PHYS121 Integrated Science-Physics

W1T2 Freely falling object, Vectors

References:

[1] David Halliday, Jearl Walker, Resnick Jearl, ‘Fundamentals of Physics’, (Wiley, 2018)
[2] Doug Giancoli, ‘Physics for Scientists and Engineers with modern physics’, (Pearson, 2009)
[3] Hugh D. Young, Roger A. Freedman, ‘University Physics with Modern Physics’, (Pearson, 2012)
And others specified when needed.

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2.6.8. Two cars travel along a level highway. An observer on the ground notices that the distance between the cars is *increasing*. Which one of the following statements concerning this situation is *necessarily* true?

a) Both cars could be accelerating at the same rate.

b) The leading car has the greater acceleration.

c) The trailing car has the smaller acceleration.

d) The velocity of each car is increasing.

e) At least one of the cars has a *non-zero* acceleration.

Slide based on Ref. [1]

Learning outcomes

- Analyze the freely falling object motion.
- Solve problems with changing acceleration in 1-D using calculus.
- Explain the difference between a scalar and a vector.
- Perform vector calculation in physics problems.

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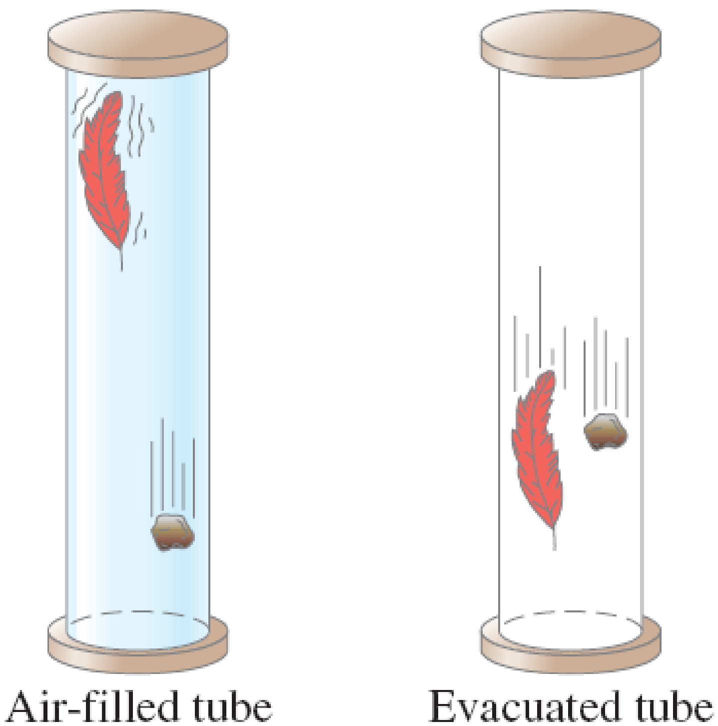
Freely Falling Objects



The acceleration due to gravity at the Earth's surface is approximately 9.80 m/s^2. At a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

Equations for constant acceleration motion apply.

Pay attention to the positive direction!

Slide based on Ref. [2]

Free-Fall Acceleration

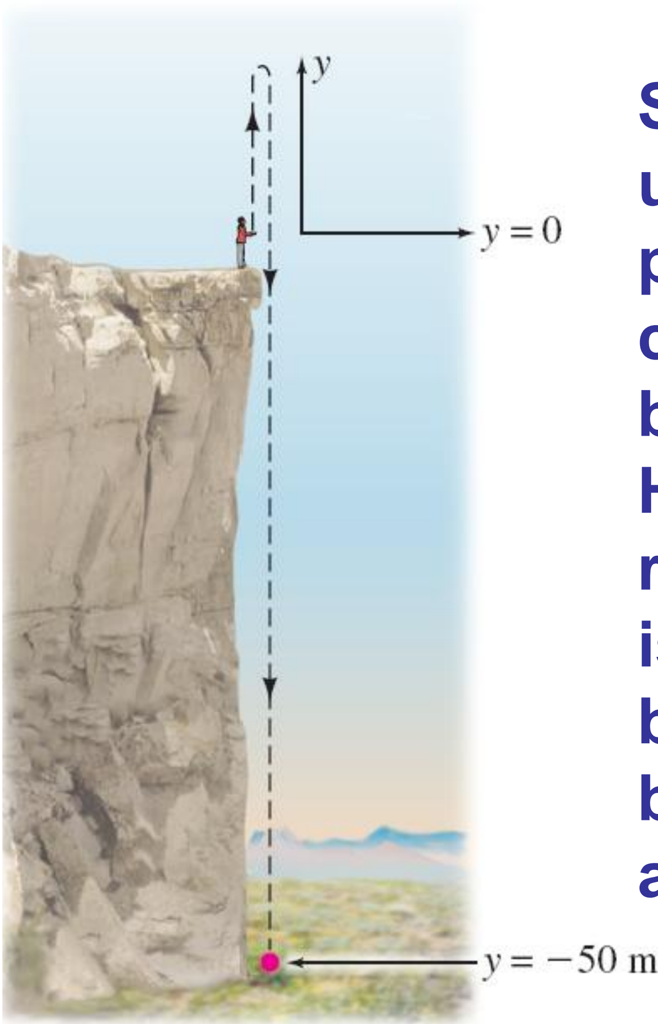
Checkpoint 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

Answers:

(a) The sign is positive (the ball moves upward); (b) The sign is negative (the ball moves downward); (c) The ball's acceleration is always $-9.8\ \text{m/s}^2$ at all points along its trajectory

Example 2-20: Ball thrown upward at edge of cliff. (self practice)



Suppose that a ball is thrown upward at a speed of 15.0 m/s by a person standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below. (a) How long does it take the ball to reach the base of the cliff? (b) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

Slide based on Ref. [2]

Variable Acceleration; Integral Calculus

Deriving the kinematic equations through integration:

$$dv = a\,dt$$

$$\int_{v=v_0}^{v} dv = \int_{t=0}^{t} a\,dt.$$

For constant acceleration,

$$v - v_0 = at.$$

Slide based on Ref. [2]

Then:

$$dx = v\,dt$$
$$= (v_0 + at)\,dt$$

$$\int_{x=x_0}^{x} dx = \int_{t=0}^{t} (v_0 + at)\,dt.$$

For constant acceleration,

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2.$$

Slide based on Ref. [2]

Variable Acceleration; Integral Calculus

Example 2-21: Integrating a time-varying acceleration.

An experimental vehicle starts from rest ($v_0 = 0$) at $t = 0$ and accelerates at a rate given by $a = (7.00\ \mathrm{m/s^3})t$. What is (a) its velocity and (b) its displacement 2.00 s later?

Slide based on Ref. [2]

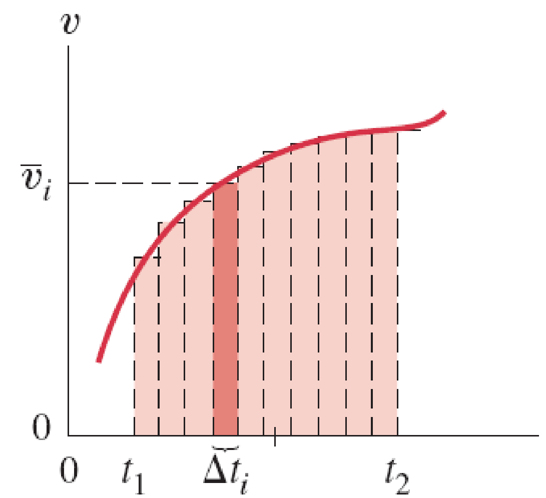
Graphical Analysis and Numerical Integration

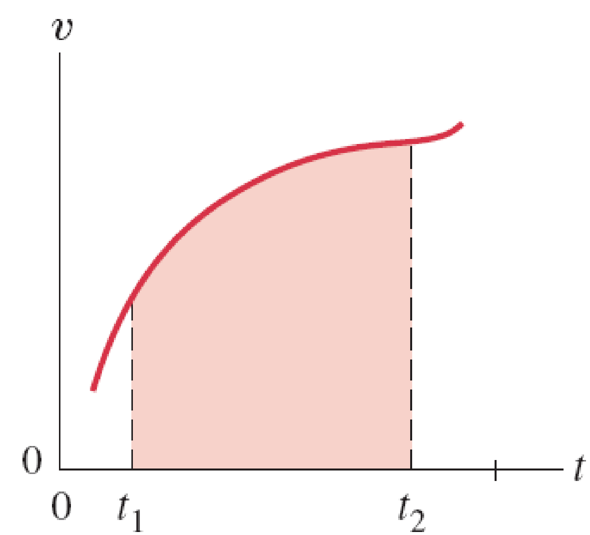
The total displacement of an object can be described as the area under the *v-t* curve:

$$x_2 - x_1 = \lim_{\Delta t \to 0} \sum_{t_1}^{t_2} \bar{v}_i \Delta t_i$$

$$= \int_{t_1}^{t_2} v(t)\, dt.$$

$$\int_{t_0}^{t_1} v\, dt = \begin{pmatrix} \text{area between velocity curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$





Slide based on Ref. [2]

Graphical Analysis and Numerical Integration

Similarly, the velocity may be written as the area under the *a-t* curve.

$$v_1 - v_0 = \int_{t_0}^{t_1} a \, dt$$

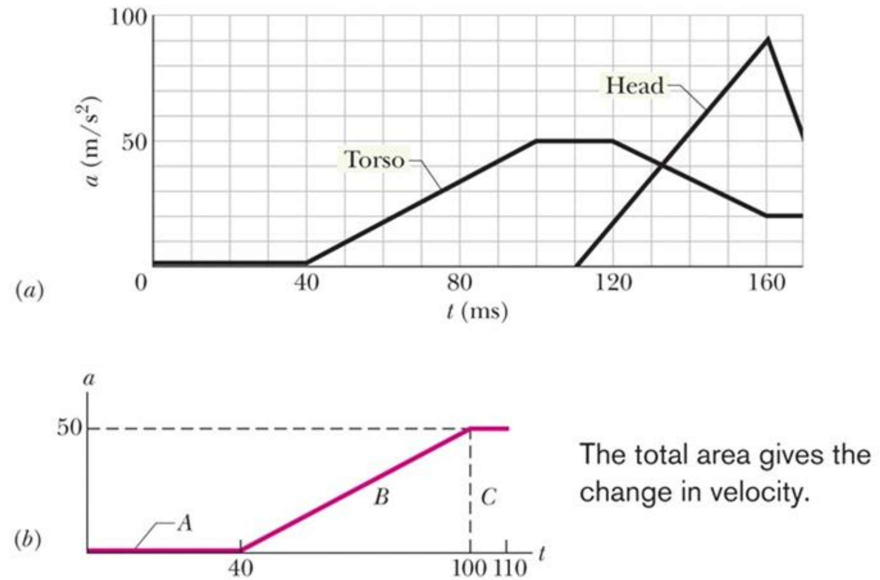
$$\int_{t_0}^{t_1} a \, dt = \begin{pmatrix} \text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

However, if the velocity or acceleration is not integrable, or is known only graphically, numerical integration may be used instead.

Slide based on Ref. [2]

Example

The graph shows the acceleration of a person's head and torso in a whiplash incident (sit in a car rear-ended by another one).



Find the torso speed at $t = 0.110$ s (assuming an initial speed of 0).

The area under the pink curve:

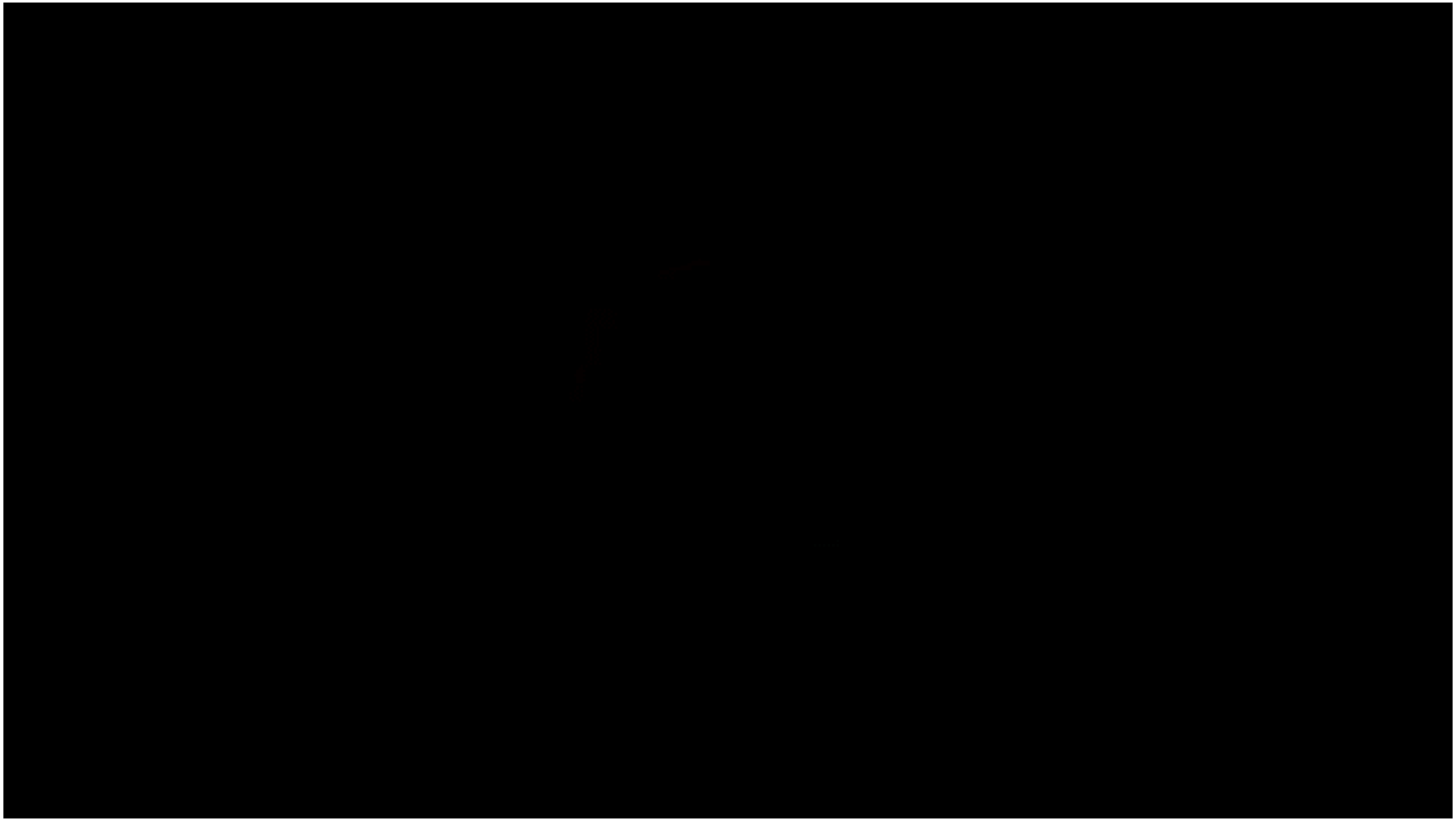
$$\text{area } A = 0$$

$$\text{area } B = 0.5(0.060\,\text{s})(50\,\text{m/s}^2) = 1.5\,\text{m/s}$$

$$\text{area } C = (0.010\ \text{s})(50\ \text{m/s}^2) = 0.50\ \text{m/s}$$

$$\text{total area} = 2.0\ \text{m/s}$$

Vectors, what even are they Essence of linear algebra, chapter 1



https://www.youtube.com/watch?v=fNk_zzaMoSs

Vectors and Their Components

- Physics deals with quantities that have both size and direction
- A **vector 向量** is a mathematical object with size and direction
- A **vector quantity** is a quantity that can be represented by a vector
  - Examples: position, velocity, acceleration
  - Vectors have their own rules for manipulation
- A **scalar 标量** is a quantity that does not have a direction
  - Examples: time, temperature, energy, mass
  - Scalars are manipulated with ordinary algebra

- The simplest example is a **displacement vector**
- If a particle changes position from *A* to *B*, we represent this by a vector arrow pointing from *A* to *B*

- In (a) we see that all three arrows have the same magnitude and direction: they are identical displacement vectors.
- In (b) we see that all three paths correspond to the same displacement vector. The vector tells us nothing about the actual path that was taken between *A* and *B*.

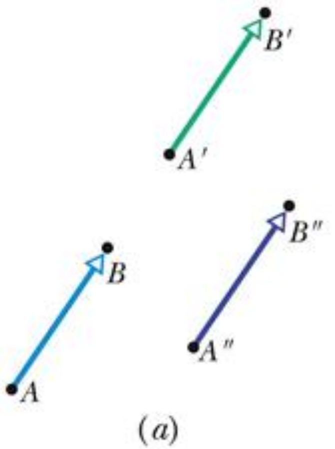
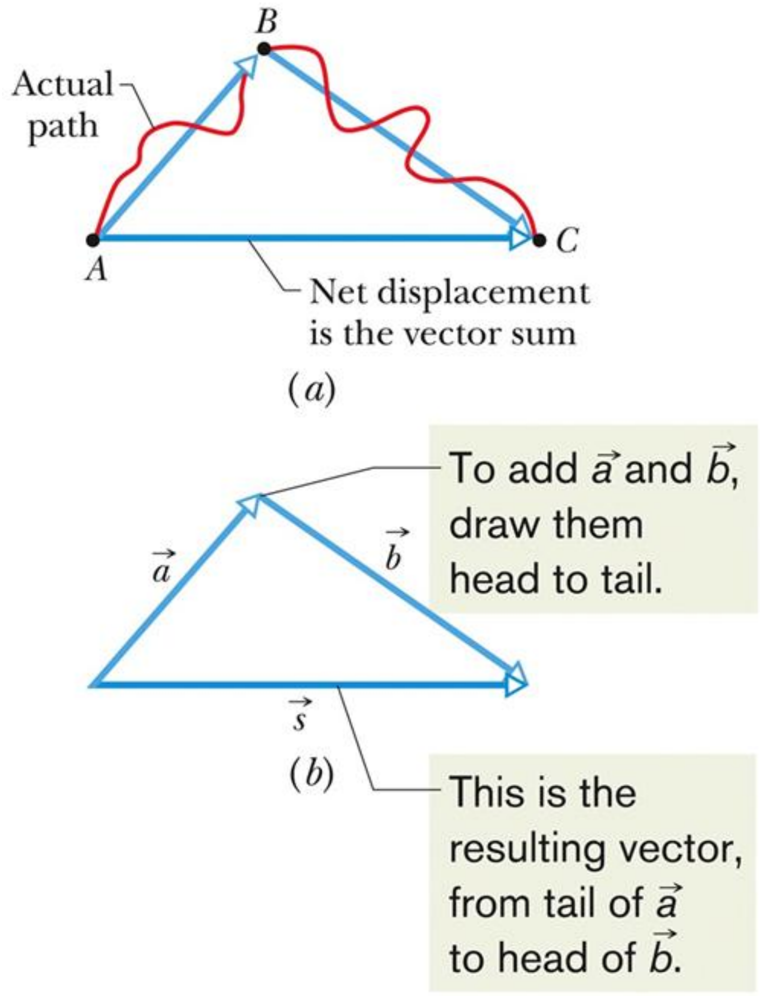


Figure 3-1

- The **vector sum**, or **resultant**
  - Is the result of performing vector addition
  - Represents the net displacement of two or more displacement vectors

$$\vec{s} = \vec{a} + \vec{b},$$ **Equation (3-1)**

  - Can be added graphically as shown: (tail-to-tip method)



- Vector addition is **commutative**
  - We can add vectors in any order

$$\vec{a}+\vec{b}=\vec{b}+\vec{a} \quad (\text{commutative law}). \qquad \textbf{Equation (3-2)}$$

- Vector addition is **associative**
  - We can group vector addition however we like

$$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c}) \quad (\text{associative law}). \qquad \textbf{Equation (3-3)}$$

- A negative sign reverses vector direction

$$\vec{b}+(-\vec{b})=0.$$

- We use this to define vector subtraction

$$\vec{d}=\vec{a}-\vec{b}=\vec{a}+(-\vec{b}) \qquad \textbf{Equation (3-4)}$$

- These rules hold for all vectors, whether they represent displacement, velocity, etc.
- Only vectors of the same kind can be added
  - (displacement) + (displacement) makes sense
  - (displacement) + (velocity) does not

The magnitudes of displacements $\vec{a}$ and $\vec{b}$ are 3 m and 4 m, respectively, and $\vec{c} = \vec{a} + \vec{b}$. Considering various orientations of $\vec{a}$ and $\vec{b}$, what are (a) the maximum possible magnitude for $\vec{c}$ and (b) the minimum possible magnitude?

Answer:

(a) 3 m + 4 m = 7 m

(b) 4 m − 3 m = 1 m

- Rather than using a graphical method, vectors can be added by **components**
  - A component is the projection of a vector on an axis
- The process of finding components is called **resolving the vector**

- The components of a vector can be positive or negative.
- They are unchanged if the vector is shifted in any direction (but not rotated).

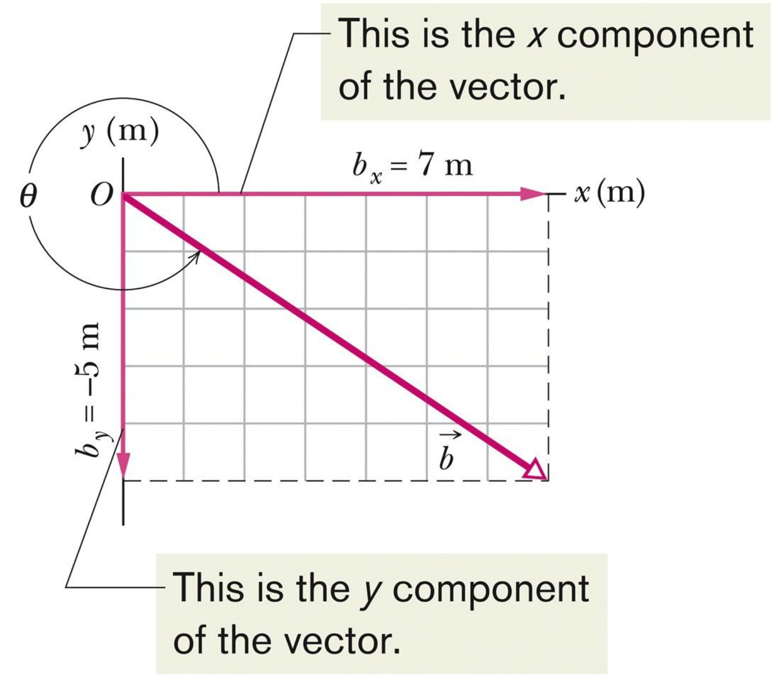


Figure (3-8)

- Components in two dimensions can be found by:

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta, \qquad \textbf{Equation (3-5)}$$

- Where θ is the angle the vector makes with the positive x axis, and a is the vector length
- The length and angle can also be found if the components are known

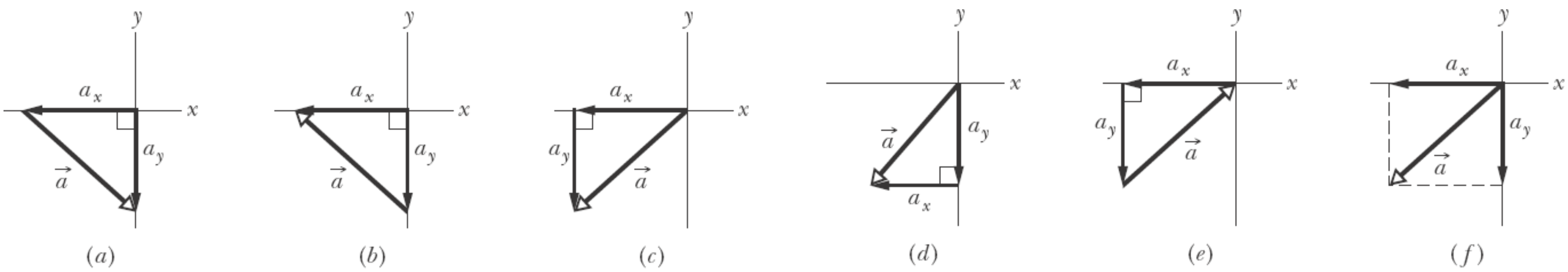
$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x} \qquad \textbf{Equation (3-6)}$$

- Therefore, components fully define a vector
- In the three dimensional case we need more components to specify a vector

$$(a, \theta, \phi) \text{ or } (a_x, a_y, a_z)$$

Checkpoint 2

In the figure, which of the indicated methods for combining the *x* and *y* components of vector to determine that vector?



Answer: choices (c), (d), and (f) show the components properly arranged to form the vector

- Angles may be measured in degrees or radians
- Recall that a full circle is 360°, or 2π rad

$$40^\circ \frac{2\pi \text{ rad}}{360^\circ} = 0.70 \text{ rad.}$$

- Know the three basic trigonometric functions

$$\sin\theta = \frac{\text{leg opposite } \theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{leg adjacent to } \theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{leg opposite } \theta}{\text{leg adjacent to } \theta}$$

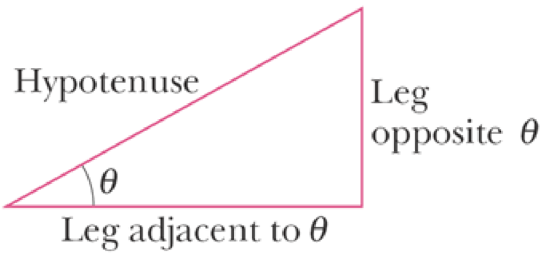


Figure (3-11)

Unit Vectors, Adding Vectors by Components

- A **unit vector**
  - Has magnitude 1
  - Has a particular direction
  - Lacks both dimension and unit
  - Is labeled with a hat: ^
- We use a **right-handed coordinate system**
  - Remains right-handed when rotated

$$\vec{a} = a_x \hat{\mathrm{i}} + a_y \hat{\mathrm{j}}$$ **Equation (3-7)**

$$\vec{b} = b_x \hat{\mathrm{i}} + b_y \hat{\mathrm{j}}.$$ **Equation (3-8)**

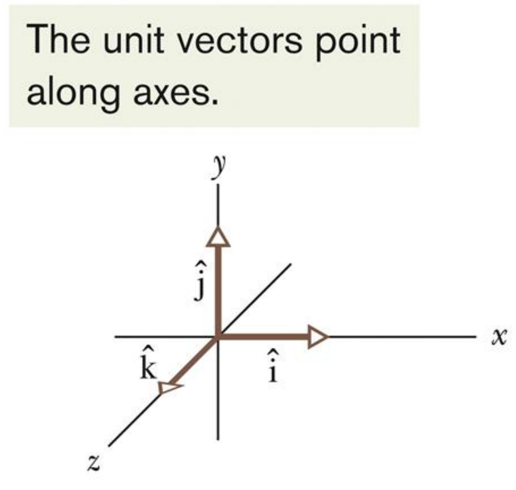
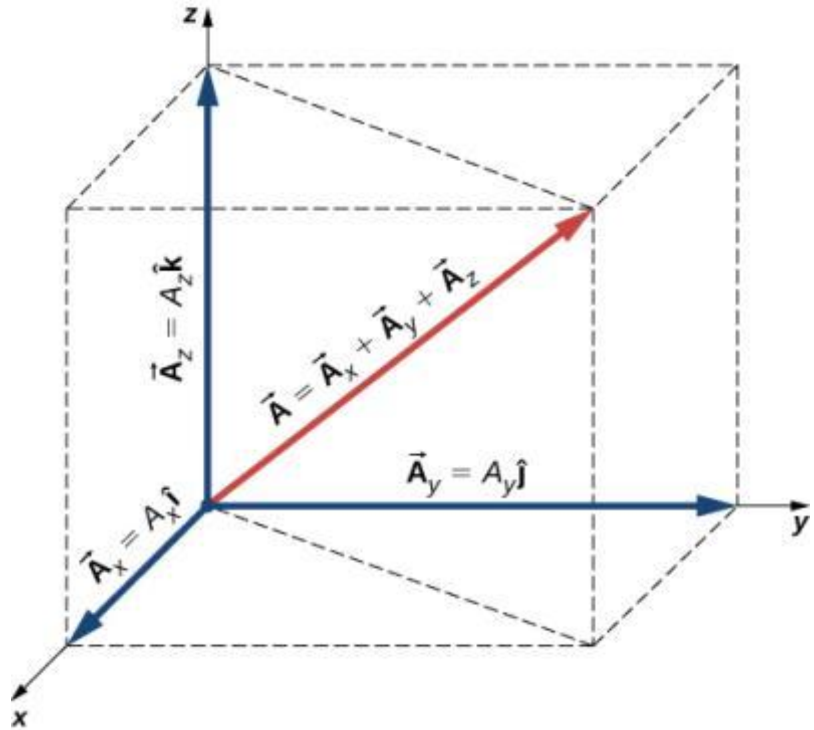


Figure (3-13)

FIGURE 2.22



A vector in three-dimensional space is the vector sum of its three vector components.

- The quantities $a_x\mathrm{i}$ and $a_y\mathrm{j}$ are **vector components**

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}$$ **Equation (3-7)**

$$\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}.$$ **Equation (3-8)**

- The quantities a_x and a_y alone are **scalar components**
  - Or just "components" as before
- Vectors can be added using components

Equation (3-9) $$\vec{r} = \vec{a} + \vec{b}, \quad \rightarrow \quad r_x = a_x + b_x$$ **Equation (3-10)**

$$r_y = a_y + b_y$$ **Equation (3-11)**

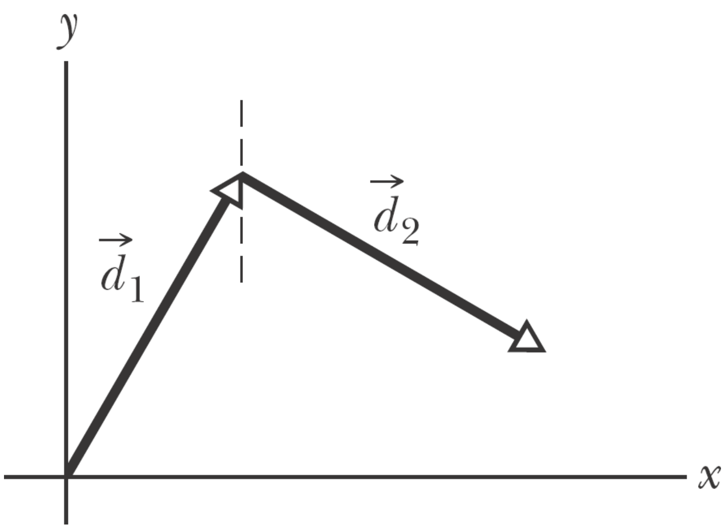
$$r_z = a_z + b_z.$$ **Equation (3-12)**

- To subtract two vectors, we subtract components

$$d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z,$$

$$\vec{d} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}} + d_z\hat{\mathrm{k}}. \qquad \textbf{Equation (3-13)}$$

(a) In the figure here, what are the signs of the x components of $\vec{d}_1$ and $\vec{d}_2$? (b) What are the signs of the y components of $\vec{d}_1$ and $\vec{d}_2$? (c) What are the signs of the x and y components of $\vec{d}_1 + \vec{d}_2$?



Answer:

(a) positive, positive

(b) positive, negative

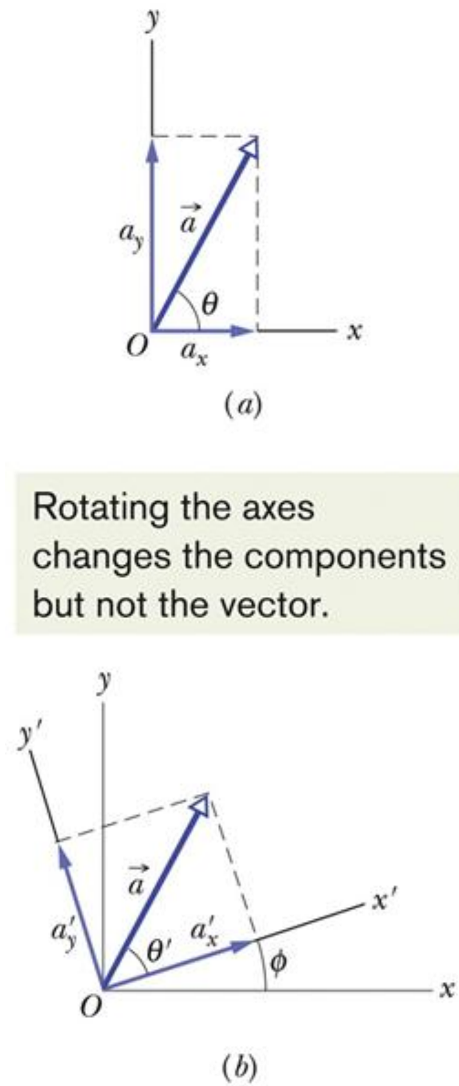
(c) positive, positive

- Vectors are independent of the coordinate system used to measure them
- We can rotate the coordinate system, without rotating the vector, and the vector remains the same

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a'^2_x + a'^2_y}$$ **Equation (3-14)**

$$\theta = \theta' + \phi.$$ **Equation (3-15)**

- All such coordinate systems are equally valid.



Multiplying Vectors

- Multiplying a vector ***a*** by a scalar

$$c\vec{a} = ca_x\hat{\mathrm{i}} + ca_y\hat{\mathrm{j}}$$

- Multiplying two vectors: the **scalar product**
  - Also called the **dot product**
  - Results in a scalar, where a and b are magnitudes and φ is the angle between the directions of the two vectors:

$$\vec{a}\cdot\vec{b} = ab\cos\phi,$$ **Equation (3-20)**

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}.$$ **Equation (3-23)**

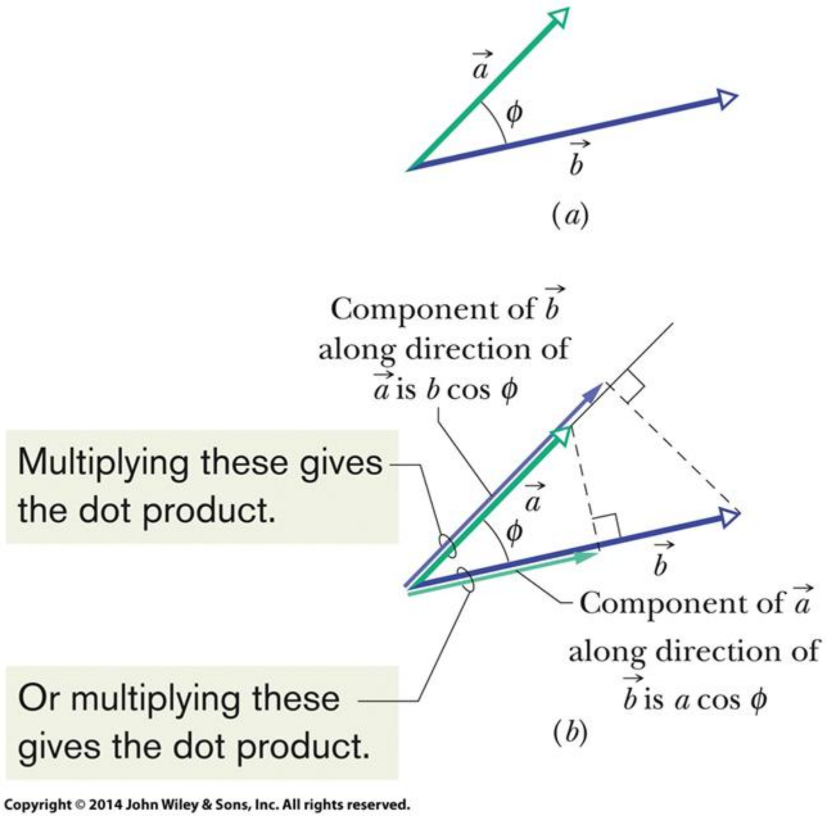
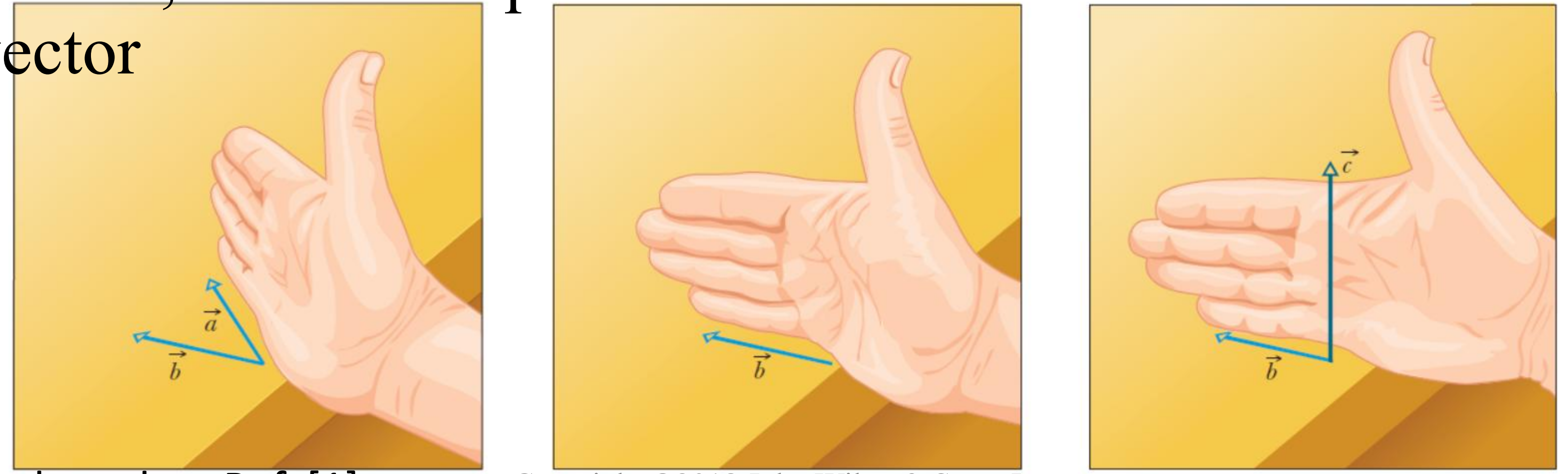


Figure (3-18)

- Multiplying two vectors: the **vector product** $\vec{c} = \vec{a} \times \vec{b}$
  - The **cross product** of two vectors with magnitudes *a* & *b*, separated by angle φ, produces a vector with magnitude:

$$c = ab \sin \phi, \qquad \textbf{Equation (3-24)}$$

  - And a direction perpendicular to both original vectors
- Direction is determined by the **right-hand rule**
- Place vectors tail-to-tail, sweep fingers from the first to the second, and thumb points in the direction of the resultant vector



- The cross product is not commutative

$$\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right). \qquad \textbf{Equation (3-25)}$$

Checkpoint 5

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if the magnitude of the vector product $\vec{C} \times \vec{D}$ is (a) zero and (b) 12 units?

Answer:

(a) 0 degrees

(b) 90 degrees

- To evaluate, we distribute over components:

$$\vec{a}\times\vec{b}=\left(a_x\hat{\mathrm{i}}+a_y\hat{\mathrm{j}}+a_z\hat{\mathrm{k}}\right)\times\left(b_x\hat{\mathrm{i}}+b_y\hat{\mathrm{j}}+b_z\hat{\mathrm{k}}\right),$$ **Equation (3-26)**

$$a_x\hat{\mathrm{i}}\times b_x\hat{\mathrm{i}}=a_xb_x\left(\hat{\mathrm{i}}\times\hat{\mathrm{i}}\right)=0,$$

$$a_x\hat{\mathrm{i}}\times b_y\hat{\mathrm{j}}=a_xb_y\left(\hat{\mathrm{i}}\times\hat{\mathrm{j}}\right)=a_xb_y\hat{\mathrm{k}}.$$

- Therefore, by expanding (3-26):

$$\vec{a}\times\vec{b}=\left(a_yb_z-b_ya_z\right)\hat{\mathrm{i}}+\left(a_zb_x-b_za_x\right)\hat{\mathrm{j}}+\left(a_xb_y-b_xa_y\right)\hat{\mathrm{k}}.$$ **Equation (3-27)**

Sample Problem 3.07 **Cross product, unit-vector notation**

If $\vec{a}=3\hat{\mathrm{i}}-4\hat{\mathrm{j}}$ and $\vec{b}=-2\hat{\mathrm{i}}+3\hat{\mathrm{k}}$, what is $\vec{c}=\vec{a}\times\vec{b}$?

Learning outcomes

- ✓ Analyze the freely falling object motion.
- ✓ Solve problems with changing acceleration in 1-D using calculus.
- ✓ Explain the difference between a scalar and a vector.
- ✓ Perform vector calculation in physics problems.

$$dv = a\,dt \qquad dx = v\,dt$$

$$\int_{v=v_0}^{v} dv = \int_{t=0}^{t} a\,dt. \qquad \int_{x=x_0}^{x} dx = \int_{x=t_0}^{t} \upsilon dt$$

Questions