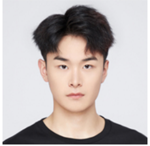
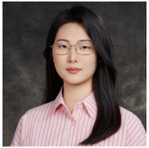
Group Meeting: Perfusion Model (Convolution and ODE)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Outline

Problem Description

Goal: to obtain quantitative **perfusion indices** from pwiMRI images

- **CBF**: Cerebral Blood Flow. The rate at which blood is delivered. $Q[mL/100g]$
- **CBV**: Cerebral Blood Volume. The total volume of blood present. $V[mL/100g]$
- **MTT**: Mean Transit Time. The average duration that blood spends transiting. $MTT[s]$
- $\mathbf{T_{max}}$: Time to Maximum of the Residue Function. The time required for the residue function of a tracer to reach its peak, indicative of cerebral perfusion efficiency. $T_{max}[s]$

Assumptions

zierlerTheoreticalBasisIndicatorDilution1962

- Single inflow and a single outflow orifice
- Recirculation does not occur
- Flow and volume be constant
- The system must exhibit stationarity (constant distribution of transit times)
- Distribution of transit times of indicator particles be identical with the distribution of transit times of the native fluid

Gamma Variate Function

thompsonIndicatorTransitTime1964

- GVF & Adj. Sheppard's model:

$$C(t) = k(t - AT)^{\alpha} e^{-\frac{(t-AT)}{\sigma}} \tag{1}$$

$$C(t) = \frac{A(t - AT)^{\alpha}}{\Gamma(1+\alpha)\sigma^{1+\alpha}} e^{-\frac{(t-AT)}{\sigma}} \tag{2}$$

$t =$ time after injection

$C(t) =$ indicator concentration at time, t

$k =$ constant scale factor

$AT =$ appearance time

$\alpha, \sigma =$ arbitrary parameters, $1/\sigma = Q/V$

$A =$ total area under the curve, I/Q

Indicator transit time has been shown to exhibit the mathematical properties of a general class of random variables, known as "gamma variates." Curve-fitting techniques were employed to show that the arterial indicator curves are equivalent to frequency distribution functions for this class of variables.