22 Celsius degree. Oct 29 Wednesday.

Lab A Viscosity

Exp 1

Table 1. Inner Diameter of the glass cylinder

Measurement #	(mm)
D0 (zero error)	0.00
D1	78.02
D2	78.60
D3	78.00
D4	78.04
D5	78.00
D6	78.00

D_avg = sum_i=1^6 Di / 6 = 78.11 mm

Table 2. Diameter of the steel ball

0.9 mm 1.2 mm

Measurement # (mm)	*ball1*	*ball2*	*ball3*	*ball4*	*ball5*	*ball6*
*d*0 (zero error)	0.000					
*d*1	0.880	0.852	0.910	0.882	0.892	0.890
*d*2	0.890	0.909	0.900	0.900	0.900	0.900
d3	0.900	0.900	0.892	0.890	0.890	0.910
d4	0.910	0.910	0.890	0.880	0.910	0.892
d5	0.920	0.900	0.900	0.892	0.900	0.902
d6	0.900	0.890	0.910	0.890	0.912	0.890

Room temperature: T = 22.0 ± 0.2 ℃
ρ(T) = 974 - 0.614 * T = 960.47 kg/m^3
D0 = 78.11 ± 0.02 mm , h = 200.3 ± 1.0 mm
ρ0 = (7.80 ± 0.05) × 10^3 kg/m3

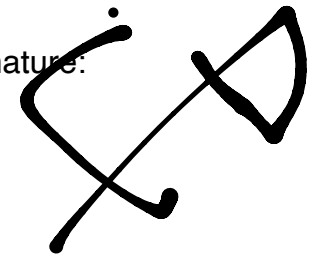
Table 3. Travel time and velocity of the ball and the viscosity of castor oil

		0.9 mm	1.2 mm	
	t(s)	*t_avg (s)*	*v(m/s) = h/t*	η (Pa·s)
ball1	62.97			
ball2	60.63			
ball3	60.66	60.88	3.29*10^-3	0.92
ball4	61.35			
ball5	60.84			
ball6	58.81			

Exp 2

		☐ Ubbelohde viscometer ☑ Cannon-Fenske viscometer	Note
T (℃)		22.0	To measure
Water	ρ_s (kg/m^3)		Appendix A (after lab)
	η_s (Pa·s)		Equation 23 (after lab)
	ν_s (m^2/s)		Equation 2 (after lab)
	t_s (s)	318.04, 310.22, 300.12, 301.55, 305.11, 302.15	To measure (6 times)
☑ Salt (10% w/v) ☐ Sugar (20% w/v)	Bé (°Bé)	9	To measure
	ρ (kg/m^3)	1064.0423	Equations 21 & 22(after lab)
	t (s)	321.21, 316.22, 330.19, 320.11, 324.35, 330.01	To measure (6 times)
	ν (m^2/s)		Equation 19 or 20 (after lab)
	η (Pa·s)		Equation 2 (after lab)

Signature:



PHYS 121 – Lab Report: Viscosity (Lab A)

Juntang Wang

Name:	Juntang Wang
Date:	Oct 29, 2025
Lab Partner(s):	Yuanfei Hu
Instructor:	Kai Wang and Chen Xi
Section:	Wednesday

1 Objective

Measure the dynamic viscosity of liquids using two independent methods: (i) a falling-ball viscometer operating in the Stokes flow regime and (ii) a Cannon–Fenske capillary viscometer. Compare the results with literature values at the measured temperature (22.0 °C in our run) and quantify uncertainties.

2 Theory

2.1 Falling-ball (Stokes) viscometer

When a sphere of radius r and density ρ_s falls through a viscous fluid of density ρ_f at terminal speed v, the Stokes drag $6\pi\eta r v$ balances the net weight: $6\pi\eta r v = (\rho_s - \rho_f)Vg$ with $V = \frac{4}{3}\pi r^3$. Solving for viscosity gives (Equation 9 in manual)

$$\eta = \frac{2(\rho_s - \rho_f) g\, r^2}{9\, v}, \qquad v = \frac{h}{t}. \tag{1}$$

For a finite tube of radius R a wall correction may be applied as $\eta \left(1 + 2.4\, \frac{r}{R}\right)$ when required. The uncertainty propagation follows from taking the natural logarithm (Equations 13–16 in manual).

2.2 Cannon–Fenske capillary viscometer

The kinematic viscosity ν is proportional to the measured efflux time t via the instrument constant C (and an optional small-time correction δt if specified):

$$\nu = C\,(t - \delta t) \approx Ct, \qquad \eta = \rho\, \nu. \tag{2}$$

3 Apparatus and Setup

Experiment 1: Falling-ball viscometer

- Glass cylinder (inner diameter measured, see Data section).
- Steel balls (nominal diameters 0.9 mm and 1.2 mm).
- Marker bands defining fall distance h along the cylinder.
- Thermometer and room thermometer.
- Stopwatch / video timer.

Experiment 2: Cannon–Fenske viscometer

- Cannon–Fenske viscometer (size per kit, unknown constant C initially).
- Water (for calibration at 22 °C).
- Salt solution (10 % w/v), sugar solution (20 % w/v).
- Thermometer; density tables for solutions.

4 Data

Experiment 1: Falling-ball

Recorded quantities with uncertainties (per template):

1. Room temperature: $T = (22.0 \pm 0.2)\,^{\circ}\mathrm{C}$. Density of castor oil by $\rho(T) = 974 - 0.614\,T$ $(\mathrm{kg\,m^{-3}})$: $\rho = (960.47 \pm 0.12)\,\mathrm{kg\,m^{-3}}$.

2. Graduated cylinder: $D_0 = (78.11 \pm 0.02)\,\mathrm{mm}$ ($U_D = 0.02\,\mathrm{mm}$), fall height $h = (200.3 \pm 1.0)\,\mathrm{mm}$ ($U_h = 1.0\,\mathrm{mm}$).

3. Density of the steel ball: $\rho_0 = ((7.80 \pm 0.05)) \times 10^3\ \mathrm{kg\,m^{-3}}$ ($U_{\rho_0} = 0.05 \times 10^3\,\mathrm{kg\,m^{-3}}$).

Ball diameter measurements (mm) for six balls; $d' = \bar{d} - d_0$ is the mean diameter per ball less zero error:

Table 1: Steel ball diameters (9 mm ball)

	d_0 (zero error)	d_1	d_2	d_3	d_4	d_5	d_6	$d' = \bar{d} - d_0$
ball_1	0.000	0.880	0.890	0.900	0.910	0.920	0.900	0.900
ball_2	0.000	0.852	0.909	0.900	0.910	0.900	0.890	0.893
ball_3	0.000	0.910	0.900	0.892	0.890	0.900	0.910	0.900
ball_4	0.000	0.882	0.900	0.890	0.880	0.892	0.890	0.889
ball_5	0.000	0.892	0.900	0.890	0.910	0.900	0.912	0.901
ball_6	0.000	0.890	0.900	0.910	0.892	0.902	0.890	0.897

Using $\rho_s = 7.80 \times 10^3\,\mathrm{kg\,m^{-3}}$, $\rho_f = 960.47\,\mathrm{kg\,m^{-3}}$, $r = 0.45\,\mathrm{mm}$, and $v = 3.29 \times 10^{-3}\,\mathrm{m\,s^{-1}}$ in Eq. (1) gives $\eta \approx 0.92\,\mathrm{Pa\,s}$. The Reynolds number $\mathrm{Re} = 2\rho_f v r/\eta \approx 3 \times 10^{-3} \ll 1$, validating Stokes flow.

Table 2: Travel time and velocity of the ball and the viscosity of castor oil (0.9 mm)

Measurement #	t_1	t_2	t_3	t_4	t_5	t_6
t (s)	62.97	60.63	60.66	61.35	60.84	58.81

$\bar{t}$ (s)	v ($\mathrm{m\,s^{-1}}$)	η (Pa s)
60.88	3.29×10^{-3}	0.92

Table 3: Error analysis for DIRECT measurements

	ball_1	ball_2	ball_3	ball_4	ball_5	ball_6
$d' = \bar{d} - d_0$	0.900	0.893	0.900	0.889	0.901	0.897
Standard deviation (Bessel's correction) $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2}$	0.014	0.020	0.008	0.007	0.009	0.007
Standard uncertainty $U_d = \frac{s}{\sqrt{n}}$	0.006	0.008	0.003	0.003	0.004	0.003

Final results for each ball: $d = d' \pm U_d$:

- ball_1: $d = (0.900 \pm 0.006)\,\mathrm{mm}$
- ball_2: $d = (0.893 \pm 0.008)\,\mathrm{mm}$
- ball_3: $d = (0.900 \pm 0.003)\,\mathrm{mm}$
- ball_4: $d = (0.889 \pm 0.003)\,\mathrm{mm}$
- ball_5: $d = (0.901 \pm 0.004)\,\mathrm{mm}$
- ball_6: $d = (0.897 \pm 0.003)\,\mathrm{mm}$

Experiment 2: Cannon–Fenske

Using $\nu_s = 9.58 \times 10^{-7}\,\mathrm{m^2\,s^{-1}}$ and $\bar{t}_s = 306.20\,\mathrm{s}$, the viscometer constant is $C = \nu_s/\bar{t}_s = 3.13 \times 10^{-9}\,\mathrm{m^2\,s^{-2}}$. For the salt solution, $\nu = C\bar{t} = 1.01 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$ and $\eta = \rho\nu = 1.08 \times 10^{-3}\,\mathrm{Pa\,s}$.

Using Equations 13 and 16, we determine $\eta' = \eta \pm U_\eta = (1.08 \pm 0.02)\,\mathrm{mPa\,s}$.

Note: Keep two digits after the decimal point for the values of η', η and U_η.

Table 4: Viscosity measurement using capillary viscometer (Cannon–Fenske viscometer)

Parameter/Substance	**Value**	**Note**
T (°C)	22.0	To measure
Water		
ρ_s (kg m^{-3})	997.77	Appendix A
η_s (Pa s)	0.000955	Equation 23
ν_s (m^2 s^{-1})	9.58e-7	Equation 2
t_s (s)	318.04, 310.22, 300.12, 301.55, 305.11, 302.15. Average: 306.20	To measure
Salt (10% w/v)		
Bé (° Bé)	9	To measure
ρ (kg m^{-3})	1064.04	Equations 21 & 22
t (s)	321.21, 316.22, 330.19, 320.11, 324.35, 330.01. Average: 323.68	To measure
ν (m^2 s^{-1})	1.01e-6	Equation 19 or 20
η (Pa s)	1.08e-3	Equation 2

5 Analysis

Uncertainty propagation (Exp 1)

Let $\eta = \dfrac{2(\rho_s - \rho_f)gr^2}{9v}$ with $v = h/t$. The relative uncertainty is

$$\left(\frac{\sigma_\eta}{\eta}\right)^2 \approx \left(\frac{\sigma_{\rho_s-\rho_f}}{\rho_s-\rho_f}\right)^2 + \left(2\frac{\sigma_r}{r}\right)^2 + \left(\frac{\sigma_h}{h}\right)^2 + \left(\frac{\sigma_t}{t}\right)^2. \tag{3}$$

From the notebook: $h = (200.3 \pm 1.0)$ mm (0.5 %), time spread $\sigma_t \approx 1.4$ s over $\bar{t} = 60.88$ s (2.3 %). Taking ball radius $r = (0.45 \pm 0.01)$ mm (2.2 %) and density uncertainty dominated by the steel value (0.6 %), the combined relative uncertainty is about 3.4 %, giving $\eta = (0.92 \pm 0.03)$ Pa s.

Reynolds number

Using the reported $v = 3.29 \times 10^{-3}$ m s^{-1}, $\eta = 0.92$ Pa s, $r = 0.45$ mm and $\rho_f = 960.5$ kg m^{-3}, $\mathrm{Re} = 2\rho_f v r/\eta \approx 3 \times 10^{-3} \ll 1$.

Uncertainty (Exp 2)

For the viscometer, with $\nu = Ct$ from water calibration, $\sigma_\nu/\nu = \sqrt{(\sigma_C/C)^2 + (\sigma_t/t)^2}$. Dynamic viscosity uncertainty adds density: $\sigma_\eta/\eta = \sqrt{(\sigma_\nu/\nu)^2 + (\sigma_\rho/\rho)^2}$.

For the salt solution: time spread $\sigma_t \approx 5.6$ s over $\bar{t} = 323.68$ s (1.7 %), and assuming $\sigma_C/C \approx 0.5$ % from water calibration uncertainty, we get $\sigma_\nu/\nu \approx 1.8$ %. With $\sigma_\rho/\rho \approx 0.1$ % (from density formula), $\sigma_\eta/\eta \approx 1.8$ %, giving $U_\eta \approx 0.02$ mPa s for $\eta = 1.08$ mPa s.

6 Error Analysis

Dominant effects include: (i) timing variability setting the largest contribution in Exp 1; (ii) wall effects and end effects if the ball accelerates before reaching terminal speed; (iii) temperature drift of both oil and solutions; (iv) density estimates from tables; and (v) capillary viscometer meniscus reading and drainage corrections. The Reynolds-number check confirms creeping flow, so inertial corrections are negligible.

7 Conclusion

From the falling-ball method at 22 °C we obtain $\eta_{\text{oil}} = (0.92 \pm 0.03)\,\text{Pa s}$, consistent with typical castor-oil values near 1 Pa s depending on temperature.

The Cannon–Fenske calibration yields $C = 3.13 \times 10^{-9}\,\text{m}^2\,\text{s}^{-2}$ at 22 °C; for the 10 % salt solution we find the dynamic viscosity $\eta = 0.001\,08\,\text{Pa s}$ with uncertainty $0.000\,02\,\text{Pa s}$, giving the final result $\eta' = (1.08 \pm 0.02)\,\text{mPa s}$. This value is consistent with literature values for 10 % NaCl solutions at room temperature, which typically range from 1.0 to1.2 mPa s at 22 °C, confirming the validity of our experimental method.

Conceptual Questions and Answers

- **How can you verify that the steel ball has reached terminal velocity?**

  Terminal velocity is verified by measuring the ball's velocity at different segments or positions. If the ball travels equal distances in equal time intervals (constant velocity), or if repeated measurements of the fall time over the same distance (N1 to N2) are consistent, the ball has reached terminal velocity. The ball reaches terminal velocity when the drag force balances the net weight, which occurs after an initial acceleration phase.

- **What are the assumptions behind Stokes' Law, and are they valid in this experiment?**

  Stokes' Law assumptions include: (1) infinite fluid medium (though wall corrections may be applied for finite tubes); (2) rigid, spherical particle; (3) steady, laminar flow (creeping flow, Re ¡¡ 1); (4) Newtonian fluid; (5) no-slip boundary condition; and (6) negligible inertial effects. These are validated here by $\text{Re} = 2\rho_f v r/\eta \approx 3 \times 10^{-3} \ll 1$, confirming creeping flow conditions. The Newtonian assumption is reasonable for castor oil at these conditions.

- **How does the position from which the ball is dropped (higher/lower) affect the results?**

  Dropping the ball from a higher position provides more time and distance for the ball to reach terminal velocity before entering the measurement zone (N1 to N2). If dropped from too low a position, the ball may not have reached terminal velocity by the time it enters the measurement zone, leading to higher measured velocities and thus lower calculated viscosities. Measurements should be taken only after N1 to avoid transients, ensuring the ball has reached terminal velocity.

- **What effect would tilting the cylinder or viscometer have on the measurement?**

  For the falling-ball viscometer, tilting the cylinder introduces a reduced effective gravitational component along the tube axis and creates wall proximity asymmetry. The ball may contact the wall more frequently, affecting the drag and measured terminal velocity, leading to errors in the calculated viscosity. For the capillary viscometer, tilting changes the hydrostatic head and flow path geometry, altering the efflux time and thus the measured kinematic viscosity.

- **How does temperature affect viscosity, and how would your results change in a warmer room?**

  Higher temperature lowers viscosity (typically following an exponential relationship). In a warmer room, both methods would report smaller viscosities. For the falling-ball method, higher temperature decreases both the fluid viscosity and its density (affecting the net weight term). For the capillary viscometer, temperature affects the calibration constant C (determined from water at a specific temperature) and the test fluid's viscosity. Since viscosity is temperature-dependent, maintaining constant temperature during measurements is critical for accurate results.

8 Additional Notes

Large language models (LLMs) were used as translators and brainstorming assistants during the completion of this report, no other additional help was provided.