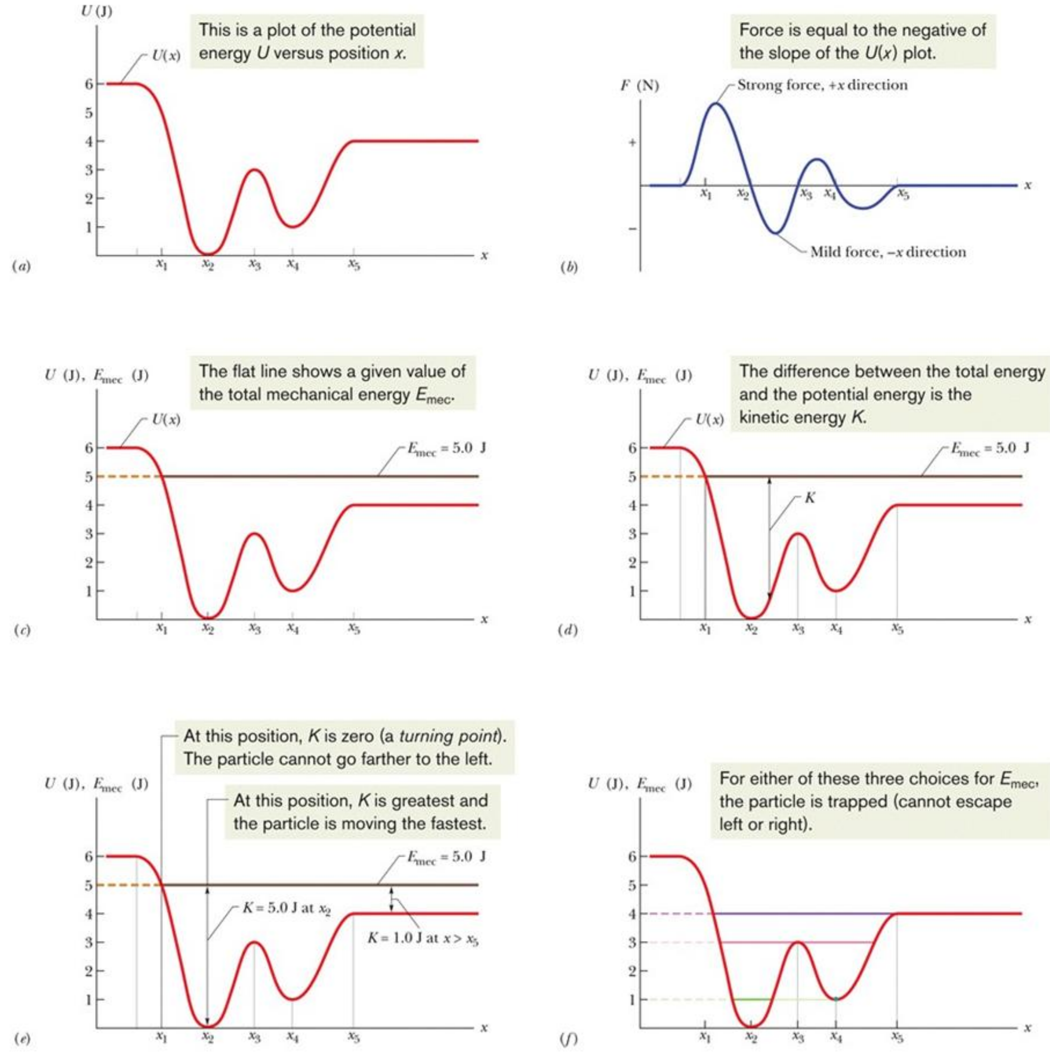
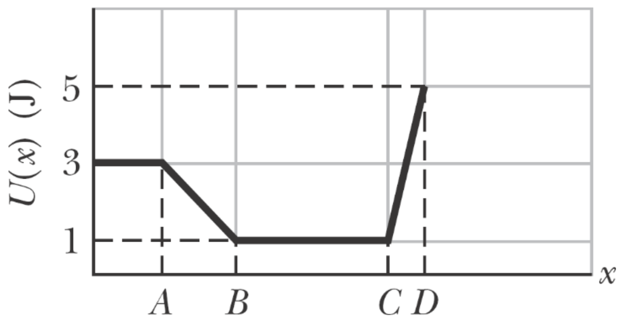


Figure 8-9

Checkpoint 4

The figure gives the potential energy function $U(x)$ for a system in which a particle is in one dimensional motion. (a) Rank regions AB, BC, and CD according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region AB?



Answer:

(a) CD, AB, BC

(b) to the right

Work Done on a System by an External Force

- We can extend work on an object to work on a system:

Work is energy transferred to or from a system by means of an external force acting on that system.

- For a system of more than 1 particle, work can change both *K* and *U*, or other forms of energy of the system
- For a frictionless system:

$$W = \Delta K + \Delta U,$$ **Equation (8-25)**

$$W = \Delta E_{\mathrm{mec}}$$ **Equation (8-26)**

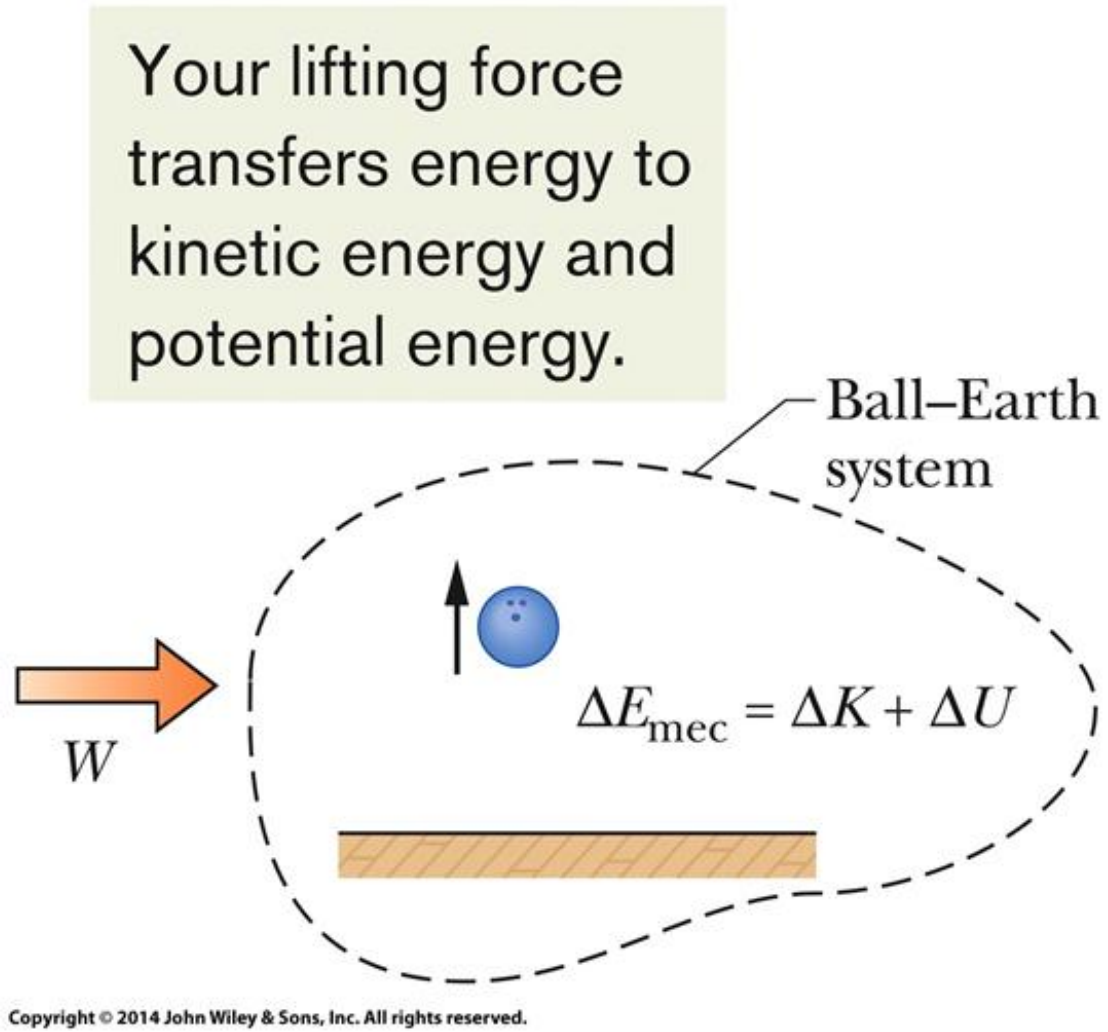

Your lifting force transfers energy to kinetic energy and potential energy.
Ball–Earth system
W
$\Delta E_{mec} = \Delta K + \Delta U$
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Figure 8-12

- For a system with friction:

$$\Delta E_{\text{th}} = f_k d \quad (\text{increase in thermal energy by sliding}).$$ **Equation (8-31)**

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$ **Equation (8-33)**

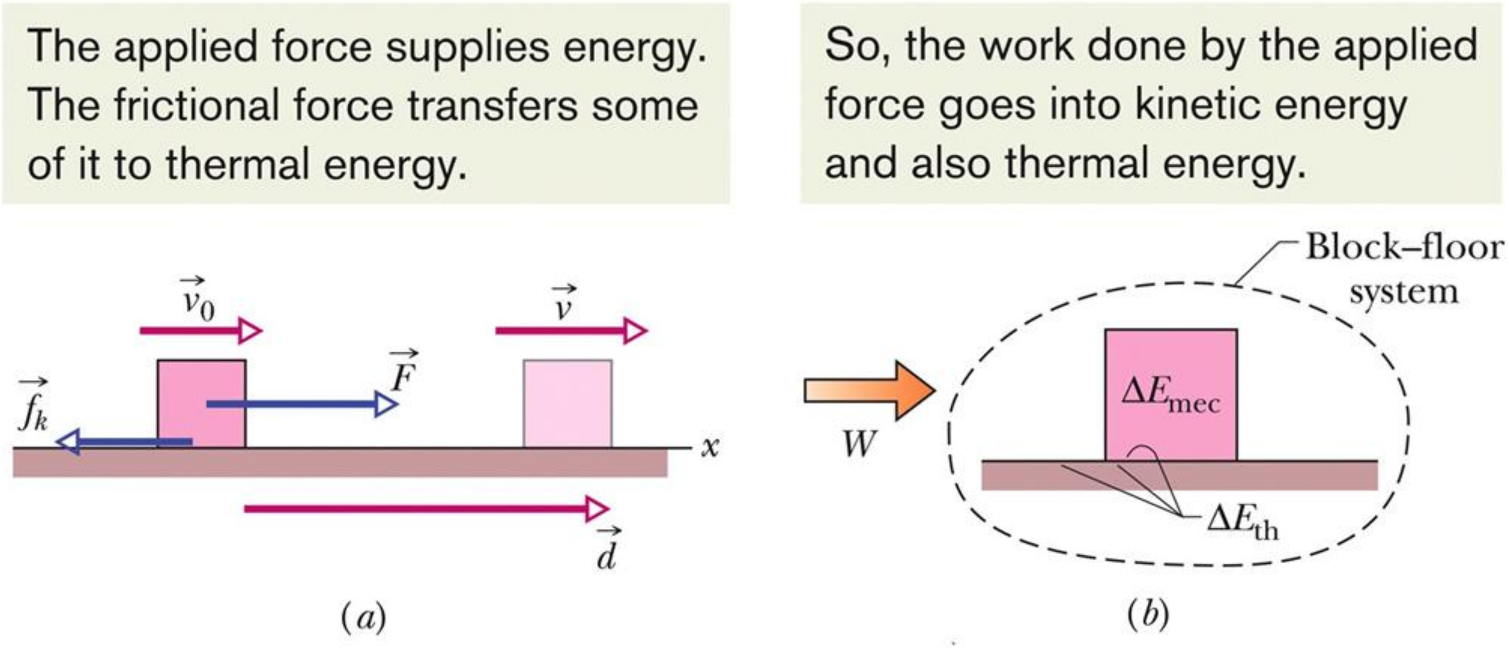

Figure 8-13

Checkpoint 5

In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless, as in Figure 8-13*a*.The magnitudes F of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance d. Rank the three trials according to the change in the thermal energy of the block and floor that occurs in that distance d, greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	increases

Answer:

All trials result in equal thermal energy change. The value of f_k is the same in all cases, since μ_k has only 1 value.

Conservation of Energy

- Energy transferred between systems can always be accounted for
- The **law of conservation of energy** concerns
  - The **total energy** E of a system
  - Which includes mechanical, thermal, and other internal energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

- Considering only energy transfer through work:

$$W = \Delta E = \Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}},$$

Equation (8-35)

- An *isolated system* is one for which there can be no external energy transfer

The total energy E of an isolated system cannot change.

- Energy transfers may happen internal to the system
- We can write: $$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + E_{\text{int}} = 0$$ **Equation (8-36)**
- Or, for two instants of time:

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}.$$ **Equation (8-37)**

In an isolated system, we can relate the total energy at one instant to the total energy at another instant without considering the energies at intermediate times.

- External forces can act on a system without doing work:

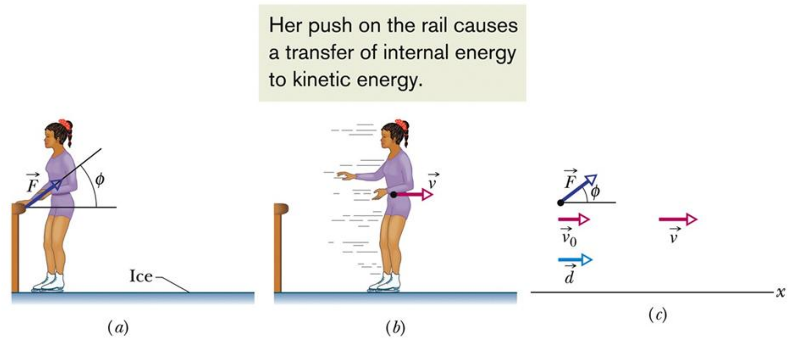

Figure 8-15

- The skater pushes herself away from the wall
- She turns internal chemical energy in her muscles into kinetic energy
- Her *K* change is caused by the force from the wall, but the wall does not provide her any energy

- We can expand the definition of power
- In general, power is the rate at which energy is transferred by a force from one type to another
- If energy ΔE is transferred in time Δt, the **average power** is:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}.$$

Equation (8-40)

- And the **instantaneous power** is:

$$P = \frac{dE}{dt}.$$

Equation (8-41)

Learning outcomes

- ✓ Calculate the potential energy of a conservative force, such as gravitational force and elastic force.
- ✓ Find the mechanical energy of objects.
- ✓ Apply conservation of mechanical energy.
- ✓ Explain the status of motion with regarding to potential energy curves.
- ✓ Apply the general conservation of energy.

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx.$$

$$F(x) = -\frac{dU(x)}{dx}$$

$$\Delta E_{\mathrm{mec}} = \Delta K + \Delta U = 0.$$

$$\Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + E_{\mathrm{int}} = 0$$

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CHAPTER REVIEW AND EXAMPLES, DEMOS

https://openstax.org/books/university-physics-volume-1/pages/8-summary

Most importantly, whatever choice is made should be stated and kept consistent throughout the given problem. There are some well-accepted choices of initial potential energy. For example, the lowest height in a problem is usually defined as zero potential energy, or if an object is in space, the farthest point away from the system is often defined as zero potential energy.
It is important to remember that potential energy is a property of the interactions between objects in a chosen system, and not just a property of each object. This is especially true for electric forces, although in the examples of potential energy we consider below, parts of the system are either so big (like Earth, compared to an object on its surface) or so small (like a massless spring), that the changes those parts undergo are negligible when included in the system.

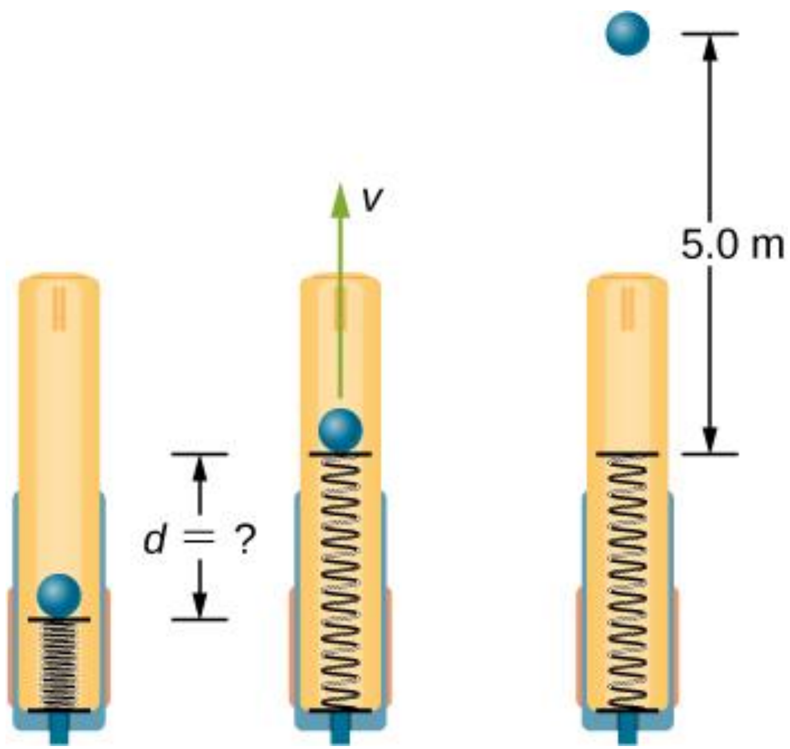
https://openstax.org/books/university-physics-volume-1/pages/8-1-potential-energy-of-a-system

Since energy in an isolated system is not destroyed, created, or generated, you might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat, that is, work that has been "degraded" in the energy transformation. We will discuss this idea in more detail in the chapters on thermodynamics.

https://openstax.org/books/university-physics-volume-1/pages/8-5-sources-of-energy

EXERCISE 43

The massless spring of a spring gun has a force constant k=12N/cm. When the gun is aimed vertically, a 15-g projectile is shot to a height of 5.0 m above the end of the expanded spring. (See below.) How much was the spring compressed initially?



Example 8.5

Conservative or Not?

Which of the following two-dimensional forces are conservative and which are not? Assume a and b are constants with appropriate units:

(a) $axy^3\,\hat{\mathbf{i}} + ayx^3\,\hat{\mathbf{j}}$, (b) $a\left[\left(y^2/x\right)\hat{\mathbf{i}} + 2y\ln(x/b)\,\hat{\mathbf{j}}\right]$, (c) $\dfrac{ax\,\hat{\mathbf{i}} + ay\,\hat{\mathbf{j}}}{x^2+y^2}$

‘exact differential’

$$\frac{dF_x}{dy} = \frac{dF_y}{dx}. \tag{8.10}$$

‘optional’

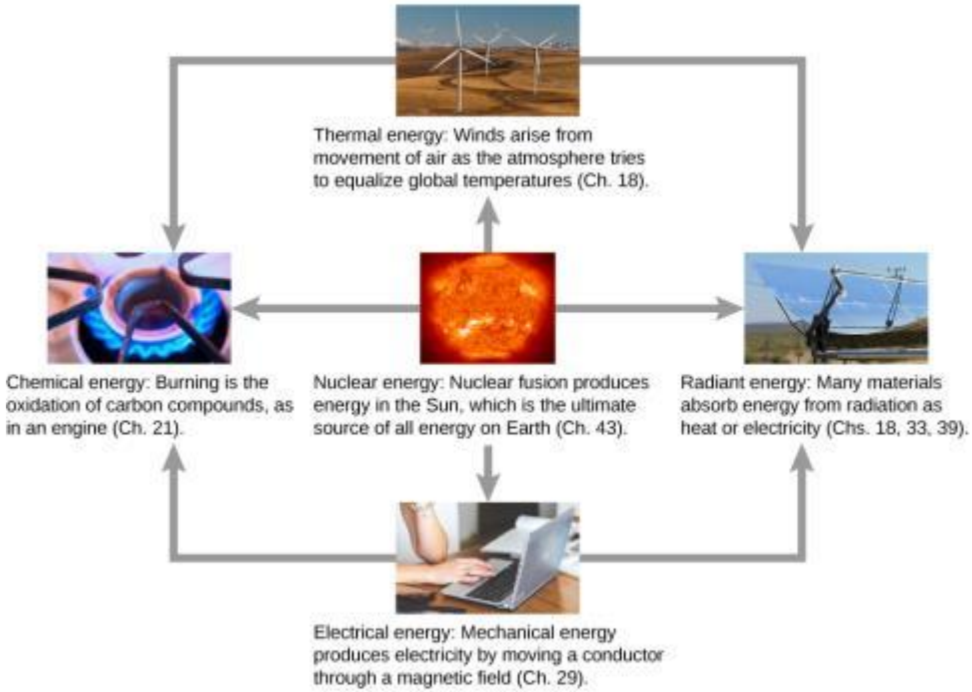
Example 8.10

Quartic and Quadratic Potential Energy Diagram

The potential energy for a particle undergoing one-dimensional motion along the x-axis is $U(x) = 2(x^4 - x^2)$, where U is in joules and x is in meters. The particle is not subject to any non-conservative forces and its mechanical energy is constant at $E = -0.25\ \mathrm{J}$. (a) Is the motion of the particle confined to any regions on the x-axis, and if so, what are they? (b) Are there any equilibrium points, and if so, where are they and are they stable or unstable?

https://www.desmos.com/calculator

FIGURE 8.13



Energy that we use in society takes many forms, which be converted from one into another depending on the process involved. We will study many of these forms of energy in later chapters in this text. (credit “sun”: EIT SOHO Consortium, ESA, NASA; credit “solar panels”: “kjkolb”/Wikimedia Commons; credit “gas burner”: Steven Depolo)

Questions