PHYS121 Integrated Science-Physics

W4 Ch15 Oscillation

References:
[1] David Halliday, Jearl Walker, Resnick Jearl, 'Fundamentals of Physics', (Wiley, 2018)
[2] Doug Giancoli, 'Physics for Scientists and Engineers with modern physics', (Pearson, 2009)
[3] Hugh D. Young, Roger A. Freedman, 'University Physics with Modern Physics', (Pearson, 2012)
And others specified when needed.

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Learning Outcomes

- Analyze the motion of simple harmonic motion (spring-mass system, simple pendulum).
- Apply Newton's second law to torsion pendulum and physical pendulum.
- Describe and analyze damped/forced oscillations and resonance.

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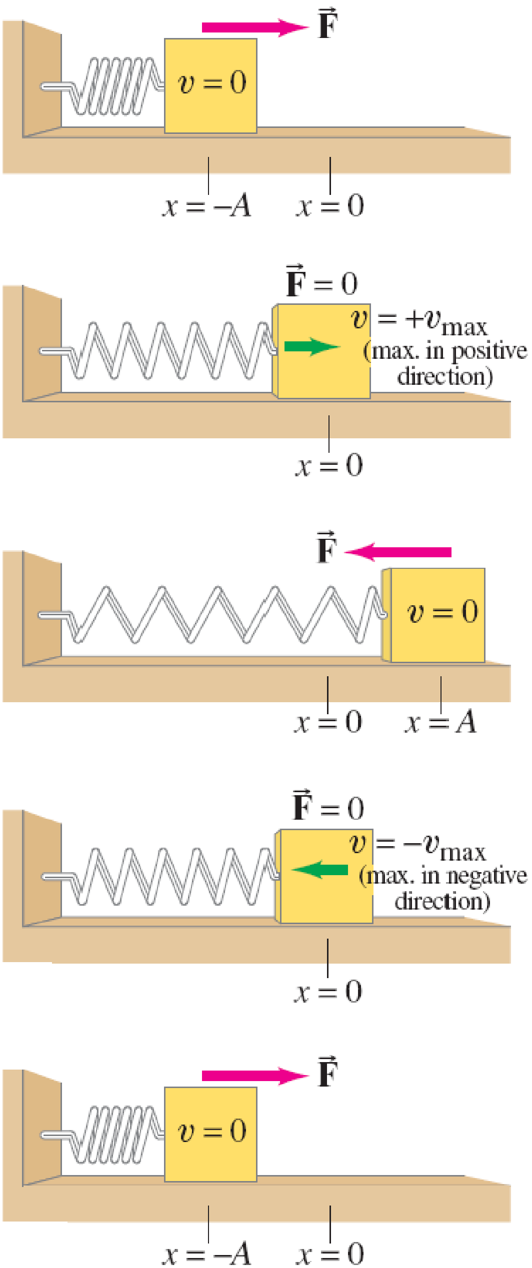
Oscillations of a Spring



- **Displacement** is measured from the **equilibrium** point.
- **Amplitude** is the **maximum** displacement.
- A **cycle** is a full to-and-fro motion.
- **Period** is the **time** required to complete one cycle.
- **Frequency** is the **number** of cycles completed per second.

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Simple Harmonic Motion

Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator (SHO).

Substituting $F = -kx$ into Newton's second law gives the equation of motion:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0,$$

with trial solutions of the form:

$$\overbrace{x(t)}^{\text{Displacement at time } t} = \underbrace{x_m}_{\text{Amplitude}} \cos\overbrace{(\underset{\text{Angular frequency}}{\omega}\, \underset{\text{Time}}{t} + \underset{\text{Phase constant or phase angle}}{\phi})}^{\text{Phase}}$$

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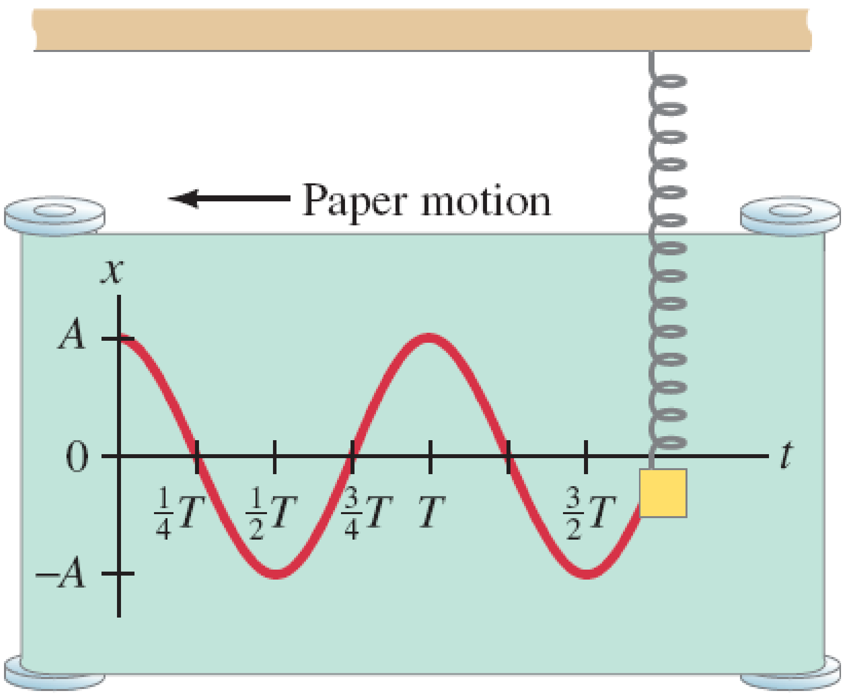
Substituting, we verify that this solution does indeed satisfy the equation of motion, with:

$$\omega^2 = \frac{k}{m}.$$



The constants A (x_m) and ϕ will be determined by initial conditions; A (x_m) is the amplitude, and ϕ gives the phase of the motion at $t = 0$.

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- The angular frequency is also:

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

Equation (15-5)

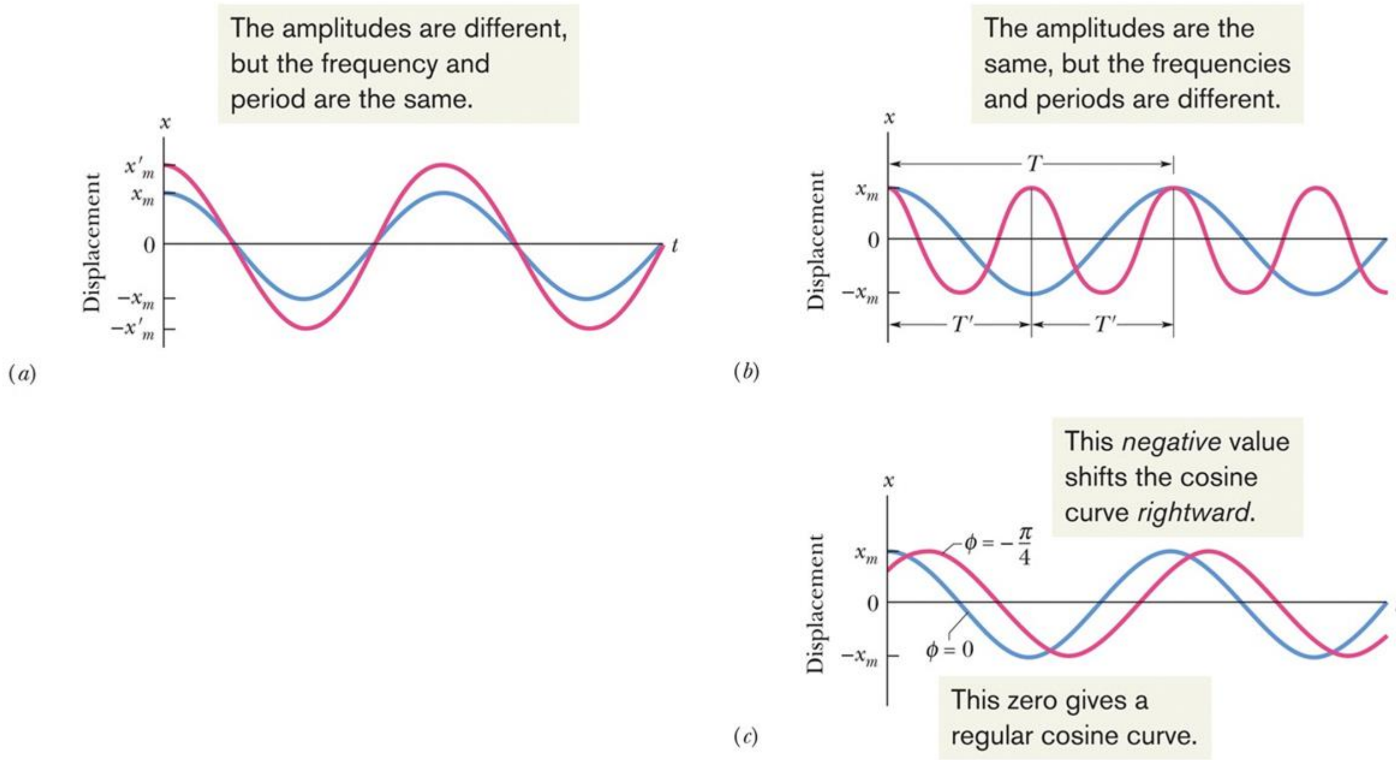

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Figure 15-5

- The velocity can be found by the time derivative of the position function:

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

- The value ωx_m is the **velocity amplitude** v_m
- The acceleration can be found by the time derivative of the velocity function, or 2nd derivative of position:

$$a(t) = -\omega^2 x_m \cos(\omega + \phi)$$

- The value $\omega^2 x_m$ is the **acceleration amplitude** a_m
- Acceleration related to position:

$$a(t) = -\omega^2 x(t).$$

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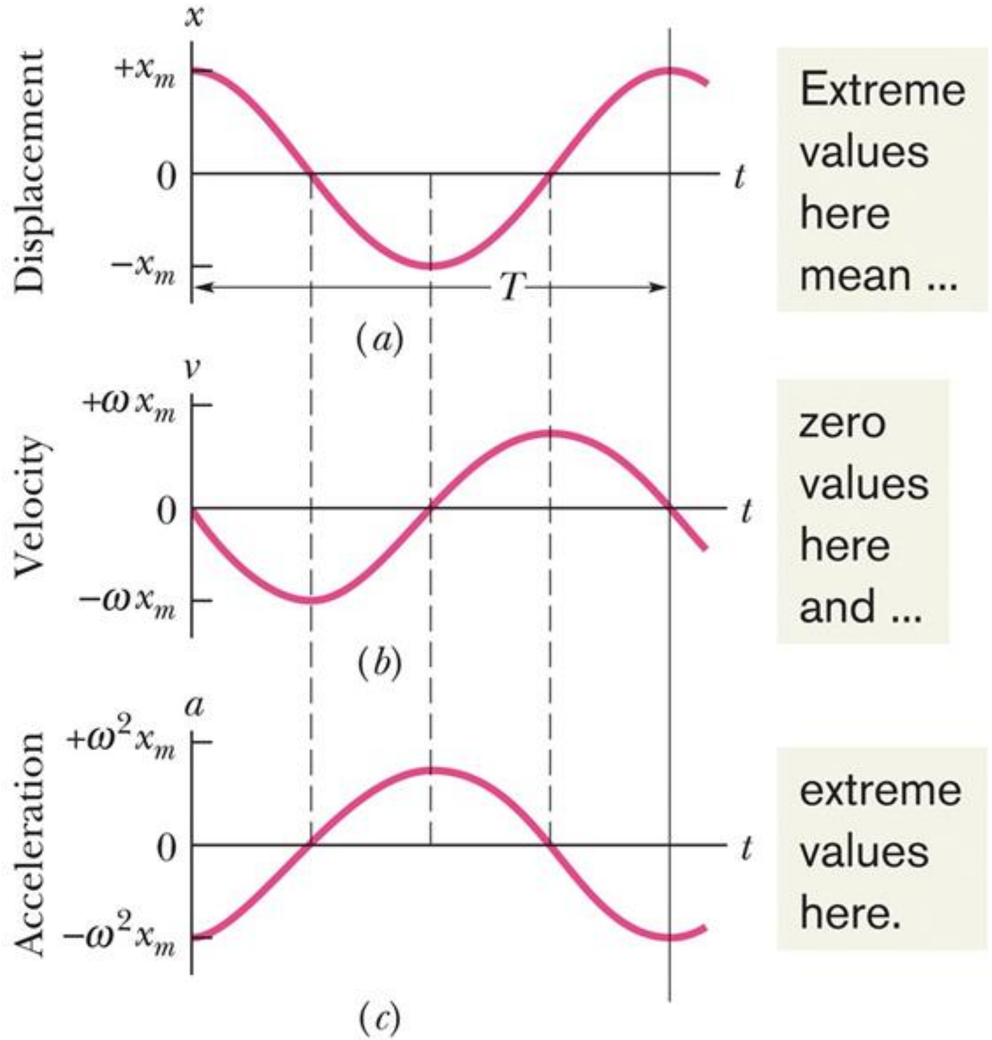

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Figure 15-6

In SHM, the acceleration a is proportional to the displacement x but opposite in sign, and the two quantities are related by the square of the angular frequency ω.

Checkpoint 2

Which of the following relationships between a particle's acceleration a and its position x indicates simple harmonic oscillation: (a) $a = 3x^2$, (b) $a = 5x$, (c) $a = -4x$, (d) $a = \frac{-2}{x}$?

For the SHM, what is the angular frequency (assume the unit of rad/s)?

Answer:

(c) where the angular frequency is 2

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Checkpoint 3

Which of the following relationships between the force F on a particle and the particle's position x gives SHM: (a) $F = -5x$,

(b) $F = -400x^2$, (c) $F = 10x$, (d) $F = 3x^2$?

Answer:

only (a) is simple harmonic motion (note that b is harmonic motion, but nonlinear and not SHM)

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Energy in Simple Harmonic Motion

- Write the functions for kinetic and potential energy:

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}kx_m{}^2 \sin^2(\omega t + \phi).$$

- Their sum is defined by:

$$E = U + K = \frac{1}{2}kx_m^2.$$

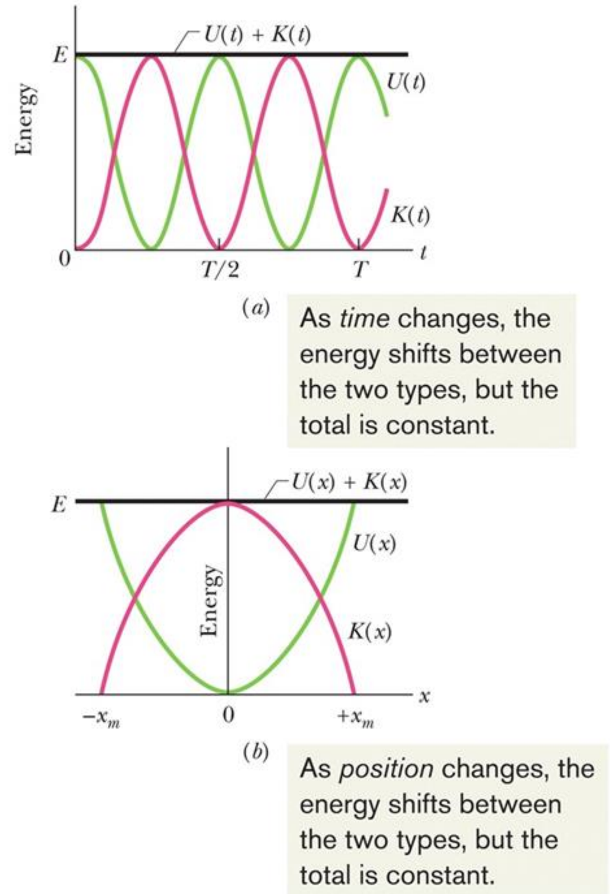

Figure 15-8

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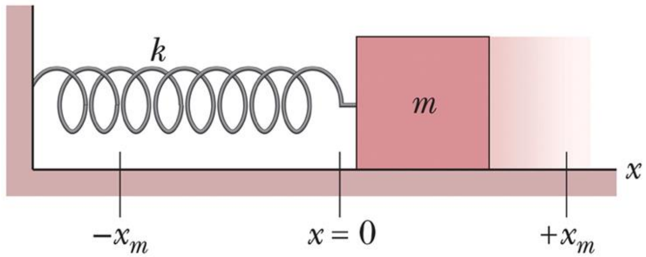


Figure 15-7

Checkpoint 4

In Fig. 15-7, the block has a kinetic energy of 3 J and the spring has an elastic potential energy of 2 J when the block is at $x = +2.0$ cm. (a) What is the kinetic energy when the block is at $x = 0$? What is the elastic potential energy when the block is at (b) $x = -2.0$ cm and (c) $x = -x_m$?

Answer: (a) 5 J (b) 2 J (c) 5 J

Pendulums, Circular Motion

- A **simple pendulum**: a bob of mass m suspended from an unstretchable, massless string
- Bob feels a restoring torque:

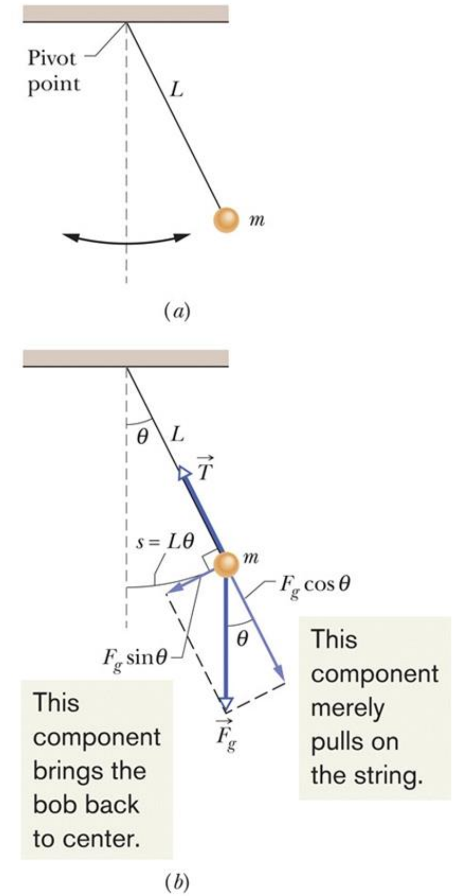
$$\tau = -L\left(F_g \sin\theta\right),$$

- Relating this to moment of inertia:

$$\alpha = -\frac{mgL}{I}\theta.$$

If the angle is small, $\sin\theta \approx \theta$.

- Angular acceleration proportional to position but opposite in sign



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- **Angular amplitude** θ_m of the motion must be <u>*small*</u>

$$\alpha = -\frac{mgL}{I}\theta = \frac{d^2\theta}{dt^2} \qquad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0,$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{mgL}{I}\right)\theta = 0, \quad \theta = \theta_{\max}\cos(\omega t + \phi),$$

- The angular frequency is: $\omega = \sqrt{\dfrac{mgL}{I}}.$

- The period is (for simple pendulum, $I = mL^2$):

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Equation (15-28)

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Pendulums, Circular Motion

- A **physical pendulum** has a complicated mass distribution

$$\tau = -mgh\sin\theta.$$

$$I\frac{d^2\theta}{dt^2} = -mgh\sin\theta.$$

For small angles, this becomes:

$$\frac{d^2\theta}{dt^2} + \left(\frac{mgh}{I}\right)\theta = 0,$$

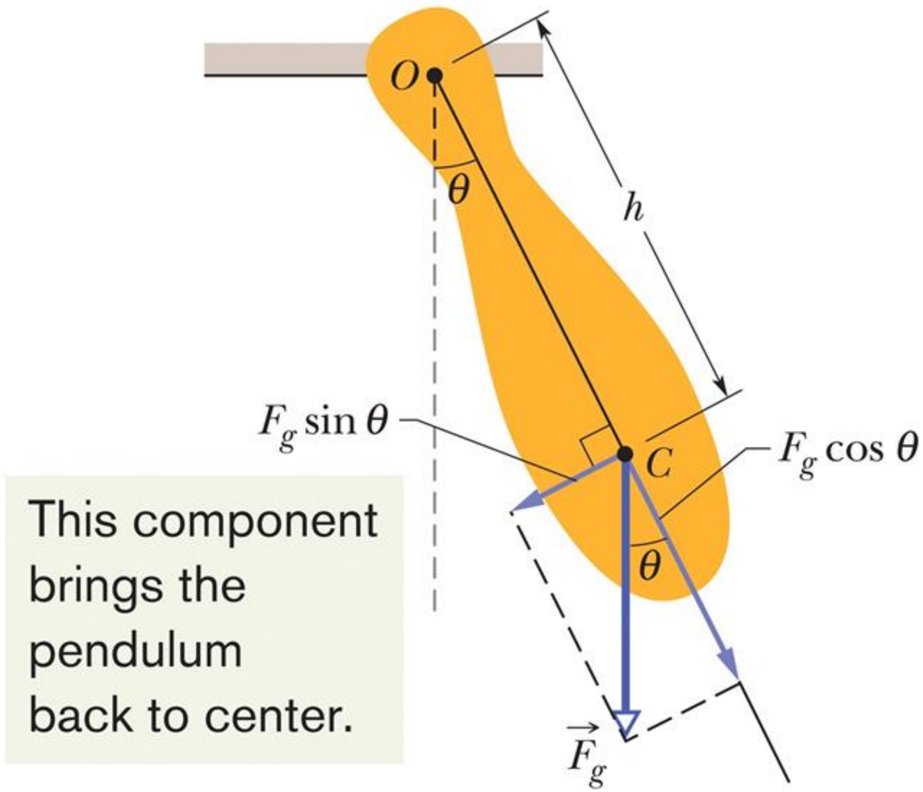

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Figure 15-12

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- An analysis is the same except rather than length L we have distance h to the com, and I will be particular to the mass distribution $\theta = \theta_{\max} \cos(\omega t + \phi),$
- The period is:

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Equation (15-29)

- A physical pendulum will not show SHM if pivoted about its com
- The center of oscillation of a physical pendulum is the length L_0 of a simple pendulum with the same period

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- A physical pendulum can be used to determine free-fall acceleration g
- Assuming the pendulum is a uniform rod of length L:

$$I = I_{\text{com}} + mh^2 = \frac{1}{12}mL^2 + m\left(\frac{1}{2}L\right)^2 = \frac{1}{3}mL^2. \qquad \textbf{Equation (15-30)}$$

- Then solve Eq. 15-29 for g:

$$g = \frac{8\pi^2 L}{3T^2}. \qquad \textbf{Equation (15-31)}$$

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Checkpoint 5

Three physical pendulums, of masses m_0, $2m_0$, and $3m_0$, have the same shape and size and are suspended at the same point. Rank the masses according to the periods of the pendulums, greatest first.

Answer:

All the same: mass does not affect the period of a pendulum

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An Angular Simple Harmonic Oscillator

- A **torsion pendulum**: elasticity from a twisting wire
- Moves in **angular simple harmonic motion**

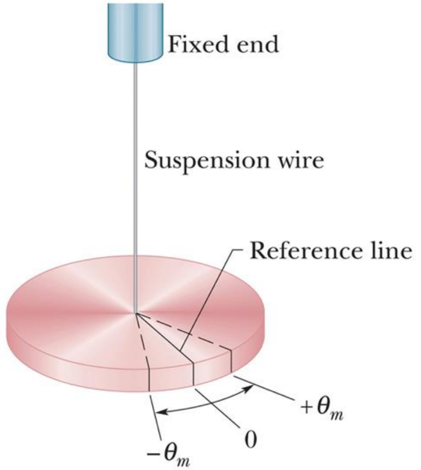

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Figure 15-9

$$\tau = -\kappa\theta = I\alpha$$ **Equation (15-22)**

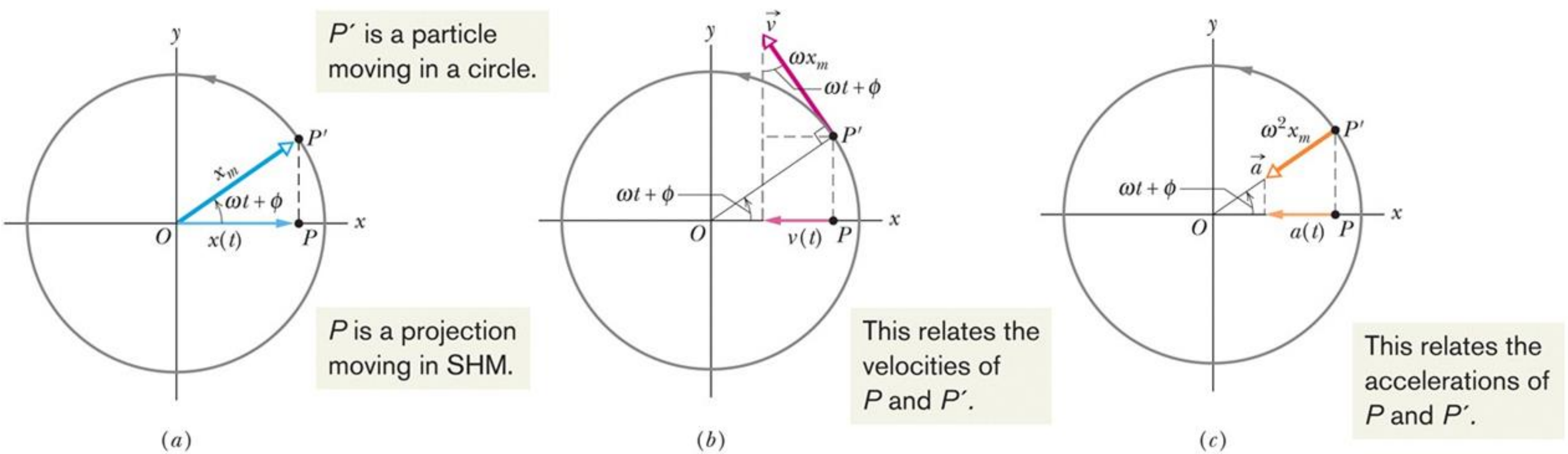
- κ is called the torsion constant
- Angular form of Hooke's law
- Replace linear variables with their angular analogs and
- We find:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$ **Equation (15-23)**

- Simple harmonic motion is circular motion viewed edge-on

  Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

- Figure 15-15 shows a reference particle moving in uniform circular motion
- Its angular position at any time is $\omega t + \phi$



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- Projecting its position onto x:

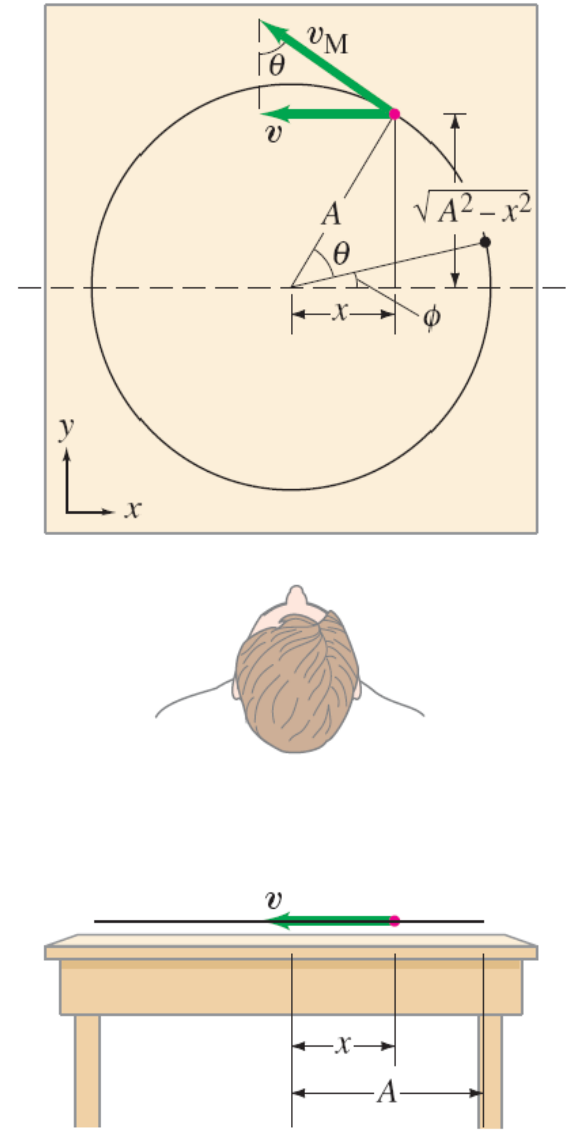
$$x(t) = x_m \cos(\omega t + \phi),$$

- Similarly with velocity and acceleration:

$$v(t) = -\omega x_m \sin(\omega t + \phi),$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi),$$

- We indeed find this projection is simple harmonic motion



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Damped Simple Harmonic Motion

- When an external force reduces the motion of an oscillator, its motion is **damped**
- Assume the liquid exerts a **damping force** proportional to velocity (accurate for slow motion)

$$F_d = -bv,$$ **Equation (15-39)**

- b is a damping constant, depends on the vane and the viscosity of the fluid

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- We use Newton's second law and rearrange to find:

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0.$$

- The solution to this differential equation is:

$$x(t)=x_m e^{\frac{-bt}{2m}}\cos(\omega' t+\phi),$$

- With angular frequency:

$$\omega'=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}.$$

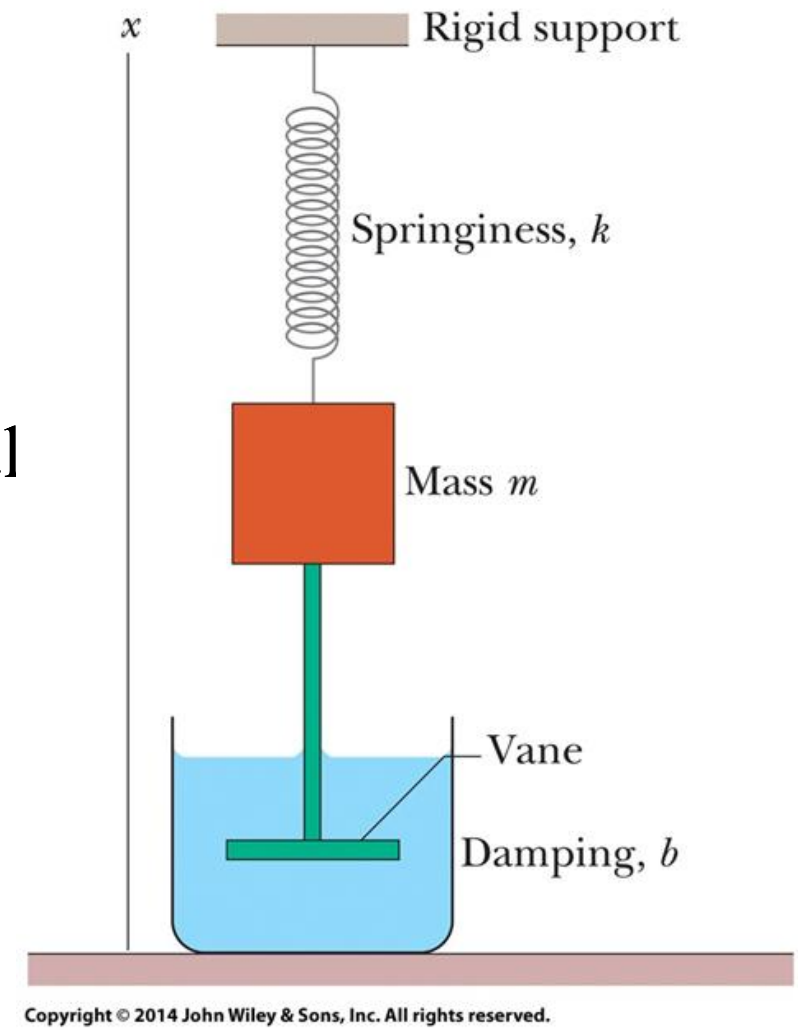

Figure 15-16

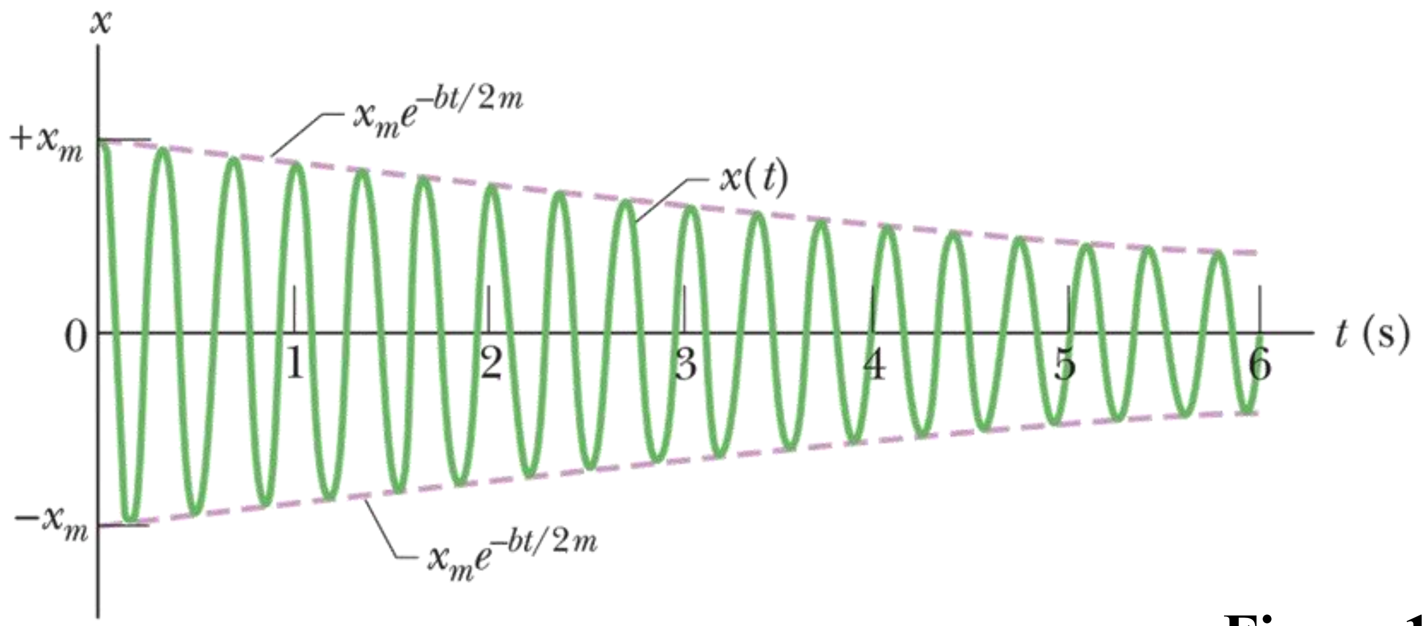

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Figure 15-17

- If the damping constant is small, $\omega' \approx \omega$
- For small damping we find mechanical energy by substituting our new, decreasing amplitude:

$$E(t) \approx \frac{1}{2} k x_m^2 e^{\frac{-bt}{m2}},$$

Equation (15-44)

Forced Oscillations and Resonance

- Forced, or driven, oscillations are subject to a periodic applied force
- The natural angular frequency ω of the system, which is the angular frequency at which it would oscillate when left to.
- A forced oscillator oscillates at the angular frequency of its driving force ω_d:

$$x(t) = x_m \cos(\omega_d t + \phi), \qquad \textbf{Equation (15-45)}$$

- The displacement amplitude is a complicated function of ω_d and ω
- The velocity amplitude of the oscillations is greatest when:

$$\omega_d = \omega \qquad \textbf{Equation (15-46)}$$

- This condition is called **resonance** $\omega_d = \omega$
- This is also *approximately* when the displacement amplitude is largest
- Resonance has important implications for the stability of structures
- Forced oscillations at resonant frequency may result in rupture or collapse

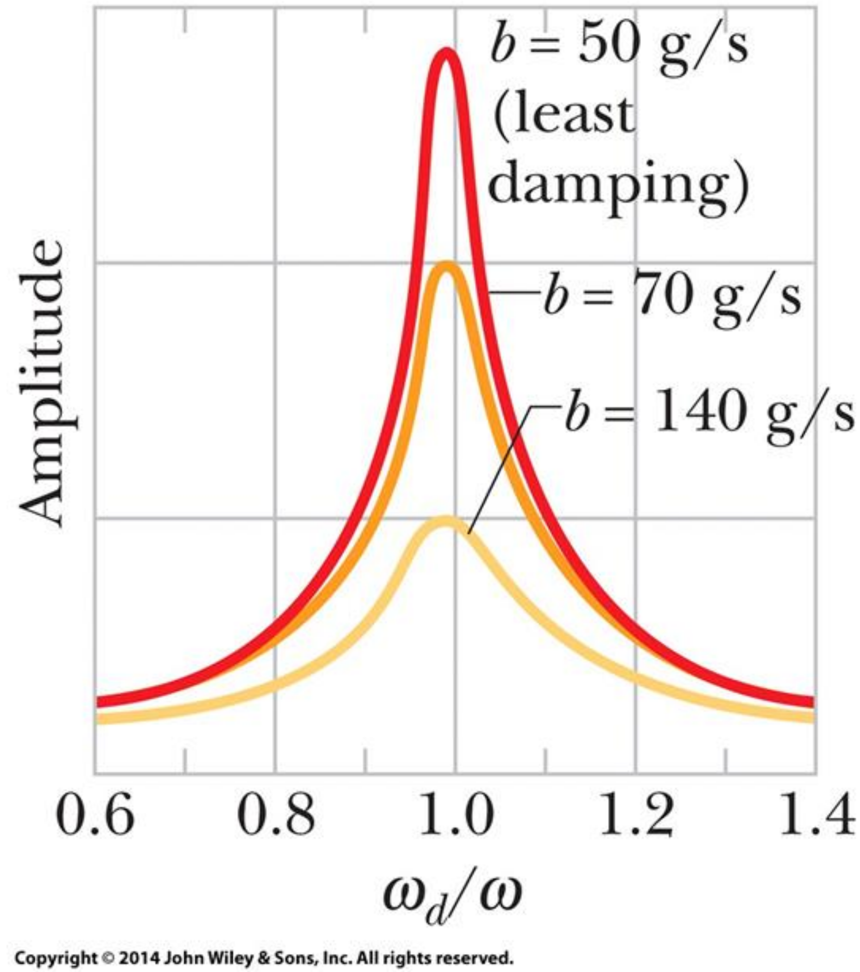

Figure 15-18

Learning Outcomes

- ✓ Analyze the motion of simple harmonic motion (spring-mass system, simple pendulum).
- ✓ Apply Newton's second law to torsion pendulum and physical pendulum.
- ✓ Describe and analyze damped/forced oscillations and resonance. $\omega_d = \omega$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$x(t) = x_m \cos(\omega t + \phi) \qquad \omega = \frac{2\pi}{T} = 2\pi f. \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad \omega = \sqrt{\frac{k}{m}}$$

$$x(t) = x_m e^{\frac{-bt}{2m}} \cos(\omega' t + \phi), \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

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CHAPTER REVIEW AND EXAMPLES, DEMOS

https://openstax.org/books/university-physics-volume-1/pages/15-summary

What is so significant about SHM? For one thing, *the period T and frequency f of a simple harmonic oscillator are independent of amplitude*. The string of a guitar, for example, oscillates with the same frequency whether plucked gently or hard.

This (the vertical spring) is just what we found previously for a horizontally sliding mass on a spring. The constant force of gravity only served to shift the equilibrium location of the mass.

The lab about Pohl's pendulum.

APPENDIX E

Series expansions

1. Binomial theorem: $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \cdots$

2. $(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \cdots \left(x^2 < 1\right)$

3. $(1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \cdots \left(x^2 < 1\right)$

4. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

5. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

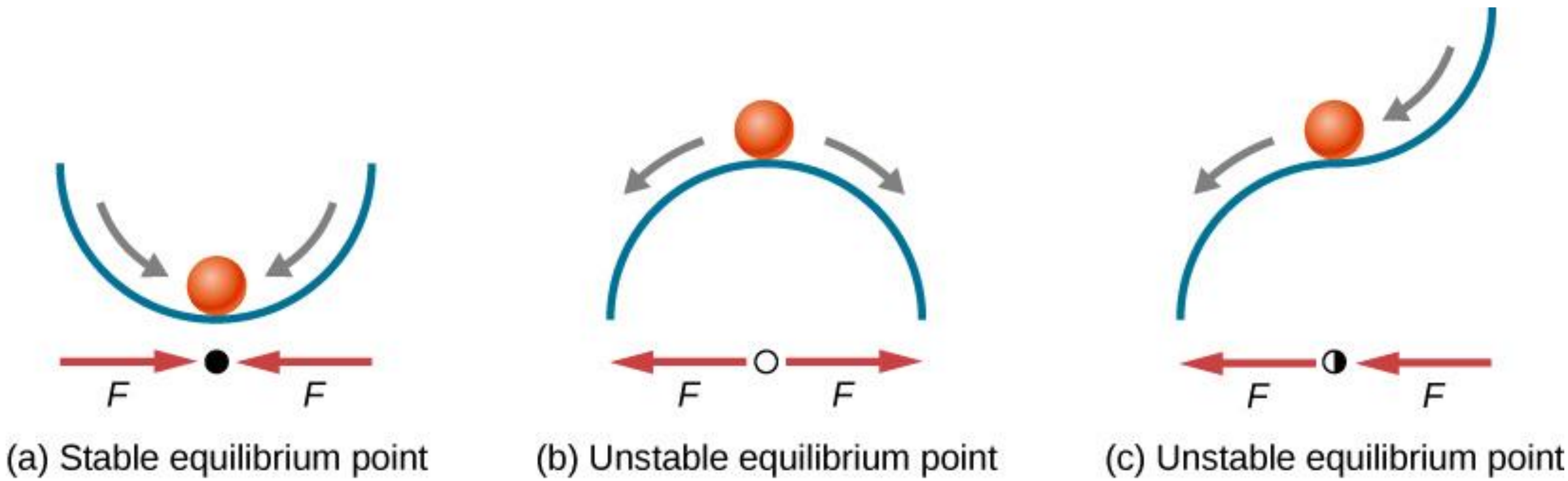
6. $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$

7. $e^x = 1 + x + \frac{x^2}{2!} + \cdots$

8. $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots (|x| < 1)$

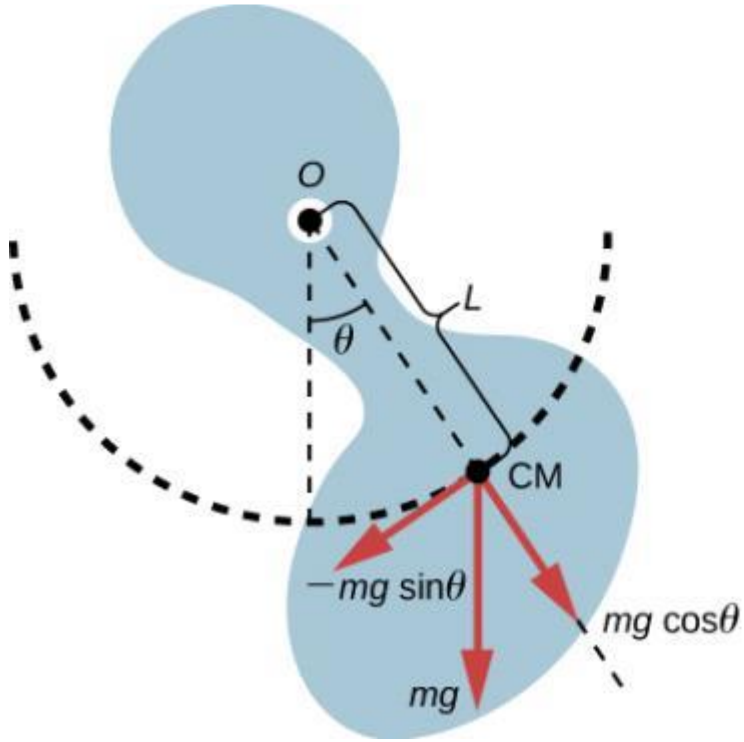
This OpenStax book is available for free at http://cnx.org/content/col12031/1.10

FIGURE 15.14



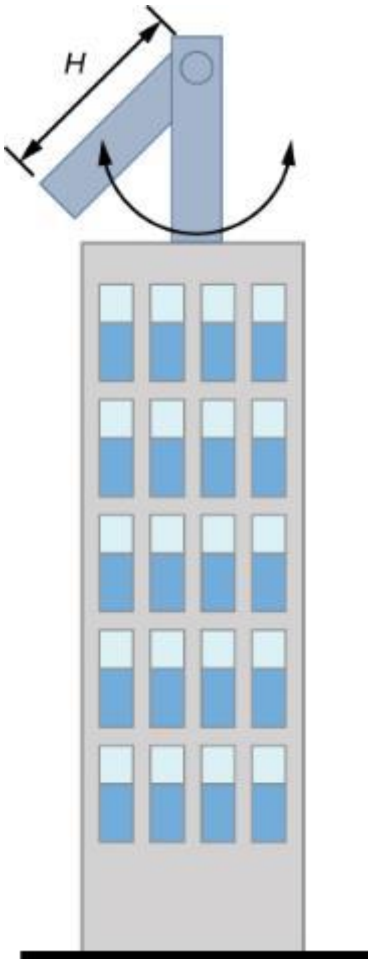
Examples of equilibrium points. (a) Stable equilibrium point; (b) unstable equilibrium point; (c) unstable equilibrium point (sometimes referred to as a half-stable equilibrium point).

FIGURE 15.21



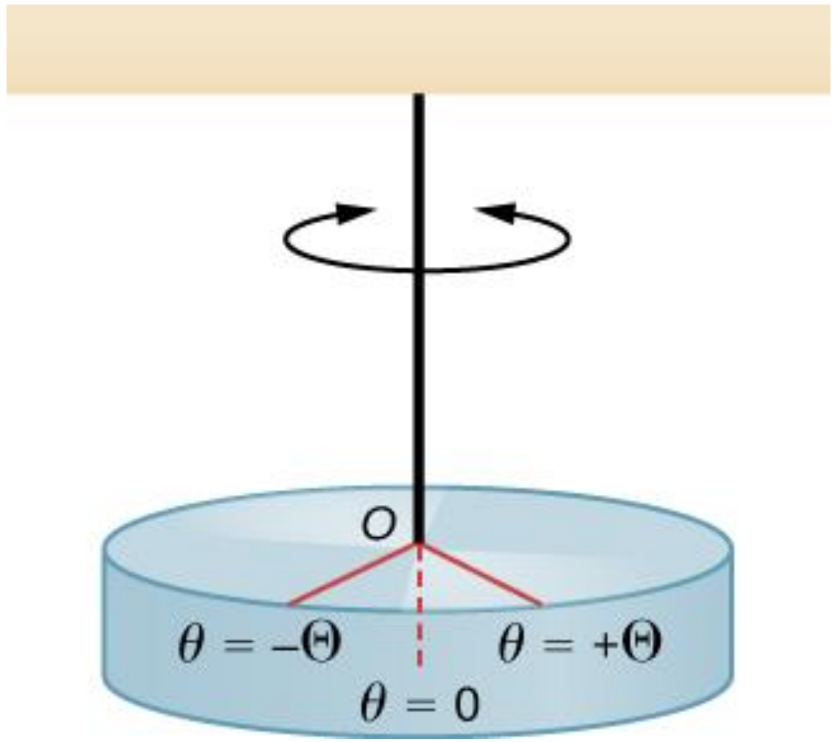
A physical pendulum is any object that oscillates as a pendulum, but cannot be modeled as a point mass on a string. The force of gravity acts on the center of mass (CM) and provides the restoring force that causes the object to oscillate. The minus sign on the component of the weight that provides the restoring force is present because the force acts in the opposite direction of the increasing angle θ.

EXAMPLE 15.4



In extreme conditions, skyscrapers can sway up to two meters with a frequency of up to 20.00 Hz due to high winds or seismic activity. Several companies have developed physical pendulums that are placed on the top of the skyscrapers. As the skyscraper sways to the right, the pendulum swings to the left, reducing the sway.

FIGURE 15.22



A torsional pendulum consists of a rigid body suspended by a string or wire. The rigid body oscillates between $\theta = +\Theta$ and $\theta = -\Theta$.

Questions