PHYS 121 — HW1 (Redesigned by ChatGPT 5.0)

Learning-by-doing version with sketches, checks, and clear scaffolding.

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Submission & Format (read me first)

• Show your reasoning. Every part must include a labeled sketch/diagram when asked.
• Box final answers with units and significant figures.
• If you use any AI tools, append a 1–3 sentence "Assistance note" describing exactly what it did for you at the end of your submission.
• Upload to Gradescope as a single PDF; scan your hand sketches clearly.

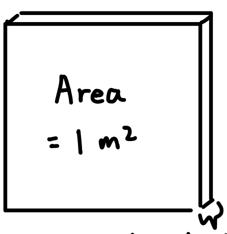
Tip: For force/friction problems, build a clean free-body diagram before any algebra.

Q1. Units & Measurement (8 pts)

A paint's coverage is 435 ft²/gal. (feet²/gallon, 1 gal = 3.785 L)

1) Sketch a tiny "unit-tile" of paint: annotate area and film thickness.
2) Convert ft² → m² and gal → L with a factor-label chain.
3) Report in m²/L, then convert to pure SI (m⁻¹).
4) Invert the quantity; interpret physically as required volume per area (L/m²) and relate it to film thickness.
5) Sanity check: if you need ~0.1 L per m², what room size does 1 gallon cover?

1)



Film thickness

$$t = \frac{V}{A} = \frac{0.0935\ L/m^2 \times 1 m^2}{1 m^2} = 9.35 \times 10^{-5} m$$

2)

$$435\ ft^2 \times 0.0929003 \frac{m^2}{ft^2} = 40.6 m^2$$

$$435\ ft^2/gal \times \frac{0.092903 m^2}{1 ft^2} \times \frac{1 gal}{3.785 L}$$

$$= 435 \times \frac{0.092903}{3.785} m^2/L$$

$$= 435 \times 0.02455 = \boxed{10.7\ m^2/L}$$

3) $1\ L = 10^{-3}\ m^3$

$$10.7 \frac{m^2}{L} = 10.7 \times 10^3 \frac{m^2}{m^3} = \boxed{1.07 \times 10^4\ m^{-1}}$$

4)

$$\frac{1}{10.7} = 0.0935\ L/m^2$$

That is $\boxed{0.0935\ L \text{ per } m^2}$

$$t = 0.0935\ L/m^2 \times 10^{-3} \frac{m^3}{L}$$

$$= \boxed{9.35 \times 10^{-5}\ m}$$

So the film thickness around 90 μm

5)

$$\frac{3.785 L}{0.1 L/m^2} = 37.85 m^2$$

close to 40 m²

consistent with earlier number

sanity check passed

Q2. Vectors (8 pts)

Here are three vectors in *meters*:

$$\vec{d}_1 = -3.0\hat{\imath} - 3.0\hat{\jmath} + 2.0\hat{k}$$
$$\vec{d}_2 = -2.0\hat{\imath} - 4.0\hat{\jmath} + 2.0\hat{k}$$
$$\vec{d}_3 = 2.0\hat{\imath} + 3.0\hat{\jmath} + 1.0\hat{k}$$

What results from (a) $\vec{d}_1 \cdot \left(\vec{d}_2 + \vec{d}_3\right)$, (b) $\vec{d}_1 \cdot \left(\vec{d}_2 \times \vec{d}_3\right)$, and (c) $\vec{d}_1 \times \left(\vec{d}_2 + \vec{d}_3\right)$? Reflection [not graded]: try to draw it out and check when do drawings catch algebra mistakes?

(a) $\vec{d}_2 + \vec{d}_3 = [(-2+2)\hat{i}] + [(-4+3)\hat{j}] + [(2+1)\hat{k}] = -\hat{j} + 3\hat{k}$

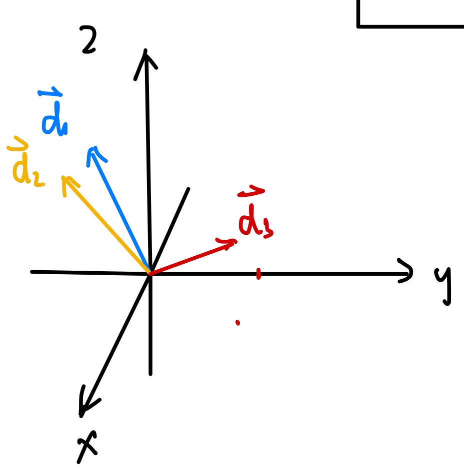
$\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3) = (-3)0 + (-3)(-1) + (2)(3) = 0 + 3 + 6 = \boxed{9}$

(b) $\vec{d}_2 \times \vec{d}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \hat{i}(-4-6) - \hat{j}(-2-4) + \hat{k}(-6+8) = -10\hat{i} + 6\hat{j} + 2\hat{k}$

$\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3) = (-3)(-10) + (-3)(6) + (2)(2) = 30 - 18 + 4 = \boxed{16}$

(c) $\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -3 & 2 \\ 0 & -1 & 3 \end{vmatrix} = \hat{i}(-9+2) - \hat{j}(-9-0) + \hat{k}(3-0)$

$= \boxed{-7\hat{i} + 9\hat{j} + 3\hat{k}}$



Q3. 1D Kinematics (10 pts)

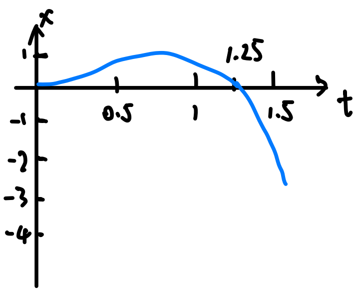
$x(t) = 5.0\,t^2 - 4.0\,t^3$ (meters).

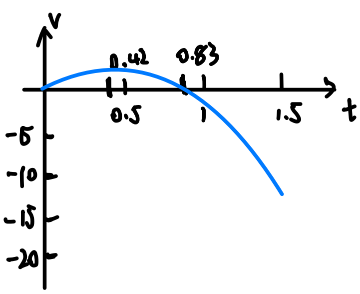
1) Differentiate to get v(t), a(t).
2) Sketches (required): qualitative x(t), v(t), a(t); mark key times.
3) Evaluate v(2.5 s), a(2.5 s).
4) Extremum: set v=0 → candidates; confirm max via a.
5) Compute x_{max} and the time when velocity is zero; annotate on your sketches.
6) Check: units and limiting behavior.

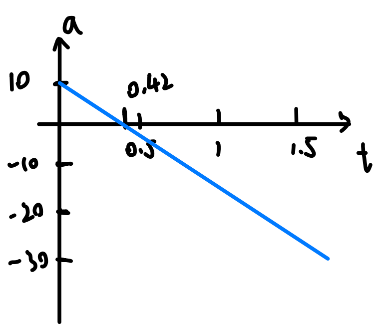
1) $v(t) = x'(t) = 10t - 12t^2$ m/s

$a(t) = v'(t) = 10 - 24t$ m/s^2

2)







3) $v(2.5) = 10(2.5) - 12(2.5)^2 = \boxed{-50\ m/s}$ towards negative direction

$a(2.5) = 10 - 24(2.5) = \boxed{-50\ m/s^2}$ towards negative direction

4) $v(t) = 0$, $t(10-12t) = 0 \Rightarrow t = \boxed{0 \text{ or } \frac{5}{6}\ s}$

Check with $a(t)$: $a(0) = 10 > 0$, x has local minimum at $t=0$ s

$a(\frac{5}{6}) = -10 < 0$, x has local maximum at $t = \frac{5}{6}$ s

5) from 4). times when $v=0$: $t=0$ and $\frac{5}{6}$ s

$x_{max} = x(\frac{5}{6}) = 5(\frac{5}{6})^2 - 4(\frac{5}{6})^3 = \frac{250}{216} \approx \boxed{1.16\ m}$

6) Units: x in m, v in m/s. a in m/s^2

Limiting behavior: as $t \to \infty$

$x(t) \to -\infty$. $v(t) \sim -12t^2 \to -\infty$. $a(t) \sim -24t \to -\infty$

Near $t = 0^+$

$v \approx 10t > 0$. $a \approx 10$

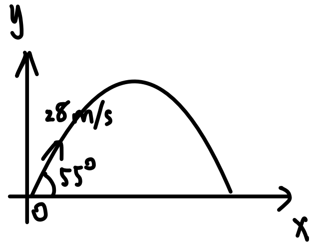
The plots are consistent.

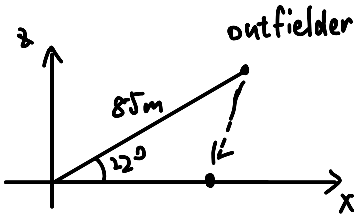
Q4. Projectile + Relative Motion in 3D (10 pts)

Baseball launched at 28 m/s, 55°. An outfielder is 85 m from the batter and 22° off the vertical plane of flight.

1) Two drawings (required): side view (projectile plane) and top view (show the 22° offset).
2) Choose axes; write $r_{ball}(t)$.
3) Time of flight to same height from y(t).
4) Fielder path: constant-speed straight line from initial position to ball at catch time; write $r_f(t)$.
5) Solve required speed and heading; report angle relative to his/fielder sightline to home plate.
6) Check: as 22°→0, do formulas reduce sensibly?

1)





2) x-y be the flight plane, with y be vertical

x-z shows the top view.

$$\vec{r}_{ball}(t) = \begin{cases} x(t) = V_0 \cos\theta\, t \\ y(t) = v_0 \sin\theta\, t - \frac{1}{2} g t^2 \\ z(t) = 0 \end{cases}$$

using $g = 9.8 m/s^2$.

numerically $v_0 \cos 55^\circ = 16.1$ m/s.

$v_0 \sin 55^\circ = 22.9$ m/s

$$\boxed{\vec{r}_{ball}(t) = \langle 16.1t,\ 22.9t - 4.9t^2,\ 0 \rangle}$$

3) set $y(t) = 0$, $t_f = \frac{2 v_0 \sin\theta}{g} = \frac{2(22.9)}{9.8} = \boxed{4.7\ s}$

4) $\vec{r}_f(0) = \langle 85 \cos 22^\circ, 0, 85 \sin 22^\circ \rangle$

$= \langle 78.8, 0, 31.8 \rangle$

$\vec{r}_f(t_f) = \langle 16.1 \times 4.7, 0, 0 \rangle$

$= \langle 75.3, 0, 0 \rangle$

$$\boxed{\begin{aligned} \vec{r}_f(t) &= \vec{r}_f(0) + \frac{t}{t_f}\left(\vec{r}_f(t_f) - \vec{r}_f(0)\right) \\ &= \langle 78.8, 0, 31.8 \rangle + \frac{t}{4.7} \langle -3.5, 0, -31.8 \rangle \end{aligned}}$$

5) $s = \frac{\| \vec{r}_f(t_f) - \vec{r}_f(0) \|}{t_f}$

$\theta = \tan^{-1}\left(\frac{85 \sin 22^\circ}{85 \cos 22^\circ - (v_0 \cos 55^\circ) t_f} \right)$.

plugging in

$x_{ball}(t_f) = (28 \cos 55^\circ)\, 4.7 = 75.8$ m

$x_{fielder}(0) = 78.8$ m, $z_{fielder}(0) = 31.8$m

so fielder moves $\sqrt{(78.8 - 75.8)^2 + 31.8^2} \approx 32.0$m

$\theta \approx \tan^{-1}\left(\frac{31.8}{78.8 - 75.8}\right) \approx \boxed{85^\circ}$

$s = \frac{32.0}{4.7} \approx \boxed{6.8\ m/s}$

6.) as $22^\circ \to 0^\circ$

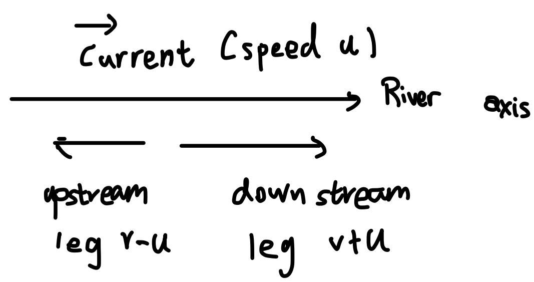
- Fielder is directly in the plane of flight ($z=0$)
- The problem reduces to 2D projectile motion formulas simplify correctly

Q5. Relative Motion—Boat vs Current (9 pts)

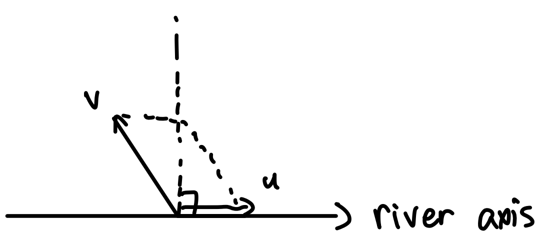
Round trip total distance D; boat speed v in still water; current u. Compare (a) up & downstream vs (b) straight across & back. Assume $u < v$.

1) Diagrams (required): (a) river axis with upstream/downstream legs; (b) cross-current headings with velocity triangles.
2) Derive times needed for (a) and (b).
3) State clearly why u<v is needed.
4) Check: as u→0, times must match.

1) (a)



(b)



2) (a) For one-way distance $\frac{D}{2}$

$$t_{up} = \frac{D/2}{v-u}, \quad t_{down} = \frac{D/2}{v+u}$$

$$T_a = t_{up} + t_{down} = \boxed{\frac{Dv}{v^2-u^2}}$$

(b) $$t_{across} = \frac{D/2}{\sqrt{v^2-u^2}}$$

$$T_b = 2t_{across} = \boxed{\frac{D}{\sqrt{v^2-u^2}}}$$

3) if $u \geq v$

- For the upstream case, $v-u \leq 0 \rightarrow$ boat can't make headway
- For the cross-current case, $\sin\theta = \frac{u}{v} > 1 \rightarrow$ no real angle exsist.

Hence, $u < v$ is requlred for motion.

4). as $u \rightarrow 0$, $t_a \rightarrow \frac{D}{v}$, $t_b \rightarrow \frac{D}{v}$

times must match

5) $t_a = \frac{Dv}{v^2-u^2}$. $t_b = \frac{D}{\sqrt{v^2-u^2}}$

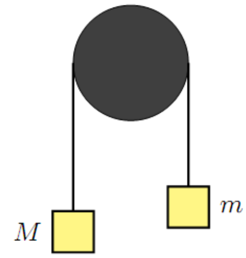
since $v < \sqrt{v^2-u^2} + u$

it follows that $t_a > t_b$

The cross-curret trip is faster

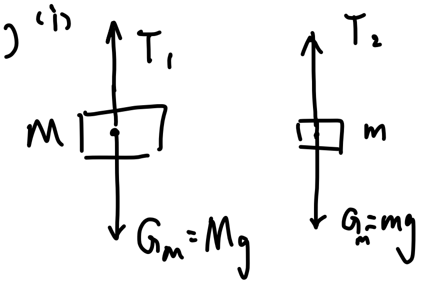
Q6. Capstan-Style Friction over a Cylinder (8 pts)

String over top of a fixed cylinder (R = 10 cm). Masses m = 10 kg and M at the two ends; $\mu_s = 0.45$. Find max/min M so the string does not slip.

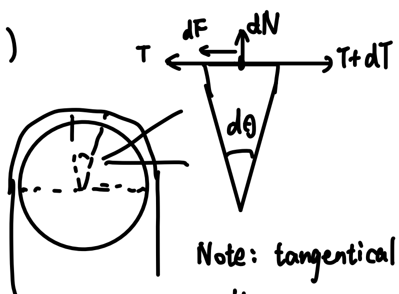


1) Two drawings (required): (i) FBDs of masses; (ii) infinitesimal string element on cylinder.
2) Use wrap angle $\theta = \pi$ rad (half-wrap) and justify.
3) Derive the capstan relation: $\int dT/T = \mu_s \int d\theta$.
4) Two impending-motion cases → bounds on M.
5) Check: as $\mu_s \to 0$, window collapses to $M \approx m$.

1) (i)



(ii)



Note: tangential direction would be reverse for the case M slide down.

2) The string wraps halfway over the top of the cylinder — from vertical left to vertical right — therefore: $\theta = \pi$ radians.

3) $dT = \mu_s T \, d\theta$, assuming $T_2 < T_1$ integrate from T_2 to T_1

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu_s \int_0^{\theta} d\theta \Rightarrow \ln\left(\frac{T_1}{T_2}\right) = \mu_s \theta, \quad T_1 = T_2 e^{\mu_s \theta}$$

otherwise $T_2 = T_1 e^{\mu_s \theta}$

either case $\int dT/T = \mu_s \int d\theta$ holds.

4) Case A: M heavier (tends down)

$$\frac{Mg}{mg} = e^{\mu_s \theta}$$

$$\Rightarrow M_{max} = m e^{\mu_s \theta}$$

Case B: M lighter (tends up)

$$\frac{mg}{Mg} = e^{\mu_s \theta}$$

$$\Rightarrow M_{min} = m e^{-\mu_s \theta}$$

Numerically.

$M_{max} = 10(4.11) = \boxed{41.1 \text{ kg}}$

$M_{min} = 10(0.243) = \boxed{2.43 \text{ kg}}$

5) $\mu_s \to 0 \Rightarrow e^{\mu_s \theta} \to 1$

$M_{max} = M_{min} = M$

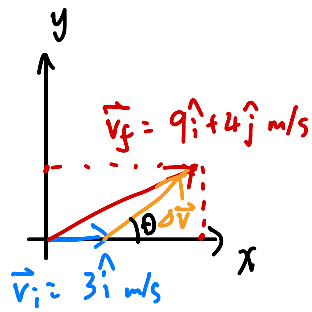
correct physical limit.

Q7. Newton's Laws on Ice (8 pts)

Drone, with m = 2.00 kg, goes from 3.00 ***i*** m/s to (9.00 ***i*** + 4.00 ***j***) m/s in 10.0 s due to a constant horizontal force.

1) Drawing (required): velocity triangle (initial, final, change).
2) Δv components → F = m(Δv/Δt) → F_x, F_y.
3) Magnitude & direction; include small orientation sketch (angle from +x).

1)



2) $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = (9-3)\hat{i} + 4\hat{j} = 6\hat{i} + 4\hat{j}$ m/s

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{6\hat{i} + 4\hat{j}}{10} = 0.6\hat{i} + 0.4\hat{j} \text{ m/s}^2$$

$$\vec{F} = m\vec{a} = 2.00(0.6\hat{i} + 0.4\hat{j}) = 1.2\hat{i} + 0.8\hat{j} \text{ N}$$

So $\boxed{F_x = 1.2\text{N}, \quad F_y = 0.8 \text{ N} \text{ towards positive of x, y axis}}$

3). $F = \sqrt{F_x^2 + F_y^2} = \sqrt{1.2^2 + 0.8^2} = \sqrt{2.08} = \boxed{1.44\text{N}}$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{0.8}{1.2}\right) = \boxed{33.7^\circ}$ above +x-axis

Bonus (0–2 pts): Timekeeping today

What's the current definition of the second and any imminent redefinition plans? Cite an authoritative source, 2–5 sentences.

The SI second is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom (BIPM, 2019). The BIPM and CCTF plan to refine the second using optical atomic clocks, which are far more precise than cesium clocks. A proposal for this redefinition is expected around 2026, with possible adoption later in the 2030s (BIPM, 2024).

Assistance notes:

References.

BIPM. 2019. "SI base unit: second (s)". Available at: https://www.bipm.org/en/si-base-units/second

BIPM. 2024. "FAQ concerning the redefinition of the second". Available at: https://www.bipm.org/en/faq-redefinition-second

LLMs are used as translators, brainstorm-assistants during the work. No direct words from LLM was taken.