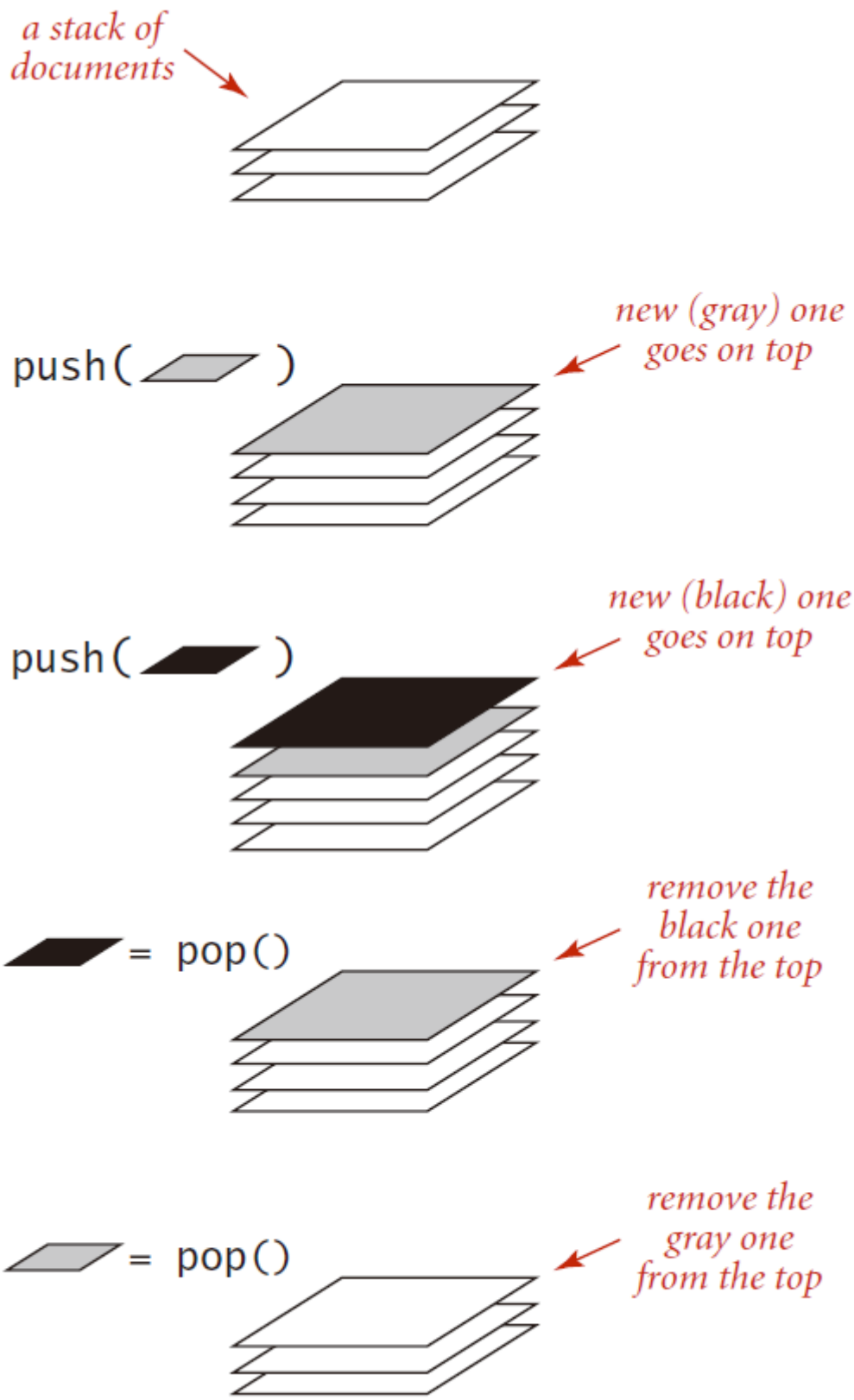
> 2018年10月31日

栈是一种后进先出(LIFO)的数据结构，如同摞书本一样，最先放的书本是最后才会拿到：



# 栈的数组实现

```java
public class ArrayStack<Item> implements Iterable<Item> {
    private Item[] a = (Item[]) new Object[1];
    private int N;

    public boolean isEmpty() {
        return N == 0;
    }

    public int size() {
        return N;
    }

    public void push(Item item) {
        if (N == a.length) {
            resize(2 * a.length);
        }
        a[N++] = item;
    }

    public Item pop() {
        if (isEmpty()) {
            throw new NoSuchElementException("Stack underflow");
        }
        Item item = a[--N];
        a[N] = null;
        if (N > 0 && N == a.length / 4) {
            resize(a.length / 2);
        }
        return item;
    }

    private void resize(int max) {
        Item[] temp = (Item[]) new Object[max];
        for (int i = 0; i < N; i++) {
            temp[i] = a[i];
        }
        a = temp;
    }

    @Override
    public Iterator<Item> iterator() {
        return new ReverseArrayIterator();
    }

    private class ReverseArrayIterator implements Iterator<Item> {
        private int i = N;

        @Override
        public boolean hasNext() {
            return i > 0;
```

```
51          }
52
53          @Override
54          public Item next() {
55              if (!hasNext()) {
56                  throw new NoSuchElementException();
57              }
58              return a[--i];
59          }
60
61          @Override
62          public void remove() {
63
64          }
65      }
66  }
```

以上的实现中当栈中容量与数组容量相等时，会进行扩容；当栈中容量为数组容量四分之一时，会进行缩容，具体操作如下图：

| push() | pop() | N | a.length | a[] 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | *null* | | | | | | | |
| to | | 1 | 1 | to | | | | | | | |
| be | | 2 | 2 | to | be | | | | | | |
| or | | 3 | 4 | to | be | or | *null* | | | | |
| not | | 4 | 4 | to | be | or | not | | | | |
| to | | 5 | 8 | to | be | or | not | to | *null* | *null* | *null* |
| - | to | 4 | 8 | to | be | or | not | *null* | *null* | *null* | *null* |
| be | | 5 | 8 | to | be | or | not | be | *null* | *null* | *null* |
| - | be | 4 | 8 | to | be | or | not | *null* | *null* | *null* | *null* |
| - | not | 3 | 8 | to | be | or | *null* | *null* | *null* | *null* | *null* |
| that | | 4 | 8 | to | be | or | that | *null* | *null* | *null* | *null* |
| - | that | 3 | 8 | to | be | or | *null* | *null* | *null* | *null* | *null* |
| - | or | 2 | 4 | to | be | *null* | *null* | | | | |
| - | be | 1 | 2 | to | *null* | | | | | | |
| is | | 2 | 2 | to | is | | | | | | |

- **最好时间复杂度**：最理想的情况下，当前栈中元素数量比数组的容量小，此时就直接执行代码块`a[N++] = item;`，即此时的时间复杂度为 $O(1)$。
- **最坏时间复杂度**：最糟糕的情况下，当前栈中元素数量与数组的容量相等，此时就要执行`resize`方法进行扩容了，进入循环体，执行`N`次复制操作，此时的时间复杂度为 $O(N)$。
- **平均时间复杂度**：
  - 当栈中元素小于数组容量时，此时进行压栈就有`N`种情况，且每种情况的时间复杂度为 $O(1)$；当栈中元素与数组容量相等时，此时进行压栈就只有一种情况了，要进行扩容操作，这种情况的时间复杂度为 $O(N)$；则总共有`N+1`中情况，对其取平均值：
    $\frac{1+1+1+\ldots+1+N}{N+1}=\frac{2N}{N+1}$ 在大 $O$标记法中，可以省略系数与低阶项，所以其平均时间复杂度为 $O(1)$

- 下面使用概率来分析，由于有`N+1`中情况，每种情况的发生概率为 $\frac{1}{N+1}$，则其平均时间复杂度为：$1 \times \frac{1}{N+1} + 1 \times \frac{1}{N+1} + \ldots + 1 \times \frac{1}{N+1} + N \times \frac{1}{N+1} = O(1)$

- **均摊时间复杂度**：根据上述代码，每出现一次扩容操作时，即此时压栈的时间复杂度为 $O(N)$，那么后面的`N`次压栈操作的时间复杂度均为 $O(1)$，前后是连贯的，因此将 $O(N)$平摊到前`N`次上，得出均摊时间复杂度为 $O(1)$。

# 栈的链表实现

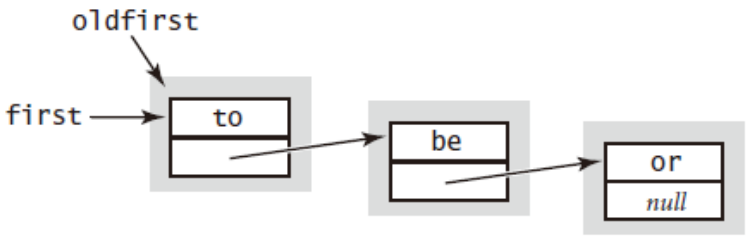
```
public class ListStack<Item> implements Iterable<Item> {
    private Node first;
    private int N;

    private class Node {
        Item item;
        Node next;
    }

    public void push(Item item) {
        Node oldFirst = first;
        first = new Node();
        first.item = item;
        first.next = oldFirst;
        N++;
    }

    public Item pop() {
        if (isEmpty()) {
            throw new NoSuchElementException();
        }
        Item item = first.item;
        first = first.next;
        N--;
        return item;
    }

    public Item peek() {
        if (isEmpty()) {
            throw new NoSuchElementException();
        }
        return first.item;
    }

    public boolean isEmpty() {
        return first == null;
    }

    public int size() {
        return N;
    }

    @Override
```

```
    public Iterator<Item> iterator() {
        return new ListIterator();
    }

    private class ListIterator implements Iterator<Item> {
        private Node current = first;

        @Override
        public boolean hasNext() {
            return current != null;
        }

        @Override
        public Item next() {
            Item item = current.item;
            current = current.next;
            return item;
        }

        @Override
        public void remove() {

        }
    }
}
```

`push`操作如图：

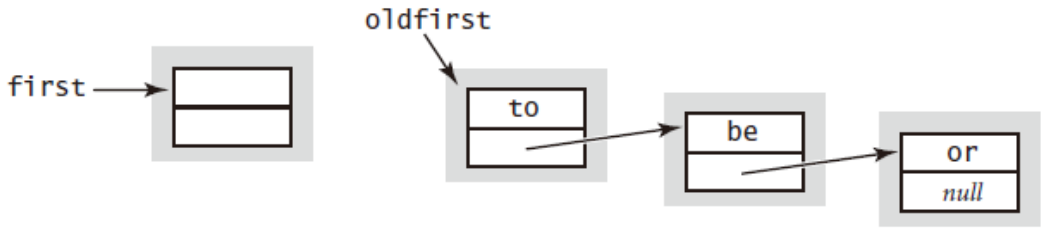
save a link to the list

```
Node oldfirst = first;
```



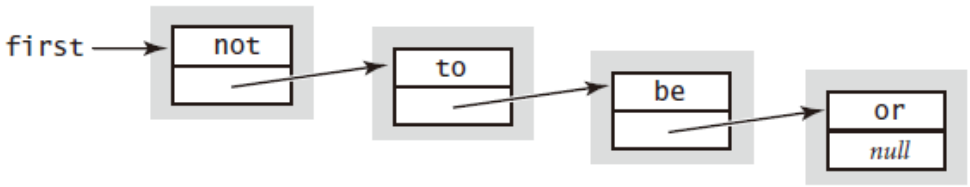

create a new node for the beginning

```
first = new Node();
```

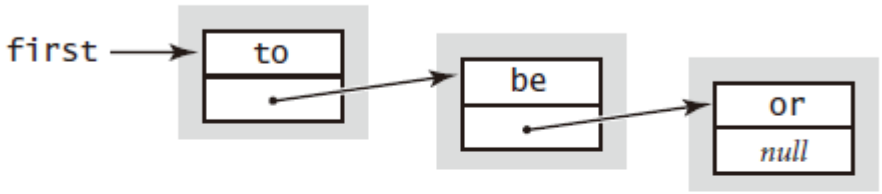



set the instance variables in the new node

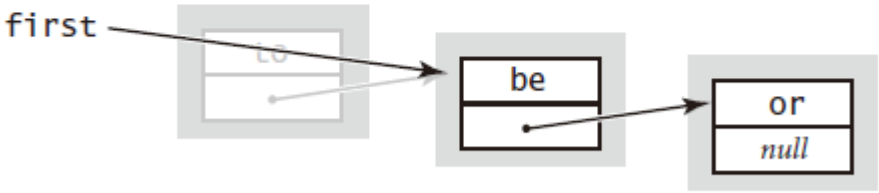
```
first.item = "not";
first.next = oldfirst;
```



```
first = first.next;
```

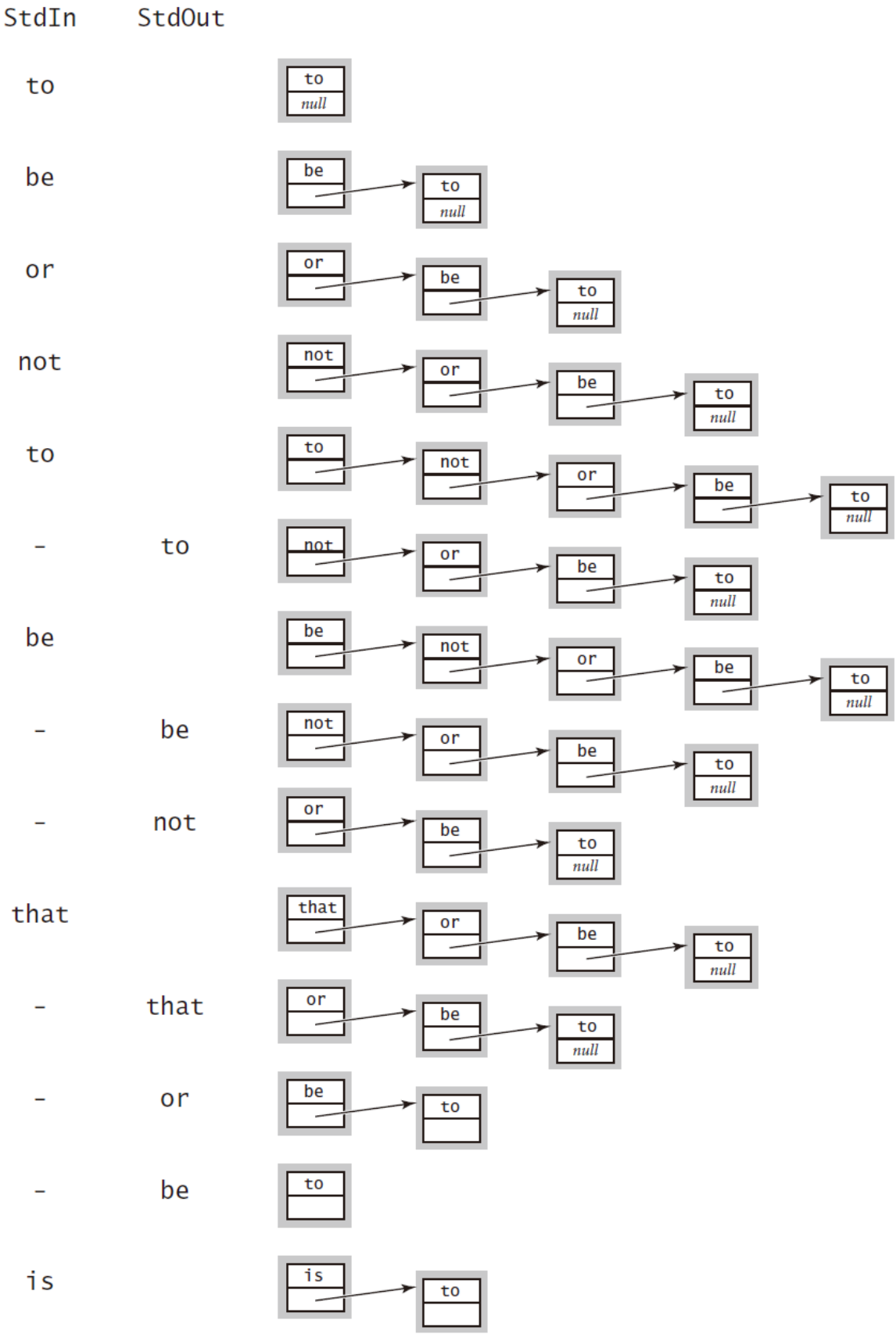



pop 操作如图：




**Removing the first node in a linked list**

整体操作如下：


StdIn StdOut
to
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or
not
to
- to
be
- be
- not
that
- that
- or
- be
is
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be
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# 栈的应用

# 1，括号匹配

每个左括号必然对应其右括号，如`[()]`就是合法的，而`[(])`就是错误的；这个程序可以用栈来实现：

> 定义一个空栈，读入字符，如果字符是一个左括号，则将其压入栈中。如果字符是一个右括号，则当栈空时报错，否则，将栈顶元素弹出，如果弹出的字符不是对应的左括号，则报错。若全部字符读完后，栈不为空则报错。

```
    public static boolean isComplete(String str) {
        if (str.length() == 0) {
            return false;
        }
        Deque<Character> sta = new ArrayDeque<>();
        for (int i = 0; i < str.length(); i++) {
            if (str.charAt(i) == '[') {
                sta.push('[');
            }
            if (str.charAt(i) == '(') {
                sta.push('(');
            }
            if (str.charAt(i) == '{') {
                sta.push('{');
            }
            if (str.charAt(i) == ']') {
                if (sta.isEmpty()) {
                    return false;
                }
                if (sta.pop() != '[') {
                    return false;
                }
            }
            if (str.charAt(i) == ')') {
                if (sta.isEmpty()) {
                    return false;
                }
                if (sta.pop() != '(') {
                    return false;
                }
            }
            if (str.charAt(i) == '}') {
                if (sta.isEmpty()) {
                    return false;
                }
                if (sta.pop() != '{') {
                    return false;
                }
            }
        }
```

# 2，后缀(逆波兰)表达式

中缀表达式就是我们常见的表达式，如：$6*(5+(2+3)*8+3)$

而后缀表达式的表示形式为：$6\ 5\ 2\ 3+8*+3+*$

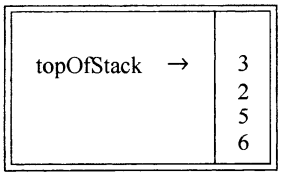
前缀表达式的表示形式为：$*6++5*+2\ 3\ 8\ 3$

在计算机中，利用后缀表达式进行计算没有必要知道任何优先规则，使用后缀表达式计算的过程及其程序：

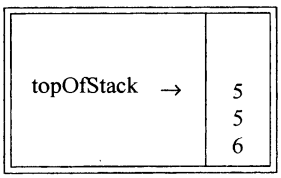
后缀表达式

6 5 2 3 + 8 * + 3 + *

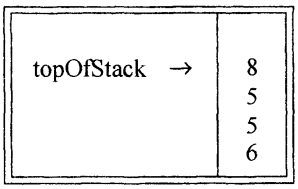
计算如下：前四个字符放入栈中，此时栈变成



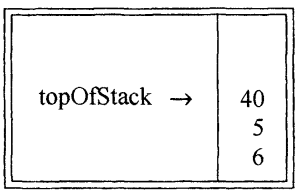

下面读到一个‘ + ’号，所以 3 和 2 从栈中弹出并且它们的和 5 被压入栈中



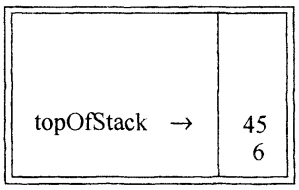

接着，8 进栈



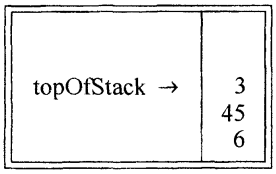

现在见到一个‘ * ’号，因此 8 和 5 弹出并且 5 * 8 = 40 进栈



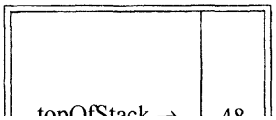

接着又见到一个‘ + ’号，因此 40 和 5 被弹出并且 5 + 40 = 45 进栈

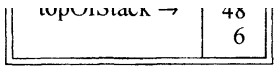



现在将 3 压入栈中

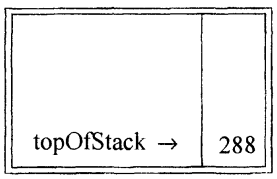



然后‘ + ’使得 3 和 45 从栈中弹出并将 45 + 3 = 48 压入栈中






最后，遇到一个‘ * ’号，从栈中弹出 48 和 6；将结果 6 * 48 = 288 压进栈中




```
    private static Pattern ISNUMBER = Pattern.compile("[0-9]+");

    /**
     * 计算后缀表达式: 2 3 * 2 1 - / 3 4 1 - * +
     * 其中缀表达式: 2 * 3 / ( 2 - 1 ) + 3 * ( 4 - 1 )
     * 将数字压栈，一遇到运算符就将其取出运算，结果再压入栈
     */
    public static void evaluatePostFix() {
        String str = "2 3 * 2 1 - / 3 4 1 - * +";
        String[] strings = str.split("\\s+");
        Stack<Integer> sta = new Stack<>();
        for (String s : strings) {
            if (ISNUMBER.matcher(s).matches()) {
                sta.push(Integer.parseInt(s));
            } else {
                int n1 = sta.pop();
                int n2 = sta.pop();
                int n3 = 0;
                if (s.equals("+")) {
                    n3 = n2 + n1;
                } else if (s.equals("-")) {
                    n3 = n2 - n1;
                } else if (s.equals("*")) {
                    n3 = n2 * n1;
                } else if (s.equals("/")) {
                    n3 = n2 / n1;
                }
                sta.push(n3);
            }
        }
        System.out.println(sta.pop());
    }
```

前缀式求值是先将前缀式逆序，后通过后缀式求值的方法，求值。但要注意的是操作数的运算顺序是与后缀表达式相反的。举个例子： 中缀表达式8-7；后缀表达式为87-，前缀表达式为-87，前缀表达式逆序78-，左操作数为8，右操作数为7

```
    private static Pattern ISNUMBER = Pattern.compile("[0-9]+");
    /**
     * 计算前缀表达式: + / * 2 3 - 2 1 * 3 - 4 1
     * 其中缀表达式为: 2 * 3 / ( 2 - 1 ) + 3 * ( 4 - 1 )
     * 先将其反转，将数字压入栈，一遇到运算符就取出数字计算，将计算结果压入栈
```

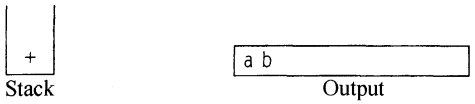
```java
     */
    public static void evaluatePreFix() {
        String str = "+ / * 2 3 - 2 1 * 3 - 4 1";
        String[] strings = str.split("\\s+");
        for (int i = 0; i < strings.length / 2; i++) {
            String temp = strings[i];
            strings[i] = strings[strings.length - i - 1];
            strings[strings.length - i - 1] = temp;
        }
        Stack<Integer> sta = new Stack<>();
        for (String s : strings) {
            if (ISNUMBER.matcher(s).matches()) {
                sta.push(Integer.parseInt(s));
            } else {
                int n1 = sta.pop();
                int n2 = sta.pop();
                int n3 = 0;
                if (s.equals("+")) {
                    n3 = n1 + n2;
                } else if (s.equals("-")) {
                    n3 = n1 - n2;
                } else if (s.equals("*")) {
                    n3 = n1 * n2;
                } else if (s.equals("/")) {
                    n3 = n1 / n2;
                }
                sta.push(n3);
            }
        }
        System.out.println(sta.pop());
    }
```

如何将中缀表达式转化为后缀表达式？ 假设中缀表达式： $a+b*c+(d*e+f)*g$

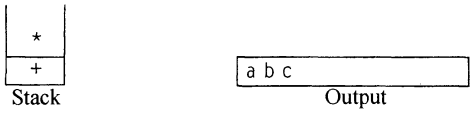
后缀表达式为： $abc*+de*f+g*+$

过程如下：

为了理解这种算法的运行机制，我们将把上面长的中缀表达式转换成后缀形式。首先，符号 a 被读入，于是它被传向输出。然后，‘ + ’被读入并被放入栈中。接下来 b 读入并流向输出。这一时刻的状态如下：

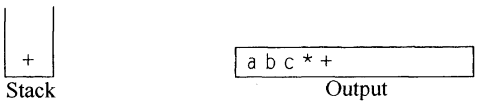



接着 * 号被读入。操作符栈的栈顶元素比 * 的优先级低，故没有输出且 * 进栈。接着，c 被读入并输出。至此，我们有

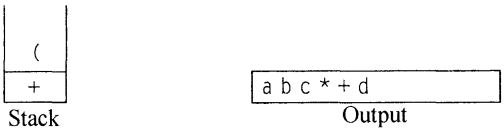



]

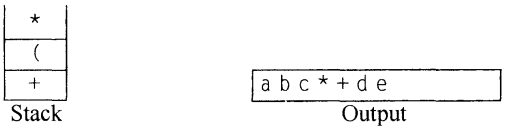
后面的符号是一个 + 号。检查一下栈我们发现，需要将 * 从栈弹出并把它放到输出中；弹出栈中剩下的 + 号，该算符不比刚刚遇到的 + 号优先级低而是有相同的优先级；然后，将刚刚遇到的 + 号压入栈中



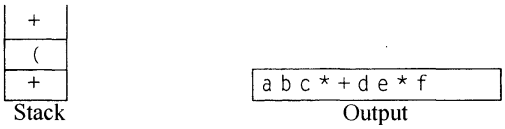

下一个被读到的符号是一个(，由于有最高的优先级，因此它被放进栈中。然后，d 读入并输出



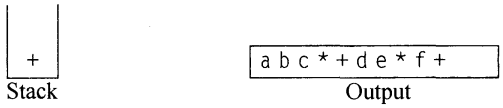

继续进行，我们又读到一个 * 。由于除非正在处理闭括号否则开括号不会从栈中弹出，因此没有输出。下一个是 e，它被读入并输出



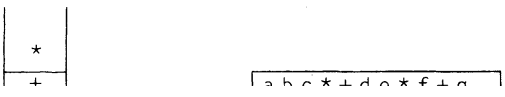

再往后读到的符号是 + 。我们将 * 弹出并输出，然后将 + 压入栈中。这以后，我们读到 f 并输出

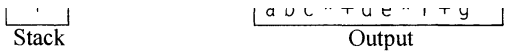



现在，我们读到一个)，因此将栈元素直到(弹出，我们将一个 + 号输出

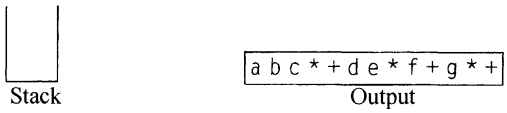



下面又读到一个 * ；该算符被压入栈中。然后，g 被读入并输出






现在输入为空，因此我们将栈中的符号全部弹出并输出，直到栈变成空栈




程序如下：

```
    /**
     * 利用正则表达式判断是否为整数
     * 以下只能匹配非负整数
     */
    private static Pattern ISNUMBER = Pattern.compile("[0-9]+");

    /**
     * 将中缀表达式转为后缀表达式
     * 结果为: 2 3 * 2 1 - / 3 4 1 - * +
     */
    public static void infixToPostfix() {
        String str = "2 * 3 / ( 2 - 1 ) + 3 * ( 4 - 1 )";
        String[] strings = str.split("\\s+");
        Stack<String> sta = new Stack<>();
        StringBuffer sb = new StringBuffer();
        for (String s : strings) {
            if (ISNUMBER.matcher(s).matches()) {
                sb.append(s + " ");
            } else {
                switch (s) {
                    case ")":
                        while (!sta.isEmpty() && (!"(".equals(sta.peek()))) {
                            sb.append(sta.pop() + " ");
                        }
                        sta.pop();
                        break;
                    case "(":
                        sta.push(s);
                        break;
                    case "^":
                        while (!sta.isEmpty() && (!("^".equals(sta.peek()) ||
                                "(".equals(sta.peek())))) {
                            sb.append(sta.pop());
                        }
                        sta.push(s);
                        break;
                    case "*":
                    case "/":
                        while (!sta.isEmpty() && (!"+".equals(sta.peek())) &&
                                (!"-".equals(sta.peek())) && (!"
("".equals(sta.peek()))) {
                            sb.append(sta.pop()+" ");
                        }
                        sta.push(s);
```

```
                        break;
                    case "+":
                    case "-":
                        while (!sta.isEmpty() && (!"(".equals(sta.peek()))) {
                            sb.append(sta.pop() + " ");
                        }
                        sta.push(s);
                        break;
                    default:
                }
            }
        }
        while (!sta.isEmpty()) {
            sb.append(sta.pop()+" ");
        }
        System.out.println(sb.toString());
    }
```

将中缀表达式转为前缀表达式，要先将其反转，当栈中数据全部弹出后，再将其反转即可：

```
    /**
     * 将中序表达式转为前序表达式
     * 首先将其反转，将运算符压栈，遇到（就弹出
     * 结果为：+/*23-21*3-41
     */
    public static void infixToPrefix() {
        String str = "2 * 3  / ( 2 - 1 ) + 3 * ( 4 - 1 )";

        String[] strings = str.split("\\s+");
        for (int i = 0; i < strings.length / 2; i++) {
            String temp = strings[i];
            strings[i] = strings[strings.length - i - 1];
            strings[strings.length - i - 1] = temp;
        }
        System.out.println(Arrays.toString(strings));
        Stack<String> sta = new Stack<>();
        StringBuffer sb = new StringBuffer();
        for (String s : strings) {
            if (ISNUMBER.matcher(s).matches()) {
                sb.append(s + " ");
            } else {
                switch (s) {
                    case "(":
                        while (!sta.isEmpty() && (!")".equals(sta.peek()))) {
                            sb.append(sta.pop() + " ");
                        }
                        sta.pop();
                        break;
                    case ")":
                        sta.push(s);
                        break;
                    case "^":
                        while (!sta.isEmpty() && (!("^".equals(sta.peek()) ||
```

```
                                "(".equals(sta.peek()))) {
                            sb.append(sta.pop());
                        }
                        sta.push(s);
                        break;
                    case "*":
                    case "/":
                        sta.push(s);
                        break;
                    case "+":
                    case "-":
                        while (!sta.isEmpty() && (!")".equals(sta.peek()))) {
                            sb.append(sta.pop() + " ");
                        }
                        sta.push(s);
                        break;
                    default:
                }
            }
        }
        while (!sta.isEmpty()) {
            sb.append(sta.pop() + " ");
        }
        System.out.println(sb.reverse().toString());
    }
```